Non-perturbative particle production mechanism in time-dependent strong non-Abelian fields

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Abstract. Non-perturbative production of quark-antiquarks is investigated in the early stage of heavy-ion collisions. The time-dependent study is based on a kinetic description of the fermion-pair production in strong non-Abelian fields. We introduce time-dependent chromo-electric external field with a pulse-like time evolution to simulate the overlap of two colliding heavy ions. We have found that the small inverse duration time of the field pulse determines the efficiency of the quark-pair production. The expected suppression for heavy quark production, as follows from the Schwinger formula for a constant field, is not seen, but an enhanced heavy quark production appears at ultrarelativistic energies. We convert our pulse duration time-dependent results into collisional energy dependence and introduce energy and flavour-dependent string tensions, which can be used in string based model calculations at RHIC and LHC energies.

Keywords: Quark-antiquark pair, Schwinger mechanism, strong fields, non-Abelian field
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INTRODUCTION

The main aim of ultrarelativistic heavy-ion collisions is to create extreme high energy densities and study the deconfinement phase transition of colored quarks and gluons. Experiments at the BNL Relativistic Heavy Ion Collider (RHIC) have been investigated the center of mass colliding energy region up to $\sqrt{s} = 200$ AGeV and detectors at CERN Large Hadronic Collider (LHC) are exploring the energy range up to $\sqrt{s} = 5500$ AGeV. At such high energies the colliding nuclei are two colliding sheets of nucleons with a huge Lorentz-contraction ($\gamma_{cm} = 100$ at RHIC and $\gamma_{cm} = 2750$ at LHC), surrounded by a gluon cloud. Their overlap results in a strong chromo-electric and chromo-magnetic field to be built up. Particles, namely gluons and quark-antiquark pairs are produced from this strong field, similarly to the Schwinger mechanism in quantum electrodynamics (QED) [1]. The particle production rate depends on the field strength, which is varying in time. The soft particles produced in such a non-perturbative way form the bulk of the quark-gluon plasma, after they successfully thermalized. Here we investigate the primordial non-perturbative production of light and heavy quark-antiquark pairs.

Theoretical descriptions of particle production in high energy pp collisions are based on the introduction of chromoelectric flux tube ('string') models [2, 3, 4, 5, 6, 7]. String picture is a good example of how to convert the kinetic energy of a collision into field energy, than later on gain the stored kinetic energy back. At RHIC and LHC energies the string density is expected to be so large that a strong collective gluon field will be formed in the whole available transverse volume. Furthermore, the gluon number could be so high that a classical gluon field as the expectation value of the quantum field can be considered in the reaction volume [8, 9, 10]. Alternatively, at extremely high energies, nucleus nucleus collisions can be described as two colliding sheets of Colored Glass Condensate. In the framework of this model, it was shown that in the early stage of collision longitudinal color-electric and color-magnetic fields are created. The properties of such non-Abelian classical fields and gluon production were studied very intensively during the last years, especially asymptotic solutions (see Refs. [11, 12, 13]).

Fermion and boson pair production was investigated by different models of particle production from strong Abelian [14, 15, 16, 17, 18], and non-Abelian [19, 20, 21] fields. These calculations concentrated mostly on the bulk properties of the gluon and quark matter, the time evolution of the system, the time dependence of energy and particle number densities, and the appearance of fast thermalization.

In our related works (see Refs. [22, 23, 24, 25]), we investigated massless fermion and boson production in strong Abelian and non-Abelian external electric field with a pulse-like time dependence. We have described light, strange, charm and bottom quark-pair production. We have realized, that the role of mass becomes unimportant when the collisional energy is increasing and the the pulse duration time becomes comparable to the inverse quark mass [24, 25]. Motivated by the problems raised in Refs. [9, 10] we investigated the energy dependence of effective string tensions introduced to describe the obtained quark-antiquark pair production. Instead of the usual $\kappa \sim 1$ GeV/fm value, much higher effective string tensions appeared in our calculations.

THE KINETIC EQUATION FOR THE WIGNER FUNCTION

The equation of motion for color Wigner function of a homogeneous system $W(\mathbf{k},t)$ (see Refs. [22, 23, 24, 25] for details)

$$\partial_t W + \frac{g}{8}\frac{\partial}{\partial k_i}\left(4\{W,F_{0i}\} + 2\left\{F_{i\nu},[W,\gamma^0\gamma^\nu]\right\} - \left[F_{i\nu},\{W,\gamma^0\gamma^\nu\}\right]\right) = \\ = ik_i\{\gamma^0\gamma^i,W\} - im[\gamma^0,W] + ig\left[A_i,[\gamma^0\gamma^i,W]\right]. \quad (1)$$

Here m denotes the current mass of the fermions, g is the coupling constant, A_μ is the 4-potential of an external space-homogeneous color field and $F_{\mu\nu}$ is the corresponding field tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]. \quad (2)$$

The color decomposition of the Wigner function with SU(N_c) generators in the fundamental representation is given by

$$W = W^s + W^a t^a, \quad a = 1,2,...,N_c^2-1\ , \quad (3)$$

where W^s is the color singlet part and W^a is the color multiplet components. It is also convenient to perform spinor decomposition separating scalar a, vector b_μ, tensor $c_{\mu\nu}$, axial vector d_μ and pseudo-scalar parts e:

$$W^{s|a} = a^{s|a} + b_\mu^{s|a}\gamma^\mu + c_{\mu\nu}^{s|a}\sigma^{\mu\nu} + d_\mu^{s|a}\gamma^\mu\gamma^5 + ie^{s|a}\gamma^5. \quad (4)$$

After the color and spinor decomposition of the equations for the Wigner function in case of pure longitudinal external color field with fixed color direction $A_z^a = A_z^o n^a$, where $n^a n^a = 3$ and $\partial_t n^a = 0$ [23], we obtain system of differential equations for the singlet and triplet components, as we described in Ref. [25].

The distribution function for massive fermions is defined by the components a, b [23]:

$$f_q(\mathbf{k},t) = \frac{m a^s(\mathbf{k},t) + \mathbf{k}\,\mathbf{b}^s(\mathbf{k},t)}{\omega(\mathbf{k})} + \frac{1}{2}, \tag{5}$$

where $\omega(\mathbf{k}) = \sqrt{\mathbf{k}^2 + m^2}$. For time- and momentum-dependent distribution functions scalar a, vector b_μ, axial vector d_μ, and tensor c components of the Wigner function are needed, only. The initial conditions for the Wigner function in vacuum reads as $a^s = -m/2\omega$ and $\mathbf{b}^s = -\mathbf{k}/2\omega$.

NUMERICAL RESULTS FROM THE KINETIC EQUATION

In the numerical calculation we have used the following parameters: the maximal magnitude of the field $E_0 = 0.68$ GeV/fm; the strong coupling constant $g = 2$; the current quark masses $m_{u,d} = 8$ MeV, $m_s = 150$ MeV, $m_c = 1.2$ GeV, $m_b = 4.2$ GeV for light, strange, charm and bottom quarks, correspondingly. The value of maximal magnitude of the field corresponds to the effective string tension $\kappa \sim 1.17$ GeV/fm [25].

The particle production is ignited by a pulse-like color field simulating a heavy-ion collision [23]:

$$E^o(t) = E_0 \cdot \left[1 - \tanh^2(t/\tau)\right], \tag{6}$$

where τ is a pulse duration time.

The suppression factor of heavier quark Q to light one u is defined in the asymptotic future (c.f. [1]), $t \gg \tau$, as

$$\gamma^Q = \lim_{t\to\infty} n_Q(t)/n_u(t). \tag{7}$$

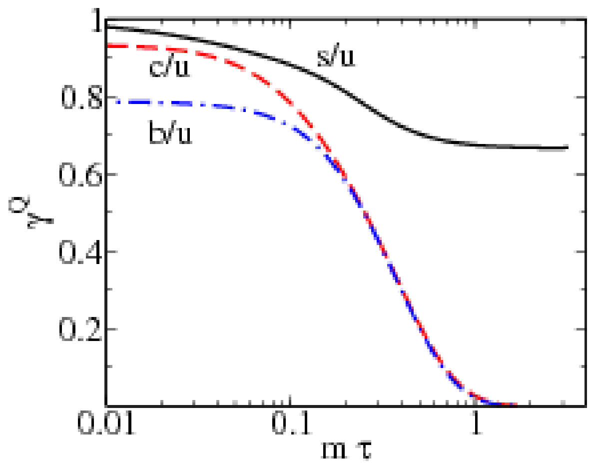

FIGURE 1. The suppression factor for heavy flavours at different values of $m\tau$.

Here $n_q(t)$ is the number density of corresponding quarks given by

$$n_q(t) = 4N_c \int \frac{d^3k}{(2\pi)^3} f_q(\mathbf{k},t) \,, \tag{8}$$

where q denotes different quark flavours, q=u, d, s, c, b.

In Fig. 1 we display our numerical results in case of SU(2)-color field for the pulse duration time dependence of the suppression factor for different flavours. A dramatic change of the suppression factors for heavy quarks happens in the region $0.1 < m\tau < 1$, where m is the bare quark mass. Actually, as it was shown in Ref. [25], there are two dimensionless parameters that control the behaviour of the particle production. One is the dimensionless pulse duration, $m\tau$; the other is adiabaticity parameter, $\Gamma_K \sim m/(E_0\tau)$. The Schwinger formula is valid for the combination $m\tau \gg 1$ and $\Gamma_K \ll 1$.

To fix free parameters we use the following simple model. The field strength depends on two unknown parameters, τ and E_0. We will fix them as the best fit of the suppression factors for primordial strange and charm quarks obtained in a quark coalescence calculation [29, 30] at RHIC energy, $\sqrt{s} = 200$ AGeV. The suppression factors are $\gamma^s = 0.88$ and $\gamma^c = 6 \cdot 10^{-2}$. The best fit reads $E_0 = 0.68$ GeV/fm and $\tau_0 = 0.134$ fm$/c$. Surprisingly, our simple model provides reasonable values for these parameters.

From intuitive reasons the duration of field pulse is proportional to the time of two Lorenz-contracted heavy ions pass each other at almost speed of light, i.e.

$$\tau \simeq \alpha \frac{2R}{\gamma_{cm}}, \tag{9}$$

where R is the radius of a nuclei, $\gamma_{cm} \simeq \sqrt{s}/(2\text{GeV})$ is the gamma-factor, α is an unknown proportionality coefficient. In case of gold-gold collision at RHIC energy we obtained $\alpha = 0.96$ from the best fit values given above. We further assume that α weakly depends on the collision energy and this dependence can be neglected. Thus having the value of α in hand we can transform the duration time dependence to the collision energy dependence. Although this conversion is oversimplified (e.g. it does not take into account any stopping effects), we expect to obtain results of the right order of magnitude. The extracted numbers make possible to interpret our numerical results on realistic basis, namely energy scales.

TABLE 1. The suppression factor, γ^Q, for experimentally favored energies. The calculations are done with a string tension $\kappa \simeq 1.17$. The result obtained by Schwinger formula is denoted by γ_∞.

	γ_∞	**130 AGeV**	**200 AGeV**	**1 ATeV**	**2 ATeV**	**5.5 ATeV**
s	0.74	0.84	0.88	0.96	0.98	0.99
c	$3{\cdot}10^{-9}$	$9{\cdot}10^{-3}$	0.06	0.66	0.82	0.91
b	~ 0	~ 0	10^{-6}	0.15	0.45	0.72

Effective string tension

The suppression factors, γ^Q, (see Table 1) deviate from the asymptotic Schwinger values so much, that we need to drop the idea of applying one general string tension for every quark flavour. If we want to use the results of our numerical calculations in one of the string-based Monte-Carlo models [2, 3, 4, 5, 6], then we must introduce effective flavour dependent string tension values. This way we can investigate the consequences of the enhanced heavy flavour production in full scale Monte-Carlo models, as it was demonstrated in latest HIJING-$B\overline{B}$ calculations [10]. Here we calculate these effective string tension values for different quark flavours and offer them for later use.

Let us recall the Schwinger formula for particles with mass m

$$\frac{dN}{dt\,d^3x} = \frac{\kappa^2}{4\pi^3}\exp\left(-\frac{\pi m^2}{\kappa}\right). \tag{10}$$

Here κ is a string tension. According to this formula the suppression factor of heavier (Q) to light quarks (q) is given by

$$\gamma_\infty^Q = \exp\left(-\frac{\pi(m_Q^2 - m_q^2)}{\kappa}\right). \tag{11}$$

This formula is valid for the case of arbitrary N in SU(N), see Ref. [31].

In parallel, on the basis of Eq.(10), we can derive a similar suppression factor by introducing "flavour specific" effective string tensions for heavier flavours.

$$\hat{\gamma}_\infty^Q = \left(\frac{\hat{\kappa}_{\rm eff}^Q}{\hat{\kappa}_{\rm eff}^u}\right)^2 \exp\left(-\pi\frac{m_Q^2}{\hat{\kappa}_{\rm eff}^Q} + \pi\frac{m_u^2}{\hat{\kappa}_{\rm eff}^u}\right). \tag{12}$$

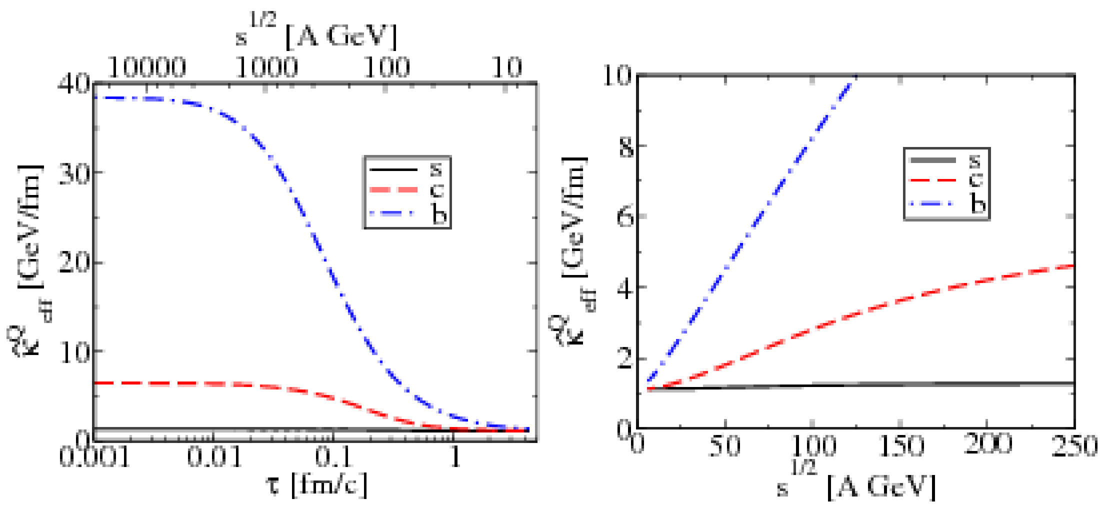

FIGURE 2. The "flavour specific" effective string tension, $\hat{\kappa}_{eff}^Q$ as a function of pulse duration time, τ, and collision energy, $\sqrt{s}$, extracted from the values of Fig. 1 (left panel). The same $\hat{\kappa}_{eff}^Q$ values in linear scales, but zoomed to RHIC energy range (right panel).

Here we will keep the usual string tension for light quark, as $\hat{\kappa}^{u}_{\rm eff} \simeq 1.17$ GeV/fm. In this way we can extract the "flavour specific" effective string tensions from our numerical results displayed on Fig. 1, and we obtain Fig. 2. The left panel displays the energy dependence in a logarithmic scale. The right panel is in linear scales, and zoomed to the RHIC energy range.

As a summary, we emphasize the importance of time-dependent strong field calculations to determine quark-antiquark pair production in heavy-ion collisions. The field theoretical results can be substituted into Monte-Carlo string models through flavour (and energy) dependent effective string tensions.

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