Is the sub-eV active neutrino Dirac or Majorana ?

DPF, Mexican Physics Society, 06/12/2023

C S Kim (w/ M V N Murthy, J Rosiek and D Sahoo)

Yonsei University

practical Dirac-Majorana Confusion Theorem for sub-eV active neutrino, and How to overcome it?

DPF, Mexico Physics Society, 06/12/2023
SNU-Physics, 03/30/2023
MPI-Heidelberg, Germany, 01/23/2023
High Energy Physics in LHC Era, Chile, 01/10/2023
Apres-LHC Workshop, 08/09/2022
4th Mini WS on CUBES, 11/05/2022
WS on Physics of Dark Cosmos, 10/23/2022
KEK-IPNS, Japan, 08/22/2022
IHEP, China, 07/15/2022

C S Kim (w/ M V N Murthy, J Rosiek and D Sahoo)
Yonsei University/DongShin University

CSK,MM,JR,DS (work in progress)

CONTENTS

INTRODUCTION

Sub-eV active neutrino mass

Seesaw mechanism

$\Delta L = 2$ processes & 0-nu-Beta-Beta

(sub-eV active) neutrinos have mass

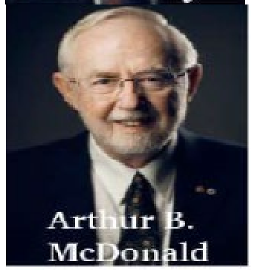
- Neutrinos are massless in SM, $m_\nu = 0$.
  All neutrinos are only left-handed (ν_L).

$$\mathcal{L}^D_{\text{mass}} = -m_\nu \left(\overline{\nu_R}\nu_L + \overline{\nu_L}\nu_R\right), \qquad m_\nu = \frac{Y_\nu \, \mathrm{v}}{\sqrt{2}},$$

  where Y_ν = Higgs-neutrino Yukawa coupling constant,
  and v = Higgs VEV.
  No way to generate mass without right-handed neutrinos (ν_R).

- But observations of neutrino oscillation imply that **neutrinos have mass**, $m_\nu \neq 0$.

  The Nobel Prize in Physics 2015 was awarded jointly to Takaaki Kajita (Super-Kamiokande) and Arthur B. McDonald (Sudbury Neutrino Observatory) "for the discovery of neutrino oscillations, which shows that neutrinos have mass".



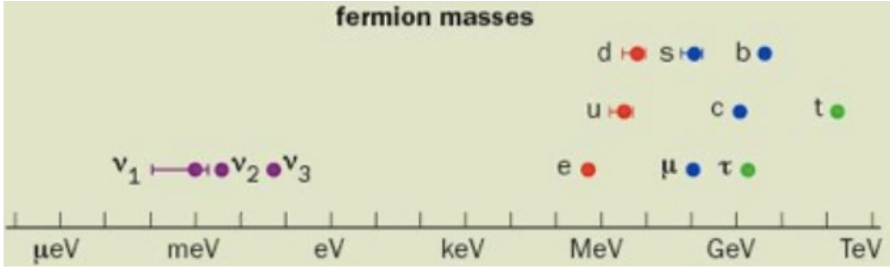
How to give neutrinos mass?

There are various suggestions as to how neutrinos can get mass.

- **Dirac mass:**
  - *Assumption:* ν_R exists.
  - *Lagrangian:*

$$\mathscr{L}^D_{\text{mass}} = -m^D_\nu \left(\overline{\nu_R}\nu_L + \overline{\nu_L}\nu_R\right).$$

  - *Disadvantage:* No reason for m^D_ν to be small.



- **Majorana mass:**
  - *Assumption:* neutrino ≡ anti-neutrino.
  - *Lagrangian:*

$$\mathscr{L}^M_{\text{mass}} = \frac{1}{2} m^M_\nu \left(\overline{\nu^C_L}\nu_L + \overline{\nu_L}\nu^C_L\right).$$

  - *Disadvantage:* $\mathscr{L}^M_{\text{mass}}$ is not invariant under $SU(2)_L \times U(1)_Y$ gauge group, so not allowed by SM.

How to give neutrinos (very small) mass ?

- **See-saw mechanism:** A simpler version of Dirac-Majorana mass, with a nice twist. [PM,PLB67(1977)421]
  - *Assumptions:* $m_\nu^L = 0$ and $m_\nu^D \ll m_\nu^R$.
  - *Lagrangian:*

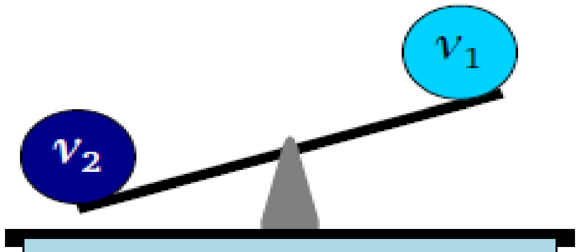
    $\mathscr{L}_{\text{mass}}^{D+M} = \frac{1}{2} m_\nu^R \left(\overline{\nu_R^C} \nu_R \right) - m_\nu^D \left(\overline{\nu_R} \nu_L \right) + \text{H.c.} = \frac{1}{2} \overline{N_L^C} M N_L + \text{H.c.}$, where

    $N_L = \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix}$ and $M = \begin{pmatrix} 0 & m_\nu^D \\ m_\nu^D & m_\nu^R \end{pmatrix}$ is the mass matrix.
  - *Mass eigenvalues:*

$$m_{2,1} = \frac{1}{2} \left(m_\nu^R \pm \sqrt{\left(m_\nu^R\right)^2 + 4 \left(m_\nu^D\right)^2} \right)$$

$$\approx \frac{1}{2} m_\nu^R \left(1 \pm 1 \pm 2 \left(\frac{m_\nu^D}{m_\nu^R} \right)^2 \right).$$

$$\Longrightarrow m_1 \approx - \frac{\left(m_\nu^D\right)^2}{m_\nu^R} \text{ and } m_2 \approx m_\nu^R.$$



As ν_2 gets heavier, ν_1 become lighter.

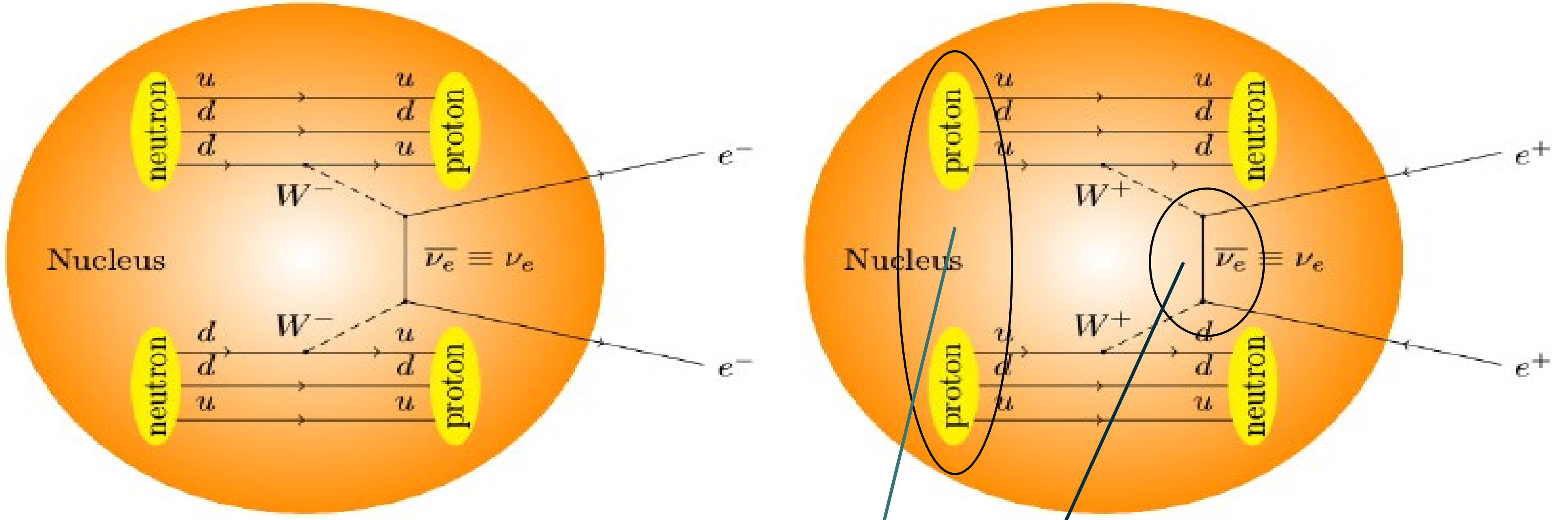
  - *Advantage:* $m_1 \ll m_2$, so light neutrinos are possible.
  - *Challenges:*
    - To find the heavy ν_2 experimentally.
    - To prove that both the light ν_1 and heavy ν_2 are Majorana neutrinos.

6 / 1

Neutrino-less double-beta decay $(0\nu\beta\beta)$ (1)

Lepton Number Violation (LNV)
➔ not allowed within SM

❖ Process:



$${}^A_Z\mathcal{N} \longrightarrow {}^A_{Z+2}\mathcal{N} + 2e^-$$

$${}^A_Z\mathcal{N} \longrightarrow {}^A_{Z-2}\mathcal{N} + 2e^+$$

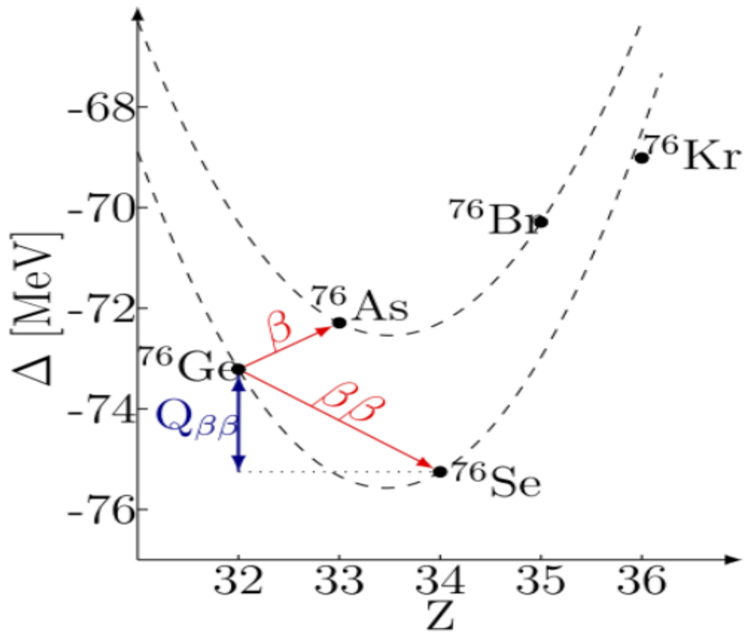
Doubly weak charged current process

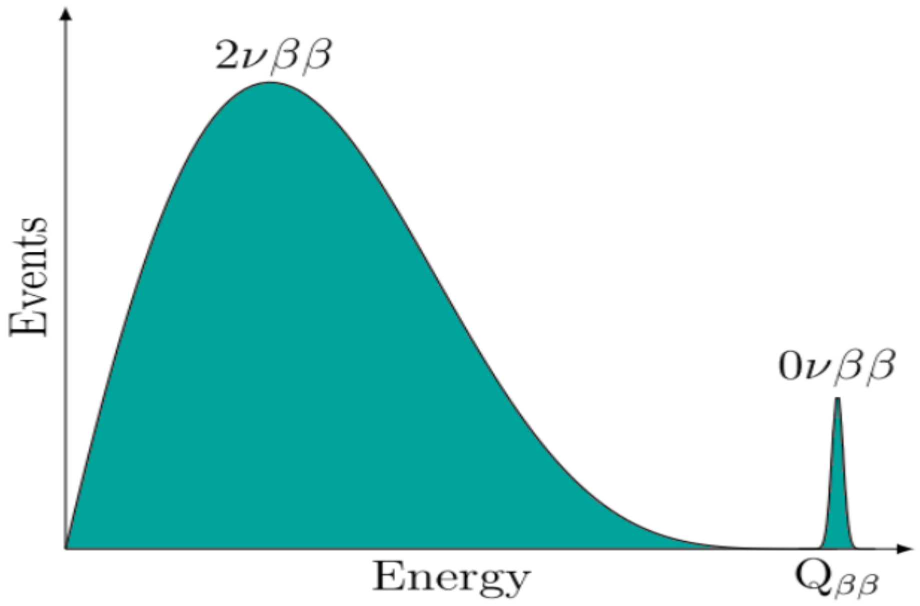
❖ The half-life of a nucleus decaying via $0\nu\beta\beta$ is,

$$\left[T^{0\nu}_{1/2}\right]^{-1} = G_{0\nu}\ \left|M_{0\nu}\right|\ \left|m_{\beta\beta}\right|^2,$$

Neutrino-less double-beta decay $(0\nu\beta\beta)$ (2)

- Double-beta $(2\nu\beta\beta)$ decay has been observed in 10 isotopes, ^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{128}Te, ^{130}Te, ^{150}Nd, ^{238}U, with half-life $T_{1/2} \approx 10^{18} - 10^{24}$ years.





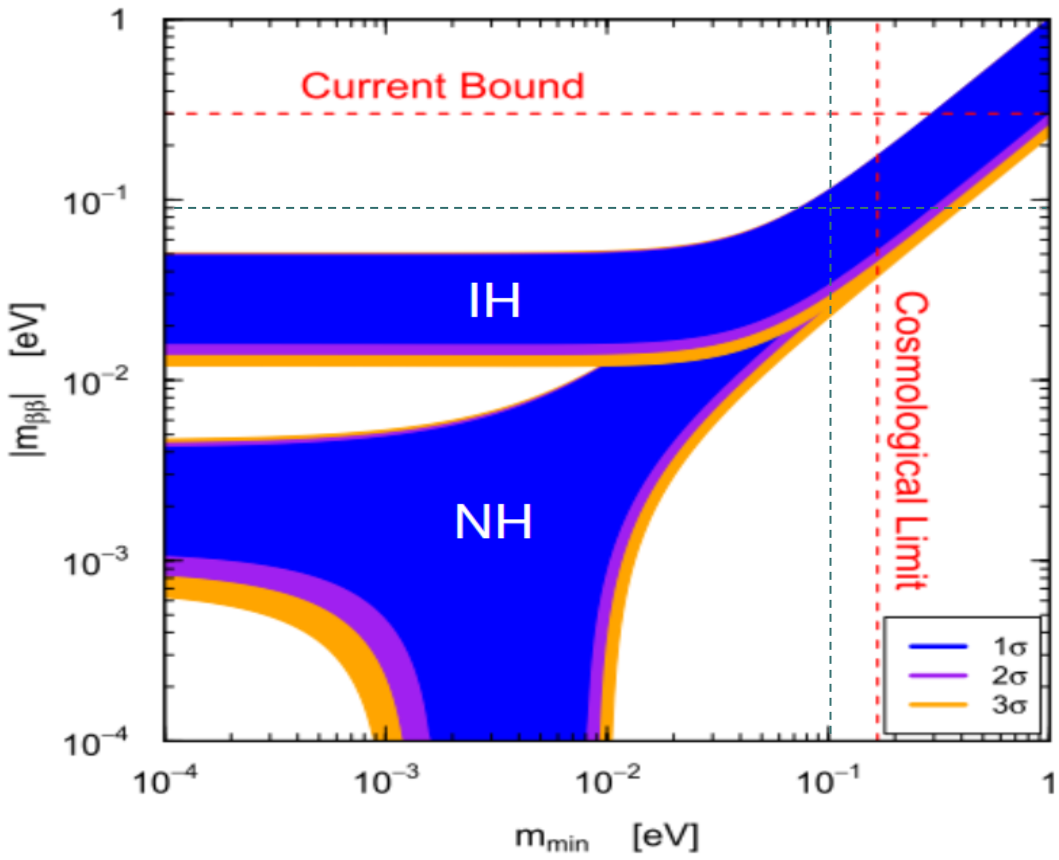
Giovanni Benato (for the GERDA collaboration), arXiv:1509.07792

- **$0\nu\beta\beta$ (forbidden in SM) is yet to be observed in any experiment.**

$$T^{0\nu}_{1/2}\,[^{76}\mathrm{Ge}] > 2.1 \times 10^{25} \text{ years}\,(90\%\ \text{C.L.}).$$

M. Agostini *et al.* (GERDA Collaboration) Phys. Rev. Lett. **111**, 122503 (2013).

Neutrino-less double-beta decay $(0\nu\beta\beta)$ (3)



NH: Normal hierarchy
IH: Inverted hierarchy

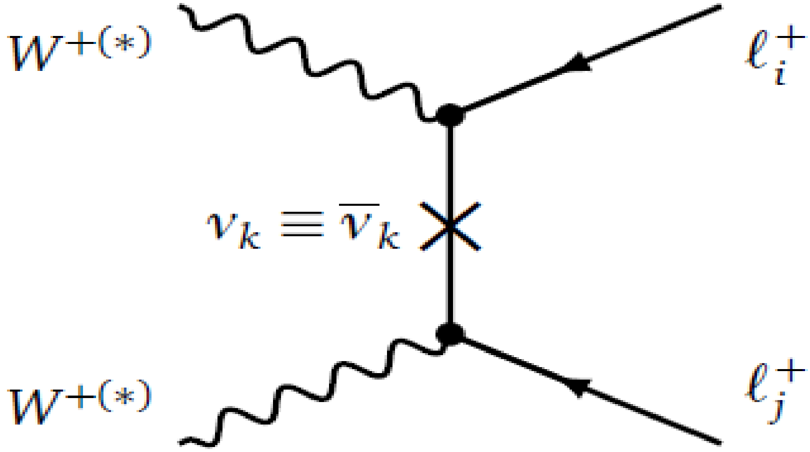
S. M. Bilenky and C. Giunti
Mod. Phys. Lett. A **27**, 1230015 (2012),
arXiv:1203.5250

- **If $m_{\beta\beta} < 10^{-2}$, only NH is viable and the $T_{1/2}^{0\nu}$ will be much larger than the current experimental lower boun**

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}\,\left|M_{0\nu}\right|\,\left|m_{\beta\beta}\right|^2$$

Looking for Majorana neutrinos via $\Delta L = 2$ processes (1)

- Neutrinos are the only *elementary fermions* known to us that *can* have Majorana nature.
- Majorana neutrinos: $\nu \equiv \overline{\nu}$.
- Majorana neutrinos violate lepton flavor number (L), they mediate $\Delta L = 2$ processes.



$$\propto \int \frac{d^4p}{(2\pi)^4} \sum_k U_{\ell_i k}\, U_{\ell_j k}\, \frac{m_k + \not{p}}{p^2 - m_k^2}$$

- $\Delta L = 2$ processes play crucial rule to probe Majorana nature of ν's.
  - neutrinoless double-beta ($0\nu\beta\beta$) decay
  - Rare meson decays with $\Delta L = 2$
  - Collider searches at LHC

Looking for Majorana neutrinos via $\Delta L = 2$ processes (2)

- **Decay rate of any $\Delta L = 2$ process with final leptons $\ell_1^+ \ell_2^+$:**

$$\Gamma_{\Delta L=2} \propto \left| \sum_k U_{\ell_1 k} U_{\ell_2 k} \frac{m_k}{p^2 - m_k^2 + i m_k \Gamma_k} \right|^2 ,$$

where we have used the fact that $(1-\gamma^5)\, \not{p}(1-\gamma^5) = 0$.

  - Light ν:

$$\Gamma_{\Delta L=2} \propto \left| \sum_k U_{\ell_1 k} U_{\ell_2 k} m_k \right|^2 = \left| m_{\ell_1 \ell_2} \right|^2 .$$

  - Heavy ν:

$$\Gamma_{\Delta L=2} \propto \left| \sum_k \frac{U_{\ell_1 k} U_{\ell_2 k}}{m_k} \right|^2 .$$

  - Resonant ν:

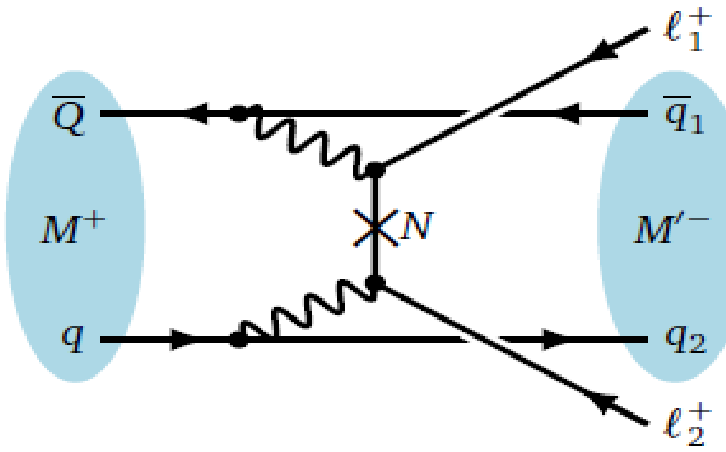
$$\Gamma_{\Delta L=2} \propto \frac{\Gamma(N \to i)\, \Gamma(N \to f)}{m_N \Gamma_N} .$$

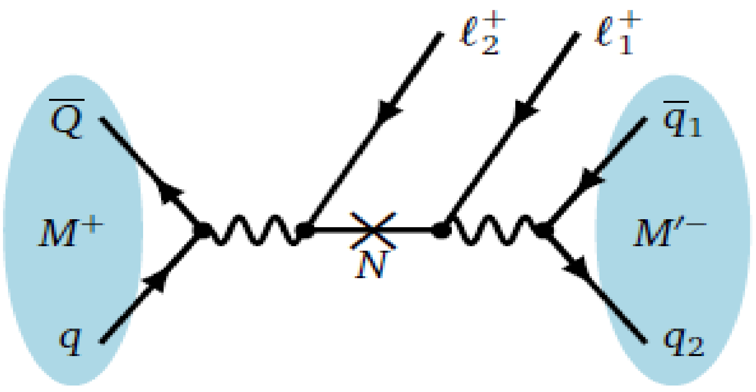
Looking for Majorana neutrinos via $\Delta L = 2$ processes (3)

(Rare meson decays for massive sterile neutrinos)

- **Processes:** $M^+ \to M'^- \ell_1^+ \ell_2^+$,
  where $M = K, D, D_s, B, B_c$ and $M' = \pi, K, D, \ldots$

G. Cvetic, C.S. Kim, arXiv:1606.04140 (PRD **94**, 053001, 2016)
G. Cvetic, C. Dib, S. Kang, C. S. Kim, arXiv:1005.4282 (PRD **82**, 053010, 2010)





- **No nuclear matrix element unlike** $0\nu\beta\beta$, but probes Majorana nature of massive neutrino(s) N.

Looking for heavy Majorana neutrinos via $\Delta L = 2$ processes (4)

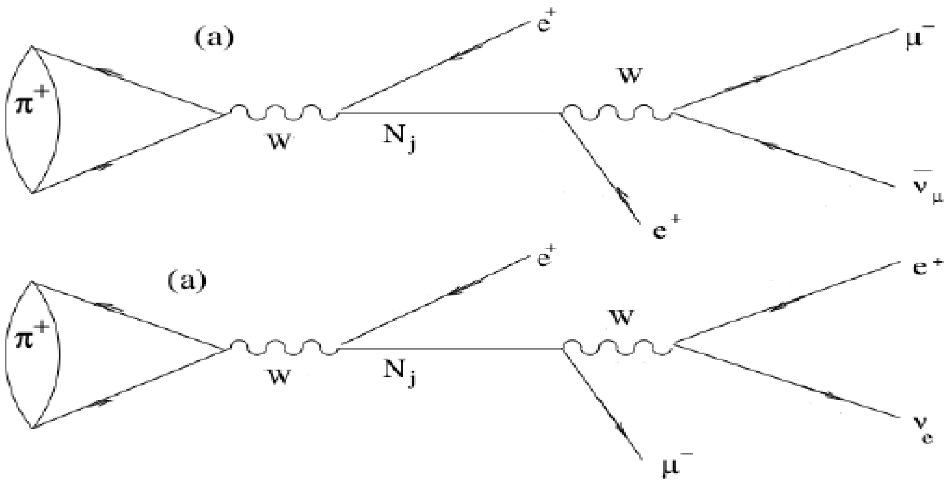
(tau lepton decays & pion decays)

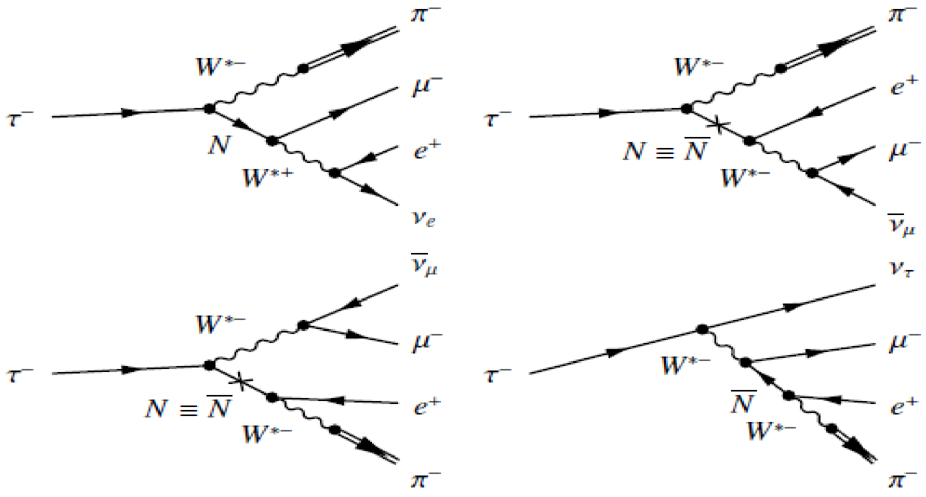
- **Process:**
  $\pi^{\pm} \to e^{\pm}N \to e^{\pm}e^{\pm}\mu^{\mp}\nu$
  G. Cvetič, C. S. Kim and J. Zamora-Saá,
  arXiv:1311.7554 [hep-ph]
  (J. Phys. G **41**, 075004 (2014))

- **Mass range:**
  106 MeV $\leqslant m_N \leqslant$ 139 MeV

- **Process:** $\tau^- \to \pi^-\mu^-e^+\nu/\overline{\nu}$
  C.S. Kim, G. L. Castro and D. Sahoo,
  arXiv:1708.00802 [hep-ph]
  (PRD **96**, 075016 (2017))

- **Mass range:**
  106 MeV $\leqslant m_N \leqslant$ 1637 MeV.



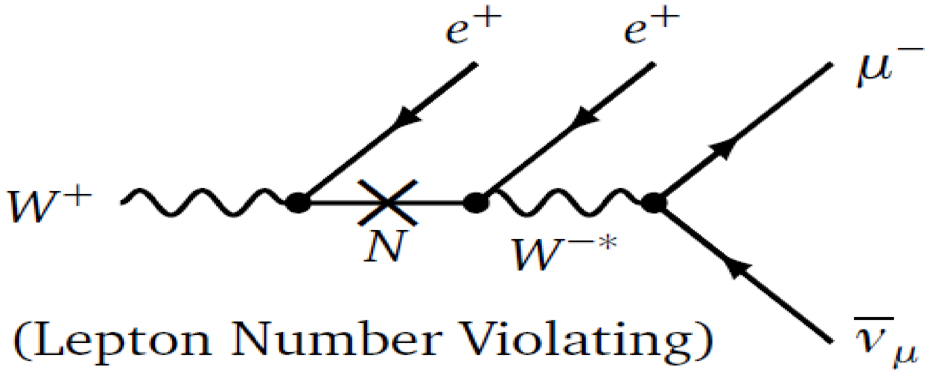


Looking for Majorana neutrinos via $\Delta L = 2$ processes (5)

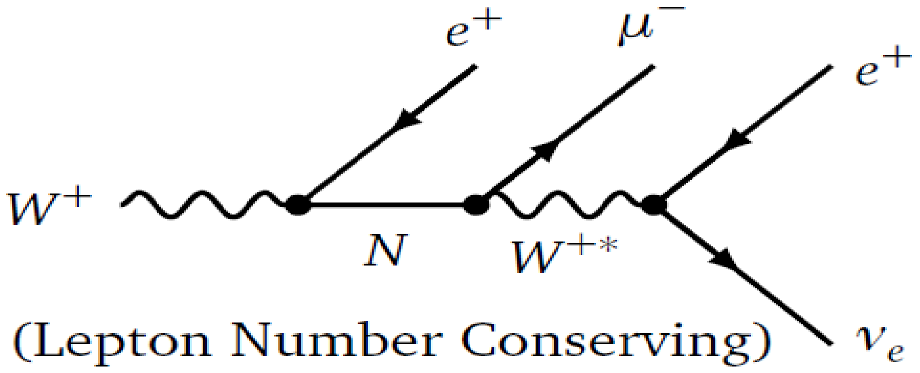
(Collider searches at LHC)

- **Processes:** $W^+ \to e^+ e^+ \mu^- \overline{\nu}_\mu$, $W^+ \to \mu^+ \mu^+ e^- \overline{\nu}_e$. Involves heavy neutrino N which can have Majorana nature as well.

C. Dib, C.S. Kim, arXiv:1509.05981 (PRD **92**, 093009, 2015);
C. Dib, C.S. Kim, K. Wang, J. Zhang,
arXiv:1605.01123 (PRD **94**, 013005, 2016)



(Lepton Number Violating)



(Lepton Number Conserving)

- **Decay widths:**
  - LNV: $\Gamma\left(W^+ \to e^+ e^+ \mu^- \overline{\nu}_\mu\right) = \left|U_{Ne}\right|^4 \hat{\Gamma}$,
  - LNC: $\Gamma\left(W^+ \to e^+ e^+ \mu^- \overline{\nu}_\mu\right) = \left|U_{Ne} U_{N\mu}\right|^2 \hat{\Gamma}$,

where $\hat{\Gamma} = \dfrac{G_F^3 M_W^3}{12 \times 96\sqrt{2}\pi^4} \dfrac{m_N^5}{\Gamma_N} \left(1 - \dfrac{m_N^2}{M_W^2}\right)^2 \left(1 - \dfrac{m_N^2}{2M_W^2}\right)$.

Looking for eV-scale sterile neutrino, not via Oscillation

(compared to LSND, miniBoone, …. searches light neutrino via neutrino Oscillation)

- If an eV scale sterile neutrino is present, its mixing with active flavor neutrinos would affect,
  1. muon decay $\longrightarrow$ extraction of Fermi constant,
  2. leptonic decays of tau $\longrightarrow$ testing unitarity of neutrino mixing matrix,
  3. semi-leptonic decays of tau & leptonic decays of pion and kaon
     $\longrightarrow$ additional tests of unitarity of neutrino mixing matrix,
  4. invisible width of the Z boson & number of light active neutrinos,
     $\longrightarrow$ extract individual active-sterile mixing parameters.

- Our analysis, taking precision measurements into account, supports the hypothesis that there are no such light sterile neutrinos.

C. S. Kim, G. L. Castro and D. Sahoo,
arXiv:1809.02265 [hep-ph]
(PRD **98** 11, 115021 (2018))

practical Dirac-Majorana Confusion Theorem

Neutrino Casimir Force

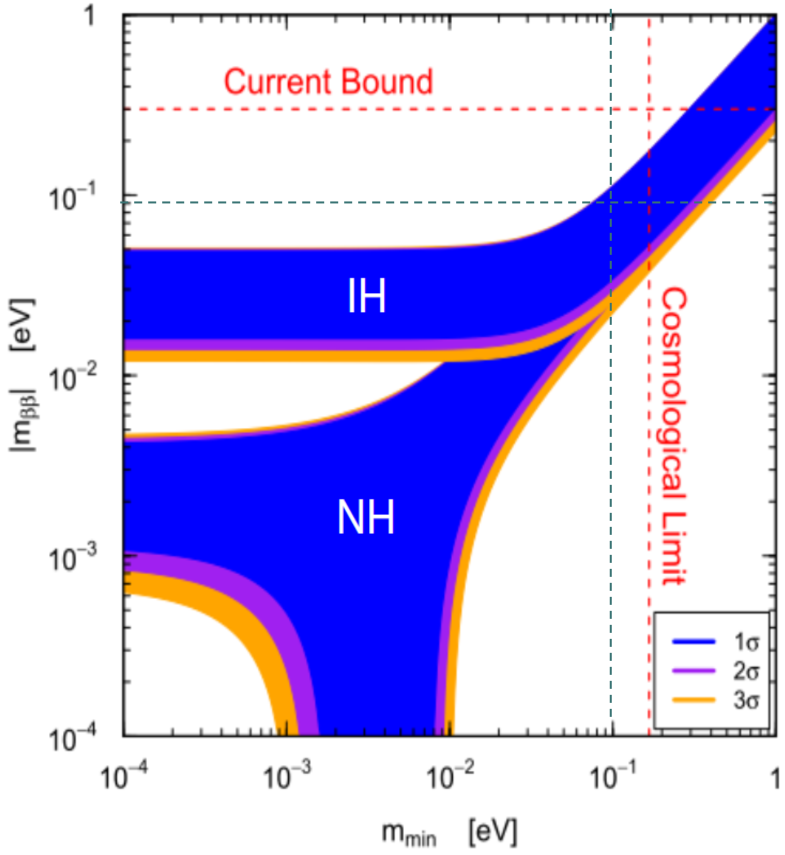
EDM & MDM of neutrinos

Fermi-Dirac Statistics for Fermion (nu)

practical Dirac-Majorana Confusion Theorem (pDMCT)

Discussions on pDMCT

Neutrino-less Double Beta Decay 0nuBB ($\Delta L = 2$ process)



NH: Normal hierarchy
IH: Inverted hierarchy

S. M. Bilenky and C. Giunti
Mod. Phys. Lett. A **27**, 1230015 (2012),
arXiv:1203.5250

Lepton Number Violation (LNV)
➔ not allowed within SM

The half-life of a nucleus decaying via $0\nu\beta\beta$ is,

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}\ |M_{0\nu}|\ |m_{\beta\beta}|^2$$

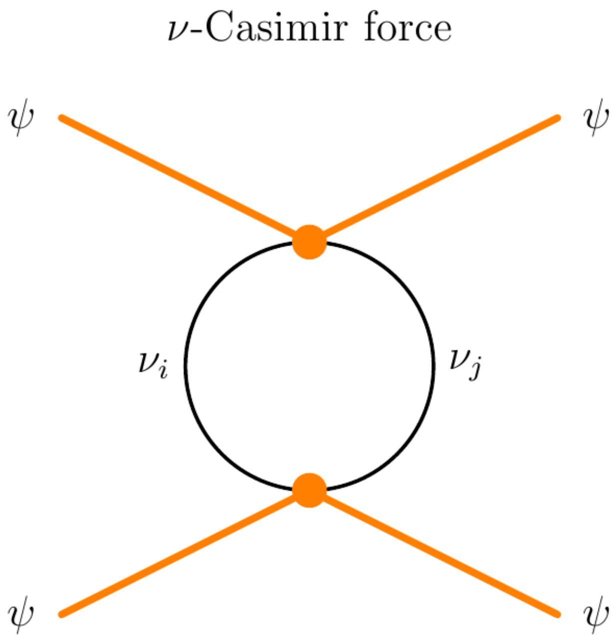
Possibility of very small mass($m_{\nu_e} \sim m_{\beta\beta}$)
May fail to observe !!

❖ **If $m_{\beta\beta} < 10^{-2}$, only NH is viable and the $T_{1/2}^{0\nu}$ will be much larger than the current experimental lower bound.**

Alternative to 0nuBB (1) – Neutrino Casimir force

Principle: Exchange of pair of neutrinos can give rise to long-range quantum force (aka neutrino Casimir force or the neutrino exchange force) between macroscopic objects, and the effective potential can differentiate Dirac and Majorana neutrinos.

G Feinberg, J Sucher, PRD166(1968)
Phys. Rev. D 101, no.11, 116006 (2020); JHEP 09, 122 (2020)
arXiv:2209.07082 [hep-ph]



Issue: The potential (and hence the force) is proportional to product of the tiny neutrino masses in the loop.

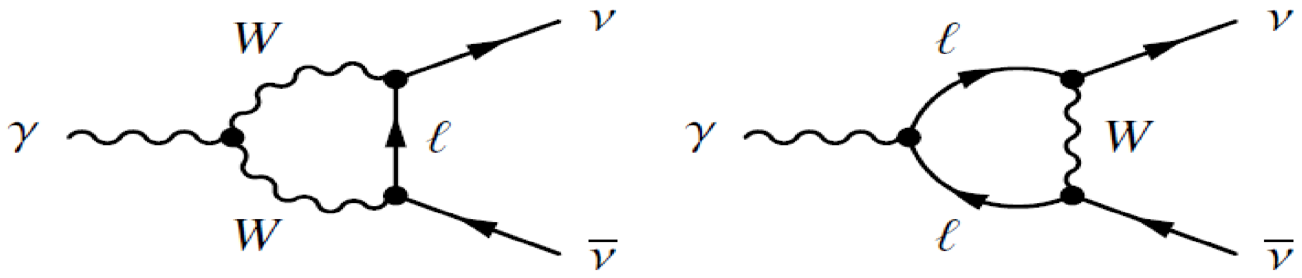
** Thermal fluctuation, van der Waals force

** Very weak force. For $r > 1$ nm, Gravitational force between two protons is bigger than this force.

Status: Experimental study is still awaited.

Alternative to 0nuBB (2) – e.d.m. & m.d.m. of Neutrino

- Neutrinos: electrically neutral ($Q_\nu = 0$).
  But might carry non-zero electric and magnetic dipole moments.
- Neutrino has no interaction with photon at tree-level.
  At one loop-level involving charged lepton and W boson in loop such interaction is allowed.



- CPT invariance prohibits Majorana neutrinos to have any electric and magnetic dipole moment. Loop effects also confirm this, following quantum statistics in $\gamma^* \to \nu\overline{\nu}$.

$$\text{edm} = 0 = \text{mdm}, \quad \text{(Majorana)}$$
$$\text{edm} \neq 0 \neq \text{mdm}. \quad \text{(Dirac)}$$

- Such striking differences in electromagnetic properties of Dirac and Majorana neutrinos vanishes as $m_\nu \to 0 \implies$ DMCT.
  [B. Kayser, Phys. Rev. D **26**, 1662 (1982).]

Alternative to 0nuBB (3) – Quantum Statistics

[CSK,MM,DS, PRD105,113005(2022)]

Allowed within SM

Minimal final state: $\ell^-\ell^+\nu_\ell\overline{\nu}_\ell$

Preference: $\ell = \mu$. $\ell = e$ has shorter track and $\ell = \tau$ has low reconstruction efficiency.

Decay mode: $B^0 \to \mu^-\mu^+\nu_\mu\overline{\nu}_\mu$

Experiment: Belle II.
$\overline{B}$ is hadronically reconstructed.

Majorana neutrinos

Quantum statistics

Final state

$\nu\ \nu$ $\quad \overline{\nu}\ \overline{\nu}$ $\quad \nu\ \overline{\nu}$

Anti-symmetrization for both Dirac and Majorana cases

Can not be used to distinguish between Dirac and Majorana cases

Anti-symmetrization for only Majorana case

Can be used to distinguish between Dirac and Majorana cases

practical Dirac-Majorana Confusion Theorem (1)

Consider the SM allowed decay, e.g.

$$B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\,\bar{\nu}_\mu(p_1)\,\nu_\mu(p_2),$$

Amplitude for Dirac case

$$\mathscr{M}^D = \mathscr{M}(p_1, p_2),$$

not helicity flip, but momentum exchange

For Majorana case

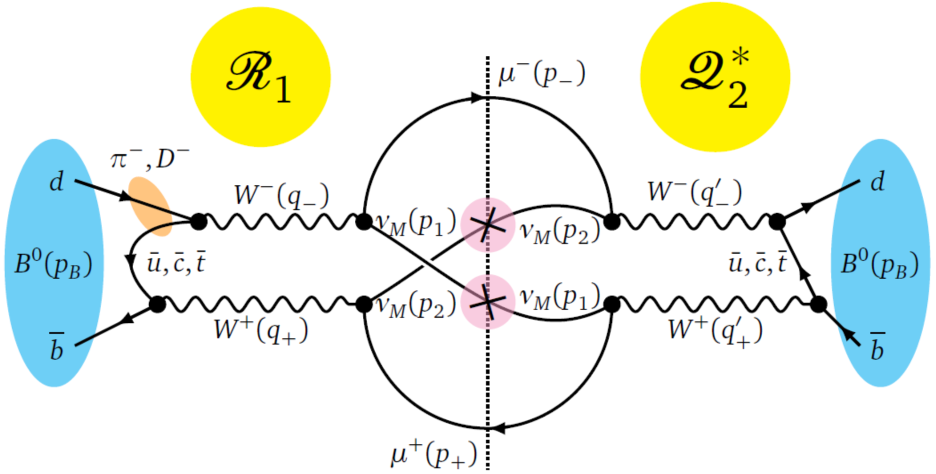
$$\mathscr{M}^M = \frac{1}{\sqrt{2}}\Big(\mathscr{M}(p_1, p_2) - \mathscr{M}(p_2, p_1)\Big).$$

required to know 4-momenta of p_1 and p_2, to be useful

Difference between D and M

$$|\mathscr{M}^D|^2 - |\mathscr{M}^M|^2 = \frac{1}{2}\Big(\underbrace{|\mathscr{M}(p_1, p_2)|^2}_{\text{Direct term}} - \underbrace{|\mathscr{M}(p_2, p_1)|^2}_{\text{Exchange term}}\Big) + \underbrace{\mathrm{Re}\Big(\mathscr{M}(p_1, p_2)^* \mathscr{M}(p_2, p_1)\Big)}_{\text{Interference term}}.$$

Dirac-Majorana Confusion Theorem (2)



Interference term

$$\mathrm{Re}\left(\mathscr{M}(p_1,p_2)^* \mathscr{M}(p_2,p_1)\right) \propto m_\nu^2.$$

In general
(useful if p_1 and/or p_2 are known)

$$\underbrace{|\mathscr{M}(p_1,p_2)|^2}_{\text{Direct term}} \neq \underbrace{|\mathscr{M}(p_2,p_1)|^2}_{\text{Exchange term}}.$$

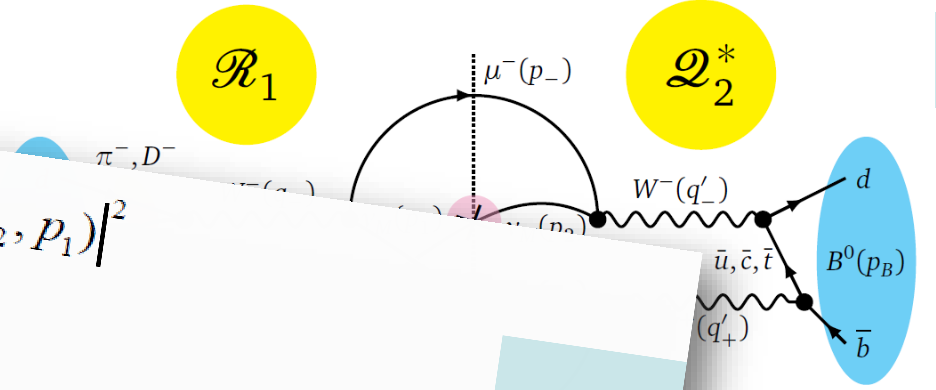
**For special case, such as Z → nu nubar

However, after integration
(required if momenta p_1 and p_2 are unobservable)

$$\iint \underbrace{|\mathscr{M}(p_1,p_2)|^2}_{\text{Direct term}} \mathrm{d}^4p_1\,\mathrm{d}^4p_2 = \iint \underbrace{|\mathscr{M}(p_2,p_1)|^2}_{\text{Exchange term}} \mathrm{d}^4p_1\,\mathrm{d}^4p_2,$$

Dirac-Majorana Confusion Theorem (2)

Interference term



** Special case : $|M(p_1,p_2)|^2 = |M(p_2,p_1)|^2$

$$|\mathscr{M}^D|^2 - |\mathscr{M}^M|^2 = \frac{1}{2}\Big(\underbrace{|\mathscr{M}(p_1,p_2)|^2}_{\text{Direct term}} - \underbrace{|\mathscr{M}(p_2,p_1)|^2}_{\text{Exchange term}}\Big) + \underbrace{\mathrm{Re}\big(\mathscr{M}(p_1,p_2)^* \mathscr{M}(p_2,p_1)\big)}_{\text{Interference term}}. \longrightarrow m_\nu^2$$

<u>pDMCT holds at the Amplitude-Squared level</u>

➔ Difference between Dirac and Majorana neutrinos becomes 0 at every observable

➔ e.g. Within the SM, weak neutral current decays : $Z^{(*)} \to \nu\bar{\nu}$

3/1/2023 practical Dirac-Majorana Confusion Theorem 32

→ nu nubar

<u>In general</u>

(useful if p_1 and/or p_2 are known)

However, after integ[...] p_2,

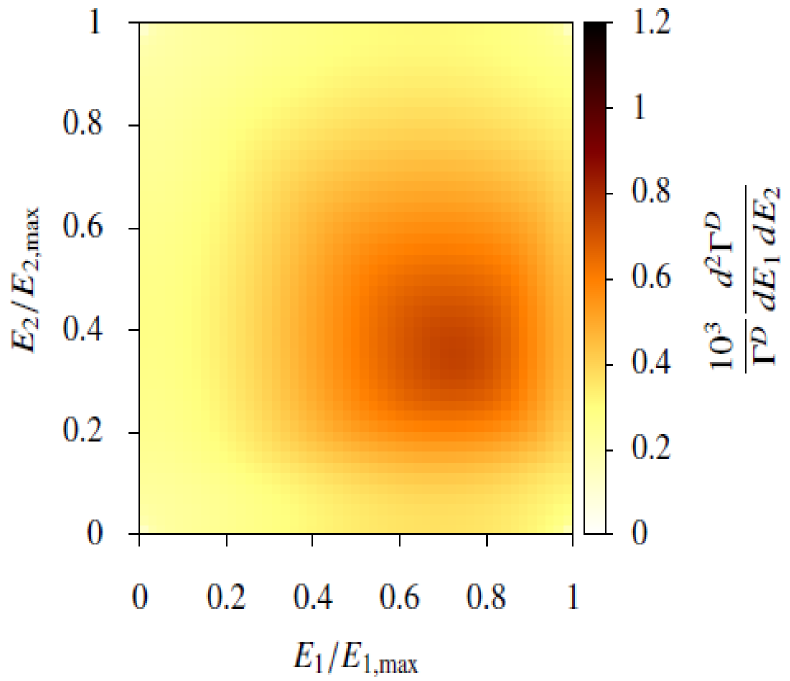
(required if momenta p_1 and p_2 are unobservable)

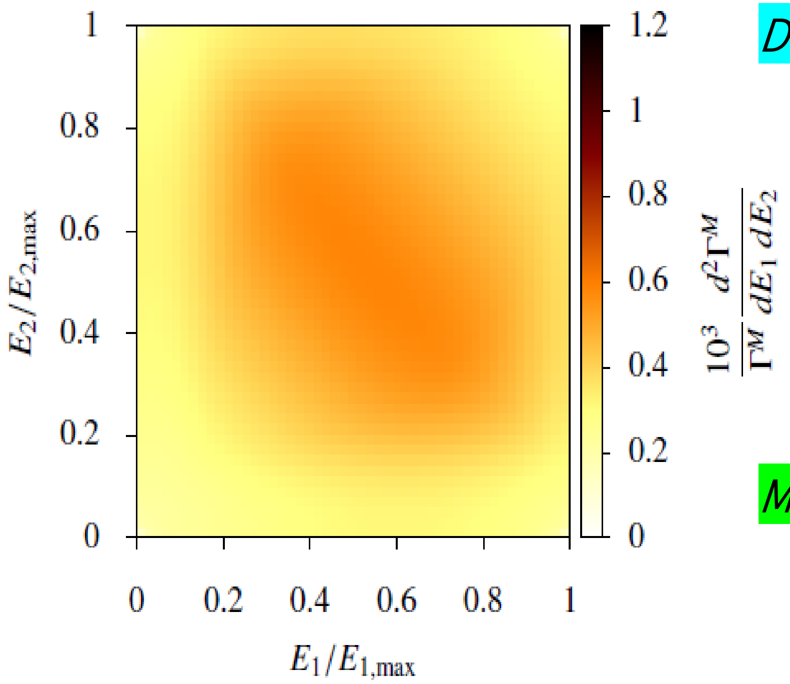
Different distribution, but the same total rate (DMCT)

$$\mathcal{M}^D = \mathcal{M}(p_1, p_2), \qquad \mathcal{M}^M = \frac{1}{\sqrt{2}}\Big(\mathcal{M}(p_1, p_2) - \mathcal{M}(p_2, p_1)\Big).$$

Dirac case

Majorana case





D=in general, no reason to be symmetric

M=always, must be symmetric

$$\underbrace{|\mathcal{M}(p_1, p_2)|^2}_{\text{Direct term}} \neq \underbrace{|\mathcal{M}(p_2, p_1)|^2}_{\text{Exchange term}}.$$

$$\iint \left(\left|\mathcal{M}^D\right|^2 - \left|\mathcal{M}^M\right|^2\right) \mathrm{d}^4p_1\, \mathrm{d}^4p_2 \quad \propto m_\nu^2.$$

Dirac-Majorana Confusion Theorem (3)

Therefore,
(if momenta p_1 and p_2 are unobservable)

$$\iint \left(|\mathscr{M}^D|^2 - |\mathscr{M}^M|^2 \right) d^4p_1\, d^4p_2$$

$$= 2 \iint \underbrace{\mathrm{Re}\left(\mathscr{M}(p_1, p_2)^* \mathscr{M}(p_2, p_1) \right)}_{\text{Interference term}} d^4p_1\, d^4p_2$$

In general,

$$\propto m_\nu^2.$$

Practical Dirac-Majorana confusion theorem: By looking at the total decay rate or any other kinematic test of a process allowed in the SM, it is practically impossible to distinguish between the Dirac and Majorana neutrinos in the limit neutrino mass goes to zero.

Weak neutral current process in SM

B. Kayser, Phys. Rev. D **26**, 1662 (1982).

No general proof independent of process or observable

[J Schechter, JW Valle, PRD23 (1981)]

History trying to overcome DMCT, but only confirming

All for weak neutral current process in SM

$$\gamma^* \to \nu\bar{\nu}$$ [B Kayser, PRD26(1982)] ** [0]

[1] S.P.Rosen,PRL48(1982);
W.Rodejohann etal,JHEP05(2017).

$$Z \to \nu\bar{\nu}$$ [RE Shrock, eConf(1982)]

$$e^+e^- \to \nu\bar{\nu}$$ [E Ma, JT Pantaleone, PRD40(1989)]

$$K^+ \to \pi^+\nu\bar{\nu}$$ [JF Nieves, PB Pal, PRD32(1985)]

$$e^+e^- \to \nu\bar{\nu}\gamma$$ [T Chabra, PR Babu, PRD46(1992)] ** [2]

$$|es> \to |gs> + \gamma\nu\bar{\nu}$$ [Y Yoshimura, PRD75(2007)], ….

$$e^-\gamma \to e^-\nu\bar{\nu}$$ [JM Berryman etal, PRD98(2018)]

** All practically impossible to measure momenta of nu-nubar ➔ Need integrate out ➔ pDMCT

Discussion on pDMCT (1) – Is it 'Theorem' ?

** It is believed to suggest that all difference between Dirac and Majorana neutrinos must be proportional to some power of neutrino mass (m_ν).

(a) There is no model-independent, process-independent and observable-independent proof of this so-called "theorem". All processes where it was shown to hold involved full integration over the 4-momenta of missing neutrinos and/or only for Z^(*) → nu nubar case .

(b) We prove the fact that this "theorem" can be overcome, if energy and/or momentum of neutrino can be inferred or measured. The interesting question is to find out a way to realize this, which we do by measuring muon energy in the back-to-back muons configuration in the B rest frame.

[CSK,MM,DS, PRD105,113005(2022)]

$$B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2), \qquad E_\nu = m_B/2 - E_\mu$$

on pDMCT (4-4) – Is the final state $\nu\bar{\nu}$ the same as $\nu_L\bar{\nu}_R$?

We see that chirality and helicity have different characteristics: Helicity is conserved for a free particle but is not Lorentz invariant, whereas chirality is Lorentz invariant but not conserved. Therefore, none of these properties is appropriate for characterizing a fermion that has mass. If a particle

[Reference: P. B. Pal, Am. J. Phys. **79**, 485-498 (2011).]

is the only way we can observe them. There is thus no empirical evidence for the existence of ν_R (and $\bar{\nu}_L$), and it could well be that they do not exist in nature. Of course, the assertion that only ν_L (and $\bar{\nu}_R$) occur can only be made if the mass is strictly zero. Otherwise, we could perform a Lorentz transformation which would change a ν_L into a ν_R.

[Reference: F. Halzen and A. D. Martin, Joh Wiley & Sons, Inc. (page 115).]

[1]Helicity is identical to handedness (or chirality) for massless neutrinos and nearly identical to handedness for particles traveling near the speed of light. For that reason, helicity is sometimes loosely referred to as "handedness." For massive particles, however, the two quantities are quite different. Massive particles must exist in right- and left-helicity and in right- and left-handed states. As illustrated in the box on page 32 and

[Reference: Richard Slansky *et al.*, "The Oscillating neutrino," Los Alamos Science, Number 25, 1997.]

state containing two fermions. If the fermions are Dirac particles, the Feynman amplitude for the process is

$$\boxed{\mathcal{M} = g\sum_{s_1,s_2} \bar{u}_{s_1}(p_1)Fv_{s_2}(p_2), \qquad (146)}$$

If, in contrast, a Majorana pair is produced in the final

state, the amplitude is different because the operator ψ can create either of the two, and so can $\bar{\psi}$. We therefore write

$$\boxed{\mathcal{M} = g\sum_{s_1,s_2} \left(\bar{u}_{s_1}(p_1)Fv_{s_2}(p_2) - \bar{u}_{s_2}(p_2)Fv_{s_1}(p_1)\right), \qquad (147)}$$

[Reference: P. B. Pal, Am. J. Phys. **79**, 485-498 (2011).]

teractions as well. If we take them into account, the effective leptonic current, which is simply $\bar{\nu}\gamma^\mu(1-\gamma_5)\nu$ for the Dirac case, becomes

$$\bar{\nu}\gamma^\mu(1-\gamma_5)\nu - \bar{\nu}\gamma^\mu(1+\gamma_5)\nu = -2\bar{\nu}\gamma^\mu\gamma_5\nu$$

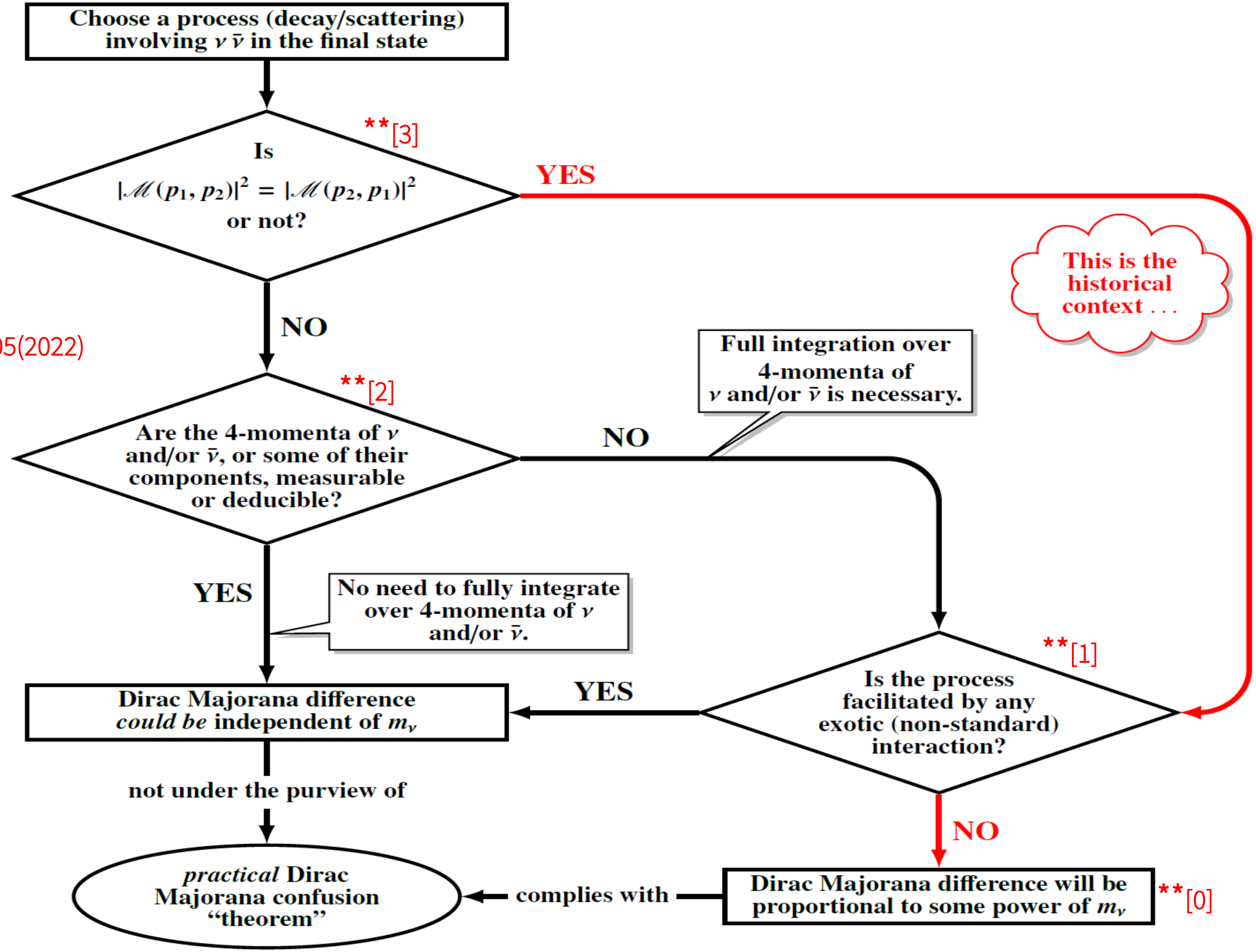
for the Majorana case.

[Reference: J. F. Nieves and P. B. Pal, Phys. Rev. D **32**, 1849-1852 (1985).]

A Majorana-neutrino pair can coherently interfere with each other since particle and antiparticle are identical while a Dirac pair will not interfere. The Feynman diagrams for the Majorana-neutrino production process are shown in Fig. 1. For an electron (or quark) coupling of

[Reference: E. Ma and J. T. Pantaleone, Phys. Rev. D **40**, 2172-2176 (1989).]

[0] B.Kayser,PRD26(1982)
[1] S.P.Rosen,PRL48(1982); CSK etal. EPJC83(2023)
[2] T.Chabra,PR.Babu,PRD46(1992); CSK etal. PRD105(2022)
[3] CS Kim, Work in Progress(2023)



How to overcome pDMCT ?

New physics effects in neutrino interaction

$B^0(p_B) \to \mu^-(p_-)\,\mu^+(p_+)\,\bar{\nu}_\mu(p_1)\,\nu_\mu(p_2),$ and deduction of neutrino energy-momenta

Extra 1 [C.S.Kim, J.Rosiek, D.Sahoo, arXiv:2209.10110, EPJC83 (2023)]

New Physics Effects to distinguish Majorana from Dirac

General Comments on New Physics Scenario

Detailed study on $Z \to \nu_\ell \bar{\nu}_\ell$

Detailed study of $B \to K\nu\bar{\nu}, K \to \pi\nu\bar{\nu}$

** **Special case :** $|M(p_1,p_2)|^2 = |M(p_2,p_1)|^2$

$$|\mathscr{M}^D|^2 - |\mathscr{M}^M|^2 = \frac{1}{2}\Big(\underbrace{|\mathscr{M}(p_1,p_2)|^2}_{\text{Direct term}} - \underbrace{|\mathscr{M}(p_2,p_1)|^2}_{\text{Exchange term}}\Big)$$

$$+ \underbrace{\mathrm{Re}\big(\mathscr{M}(p_1,p_2)^* \mathscr{M}(p_2,p_1)\big)}_{\text{Interference term}}. \longrightarrow m_\nu^2$$

<u>pDMCT holds at the Amplitude-Squared level</u>

➔ Difference between Dirac and Majorana neutrinos becomes 0 at every observable

➔ e.g. Within the SM, weak neutral current decays : $Z^{(*)} \to \nu\bar{\nu}$

general Comments on New physics effects to pDMCT

Choose Process: $X(p_X) \to Y(p_Y)\, \nu(p_1)\, \overline{\nu}(p_2)$

(a) $X, Y =$ single/multi-particle states, Y can also be null,

(b) 4-momenta p_X, p_Y are well measured.

Decay Amplitudes: Showing p_1, p_2 dependencies alone for brevity of expression,

(a) DIRAC case: $\mathscr{M}^D = \mathscr{M}(p_1, p_2)$,

(b) MAJORANA case: $\mathscr{M}^M = \frac{1}{\sqrt{2}}\Big(\underbrace{\mathscr{M}(p_1,p_2)}_{\text{Direct amplitude}} - \underbrace{\mathscr{M}(p_2,p_1)}_{\text{Exchange amplitude}} \Big)$.

$$|\mathscr{M}^D|^2 - |\mathscr{M}^M|^2 = \frac{1}{2}\Big(\underbrace{|\mathscr{M}(p_1,p_2)|^2}_{\text{Direct term}} - \underbrace{|\mathscr{M}(p_2,p_1)|^2}_{\text{Exchange term}} \Big) + \underbrace{\mathrm{Re}\left(\mathscr{M}(p_1,p_2)^* \mathscr{M}(p_2,p_1)\right)}_{\text{Interference term}}.$$

In general, $\underbrace{|\mathscr{M}(p_1,p_2)|^2}_{\text{Direct term}} \neq \underbrace{|\mathscr{M}(p_2,p_1)|^2}_{\text{Exchange term}}$.

Special cases $\underbrace{|\mathscr{M}(p_1,p_2)|^2}_{\text{Direct term}} = \underbrace{|\mathscr{M}(p_2,p_1)|^2}_{\text{Exchange term}}$,

$$\underbrace{|\mathscr{M}(p_1,p_2)|^2}_{\text{Direct term}} = \underbrace{|\mathscr{M}(p_2,p_1)|^2}_{\text{Exchange term}} \implies \begin{cases} p_1 = p_2 \equiv p, & \left(\text{special scenario } \boxed{\text{A}}\right) \\ \mathscr{M}(p_1,p_2) = +\mathscr{M}(p_2,p_1), & \left(\text{special scenario } \boxed{\text{B}}\right) \\ \mathscr{M}(p_1,p_2) = -\mathscr{M}(p_2,p_1). & \left(\text{special scenario } \boxed{\text{C}}\right) \end{cases}$$

$\boxed{\text{A}}$ $\implies \boxed{\left|\mathscr{M}^D_{\text{collinear}}\right|^2 - \left|\mathscr{M}^M_{\text{collinear}}\right|^2 = |\mathscr{M}(p,p)|^2 \neq 0.}$

$\boxed{\text{B}}$ $\implies \boxed{\left|\mathscr{M}^D_{\text{symmetric}}\right|^2 - \left|\mathscr{M}^M_{\text{symmetric}}\right|^2 = |\mathscr{M}(p_1,p_2)|^2 \neq 0.}$

$$\mathscr{M}(p_1,p_2) \propto \begin{cases} \left[\overline{u}(p_1)\gamma^\alpha v(p_2)\right], & \text{(neutral vector current)} \\ \left[\overline{u}(p_1)\sigma^{\alpha\beta} v(p_2)\right], & \text{(neutral tensor current)} \end{cases}$$

$\boxed{\text{C}}$ $\implies \boxed{\left|\mathscr{M}^D_{\text{anti-symm}}\right|^2 - \left|\mathscr{M}^M_{\text{anti-symm}}\right|^2 = -|\mathscr{M}(p_1,p_2)|^2 \neq 0.}$

$$\mathscr{M}(p_1,p_2) \propto \begin{cases} \left[\overline{u}(p_1) v(p_2)\right], & \text{(neutral scalar current)} \\ \left[\overline{u}(p_1)\gamma^5 v(p_2)\right], & \text{(neutral pseudo-scalar current)} \\ \left[\overline{u}(p_1)\gamma^\alpha \gamma^5 v(p_2)\right], & \text{(neutral axial-vector current)} \end{cases}$$

$Z \to \nu\,\overline{\nu}$

$$\boxed{|\mathscr{M}^D|^2 - |\mathscr{M}^M|^2 = \frac{g_Z^2}{3}\left(\left(C_V^2 - C_A^2\right)\left(m_Z^2 + 2m_\nu^2\right) + 6\,C_A^2 m_\nu^2\right).}$$

$C_V = C_A = \frac{1}{2}$ $\qquad |\mathscr{M}^D|^2 - |\mathscr{M}^M|^2 = \frac{g_Z^2}{2} m_\nu^2$,

General New Physics Scenario for $X(p_X) \to Y(p_Y)\,\nu(p_1)\,\bar{\nu}(p_2)$ (1)

- In general,

$$\mathcal{M}^D = \mathcal{M}(p_1, p_2) = \mathcal{M}_{\text{symm}}(p_1, p_2) + \mathcal{M}_{\text{anti-symm}}(p_1, p_2),$$

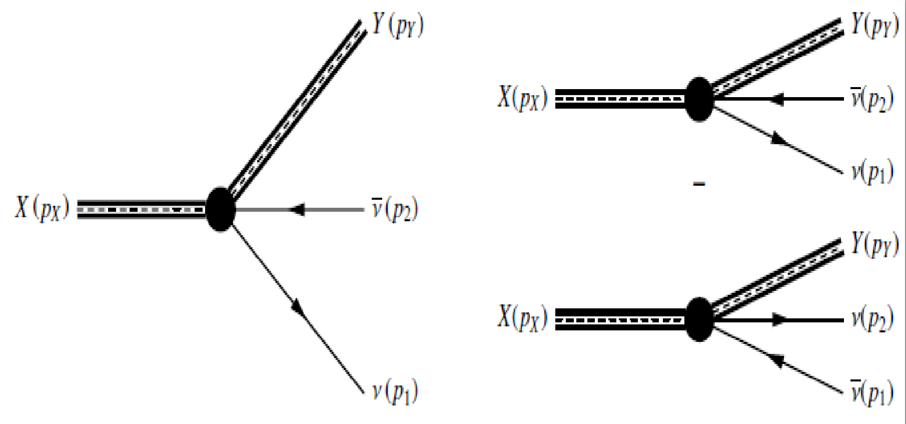
where

$$\mathcal{M}_{\text{symm}}(p_1, p_2) = \frac{1}{2}\Big(\mathcal{M}(p_1, p_2) + \mathcal{M}(p_2, p_1)\Big) = \mathcal{M}_{\text{symm}}(p_2, p_1),$$

$$\mathcal{M}_{\text{anti-symm}}(p_1, p_2) = \frac{1}{2}\Big(\mathcal{M}(p_1, p_2) - \mathcal{M}(p_2, p_1)\Big) = -\,\mathcal{M}_{\text{anti-symm}}(p_2, p_1),$$

then,

$$\mathcal{M}^M = \sqrt{2}\,\mathcal{M}_{\text{anti-symm}}(p_1, p_2).$$



(a) Dirac neutrinos (b) Majorana neutrinos (with exchange diagram)

Therefore,

$$\left|\mathcal{M}^D\right|^2 - \left|\mathcal{M}^M\right|^2 = \left|\mathcal{M}_{\text{symm}}(p_1, p_2) + \mathcal{M}_{\text{anti-symm}}(p_1, p_2)\right|^2 - 2\left|\mathcal{M}_{\text{anti-symm}}(p_1, p_2)\right|^2$$
$$= \left|\mathcal{M}_{\text{symm}}(p_1, p_2)\right|^2 - \left|\mathcal{M}_{\text{anti-symm}}(p_1, p_2)\right|^2$$
$$+\, 2\,\mathrm{Re}\left(\mathcal{M}_{\text{symm}}(p_1, p_2)^*\,\mathcal{M}_{\text{anti-symm}}(p_1, p_2)\right).$$

So, after full integration over nu and nu-bar momenta,

$$\iint\left(\left|\mathcal{M}^D\right|^2 - \left|\mathcal{M}^M\right|^2\right) d^4p_1\, d^4p_2 = \iint\left(\left|\mathcal{M}_{\text{symm}}(p_1, p_2)\right|^2 - \left|\mathcal{M}_{\text{anti-symm}}(p_1, p_2)\right|^2\right) d^4p_1\, d^4p_2.$$

which may not vanish with NP effects (overcoming pDMCT).

General New Physics Scenario for $X(p_X) \to Y(p_Y)\,\nu(p_1)\,\bar{\nu}(p_2)$ (2)

- ** Important point for Majorana neutrino:

- For spinor of Majorana neutrinos, consider $\mathscr{M}^M = \sqrt{2}\,\mathscr{M}_{\text{anti-symm}}(p_1, p_2).$

$$\bar{u}(k_1)\,\Gamma\,v(k_2) = (-1)\,\bar{u}(k_2)\,C\,\Gamma^T\,C^{-1}\,v(k_1),$$

[Denner et al, NPB387, 467 (1992)]

Γ	$C\,\Gamma^T\,C^{-1}$
$\mathbf{1}$	$\mathbf{1}$
γ^5	γ^5
γ^α	$-\gamma^\alpha$
$\gamma^\alpha\,\gamma^5$	$\gamma^\alpha\,\gamma^5$
$\sigma^{\alpha\beta}$	$-\sigma^{\alpha\beta}$
$\sigma^{\alpha\beta}\,\gamma^5$	$-\sigma^{\alpha\beta}\,\gamma^5$

where (-1) from anticommutation

C for charge conjugation

$\mathscr{M}_{\text{symm}}(p_1, p_2)$ gets contributions only from vector, tensor and axial-tensor interactions

$\mathscr{M}_{\text{anti-symm}}(p_1, p_2)$ gets contributions from scalar, pseudo-scalar and axial-vector interactions

Detailed study of $Z \to \nu_\ell \bar{\nu}_\ell$ (1)

Most general Z → nu nubar amplitude for Dirac neutrino

$$\mathcal{M}^D = -i\,\epsilon_\alpha(p)\,\overline{u}(p_1)\Big[\big(g_S^+ + g_P^+\gamma^5\big)p^\alpha + \big(g_S^- + g_P^-\gamma^5\big)q^\alpha + \gamma^\alpha\big(g_V + g_A\gamma^5\big) + \sigma^{\alpha\beta}\big(g_{T_{\rm md}}^+ + g_{T_{\rm ed}}^+\gamma^5\big)p_\beta + \sigma^{\alpha\beta}\big(g_{T_{\rm md}}^- + g_{T_{\rm ed}}^-\gamma^5\big)q_\beta\Big]v(p_2),$$

In the SM, $g_S^\pm = g_P^\pm = g_{T_{\rm md}}^\pm = g_{T_{\rm ed}}^\pm = 0$ and $g_V = -g_A = \frac{g_Z}{4}$ where $g_Z = e/(\sin\theta_W \cos\theta_W)$

After using $p^\alpha\,\epsilon_\alpha(p) = 0,$ CP and CPT conservation, and w/ Gordon identity,
we get the most general decay amplitude for Z → nu nubar becomes:

$$\mathcal{M}^D = \mathcal{M}(p_1, p_2) = -\frac{i\,g_Z}{2}\epsilon_\alpha(p)\left[\overline{u}(p_1)\,\gamma^\alpha\left(C_V^\ell - C_A^\ell\,\gamma^5\right)v(p_2)\right],$$

with $C_{V,A}^\ell = \frac{1}{2} + \varepsilon_{V,A}^\ell,$

Detailed study of $Z \to \nu_\ell \bar{\nu}_\ell$ (2)

Most general amplitude for Dirac neutrino

$$\mathscr{M}^D = \mathscr{M}(p_1, p_2) = -\frac{i\, g_Z}{2}\epsilon_\alpha \left[\overline{u}(p_1)\,\gamma^\alpha \left(C_V^\ell - C_A^\ell\,\gamma^5\right) v(p_2)\right],$$

$$\boxed{C_{V,A}^\ell = \frac{1}{2} + \varepsilon_{V,A}^\ell,} \qquad \varepsilon_{V,A}^\ell = 0. \quad \text{(SM)}$$

Then,
$$\mathscr{M}^M = \frac{1}{\sqrt{2}}\Big(\mathscr{M}(p_1, p_2) - \mathscr{M}(p_2, p_1)\Big) = \frac{i\, g_Z\, C_A^\ell}{\sqrt{2}}\epsilon_\alpha \left[\overline{u}(p_1)\,\gamma^\alpha\,\gamma^5\, v(p_2)\right].$$

$$\left|\mathscr{M}^D\right|^2 = \frac{g_Z^2}{3}\left(\left((C_V^\ell)^2 + (C_A^\ell)^2\right)\left(m_Z^2 - m_\nu^2\right) + 3\left((C_V^\ell)^2 - (C_A^\ell)^2\right) m_\nu^2\right),$$

$$\left|\mathscr{M}^M\right|^2 = \frac{2\, g_Z^2\,(C_A^\ell)^2}{3}\left(m_Z^2 - 4\, m_\nu^2\right),$$

such that
$$\left|\mathscr{M}^D\right|^2 - \left|\mathscr{M}^M\right|^2 = \frac{g_Z^2}{3}\left(\left((C_V^\ell)^2 - (C_A^\ell)^2\right)\left(m_Z^2 + 2\, m_\nu^2\right) + 6\,(C_A^\ell)^2\, m_\nu^2\right)$$
$$= \begin{cases} \dfrac{g_Z^2}{2} m_\nu^2, & \text{(for SM alone)} \\[2ex] \dfrac{g_Z^2}{3}\left(\varepsilon_V^\ell - \varepsilon_A^\ell\right) m_Z^2, & \text{(with NP but neglecting } m_\nu) \end{cases}$$

[DMCT at amplitude^2]

[DMCT x with NP]

Detailed study of $Z \to \nu_\ell \bar{\nu}_\ell$ (3)

Therefore, neglecting neutrino mass,

$$\Gamma^D(Z \to \nu_\ell \bar{\nu}_\ell) = \Gamma_Z^0\left(1 + 2\,\varepsilon_V^\ell + 2\,\varepsilon_A^\ell\right),$$
$$\Gamma^M(Z \to \nu_\ell \bar{\nu}_\ell) = \Gamma_Z^0\left(1 + 4\,\varepsilon_A^\ell\right),$$

$$\Gamma_Z^0 = \frac{G_F\, m_Z^3}{12\sqrt{2}\,\pi},$$

Then,

$$\Gamma_{Z,\text{inv}} = \begin{cases} \Gamma_Z^0\left(3 + 2\displaystyle\sum_{\ell=e,\mu,\tau}\left(\varepsilon_V^\ell + \varepsilon_A^\ell\right)\right), & \text{(for Dirac neutrinos)} \\ \Gamma_Z^0\left(3 + 4\displaystyle\sum_{\ell=e,\mu,\tau}\varepsilon_A^\ell\right). & \text{(for Majorana neutrinos)} \end{cases}$$

$$N_\nu = \Gamma_{Z,\text{inv}}/\Gamma_Z^0 = 2.9963 \pm 0.0074.$$

[pdg(2021), PLB803(2020)]

$$\sum_{\ell=e,\mu,\tau}\left(\varepsilon_V^\ell + \varepsilon_A^\ell\right) = -0.0018 \pm 0.0037, \qquad \text{(for Dirac neutrinos)}$$

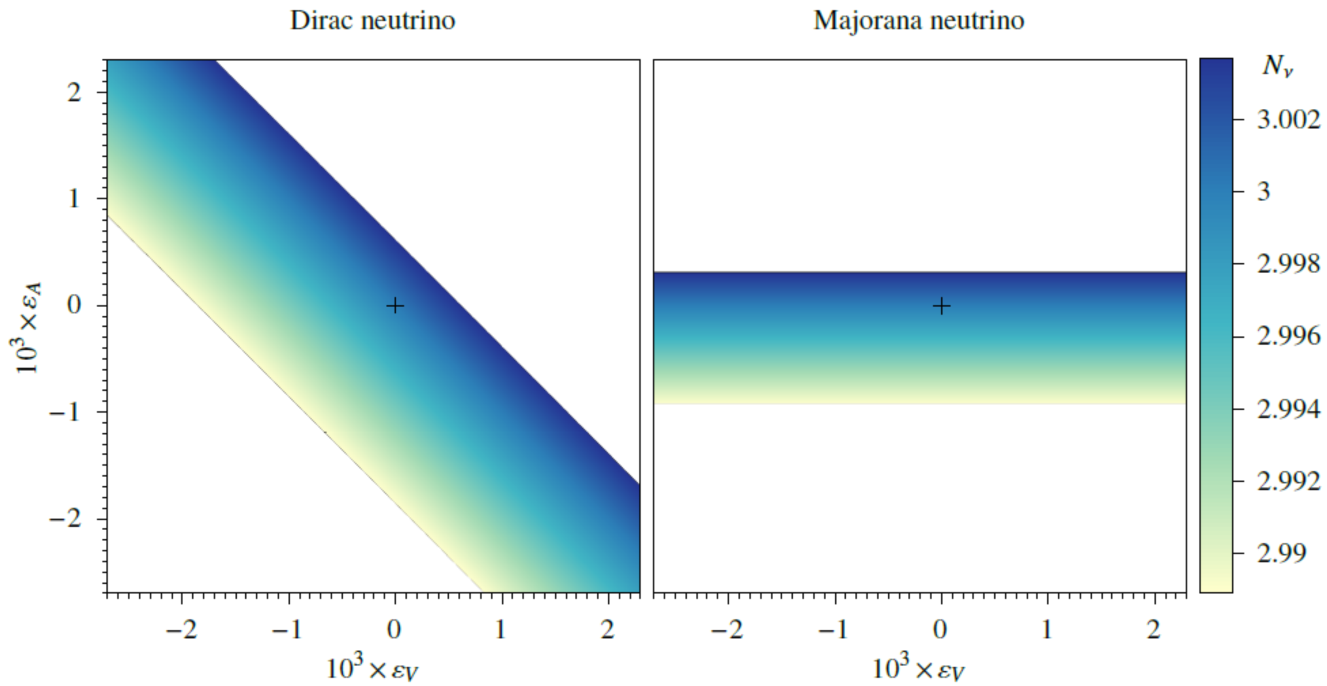
$$\sum_{\ell=e,\mu,\tau}\varepsilon_A^\ell = -0.0009 \pm 0.0018, \qquad \text{(for Majorana neutrinos)}$$

Detailed study of $Z \to \nu_\ell \bar{\nu}_\ell$ (4)

$$N_\nu = \Gamma_{Z,\mathrm{inv}}/\Gamma_Z^0 = 2.9963 \pm 0.0074.$$

$$\sum_{\ell=e,\mu,\tau} \left(\varepsilon_V^\ell + \varepsilon_A^\ell\right) = -0.0018 \pm 0.0037, \qquad \text{(for Dirac neutrinos)}$$

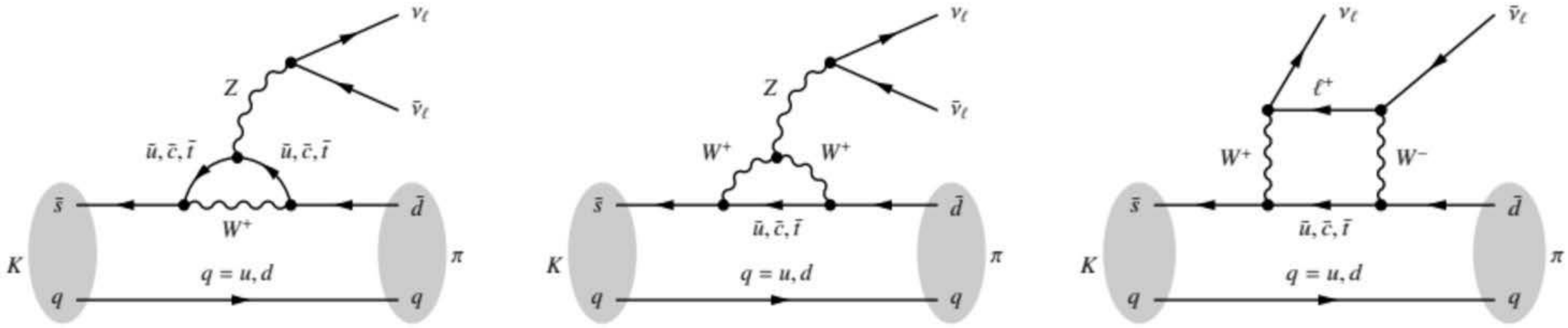
$$\sum_{\ell=e,\mu,\tau} \varepsilon_A^\ell = -0.0009 \pm 0.0018, \qquad \text{(for Majorana neutrinos)}$$



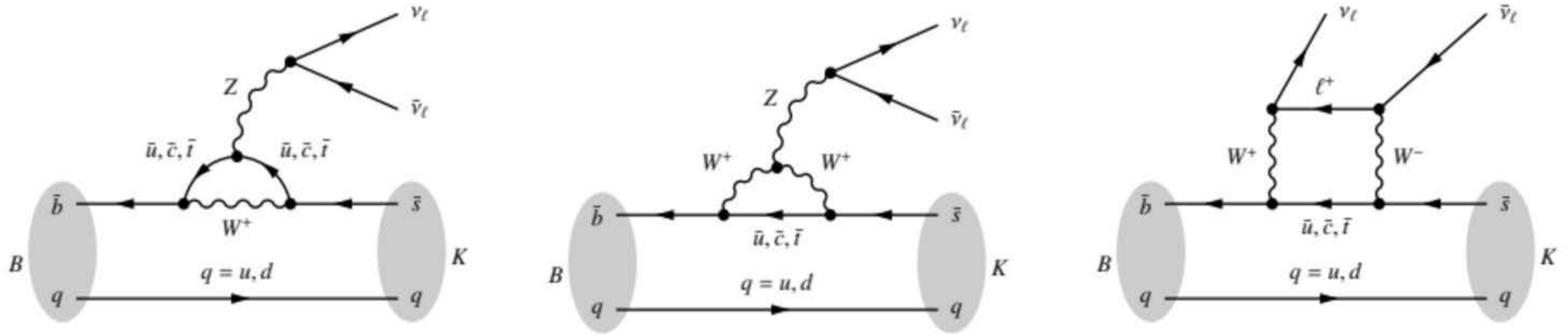
$$\varepsilon_{V,A}^e = \varepsilon_{V,A}^\mu = \varepsilon_{V,A}^\tau \equiv \varepsilon_{V,A}$$

+ = SM

Detailed study of $B \to K\nu\bar{\nu}, K \to \pi\nu\bar{\nu}$ (1) (within SM)



Feynman Diagrams in the SM (Weak penguin and Box diagrams)



Detailed study of $B \to K\nu\bar{\nu}, K \to \pi\nu\bar{\nu}$ (2) (beyond SM)

For general process, $P_i \to P_f \nu\bar{\nu}$ where $P_i = B, K$ and $P_f = K, \pi$

(most general effective) Lagrangian:

$$\mathcal{L} = J_{SL}\left(\overline{\psi}_\nu P_L \psi_{\bar{\nu}}\right) + J_{SR}\left(\overline{\psi}_\nu P_R \psi_{\bar{\nu}}\right) + (J_{VL})_\alpha \left(\overline{\psi}_\nu \gamma^\alpha P_L \psi_{\bar{\nu}}\right) + (J_{VR})_\alpha \left(\overline{\psi}_\nu \gamma^\alpha P_R \psi_{\bar{\nu}}\right)$$
$$+ (J_{TL})_{\alpha\beta} \left(\overline{\psi}_\nu P_L \sigma^{\alpha\beta} \psi_{\bar{\nu}}\right) + (J_{TR})_{\alpha\beta} \left(\overline{\psi}_\nu \sigma^{\alpha\beta} P_R \psi_{\bar{\nu}}\right) + \text{h.c.},$$

where J_x denotes various effective hadronic transition currents

Most general Decay amplitude: decay amplitude for $\mathcal{P}_i(p_i) \to \mathcal{P}_f(p_f)\, \nu(p_1)\, \overline{\nu}(p_2)$

Dirac case:

$$\mathcal{M}^D = \mathcal{M}(p_1, p_2) = \overline{u}(p_1)\Big[F_{SL} P_L + F_{SR} P_R$$
$$+ (F_{VL}^+ p_\alpha + F_{VL}^- q_\alpha)\, \gamma^\alpha P_L + (F_{VR}^+ p_\alpha + F_{VR}^- q_\alpha)\, \gamma^\alpha P_R$$
$$+ F_{TL}\, p_\alpha q_\beta \sigma^{\alpha\beta} P_L\, v(p_2) + F_{TR}\, p_\alpha q_\beta \sigma^{\alpha\beta} P_R\Big] v(p_2),$$

where $p = p_i + p_f$, $q = p_i - p_f = p_1 + p_2$ and

various form factors F_{SX}^ℓ, $F_{VX}^{\ell\pm}$, F_{TX}^ℓ are defined as

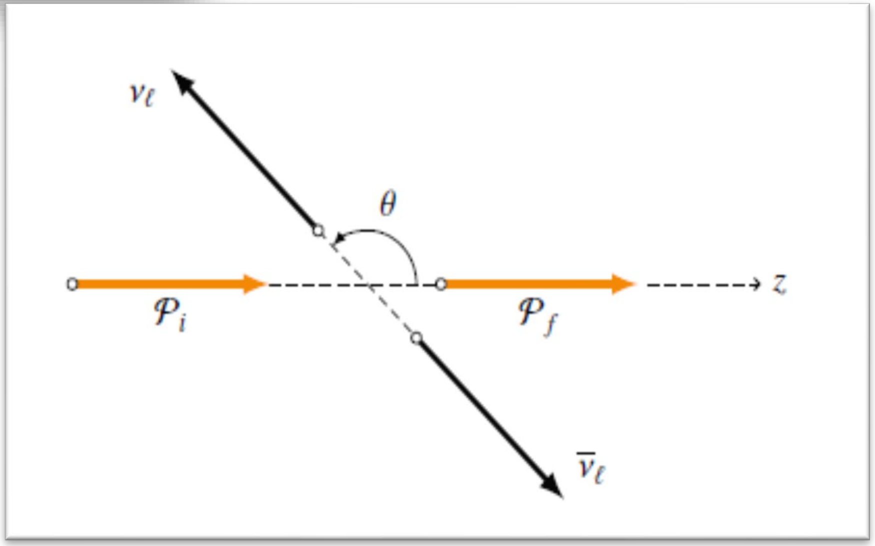
$$\left\langle \mathcal{P}_f \middle| J_{SX}^\ell \middle| \mathcal{P}_i \right\rangle = F_{SX}^\ell,$$
$$\left\langle \mathcal{P}_f \middle| \left(J_{VX}^\ell\right)_\alpha \middle| \mathcal{P}_i \right\rangle = F_{VX}^{\ell+} p_\alpha + F_{VX}^{\ell-} q_\alpha$$
$$\left\langle \mathcal{P}_f \middle| \left(J_{TX}^\ell\right)_{\alpha\beta} \middle| \mathcal{P}_i \right\rangle = F_{TX}^\ell\, p_\alpha q_\beta,$$

Detailed study of $B \to K\nu\bar{\nu}, K \to \pi\nu\bar{\nu}$ (3)

For general process, $P_i \to P_f \nu\bar{\nu}$ where $P_i = B, K$ and $P_f = K, \pi$

Majorana case (after antisymmetrization):

$$\mathcal{M}^M = \frac{1}{\sqrt{2}}\left(\mathcal{M}(p_1, p_2) - \mathcal{M}(p_2, p_1)\right)$$
$$= \sqrt{2}\,\bar{u}(p_1)\left[F_{SL} P_L + F_{SR} P_R + \left(\frac{F^+_{VR} - F^+_{VL}}{2} p_\alpha + \frac{F^-_{VR} - F^-_{VL}}{2} q_\alpha\right)\gamma^\alpha \gamma^5\right] v(p_2).$$



Form Factors, F(as a function of s=q^2), including CKM.

Phase space can be fully described by s=q^2=(p_1+p_2)^2, and the angle theta, defined as:

Kinematic configuration of the decay $\mathcal{P}_i \to \mathcal{P}_f \nu\bar{\nu}$ in center-of-momentum frame of the $\nu\bar{\nu}$ pair.

Detailed study of $B \to K\nu\bar{\nu}, K \to \pi\nu\bar{\nu}$ (4)

With futuristic detectors, such as FASER, SHiP, Mathusla, Gazelle, …

$$\left|\mathscr{M}^{D/M}\right|^2 = C_0^{D/M} + C_1^{D/M}\cos\theta + C_2^{D/M}\cos^2\theta,$$

where

$$\begin{aligned}
C_0^D &= s\left(\left|F_{SL}^\ell\right|^2 + \left|F_{SR}^\ell\right|^2\right) + \lambda\left(\left|F_{VL}^{\ell+}\right|^2 + \left|F_{VR}^{\ell+}\right|^2\right),\\
C_1^D &= 2\,s\,\sqrt{\lambda}\left(\mathrm{Im}\left(F_{SL}^\ell F_{TL}^{\ell*}\right) + \mathrm{Im}\left(F_{SR}^\ell F_{TR}^{\ell*}\right)\right),\\
C_2^D &= -\lambda\left(\left|F_{VL}^{\ell+}\right|^2 + \left|F_{VR}^{\ell+}\right|^2 - s\left(\left|F_{TL}^\ell\right|^2 + \left|F_{TR}^\ell\right|^2\right)\right),
\end{aligned}$$

$$\begin{aligned}
C_0^M &= 2\,s\left(\left|F_{SL}^\ell\right|^2 + \left|F_{SR}^\ell\right|^2\right) + \lambda\left|F_{VL}^{\ell+} - F_{VR}^{\ell+}\right|^2,\\
C_1^M &= 0,\\
C_2^M &= -\lambda\left|F_{VL}^{\ell+} - F_{VR}^{\ell+}\right|^2,
\end{aligned}$$

with $\lambda = M_i^4 + M_f^4 + s^2 - 2\left(M_i^2 M_f^2 + s\,M_i^2 + s\,M_f^2\right).$

Once integrated over invisible nu, nu-bar (integrating cos(theta))

$$\frac{\mathrm{d}\Gamma^{D/M}}{\mathrm{d}s} = \frac{1}{(2\pi)^3}\frac{b}{16\,M_i^3}\left(C_0^{D/M} + \frac{1}{3}C_2^{D/M}\right),$$

$$b = \frac{\sqrt{\lambda}}{2}\sqrt{1 - \frac{4\,m_\nu^2}{s}}.$$

Detailed study of $B \to K\nu\bar{\nu}, K \to \pi\nu\bar{\nu}$ (5)

If NP effects are significant, they can not only be probed from the missing mass square ("s") distributions, but they would also be helpful in distinguishing Dirac and Majorana neutrinos.

Possible observables: missing-mass-square (s), number of event (BR), momenta of final meson

To probe NP effects and distinguish D-M neutrino, here we consider a simplified model independent study

$$
\begin{aligned}
F_{SX} &= M_i F_{\rm SM}\,\varepsilon_{SX},\\
F^+_{VL} &= F_{\rm SM}\left(1+\varepsilon_{VL}\right),\\
F^+_{VR} &= F_{\rm SM}\,\varepsilon_{VR},\\
F_{TX} &= \frac{1}{M_i} F_{\rm SM}\,\varepsilon_{TX},
\end{aligned}
$$

$\Rightarrow$

$$\frac{d\Gamma^D}{ds} = \frac{d\Gamma^{\rm SM}}{ds}\left(1 + 2\,{\rm Re}\,\varepsilon_{VL} + |\varepsilon_{VL}|^2 + |\varepsilon_{VR}|^2 + \frac{3sM_i^2}{2\lambda}\left(|\varepsilon_{SL}|^2 + |\varepsilon_{SR}|^2\right) + \frac{s}{2M_i^2}\left(|\varepsilon_{TL}|^2 + |\varepsilon_{TR}|^2\right)\right)$$

$$\frac{d\Gamma^M}{ds} = \frac{d\Gamma^{\rm SM}}{ds}\left(1 + 2\,{\rm Re}\,\varepsilon_{VL} - 2\,{\rm Re}\,\varepsilon_{VR} + |\varepsilon_{VL} - \varepsilon_{VR}|^2 + \frac{3sM_i^2}{\lambda}\left(|\varepsilon_{SL}|^2 + |\varepsilon_{SR}|^2\right)\right)$$

F_SM from Lattice (MILC) [PRD93,025026(2016)]

where $\frac{d\Gamma^{\rm SM}}{ds} = \frac{1}{(2\pi)^3}\frac{\lambda b\,|F_{\rm SM}|^2}{24\,M_i^3}$ $\quad \lambda = M_i^4 + M_f^4 + s^2 - 2\left(M_i^2 M_f^2 + s M_i^2 + s M_f^2\right)$ $\quad b = \frac{\sqrt{\lambda}}{2}\sqrt{1 - \frac{4m_\nu^2}{s}}.$

Detailed study of $B \to K\nu\bar{\nu}, K \to \pi\nu\bar{\nu}$ (6)

NP effects can be probed in three steps :

i) 1st step : Dominant contribution only

$$\frac{d\Gamma^D}{ds} = \frac{d\Gamma^{SM}}{ds}\left(1 + 2\,\mathrm{Re}\,\varepsilon_{VL} + |\varepsilon_{VL}|^2 + |\varepsilon_{VR}|^2 + \frac{3sM_i^2}{2\lambda}\left(|\varepsilon_{SL}|^2 + |\varepsilon_{SR}|^2\right) + \frac{s}{2M_i^2}\left(|\varepsilon_{TL}|^2 + |\varepsilon_{TR}|^2\right)\right) \approx \frac{d\Gamma^{SM}}{ds}\left(1 + 2\,\mathrm{Re}\,\varepsilon_{VL}\right),$$

$$\frac{d\Gamma^M}{ds} = \frac{d\Gamma^{SM}}{ds}\left(1 + 2\,\mathrm{Re}\,\varepsilon_{VL} - 2\,\mathrm{Re}\,\varepsilon_{VR} + |\varepsilon_{VL} - \varepsilon_{VR}|^2 + \frac{3sM_i^2}{\lambda}\left(|\varepsilon_{SL}|^2 + |\varepsilon_{SR}|^2\right)\right) \approx \frac{d\Gamma^{SM}}{ds}\left(1 + 2\,\mathrm{Re}\,\varepsilon_{VL} - 2\,\mathrm{Re}\,\varepsilon_{VR}\right),$$

$$\frac{d\Gamma^D}{ds} - \frac{d\Gamma^M}{ds} \approx 2\frac{d\Gamma^{SM}}{ds}\mathrm{Re}\varepsilon_{VR},$$

non-zero difference can, in principle, arise only if $\mathrm{Re}\varepsilon_{VR} \neq 0$.
can not be distinguished using $d\Gamma^{D/M}/ds$ alone, if only vector NP contributions are present.

left-chiral

$$\mathrm{Br}^{\mathrm{Exp}}(K^+ \to \pi^+ \nu\bar{\nu}) = \left(10.6^{+4.0}_{-3.4}\big|_{\mathrm{stat}} \pm 0.9_{\mathrm{syst}}\right) \times 10^{-11},$$ [NA62 Coll. JHEP06,093(2021)]

$$\mathrm{Br}^{\mathrm{SM}}(K^+ \to \pi^+ \nu\bar{\nu}) = (9.11 \pm 0.72) \times 10^{-11}.$$ [A. Buras et al, JHEP11,033(2015)]

$\mathrm{Re}\,\varepsilon_{VL} = 0.082 \pm 0.214,$ (for Dirac neutrinos)

$\mathrm{Re}\,\varepsilon_{VL} - \mathrm{Re}\,\varepsilon_{VR} = 0.082 \pm 0.214,$ (for Majorana neutrinos)

Detailed study of $B \to K\nu\bar{\nu}, K \to \pi\nu\bar{\nu}$ (7)

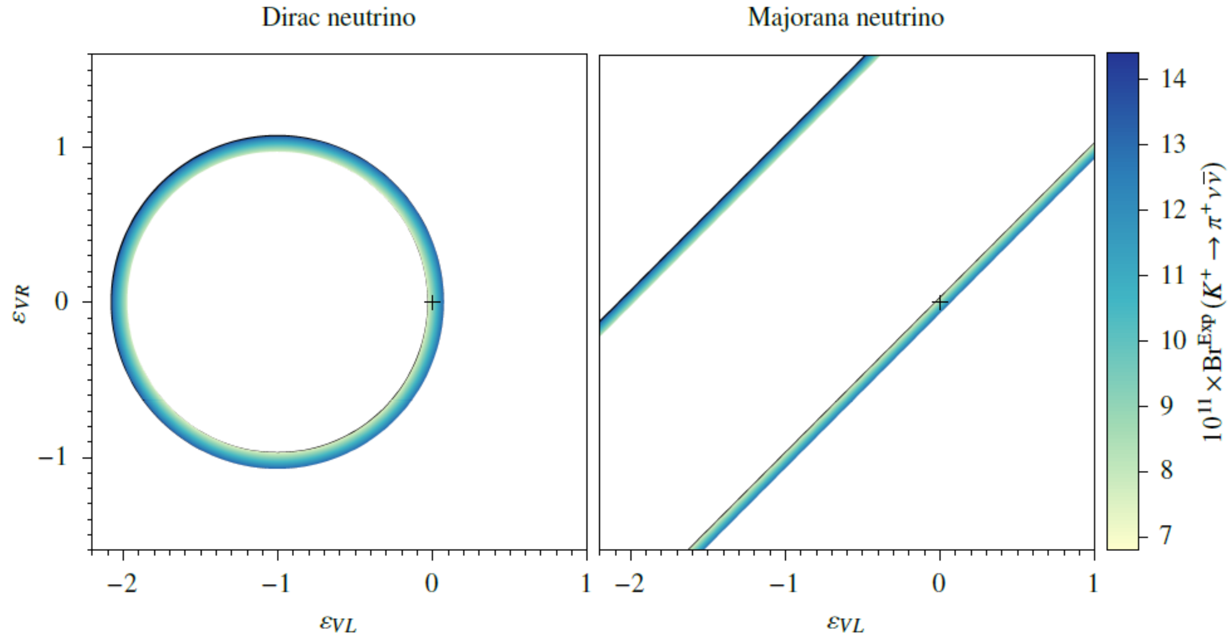
ii) 2^{nd} step : Dominant contribution + sub-dominant in Branching Fraction

$$\frac{d\Gamma^D}{ds} = \frac{d\Gamma^{SM}}{ds}\left(1 + 2\,\mathrm{Re}\,\varepsilon^{\ell}_{VL} + |\varepsilon^{\ell}_{VL}|^2 + |\varepsilon^{\ell}_{VR}|^2 + \right.$$

$$\frac{d\Gamma^M}{ds} = \frac{d\Gamma^{SM}}{ds}\left(1 + 2\,\mathrm{Re}\,\varepsilon^{\ell}_{VL} - 2\,\mathrm{Re}\,\varepsilon^{\ell}_{VR} + |\varepsilon^{\ell}_{VL} - \varepsilon^{\ell}_{VR}|^2 + \right.$$

$$\mathrm{Br}^{\mathrm{Exp}}(K^+ \to \pi^+ \nu\bar{\nu}) = \left(10.6^{+4.0}_{-3.4}\Big|_{\mathrm{stat}} \pm 0.9_{\mathrm{syst}}\right) \times 10^{-11},$$

$$\mathrm{Br}^{\mathrm{SM}}(K^+ \to \pi^+ \nu\bar{\nu}) = (9.11 \pm 0.72) \times 10^{-11}.$$



$$\mathrm{Br}^{\mathrm{SM}}(B^+ \to K^+ \nu\bar{\nu}) = (4.6 \pm 0.5) \times 10^{-6},$$

the current experimental upper limit by Belle II [39] is 4.1×10^{-5} at 90% confidence level.

Detailed study of $B \to K\nu\bar{\nu}, K \to \pi\nu\bar{\nu}$ (8)

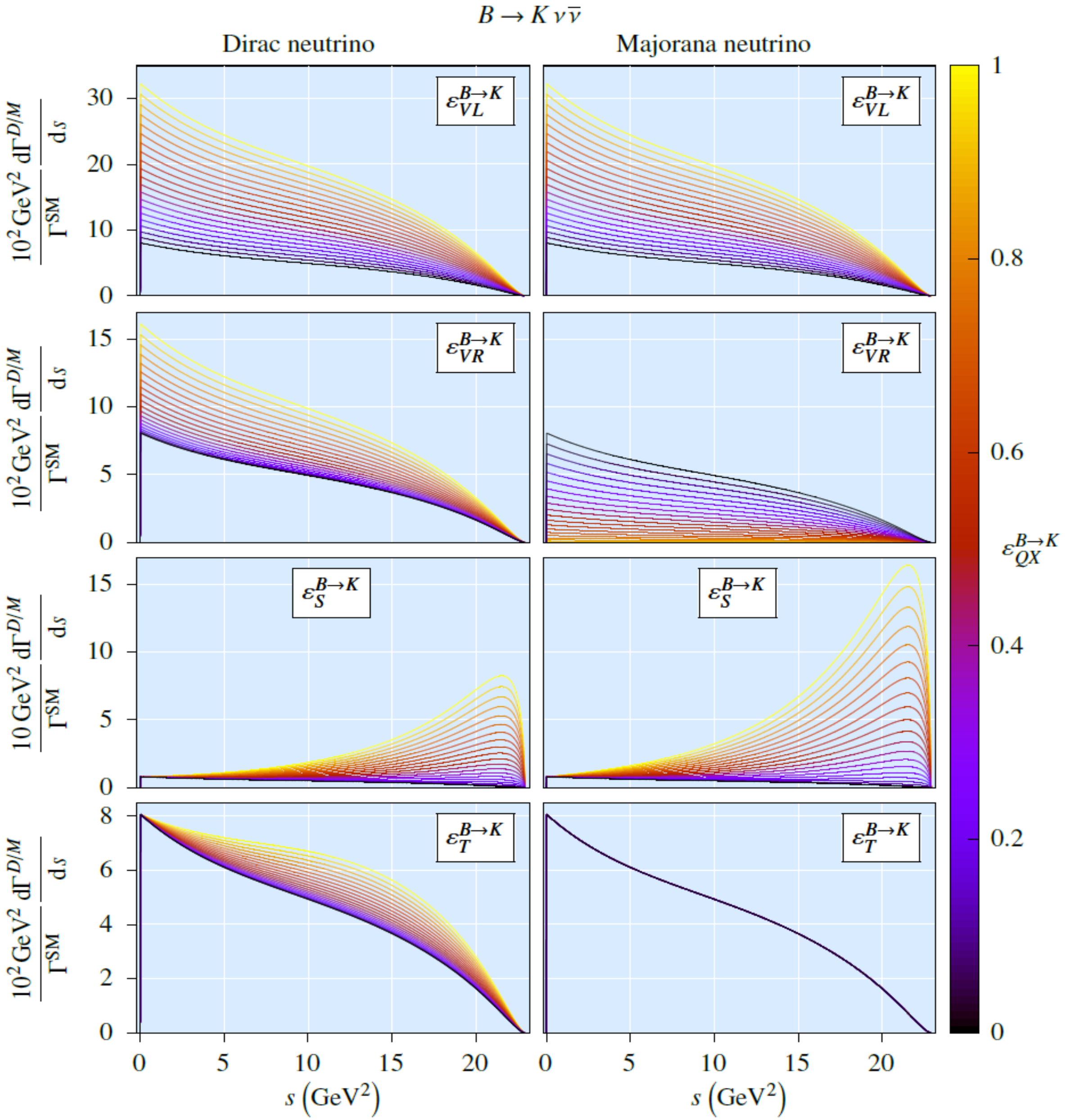
iii) 3rd step : Real pattern in s-distribution

Patterns in s distributions of $B \to K\,\nu\,\bar{\nu}$ decay in presence of NP

WITH FULL SM form factors **F_SM(s),**
INCLUDING **(M_f/M_i)^2** terms,
INCLUDING **m_nu** dependent terms.

F_SM from Lattice (MILC) [PRD93,025026(2016)]

1) (V-A) or W_L NP interaction just scaling the SM.
2) **(V+A) or W_R / Z' NP interaction gives DRASTIC difference between Dirac and Majorana nu.**
3) **V_L / Scalar NP interactions** give strong enhancement for **small s / large s region.**
4) Tensor NP interaction gives only small changes.



Discussions on new physics effects to neutrino property

(1) Probable existence of non-standard interactions can be probed in 2, 3-body meson decays such as using $Z \to \nu_\ell \bar{\nu}_\ell$ and $B \to K\nu\bar{\nu}, K \to \pi\nu\bar{\nu}$

(2) If such NP effects are experimentally observed, one can utilize them to distinguish between Dirac and Majorana neutrino possibilities.

(3) Within the SM, Z^(*) → nu nu-bar, pDMCT holds at amplitude^2 level.

→ Within SM, processes only for weak neutral current, pDMCT at any observables !!

(4) Our discussion very clearly illustrates the applicability and non-applicability of the practical Dirac Majorana confusion "theorem" (pDMCT).

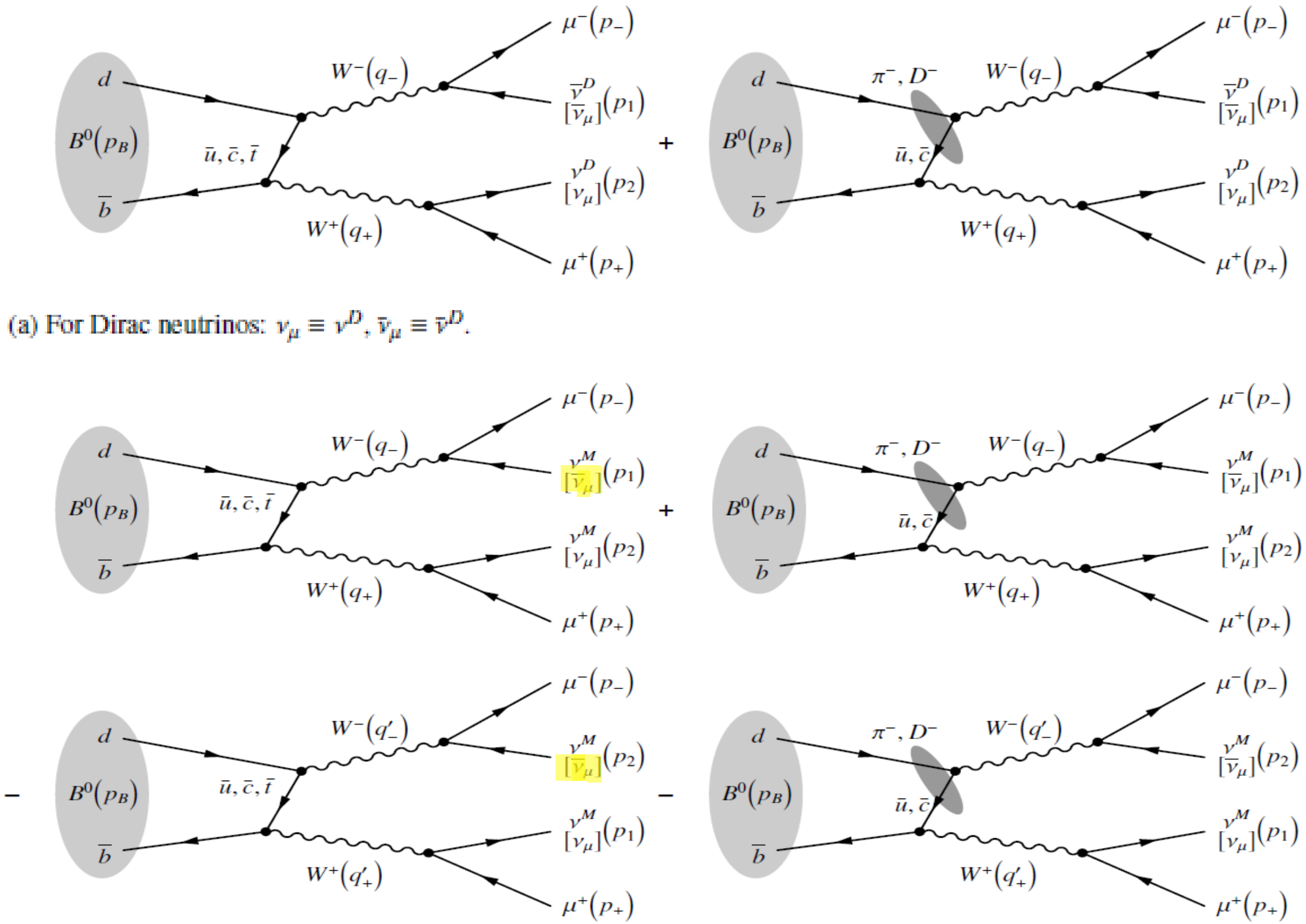
Extra 2 C. S. Kim, M. Murthy, D. Sahoo, PRD105(2022)

BACK-TO-BACK muons (ie. B2B $\nu - \bar{\nu}$)
- experimentally observable
  exception to DMCT

Study on $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$,

Doubly weak charged current process in SM

We can also consider
(B^0 → mu mu)(Z^* → nu nu)



(a) For Dirac neutrinos: $\nu_\mu \equiv \nu^D$, $\bar{\nu}_\mu \equiv \bar{\nu}^D$.

(b) For Majorana neutrinos: $\nu_\mu = \bar{\nu}_\mu \equiv \nu^M$.

$$\mathscr{M}^D = \frac{G_F^2}{2} H^{\alpha\beta} L_{\alpha\beta} \equiv \mathscr{Q}_{12} + \mathscr{R}_{12},$$

$$\mathscr{M}^M = \frac{G_F^2}{2\sqrt{2}} \left(H^{\alpha\beta} L_{\alpha\beta} - H'^{\alpha\beta} L'_{\alpha\beta} \right)$$

$$\equiv \frac{1}{\sqrt{2}} \left(\mathscr{Q}_{12} - \mathscr{Q}_{21} + \mathscr{R}_{12} - \mathscr{R}_{21} \right),$$

not helicity flip, but momentum exchange

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, (1)

$$L_{\alpha\beta} = \left[\bar{u}(p_-)\gamma_\alpha\left(1-\gamma^5\right)v(p_1)\right]\left[\bar{u}(p_2)\gamma_\beta\left(1-\gamma^5\right)v(p_+)\right] \qquad L'_{\alpha\beta} = \left[\bar{u}(p_-)\gamma_\alpha\left(1-\gamma^5\right)v(p_2)\right]\left[\bar{u}(p_1)\gamma_\beta\left(1-\gamma^5\right)v(p_+)\right]$$

$$\mathscr{R}_{12} = \frac{G_F^2}{2}\mathrm{H}^{\alpha\beta}L_{\alpha\beta}, \quad \mathrm{H}^{\alpha\beta} \equiv V_{ub}^*V_{ud}\mathbf{V}_\pi^{\alpha\beta} + V_{cb}^*V_{cd}\mathbf{V}_D^{\alpha\beta} = \left(\mathrm{F}_+q_+^\beta + \mathrm{F}_-q_-^\beta\right)q_-^\alpha, \quad \mathbf{V}_R^{(\prime)\alpha\beta} = \frac{f_R}{q_-^{(\prime)2} - m_R^2 + im_R\Gamma_R}q_-^{(\prime)\alpha}\left(F_{R+}^{(\prime)}q_+^{(\prime)\beta} + F_{R-}^{(\prime)}q_-^{(\prime)\beta}\right),$$

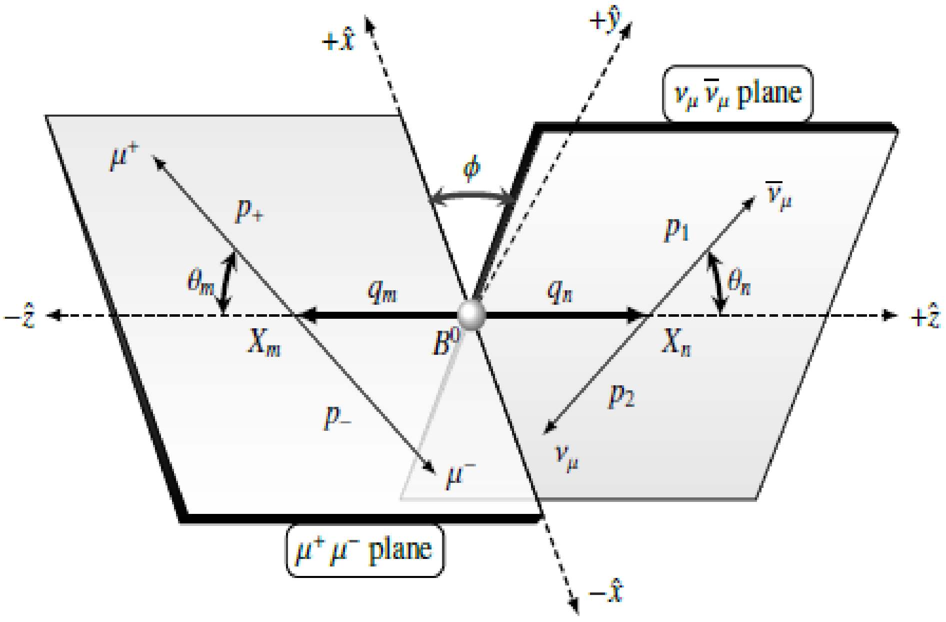
$$\mathscr{R}_{21} \equiv \frac{G_F^2}{2}\mathrm{H}'^{\alpha\beta}L'_{\alpha\beta}, \quad \mathrm{H}'^{\alpha\beta} \equiv V_{ub}^*V_{ud}\mathbf{V}'^{\alpha\beta}_\pi + V_{cb}^*V_{cd}\mathbf{V}'^{\alpha\beta}_D = \left(\mathrm{F}'_+q'^\beta_+ + \mathrm{F}'_-q'^\beta_-\right)q'^\alpha_-,$$

$$\mathscr{Q}_{12} = \frac{G_F^2}{2}\left(\sum_{Q=u,c,t}V_{Qb}^*V_{Qd}\mathbb{V}_Q^{\alpha\beta}\right)L_{\alpha\beta} = \frac{G_F^2}{2}\mathbb{H}^{\alpha\beta}L_{\alpha\beta}, \qquad \mathbb{V}_Q^{(\prime)\alpha\beta} = F_a^{(\prime)Q}g^{\alpha\beta} + F_b^{(\prime)Q}p_B^\alpha p_B^\beta + i\,F_c^{(\prime)Q}\epsilon^{\alpha\beta\mu\nu}q_{+\mu}^{(\prime)}q_{-\nu}^{(\prime)},$$

$$\mathbb{H}^{\alpha\beta} = \mathbb{F}_a\,g^{\alpha\beta} + \mathbb{F}_b\,p_B^\alpha p_B^\beta + i\,\mathbb{F}_c\,\epsilon^{\alpha\beta\mu\nu}q_{+\mu}q_{-\nu}, \qquad \mathbb{F}_i \equiv \mathbb{F}_i\left(q_+^2, q_-^2\right) = \sum_{Q=u,c,t}V_{Qb}^*V_{Qd}\,F_i^Q\left(q_+^2, q_-^2\right),$$

$$\mathscr{Q}_{21} = \frac{G_F^2}{2}\mathbb{H}'^{\alpha\beta}L'_{\alpha\beta} \qquad \mathbb{H}'^{\alpha\beta} = \mathbb{F}'_a\,g^{\alpha\beta} + \mathbb{F}'_b\,p_B^\alpha p_B^\beta + i\,\mathbb{F}'_c\,\epsilon^{\alpha\beta\mu\nu}q'_{+\mu}q'_{-\nu},$$

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, (2)



$$\frac{d^5\Gamma^{D/M}}{dm_{\mu\mu}^2\, dm_{\nu\nu}^2\, d\cos\theta_m\, d\cos\theta_n\, d\phi} = \frac{Y\, Y_m\, Y_n \left\langle \left|\mathscr{M}^{D/M}\right|^2 \right\rangle}{(4\pi)^6\, m_B^2\, m_{\mu\mu}\, m_{\nu\nu}},$$

$$Y = \frac{\sqrt{\lambda\left(m_B^2, m_{\mu\mu}^2, m_{\nu\nu}^2\right)}}{2m_B},$$

$$Y_m = \frac{\sqrt{m_{\mu\mu}^2 - 4m_\mu^2}}{2},$$

$$Y_n = \frac{\sqrt{m_{\nu\nu}^2 - 4m_\nu^2}}{2},$$

After integrating out unobservable neutrino phase space

$$\frac{d^3\Gamma^M}{dm_{\mu\mu}^2\, dm_{\nu\nu}^2\, d\cos\theta_m} - \frac{d^3\Gamma^D}{dm_{\mu\mu}^2\, dm_{\nu\nu}^2\, d\cos\theta_m} \propto m_\nu^2,$$

(confirming pDMCT)

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, (3)

$$\frac{d^3\Gamma^M}{dm_{\mu\mu}^2\, dm_{\nu\nu}^2\, d\cos\theta_m} - \frac{d^3\Gamma^D}{dm_{\mu\mu}^2\, dm_{\nu\nu}^2\, d\cos\theta_m} = \frac{G_F^4\, Y\, Y_m\, Y_n}{2\,(4\,\pi)^6\, m_B^2\, m_{\mu\mu}\, m_{\nu\nu}} \int_{-1}^{1}\int_0^{2\pi} d\cos\theta_n\, d\phi$$

$$\times\Big(-|\mathbb{F}_a|^2 S^M_{aa} - |\mathbb{F}_b|^2 S^M_{bb} - |\mathbb{F}_c^2| S^M_{cc} - |\mathbf{F}_+|^2 S^M_{pp} - |\mathbf{F}_-|^2 S^M_{mm} + |\mathbb{F}'_a|^2 S^M_{a'a'} + |\mathbb{F}'_b|^2 S^M_{b'b'} + |\mathbb{F}'_c|^2 S^M_{c'c'} + |\mathbf{F}'_+|^2 S^M_{p'p'} + |\mathbf{F}'_-|^2 S^M_{m'm'}$$

$$- \mathrm{Re}(\mathbb{F}_a\mathbb{F}_b^*) R^M_{ab} - \mathrm{Re}(\mathbb{F}_a\mathbb{F}_c^*) R^M_{ac} - \mathrm{Re}(\mathbb{F}_a\mathbf{F}_+^*) R^M_{ap} - \mathrm{Re}(\mathbb{F}_a\mathbf{F}_-^*) R^M_{am} - \mathrm{Re}(\mathbb{F}_b\mathbb{F}_c^*) R^M_{bc} - \mathrm{Re}(\mathbb{F}_b\mathbf{F}_+^*) R^M_{bp} - \mathrm{Re}(\mathbb{F}_b\mathbf{F}_-^*) R^M_{bm}$$

$$- \mathrm{Re}(\mathbb{F}_c\mathbf{F}_-^*) R^M_{cm} + \mathrm{Re}(\mathbb{F}'_a\mathbb{F}'^*_b) R^M_{a'b'} + \mathrm{Re}(\mathbb{F}'_a\mathbb{F}'^*_c) R^M_{a'c'} + \mathrm{Re}(\mathbb{F}'_a\mathbf{F}'^*_+) R^M_{a'p'} + \mathrm{Re}(\mathbb{F}'_a\mathbf{F}'^*_-) R^M_{a'm'} + \mathrm{Re}(\mathbb{F}'_b\mathbb{F}'^*_c) R^M_{b'c'}$$

$$+ \mathrm{Re}(\mathbb{F}'_b\mathbf{F}'^*_+) R^M_{b'p'} + \mathrm{Re}(\mathbb{F}'_b\mathbf{F}'^*_-) R^M_{b'm'} + \mathrm{Re}(\mathbb{F}'_c\mathbf{F}'^*_-) R^M_{c'm'} - \mathrm{Re}(\mathbf{F}_+\mathbf{F}_-^*) R^M_{pm} + \mathrm{Re}(\mathbf{F}'_+\mathbf{F}'^*_-) R^M_{p'm'}$$

$$- \mathrm{Im}(\mathbb{F}_a\mathbb{F}_b^*) I^M_{ab} - \mathrm{Im}(\mathbb{F}_a\mathbb{F}_c^*) I^M_{ac} - \mathrm{Im}(\mathbb{F}_a\mathbf{F}_-^*) I^M_{am} - \mathrm{Im}(\mathbb{F}_b\mathbb{F}_c^*) I^M_{bc} - \mathrm{Im}(\mathbb{F}_c\mathbf{F}_+^*) I^M_{cp} - \mathrm{Im}(\mathbb{F}_c\mathbf{F}_-^*) I^M_{cm}$$

$$+ \mathrm{Im}(\mathbb{F}'_a\mathbb{F}'^*_b) I^M_{a'b'} + \mathrm{Im}(\mathbb{F}'_a\mathbb{F}'^*_c) I^M_{a'c'} + \mathrm{Im}(\mathbb{F}'_a\mathbf{F}'^*_-) I^M_{a'm'} + \mathrm{Im}(\mathbb{F}'_b\mathbb{F}'^*_c) I^M_{b'c'} + \mathrm{Im}(\mathbb{F}'_c\mathbf{F}'^*_+) I^M_{c'p'} + \mathrm{Im}(\mathbb{F}'_c\mathbf{F}'^*_-) I^M_{c'm'}$$

$$+ m_\nu^2\Big(\mathrm{Re}(\mathbb{F}_a\mathbb{F}'^*_a) R^M_{aa'} + \mathrm{Re}(\mathbb{F}_a\mathbb{F}'^*_b) R^M_{ab'} + \mathrm{Re}(\mathbb{F}_a\mathbf{F}'^*_+) R^M_{ap'} + \mathrm{Re}(\mathbb{F}_b\mathbb{F}'^*_a) R^M_{ba'} + \mathrm{Re}(\mathbb{F}_b\mathbb{F}'^*_b) R^M_{bb'} + \mathrm{Re}(\mathbb{F}_b\mathbb{F}'^*_c) R^M_{bc'}$$

$$+ \mathrm{Re}(\mathbb{F}_b\mathbf{F}'^*_+) R^M_{bp'} + \mathrm{Re}(\mathbb{F}_b\mathbf{F}'^*_-) R^M_{bm'} + \mathrm{Re}(\mathbb{F}_c\mathbb{F}'^*_b) R^M_{cb'} + \mathrm{Re}(\mathbb{F}_c\mathbb{F}'^*_c) R^M_{cc'} + \mathrm{Re}(\mathbb{F}_c\mathbf{F}'^*_+) R^M_{cp'} + \mathrm{Re}(\mathbb{F}_c\mathbf{F}'^*_-) R^M_{cm'}$$

$$+ \mathrm{Re}(\mathbb{F}'_a\mathbf{F}^*_+) R^M_{a'p} + \mathrm{Re}(\mathbb{F}'_a\mathbf{F}^*_-) R^M_{a'm} + \mathrm{Re}(\mathbb{F}'_b\mathbf{F}^*_+) R^M_{b'p} + \mathrm{Re}(\mathbb{F}'_b\mathbf{F}^*_-) R^M_{b'm} + \mathrm{Re}(\mathbb{F}'_c\mathbf{F}^*_+) R^M_{c'p} + \mathrm{Re}(\mathbb{F}'_c\mathbf{F}^*_-) R^M_{c'm}$$

$$+ \mathrm{Im}(\mathbb{F}_a\mathbb{F}'^*_c) I^M_{ac'} + \mathrm{Im}(\mathbb{F}_a\mathbf{F}'^*_-) I^M_{am'} + \mathrm{Im}(\mathbb{F}_b\mathbf{F}'^*_+) I^M_{bp'} + \mathrm{Im}(\mathbb{F}_b\mathbf{F}'^*_-) I^M_{bm'} + \mathrm{Im}(\mathbb{F}_c\mathbb{F}'^*_a) I^M_{ca'} + \mathrm{Im}(\mathbb{F}_c\mathbb{F}'^*_c) I^M_{cc'}$$

$$+ \mathrm{Im}(\mathbb{F}_c\mathbf{F}'^*_+) I^M_{cp'} + \mathrm{Im}(\mathbb{F}_c\mathbf{F}'^*_-) I^M_{cm'} + \mathrm{Im}(\mathbb{F}'_b\mathbf{F}^*_+) I^M_{b'p} + \mathrm{Im}(\mathbb{F}'_b\mathbf{F}^*_-) I^M_{b'm} + \mathrm{Im}(\mathbb{F}'_c\mathbf{F}^*_+) I^M_{c'p} + \mathrm{Im}(\mathbb{F}'_c\mathbf{F}^*_-) I^M_{c'm}\Big)\Big). \quad \propto m_\nu^2,$$

Back-to-back muons, (easily measurable exception to DMCT)

$$B^0(p_B) \to \mu^-(p_-)\,\mu^+(p_+)\,\bar{\nu}_\mu(p_1)\,\nu_\mu(p_2),$$

In the rest frame
IF muon- and m
➔ nu and nu-b

t opposite direction

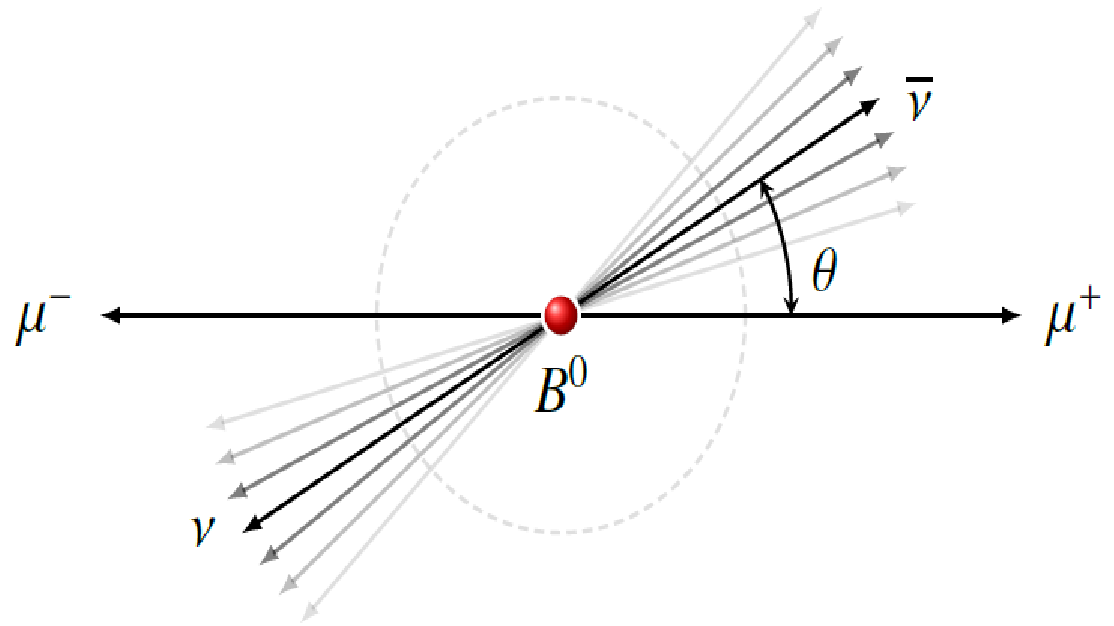
$E_1 = E_2$

$m^2_{\mu\mu}$

$Y_m = \sqrt{(}$

Y_n

Back-to-back geometry: $\vec{p}_1 = -\vec{p}_2$



See Sec.IV of Phys. Rev. D **105**, no.11, 113006 (2022).

ble or measurable ,

and $\mu_- - \mu_+$)

$$\frac{d\Gamma}{dE_\nu^2\, d\sin\theta}$$

Need not
only unmeasurable angle (θ) integrate out ➔ Overcoming DMCT constraint

Back-to-back muons, (easily measurable exception to DMCT)

$$B^0(p_B) \to \mu^-(p_-)\,\mu^+(p_+)\,\bar{\nu}_\mu(p_1)\,\nu_\mu(p_2),$$

In the rest frame of parent B meson,
IF muon- and muon+ are back-to back, ie. flying with 3 momenta of equal magnitude but opposite direction
➔ nu and nu-bar also back-to-back

$$E_1 = E_2 = E_\nu = m_B/2 - E_\mu$$
$$m_{\nu\nu}^2 = 4E_\nu^2$$
$$m_{\mu\mu}^2 = (m_B - 2E_\nu)^2$$
$$Y_m = \sqrt{(m_B/2 - E_\nu)^2 - m_\mu^2}$$
$$Y_n = \sqrt{E_\nu^2 - m_\nu^2}$$

➔ All kinematic variables are calculable or measurable ,

Only the angle (between $\nu - \bar{\nu}$ and $\mu_- - \mu_+$) UNKNOWN

$$\boxed{\frac{d\Gamma}{dE_\mu^2\, d\sin\theta}} \to \boxed{\frac{d\Gamma}{dE_\nu^2\, d\sin\theta}}$$

Need not integrate out nu-nubar full phase space,
only unmeasurable angle (θ) integrate out ➔ Overcoming DMCT constraint

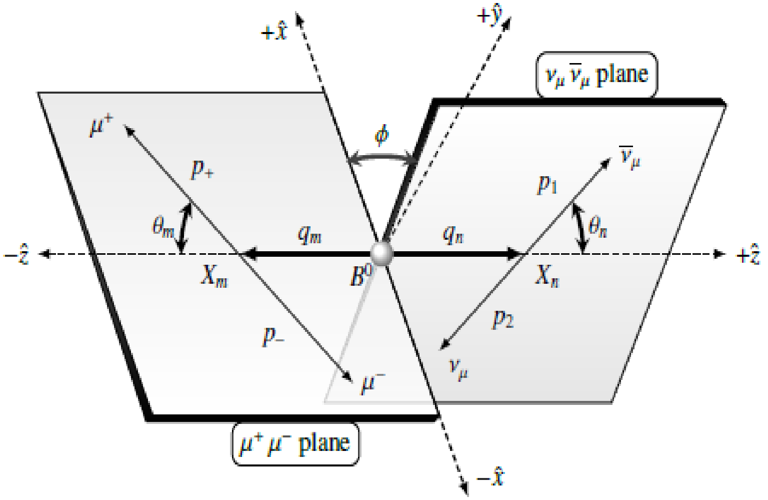
Detailed study of $B^0(p_B)\to\mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, -- B2B muons (1)

Kinematics of back-to-back muons at B-rest frame (ie. B2B nu-nubar)

$$E_1 = E_2 = E_\nu = m_B/2 - E_\mu$$
$$m_{\nu\nu}^2 = 4E_\nu^2$$
$$m_{\mu\mu}^2 = (m_B - 2E_\nu)^2$$
$$Y_m = \sqrt{(m_B/2 - E_\nu)^2 - m_\mu^2}$$
$$Y_n = \sqrt{E_\nu^2 - m_\nu^2}$$

$$\cos\theta_n = 0$$
$$\phi = 0$$
$$\theta_m = \pi/2 - \theta$$
$$(\cos\theta_m = \sin\theta)$$

** θ = angle($\mu^+, \bar{\nu}$)



➔ Only $(E_\mu, \sin\theta)$ are independent variable ➔

$$\frac{d^3\Gamma_{\leftrightarrow}^{D/M}}{dE_\mu^2\, d\sin\theta} = \frac{2\sqrt{E_\mu^2 - m_\mu^2}}{(4\pi)^6 m_B E_\mu}\left(\left(\frac{m_B}{2} - E_\mu\right)^2 - m_\nu^2\right)\langle|\mathscr{M}_{\leftrightarrow}^{D/M}|^2\rangle,$$

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, -- B2B muons (2)

$$\frac{\mathrm{d}^3\Gamma^D_{\leftrightarrow}}{\mathrm{d}E^2_\mu\,\mathrm{d}\sin\theta} - \frac{\mathrm{d}^3\Gamma^M_{\leftrightarrow}}{\mathrm{d}E^2_\mu\,\mathrm{d}\sin\theta} = \frac{G_F^4\,\sqrt{E_\mu^2 - m_\mu^2}}{(4\,\pi)^6\;m_B\,E_\mu}\left(\frac{m_B}{2} - E_\mu\right)^2$$

$$\begin{aligned}
&\times\Bigg(\left(|\mathbb{F}_a|^2 - |\mathbb{F}'_a|^2\right)\Delta_{aa} + \left(|\mathbb{F}_b|^2 - |\mathbb{F}'_b|^2\right)\Delta_{bb} + \left(|\mathbb{F}_c|^2 - |\mathbb{F}'_c|^2\right)\Delta_{cc}\\
&+ \left(|\mathbf{F}_+|^2 - |\mathbf{F}'_+|^2\right)\Delta_{pp} + \left(|\mathbf{F}_-|^2 - |\mathbf{F}'_-|^2\right)\Delta_{mm} + \left(\mathrm{Re}\left(\mathbb{F}_a\mathbb{F}_b^*\right) - \mathrm{Re}\left(\mathbb{F}'_a\mathbb{F}'^*_b\right)\right)\Delta_{ab}\\
&+ \left(\mathrm{Re}\left(\mathbb{F}_a\mathbf{F}_+^*\right) - \mathrm{Re}\left(\mathbb{F}'_a\mathbf{F}'^*_+\right)\right)\Delta_{ap} + \left(\mathrm{Re}\left(\mathbb{F}_b\mathbf{F}_+^*\right) - \mathrm{Re}\left(\mathbb{F}'_b\mathbf{F}'^*_+\right)\right)\Delta_{bp} + \left(\mathrm{Re}\left(\mathbb{F}_a\mathbf{F}_-^*\right) - \mathrm{Re}\left(\mathbb{F}'_a\mathbf{F}'^*_m\right)\right)\Delta_{am}\\
&+ \left(\mathrm{Re}\left(\mathbb{F}_b\mathbf{F}_-^*\right) - \mathrm{Re}\left(\mathbb{F}'_b\mathbf{F}'^*_-\right)\right)\Delta_{bm} + \left(\mathrm{Re}\left(\mathbb{F}_c\mathbf{F}_-^*\right) - \mathrm{Re}\left(\mathbb{F}'_c\mathbf{F}'^*_-\right)\right)\Delta_{cm} + \left(\mathrm{Re}\left(\mathbf{F}_+\mathbf{F}_-^*\right) - \mathrm{Re}\left(\mathbf{F}'_+\mathbf{F}'^*_-\right)\right)\Delta_{pm}\\
&+ \cos\theta\Bigg(\left(|\mathbb{F}_a|^2 + |\mathbb{F}'_a|^2\right)\Sigma_{aa} + \left(|\mathbb{F}_b|^2 + |\mathbb{F}'_b|^2\right)\Sigma_{bb} + \left(|\mathbf{F}_+|^2 + |\mathbf{F}'_+|^2\right)\Sigma_{pp} + \left(|\mathbf{F}_-|^2 + |\mathbf{F}'_-|^2\right)\Sigma_{mm}\\
&\qquad + \left(\mathrm{Re}\left(\mathbb{F}_a\mathbb{F}_b^*\right) + \mathrm{Re}\left(\mathbb{F}'_a\mathbb{F}'^*_b\right)\right)\Sigma_{ab} + \left(\mathrm{Re}\left(\mathbb{F}_a\mathbf{F}_-^*\right) + \mathrm{Re}\left(\mathbb{F}'_a\mathbf{F}'^*_m\right)\right)\Sigma_{am}\\
&\qquad + \left(\mathrm{Re}\left(\mathbb{F}_b\mathbf{F}_-^*\right) + \mathrm{Re}\left(\mathbb{F}'_b\mathbf{F}'^*_-\right)\right)\Sigma_{bm} + \left(\mathrm{Re}\left(\mathbf{F}_+\mathbf{F}_-^*\right) + \mathrm{Re}\left(\mathbf{F}'_+\mathbf{F}'^*_-\right)\right)\Sigma_{pm}\Bigg)\Bigg), \quad \not\propto \boldsymbol{m_\nu^2} \qquad (47)
\end{aligned}$$

$$\underbrace{|\mathscr{M}(p_1,p_2)|^2}_{\text{Direct term}} \neq \underbrace{|\mathscr{M}(p_2,p_1)|^2}_{\text{Exchange term}}.$$

$$\cos\theta = 0, (\sin\theta = \pm 1) \rightarrow \Gamma^D = \Gamma^M$$

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, -- B2B muons (3)

$$\frac{d^3\Gamma^D_{\leftrightarrow}}{dE_\mu^2\, d\sin\theta} - \frac{d^3\Gamma^M_{\leftrightarrow}}{dE_\mu^2\, d\sin\theta} = \frac{G_F^4\,\sqrt{E_\mu^2 - m_\mu^2}}{(4\,\pi)^6\; m_B\, E_\mu}\left(\frac{m_B}{2} - E_\mu\right)^2$$

$$\times\Bigg(\left(|\mathbb{F}_a|^2 - |\mathbb{F}'_a|^2\right)\Delta_{aa} + \left(|\mathbb{F}_b|^2 - |\mathbb{F}'_b|^2\right)\Delta_{bb} + \left(|\mathbb{F}_c|^2 - |\mathbb{F}'_c|^2\right)\Delta \cdots$$

$$+\left(|\mathbf{F}_+|^2 - |\mathbf{F}'_+|^2\right)\Delta_{pp} + \left(|\mathbf{F}_-|^2 - |\mathbf{F}'_-|^2\right)\Delta_{mm} + \Big(\cdots$$

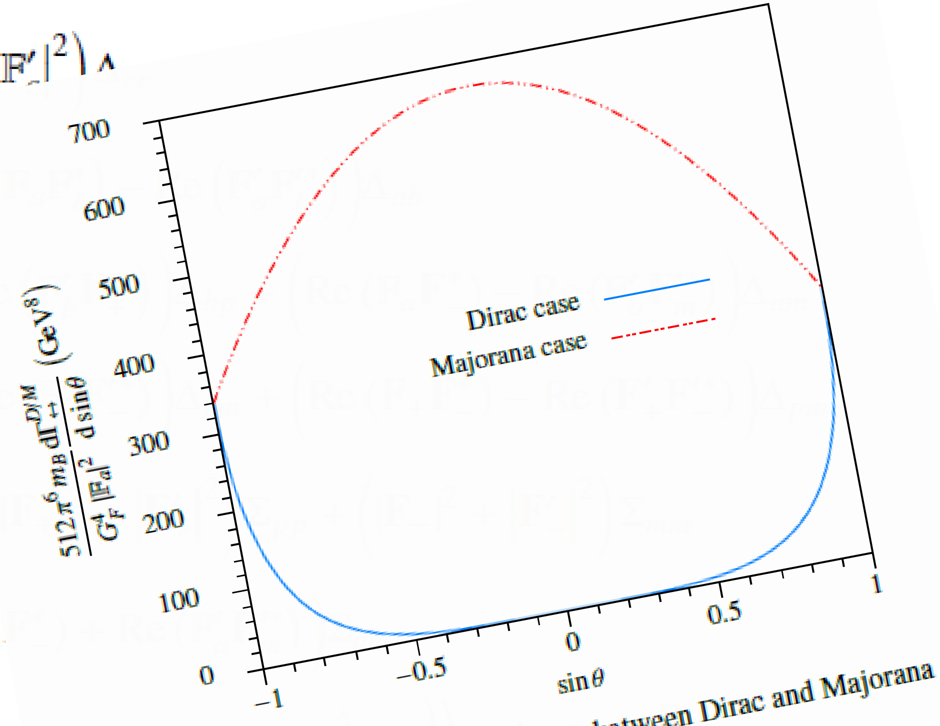
$$+\left(\mathrm{Re}\,(\mathbb{F}_a\mathbf{F}^*_+) - \mathrm{Re}\,(\mathbb{F}'_a\mathbf{F}'^*_+)\right)\Delta_{ap} + \Big(\mathrm{Re}\,(\mathbb{F}_b\mathbf{F}^*_+) - \cdots$$

$$+\left(\mathrm{Re}\,(\mathbb{F}_b\mathbf{F}^*_-) - \mathrm{Re}\left(\mathbb{F}'_b\mathbf{F}'^*_-\right)\right)\Delta_{bm} + \Big(\mathrm{Re}\,(\mathbb{F}_c\mathbf{F}^*_-) - \cdots$$

$$+\cos\theta\Bigg(\left(|\mathbb{F}_a|^2 + |\mathbb{F}'_a|^2\right)\Sigma_{aa} + \left(|\mathbb{F}_b|^2 + |\mathbb{F}'_b|^2\right)\Sigma_{bb} + \cdots$$

$$+\left(\mathrm{Re}\left(\mathbb{F}_a\mathbb{F}^*_b\right) + \mathrm{Re}\left(\mathbb{F}'_a\mathbb{F}'^*_b\right)\right)\Sigma_{ab} + \Big(\mathrm{Re}\,(\mathbb{F}_a \cdots$$

$$+\left(\mathrm{Re}\,(\mathbb{F}_b\mathbf{F}^*_-) + \mathrm{Re}\left(\mathbb{F}'_b\mathbf{F}'^*_-\right)\right)\Sigma_{bm} + \Big(\mathrm{Re}\,(\mathbf{F}_+ \cdots$$



(d) Comparison of sin θ distribution alone between Dirac and Majorana cases. Compare with Fig. 2.

$$\cos\theta = 0,\ (\sin\theta = \pm 1) \rightarrow \Gamma^D = \Gamma^M$$

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, -- B2B muons (4)

$$\frac{d^3\Gamma^{D/M}_{\leftrightarrow}}{dE_\mu^2\, d\sin\theta} = \frac{2\sqrt{E_\mu^2 - m_\mu^2}}{(4\pi)^6\, m_B E_\mu}\left(\left(\frac{m_B}{2} - E_\mu\right)^2 - m_\nu^2\right)\left\langle\left|\mathscr{M}^{D/M}_{\leftrightarrow}\right|^2\right\rangle,$$

Consider a simple case for numerical purpose only:

(1) neglect muon & neutrino mass ➔ we consider only non-resonant contributions

(2) consider only dominant form factor, ($\mathbb{H}^{\alpha\beta} = \mathbb{F}_a\, g^{\alpha\beta} + \mathbb{F}_b\, p_B^\alpha p_B^\beta + i\,\mathbb{F}_c\, \epsilon^{\alpha\beta\mu\nu} q_{+\mu} q_{-\nu}$,)

(3) assume the form factor to be a constant

$$\frac{d^3\Gamma^D_{\leftrightarrow}}{dE_\mu^2\, d\sin\theta} = \frac{G_F^4\, |\mathbb{F}_a|^2 \left(m_B - 2E_\mu\right)^4 K_\mu}{512\pi^6\, m_B E_\mu}\left(E_\mu - K_\mu \cos\theta\right)^2,$$

$$\frac{d^3\Gamma^M_{\leftrightarrow}}{dE_\mu^2\, d\sin\theta} = \frac{G_F^4\, |\mathbb{F}_a|^2 \left(m_B - 2E_\mu\right)^4 K_\mu}{512\pi^6\, m_B E_\mu}\left(E_\mu^2 + K_\mu^2 \cos^2\theta\right),$$

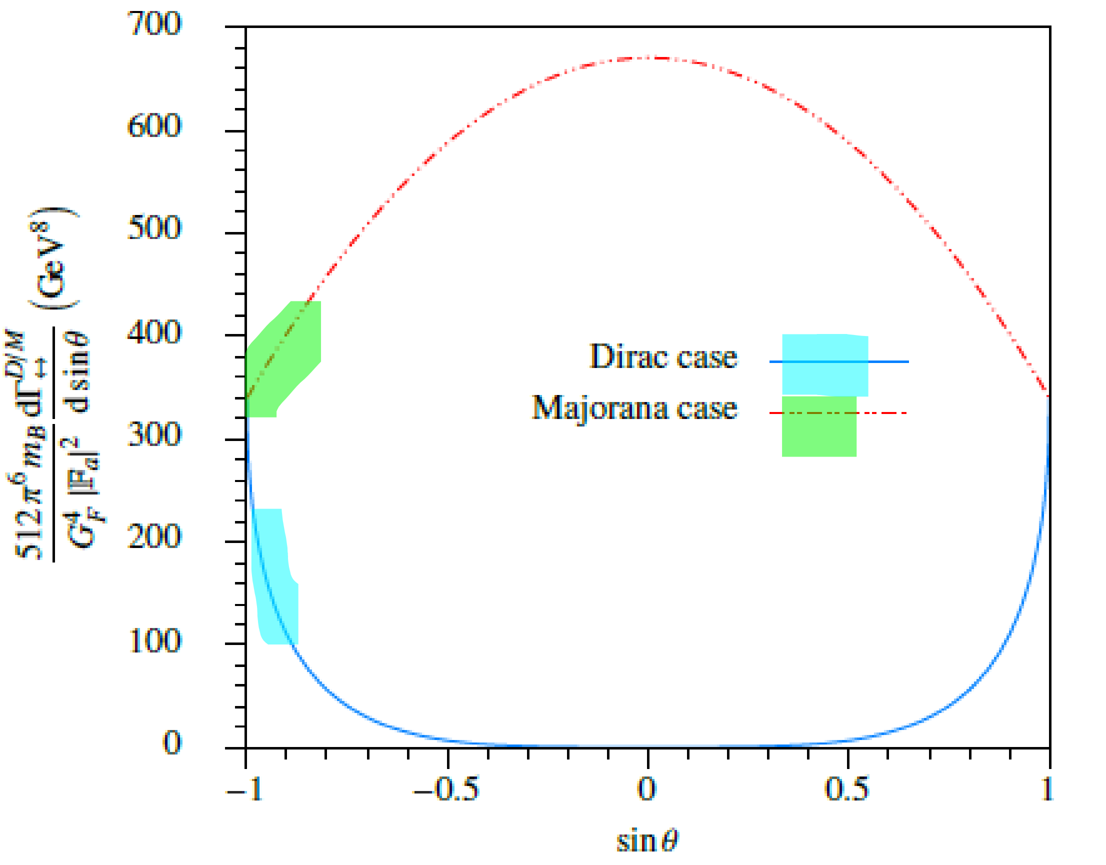
$$K_\mu = \sqrt{E_\mu^2 - m_\mu^2}$$

(4) approximate $E_\mu \approx K_\mu$

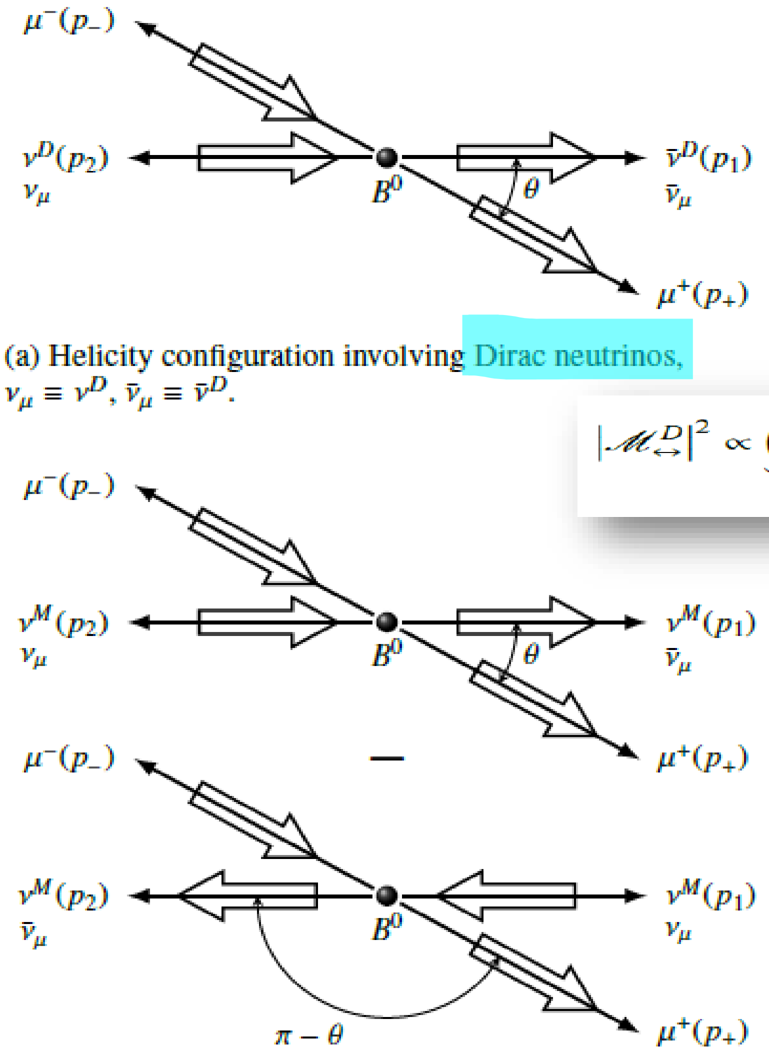
$$\frac{d^3\Gamma^D_{\leftrightarrow}}{dE_\mu^2\, d\sin\theta} = \frac{G_F^4\, |\mathbb{F}_a|^2 \left(m_B - 2E_\mu\right)^4 E_\mu^2}{512\pi^6\, m_B}\left(1 - \cos\theta\right)^2,$$

$$\frac{d^3\Gamma^M_{\leftrightarrow}}{dE_\mu^2\, d\sin\theta} = \frac{G_F^4\, |\mathbb{F}_a|^2 \left(m_B - 2E_\mu\right)^4 E_\mu^2}{512\pi^6\, m_B}\left(1 + \cos^2\theta\right),$$

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, -- B2B muons (5)



(d) Comparison of $\sin\theta$ distribution alone between Dirac and Majorana cases. Compare with Fig. 2.



(a) Helicity configuration involving Dirac neutrinos, $\nu_\mu \equiv \nu^D$, $\bar{\nu}_\mu \equiv \bar{\nu}^D$.

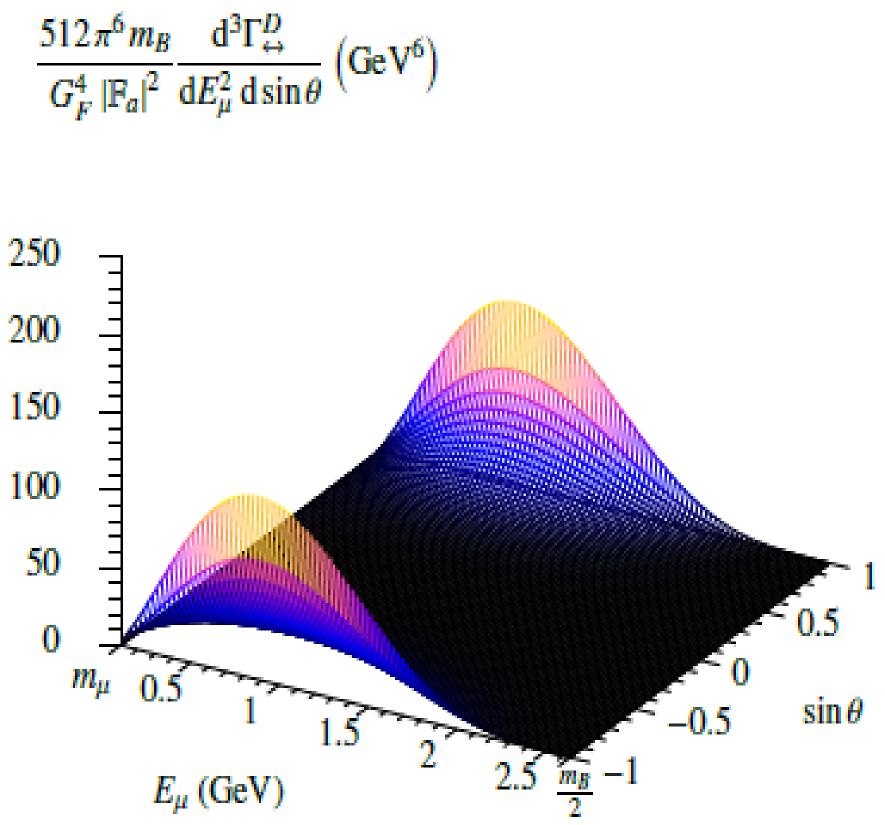
$$|\mathscr{M}_{\leftrightarrow}^D|^2 \propto \underbrace{(1-\cos\theta)^2}_{\text{Direct term}}.$$

(b) Helicity configuration involving Majorana neutrinos, $\nu_\mu = \bar{\nu}_\mu \equiv \nu^M$.

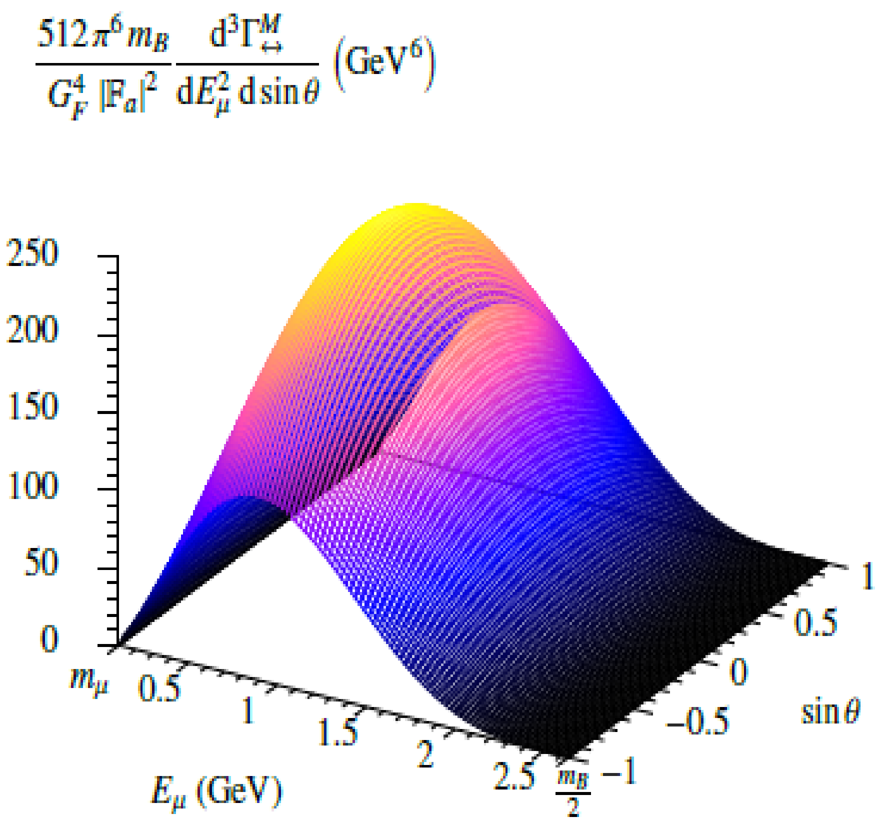
$$|\mathscr{M}_{\leftrightarrow}^M|^2 \propto \frac{1}{2}\Big[\underbrace{(1-\cos\theta)^2}_{\text{Direct term}} + \underbrace{(1-\cos(\pi-\theta))^2}_{\text{Exchange term}} - \underbrace{O(m_\nu^2)}_{\text{Interference term}}\Big] \simeq 1+\cos^2\theta.$$

** Presently nu nu-bar totally missing, the angle θ is completely unknown, therefore, need to integrate out. ➔ BR(M) >> BR(D)

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, -- B2B muons (6)



(a) Three dimensional view of the differential decay rate for Dirac case with an appropriate normalization as mentioned.



(b) Three dimensional view of the differential decay rate for Majorana case with an appropriate normalization as mentioned.

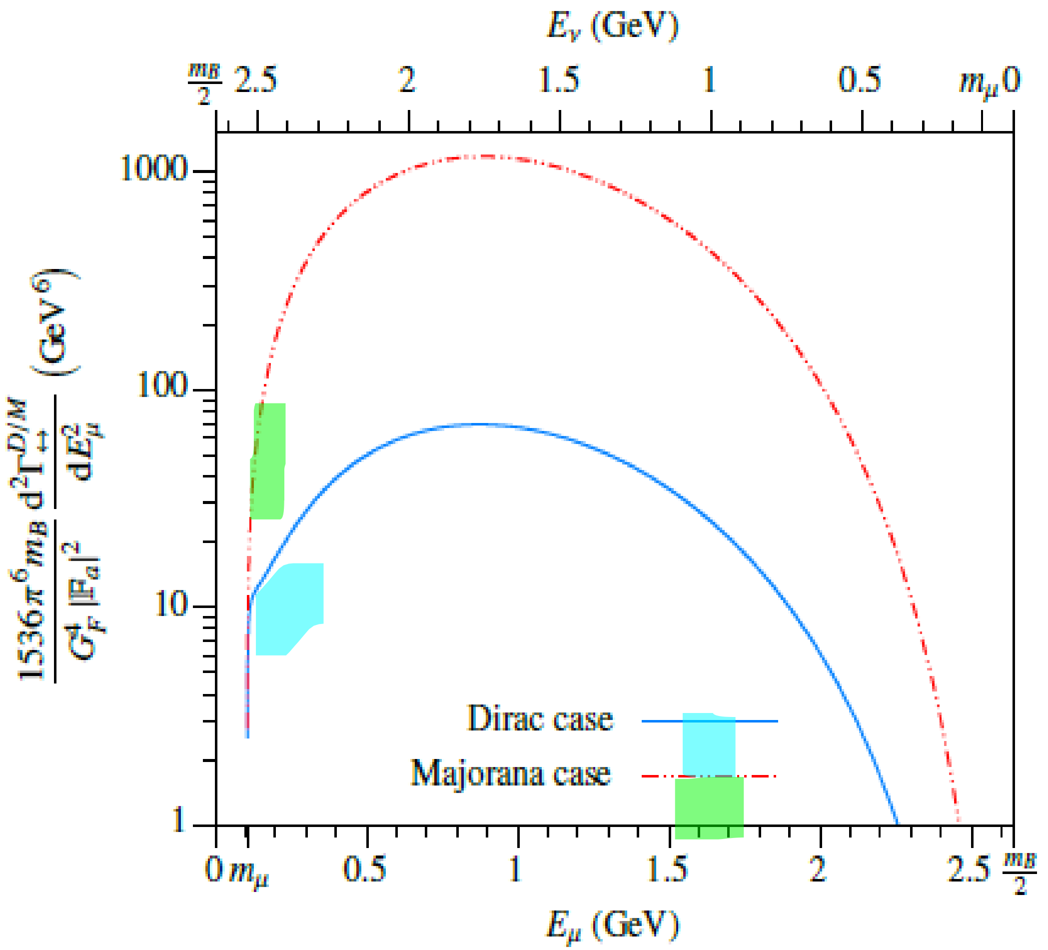
Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, -- B2B muons (7)

Integrating over currently unobservable angle θ, we get

$$\frac{d^2\Gamma^D_{\leftrightarrow}}{dE_\mu^2} = \frac{G_F^4\,|\mathbb{F}_a|^2}{1536\,\pi^6\,m_B\,E_\mu}\left(m_B - 2\,E_\mu\right)^4 K_\mu \times \left(10\,E_\mu^2 - 3\,\pi\,E_\mu\,K_\mu - 4\,m_\mu^2\right),$$

$$\frac{d^2\Gamma^M_{\leftrightarrow}}{dE_\mu^2} = \frac{G_F^4\,|\mathbb{F}_a|^2}{1536\,\pi^6\,m_B\,E_\mu}\left(m_B - 2\,E_\mu\right)^4 K_\mu \left(10\,E_\mu^2 - 4\,m_\mu^2\right),$$

$$BR(D - M) = \int \left(D_{\leftrightarrow} - M_{\leftrightarrow}\right) \neq m_\nu^2$$



(c) Comparison of muon energy distributions between Dirac and Majorana cases in the back-to-back scenario.

Discussions on $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, -- B2B muons (1)

(1) Branching ratio of B2B muons
(only w/ non-resonant contributions)

$$\mathcal{B}^D_{\leftrightarrow} = \Gamma^D_{\leftrightarrow}/\Gamma_B \approx 1.1 \times 10^{-12}\,\mathrm{GeV}^{-2} \times |\mathbb{F}_a|^2 ,$$

$$\mathcal{B}^M_{\leftrightarrow} = \Gamma^M_{\leftrightarrow}/\Gamma_B \approx 1.8 \times 10^{-11}\,\mathrm{GeV}^{-2} \times |\mathbb{F}_a|^2 ,$$

(2) Adding $B^0\,(\bar{B}^0) \to \mu^-\mu^+\nu_\mu\bar{\nu}_\mu$ $B^0\,(\bar{B}^0) \to e^+e^-\nu_e\bar{\nu}_e,$ increasing BR four-fold

(3) Futuristic ~~detectors, e.g. FASER, MATHUSLA, SHiP~~, GAZELLA, could enable
to probe the ~~angular distribution~~, $D \propto (1-\cos\theta)^2, M \propto (1+\cos\theta^2)$

(4) Bg processes, B2B muons + "missing momentum"
small due to additional vertices, phase space suppression

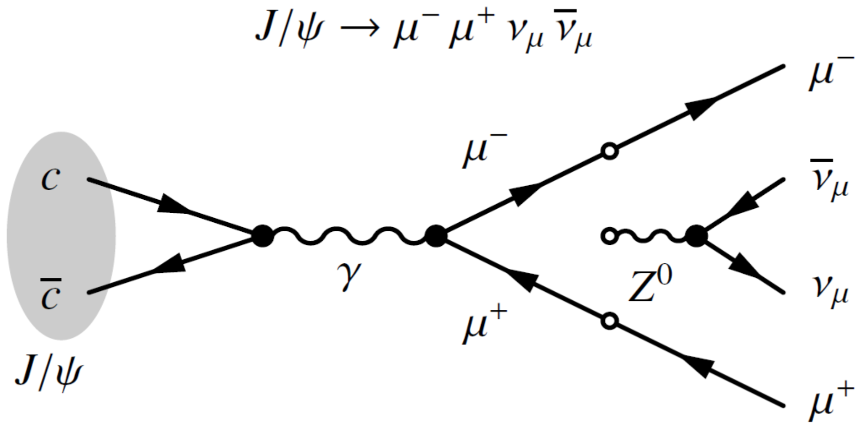
1. $B^0 \to \tau^+\nu_\tau\mu^-\bar{\nu}_\mu \to \mu^-\mu^+\nu_\mu\bar{\nu}_\mu\nu_\tau\bar{\nu}_\tau$, and
2. $B^0 \to \tau^+\tau^- \to \mu^-\mu^+\nu_\mu\bar{\nu}_\mu\nu_\tau\bar{\nu}_\tau$.

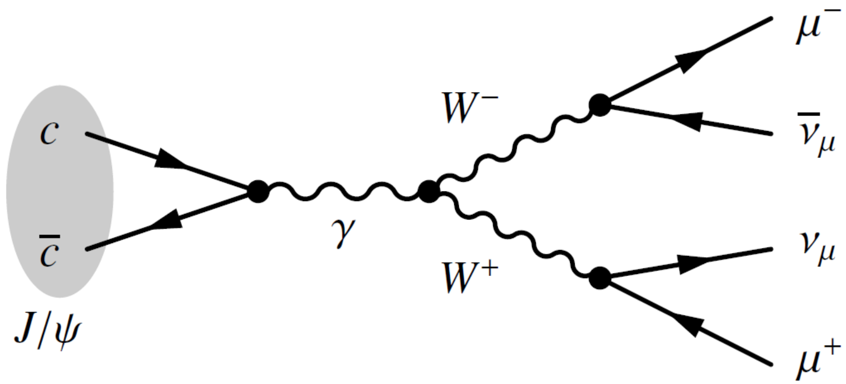
(5) Many similar processes,

$$H \to \mu^+\mu^-\nu_\mu\bar{\nu_\mu}, D \to \mu^+\mu^-\nu_\mu\bar{\nu_\mu}, J/\psi \to \mu^+\mu^-\nu_\mu\bar{\nu_\mu}$$
$$\psi(2S) \to \pi^+\pi^-\nu_\tau\bar{\nu_\tau}, K^0 \to \mu^+\mu^-\nu_\mu\bar{\nu_\mu}, ..$$

Discussions on $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, -- B2B muons (2)

(6) Cases for $J/\psi \to \mu^+\mu^-\nu_\mu\bar{\nu}_\mu$ at BES III , $\Upsilon(1s) \to \mu^+\mu^-\nu_\mu\bar{\nu}_\mu$ at Belle2





** Present measurements:

$$Br(B^0 \to \mu^+\mu^-) = (1.1 \pm 1.4) * 10^{-10}$$ FCNC decay via photonic Penguin

$B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, Doubly weak charged current decay

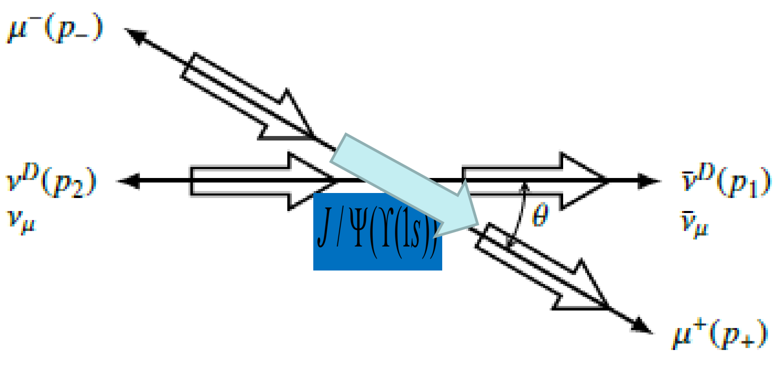
$$Br(J/\Psi \to \mu^+\mu^-) = (5.961 \pm 0.033)\%$$

$$Br(\Upsilon(1s) \to \mu^+\mu^-) = (2.48 \pm 0.05)\%$$

Discussions on $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, -- B2B muons (3)

(7) Cases for $J/\psi \to \mu^+\mu^-\nu_\mu\bar{\nu}_\mu$ at BES III, $\Upsilon(1s) \to \mu^+\mu^-\nu_\mu\bar{\nu}_\mu$ at Belle2

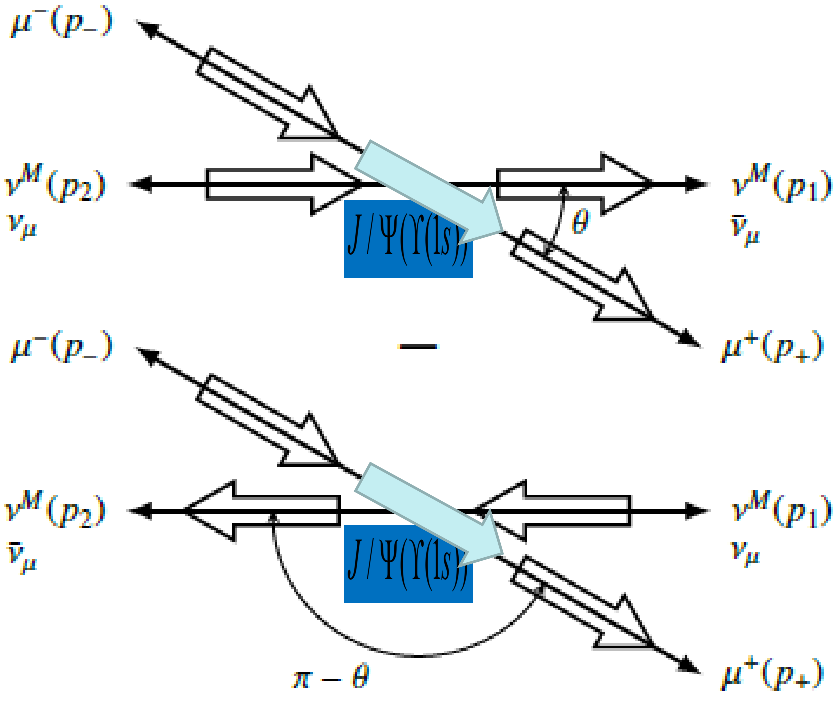
** (simple rough) Helicity Consideration



(a) Helicity configuration involving Dirac neutrinos, $\nu_\mu \equiv \nu^D$, $\bar{\nu}_\mu \equiv \bar{\nu}^D$.

$$\left|\mathfrak{M}^D_{\leftrightarrow}\right|^2 \propto (1+\cos\theta)^2$$

$$\left|\mathfrak{M}^M_{\leftrightarrow}\right|^2 \propto 1+\cos^2\theta$$

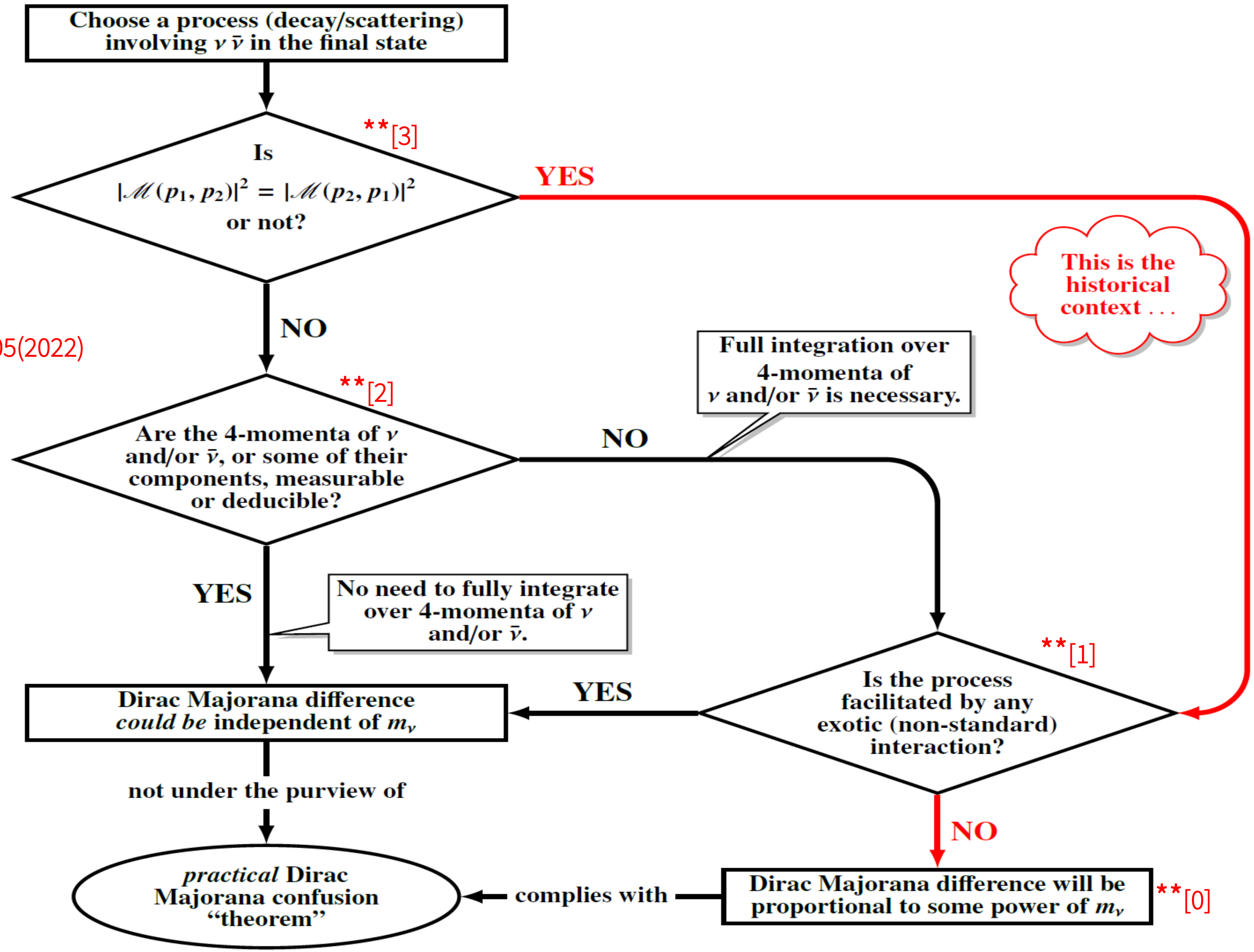


(b) Helicity configuration involving Majorana neutrinos, $\nu_\mu = \bar{\nu}_\mu \equiv \nu^M$.

SUMMARY

Conclusion

Acknowledgements



[0] B.Kayser,PRD26(1982)
[1] S.P.Rosen,PRL48(1982); CSK etal. EPJC83(2023)
[2] T.Chabra,PR.Babu,PRD46(1992); CSK etal. PRD105(2022)
[3] CS Kim, Work in Progress(2023)

Conclusion

(1) (a) We consider the B decay, $B^0 \to \mu^-\mu^+\nu_\mu\bar{\nu}_\mu$, implementing the Fermi-Dirac statistics to find the difference between Dirac and Majorana neutrino, and to test the practical Dirac-Majorana Confusion Theorem.
(b) We also consider $B \to K\nu\bar{\nu}, K \to \pi\nu\bar{\nu}$ implementing model independent new physics scenarios to find the difference between Dirac and Majorana neutrino.

(2) (a) If we consider the special kinematic configuration of back-to-back muons in the B rest frame, there exists striking difference between D and M cases, which do not depend on neutrino mass, hence, overcoming pDMCT.
(b) If we consider scalar and/or axial vector new physics, we can distinguish D from M, which do not depend on neutrino mass, hence, overcoming pDMCT.

(3) We give full details of analysis, including resonant and non-resonant contributions, tiny neutrino mass dependence, helicity consideration, etc, also confirming pDMC if we integrate out full nu nu-bar phase space.

Conclusion – Final Comment

** The neutrino-less double beta decay (NDBD) has a limitation that it is dependent on the unknown tiny mass of the neutrino. If it is too small there is no possibility of establishing the nature of the neutrino through NDBD. Our proposals are the only viable alternatives to NDBD as far as probing Majorana nature of sub-eV active neutrinos is concerned.

MEET OUR COLLABORATORS

Choong Sun Kim

Professor

M V N Murthy

Professor

Janusz Rosiek

Professor

Dibyakrupa Sahoo

Postdoctor

THANK YOU

Choong Sun Kim

cskim@yonsei.ac.kr

http://kimcs.Yonsei.ac.kr/~cskim/

Back-up (Details)

Discussions on pDMCT

Helicity & Chirality

Helicity configuration of back-to-back muons

Detailed study of $B^0(p_B) \to \mu^-(p_-)\,\mu^+(p_+)\,\bar{\nu}_\mu(p_1)\,\nu_\mu(p_2),$

Extra : Dirac vs. Majorana

Discussion on pDMCT (1) – Is it 'Theorem' ?

** It is believed to suggest that all difference between Dirac and Majorana neutrinos must be proportional to some power of neutrino mass (m_ν).

(a) There is no model-independent, process-independent and observable-independent proof of this so-called "theorem". All processes where it was shown to hold involved full integration over the 4-momenta of missing neutrinos.

(b) We prove the fact that this "theorem" can be overcome, if energy and/or momentum of neutrino can be inferred or measured. The interesting question is to find out a way to realize this, which we do by measuring muon energy in the back-to-back muons configuration in the B rest frame, $E_\nu = m_B/2 - E_\mu$. [CSK,MM,DS, PRD105,113005(2022)]

$$B^0(p_B) \to \mu^-(p_-)\,\mu^+(p_+)\,\bar{\nu}_\mu(p_1)\,\nu_\mu(p_2),$$

on pDMCT (2) – Smooth transition when m → 0 limit ?

** Is there smooth transition between Majorana to Dirac neutrinos under m → 0 limit ??

(a) When m = 0 both Dirac and Majorana neutrinos can be described as Weyl fermions. The reduction of neutrino degrees of freedom from 4 to 2 for m = 0 is a discrete jump, and not a continuous change. So the **massless neutrino is an entirely different species than a massive one even with extremely tiny mass.**

(b) Dirac neutrino and antineutrino are fully distinguishable, while Majorana neutrino and antineutrino are quantum mechanically indistinguishable. There is no smooth limit that takes indistinguishable particles and makes them distinguishable. There is no intermediate state between distinguishable and indistinguishable particles.

(c) Majorana neutrino and antineutrino pair have to obey Fermi-Dirac statistics while Dirac neutrino and antineutrino pair do not. We emphasize that **statistics of particles does not depend on a parameter like mass, but its spin.**

on pDMCT (3) – Is Helicity suppression DMCT ?

** Is Helicity suppression of $\nu \leftrightarrow \bar{\nu}$ the origin of DMCT ??

(a) For Majorana neutrinos, the antisymmetrizing second (exchanged) diagram is from the momentum exchange, not from m/E suppressed helicity flip.

(b) The pDMCT is a result of the interference between the direct and the exchanged diagram, which is only surviving term of Dirac - Majorana after full phase integral of invisible neutrinos.

(c) For the processes only with weak neutral current in the SM, the m/E suppressed helicity flip is directly related to origin of DMCT. (eg. Z → nu nu-bar, …)

on pDMCT (4-1) – Is the final state $\nu\bar{\nu}$ the same as $\nu_L\bar{\nu}_R$?

** In case of high energy scattering amplitudes, only when the two neutrinos have the same helicity, they can interfere, and hence their statistics can be studied experimentally ?

(a) For a massive fermion its helicity and chirality are not equivalent. In such a case, helicity is not Lorentz invariant, while chirality is always Lorentz invariant, however, not a conserved quantity. Depending on the type of interactions present in the process, the fermion could be produced in specific chiral states. But the physically propagating state, which is a mass eigenstate, can have both the chiral possibilities. When we consider massive Majorana nu and nu-bar, they are described by real solution of Dirac equation. This puts Majorana nu and nu-bar as identical fermions, ensuring anti-symmetrization of our decay amplitude. (Please think Moeller scattering of e- e-)

(b) Doing anti-symmetrization of amplitude and following the Feynman rules relevant for Majorana fermions, we do a QFT calculation for the process under our consideration. We find that, if we fully integrate over the allowed phase space for nu and nu-bar, only the interference term leads to the difference between Dirac and Majorana cases. Our explicit calculation shows that the interference term is proportional to m_nu^2. (Confirming DMCT)

on pDMCT (4-2) – Is the final state $\nu\bar{\nu}$ the same as $\nu_L\bar{\nu}_R$?

** In case of high energy scattering amplitudes, only when the two neutrinos have the same helicity, they can interfere, and hence their statistics can be studied experimentally ?

(1) Unlike Dirac neutrinos, the Majorana neutrinos do not have any lepton (family) number.
This is because, by definition, Majorana neutrino and antineutrino are indistinguishable.
For Majorana neutrinos (=antineutrinos) the flavor subscript and the bar might be helpful for book-keeping, to write the leptonic currents correctly, but they do not imply that Majorana neutrino is different from Majorana antineutrino.

(2) We do not describe the neutrinos in Weyl basis which utilizes 2-component spinors.
Because neutrinos have non-zero mass, we always use 4-component spinors.
The anti-symmetrization of amplitude for processes with final nu nubar pair, is done only for Majorana nu and nubar due to their quantum mechanical identical nature.
This is in fact a consequence of the fact that Majorana spinors are the real solutions of the Dirac equation.

(3) For the anti-symmetrization of the final states of nu and nu-bar, (not just nu nu and nu-bar nu-bar states) in case of Majorana neutrino, it has been known for long time, and used by all those earlier papers concerning DMCT (Dirac Majorana Confusion Theorem) from B. Kayser, R. Shrock, E. Ma, J. Nieves, P. Pal, T. Chabra, ……

on pDMCT (4-3) – Is the final state $\nu\bar{\nu}$ the same as $\nu_L\bar{\nu}_R$?

** In case of high energy scattering amplitudes, only when the two neutrinos have the same helicity, they can interfere, and hence their statistics can be studied experimentally ?

(A) Doing anti-symmetrization of amplitude and following the Feynman rules relevant for Majorana fermions, we do a QFT calculation for the process under our consideration. We find that, if we fully integrate over the allowed phase space for nu and nu-bar, only the interference term leads to the difference between Dirac and Majorana cases. Our explicit calculation shows that the interference term is proportional to m_nu^2. This matches well with the usual expectation from Dirac Majorana confusion theorem (DMCT), see Eq (35) of my paper. The most interesting result is that if the energies and momenta of the nu and nu-bar could be inferred, the difference between Dirac and Majorana cases have terms that are independent of m_nu. We have figured out that the kinematics of back-to-back muons is helpful in this regard (see Sec. IV, and Eq. (47)). For this kinematic configuration, not only the muon energy distributions but also the branching ratios are predicted to be different for Dirac and Majorana cases (see Eqs. (50) and (51)).

on pDMCT (4-4) – Is the final state $\nu\bar{\nu}$ the same as $\nu_L \bar{\nu}_R$?

We see that chirality and helicity have different characteristics: Helicity is conserved for a free particle but is not Lorentz invariant, whereas chirality is Lorentz invariant but not conserved. Therefore, none of these properties is appropriate for characterizing a fermion that has mass. If a particle

[Reference: P. B. Pal, Am. J. Phys. **79**, 485-498 (2011).]

is the only way we can observe them. There is thus no empirical evidence for the existence of ν_R (and $\bar{\nu}_L$), and it could well be that they do not exist in nature. Of course, the assertion that only ν_L (and $\bar{\nu}_R$) occur can only be made if the mass is strictly zero. Otherwise, we could perform a Lorentz transformation which would change a ν_L into a ν_R.

[Reference: F. Halzen and A. D. Martin, Joh Wiley & Sons, Inc. (page 115).]

[1]Helicity is identical to handedness (or chirality) for massless neutrinos and nearly identical to handedness for particles traveling near the speed of light. For that reason, helicity is sometimes loosely referred to as "handedness." For massive particles, however, the two quantities are quite different. Massive particles must exist in right- and left-helicity and in right- and left-handed states. As illustrated in the box on page 32 and

[Reference: Richard Slansky *et al.*, "The Oscillating neutrino," Los Alamos Science, Number 25, 1997.]

state containing two fermions. If the fermions are Dirac particles, the Feynman amplitude for the process is

$$\boxed{\mathcal{M} = g \sum_{s_1,s_2} \bar{u}_{s_1}(p_1) F v_{s_2}(p_2), \qquad (146)}$$

If, in contrast, a Majorana pair is produced in the final

state, the amplitude is different because the operator ψ can create either of the two, and so can $\bar{\psi}$. We therefore write

$$\boxed{\mathcal{M} = g \sum_{s_1,s_2} \left(\bar{u}_{s_1}(p_1) F v_{s_2}(p_2) - \bar{u}_{s_2}(p_2) F v_{s_1}(p_1)\right), \qquad (147)}$$

[Reference: P. B. Pal, Am. J. Phys. **79**, 485-498 (2011).]

teractions as well. If we take them into account, the effective leptonic current, which is simply $\bar{\nu}\gamma^\mu(1-\gamma_5)\nu$ for the Dirac case, becomes

$$\bar{\nu}\gamma^\mu(1-\gamma_5)\nu - \bar{\nu}\gamma^\mu(1+\gamma_5)\nu = -2\bar{\nu}\gamma^\mu\gamma_5\nu$$

for the Majorana case.

[Reference: J. F. Nieves and P. B. Pal, Phys. Rev. D **32**, 1849-1852 (1985).]

A Majorana-neutrino pair can coherently interfere with each other since particle and antiparticle are identical while a Dirac pair will not interfere. The Feynman diagrams for the Majorana-neutrino production process are shown in Fig. 1. For an electron (or quark) coupling of

[Reference: E. Ma and J. T. Pantaleone, Phys. Rev. D **40**, 2172-2176 (1989).]

on pDMCT (5) – the final state of phase space goes to 0 ?

** the phase space of the region where the distinction can be made goes to zero when the neutrino mass goes to zero ?

(a) It is a well-known fact that if a specific kinematics is not prohibited by the various conservation laws (namely the conservation of energy, linear momentum, angular momentum) and not ruled out by the dynamics via some symmetry principle, then that kinematics would be observed in nature.

(b) The special back-to-back kinematics we have considered in our paper is allowed for both Dirac and Majorana neutrinos and not prohibited at all.

Helicity & Chirality

Dirac equation

A free fermion of mass m is described by a fermionic field $\psi(x)$ which satisfies the Dirac equation,

$$(i\partial\!\!\!/ - m)\,\psi(x) = 0, \tag{1}$$

where $\partial\!\!\!/ \equiv \gamma^\mu \partial_\mu$ with the Dirac γ matrices having two useful representations:

Dirac representation:

$$\gamma_D^0 = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} \end{pmatrix}, \quad \gamma_D^i = \begin{pmatrix} \mathbf{0} & \sigma^i \\ -\sigma^i & \mathbf{0} \end{pmatrix}, \tag{2}$$

and

$$\gamma_D^5 \equiv i\gamma_D^0\gamma_D^1\gamma_D^2\gamma_D^3 = \begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}, \tag{3}$$

Weyl or Chiral representation:

$$\gamma_C^0 = \begin{pmatrix} \mathbf{0} & -\mathbf{1} \\ -\mathbf{1} & \mathbf{0} \end{pmatrix}, \quad \gamma_C^i = \begin{pmatrix} \mathbf{0} & \sigma^i \\ -\sigma^i & \mathbf{0} \end{pmatrix}, \tag{4}$$

and

$$\gamma_C^5 \equiv i\gamma_C^0\gamma_C^1\gamma_C^2\gamma_C^3 = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} \end{pmatrix}, \tag{5}$$

where $i = 1, 2, 3$, $\mathbf{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\mathbf{0} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, and the Pauli σ matrices are given by $\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, and $\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

Note: $\boxed{\gamma_D^i = \gamma_C^i}$, $\boxed{\gamma_D^0 = \gamma_C^5}$ and $\boxed{\gamma_D^5 = -\gamma_C^0}$.

Helicity & Chirality

Helicity / Spin projection operator

For a spin 1/2 fermion, the spin could have projection along the direction of 3-momentum (helicity $\equiv h = +1$) or opposite to it ($h = -1$). The helicity operator is given by

$$\widehat{h} \equiv \frac{\vec{S} \cdot \vec{P}}{s\,|\vec{P}|}, \tag{6}$$

where $\vec{S}$ is the spin operator and $\vec{P}$ is the 3-momentum operator and $s = 1/2$ for the spin 1/2 fermion. Thus, the field $\psi(x)$ can be split into a positive helicity part $\psi^{(+)}(x)$ and a negative helicity part $\psi^{(-)}(x)$ which are eigenfunctions of the helicity operator, i.e.

$$\widehat{h}\,\psi^{(h)}(x) = h\,\psi^{(h)}(x), \tag{7}$$

for $h = \pm 1$, and

$$\psi(x) = \psi^{(+)}(x) + \psi^{(-)}(x). \tag{8}$$

Chirality projection operator

The matrix γ^5 is the *chirality matrix*. If $\psi_R(x)$ and $\psi_L(x)$ are the right and left chiral fields, then they satisfy the following eigenvalue equations,

$$\gamma^5\,\psi_R(x) = +\psi_R(x), \tag{9}$$

$$\gamma^5\,\psi_L(x) = -\psi_L(x), \tag{10}$$

and

$$\psi(x) = \psi_R(x) + \psi_L(x). \tag{11}$$

In other words,

$$\psi_R(x) = \frac{1+\gamma^5}{2}\psi(x) \equiv P_R\,\psi(x), \tag{12}$$

$$\psi_L(x) = \frac{1-\gamma^5}{2}\psi(x) \equiv P_L\,\psi(x), \tag{13}$$

Helicity & Chirality

where, in the *chiral representation*, we have

$$P_R = \frac{1+\gamma^5}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \tag{14}$$

$$P_L = \frac{1-\gamma^5}{2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{15}$$

Writing the general 4-component Dirac spinor ψ in terms of two 2-component (Weyl) spinors χ_R and χ_L as

$$\psi = \begin{pmatrix} \chi_R \\ \chi_L \end{pmatrix}, \tag{16}$$

we get (*in the chiral representation*)

$$\psi_R = P_R\,\psi = \begin{pmatrix} \chi_R \\ 0 \end{pmatrix}, \quad \psi_L = P_L\,\psi = \begin{pmatrix} 0 \\ \chi_L \end{pmatrix}. \tag{17}$$

Thus the operators P_R and P_L are called the *chirality projection operators*. The chiral spinors ψ_R and ψ_L satisfy the field equations,

$$i\partial\!\!\!/\,\psi_R = m\,\psi_L, \tag{18}$$

$$i\partial\!\!\!/\,\psi_L = m\,\psi_R. \tag{19}$$

This shows that space-time evolution of the chiral spinors ψ_R and ψ_L are related to one another by the mass m. If we consider the case of massless fermions, i.e. $m = 0$, then we obtain the *Weyl equations*:

$$i\partial\!\!\!/\,\psi_R = 0, \tag{20}$$

$$i\partial\!\!\!/\,\psi_L = 0. \tag{21}$$

Helicity & Chirality

Dirac spinors

For both helicity projections, we can have positive and negative frequency solutions of the Dirac equation. Thus,

$$\psi^{(h)}(x) = \int \frac{d^3p}{(2\pi)^3 2E}\Big[a^{(h)}(p)\, u^{(h)}(p)\, e^{-ip\cdot x} + b^{(h)\dagger}(p)\, v^{(h)}(p)\, e^{ip\cdot x}\Big], \qquad (22)$$

where the coefficients $a^{(h)}(p)$ and $b^{(h)}(p)$ are given by,

$$a^{(h)}(p) = \int d^3x\, u^{(h)\dagger}(p)\, \psi(x)\, e^{ip\cdot x}, \qquad (23)$$

$$b^{(h)}(p) = \int d^3x\, \psi^\dagger(x)\, u^{(h)}(p)\, e^{ip\cdot x}, \qquad (24)$$

and they satisfy the condition that

$$\int \frac{d^3p}{(2\pi)^3 2E} \sum_{h=\pm1} \Big[\big|a^{(h)}(p)\big|^2 + \big|b^{(h)}(p)\big|^2\Big] = 1. \qquad (25)$$

The Dirac equations satisfied by the four 4-component Dirac spinors $u^{(h)}(p)$ and $v^{(h)}(p)$ are

$$\big(\not{p} - m\big)\, u^{(h)}(p) = 0, \qquad (26)$$

$$\big(\not{p} + m\big)\, v^{(h)}(p) = 0, \qquad (27)$$

where $\not{p} \equiv \gamma^\mu p_\mu$. For the Dirac spinor associated with either positive or negative frequency solution, we can further distinguish the left and right chiral spinors, i.e.

$$u^{(h)}(p) = u_R^{(h)}(p) + u_L^{(h)}(p), \qquad (28)$$

$$v^{(h)}(p) = v_R^{(h)}(p) + v_L^{(h)}(p). \qquad (29)$$

Let us introduce the 2-component helicity eigenstate spinors $\chi^{(h)}(\vec{p})$ which satisfy the eigenvalue equation

$$\frac{\vec{p}\cdot\vec{\sigma}}{|\vec{p}|}\chi^{(h)}(\vec{p}) = h\,\chi^{(h)}(\vec{p}). \qquad (30)$$

The explicit form of Dirac spinors can be written using these 2-component spinors. The explicit form of Dirac spinors also depends on the representation of the Dirac γ matrices.

Helicity & Chirality

(1) **In the Dirac representation** we have,

$$u_D^{(h)}(p) = \begin{pmatrix} \sqrt{E+m}\,\chi^{(h)}(\vec{p}) \\ h\sqrt{E-m}\,\chi^{(h)}(\vec{p}) \end{pmatrix}$$

$$= \sqrt{E+m}\begin{pmatrix} \chi^{(h)}(\vec{p}) \\ h\dfrac{|\vec{p}|}{E+m}\chi^{(h)}(\vec{p}) \end{pmatrix}, \qquad (31)$$

$$v_D^{(h)}(p) = \begin{pmatrix} -\sqrt{E-m}\,\chi^{(-h)}(\vec{p}) \\ h\sqrt{E+m}\,\chi^{(-h)}(\vec{p}) \end{pmatrix}$$

$$= \sqrt{E+m}\begin{pmatrix} -\dfrac{|\vec{p}|}{E+m}\chi^{(-h)}(\vec{p}) \\ h\chi^{(-h)}(\vec{p}) \end{pmatrix}. \qquad (32)$$

For *non-relativistic case* we have $|\vec{p}| \ll m$ and $E \simeq m$, such that

$$u_D^{(h)}(p) = \sqrt{2m}\begin{pmatrix} \chi^{(h)}(\vec{p}) \\ h\dfrac{|\vec{p}|}{2m}\chi^{(h)}(\vec{p}) \end{pmatrix}, \qquad (33)$$

$$v_D^{(h)}(p) = \sqrt{2m}\begin{pmatrix} -\dfrac{|\vec{p}|}{2m}\chi^{(-h)}(\vec{p}) \\ h\chi^{(-h)}(\vec{p}) \end{pmatrix}. \qquad (34)$$

Since, $\dfrac{|\vec{p}|}{2m} \ll 1$, in the non-relativistic case, the two upper components of $u^{(h)}(p)$ are called the *larger components* and the two lower components are called the *smaller components*. The opposite is true for $v^{(h)}(p)$. *This makes Dirac representation a useful choice while studying non-relativistic fermions.*

Helicity & Chirality

(2) In the Weyl or Chiral representation we have

$$u_C^{(h)}(p) = \begin{pmatrix} -\sqrt{E + h\,|\vec{p}|}\,\chi^{(h)}(\vec{p}) \\ \sqrt{E - h\,|\vec{p}|}\,\chi^{(h)}(\vec{p}) \end{pmatrix}, \qquad (35)$$

$$v_C^{(h)}(p) = -h \begin{pmatrix} \sqrt{E - h\,|\vec{p}|}\,\chi^{(-h)}(\vec{p}) \\ \sqrt{E + h\,|\vec{p}|}\,\chi^{(-h)}(\vec{p}) \end{pmatrix}. \qquad (36)$$

Thus,

$$u_C^{(+)}(p) = \begin{pmatrix} -\sqrt{E + |\vec{p}|}\,\chi^{(+)}(\vec{p}) \\ \sqrt{E - |\vec{p}|}\,\chi^{(+)}(\vec{p}) \end{pmatrix}$$

$$= \sqrt{E + |\vec{p}|} \begin{pmatrix} -\chi^{(+)}(\vec{p}) \\ \frac{m}{E + |\vec{p}|}\,\chi^{(+)}(\vec{p}) \end{pmatrix}, \qquad (37)$$

$$u_C^{(-)}(p) = \begin{pmatrix} -\sqrt{E - |\vec{p}|}\,\chi^{(-)}(\vec{p}) \\ \sqrt{E + |\vec{p}|}\,\chi^{(-)}(\vec{p}) \end{pmatrix}$$

$$= \sqrt{E + |\vec{p}|} \begin{pmatrix} -\frac{m}{E + |\vec{p}|}\,\chi^{(-)}(\vec{p}) \\ \chi^{(-)}(\vec{p}) \end{pmatrix}, \qquad (38)$$

$$v_C^{(+)}(p) = -\begin{pmatrix} \sqrt{E - |\vec{p}|}\,\chi^{(-)}(\vec{p}) \\ \sqrt{E + |\vec{p}|}\,\chi^{(-)}(\vec{p}) \end{pmatrix}$$

$$= -\sqrt{E + |\vec{p}|} \begin{pmatrix} \frac{m}{E + |\vec{p}|}\,\chi^{(-)}(\vec{p}) \\ \chi^{(-)}(\vec{p}) \end{pmatrix}, \qquad (39)$$

$$v_C^{(-)}(p) = \begin{pmatrix} \sqrt{E + |\vec{p}|}\,\chi^{(+)}(\vec{p}) \\ \sqrt{E - |\vec{p}|}\,\chi^{(+)}(\vec{p}) \end{pmatrix}$$

$$= \sqrt{E + |\vec{p}|} \begin{pmatrix} \chi^{(+)}(\vec{p}) \\ \frac{m}{E + |\vec{p}|}\,\chi^{(+)}(\vec{p}) \end{pmatrix}. \qquad (40)$$

Helicity & Chirality

For *ultra-relativistic case* we have $m \ll E$ and $\vec{p} \simeq E$, such that

$$u_C^{(+)}(p) = \sqrt{2E}\begin{pmatrix} -\chi^{(+)}(\vec{p}) \\ \frac{m}{2E}\chi^{(+)}(\vec{p}) \end{pmatrix}, \tag{41}$$

$$u_C^{(-)}(p) = \sqrt{2E}\begin{pmatrix} -\frac{m}{2E}\chi^{(-)}(\vec{p}) \\ \chi^{(-)}(\vec{p}) \end{pmatrix}, \tag{42}$$

$$v_C^{(+)}(p) = -\sqrt{2E}\begin{pmatrix} \frac{m}{2E}\chi^{(-)}(\vec{p}) \\ \chi^{(-)}(\vec{p}) \end{pmatrix}, \tag{43}$$

$$v_C^{(-)}(p) = \sqrt{2E}\begin{pmatrix} \chi^{(+)}(\vec{p}) \\ \frac{m}{2E}\chi^{(+)}(\vec{p}) \end{pmatrix}. \tag{44}$$

Since, in the chiral representation, the upper two components of the 4-component Dirac spinor form the Right Weyl spinor and the lower two components form the Left Weyl spinor, let us introduce the following notation,

$$u_C^{(h)}(p) = \sqrt{2E}\begin{pmatrix} u_{C,R}^{(h)}(p) \\ u_{C,L}^{(h)}(p) \end{pmatrix}, \quad v_C^{(h)}(p) = \sqrt{2E}\begin{pmatrix} v_{C,R}^{(h)}(p) \\ v_{C,L}^{(h)}(p) \end{pmatrix}. \tag{45}$$

Using this notation and using the fact that for ultra-relativistic case $\frac{m}{2E} \ll 1$, it is easy to show that *the larger components* are

$$u_{C,R}^{(+)}(p) = -\chi^{(+)}(\vec{p}), \tag{46a}$$

$$u_{C,L}^{(-)}(p) = +\chi^{(-)}(\vec{p}), \tag{46b}$$

$$v_{C,L}^{(+)}(p) = -\chi^{(-)}(\vec{p}), \tag{46c}$$

$$v_{C,R}^{(-)}(p) = +\chi^{(+)}(\vec{p}), \tag{46d}$$

Helicity & Chirality

and the *smaller components* are

$$u^{(+)}_{C,L}(p) = +\frac{m}{2E}\,\chi^{(+)}(\vec{p}), \tag{47a}$$
$$u^{(-)}_{C,R}(p) = -\frac{m}{2E}\,\chi^{(-)}(\vec{p}), \tag{47b}$$
$$v^{(+)}_{C,R}(p) = -\frac{m}{2E}\,\chi^{(-)}(\vec{p}), \tag{47c}$$
$$v^{(-)}_{C,L}(p) = +\frac{m}{2E}\,\chi^{(+)}(\vec{p}). \tag{47d}$$

In simple terms, these equations state that for a fermionic **particle** in ultra-relativistic case:

(i) positive helicity state is mostly right-handed, and

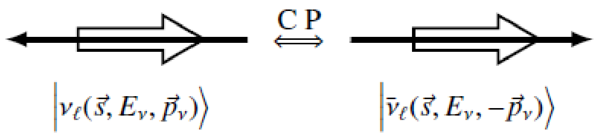
(ii) negative helicity state is mostly left-handed.

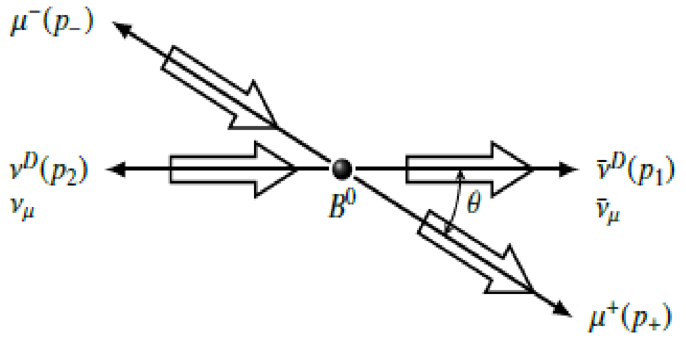
Similarly, for a fermionic **anti-particle** in ultra-relativistic case:

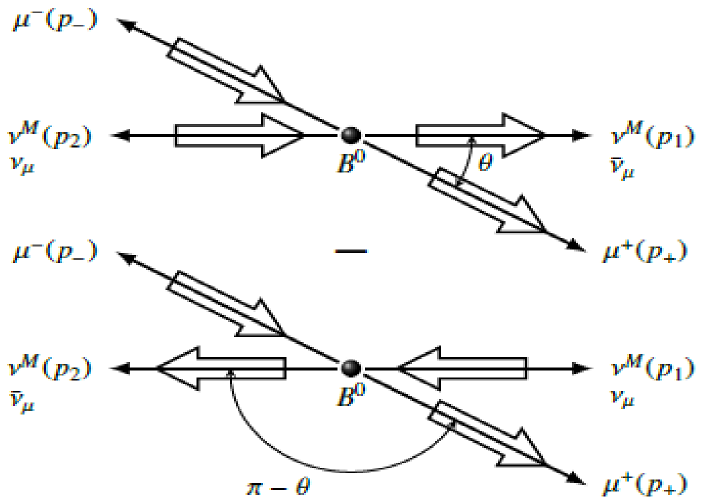
(i) positive helicity state is mostly left-handed, and

(ii) negative helicity state is mostly right-handed.

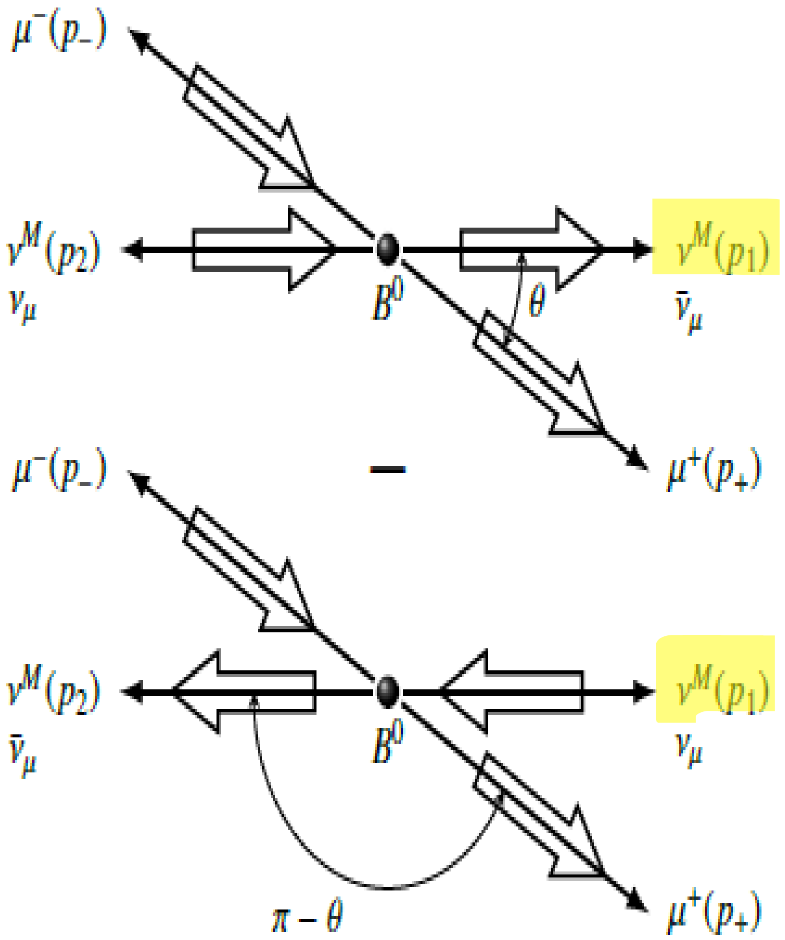
$$\mathrm{C\,P}\left|\nu_\ell(\vec{s}, E_\nu, \vec{p}_\nu)\right\rangle = \eta_P \left|\bar{\nu}_\ell(\vec{s}, E_\nu, -\vec{p}_\nu)\right\rangle ,$$





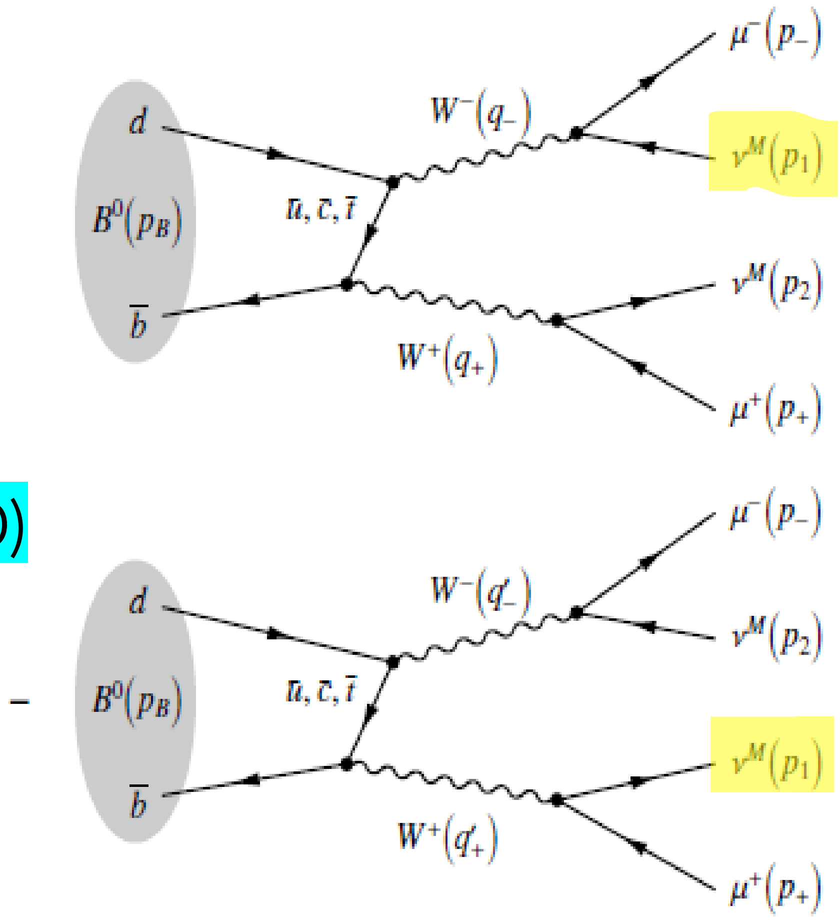


Helicity Configuration of back-to-back muons



Helicity-flip (X)

Momenta exchange (O)



Detailed study of $B^0(p_B) \to \mu^-(p_-)\,\mu^+(p_+)\,\bar{\nu}_\mu(p_1)\,\nu_\mu(p_2)$,

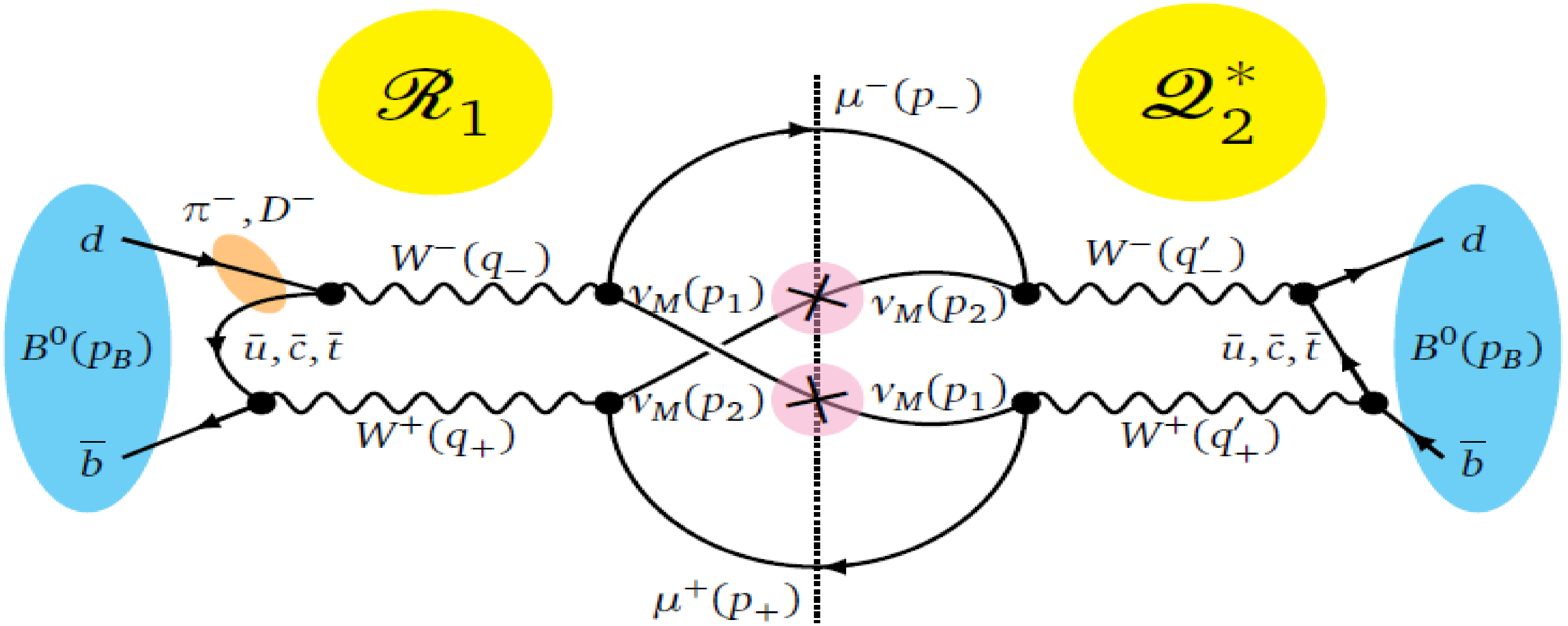
$$\frac{d^5\Gamma^{D/M}}{dm_{\mu\mu}^2\; dm_{\nu\nu}^2\; d\cos\theta_m\; d\cos\theta_n\; d\phi} = \frac{Y\, Y_m\, Y_n \left\langle \left|\mathscr{M}^{D/M}\right|^2 \right\rangle}{(4\,\pi)^6\, m_B^2\, m_{\mu\mu}\, m_{\nu\nu}},$$

$$\begin{aligned}\left\langle \left|\mathscr{M}^{D}\right|^2 \right\rangle = G_F^4 \Big(& |\mathbb{F}_a|^2 S_{aa}^D + |\mathbb{F}_b|^2 S_{bb}^D + |\mathbb{F}_c^2| S_{cc}^D + |\mathbf{F}_+|^2 S_{pp}^D + |\mathbf{F}_-|^2 S_{mm}^D + \mathrm{Re}\left(\mathbb{F}_a\mathbb{F}_b^*\right) R_{ab}^D + \mathrm{Re}\left(\mathbb{F}_a\mathbb{F}_c^*\right) R_{ac}^D + \mathrm{Re}\left(\mathbb{F}_a\mathbf{F}_+^*\right) R_{ap}^D \\ & + \mathrm{Re}\left(\mathbb{F}_a\mathbf{F}_-^*\right) R_{am}^D + \mathrm{Im}\left(\mathbb{F}_a\mathbb{F}_b^*\right) I_{ab}^D + \mathrm{Im}\left(\mathbb{F}_a\mathbb{F}_c^*\right) I_{ac}^D + \mathrm{Im}\left(\mathbb{F}_a\mathbf{F}_+^*\right) I_{ap}^D + \mathrm{Im}\left(\mathbb{F}_a\mathbf{F}_-^*\right) I_{am}^D + \mathrm{Re}\left(\mathbb{F}_b\mathbb{F}_c^*\right) R_{bc}^D \\ & + \mathrm{Re}\left(\mathbb{F}_b\mathbf{F}_+^*\right) R_{bp}^D + \mathrm{Re}\left(\mathbb{F}_b\mathbf{F}_-^*\right) R_{bm}^D + \mathrm{Re}\left(\mathbb{F}_c\mathbf{F}_+^*\right) R_{cp}^D + \mathrm{Im}\left(\mathbb{F}_b\mathbb{F}_c^*\right) I_{bc}^D + \mathrm{Im}\left(\mathbb{F}_b\mathbf{F}_+^*\right) I_{bp}^D + \mathrm{Im}\left(\mathbb{F}_b\mathbf{F}_-^*\right) I_{bm}^D \\ & + \mathrm{Im}\left(\mathbb{F}_c\mathbf{F}_+^*\right) I_{cp}^D + \mathrm{Re}\left(\mathbb{F}_c\mathbf{F}_-^*\right) R_{cm}^D + \mathrm{Re}\left(\mathbf{F}_+\mathbf{F}_-^*\right) R_{pm}^D + \mathrm{Im}\left(\mathbb{F}_c\mathbf{F}_-^*\right) I_{cm}^D + \mathrm{Im}\left(\mathbf{F}_+\mathbf{F}_-^*\right) I_{pm}^D \Big),\end{aligned}$$

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$,

$$
\begin{aligned}
\langle |\mathscr{M}^M|^2 \rangle = \frac{G_F^4}{2}\Big(& |\mathbb{F}_a|^2 S^M_{aa} + |\mathbb{F}_b|^2 S^M_{bb} + |\mathbb{F}_c^2| S^M_{cc} + |\mathbf{F}_+|^2 S^M_{pp} + |\mathbf{F}_-|^2 S^M_{mm} + |\mathbb{F}'_a|^2 S^M_{a'a'} + |\mathbb{F}'_b|^2 S^M_{b'b'} + |\mathbb{F}'_c|^2 S^M_{c'c'} + |\mathbf{F}'_+|^2 S^M_{p'p'} \\
& + |\mathbf{F}'_-|^2 S^M_{m'm'} + \mathrm{Re}\left(\mathbb{F}_a\mathbb{F}_b^*\right) R^M_{ab} + \mathrm{Re}\left(\mathbb{F}_a\mathbb{F}_c^*\right) R^M_{ac} + \mathrm{Re}\left(\mathbb{F}_a\mathbf{F}_+^*\right) R^M_{ap} + \mathrm{Re}\left(\mathbb{F}_a\mathbf{F}_-^*\right) R^M_{am} + \mathrm{Re}\left(\mathbb{F}_b\mathbb{F}_c^*\right) R^M_{bc} \\
& + \mathrm{Re}\left(\mathbb{F}_b\mathbf{F}_+^*\right) R^M_{bp} + \mathrm{Re}\left(\mathbb{F}_b\mathbf{F}_-^*\right) R^M_{bm} + \mathrm{Re}\left(\mathbb{F}_c\mathbf{F}_+^*\right) R^M_{cp} + \mathrm{Re}\left(\mathbb{F}_c\mathbf{F}_-^*\right) R^M_{cm} + \mathrm{Re}\left(\mathbb{F}'_a\mathbb{F}'^*_b\right) R^M_{a'b'} + \mathrm{Re}\left(\mathbb{F}'_a\mathbb{F}'^*_c\right) R^M_{a'c'} \\
& + \mathrm{Re}\left(\mathbb{F}'_a\mathbf{F}'^*_+\right) R^M_{a'p'} + \mathrm{Re}\left(\mathbb{F}'_a\mathbf{F}'^*_-\right) R^M_{a'm'} + \mathrm{Re}\left(\mathbb{F}'_b\mathbb{F}'^*_c\right) R^M_{b'c'} + \mathrm{Re}\left(\mathbb{F}'_b\mathbf{F}'^*_+\right) R^M_{b'p'} + \mathrm{Re}\left(\mathbb{F}'_b\mathbf{F}'^*_-\right) R^M_{b'm'} + \mathrm{Re}\left(\mathbb{F}'_c\mathbf{F}'^*_+\right) R^M_{c'p'} \\
& + \mathrm{Re}\left(\mathbb{F}'_c\mathbf{F}'^*_-\right) R^M_{c'm'} + \mathrm{Re}\left(\mathbf{F}_+\mathbf{F}_-^*\right) R^M_{pm} + \mathrm{Re}\left(\mathbf{F}'_+\mathbf{F}'^*_-\right) R^M_{p'm'} + \mathrm{Im}\left(\mathbb{F}_a\mathbb{F}_b^*\right) I^M_{ab} + \mathrm{Im}\left(\mathbb{F}_a\mathbb{F}_c^*\right) I^M_{ac} + \mathrm{Im}\left(\mathbb{F}_a\mathbf{F}_+^*\right) I^M_{ap} \\
& + \mathrm{Im}\left(\mathbb{F}_a\mathbf{F}_-^*\right) I^M_{am} + \mathrm{Im}\left(\mathbb{F}_b\mathbb{F}_c^*\right) I^M_{bc} + \mathrm{Im}\left(\mathbb{F}_b\mathbf{F}_+^*\right) I^M_{bp} + \mathrm{Im}\left(\mathbb{F}_b\mathbf{F}_-^*\right) I^M_{bm} + \mathrm{Im}\left(\mathbb{F}_c\mathbf{F}_+^*\right) I^M_{cp} + \mathrm{Im}\left(\mathbb{F}_c\mathbf{F}_-^*\right) I^M_{cm} \\
& + \mathrm{Im}\left(\mathbb{F}'_a\mathbb{F}'^*_b\right) I^M_{a'b'} + \mathrm{Im}\left(\mathbb{F}'_a\mathbb{F}'^*_c\right) I^M_{a'c'} + \mathrm{Im}\left(\mathbb{F}'_a\mathbf{F}'^*_+\right) I^M_{a'p'} + \mathrm{Im}\left(\mathbb{F}'_a\mathbf{F}'^*_-\right) I^M_{a'm'} + \mathrm{Im}\left(\mathbb{F}'_b\mathbb{F}'^*_c\right) I^M_{b'c'} + \mathrm{Im}\left(\mathbb{F}'_b\mathbf{F}'^*_+\right) I^M_{b'p'} \\
& + \mathrm{Im}\left(\mathbb{F}'_b\mathbf{F}'^*_-\right) I^M_{b'm'} + \mathrm{Im}\left(\mathbb{F}'_c\mathbf{F}'^*_+\right) I^M_{c'p'} + \mathrm{Im}\left(\mathbb{F}'_c\mathbf{F}'^*_-\right) I^M_{c'm'} + \mathrm{Im}\left(\mathbf{F}_+\mathbf{F}_-^*\right) I^M_{pm} + \mathrm{Im}\left(\mathbf{F}'_+\mathbf{F}'^*_-\right) I^M_{p'm'} \\
& + m_\nu^2\Big(\mathrm{Re}\left(\mathbb{F}_a\mathbb{F}'^*_a\right) R^M_{aa'} + \mathrm{Re}\left(\mathbb{F}_a\mathbb{F}'^*_b\right) R^M_{ab'} + \mathrm{Re}\left(\mathbb{F}_a\mathbb{F}'^*_c\right) R^M_{ac'} + \mathrm{Re}\left(\mathbb{F}_a\mathbf{F}'^*_+\right) R^M_{ap'} + \mathrm{Re}\left(\mathbb{F}_a\mathbf{F}'^*_-\right) R^M_{am'} + \mathrm{Re}\left(\mathbb{F}_b\mathbb{F}'^*_a\right) R^M_{ba'} \\
& \quad + \mathrm{Re}\left(\mathbb{F}_b\mathbb{F}'^*_b\right) R^M_{bb'} + \mathrm{Re}\left(\mathbb{F}_b\mathbb{F}'^*_c\right) R^M_{bc'} + \mathrm{Re}\left(\mathbb{F}_b\mathbf{F}'^*_+\right) R^M_{bp'} + \mathrm{Re}\left(\mathbb{F}_b\mathbf{F}'^*_-\right) R^M_{bm'} + \mathrm{Re}\left(\mathbb{F}_c\mathbb{F}'^*_a\right) R^M_{ca'} + \mathrm{Re}\left(\mathbb{F}_c\mathbb{F}'^*_b\right) R^M_{cb'} \\
& \quad + \mathrm{Re}\left(\mathbb{F}_c\mathbb{F}'^*_c\right) R^M_{cc'} + \mathrm{Re}\left(\mathbb{F}_c\mathbf{F}'^*_+\right) R^M_{cp'} + \mathrm{Re}\left(\mathbb{F}_c\mathbf{F}'^*_-\right) R^M_{cm'} + \mathrm{Re}\left(\mathbb{F}'_a\mathbf{F}_+^*\right) R^M_{a'p} + \mathrm{Re}\left(\mathbb{F}'_a\mathbf{F}_-^*\right) R^M_{a'm} + \mathrm{Re}\left(\mathbb{F}'_b\mathbf{F}_+^*\right) R^M_{b'p} \\
& \quad + \mathrm{Re}\left(\mathbb{F}'_b\mathbf{F}_-^*\right) R^M_{b'm} + \mathrm{Re}\left(\mathbb{F}'_c\mathbf{F}_+^*\right) R^M_{c'p} + \mathrm{Re}\left(\mathbb{F}'_c\mathbf{F}_-^*\right) R^M_{c'm} + \mathrm{Im}\left(\mathbb{F}_a\mathbb{F}'^*_a\right) I^M_{aa'} + \mathrm{Im}\left(\mathbb{F}_a\mathbb{F}'^*_b\right) I^M_{ab'} + \mathrm{Im}\left(\mathbb{F}_a\mathbb{F}'^*_c\right) I^M_{ac'} \\
& \quad + \mathrm{Im}\left(\mathbb{F}_a\mathbf{F}'^*_+\right) I^M_{ap'} + \mathrm{Im}\left(\mathbb{F}_a\mathbf{F}'^*_-\right) I^M_{am'} + \mathrm{Im}\left(\mathbb{F}_b\mathbb{F}'^*_a\right) I^M_{ba'} + \mathrm{Im}\left(\mathbb{F}_b\mathbb{F}'^*_b\right) I^M_{bb'} + \mathrm{Im}\left(\mathbb{F}_b\mathbb{F}'^*_c\right) I^M_{bc'} + \mathrm{Im}\left(\mathbb{F}_b\mathbf{F}'^*_+\right) I^M_{bp'} \\
& \quad + \mathrm{Im}\left(\mathbb{F}_b\mathbf{F}'^*_-\right) I^M_{bm'} + \mathrm{Im}\left(\mathbb{F}_c\mathbb{F}'^*_a\right) I^M_{ca'} + \mathrm{Im}\left(\mathbb{F}_c\mathbb{F}'^*_b\right) I^M_{cb'} + \mathrm{Im}\left(\mathbb{F}_c\mathbb{F}'^*_c\right) I^M_{cc'} + \mathrm{Im}\left(\mathbb{F}_c\mathbf{F}'^*_+\right) I^M_{cp'} + \mathrm{Im}\left(\mathbb{F}_c\mathbf{F}'^*_-\right) I^M_{cm'} \\
& \quad + \mathrm{Im}\left(\mathbb{F}'_a\mathbf{F}_+^*\right) I^M_{a'p} + \mathrm{Im}\left(\mathbb{F}'_a\mathbf{F}_-^*\right) I^M_{a'm} + \mathrm{Im}\left(\mathbb{F}'_b\mathbf{F}_+^*\right) I^M_{b'p} + \mathrm{Im}\left(\mathbb{F}'_b\mathbf{F}_-^*\right) I^M_{b'm} + \mathrm{Im}\left(\mathbb{F}'_c\mathbf{F}_+^*\right) I^M_{c'p} + \mathrm{Im}\left(\mathbb{F}'_c\mathbf{F}_-^*\right) I^M_{c'm}\Big)\Big),
\end{aligned}
$$

$\mathrm{Re}\left(\mathscr{M}(p_1,p_2)^* \mathscr{M}(p_2,p_1)\right) \propto m_\nu^2$. **of** $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$,



Since the squared diagram involves two helicity flips for the Majorana neutrinos, these contributions are directly proportional to m_ν^2.

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, -- B2B kinematics (1)

Kinematics of back-to-back muons at B-rest frame (ie. B2B nu-nubar) -- change of variables

- In general, $\cos\theta_n = \frac{m_{\nu\nu}(E_1 - E_2)}{2\,Y\,Y_n}$, which for back-to-back configuration yields $\cos\theta_n \to \pm 1$. It behaves akin to Heaviside step function $H(x)$ at $x = 0$, and this discontinuity is resolved by taking average of the two limits, i.e. $\cos\theta_n = 0$. This is also consistent with $E_1 = E_2$ for any $\Theta \neq \pi$.

- In the general kinematics the z-axis is defined for non-zero Y. For back-to-back case, $Y = 0$, which implies there is no fixed z-axis *a priori* in this case. A specific value of $\cos\theta_n$ is just a way to fix the z-axis and thus the coordinate system. For $\cos\theta_n = 0$, the z-axis is defined perpendicular to the $\nu\bar{\nu}$ direction. Thus, $\phi = 0$.

- Angle θ between neutrino and muon directions in back-to-back case is then $\theta_m = \frac{\pi}{2} - \theta$.

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, -- B2B kinematics (2)

- In terms of the five usual variables for general kinematics:

$$\frac{\mathrm{d}^5\Gamma}{\mathrm{d}m_{\mu\mu}^2\,\mathrm{d}m_{\nu\nu}^2\,\mathrm{d}\cos\theta_m\,\mathrm{d}\cos\theta_n\,\mathrm{d}\phi} = \frac{Y\,Y_m\,Y_n}{(4\,\pi)^6\,m_B^2\,m_{\mu\mu}\,m_{\nu\nu}}\left\langle|\mathscr{M}|^2\right\rangle.$$

- Using energies E_1 and E_2 of ν and $\overline{\nu}$ respectively, as well as the angle Θ between them in the rest frame of B^0:

$$\frac{\mathrm{d}^5\Gamma}{\mathrm{d}E_1\,\mathrm{d}E_2\,\mathrm{d}\cos\Theta\,\mathrm{d}\cos\theta_m\,\mathrm{d}\phi} = \frac{-Y_m\,\sqrt{\left(E_1^2-m_\nu^2\right)\left(E_2^2-m_\nu^2\right)}}{4^5\,\pi^6\,m_B\,m_{\mu\mu}}\left\langle|\mathscr{M}|^2\right\rangle,$$

where we have used

$$\mathrm{d}m_{\mu\mu}^2\,\mathrm{d}m_{\nu\nu}^2\,\mathrm{d}\cos\theta_n = -\frac{4\,m_B\,m_{\nu\nu}}{Y\,Y_n}\sqrt{\left(E_1^2-m_\nu^2\right)\left(E_2^2-m_\nu^2\right)}\,\mathrm{d}E_1\,\mathrm{d}E_2\,\mathrm{d}\cos\Theta.$$

For back-to-back configuration $E_1 = E_2 \equiv E_\nu = \frac{1}{2}m_B - E_\mu$ and $\Theta = \pi$.

➔ Only $(\boldsymbol{E_\mu}, \mathbf{sin}\boldsymbol{\theta})$ are independent variable ➔

$$\boxed{\frac{\mathrm{d}^3\Gamma_{\leftrightarrow}^{D/M}}{\mathrm{d}E_\mu^2\mathrm{d}\sin\theta} = \frac{2\sqrt{E_\mu^2-m_\mu^2}}{(4\pi)^6 m_B E_\mu}\left(\left(\frac{m_B}{2}-E_\mu\right)^2-m_\nu^2\right)\langle|\mathscr{M}_{\leftrightarrow}^{D/M}|^2\rangle,}$$

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, w/ B2B muons

Appendix B: Expressions for the various Σ_{ij} and Δ_{ij} terms

The Δ_{ij} terms appearing in Eq. (47) are given by

$$\Delta_{aa} = -16\left(m_B - 2E_\mu\right)^2\left(\left(m_\mu^2 - E_\mu^2\right)\cos^2\theta - E_\mu^2\right), \tag{B1}$$

$$\Delta_{bb} = -4\,m_B^4\left(m_B - 2E_\mu\right)^2\left(\left(m_\mu^2 - E_\mu^2\right)\cos^2\theta - E_\mu^2\right), \tag{B2}$$

$$\Delta_{cc} = -8\,m_\mu^2\left(m_\mu^2 - E_\mu^2\right)m_B^2\left(m_B - 2E_\mu\right)^2\sin^2\theta, \tag{B3}$$

$$\Delta_{pp} = -4\,m_\mu^4\left(m_B - 2E_\mu\right)^2\left(\left(m_\mu^2 - E_\mu^2\right)\cos^2\theta - E_\mu^2\right), \tag{B4}$$

$$\Delta_{mm} = 4\,m_\mu^2\left(m_B - 2E_\mu\right)^2\left(\left(m_\mu^2 - E_\mu^2\right)\left(m_B^2 - m_\mu^2\right)\cos^2\theta + E_\mu\left(E_\mu m_B^2 - 2m_\mu^2 m_B + E_\mu m_\mu^2\right)\right), \tag{B5}$$

$$\Delta_{ab} = -16\,m_B^2\left(m_B - 2E_\mu\right)^2 \times\left(\left(m_\mu^2 - E_\mu^2\right)\cos^2\theta - E_\mu^2\right), \tag{B6}$$

$$\Delta_{ap} = 16\,m_\mu^4\left(m_B - 2E_\mu\right)^2, \tag{B7}$$

$$\Delta_{bp} = 8\,m_\mu^4 m_B^2\left(m_B - 2E_\mu\right)^2, \tag{B8}$$

$$\Delta_{am} = 16\,m_\mu^2\left(m_B - 2E_\mu\right)^2\left(E_\mu m_B - m_\mu^2\right), \tag{B9}$$

$$\Delta_{bm} = 8\,m_\mu^2 m_B^2\left(m_B - 2E_\mu\right)^2\left(E_\mu m_B - m_\mu^2\right), \tag{B10}$$

$$\Delta_{cm} = 8\,m_\mu^2 m_B^2\left(E_\mu^2 - m_\mu^2\right)\left(m_B - 2E_\mu\right)^2\sin^2\theta, \tag{B11}$$

$$\Delta_{pm} = 8\,m_\mu^4\left(m_B - 2E_\mu\right)^2 \times\left(\left(m_\mu^2 - E_\mu^2\right)\cos^2\theta + E_\mu\left(m_B - E_\mu\right)\right), \tag{B12}$$

and the Σ_{ij} terms are given by,

$$\Sigma_{aa} = -32\,E_\mu\sqrt{E_\mu^2 - m_\mu^2}\left(m_B - 2E_\mu\right)^2, \tag{B13}$$

$$\Sigma_{bb} = -8\,m_B^4 E_\mu\sqrt{E_\mu^2 - m_\mu^2}\left(m_B - 2E_\mu\right)^2, \tag{B14}$$

$$\Sigma_{pp} = 8\,E_\mu m_\mu^4\sqrt{E_\mu^2 - m_\mu^2}\left(m_B - 2E_\mu\right)^2, \tag{B15}$$

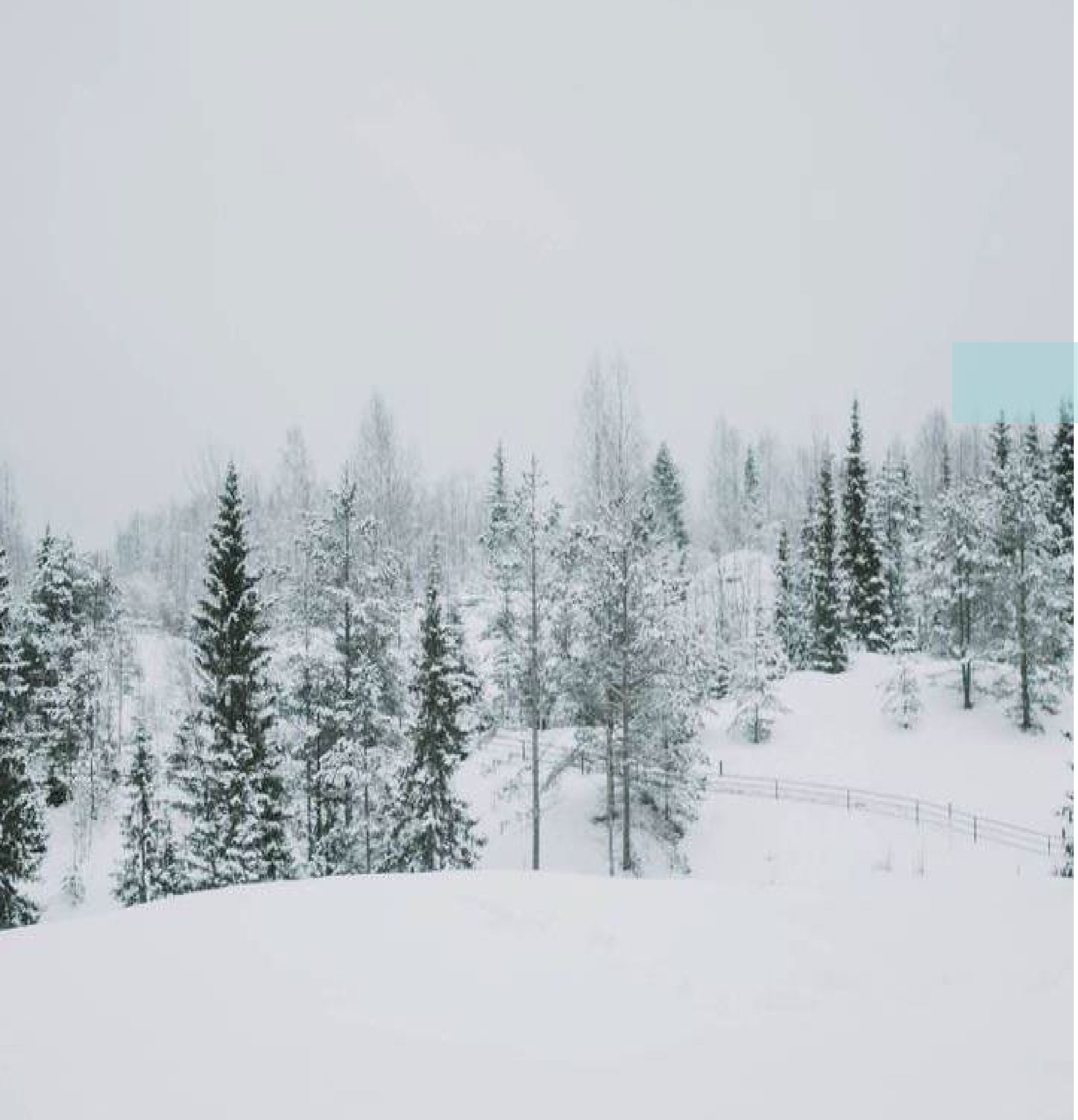
$$\Sigma_{mm} = -8\,m_\mu^4\sqrt{E_\mu^2 - m_\mu^2}\left(m_B - 2E_\mu\right)^2\left(m_B - E_\mu\right), \tag{B16}$$

$$\Sigma_{ab} = -32\,m_B^2 E_\mu\sqrt{E_\mu^2 - m_\mu^2}\left(m_B - 2E_\mu\right)^2, \tag{B17}$$

$$\Sigma_{am} = -16\,m_\mu^2 m_B\sqrt{E_\mu^2 - m_\mu^2}\left(m_B - 2E_\mu\right)^2, \tag{B18}$$

$$\Sigma_{bm} = -8\,m_\mu^2 m_B^3\sqrt{E_\mu^2 - m_\mu^2}\left(m_B - 2E_\mu\right)^2, \tag{B19}$$

$$\Sigma_{pm} = 8\,m_\mu^4\sqrt{E_\mu^2 - m_\mu^2}\left(m_B - 2E_\mu\right)^3. \tag{B20}$$



Extra:

Dirac vs. Majorana

P. Dirac vs. E. Majorana in real life

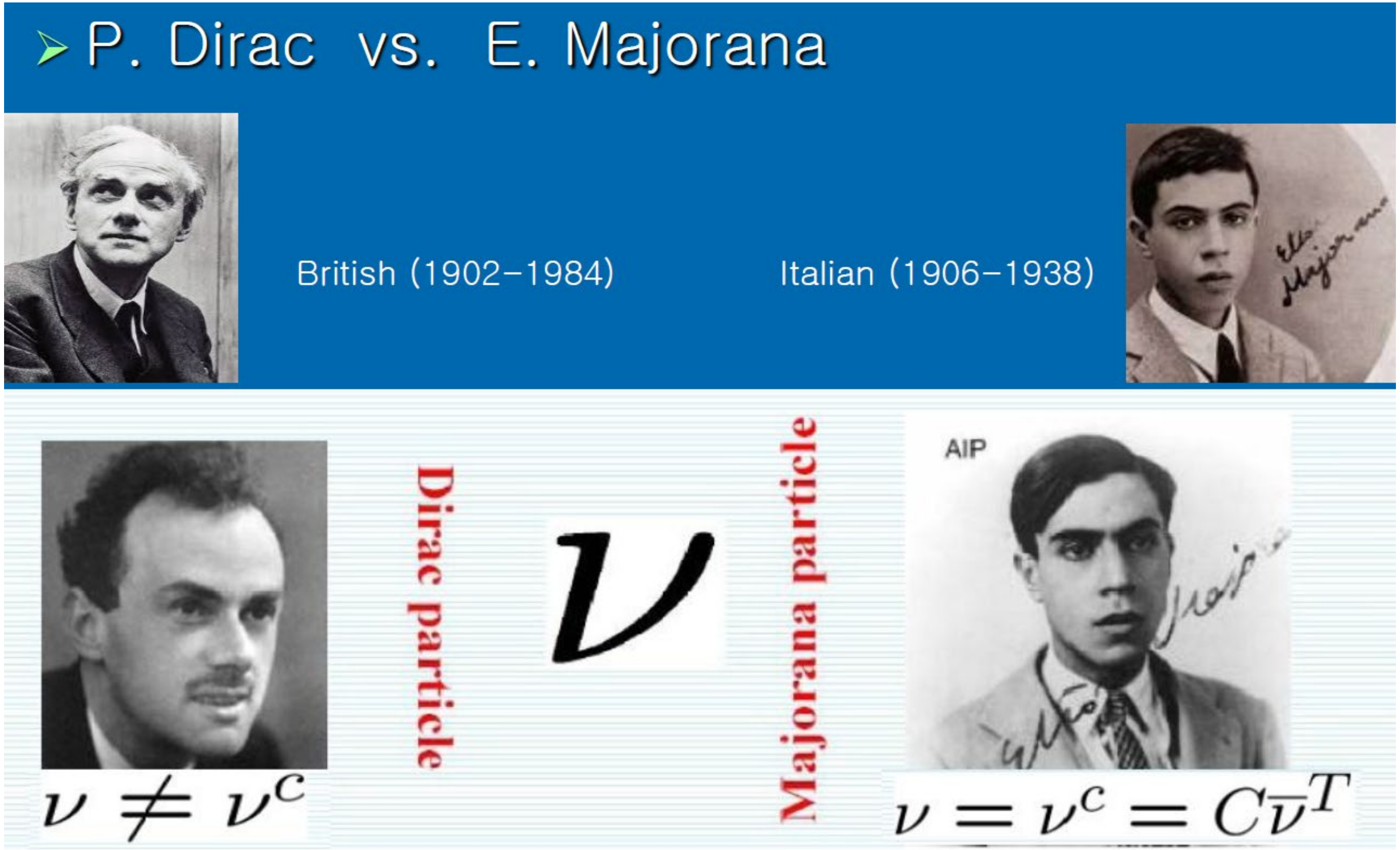
Mass generation

Lepton number violation $\Delta L = 2$

CP Violation

Quantum statistics

Dirac vs. Majorana (0) ➔ Real life



Nature of sub-eV active neutrino : Dirac or Majorana ?

Neutrinos are spin-$\frac{1}{2}$ fermions $\Longrightarrow$ they are described by 4-component Dirac spinors $\psi(x)$ which satisfy the Dirac equation ($\hbar = 1 = c$)

$$(i\gamma^{\mu}\partial_{\mu} - m_{\nu}\mathbf{I})\,\psi(x) = 0, \qquad \{\gamma^{\mu}, \gamma^{\nu}\} = 2\,g^{\mu\nu}\,\mathbf{I}, \text{ with } \mu, \nu = 0, 1, 2, 3.$$

- $m_{\nu} = 0 \Longrightarrow$ 4-component solution $\longrightarrow$ 2-component one – *Weyl basis*.
- For $m_{\nu} \neq 0$, *complex* 4-component solutions allowed – *Dirac basis*.
- For $m_{\nu} \neq 0$, one can choose γ^{μ} such that $\psi(x) = \psi^{*}(x)$, i.e. have a *real* solution – *Majorana basis*.

As neutrinos are truly neutral, $\nu = \overline{\nu}$ is possible, as suggested by Majorana.

Models for neutrino mass mostly prefer Majorana neutrinos due to theoretical elegance, but the nature must be resolved experimentally.

- For particles with *non-zero mass* handedness is not a good quantum number: chirality is not conserved & helicity is not Lorentz invariant.

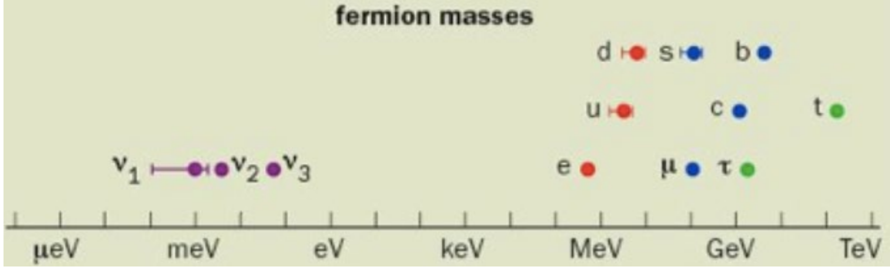
Dirac vs. Majorana (1) ➔ Mass generation

- *Assumption:* ν_R exists.
- *Lagrangian:*

$$\mathscr{L}^{D}_{\text{mass}} = -m^{D}_{\nu}\left(\overline{\nu_R}\,\nu_L + \overline{\nu_L}\,\nu_R\right).$$

- *Assumption:* neutrino $\equiv$ anti-neutrino.
- *Lagrangian:*

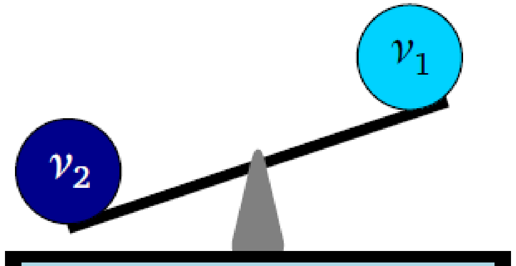
$$\mathscr{L}^{M}_{\text{mass}} = \tfrac{1}{2} m^{M}_{\nu}\left(\overline{\nu^{C}_{L}}\,\nu_L + \overline{\nu_L}\,\nu^{C}_{L}\right).$$



- *Assumptions:* $m^{L}_{\nu} = 0$ and $m^{D}_{\nu} \ll m^{R}_{\nu}$.
- *Lagrangian:* $\mathscr{L}^{D+M}_{\text{mass}} = \tfrac{1}{2} m^{R}_{\nu}\left(\overline{\nu^{C}_{R}}\,\nu_R\right) - m^{D}_{\nu}\left(\overline{\nu_R}\,\nu_L\right) + \text{H.c.} = \tfrac{1}{2}\overline{N^{C}_{L}} M N_L + \text{H.c.},$

where $N_L = \begin{pmatrix} \nu_L \\ \nu^{C}_{R} \end{pmatrix}$ and $M = \begin{pmatrix} 0 & m^{D}_{\nu} \\ m^{D}_{\nu} & m^{R}_{\nu} \end{pmatrix}$ is the mass matrix.

$$\Longrightarrow m_1 \approx -\frac{\left(m^{D}_{\nu}\right)^2}{m^{R}_{\nu}} \text{ and } m_2 \approx m^{R}_{\nu}.$$



As ν_2 gets heavier, ν_1 become lighter.

[PM,PLB67(1977)421]

- *Challenges:*
  - To find the heavy ν_2 experimentally.
  - To prove that both the light ν_1 and heavy ν_2 are Majorana neutrinos.

Dirac vs. Majorana (2) ➔ Lepton number violation

$L = 1$

	L_e	L_μ	L_τ
(ν_e, e^-)	+1	0	0
(ν_μ, μ^-)	0	+1	0
(ν_τ, τ^-)	0	0	+1

$L = -1$

	L_e	L_μ	L_τ
$(\bar{\nu}_e, e^+)$	−1	0	0
$(\bar{\nu}_\mu, \mu^+)$	0	−1	0
$(\bar{\nu}_\tau, \tau^+)$	0	0	−1

$$\nu \neq \nu^c$$

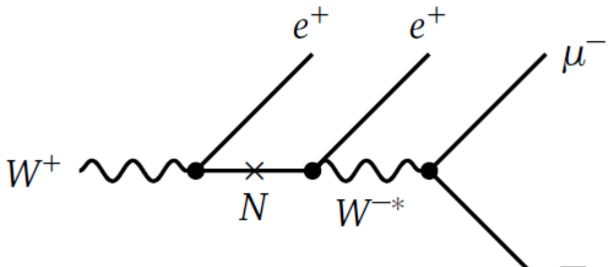
For Dirac nu, Lepton number (LN) is a good QN.

$$\nu = \nu^c = C\overline{\nu}^T$$

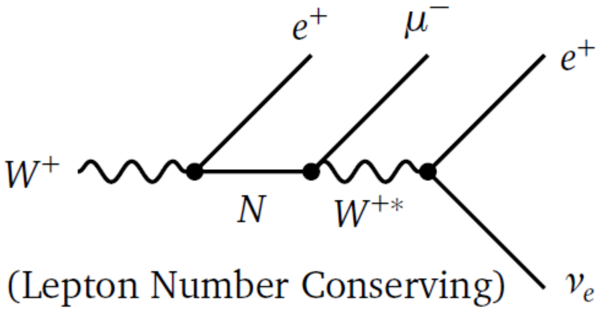
For Majorana nu, LN is not a good QN (quantum number)

LN may conserve or not conserve for Majorana nu

C. Dib, C.S. Kim, arXiv:1509.05981 (PRD **92**, 093009, 2015);
C. Dib, C.S. Kim, K. Wang, J. Zhang, arXiv:1605.01123 (PRD **94**, 013005, 2016)

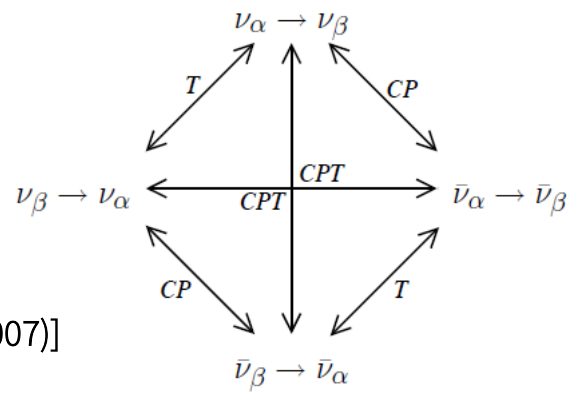


(Lepton Number Violating)

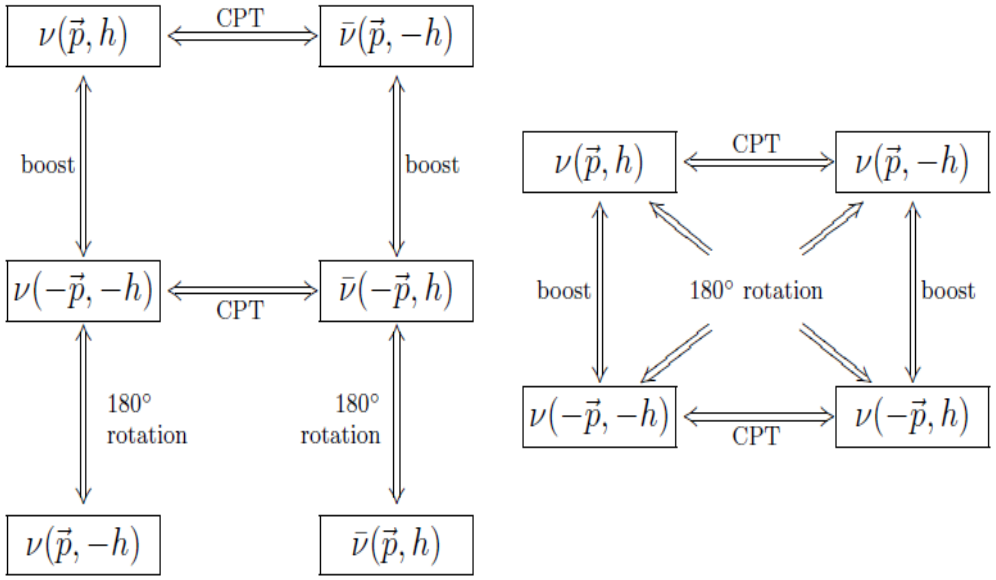


(Lepton Number Conserving)

Dirac vs. Majorana (3) ➔ CP Violation



[Guinti & Kim, Oxford Press (2007)]



(a) Dirac. (b) Majorana.

For Dirac nu :

$$\psi \xrightarrow{\mathrm{C}} \xi_{\mathrm{C}}\, \mathcal{C}\, \overline{\psi}^{T} \xrightarrow{\mathrm{C}} |\xi_{\mathrm{C}}|^2\, \psi\,,$$

$$\psi(x) \xrightarrow{\mathrm{P}} \xi_{\mathrm{P}}\, \gamma^0\, \psi(x) \xrightarrow{\mathrm{P}} \xi_{\mathrm{P}}^2\, \psi(x)\,.$$

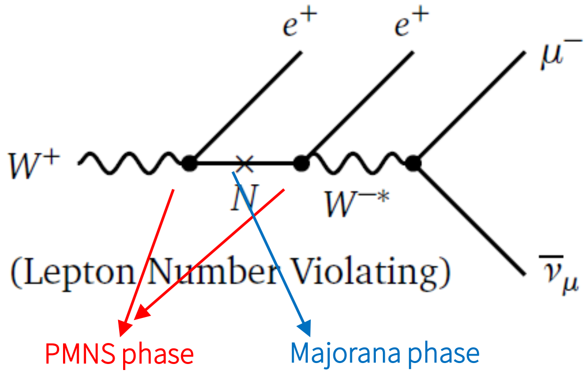
$$\Longrightarrow \qquad |\xi_{\mathrm{CP}}|^2 = 1\,,$$

If more than 2 families, CPV comes from mixing among them, ie. PMNS mixing phase.

For Majorana nu :

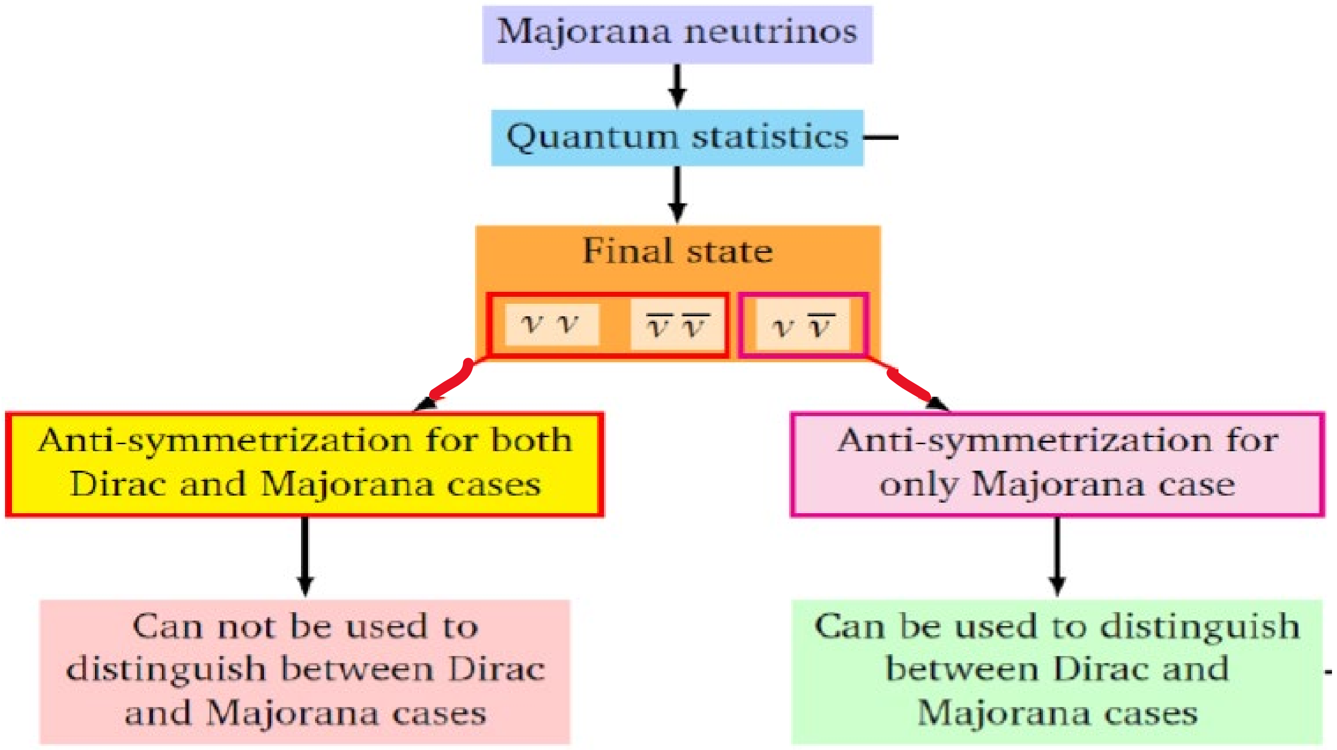
$$\psi = \mathcal{C}\, \overline{\psi}^{T} \qquad \Longrightarrow \qquad \xi_{\nu}^{\mathrm{CP}} = \pm i\,.$$

In addition to PMNS mixing phase, (n_family - 1) CPV phases.



Dirac vs. Majorana (4) ➔ Quantum statistics

C. S. Kim, M. Murthy, D. Sahoo, PRD105(2022)



For Dirac Fermion :

$$\mathcal{M} = g \sum_{s_1,s_2} \bar{u}_{s_1}(p_1) F v_{s_2}(p_2),$$

$$\bar{\nu}\gamma^\mu(1-\gamma_5)\nu$$

For Majorana Fermion :

$$\mathcal{M} = g \sum_{s_1,s_2} (\bar{u}_{s_1}(p_1) F v_{s_2}(p_2) - \bar{u}_{s_2}(p_2) F v_{s_1}(p_1)),$$

$$\bar{\nu}\gamma^\mu(1-\gamma_5)\nu - \bar{\nu}\gamma^\mu(1+\gamma_5)\nu = -2\bar{\nu}\gamma^\mu\gamma_5\nu$$

[Reference: P. B. Pal, Am. J. Phys. **79**, 485-498 (2011).]

[Reference: J. F. Nieves and P. B. Pal, Phys. Rev. D **32**, 1849-1852 (1985).]

[Reference: E. Ma and J. T. Pantaleone, Phys. Rev. D **40**, 2172-2176 (1989).]

[Reference: T. Chhabra and P. R. Babu, Phys. Rev. D **46**, 903-909 (1992).]

[0] B.Kayser,PRD26(1982)
[1] S.P.Rosen,PRL48(1982); CSK etal. EPJC83(2023)
[2] T.Chabra,PR.Babu,PRD46(1992); CSK etal. PRD105(2022)