Mini-jet thermalization and diffusion of transverse correlations in heavy ion collisions

Qun Wang (王群)
Univ of Sci & Tech of China
(中国科技大学)

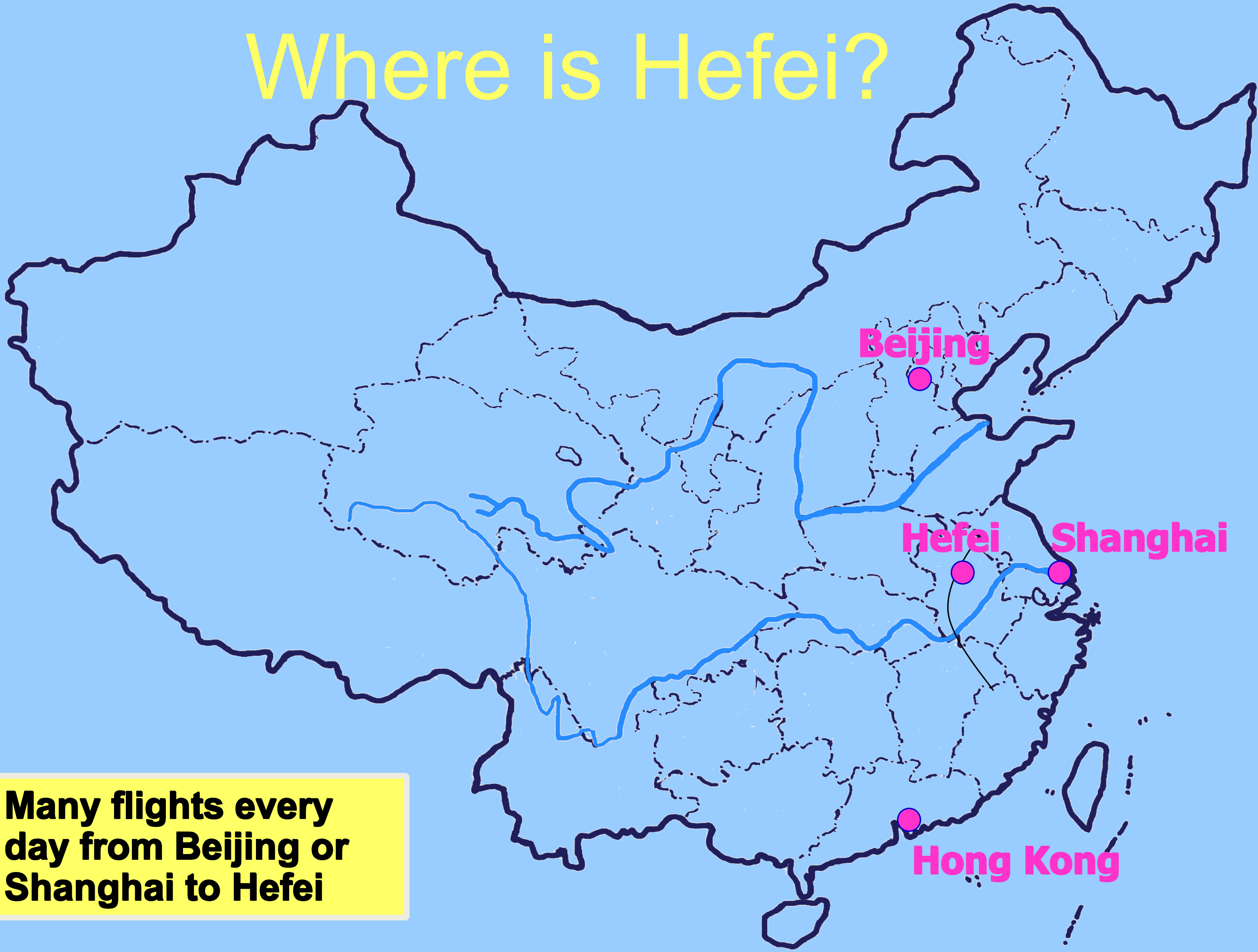
With L.G.Pang,X.N.Wang,R.Xu, PRC 81,031903(2010)

5th int. workshop on high pt physics at LHC, Sept 27-Oct 1, 2010
ICN-UNAM, Mexico City

Univ of Science & Technology of China (USTC)

- The sole university in Chinese Academy of Sciences (CAS) and is the powerhouse of talents for hundreds of CAS institutes. Top 5 in physics among 100 research universities in china.
- Hosts one of the strongest research teams in experimental and theoretical particle/nuclear physics in China. Our experimentalists have involved in major international collaborations ATLAS, BELLE, D0,STAR etc. Our theorists are actively involved in related topics for LHC physics.
- Located in Hefei, the capital city of Anhui Province, a middle-sized city in Yangtze river delta, about 350 km west of Shanghai and 800 km south of Beijing. The transportation to Hefei is convenient. It can be reached from Beijing or Shanghai by air or by newly constructed high speed railway (up to 350 km/hour).

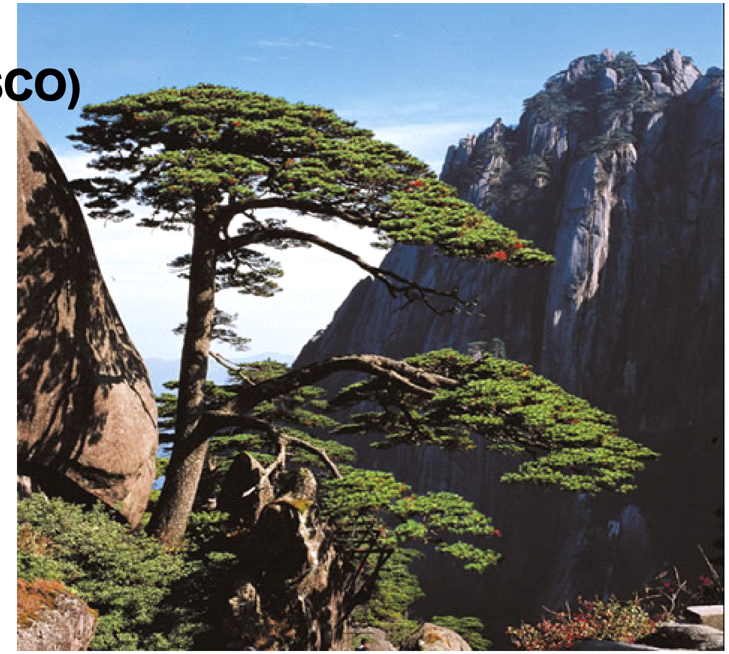
Where is Hefei?



Many flights every day from Beijing or Shanghai to Hefei

Yellow mountain

(World Natural & Cultural Heritages by UNESCO)



Xidi and Hongcun

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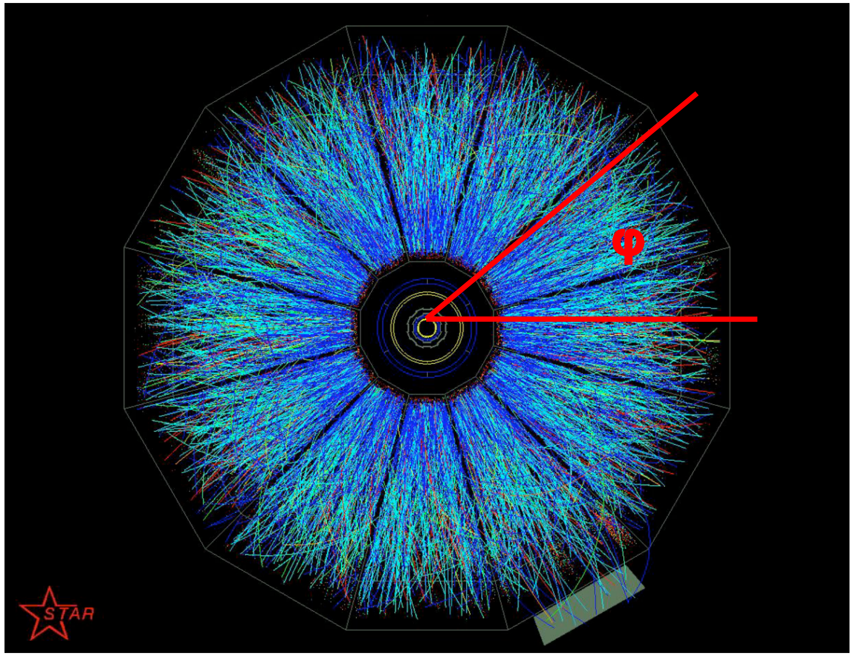
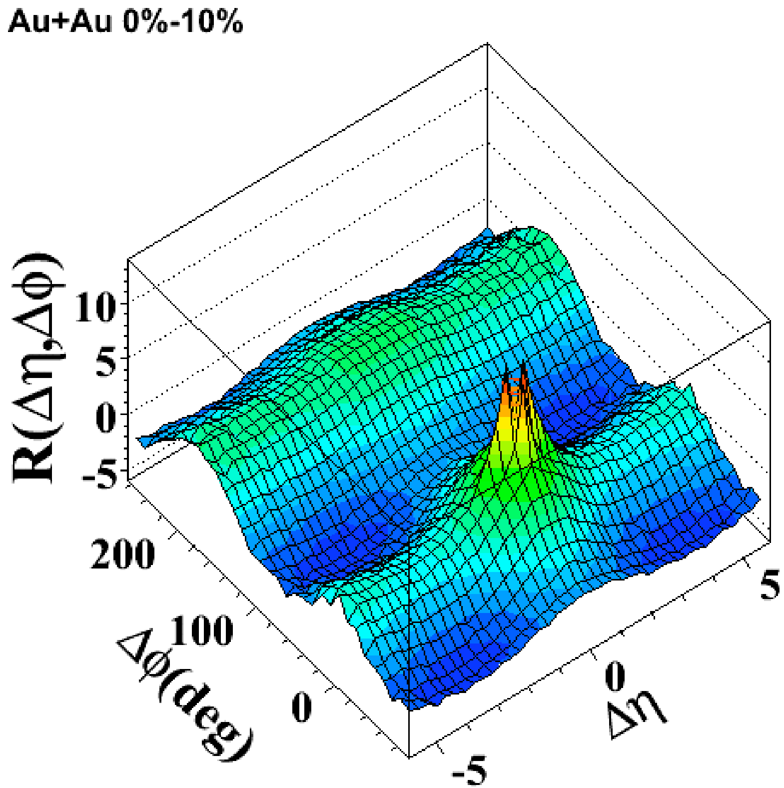
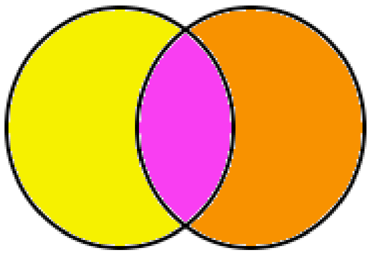
Correlation in transverse plane

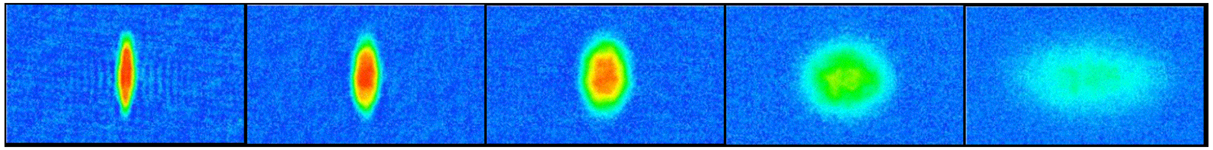




Talks: **Kettlemann, Jalilian-Marian,**
Putschke, Revol, Sawyer, Tejeda-Yeomans, et al.

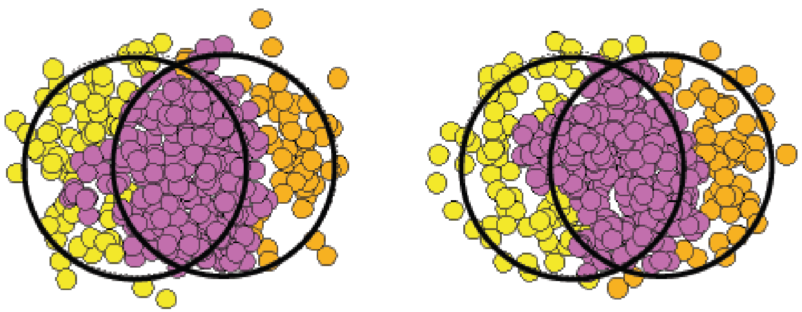
Elliptic and triangular flow

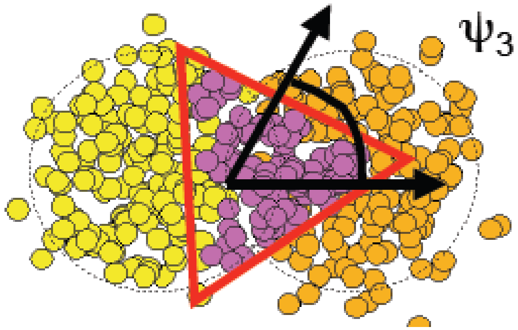




$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left\{1 + 2\sum_n v_n \cos[n(\phi - \psi_R)]\right\}$$

$$\begin{aligned} v_2 &= \langle \cos[2(\phi - \psi_R)] \rangle \\ v_3 &= 0 \end{aligned}$$





fluctuation in nucleon position leads to triangular flow

$$\begin{aligned} v_2 &= \langle \cos[2(\phi - \psi_2)] \rangle \\ v_3 &= \langle \cos[3(\phi - \psi_3)] \rangle \end{aligned}$$

Alver, Roland, PRC81, 054905 (2010)

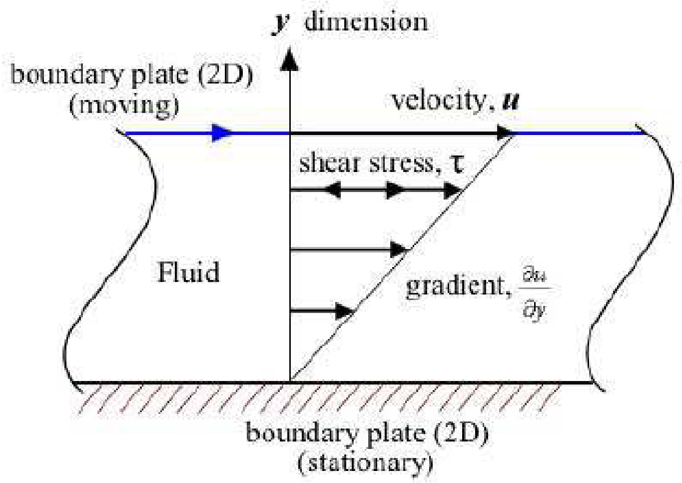
Shear viscosity

Transverse correlations

What is viscosity related to RHIC

viscosity = resistance of liquid to shear forces (and hence to flow)

Viscosity: introduced by Claude Navier in 1822 into what would be later called the Navier-Stokes equation.



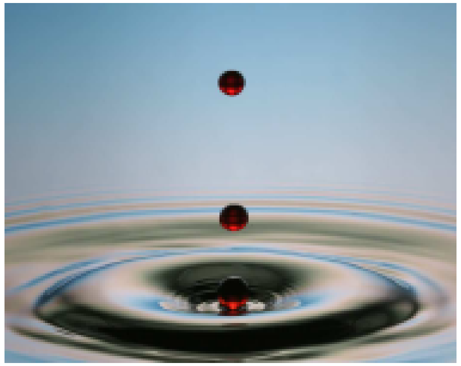
Friction force between two plates:

$$F = \eta A \frac{\partial u_x}{\partial y}$$

Shear viscosity in ideal gas and liquid

- ideal gas, high T

$$\boxed{\eta \quad = \quad \frac{1}{3} n p l_{mfp}, \quad p \sim \sqrt{T}\,(\uparrow), \quad \text{as}\, T \uparrow}$$



- liquid, low T

$$\boxed{\eta \quad \sim \quad n e^{E/T}\,(\downarrow), \quad \text{as}\, T \uparrow, \quad E \sim \text{activating energy}}$$

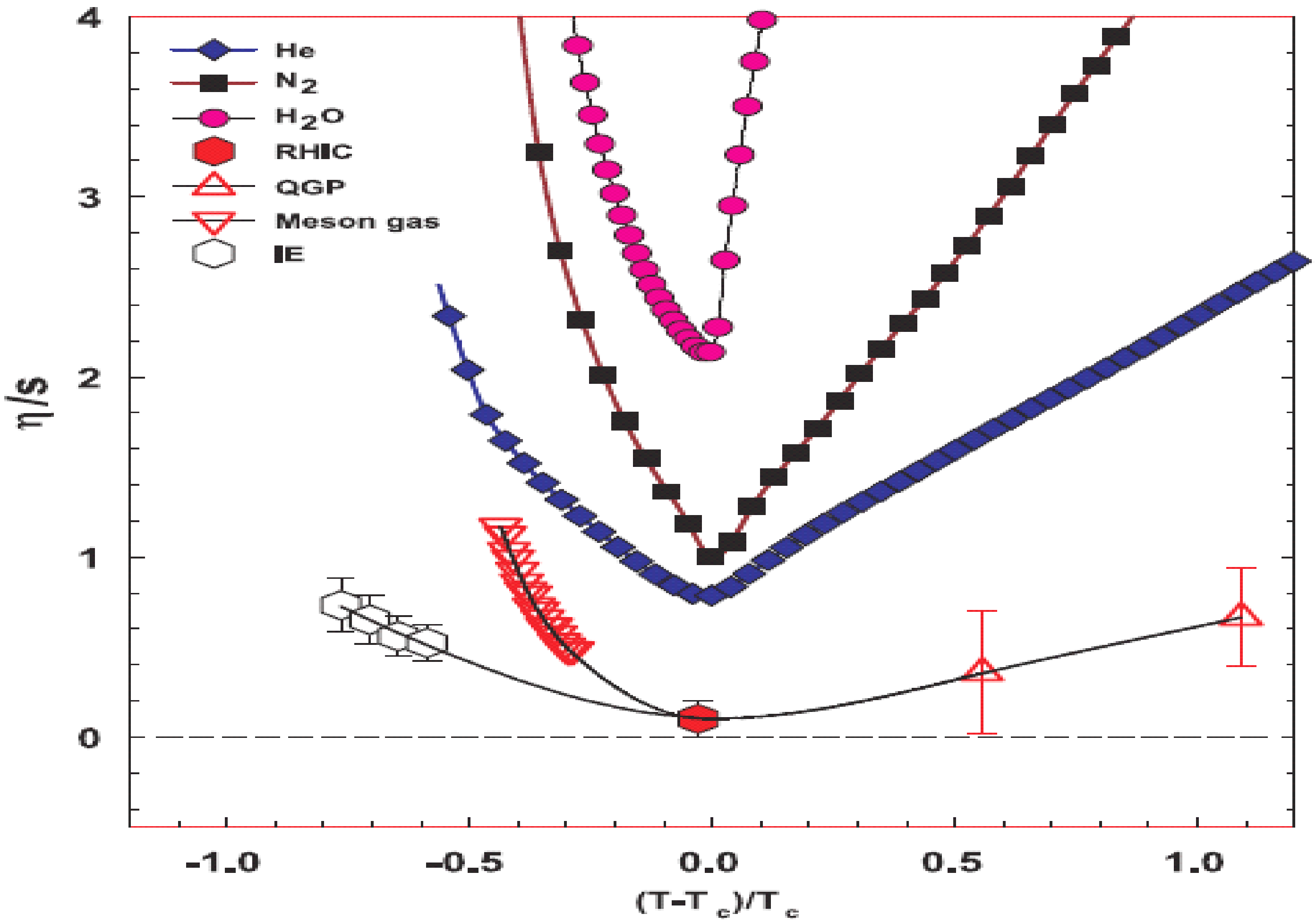
Frenkel, 1955

- lower bound by uncertainty principle

$$\boxed{l_{mfp} \quad \gtrsim \quad \frac{1}{p}} \Longrightarrow \boxed{\eta \quad = \quad \frac{1}{3} n p l_{mfp} \sim s}$$

Danielewicz, Gyulassy, 1985

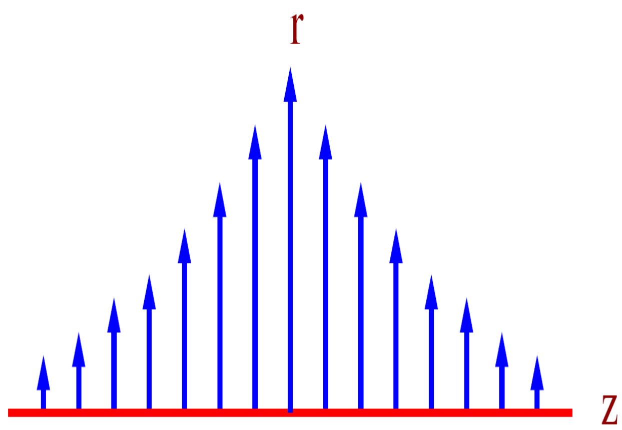
Ratio of Shear Viscosity to entropy density



Lacey et al, PRL98, 092301 (2007)

Momentum fluctuation as tool to measure shear viscosity (1)

Cylindrical coordinate $X^\mu = (t, z, r, \phi)$



Energy-Momentum tensor for fluid

$$\boxed{\begin{aligned} T^{\mu\nu} &= (\epsilon + P) u^\mu u^\nu - P g^{\mu\nu} + \pi^{\mu\nu} \\ u^\mu &= \gamma_T (1, 0, v_T(z), 0) \\ \gamma_T &= \frac{1}{\sqrt{1 - v_T^2(z)}} \end{aligned}}$$

EM conservation leads to equation for fluctuation

[Gavin, Abdel-Aziz, PRL 2006]

$$\boxed{\begin{aligned} &\partial_0 T^{0r} + \partial_z T^{zr} = 0 \\ &T^{0r} = (\epsilon + P)\gamma_T^2 v_T(z) \\ &T^{zr} = -\eta \frac{d v_T(z)}{dz} \end{aligned}} \xrightarrow{\text{local rest frame}} \boxed{\begin{aligned} \partial_0 \delta T^{0r} + \partial_z \delta T^{zr} &= 0 \\ \delta T^{0r} &= (\epsilon + P) \delta v_T \\ \delta T^{zr} &= -\eta \frac{d \delta v_T(z)}{dz} \end{aligned}}$$

Momentum fluctuation as tool to measure shear viscosity (2)

Diffusion equation for momentum density

$$\frac{\partial \delta v_T}{\partial t} = \nu \frac{\partial^2 \delta v_T}{\partial^2 z}$$

$$\frac{\partial \delta g_T}{\partial t} = \nu \frac{\partial^2 \delta g_T}{\partial^2 z}$$

$$\delta g_T = (\epsilon + P)\delta v_T$$

$$\nu \equiv \frac{\eta}{\epsilon + P} = \frac{\eta}{Ts}$$

Diffusion equation for momentum density in rapidity and proper time

$$\frac{\partial \delta g_T}{\partial \tau} = \frac{\nu}{\tau^2} \frac{\partial^2 \delta g_T}{\partial^2 y}$$

$$\begin{aligned} z &= \tau \sinh y \\ t &= \tau \cosh y \end{aligned}$$

Momentum fluctuation as tool to measure shear viscosity (3)

Covariance of momentum fluctuation

$$\boxed{r_g(x_1 - x_2) \quad = \quad \langle \delta g_T(x_1) \delta g_T(x_2) \rangle - \langle \delta g_T(x_1) \rangle \langle \delta g_T(x_2) \rangle}$$

$$\boxed{x \quad \equiv \quad (\tau, y)}$$

Diffusion equation for covariance

$$\boxed{\frac{\partial \Delta r_g}{\partial \tau} \quad = \quad \frac{\nu}{\tau^2} \frac{\partial^2 \Delta r_g}{\partial^2 y}, \qquad \Delta r_g \equiv r_g - r_g^{eq}}$$

Δr_g **is broadened by diffusion driven by shear viscosity**

Momentum fluctuation as tool to measure shear viscosity (4)

Connection to observable

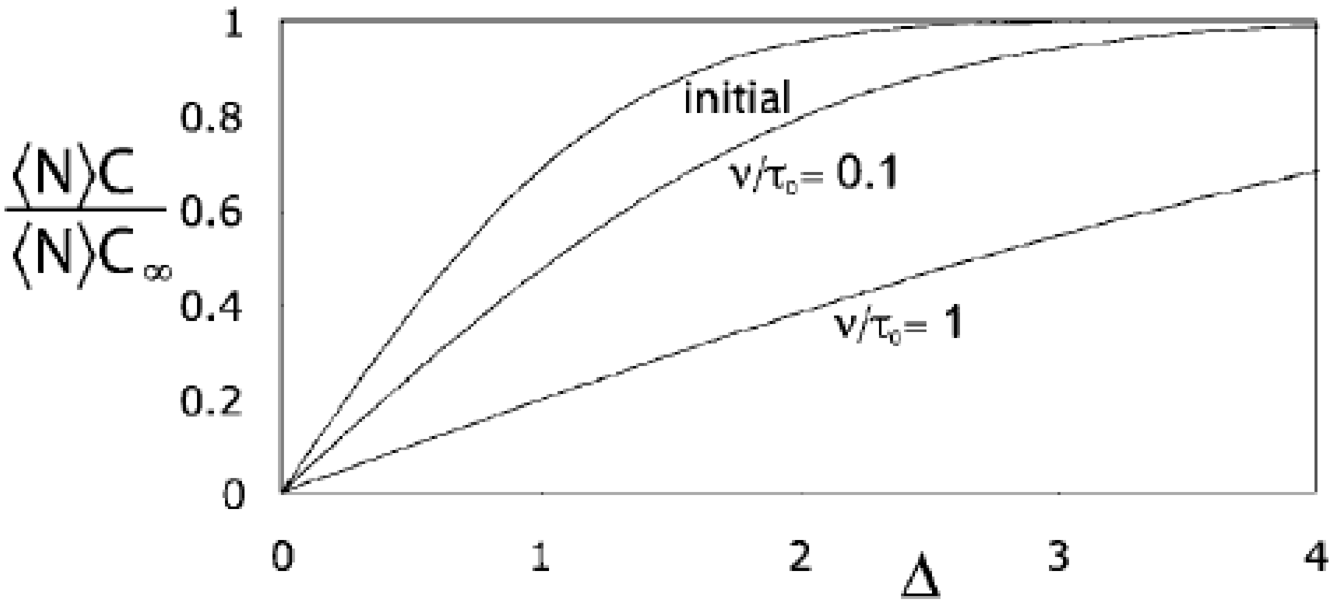
$$\boxed{\begin{aligned} C &= \frac{1}{\langle N \rangle^2} \left[\left\langle \sum_{i \neq j} p_{Ti} p_{Tj} \right\rangle - \left\langle \sum p_{Ti} \right\rangle^2 \right] \\ &= \frac{1}{\langle N \rangle^2} \int dx_1 dx_2 \Delta r_g (x_1 - x_2) \end{aligned}}$$

- **i labels particles from each event**
- **brackets represent the event average**

Shear viscosity can broaden the rapidity correlations of the momentum current. This broadening can be observed by measuring the transverse momentum covariance as a function of rapidity acceptance.

Momentum fluctuation as tool to measure shear viscosity (5)

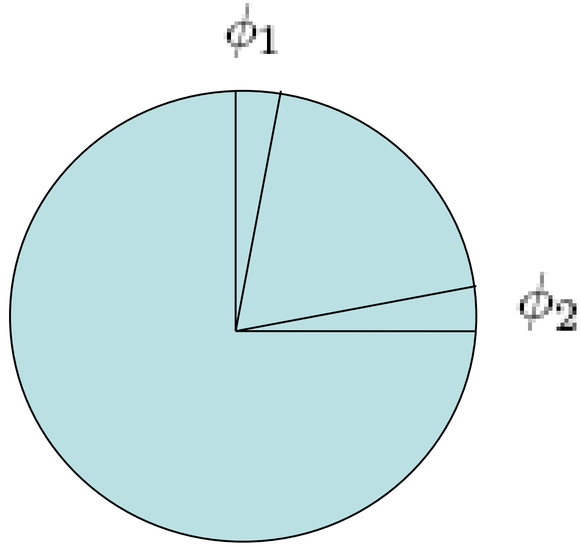
Rapidity correlations to measure the shear viscosity



[Gavin, Abdel-Aziz, PRL 2006]

Azimuthal correlation in transverse momenta

Transverse plane can be separated to ϕ-bins



Focus on two ϕ-bins 1 and 2, define correlation function

$$C_{12} = \frac{1}{\langle N_1 \rangle \langle N_2 \rangle} \left[\sum_{i,j} \langle p_{T1,i}\, p_{T2,j} \rangle - \left\langle \sum_i p_{T1,i} \right\rangle \left\langle \sum_j p_{T2,j} \right\rangle \right]$$

particle numbers in bins 1 and 2

i,j: particles in bins 1 and 2

average is taken over events

Diffusion equation for azimuthal correlation in central collisions

Cylindrical coordinates, metrics and velocity

$$\boxed{\begin{aligned} X^\mu &= (\tau, \eta, r, \phi) \\ g_{\mu\nu} &= \mathrm{diag}(-1, \tau^2, 1, r^2) \\ u^\mu &= \gamma_T(1, 0, v_T(\tau, r, \phi), 0) \end{aligned}}$$

central collision

Energy-momentum tensor

$$\begin{aligned} T^{\mu\nu} &= (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \pi^{\mu\nu} \\ &= \begin{pmatrix} -P + (\epsilon + P)\gamma_T^2 & 0 & (\epsilon + P)\gamma_T^2 v_T & 0 \\ 0 & P/\tau^2 & 0 & 0 \\ (\epsilon + P)\gamma_T^2 v_T & 0 & P + (\epsilon + P)\gamma_T^2 v_T^2 & 0 \\ 0 & 0 & 0 & P/r^2 \end{pmatrix} + \pi^{\mu\nu} \end{aligned}$$

$$\begin{aligned} T^{\phi r} &= \pi^{\phi r} = -2\eta \left\langle \nabla^\phi u^r \right\rangle = -\eta \gamma_T^3 \frac{1}{r^2} \partial_\phi v_T \\ T^{\alpha\beta} &= T_0^{\alpha\beta} + \pi^{\alpha\beta} = T_0^{\alpha\beta} - 2\eta \left\langle \nabla^\alpha u^\beta \right\rangle, \qquad \alpha\beta = \tau r, rr, \phi\phi \end{aligned}$$

Diffusion equation for azimuthal correlation in central collisions

EM conservation leads to

$$\nabla_\mu T^{\mu\nu} = 0 \quad \Longrightarrow \quad \frac{\eta}{r^2}\frac{\partial}{\partial \phi}\left(\gamma_T^3 \frac{\partial v_T}{\partial \phi}\right) = \frac{1}{\tau}\frac{\partial}{\partial \tau}\left[\tau(\epsilon+P)\gamma_T^2 v_T\right] + \frac{\partial P}{\partial r} + \frac{1}{r}(\epsilon+P)\gamma_T^2 v_T^2 + \frac{\partial}{\partial r}\left[(\epsilon+P)\gamma_T^2 v_T^2\right]$$

Diffusion equation for azimuthal covariance

$$v_T \rightarrow v_T + \delta v_T \quad \Longrightarrow \quad \frac{\eta}{r^2}\gamma_T^3 \frac{\partial^2 \delta v_T}{\partial \phi^2} = \frac{1}{\tau}\frac{\partial}{\partial \tau}\left[\tau(\epsilon+P)\gamma_T^4(1+v_T^2)\delta v_T\right] + \frac{2}{r}\frac{\partial}{\partial r}\left[r(\epsilon+P)\gamma_T^4 v_T \delta v_T\right]$$

local rest frame

$$\Longrightarrow \quad \frac{\partial}{\partial \tau}(\tau \delta v_T) = \frac{\eta}{r^2 T s}\frac{\partial^2}{\partial^2 \phi}(\tau \delta v_T)$$

Solve diffusion equation in general frame

1. Initial condition essemble given by HIJING

$$\boxed{\delta v_T(\phi, \tau_0)} \qquad \boxed{\tau_0 = 0.6 \text{ fm}}$$

2. Solve fluid equation to determine thermodynamic quantities as input to diffusion equation

$$\boxed{\epsilon_f = 0.075 \text{ GeV}/\text{fm}^3}$$

freeze-out energy density

$$\boxed{\nabla_\mu T^{\mu\nu} = 0 \qquad \Longrightarrow \qquad \epsilon(\tau, r),\ P(\tau, r),\ v_T(\tau, r)}$$

3. Sovle evolution for δv_T to obtain essemble at freeze-out

$$\begin{aligned} \frac{\eta}{r^2}\gamma_T^3 \frac{\partial^2 \delta v_T}{\partial \phi^2} &= \frac{1}{\tau}\frac{\partial}{\partial \tau}\left[\tau(\epsilon + P)\gamma_T^4(1 + v_T^2)\delta v_T\right] \\ &+ \frac{2}{r}\frac{\partial}{\partial r}\left[r(\epsilon + P)\gamma_T^4 v_T \delta v_T\right] \end{aligned}$$

$\Longrightarrow$ **essemble for δv_T at freeze-out**

Solve diffusion equation in general frame

4. Observables via average over freeze-out hyper-surface

$$\boxed{\begin{aligned}\delta E_T &= \int d\Sigma_\mu u_\nu \delta T^{\nu\mu} \\ &= \Delta\eta \int \left(v_T - \frac{\partial \tau}{\partial r} \right) \gamma_T \delta g_T r \tau dr d\phi\end{aligned}}$$

$$d\Sigma_\mu = (-1, 0, \frac{\partial \tau}{\partial r}, 0) r \tau dr d\phi d\eta$$

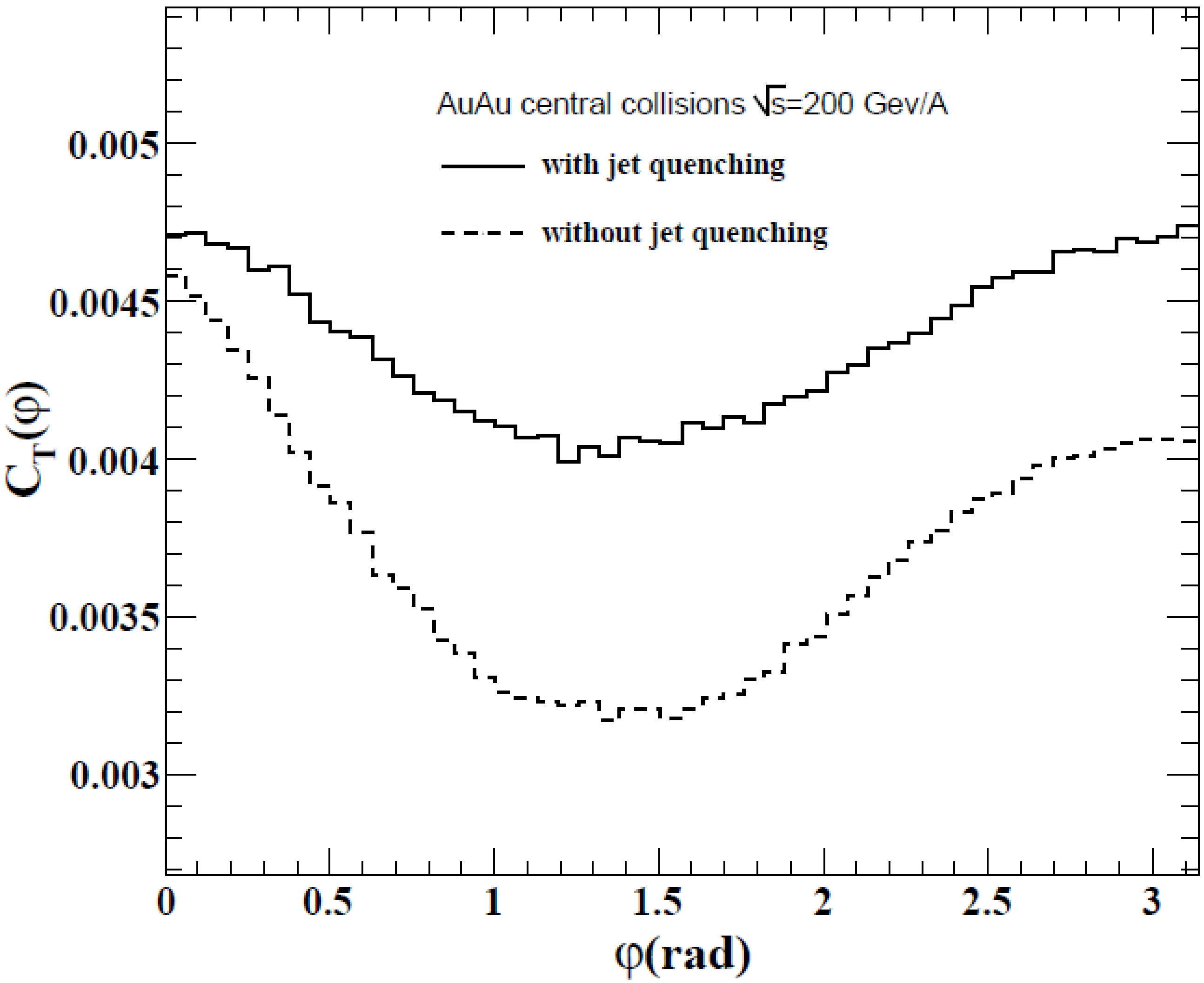
$$\delta g_T(\tau, r, \phi) = \gamma_T^2 (\epsilon + P) \delta v_T$$

5. Compute the azimuthal correlation of the transverse energy at freeze-out, average taken over essemble

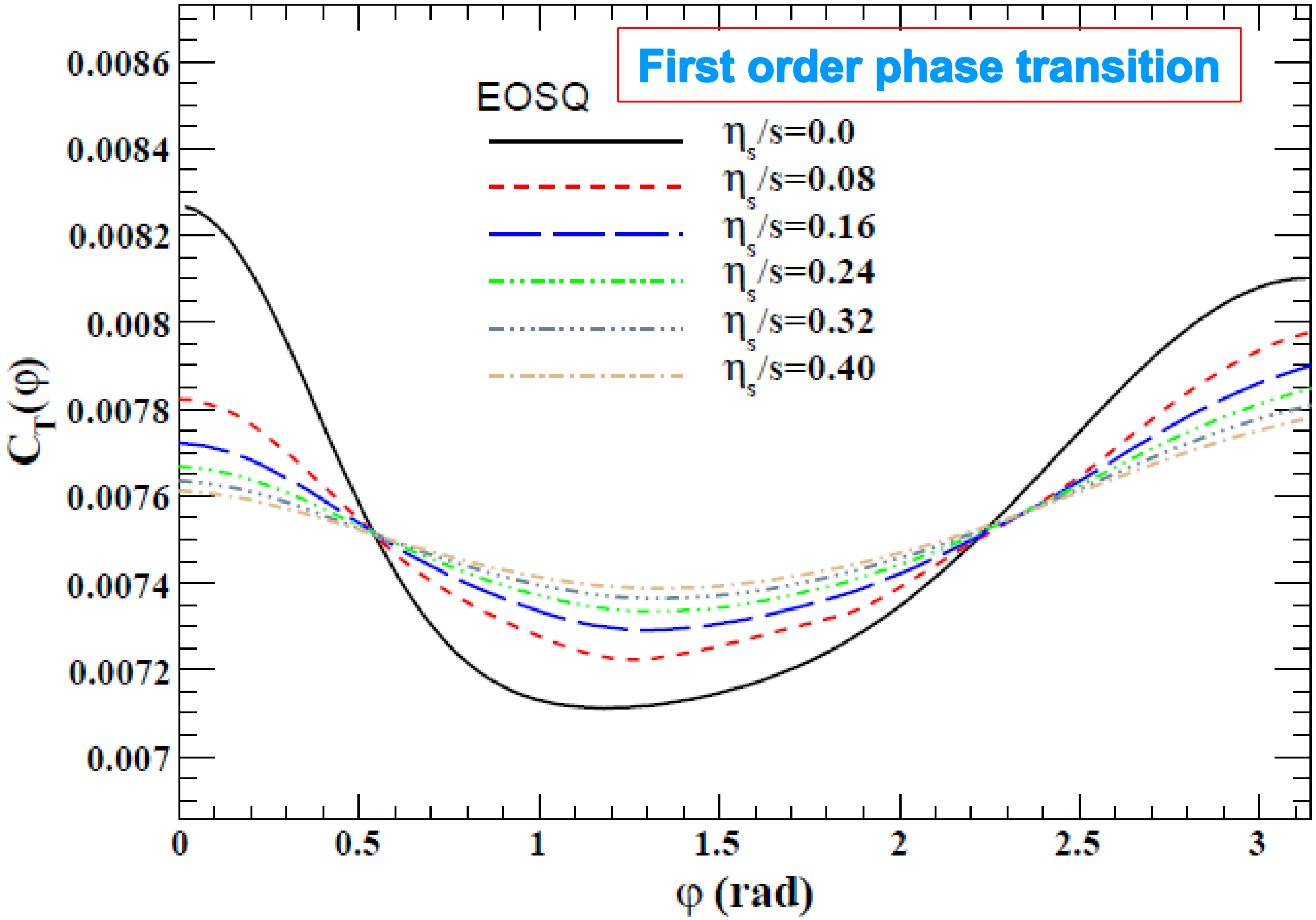
$$\boxed{\begin{aligned}C_T(\varphi) = \frac{(\Delta\eta)^2 (\delta\phi)^2}{\langle E_T(\phi)\rangle^2_{\text{out}}} \int_{\text{out}} r_1 \tau_1 dr_1 \int_{\text{out}} r_2 \tau_2 dr_2 \\ \times \langle \delta g_T(\tau_1(r_1), r_1, \phi) \delta g_T(\tau_2(r_2), r_2, \phi + \varphi) \rangle_{\text{out}} ,\end{aligned}}$$

average over essemble

Azimuthal correlation in transverse momenta result from HIJING

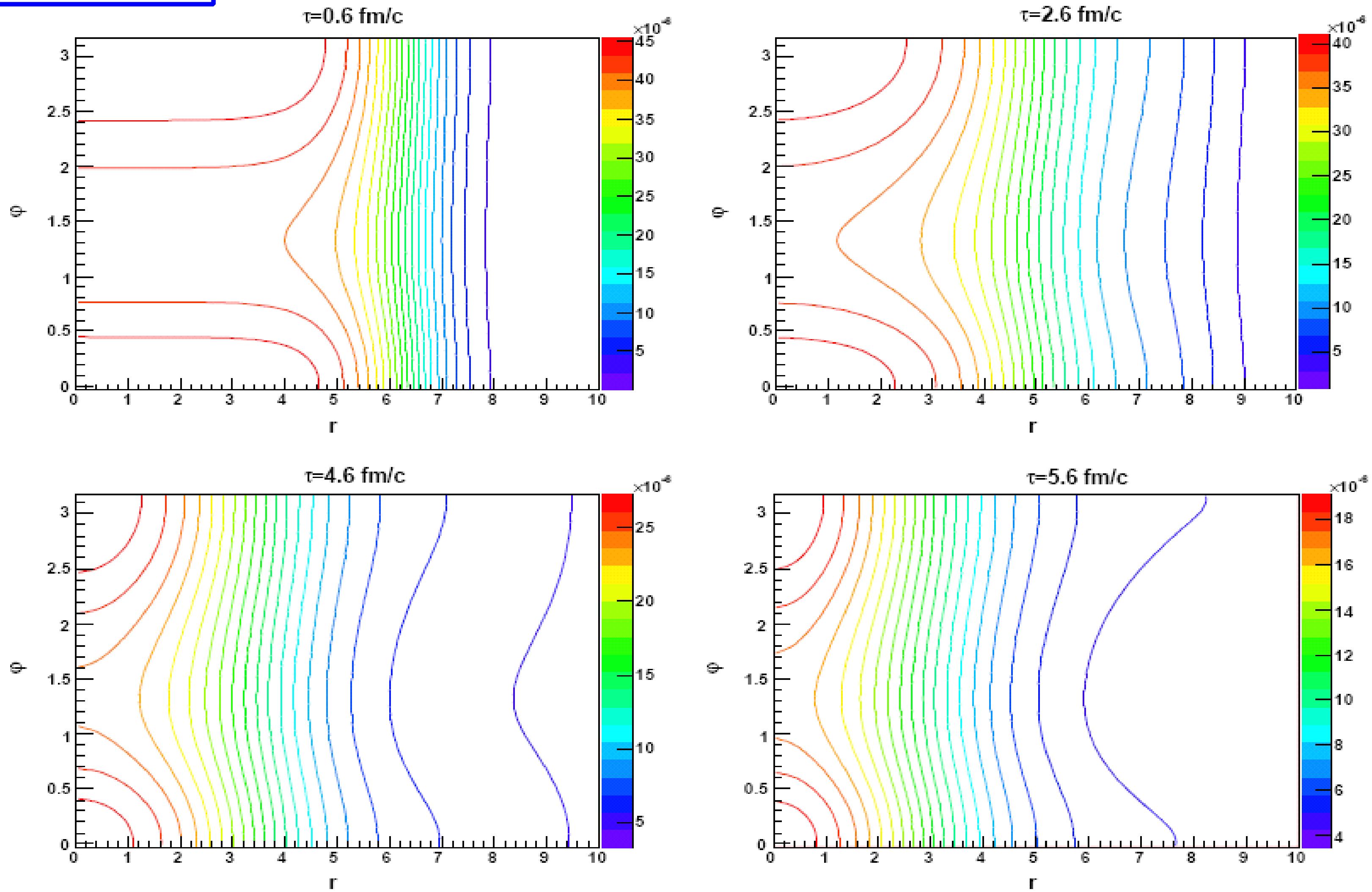


Results for azimuthal correlation(1)



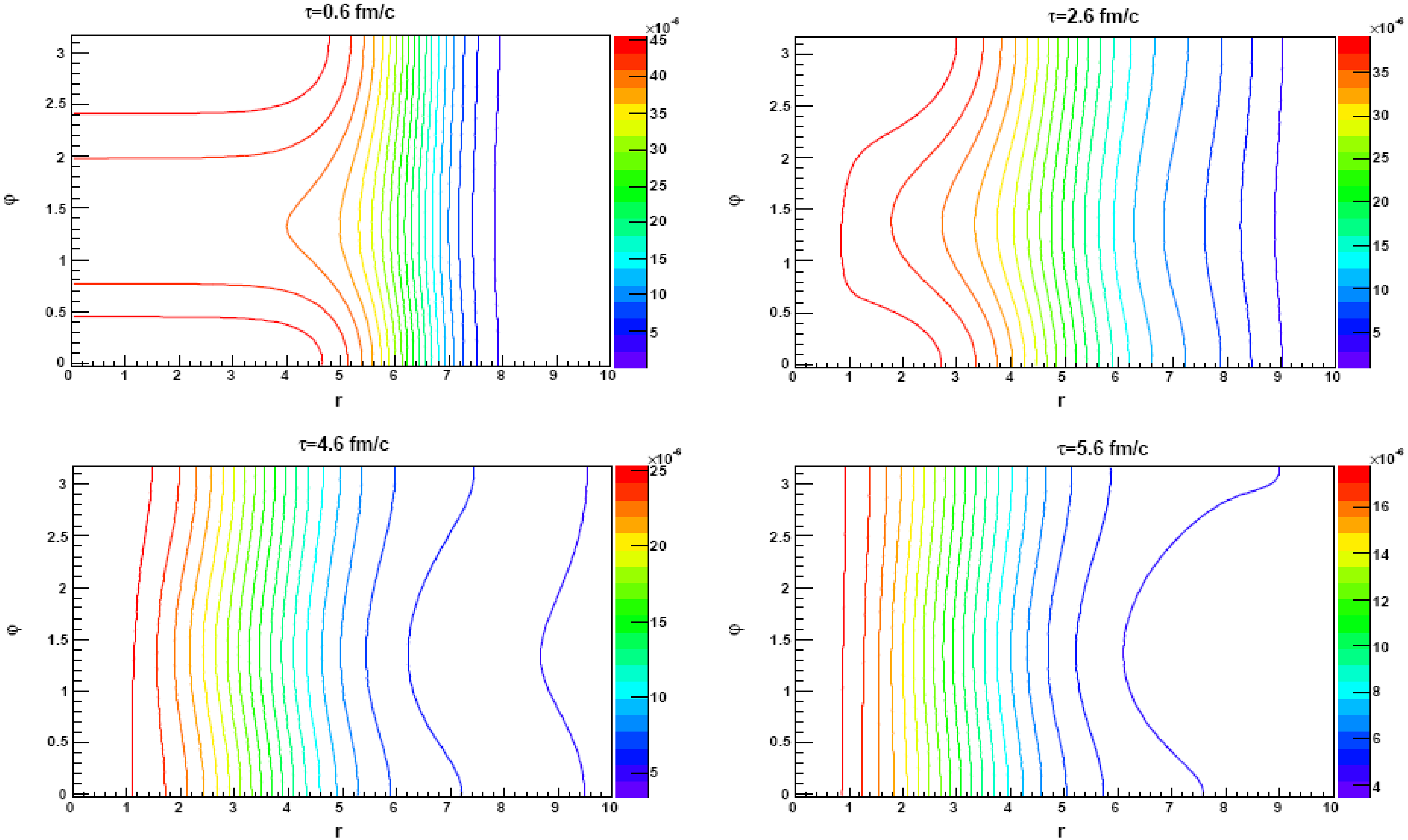
Results for azimuthal correlation (2)

$$\boxed{\eta/s \quad = \quad 0}$$



Results for azimuthal correlation (3)

$\eta/s = 0.12$



Summary and conclusion

1. A diffusion equation for azimuthal correlation for transverse momentum is derived a general Lorentz frame.

2. Mini-jet thermalization is shown in the correlation

3. Azimuthal correaltion as a measure for shear viscosity

4. For a future study, we can choose Glauber initial conditions