# ONE FLAVOR SYMMETRY

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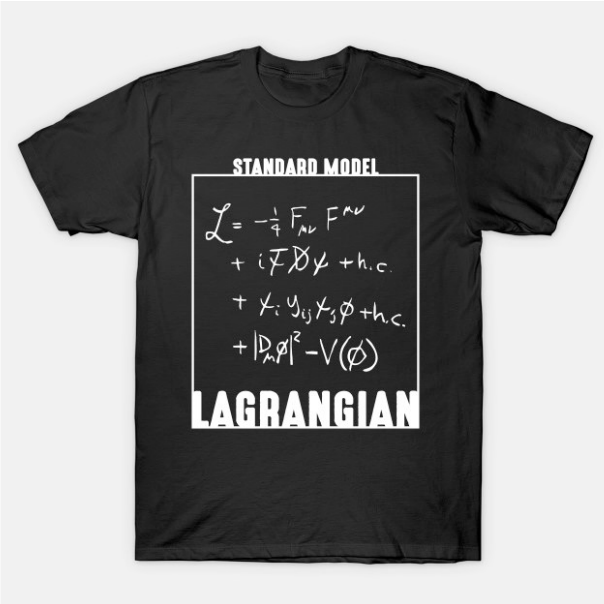
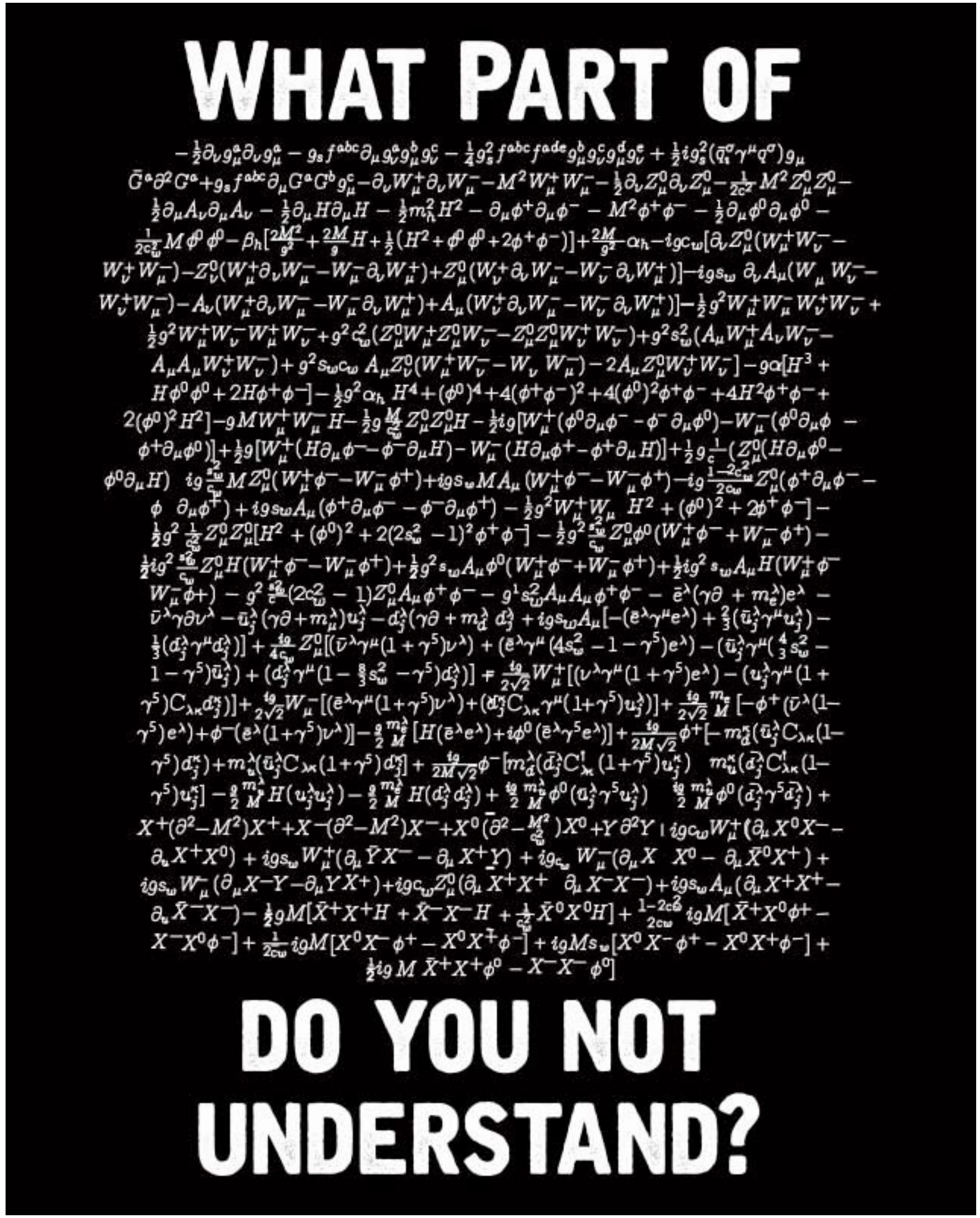
*PLB788 (2018); EPJC81 (2021); arXiv?*

*June 13, 2023 — Reunión de la DPyC-SMF*


STANDARD MODEL
L = -¼ F_μν F^μν
+ iψ̄Dψ + h.c.
+ ψ_i y_ij ψ_j φ + h.c.
+ |D_μ φ|² - V(φ)
LAGRANGIAN


WHAT PART OF
DO YOU NOT UNDERSTAND?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# HOW TO GO BSM?

- Many ways to go BSM
- Usually:  add symmetries, add particles, add interactions
- All of the above
- Messy…
- I will concentrate on masses and mixings
- And the possibility of dark matter (and perhaps leptogenesis…)

# SOME ASPECTS OF THE FLAVOUR PROBLEM

- Quark and charged lepton masses very different, very hierarchical

$$m_u : m_c : m_t \sim 10^{-6} : 10^{-3} : 1$$

$$m_d : m_s : m_b \sim 10^{-4} : 10^{-2} : 1$$

$$m_e : m_\mu : m_\tau \sim 10^{-5} : 10^{-2} : 1$$

- Neutrino masses unknown, only difference of squared masses.
- Type of hierarchy (normal or inverted) also unknown
- Higgs sector under study

- Quark mixing angles

$$\theta_{12} \approx 13.0^o$$
$$\theta_{23} \approx 2.4^o$$
$$\theta_{13} \approx 0.2^o$$

?

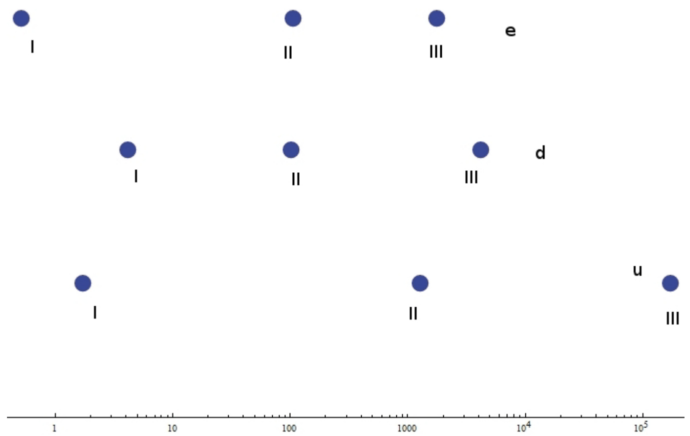
- Neutrino mixing angles

$$\Theta_{12} \approx 33.8^\circ$$
$$\Theta_{23} \approx 48.6^\circ$$
$$\Theta_{13} \approx 8.6^\circ$$

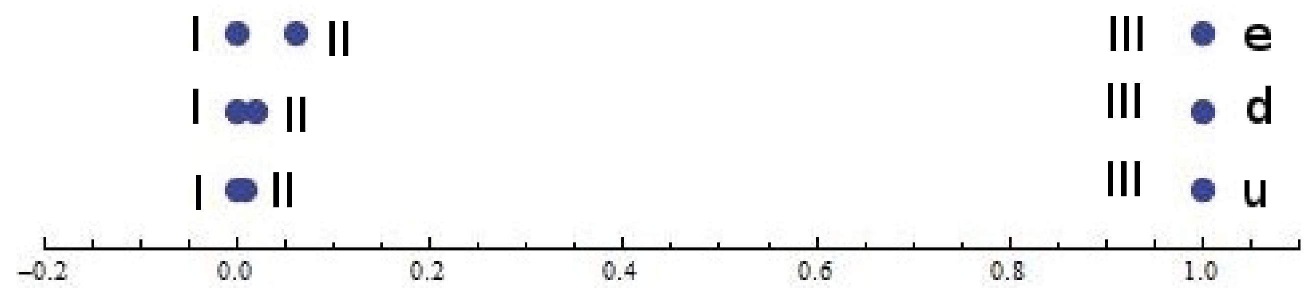
- Small mixing in quarks, large mixing in neutrinos. Very different
- Is there an underlying symmetry?

## HOW DO WE CHOOSE A FLAVOUR SYMMETRY?

- Several ways:
- Look for inspiration in a high energy extension of SM, i.e. strings or GUTs
- Look at low energy phenomenology
- At some point they should intersect…
- In here:
  - Find the smallest flavour symmetry suggested by data
  - Explore how generally it can be applied (universally)
  - Follow it to the end
  - Compare it with the data



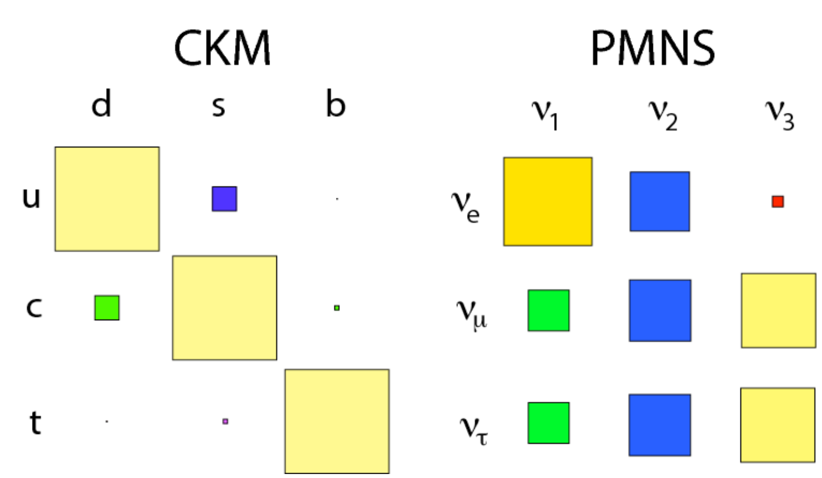

*Plot of mass ratios*




*Logarithmic plot of quark masses*

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} \approx \begin{bmatrix} 0.974 & 0.225 & 0.003 \\ 0.225 & 0.973 & 0.041 \\ 0.009 & 0.040 & 0.999 \end{bmatrix},$$

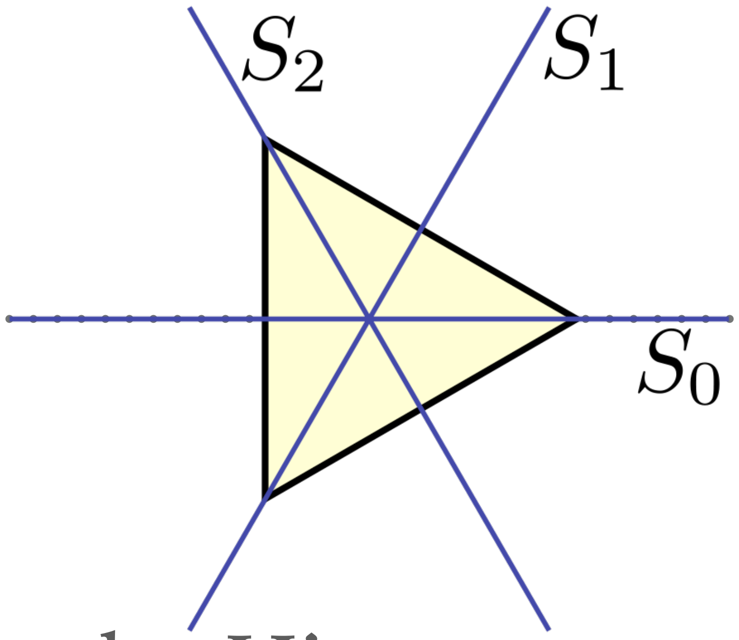
Suggests a 2⊕1 structure



# 3HDM

- Without symmetry $\Longrightarrow$ 54 real parameters in potential
- Complemented with additional symmetry(ies)
- Studies started in the 70's, hope to find global symmetry that explains the mass and mixing patterns
- The first symmetries to be added were the permutational groups S3 and S4
- Different modern versions of these models exist

# 3HDM WITH S3




- Low-energy model
- Extend the concept of flavour to the Higgs sector by adding two more eW doublets
- Add symmetry: permutation symmetry of three objects, symmetry operations (reflections and rotations) that leave an equilateral triangle invariant
- 3HDM with symmetry S3:
  8 couplings in the Higgs potential

# *A sample of S3 models*

S. Pakvasa et al, Phys. Lett. 73B, 61 (1978)

E. Derman, Phys. Rev. D19, 317 (1979)

D. Wyler, Phys. Rev. D19, 330 (1979)

R. Yahalom, Phys. Rev. D29, 536 (1984)

Y. Koide, Phys. Rev. D60, 077301 (1999)

A. Mondragon et al, Phys. Rev. D59, 093009, (1999)

J. Kubo, A. Mondragon, et al, Prog. Theor. Phys. 109, 795 (2003)

J. Kubo et al, Phys. Rev. D70, 036007 (2004)

S. Chen, M. Frigerio and E. Ma, Phys. Rev. D70, 073008 (2004)

A. Mondragon et al, Phys. Rev. D76, 076003, (2007)

S. Kaneko et al, hep-ph/0703250, (2007)

S. Chen et al, Phys. Rev. D70, 073008 (2004)

T. Teshima et al, Phys.Rev. D84 (2011) 016003 Phys.Rev. D85 105013 (2012)

F. Gonzalez Canales, A&M. Mondragon Fort. der Physik 61, Issue 4-5 (2013)

H.B. Benaoum, Phys. RevD.87.073010 (2013)

E. Ma and B. Melic, arXiv:1303.6928

F. Gonzalez Canales, A. &M Mondragon, U. Saldaña, L. Velasco, arXiv:1304.6644

R. Jora et al, Int.J.Mod.Phys. A28 (2013),1350028

A. E. Cárcamo Hernández, E. Cataño Mur, R. Martinez, Phys.Rev. D90 (2014) no.7, 073001

A.E. Cárcamo, I. de Medeiros E. Schumacheet, Phys.Rev. D93 (2016) no.1, 016003

A.E. Cárcamo, R. Martinez, F. Ochoa, Eur.Phys.J. C76 (2016)

D Das, P Pal, Pays Rev D98 (2018)

C Espinoza, E Garcés, MM, H. Reyes, PLB 788 (2019)

JC Gómez Izquierdo, MM, EPJC 79 (2019)

O. Felix-Beltran, M.M., et al, J.Phys.Conf.Ser. 171, 012028 (2009)

A. Dicus, S Ge, W Repko, Phys. Rev D82 (2010)

D. Meloni et al, Nucl. Part. Phys. 38 015003, (2011)

G. Bhattacharyya et al, Phys. Rev. D83, 011701 (2011)

D. Meloni, JHEP 1205 (2012) 124

S. Dev et al, Phys.Lett. B708 (2012) 284-289

S. Zhou, Phys.Lett. B704 (2011) 291-295

D. Meloni et al, Nucl. Part. Phys. 38 015003, (2011)

E. Ma and B. Melic, Phys.Lett. B725 (2013)

E. Barradas et al, 2014

P. Das et al, PhyrRev D89 (2014,) 2016

ZZ Zhing, D Zhang JHEP 03 2019)

S Pramanick, Phys Rev D100 (2019)

M. Gómez-Bock, A. Pérez, MM, EPJC81 (2021)

*Just a sample, there are many more…*
*I apologize for those not included*

# S3

- Smallest non-Abelian discrete group
- Has irreducible representations, 2, $1_S$ and $1_A$
- We add three right-handed neutrinos to implement the see-saw mechanism
- We apply the symmetry "universally" to quarks, leptons and Higgs-es
  - First two families in the doublet
  - Third family in symmetric singlet
- Three sectors related, we treat them simultaneously

# PREDICTIONS, ADVANTAGES?

- Possible to reparametrize mixing matrices in terms of mass ratios, successfully
- CKM has NNI and Fritzsch textures
- PMNS → fix one mixing angle, predictions for the other two within experimental range
- Reactor mixing angle $\theta_{13} \neq 0$
- Some FCNCs suppressed by symmetry
- Higgs potential has 8 couplings
- Underlying symmetry in quark, leptons and Higgs → residual symmetry of a more fundamental one?
- Lots of Higgses: 3 neutral, 4 charged, 2 pseudoscalars
- Further predictions will come from Higgs sector: decays, branching ratios

*A. Mondragón, M. M., F. González, E. Peinado, U. Saldaña, O. Félix, E. Rodríguez, A. Pérez, H. Reyes...; Das, Dey et al; Teshima et al;*

# FERMION MASSES

- The Lagrangian of the model

$$\mathcal{L}_Y = \mathcal{L}_{Y_D} + \mathcal{L}_{Y_U} + \mathcal{L}_{Y_E} + \mathcal{L}_{Y_\nu},$$

- The general form of the fermion mass matrices in the symmetry adapted basis is

$$\mathbf{M} = \begin{pmatrix} m_1 + m_2 & m_2 & m_5 \\ m_2 & m_1 - m_2 & m_5 \\ m_4 & m_4 & m_3 \end{pmatrix}.$$

where $m_{1,3} = Y_{1,3}v_3$ and $m_{1,2,4,5} = Y_{1,2,4,5}$ ($v_1$ or $v_2$)

# QUARKS

3HDM: $G_{SM} \otimes S_3$

| | $\psi_L^f$ | $\psi_R^f$ | Mass matrix | Possible mass textures | |
|---|---|---|---|---|---|
| $A$ | $\mathbf{2}$, $1_{\mathbf{S}}$ | $\mathbf{2}$, $1_{\mathbf{S}}$ | $\begin{pmatrix} \mu_1^f + \mu_2^f & \mu_4^f & \mu_6^f \\ \mu_4^f & \mu_1^f - \mu_2^f & \mu_7^f \\ \mu_8^f & \mu_9^f & \mu_3^f \end{pmatrix}$ | $\begin{pmatrix} 0 & \mu_2^f sc\left(3-t^2\right) & 0 \\ \mu_2^f sc\left(3-t^2\right) & -2\mu_2^f c^2\left(1-3t^2\right) & \mu_7^f/c \\ 0 & \mu_7^{f*}/c & \mu_3^f - \mu_1^f - \mu_2^f c^2(1-3t^2) \end{pmatrix}$ | |
| $A^{'}$ | | | | $\begin{pmatrix} 0 & \frac{2}{\sqrt{3}}\mu_2^f & 0 \\ \frac{2}{\sqrt{3}}\mu_2^f & 0 & \frac{2}{\sqrt{3}}\mu_7^f \\ 0 & \frac{2}{\sqrt{3}}\mu_9^f & \mu_3^f - \mu_1^f \end{pmatrix}$ | *NNI* |
| $B$ | $\mathbf{2}$, $1_{\mathbf{A}}$ | $\mathbf{2}$, $1_{\mathbf{A}}$ | $\begin{pmatrix} \mu_1^f + \mu_2^f & \mu_4^f & \mu_7^f \\ \mu_4^f & \mu_1^f - \mu_2^f & -\mu_6^f \\ -\mu_9^f & \mu_8^f & \mu_3^f \end{pmatrix}$ | $\begin{pmatrix} 0 & -\mu_4^f c^2\left(1-3t^2\right) & 0 \\ -\mu_4^f c^2\left(1-3t^2\right) & 2\mu_4^f sc\left(3-t^2\right) & -\mu_6^f/c \\ 0 & -\mu_6^{f*}/c & \mu_3^f - \mu_1^f + \vert\mu_4^f sc(3-t^2) \end{pmatrix}$ | |
| $B^{'}$ | | | | $\begin{pmatrix} 0 & -2\mu_4^f & 0 \\ -2\mu_4^f & 0 & -2\mu_6^f \\ 0 & 2\mu_8^f & \mu_3^f - \mu_1^f \end{pmatrix}$ | *NNI* |

Table 2: Mass matrices in $S_3$ family models with three Higgs $SU(2)_L$ doublets: $H_1$ and $H_2$, which occupy the $S_3$ irreducible representation $\mathbf{2}$, and $H_S$, which transforms as $1_{\mathbf{S}}$ for the cases when both the left- and right-handed fermion fields are in the same assignment. The mass matrices shown here follow a normal ordering of their mass eigenvalues $(m_1^f, m_2^f, m_3^f)$. We have denoted $s = \sin\theta$, $c = \cos\theta$ and $t = \tan\theta$. The third column of this table corresponds to the general case, while the fourth column to a case where we have rotated the matrix to a basis where the elements $(1,1)$, $(1,3)$ and $(3,1)$ vanish. The primed cases, A' or B', are particular cases of the unprimed ones, A or B, with $\theta = \pi/6$ or $\theta = \pi/3$, respectively.

*Mass matrices reproduce the NNI or the Fritzsch forms (rotation + shift)*

*F. González et al, Phys.Rev. D88 (2013) 096004*

# HIGGS SECTOR – TESTS FOR THE MODEL

*General Potential:*

$$
\begin{aligned}
V \;=\;& \mu_1^2\left(H_1^\dagger H_1 + H_2^\dagger H_2\right) + \mu_0^2\left(H_s^\dagger H_s\right) + a\left(H_s^\dagger H_s\right)^2 + b\left(H_s^\dagger H_s\right)\left(H_1^\dagger H_1 + H_2^\dagger H_2\right) \\
&+c\left(H_1^\dagger H_1 + H_2^\dagger H_2\right)^2 + d\left(H_1^\dagger H_2 - H_2^\dagger H_1\right)^2 + e f_{ijk}\left(\left(H_s^\dagger H_i\right)\left(H_j^\dagger H_k\right) + h.c.\right) \\
&+f\left\{\left(H_s^\dagger H_1\right)\left(H_1^\dagger H_s\right) + \left(H_s^\dagger H_2\right)\left(H_2^\dagger H_s\right)\right\} + g\left\{\left(H_1^\dagger H_1 - H_2^\dagger H_2\right)^2 + \left(H_1^\dagger H_2 + H_2^\dagger H_1\right)^2\right\} \\
&+h\left\{\left(H_s^\dagger H_1\right)\left(H_s^\dagger H_1\right) + \left(H_s^\dagger H_2\right)\left(H_s^\dagger H_2\right) + \left(H_1^\dagger H_s\right)\left(H_1^\dagger H_s\right) + \left(H_2^\dagger H_s\right)\left(H_2^\dagger H_s\right)\right\} \qquad (1)
\end{aligned}
$$

*Derman and Tsao (1979); Sugawara and Pawasa (1978); Kubo et al (2004); Felix-Beltrán, Rodríguez-Jáuregui, M.M (2009), Das and Dey (2014), Barradas et al (2014), Costa, Ogreid, Osland and Rebelo (2016), etc*

$$v_1 = v\cos\varphi\sin\theta, \quad v_2 = v\sin\varphi\sin\theta \quad v_3 = v\cos\theta.$$

$$\tan\varphi = 1/\sqrt{3} \;\Rightarrow\; \sin\varphi = \frac{1}{2} \;\;\&\;\; \cos\varphi = \frac{\sqrt{3}}{2}$$

$$\tan\theta = \frac{2v_2}{v_3} \;\Rightarrow\; \sin\theta = \frac{2v_2}{v} \;\;\&\;\; \cos\theta = \frac{v_3}{v}$$

- *The minimum of potential can be parameterised in spherical coordinates, two angles and v*
- *Minimisation fixes* $v_1^2 = 3v_2^2$
- *e = 0 massless scalar, residual continuous S2 symmetry*
- *Conditions for normal vacuum already studied, also for CP breaking ones*
  *Felix-Beltrán, Rodríguez-Jáuregui, M.M (2007); Barradas et al (2015); Costa et al (2016)*

# STABILITY CONDITIONS

$$\lambda_8 > 0$$
$$\lambda_1 + \lambda_3 > 0$$
$$\lambda_5 > -2\sqrt{(\lambda_1 + \lambda_3)\lambda_8}$$
$$\lambda_5 + \lambda_6 - 2|\lambda_7| > \sqrt{(\lambda_1 + \lambda_3)\lambda_8}$$
$$\lambda_1 - \lambda_2 > 0$$
$$\lambda_1 + \lambda_3 + \mid 2\lambda_4 \mid + \lambda_5 + 2\lambda_7 + \lambda_8 > 0$$
$$\lambda_{13} > 0$$
$$\lambda_{10} > -2\sqrt{(\lambda_1 + \lambda_3)\lambda_{13}}$$
$$\lambda_{10} + \lambda_{11} - 2|\lambda_{12}| > \sqrt{(\lambda_1 + \lambda_3)\lambda_{13}}$$
$$\lambda_{14} > -2\sqrt{\lambda_8\lambda_{13}}.$$

Das and Dey (2014)

# UNITARITY CONDITIONS

$$a_1^{\pm} = (\lambda_1 - \lambda_2 + \frac{\lambda_5 + \lambda_6}{2}) \pm \sqrt{(\lambda_1 - \lambda_2 + \frac{\lambda_5 + \lambda_6}{2})^2 - 4[(\lambda_1 - \lambda_2)(\frac{\lambda_5 + \lambda_6}{2}) - \lambda_4^2]}$$
$$a_2^{\pm} = (\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_8) \pm \sqrt{(\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_8)^2 - 4[\lambda_8(\lambda_1 + \lambda_2 + 2\lambda_3) - 2\lambda_7^2]}$$
$$a_3^{\pm} = (\lambda_1 - \lambda_2 + 2\lambda_3 + \lambda_8) \pm \sqrt{(\lambda_1 - \lambda_2 + 2\lambda_3 + \lambda_8)^2 - 4[\lambda_8(\lambda_1 + \lambda_2 + 2\lambda_3) - \frac{\lambda_6^2}{2}]}$$
$$a_4^{\pm} = (\lambda_1 + \lambda_2 + \frac{\lambda_5}{2} + \lambda_7) \pm \sqrt{(\lambda_1 + \lambda_2 + \frac{\lambda_5}{2} + \lambda_7)^2 - 4[(\lambda_1 - \lambda_2)(\frac{\lambda_5}{2} + \lambda_7) - \lambda_4^2]}$$

$$a_5^{\pm} = (5\lambda_1 - \lambda_2 + 2\lambda_3 + 3\lambda_8) \pm \sqrt{(5\lambda_1 - \lambda_2 + 2\lambda_3 + 3\lambda_8)^2 - 4[3\lambda_8(5\lambda_1 - \lambda_2 + 2\lambda_3) - \frac{1}{2}(2\lambda_5 + \lambda_6)^2]}$$
$$a_6^{\pm} = (\lambda_1 + \lambda_2 + 4\lambda_3 + \frac{\lambda_5}{2} + \lambda_6 + 3\lambda_7) \pm ((\lambda_1 + \lambda_2 + 4\lambda_3 + \frac{\lambda_5}{2} + \lambda_6 + 3\lambda_7)^2 - 4[(\lambda_1 + \lambda_2 + 4\lambda_3)(\frac{\lambda_5}{2} + \lambda_6 + 3\lambda_7) - 9\lambda_4^2])^{1/2}$$

$$b_1 = \lambda_5 + 2\lambda_6 - \lambda_7$$
$$b_2 = \lambda_5 - 2\lambda_7$$
$$b_3 = 2(\lambda_1 - 5\lambda_1 - 2\lambda_3)$$
$$b_4 = 2(\lambda_1 - \lambda_1 - 2\lambda_3)$$
$$b_5 = 2(\lambda_1 + \lambda_1 - 2\lambda_3)$$
$$b_6 = \lambda_5 - \lambda_6.$$

# HIGGS MASSES

- After electroweak symmetry breaking (Higgs mechanism) we are left with 9 massive particles

*doesn't couple to gauge bosons: Z2 symmetry*
*massless when e=0, S2 symmetry*

$$
\begin{aligned}
m_{h_0}^2 &= -9ev^2 \sin\theta\cos\theta \\
m_{H_1,H_2}^2 &= (M_a^2 + M_c^2) \pm \sqrt{(M_a^2 - M_c^2)^2 + (M_b^2)^2}
\end{aligned}
$$

$$
\begin{aligned}
M_a^2 &= \left[2(c+g)v^2\sin^2\theta + \frac{3}{2}ev^2\sin\theta\cos\theta\right] \\
M_b^2 &= \left[3ev^2\sin^2\theta + 2(b+f+2h)v^2\sin\theta\cos\theta\right] \\
M_c^2 &= 2av^2\cos^2\theta - \frac{ev^2\tan\theta\sin^2\theta}{2}
\end{aligned}
$$

*H1 or H2 can be the SM Higgs boson*

$$
\begin{aligned}
m_{A_1}^2 &= -v^2\left[2(d+g)\sin^2\theta + 5e\cos\theta\sin\theta + 2h\cos^2\theta\right] \\
m_{A_2}^2 &= -v^2(e\tan\theta + 2h)
\end{aligned}
$$

*Das and Dey (2014)*
*Barradas,Félix, González (2014)*
*Gómez-Bock, MM, Perez-Martínez (2022)*

$$
\begin{aligned}
m_{H_1^\pm}^2 &= -v^2\left[5e\sin\theta\cos\theta + (f+h)\cos^2\theta + 2g\sin^2\theta\right] \\
m_{H_2^\pm}^2 &= -v^2\left[e\tan\theta + (f+h)\right]
\end{aligned}
$$

# RESIDUAL Z2 SYMMETRY

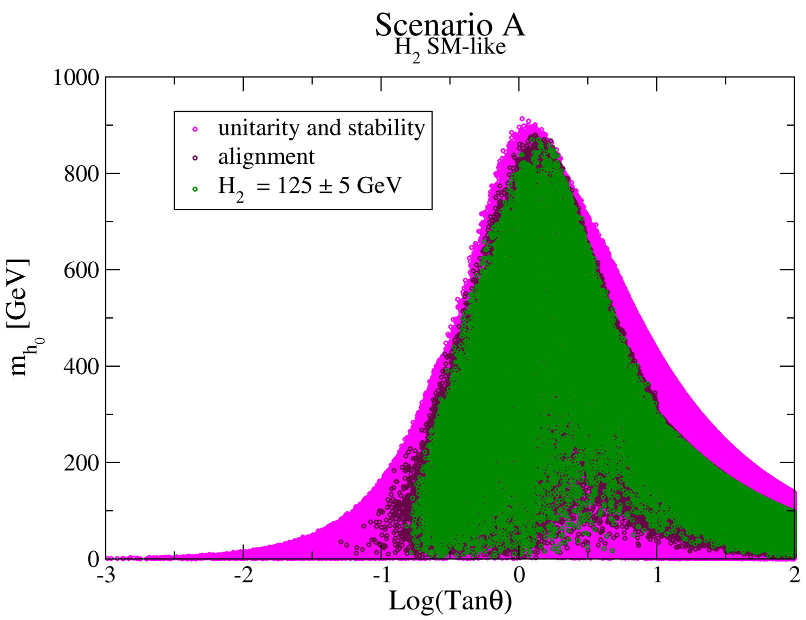
- After eW symmetry breaking, S3 breaks -> residual Z2 symmetry
  Das and Dey (2014), Ivanov (2017)
- h0 decoupled from gauge bosons
- There are 2 “alignment” limits 👀
  - H2 is the SM Higgs → H1 decoupled from gauge bosons
  - H1 is the SM Higgs → H2 decoupled from gauge bosons
    mH2 < mH1
- Z2 parity:

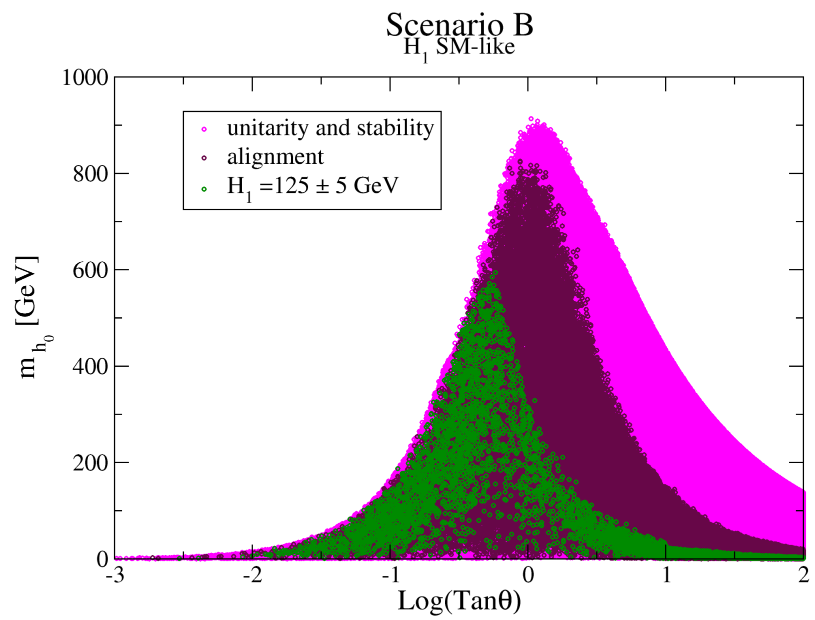
$h_0$, $A_1$, $H_1^\pm$ parity -1,
$H_1$, $H_2$ parity +1
$H_2^\pm$, $A_2$ parity +1

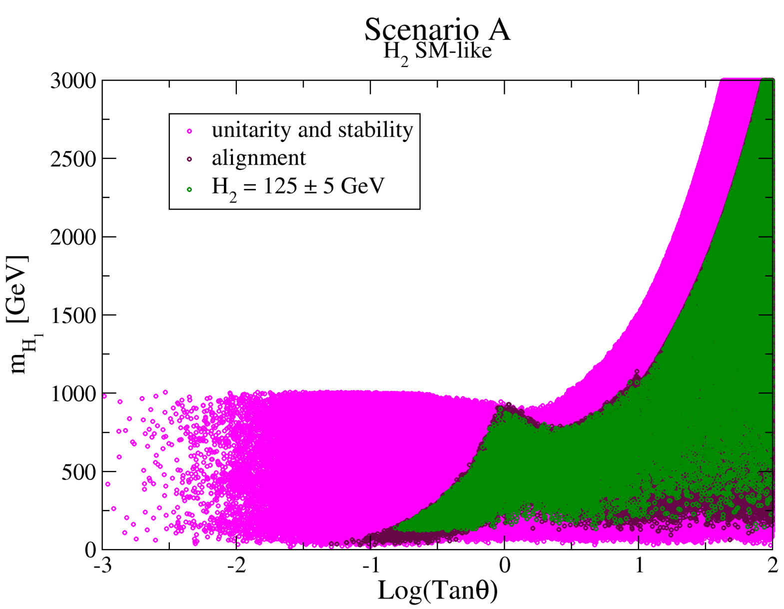
Das and Dey (2014)

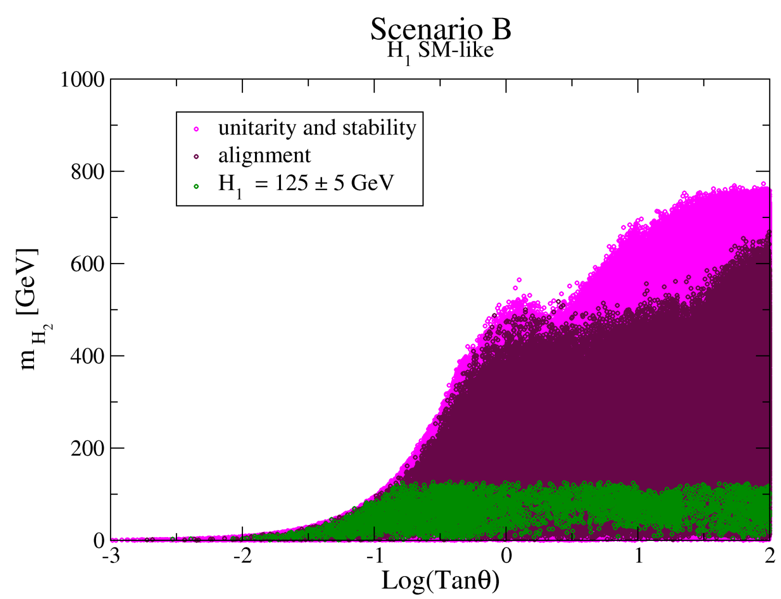
- This forbids certain couplings

# NEUTRAL SCALAR MASSES



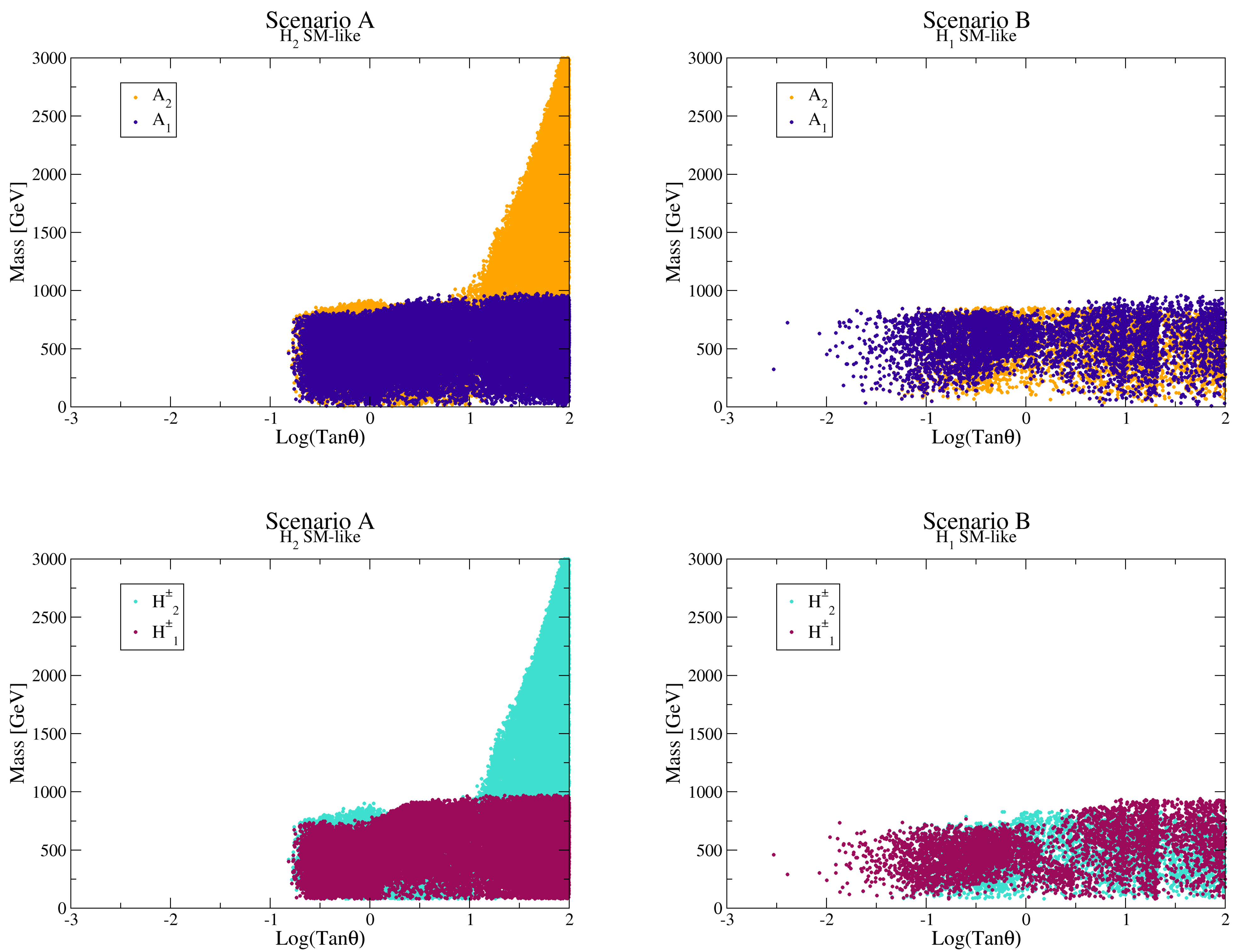



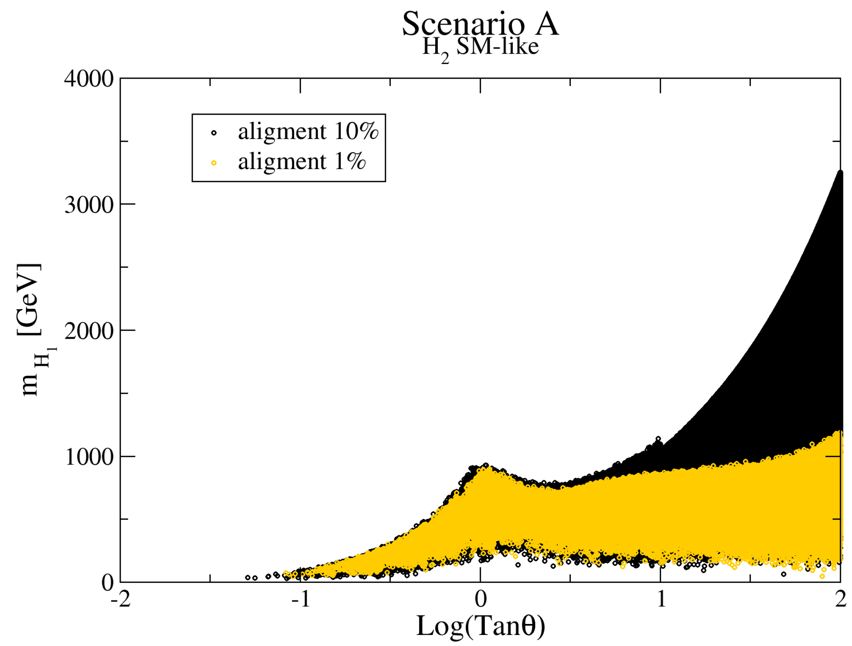

- **Magenta** satisfy stability and unitarity bounds
- **Maroon** satisfy alignment limit at 10%
  → upper bound to the scalar masses
  consistent with Das & Day (2014)
- **Green** restricted to
  A: mH2 = 125±5 GeV
  B: mH1 = 125±5 GeV

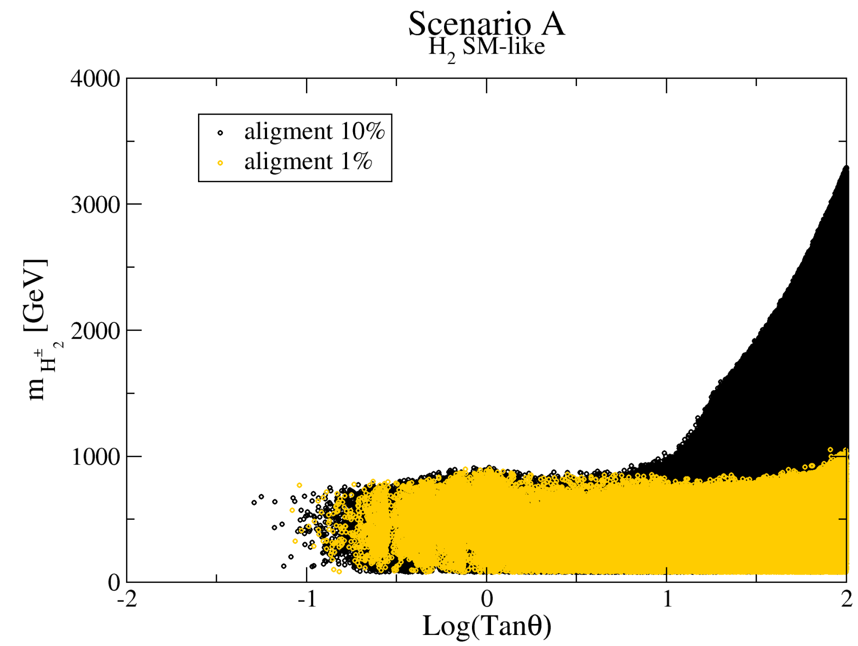
# PSEUDO SCALARS AND CHARGED SCALARS



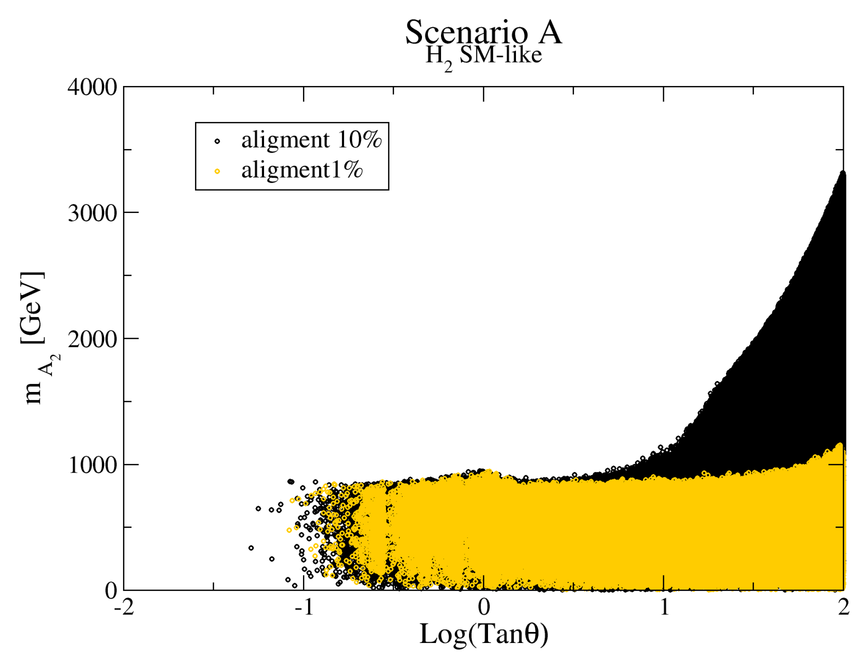

*Points shown pass all constraints*
*we assume conservative limit $mH^{\pm} > 80$ GeV*

# SCENARIO A AT 1% AND 10%




- **Black** alignment limit at 10%
- Yellow satisfy alignment limit at 1% on $(\alpha\text{-}\theta)$




- Experimental limit at 10%
- → upper bound to the scalar masses
- Other masses not affected
- Scenario B not affected

# MASSES — TREE LEVEL

- Scenario A, H2 SM Higgs
  - Upper bound for masses
    mh0 ≲ 900 GeV , mH1 ≲ 3 TeV
    mA1 ≲ 1 TeV, mA2 ≲ 3 TeV
    mH1 ≲ 1 TeV, mH2 ≲ 3 TeV
  - Taking $(\alpha\text{-}\theta)$ 1% lowers mH1, mA2, MH2 ≲ 1 TeV
- Scenario B, H1 SM Higgs
  - Upper bound for masses
    mh0 ≲ 600 GeV, mH1 ≲ 120 GeV (by construction)
    mA1, mA2, mH1, mH2 ≲ 1 TeV
- **Both scenarios allow for a neutral scalar lighter than SM Higgs**
  **h0 in A, H2 in B**
- Some of scalar masses are almost degenerate → oblique parameters

# HIGGS BASIS AND TRILINEAR COUPLINGS

- In the Higgs basis, only one Higgs has vev

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = \begin{pmatrix} \cos\varphi\sin\theta & -\sin\varphi & -\cos\varphi\cos\theta \\ \sin\varphi\sin\theta & \cos\varphi & -\sin\varphi\cos\theta \\ \cos\theta & 0 & \sin\theta \end{pmatrix} \begin{pmatrix} \phi_{vev} \\ \psi_1 \\ \psi_2 \end{pmatrix}$$

$$\phi_{vev} = \begin{pmatrix} G^{\pm} \\ \frac{1}{\sqrt{2}}(v + \widetilde{h} + iG_0) \end{pmatrix}, \quad \psi_1 = \begin{pmatrix} H_1^{\pm} \\ \frac{1}{\sqrt{2}}(\widetilde{H}_1 + iA_1) \end{pmatrix}, \quad \psi_2 = \begin{pmatrix} H_2^{\pm} \\ \frac{1}{\sqrt{2}}(\widetilde{H}_2 + iA_2) \end{pmatrix}$$

$$\begin{pmatrix} \widetilde{h} \\ \widetilde{H}_1 \\ \widetilde{H}_2 \end{pmatrix} = \begin{pmatrix} \cos(\alpha-\theta) & 0 & \sin(\alpha-\theta) \\ 0 & 1 & 0 \\ -\sin(\alpha-\theta) & 0 & \cos(\alpha-\theta) \end{pmatrix} \begin{pmatrix} H_1 \\ h_0 \\ H_2 \end{pmatrix}$$

# TRILINEAR HIGGS-GAUGE COUPLINGS

- In the exact alignments limits only H2 (H1) has couplings to the gauge bosons

| $\cos(\alpha-\theta)$ | $\sin(\alpha-\theta)$ |
|---|---|
| $H_1W^+W^-$ | $H_2W^+W^-$ |
| $H_1ZZ$ | $H_2ZZ$ |
| $ZA_2H_2$ | $ZA_2H_1$ |
| $W^\pm H_2^\mp H_2$ | $W^\pm H_2^\mp H_1$ |
| $ZW^\pm H_2^\mp H_2$ | $ZW^\pm H_2^\mp H_1$ |
| $\gamma W^\pm H_2^\mp H_2$ | $\gamma W^\pm H_2^\mp H_1$ |

- **h0 has no trilinear gauge couplings, only:**

*In accordance with Z2 symmetry*

$$ZA_1h_0,\ ZW^\pm H_1^\mp h_0,\ W^\pm H_1^\mp h_0 \text{ y } \gamma W^\pm H_1^\mp h_0$$

- **h0 has no Yukawa couplings: Dark Matter candidate!**

# SCALAR-GAUGE COUPLINGS

$$g_{h_0 W^\pm W^\mp} = 0, \quad g_{h_0 ZZ} = 0;$$

$$g_{H_1 W^\pm W^\mp} = \frac{2M_W^2 \cos(\alpha - \theta) g^{\mu\nu}}{v}, \quad g_{H_2 W^\pm W^\mp} = \frac{2M_W^2 \sin(\alpha - \theta) g^{\mu\nu}}{v};$$

$$g_{H_1 ZZ} = \frac{M_Z^2 \cos(\alpha - \theta) g^{\mu\nu}}{v}, \quad g_{H_2 ZZ} = \frac{M_Z^2 \sin(\alpha - \theta) g^{\mu\nu}}{v};$$

$$g_{h_0 h_0 W^\pm W^\mp} = \frac{M_W^2 g^{\mu\nu}}{v^2}, \quad g_{h_0 h_0 ZZ} = \frac{M_Z^2 g^{\mu\nu}}{2v^2};$$

$$g_{H_1 H_1 W^\pm W^\mp} = \frac{M_W^2 g^{\mu\nu}}{v^2}, \quad g_{H_2 H_2 W^\pm W^\mp} = \frac{M_W^2 g^{\mu\nu}}{v^2};$$

*Differs from Barradas et al, consistent with Z2 symmetry*

$$g_{H_1 H_1 ZZ} = \frac{M_Z^2 g^{\mu\nu}}{2v^2}, \quad g_{H_2 H_2 ZZ} = \frac{M_Z^2 g^{\mu\nu}}{2v^2}.$$

# SCALAR-SCALAR COUPLINGS

$$g_{h_0h_0h_0} = 0,$$

$$g_{H_2H_2H_2} = -\frac{1}{v\, s_{2\theta}}\left[m_{h_0}^2 \frac{c_{\alpha-\theta}^3}{9c_\theta^2} + m_{H_2}^2\left(c_\alpha^2 c_{\alpha-\theta} - s_\alpha s_\theta\right)\right],$$

$$g_{H_1H_1H_1} = \frac{1}{v\, s_{2\theta}}\left[m_{h_0}^2 \frac{s_{\alpha-\theta}^3}{9c_\theta^2} - m_{H_1}^2\left(c_\alpha^2 s_{\alpha-\theta} - s_\alpha c_\theta\right)\right],$$

$$g_{h_0h_0H_1} = \frac{1}{v\, s_{2\theta}}(m_{h_0}^2 s_{\alpha+\theta} + m_{H_1}^2 s_\alpha c_\theta),$$

$$g_{h_0h_0h_0h_0} = \frac{1}{24v^2 s_\theta^2}\left(m_{h_0}^2 + 3m_{H_1}^2 s_\alpha^2 + 3m_{H_2}^2 c_\alpha^2\right),$$

$$g_{H_1H_1H_1H_1} = \frac{1}{2v^2 s_{2\theta}^2}\left(m_{h_0}^2 s_{\alpha-\theta}^3 \frac{(s_{\alpha-\theta} + 2s_{\alpha+\theta})}{9c_\theta^2} + m_{H_1}^2 (s_\alpha^2 s_{\alpha-\theta} + c_\alpha s_\theta)^2 + m_{H_2}^2 \frac{s_{2\alpha}^2 s_{\alpha-\theta}^2}{4}\right),$$

$$g_{h_0h_0H_2} = -\frac{1}{v\, s_{2\theta}}(m_{h_0}^2 c_{\alpha+\theta} + m_{H_2}^2 c_\alpha c_\theta),$$

$$g_{H_1H_1H_2} = -\frac{s_{\alpha-\theta}}{v s_{2\theta}}\left(m_{h_0}^2\left(\frac{s_{2(\alpha-\theta)}}{6c_\theta^2}\right) + m_{H_1}^2 s_{2\alpha} + \frac{m_{H_2}^2 s_{2\alpha}}{2}\right),$$

$$g_{H_1H_2H_2} = \frac{c_{\alpha-\theta}}{v s_{2\theta}}\left(m_{h_0}^2\left(\frac{s_{2(\alpha-\theta)}}{6c_\theta^2}\right) + \frac{m_{H_1}^2 s_{2\alpha}}{2} + m_{H_2}^2 s_{2\alpha}\right),$$

$$g_{H_2H_2H_2H_2} = \frac{1}{2v^2 s_{2\theta}^2}\left(m_{h_0}^2 c_{\alpha-\theta}^3 \frac{(c_{\alpha-\theta} + 2c_{\alpha+\theta})}{9c_\theta^2} + m_{H_1}^2 \frac{s_{2\alpha}^2 c_{\alpha-\theta}^2}{4} + m_{H_2}^2 (c_\alpha^2 c_{\alpha-\theta} - s_\alpha s_\theta)^2\right).$$

# EXACT ALIGNMENT LIMIT A

- In the exact alignment limit A (SM Higgs the lightest scalar)

$$\sin(\alpha-\theta) = 1,\ \cos(\alpha-\theta) = 0,$$

- "Our" SM Higgs trilinear and quartic couplings reduce exactly to SM real ones

$$g_{H_2H_2H_2} = \frac{1}{v\, s_{2\theta}}\left[m^2_{H_2} s_\alpha s_\theta\right] = \frac{1}{2v}\frac{s_\alpha}{c_\theta} m^2_{H_2} = \frac{m^2_{H_2}}{2v} \equiv \lambda_{SM}.$$

$$g_{H_1H_1H_1} = \frac{1}{v\, s_{2\theta}}\left[\frac{1}{9c^2_\theta} m^2_{h_0} - s^2_\theta m^2_{H_1}\right] = \frac{1}{v\, s_{2\theta} c^2_\theta}\left[\frac{1}{9} m^2_{h_0} - \frac{1}{2} s_{2\theta} m^2_{H_1}\right].$$

$$g_{H_2H_2H_2H_2} = \frac{1}{2v^2 s^2_{2\theta}} m^2_{H_2}(-s^3_\theta c_\theta - c^3_\theta s_\theta)^2 = \frac{m^2_{H_2}}{8v^2}.$$

$$g_{H_2H_2h_0h_0} = \frac{1}{v^2 s_{2\theta}}\left(\frac{1}{6} m^2_{h_0} 3 s_{2\theta} + \frac{1}{4} m^2_{H_2} s_{2\theta}\right) = \frac{1}{4v^2}(2m^2_{h_0} + m^2_{H_2})\ .$$

# LIMITS ON MASSES — TREE LEVEL

- Some couplings depend only on masses in alignment limit
- Allows to put lower bounds on these masses, through the absence of corresponding decays

$$g_{H_2 h_0 h_0} = \frac{1}{2v}(m_{H_2}^2 + 2m_{h_0}^2), \quad g_{H_2 A_1 A_1} = \tfrac{1}{2v}(m_{H_2}^2 + 2m_{A_1}^2), \quad g_{H_2 A_2 A_2} = \frac{1}{2v}(m_{H_2}^2 + 2m_{A_2}^2),$$

$$g_{H_2 H_1^{\pm} H_1^{\mp}} = \frac{1}{v}(m_{H_2}^2 + 2m_{H_1^{\pm}}^2), \quad g_{H_2 H_2^{\pm} H_2^{\mp}} = \tfrac{1}{v}(m_{H_2}^2 + 2m_{H_2^{\pm}}^2), \quad g_{H_2 H_2 H_2 H_1} = g_{H_1 H_1 H_1 H_2} = 0.$$

- Sets a limit for all scalar masses (other than H1 and H2) at tree level of

$$m_{Hi} \gtrsim 63 \text{ GeV}$$

# ALIGNMENT NOT EXACT — LIMITS ON PARAMETERS

- Higgs-gauge couplings have been determined with 5% precision → $\kappa_\lambda$ scaling factor
- -1.8 < $\kappa_\lambda$ < 9.2

Degrassi, Di Micco, Giardino, Rossi (2021)

- If the alignment limit is not exact we can parameterize deviations from SM

$$g_{H_2H_2H_2} \equiv \lambda_{SM}\kappa_\lambda = \frac{m_{H_2}^2}{2v}\left[(1+2\delta^2)\sqrt{1-\delta^2} + \delta^3(\tan\theta - \cot\theta) - \frac{m_{h_0}^2}{m_{H_2}^2}\frac{\delta^3}{9 s_\theta c_\theta^3}\right]$$

$$\cos(\alpha-\theta) = \cos(\frac{\pi}{2} - \epsilon) = \sin\epsilon \equiv \delta,$$

- The max value for $m_{h0}$ sets constraints on tan$\theta$
e.g. for $\delta \sim 0.1 \rightarrow \tan\theta \leq 15$

# FORM OF ONE-LOOP CORRECTIONS TO MASSES

$$\Sigma^{\phi}(s)+\Sigma^{V}(s)=\begin{pmatrix}\Sigma^{\phi,V}_{h_0}(s) & 0 & 0\\ 0 & \Sigma^{\phi,V}_{H_1}(s) & \Sigma^{\phi,V}_{H_1H_2}(s)\\ 0 & \Sigma^{\phi,V}_{H_2H_1}(s) & \Sigma^{\phi,V}_{H_2}(s)\end{pmatrix}$$

$$\begin{aligned}\Sigma^{\phi,V}_{H_n} &= \sum_i \frac{g_{H_nH_n\phi^0_i\phi^0_i}}{16\pi^2}A0(m^2_{\phi^0_i})+\sum_{i,j}\frac{g^2_{H_n\phi^0_i\phi^0_j}}{8\pi^2}B0(p^2,m^2_{\phi^0_i},m^2_{\phi^0_j})+\sum_k\frac{g^2_{H_n\phi^\pm_k\phi^\mp_k}}{8\pi^2}B0(p^2,m^2_{\phi^\pm_k},m^2_{\phi^\pm_k})\\ &+ \sum_i\frac{g_{H_nH_nV_iV_i}}{16\pi^2}A0(m^2_{V_i})+\sum_i\frac{g^2_{H_nV_iV_i}}{8\pi^2}B0(p^2,m^2_{V_i},m^2_{V_i}),\end{aligned}$$

with $n=1,2$.‡ For the mixing term $H_{12}$ we get

$$\begin{aligned}\Sigma^{\phi,V}_{H_1H_2} &= \sum_i\frac{g_{H_1H_2\phi^0_i\phi^0_i}}{16\pi^2}A0(m^2_{\phi^0_i})+\sum_{i,j}\frac{g_{H_1\phi^0_i\phi^0_j}g_{H_2\phi^0_i\phi^0_j}}{8\pi^2}B0(p^2,m^2_{\phi^0_i},m^2_{\phi^0_j})\\ &+ \sum_k\frac{g_{H_1\phi^\pm_k\phi^\mp_k}g_{H_2\phi^\pm_k\phi^\mp_k}}{8\pi^2}B0(p^2,m^2_{\phi^\pm_k},m^2_{\phi^\pm_k})+\sum_i\frac{g_{H_1V_iV_i}g_{H_2V_iV_i}}{8\pi^2}B0(p^2,m^2_{V_i},m^2_{V_i})\\ &+ \sum_k\frac{g_{H_1\phi^\pm_kW^\mp}g_{H_2\phi^\pm_lW^\mp}}{8\pi^2}B0(p^2,m^2_{\phi^\pm_l},m^2_W),\end{aligned}$$

where $\phi^0_{i(j)}=h_0,H_1,H_2,A_1,A_2,G^0$, $\phi^\pm_k=H^\pm_{1,2},G^\pm$ and $V_i=W^\pm,Z^0$.

# ONE-LOOP POSSIBILITIES…

- Check for benchmarks where off-diagonal terms vanish, i.e. loop contributions extremely small (gauge and Higgs only)

| Scalar benchmarks | Masses (GeV) | $\tan\theta$ |
|---|---|---|
| light spectrum | $m_{h_0} = 80$, $m_{H_1} = 200$, $m_{A_{1,2}} = 80$, $m_{H^\pm_{1,2}} = 100$ | 1 |
| heavy spectrum | $m_{h_0} = 800$, $m_{H_1} = 800$, $m_{A_{1,2}} = 800$, $m_{H^\pm_{1,2}} = 800$ | 2.1 🙂 |

Table 2: Parameter values in scenario A that make the one-loop mixing parameter vanish, $\Sigma^{\phi}_{H_1 H_2} = 0$, taking into account only the scalar and gauge contributions.

- For N-Higgs doublet models: oblique parameters OK in compact almost degenerate spectrum Grimus et al (2008); Cárcamo et al (2015)
- You can also fix $m_{H\_SM}$ mass as finite at tree level and renormalize the rest (on-shell ran)

*Work in progress*

# IN YUKAWA SECTOR

- The Higgs Z2 residual symmetry will lead to zeroes in the CKM and PMNS matrices 😱 Das, Dey, Pal (2015), Ivanov (2017)
- To recover the good features of the symmetry:
  - Add S3 singlet Brown, Deshpande,Sugawara, Pakwasa (1984)
  - Break very softly the S3 symmetry with mass terms, recover original structure e.g., Kubo, Okada, Sakamaki (2004), Das, Dey, Pal (2015)
  - Consider CP violation Costa, Ogreid, Osland, Rebelo(2014,2021)
  - Make S3 modular Cerón, MM (2021), M.Sc. Thesis
  - Second B-L sector at higher scale with some interaction Gómez-Izquierdo, MM (2018), and L.E. Gutiérrez (now)
  - Add a fourth Higgs doublet Espinoza, Garcés, MM, Reyes (2019)
  - Combinations of the above: all introduce more parameters

# 4HDM –S3 WITH DM

- We add another doublet, inert, to have a DM candidate. We assign it to the $1^A$, and thus "saturate" the irreps
- First two generations in a flavour doublet, third in a singlet, extra anti-symmetric singlet is inert → DM candidates
- A lot of Higgses (13), but the good features of 3H-S3 remain
  Quark and lepton sectors remain unchanged
  DM candidate in inert sector
- Add a Z2 symmetry to prevent the DM candidate to decay
- S3 symmetry constrains strongly the allowed couplings

C. Espinoza, E. Garcés, M.M., H. Reyes (2019)

# HIGGS POTENTIAL 4H-S3

- We need to find the minima of the potential S3xZ2, which satisfy the stability and unitarity conditions

$$
\begin{aligned}
V_4 = {} & \mu_0^2 H_s^\dagger H_s + \mu_1^2 (H_1^\dagger H_1 + H_2^\dagger H_2) + \mu_2^2 H_a^\dagger H_a \\
& + \lambda_1 (H_1^\dagger H_1 + H_2^\dagger H_2)^2 + \lambda_2 (H_1^\dagger H_2 - H_2^\dagger H_1)^2 \\
& + \lambda_3 [(H_1^\dagger H_1 - H_2^\dagger H_2)^2 + (H_1^\dagger H_2 + H_2^\dagger H_1)^2] \\
& + \lambda_4 [(H_s^\dagger H_1)(H_1^\dagger H_2 + H_2^\dagger H_1) + (H_s^\dagger H_2)(H_1^\dagger H_1 - H_2^\dagger H_2) + \text{h.c.}] \\
& + \lambda_5 (H_s^\dagger H_s)(H_1^\dagger H_1 + H_2^\dagger H_2) \\
& + \lambda_6 [(H_s^\dagger H_1)(H_1^\dagger H_s) + (H_s^\dagger H_2)(H_2^\dagger H_s)] \\
& + \lambda_7 [(H_s^\dagger H_1)(H_s^\dagger H_1) + (H_s^\dagger H_2)(H_s^\dagger H_2) + \text{h.c.}] \\
& + \lambda_8 (H_s^\dagger H_s)^2 \\
& \cancel{+ \lambda_9 [(H_a^\dagger H_2)(H_1^\dagger H_2 + H_2^\dagger H_1) - (H_a^\dagger H_1)(H_1^\dagger H_1 - H_2^\dagger H_2) + h.c.]} \\
& + \lambda_{10} (H_a^\dagger H_a)(H_1^\dagger H_1 + H_2^\dagger H_2) \\
& + \lambda_{11} [(H_a^\dagger H_1)(H_1^\dagger H_a) + (H_a^\dagger H_2)(H_2^\dagger H_a)] \\
& + \lambda_{12} [(H_a^\dagger H_1)(H_a^\dagger H_1) + (H_a^\dagger H_2)(H_a^\dagger H_2) + \text{h.c.}] \\
& \cancel{+ \lambda_{13} (H_a^\dagger H_a)^2 + \lambda_{14} (H_s^\dagger H_a H_a^\dagger H_s) + \lambda_{15} [(H_1^\dagger H_s)(H_2^\dagger H_a) + \text{h.c.}].}
\end{aligned}
$$

# MASSES

- After electroweak symmetry breaking (Higgs mechanism) we are left with

  13 massive particles!

- One has to be the SM Higgs boson, same as in S3-3H
- Two can be DM particles
- Check lightest one of neutral scalars

$$m^2_{H^S} = \begin{pmatrix} m_{h^n_s h^n_s} & m_{h^n_1 h^n_s} & m_{h^n_2 h^n_s} & 0 \\ m_{h^n_s h^n_1} & m_{h^n_1 h^n_1} & m_{h^n_2 h^n_1} & 0 \\ m_{h^n_s h^n_2} & m_{h^n_1 h^n_2} & m_{h^n_2 h^n_2} & 0 \\ 0 & 0 & 0 & m_{h^n_a h^n_a} \end{pmatrix}$$

# And the corresponding eigenvalues are:

$$m^2_{h^n_s} = -18\lambda_4 v_0 v_2$$

$$m^2_{h^n_a} = \mu_2^2 + \lambda_{14} v_0^2 + 4(\lambda_{10} + \lambda_{11} + 2\lambda_{12}) v_2^2$$

$$m^2_{h^n_1} = (\frac{1}{v_0})(2\lambda_8 v_0^3 + v_2(3\lambda_4 v_0^2 + 8(\lambda_1 + \lambda_3) v_0 v_2 - 4\lambda_4 v_2^2) + ((4\lambda_8^2 v_0^6 - 12\lambda_4\lambda_8 v_0^5 v_2 + (9\lambda_4^2 + 16((\lambda_5 + \lambda_6 + 2\lambda_7)^2 - 2(\lambda_1 + \lambda_3)\lambda 8)) v_0^4 v_2^2 + 16\lambda_4(3(\lambda_1 + \lambda_3 + 2(\lambda_5 + \lambda_6 + 2\lambda_7)) - \lambda_8) v_0^3 v_2^3 + 8(8(\lambda_1 + \lambda_3)^2 + 21\lambda_4^2) v_0^2 v_2^4 + 64(\lambda_1 + \lambda_3)\lambda_4 v_0 v_2^5 + 16\lambda_4^2 v_2^6))^{1/2}$$

$$m^2_{h^n_2} = (\frac{1}{v_0})(2\lambda_8 v_0^3 + v_2(3\lambda_4 v_0^2 + 8(\lambda_1 + \lambda_3) v_0 v_2 - 4\lambda_4 v_2^2) - (4\lambda_8^2 v_0^6 - 12\lambda_4\lambda_8 v_0^5 v_2 + (9\lambda_4^2 + 16((\lambda_5 + \lambda_6 + 2\lambda_7)^2 - 2(\lambda_1 + \lambda_3)\lambda_8)) v_0^4 v_2^2 + 16\lambda_4(3(\lambda_1 + \lambda_3 + 2(\lambda_5 + \lambda_6 + 2\lambda_7)) - \lambda_8) v_0^3 v_2^3 + 8(8(\lambda_1 + \lambda_3)^2 + 21\lambda_4^2) v_0^2 v_2^4 + 64(\lambda_1 + \lambda_3)\lambda_4 v_0 v_2^5 + 16\lambda_4^2 v_2^6))^{1/2}.$$

$$m^2_{h^p_s} = 0$$

$$m^2_{h^p_a} = \mu_2^2 + \lambda_{14} v_0^2 + 4(\lambda_{10} + \lambda_{11} - 2\lambda_{12}) v_2^2$$

$$m^2_{h^p_1} = -\frac{2(2\lambda_7 v_0^3 + 5\lambda_4 v_0^2 v_2 + 8\lambda_2 v_0 v_2^2 + 8\lambda_3 v_0 v_2^2)}{v_0}$$

$$m^2_{h^p_2} = -\frac{2(2\lambda_7 v_0 + \lambda_4 v_2)(v_0^2 + 4v_2^2)}{v_0}.$$

$$m_{h^\pm_s} = 0$$

$$m_{h^\pm_a} = \mu_2^2 + 4\lambda_{10} v_2^2$$

$$m_{h^\pm_1} = -(\lambda_6 + 2\lambda_7) v_0^2 - 10\lambda_4 v_0 v_2 - 16\lambda_3 v_2^2$$

$$m_{h^\pm_2} = -\frac{(\lambda_6 v_0 + 2\lambda_7 v_0 + 2\lambda_4 v_2)(v_0^2 + 4v_2^2)}{v_0}.$$

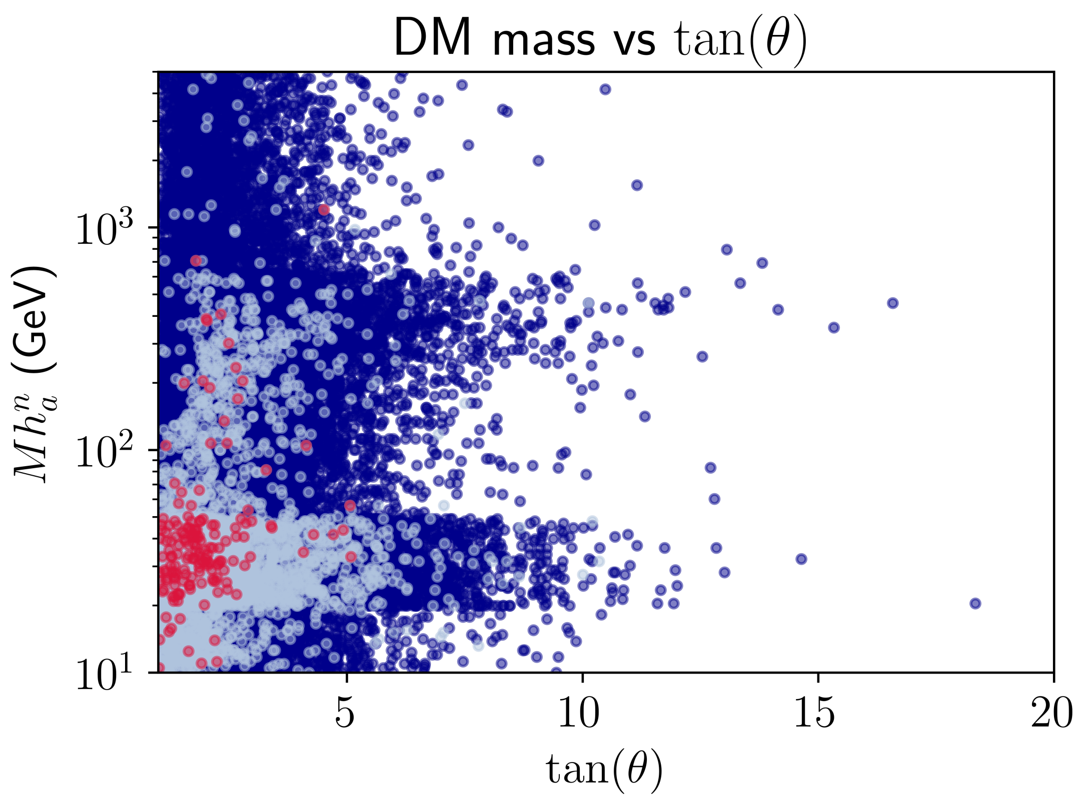
We choose the neutral scalar
Pseudoscalar also possible

# CONSTRAINTS

- In the Yukawa sector assume SM limit
- Several constraints are imposed over the parameter space:
- Usual vacuum stability conditions
- Unitarity conditions for large s (LQT conditions)
- Unitarity conditions for finite s
- SM Higgs boson mass within 125 ± 3 GeV
- Limits for Higgs searches at LEP, Tevatron and LHC

Constraints implemented using FeynArts, FormCalc, SARAH+SPheno, HiggsBounds, MicrOmegas

# DM MASS AND RELIC DENSITY

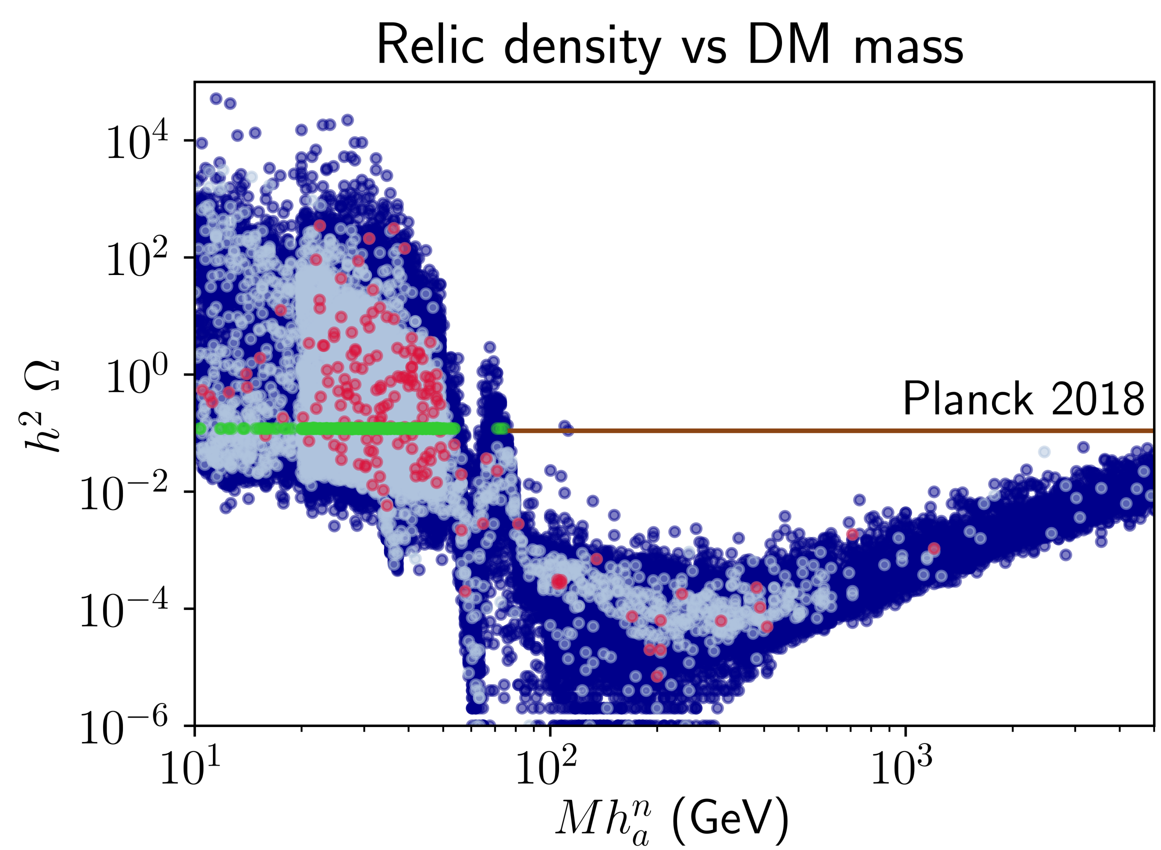



Blue points → stability and unitarity

Light blue → also Higgs bounds

Red points → also alignment limit

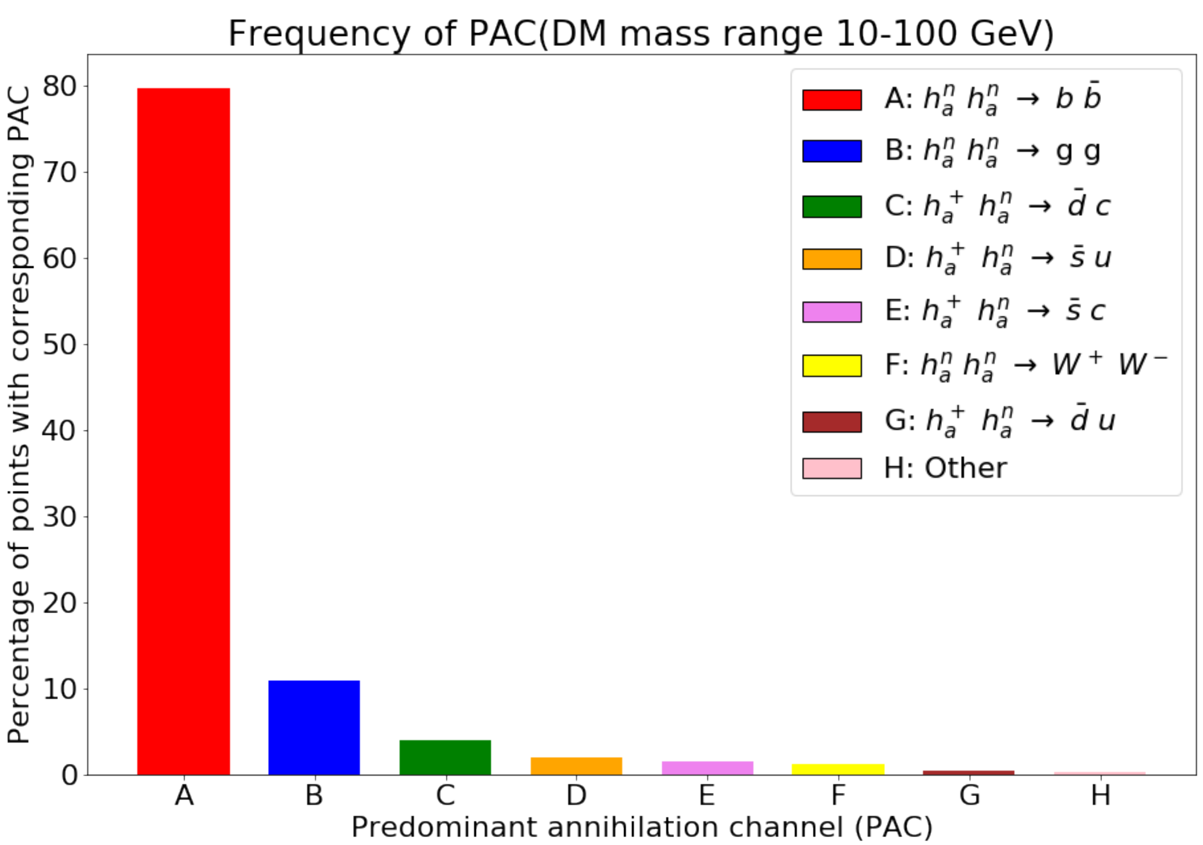
*The bounds apply to S3-3H too*

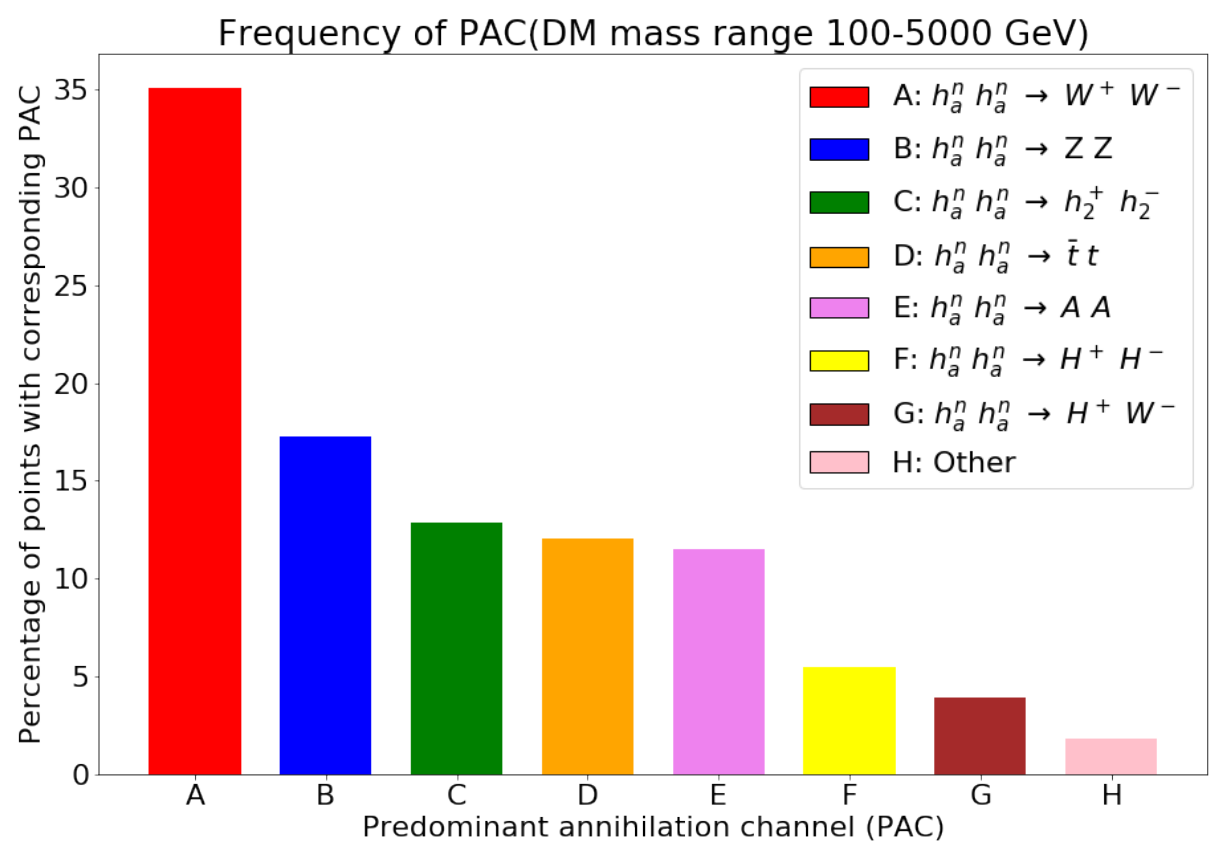



Green points → DM Planck limits

Small values of tanθ preferred

# DM ANNIHILATION CHANNELS

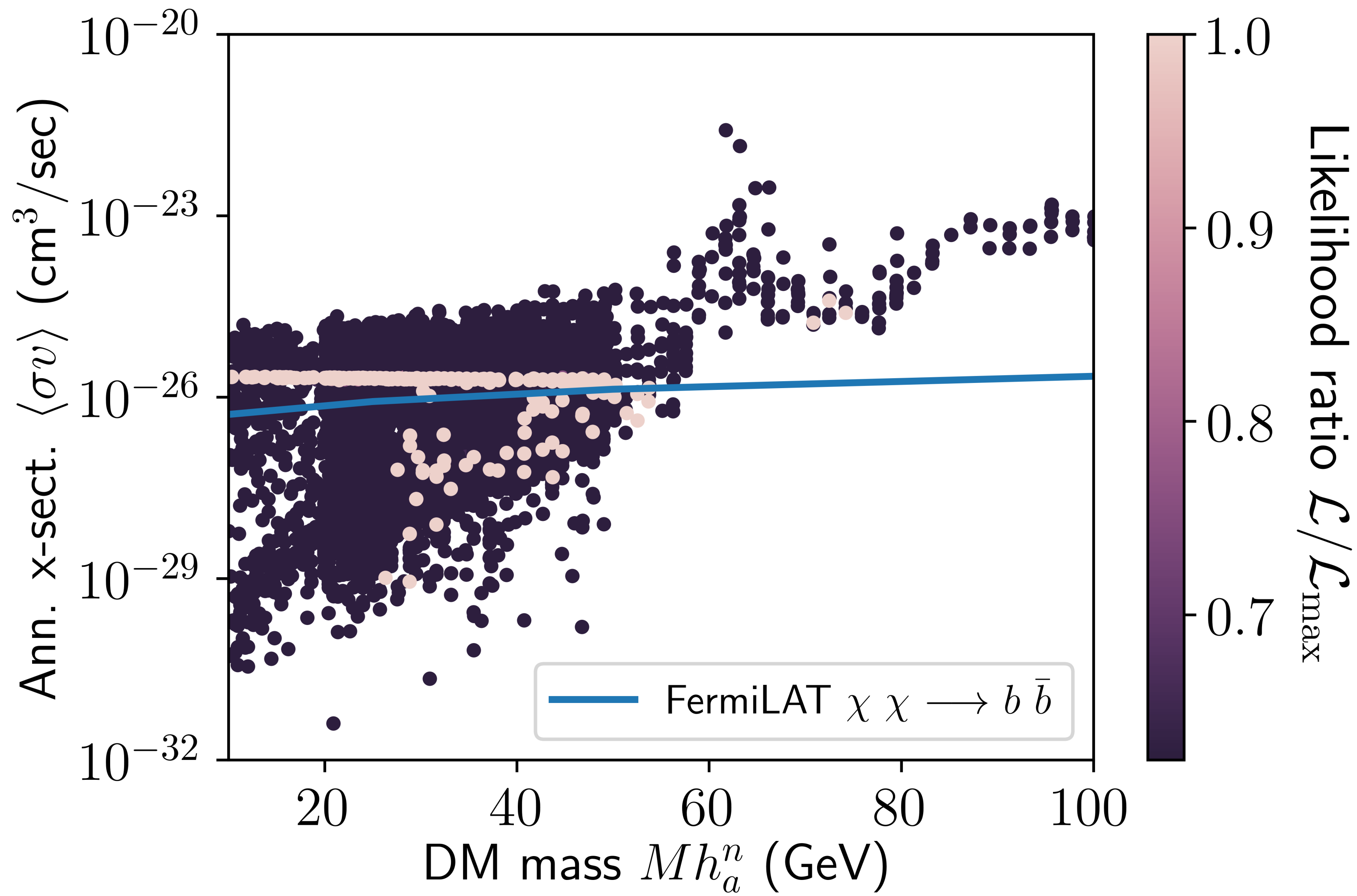



- Frequency of dominant annihilation channels that contribute to DM relic density
- All points below or at Planck limit
- Similar to i2HDM

# ANNIHILATION CROSS SECTION

- Annihilation cross section vs DM mass → relevant for indirect detection experiments
- Likelihood function with respect to Planck limits
- Pink points have relic density within experimental bounds




*There are points in parameter space which survive all constrains*
*Tree level: results will shift with radiative corrections*

# OR MAKE IT MODULAR… (FANCY, BUT DOES IT HELP??) MODULAR SYMMETRIES

- Related to moduli spaces, geometric spaces: solutions of geometric classification problems. Objects are identified (isomorphic) if they are the same geometrically.

- Using modular symmetries as flavor symmetries:
  Inspiration from supersymmetric theories, initially with extra dimensions Feruglio, Altarelli (2006-2022); Petcov et al (2019, 2021, 2022)
  Magnetized branes, superstring theories
  Cremades et al (2004); Kobayashi et al (2018)
  Superstring compactifications, especially from orbifold compactifications
  e.g. Kobayashi et al (2018, 2019); Chen, Ramos-Sánchez, Ratz (2022)

- Usually applied in supersymmetric models, but now also in non-supersymmetric models e.g. Nomura, Okada et al, (2019,2020)

# MODULAR GROUP

- Projective special linear group of 2x2 matrices and determinant; linear fractional transformations of upper half of complex plane

$$\Gamma = SL_2(\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} | a, b, c, d \in \mathbb{Z}, ad - bc = 1 \right\}.$$

The transformation $\gamma$ over a parameter $\tau$

$$\gamma(\tau) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} (\tau) \to \frac{a\tau + b}{c\tau + d}. \qquad \gamma \in \Gamma$$

- Modular forms of weight k, functions that transform under $\Gamma$ with weight k

$$f(\gamma\tau) = (c\tau + d)^k f(\tau)$$

# GAMMA AND POLYGONS

- Isomorphism between some finite modular groups and some groups associated to polygons (invariance under rotations and reflections)

$$\begin{aligned}\Gamma_2 &\simeq S_3\\ \Gamma_3 &\simeq A_4\\ \Gamma_4 &\simeq S_4\\ \Gamma_5 &\simeq A_5\end{aligned}$$

- Yukawa couplings expressed in terms of modular forms, i.e. functions of a complex scalar field

$$Y(\alpha,\beta,\gamma|\tau) = \frac{d}{d\tau}\left(\alpha \log \eta \left(\frac{\tau}{2}\right) + \beta \log \eta \left(\frac{\tau+1}{2}\right) + \gamma \log \eta\,(2\tau)\right)$$

- Fermions and scalar fields transform with a weight

$$\phi \to (c\tau + d)^{k_\phi}\phi,$$

# S3 MODULAR SYMMETRY

- We will impose a modular S3 or $\Gamma_2$ to a non-supersymmetric Lagrangian

$$SU(3)_C \times SU_L(2) \times U_y(1) \times \Gamma_2$$

- 3HDM, 3 $\nu_R$, quarks and leptons:

  first two generations in a doublet
  third generation in a singlet

  same for 3 Higgses: 2 of them in a doublet, third in a singlet

- We assign specific modular weights (again, some liberty there…) to get a NNI texture

- We'll take a **big** leap of faith and assume it stayed unbroken at low energies (problems with kinetic form and others…)

# THE ASSIGNMENT FOR THE MODEL

- We assign the fields the following weights

| | $(Q_1, Q_2)$ | $(q_1, q_2)$ | $Q_3$ | $q_3$ | $(H_1, H_2)$ | $H_s$ | $(Y_1^{(2,4)}(\tau), Y_2^{(2,4)}(\tau))$ | $Y_s^{(4)}(\tau)$ |
|---|---|---|---|---|---|---|---|---|
| $SU(2)$ | 2 | 1 | 2 | 1 | 2 | 2 | 1 | 1 |
| $S_3$ | 2 | 2 | 1 | 1 | 2 | 1 | 2 | 1 |
| $k$ | $-2$ | $-2$ | 0 | 0 | 0 | 0 | $(2,4)$ | 4 |

Table 2: charges, assignments, and modular weights of $SU(2)$ and $S_3$. The superscript $(2,4)$ on the modular forms indicates that they are of modular weight 2 or 4. The subscript $s$ indicates the symmetric singlet of the modular form of weight 4.

- The Yukawa part of the Lagrangian is

$$\begin{aligned}
\mathcal{L}_y^{(u)} &= C_1\overline{Q}\otimes u\otimes\tilde{H}\otimes Y^{(4)} + C_2\overline{Q}\otimes u\otimes\tilde{H}\otimes Y_s^{(4)} + C_3\overline{Q}\otimes u\otimes\tilde{H}_s\otimes Y^{(4)} \\
&+ C_4\overline{Q}\otimes u\otimes\tilde{H}_s\otimes Y_s^{(4)} + C_5\overline{Q}\otimes u_{3R}\otimes\tilde{H}\otimes Y^{(2)} + C_6\overline{Q}\otimes u_{3R}\otimes\tilde{H}_s\otimes Y^{(2)} \\
&+ C_7\overline{Q}_3\otimes u\otimes\tilde{H}\otimes Y^{(2)} + C_8\overline{Q}_3\otimes u\otimes\tilde{H}_s\otimes Y^{(2)} + C_9\overline{Q}_3\otimes u_{3R}\otimes\tilde{H}_s + \text{h.c.}
\end{aligned}$$

# WHAT CAN WE DO?

- A lot of freedom! too many parameters…
- Can we do something about it?
- But, look at the symmetries — geometry, of the problem
- In the modular symmetry points parameters are identified or related:
  **only few parameters remain**
- This way: possible to explain mixings, S4 and A5 studied
  Novichkov, Penedo, Petcov (2021)
- S3 studied too, but so far without exploiting these symmetric points
  Kobayashi et al (2019,2020)

# MODULAR SYMMETRIC POINTS

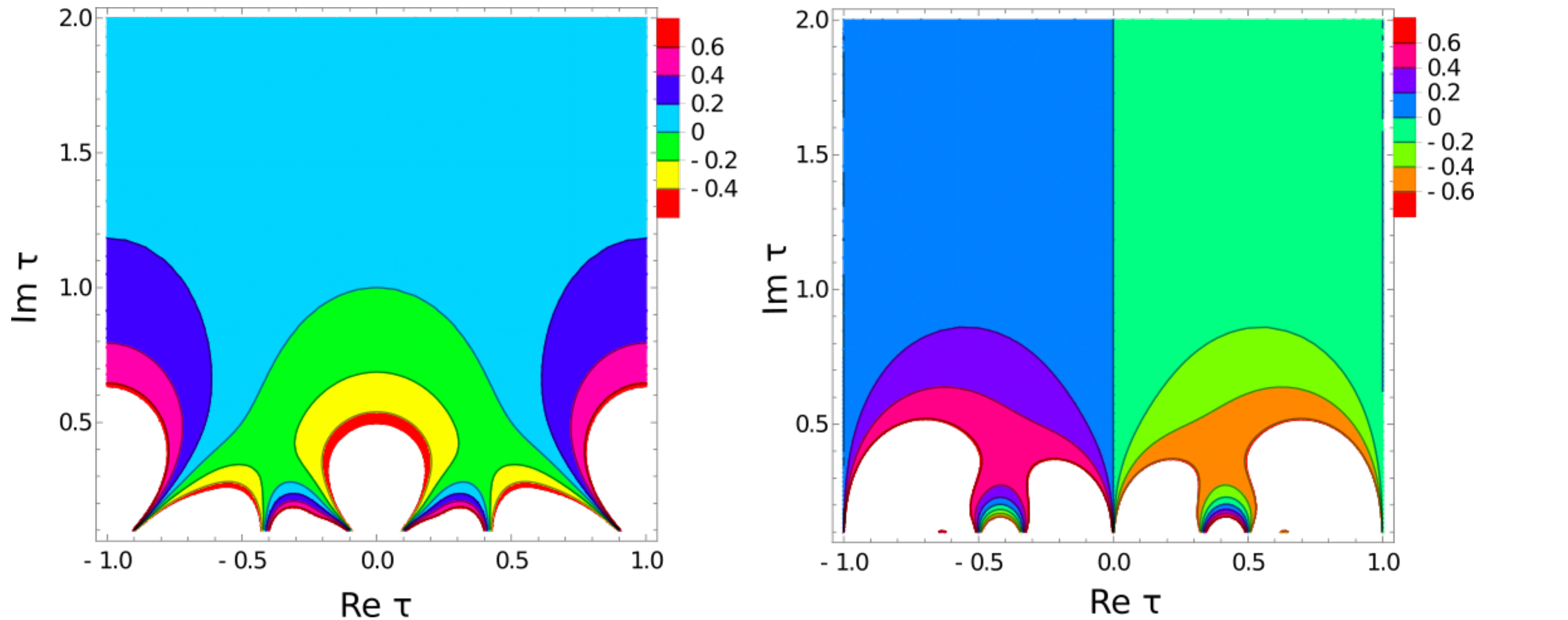



Figure 3: Real (left) and imaginary (right) part of the given expression in $M_{13}$ y $M_{31}$, that is, $Y_2^{(2)}(\tau) - \sqrt{3}Y_1^{(2)}(\tau)$. It is observed that $Y_2^{(2)}(\tau) - \sqrt{3}Y_1^{(2)}(\tau) = 0$, for both its real and imaginary parts, at the point $\tau = i$, which guarantees that $M_{13} = M_{31} = 0$.

## $V_{CKM}$ MATRIX

- Assuming the NNI form and a hierarchical structure for the mass matrices u and d, we can reparameterize them in terms of mass ratios $\widetilde{\sigma}_i = m_i/m_3$
- Exact analytical expression for the $V_{CKM}$ corresponding to the symmetry S3 with the NNI structure
- Without loss of generality we can fix the values of 2 phases

$$\phi_{1d} = \phi_{2d} = 0$$

- Now only 4 free parameters to fit the $V_{CKM}$
- We perform a $\chi^2$ analysis to find the numerical values of our parameters

# $V_{CKM}$ FIT

- Excellent fit (too excellent…overfitted?)
- Probably we have correlations among parameters → one too many?
- Analytical expression successful

| | Center value and error |
|---|---|
| $\widetilde{\sigma}_u$ | $7.032 \times 10^{-6}$ |
| $\widetilde{\sigma}_d$ | $9.44 \times 10^{-4}$ |
| $\widetilde{\sigma}_s$ | $0.0190 \pm 0.00046$ |
| $\widetilde{\sigma}_c$ | $0.00375 \pm 0.00023$ |

| | Values in the fit |
|---|---|
| $C'_{9u}$ | 0.816393 |
| $C'_{9d}$ | 0.828604 |
| $\phi_{1u}$ | 1.63797 |
| $\phi_{1d}$ | 0 |
| $\phi_{2u}$ | 0.0981477 |
| $\phi_{2d}$ | 0 |
| $\chi^2$ | 0.00070 |

👍👍

$$V_{CKM}^{th} = \begin{pmatrix} 0.97435 & 0.2250 & 0.00369 \\ 0.22486 & 0.97349 & 0.04182 \\ 0.00857 & 0.04110 & 0.999118 \end{pmatrix}$$

$$\mathcal{J}^{th} = 3.07 \times 10^{-5}.$$

# NICE, BUT…

- Modular approach might be too unrealistic, although the role of the symmetries certainly very interesting
- Now, break softly the S3-3H:
  - Introduce a soft breaking term in scalar potential V
  - Residual Z2 symmetry is broken
  - Recover the form of the mass matrices and $V_{CKM}$
  - Re-do the analysis of V
  - BUT, possibility of testing realistically the model in HL-LHC through exotic Higges

*Work in progress: Espinoza, Gómez-Bock, Heinemeyer, MM, Pérez-Martínez*

# GOING UP?

- You can embed the model (or a version of it) in a SUSY model with Q6 symmetry
- Grand Unified SU(5) x Q6 model already studied, preserves the nice features of S3 in quarks and leptons. Mixing angles in good agreement with experiment, both hierarchies allowed.

  J.C. Gómez-Izquierdo, F. González-Canales, M.M. (2014)

  Neutrino masses: add singlets or non-renormalizable interactions or radiatively
- Possible to have different assignments of Q6 in leptonic sector $\Longrightarrow$ breaking of mu-tau symmetry

  J.C. Gómez-Izquierdo, M.M. (2017)
- Flavour structure in trilinear soft SUSY breaking terms $\rightarrow$ LFV $\tau\rightarrow\mu+\gamma$, g-2 contributions through LFV in leptonic sector

  F. Flores-Báez, M. Gómez-Bock, M.M. (2018)
- Non-SUSY B-L model with S3, also breaking of mu-tau symmetry

  J.C. Gómez-Izquierdo, M.M. (2019)

# CONCLUSIONS

- S3 is a small symmetry that goes a long way
- S3-3H models consistent with CKM and PMNS
  $\theta_{13} \neq 0$ naturally
  Possible to calculate all neutrino masses and mixings
- In Higgs sector:
    - **masses bounded from above and below**
    - **trilinear and quartic Higgs coupling are SM ones in alignment limits**
    - **Possible to have light "semi-invisible" Higgs in both scenarios,**
      **with different signals/characteristics**
- Simultaneous study of Higgs, fermionic sector and DM shows model is self-consistent:
  tanθ small solutions appear both in Higgs and DM sectors

# CONCLUSIONS

- Regions of parameter space that pass all Higgs bounds:
  Extra Higgses sufficiently decoupled or inert possible
- Good DM candidate(s)
    - 4th inert Higgs
    - h0 as DM candidate
    - possible to add R-handed neutrino as DM
- Leptogenesis possible
- Vacuum much more complicated than in SM, all checks necessary:
  Need to add one-loop corrections
- Above all:
  **Consistent with known physics**
  **New predictions**
  **Testable**

# (PART OF) THE TEAM…

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**THANKS!!**