On the Dirac-Majorana neutrinos distinction in four-body decays

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arXiv:2305.14140

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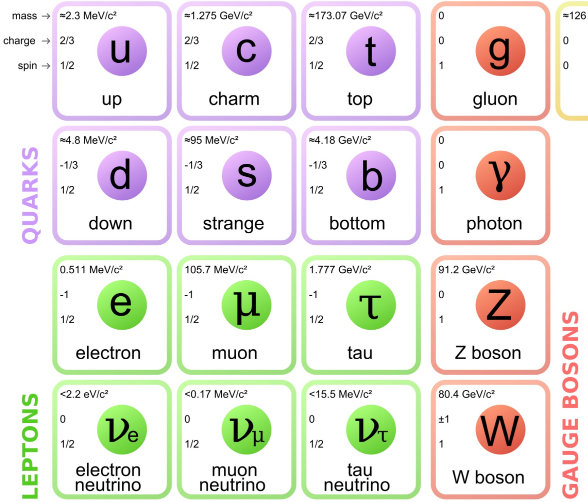
Dirac vs Majorana neutrinos



- Neutrino masses?
- Dirac vs Majorana

-Neutrinoless double beta decay
-Neutrino-electron scattering
-Lepton number violating processes
-etc.

$$D - M \propto (m_\nu / E)^n$$

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Practical Dirac-Majorana confusion theorem (DMCT)

Boris Kayser, Phys.Rev.D 26 (1982) 1662:

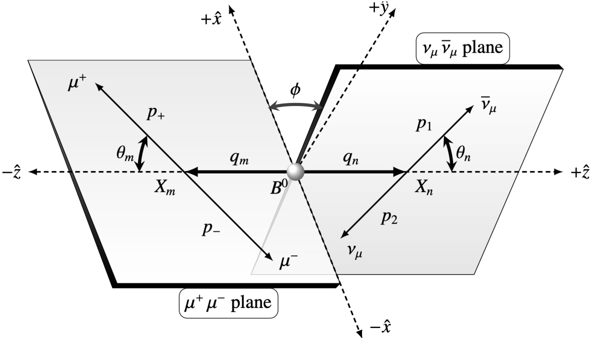
The difference between Dirac and Majorana neutrinos in SM processes (SM gauge group + massive neutrinos) is proportional to some power of the neutrino mass (if neutrino variables are not measured).

Alternatives:
- New physics effects[1]
- Measure neutrinos variables[2]

[1] R.E. Shrock, Phys. Lett. B 112 (1982) 382
M. Doi, T. Kotani and H. Nishiura, Prog. Theor. Phys. 114 (2005) 845.
J. M. Márquez, G. L. Castro and P. Roig, JHEP 11 (2022) 117.
[2] C. S. Kim, M. V. N. Murthy and D. Sahoo, Phys. Rev. D 105 (2022), 113006.

Back-to-back in four-body decays

Novel method proposed for $B^0 \to \mu^+\mu^-\bar{\nu}_\mu\nu_\mu$ (KMS method).
In the back-to-back kinematic configuration, the neutrinos energies can be inferred, since they are related by: $E_\nu = E_{\bar{\nu}} = \frac{m_B}{2} - E_\mu$.



Leads to a difference between Dirac and Majorana cases independently of neutrino masses.

Radiative leptonic decays

• Study the process $\ell \to \ell' \overline{\nu}_{\ell'} \nu_\ell \gamma$
Working in the mass basis:

$$\nu_{\ell L} = \sum_j U_{\ell j} N_{jL}, \tag{1}$$

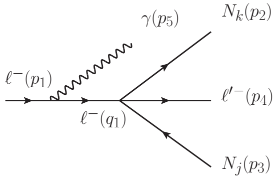
where $j = \{1, 2, 3, .., n\}$ is tagging mass-eigenstate neutrinos.
Then

$$\Gamma\left(\ell \to \ell' \overline{\nu}_{\ell'} \nu_\ell \gamma\right) = \sum_{jk} \Gamma\left(\ell \to \ell' \overline{N}_j N_k \gamma\right) \tag{2}$$

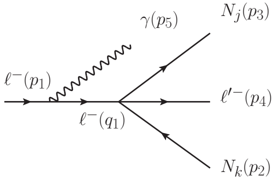
Note that $\overline{N}$ represents an antineutrino for the Dirac neutrino case, but should be identified with N for the Majorana neutrino case ($N{=}N^c{=}C\overline{N}^T$).

Dirac and Majorana amplitude

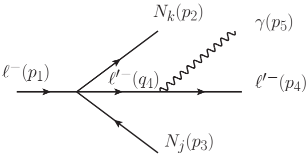
For Dirac (Majorana) neutrinos we will have two (four) first-order Feynman diagrams.



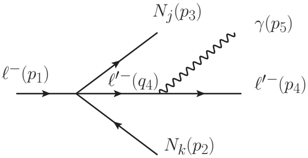
(a)



(b)



(c)



(d)

$$\mathcal{M}^D = \mathcal{M}_{(a)} + \mathcal{M}_{(c)} \equiv \mathcal{M}_{jk}(p_2, p_3) \quad \text{and} \quad \mathcal{M}^M = \mathcal{M}_{jk}(p_2, p_3) - \mathcal{M}_{kj}(p_3, p_2).$$

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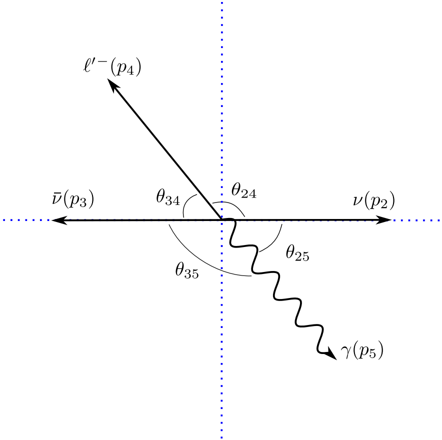
B2b configuration

For the b2b configuration: $E_\gamma = E_{\ell'}$ and $E_\nu = E_{\bar{\nu}} = \frac{m_\ell}{2} - E_{\ell'}$.



Now we do not integrate over neutrinos momenta.

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B2b differential decay rates

In the b2b case, the corresponding differential decay rates are:

$$\left.\frac{d\Gamma^D}{dE_\nu dE_{\bar\nu} d\cos\Theta_{\nu\bar\nu} d\cos\theta_{\ell'} d\phi}\right|_{b2b} = \frac{4\alpha G_F^2 (m_\ell - 2E_e)^4}{(4\pi)^5 m_\ell^2 E_{\ell'}} \left(8E_{\ell'}^2 \sin^4\frac{\theta}{2} + (1+\cos\theta) m_\ell^2\right),$$
$$\left.\frac{d\Gamma^M}{dE_\nu dE_{\bar\nu} d\cos\Theta_{\nu\bar\nu} d\cos\theta_{\ell'} d\phi}\right|_{b2b} = \frac{4\alpha G_F^2 (m_\ell - 2E_{\ell'})^4}{(4\pi)^5 m_\ell^2 E_{\ell'}} \left(E_{\ell'}^2 (3+\cos 2\theta) + m_\ell^2\right). \quad (3)$$

Then, the difference between Dirac and Majorana cases is precisely:

$$d\Gamma^D_{\nu\nu}|_{b2b} - d\Gamma^M_{\nu\nu}|_{b2b} = \frac{4\alpha G_F^2 (m_\ell - 2E_{\ell'})^5}{(4\pi)^5 m_\ell^2 E_{\ell'}} \left(m_\ell + 2E_{\ell'}\right)\cos\theta \quad (4)$$

Where $\theta_{24} = \theta$ is the angle between the neutrino and final charged-lepton directions.

Angular treatment

If we work in the system where the neutrinos define the x-axis ($\theta_\nu = \pi/2$), as done in the KMS method, we get $\cos\theta = \cos\phi \sin\theta_{\ell'}$.
Then, the difference between Dirac and Majorana cases is precisely:

$$d\Gamma^D_{\nu\nu}|_{b2b} - d\Gamma^M_{\nu\nu}|_{b2b} = \frac{4\alpha G_F^2 (m_\ell - 2E_{\ell'})^5}{(4\pi)^5 m_\ell^2 E_{\ell'}} \left(m_\ell + 2E_{\ell'}\right) \cos\phi \sin\theta_{\ell'} \quad (5)$$

After integrating over the inaccessible neutrino angle, the difference vanishes identically (unexpected considering the motivation):

$$\int \left(d\Gamma^D_{\nu\nu}|_{b2b} - d\Gamma^M_{\nu\nu}|_{b2b}\right) d\cos\theta_{\ell'} d\phi = 0$$

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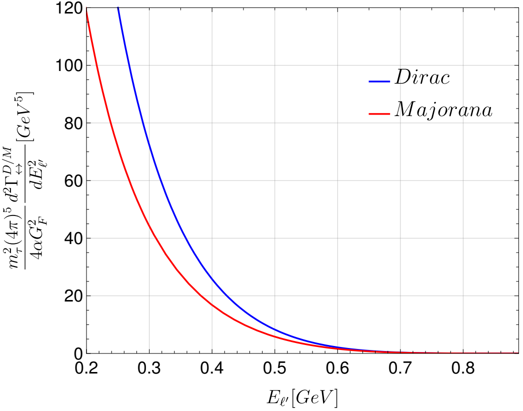
Angular treatment

We track this difference from the angular integration, specifically from the ϕ variable considerations.
From the KMS method, **the condition $\phi = 0$ is taken**, then $\cos\theta = \sin\theta_{\ell'}$.
Using this angular dependence, the corresponding energy spectra are:



Angular treatment

¿Why ϕ is not fixed by kinematics?

- In order to obtain the back-to-back scenario, we need to apply some restrictions concerning three of the five phase-space kinematic independent variables: these conditions follow as $E_\nu = E_{\bar{\nu}}$ and $\Theta_{\nu\bar{\nu}} = \pi$. Therefore, the remaining two angular variables must run over all their possible configurations, meaning ϕ is not fixed.
- Even if it is true that in the back-to-back configuration the $\nu\bar{\nu}$ and $\ell'\gamma$ systems define a plane (since they are two independent vectors), this plane is independent of the ϕ value, being $\phi = 0$ just an allowed specific configuration. Then ϕ remains as an independent variable.
- All possible configurations allowed by energy-momentum conservation must be considered.
- Many other consistency tests...

(More in arXiv:2305.14140)

Summary and perspectives

- In this work we have studied the radiative leptonic lepton-decay $\ell \to \ell' \overline{\nu} \nu \gamma$, developing our own approach, generalizing the application of the method put forward in Phys. Rev. D **105** (2022) 113006 to final state neutrinos of different flavours.
- We have computed the matrix element for the back-to-back configuration in the decaying lepton rest frame for Dirac and Majorana cases.
- We discussed in detail the angular treatment, with quantitative and qualitative arguments, which is very important, since its inaccurate interpretation could lead to very attractive results.
- Unfortunately we found that there is no difference between Dirac and Majorana distribution in $\ell \to \ell' \overline{\nu} \nu \gamma$ once the inaccessible neutrino angle is integrated out.

Summary and perspectives

- Finally, we wish to emphasize that the idea proposed by Kim et al. is very appealing in order to avoid the DMCT. This fact highlights the necessity to study other types of processes and specific kinematic scenarios within this approach, where angular or energy dependencies could lead to a non-zero difference between the Dirac and Majorana distributions, hopefully observable in current or forthcoming experiments.

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BACKUP

Amplitude

The decay modes $\ell^- \to \ell'^- \overline{N}_j N_k \gamma$ and $\ell^- \to \ell'^- \overline{N}_k N_j \gamma$ yield the same final states for $k \neq j$ as well as for $k = j$ (since $\overline{N}_i = N_i$). Then, the Majorana amplitude is of the form:

$$\mathcal{M}^M = \mathcal{M}_{jk}(p_2, p_3) - \mathcal{M}_{kj}(p_3, p_2)\,, \tag{6}$$

It can be shown that $Re\left(\mathcal{M}(p_2, p_3)\mathcal{M}^*(p_3, p_2)\right) \propto m_\nu^2 \approx 0$ due to the smallness of neutrino masses.

Thus

$$\overline{|\mathcal{M}^M|^2} = \overline{|\mathcal{M}}_{jk}(p_2, p_3)|^2 + \overline{|\mathcal{M}}_{kj}(p_3, p_2)|^2. \tag{7}$$

Remember also

$$\mathcal{M}^D = \mathcal{M}_{jk}(p_2, p_3) \to \overline{|\mathcal{M}^D|^2} = \overline{|\mathcal{M}}_{jk}(p_2, p_3)|^2. \tag{8}$$

If neutrino variables were measured, we might have D-M differences due to $p_2 \leftrightarrow p_3$ exchange in the Majorana amplitude.

Differential decay rate

For the differential decay rate (neglecting the masses of the final fermions):

$$\frac{d\Gamma^{D,M}}{dE_\nu dE_{\bar{\nu}} d\cos\Theta_{\nu\bar{\nu}} d\cos\theta_{\ell'} d\phi} = \frac{2}{m_\ell (4\pi)^6} \frac{E_\nu E_{\bar{\nu}} E_{\ell'}}{E_\gamma} \frac{1}{\epsilon} \sum_{j,k} |\overline{\mathcal{M}^{D,M}}|^2, \tag{9}$$

where $\epsilon = 1(2)$ for Dirac (Majorana) neutrinos.
If we integrate over neutrinos momenta:

$$\int \sum_{j,k} |\overline{\mathcal{M}}_{jk}(p_2, p_3)|^2 dp_2 dp_3 = \int \sum_{j,k} |\overline{\mathcal{M}}_{kj}(p_3, p_2)|^2 dp_2 dp_3. \tag{10}$$

Then

$$d\Gamma^D = d\Gamma^M \longrightarrow \boxed{d\Gamma^D - d\Gamma^M = 0} \quad \text{(DMCT result)} \tag{11}$$

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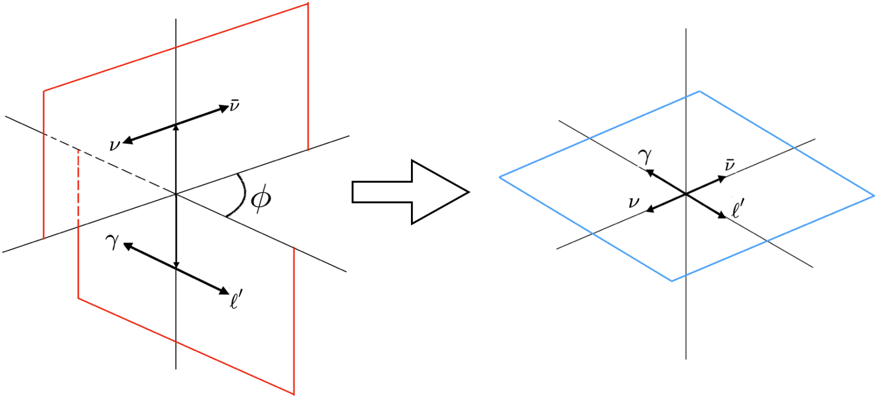
Angular treatment



Angular treatment

Again, if ϕ is not fixed, there is not difference between Dirac and Majorana cases once the unobserved neutrino angle is integrated out.