El reto de sabor con cuerdas

Saúl Ramos-Sánchez

BSM & Astroparticles

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De colaboraciones con

M-C. Chen, V. Knapp-Pérez, M. Ramos-Hamud, M. Ratz & S. Shukla: 1909.06910 & 2108.02240

A. Baur, M. Kade, H.P. Nilles & P. Vaudrevange: 2001.01736, 2008.07534, 2010.13798, 2104.03981, 2107.10677,...

Y. Olguín-Trejo & R. Pérez-Martínez: 1808.06622 & 2105.03460

Flavor puzzle

Despite the great success of the SM

- Need to explain
  - three flavors of SM particles
  - observed mass hierarchies
  - observed quark and lepton mixing *textures*
  - CP violation in CKM and PMNS
  - neutrino physics
  - ...

$$\begin{pmatrix} 0.974 & 0.224 & 0.0039 \\ 0.218 & 0.997 & 0.042 \\ 0.008 & 0.039 & 1.019 \end{pmatrix}_{CKM}, \qquad \begin{pmatrix} 0.829 & 0.539 & 0.147 \\ 0.493 & 0.584 & 0.645 \\ 0.262 & 0.607 & 0.75 \end{pmatrix}_{PMNS}$$

$m_{u_i} \sim 2.16, 1270, 172900$ MeV

$m_{d_i} \sim 4.67, 93, 4180$ MeV

$\Delta m^2_{21} = 7.4 \cdot 10^{-5}, \Delta m^2_{31(23)} \approx 2.5 \cdot 10^{-3}$ eV^2

$m_{e_i} \sim 0.511, 105.7, 1776.9$ MeV

normal ordering

[Talks by Myriam, Enrique, Antonio (yesterday)]

Approaches towards solving the flavor puzzle

Traditional: discrete non-Abelian flavor symmetries G_{flavor} lead to models for quarks and leptons with great fits, $\theta_{13} \neq 0$,… requiring careful choice of flavon sector and flavon vevs

see reviews by Ishimori, Kobayashi, Ohki, Okada, Shimizu, Tanimoto (2010); Feruglio, Romanino (2019)

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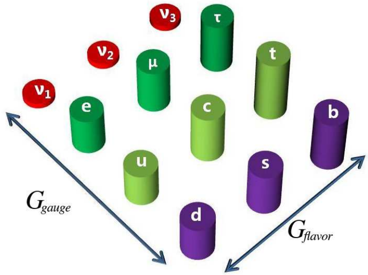

Matter fields transform as $\phi \to \underbrace{\rho_\phi(g)}_{\text{rep of } g}\,\phi, \quad g \in G_{flavor} = S_3, A_4, \ldots$

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flavon vev *alignment* is very challenging ☹

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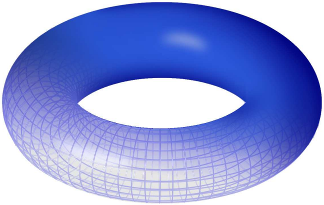

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- $\Gamma_N \cong S_3, A_4, S_4, A_5$ for $N = 2,3,4,5$

$n_Y \in 2\mathbb{Z}$

$\Rightarrow$ 9 ν observables (m_ν, θ_{ij}, phases) by fixing 3 parameters!

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- 4-fold cover $\widetilde{\Gamma}_4 \cong [96,67], \widetilde{\Gamma}_8 \cong [768, 1085324], \widetilde{\Gamma}_{12} \cong [2304, \ldots]$

$n_Y \in \mathbb{Z}/2 \quad \to \quad$ metaplectic

Liu,Ding(2019); Liu,Yau,Qu,Ding(2020)

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- $\Gamma/\ker(\varrho)$ with vector-valued modular forms

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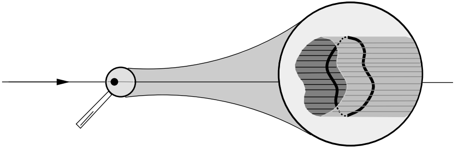
All about
STRINGS
We have resources for all
string-related topics

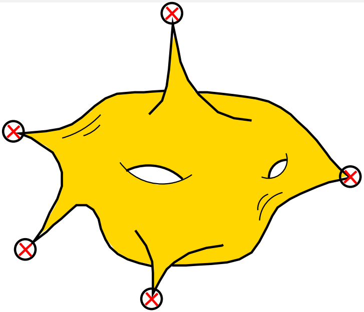
Stringy ingredients

particles $\longleftrightarrow$ strings



- Supersymmetry (in fact, supergravedad - SUGRA) & 10D space-time
  $\rightarrow$ compactify 6D on spaces with shapes and sizes set by moduli
- matter fields get all their properties from string features
  $\rightarrow$ all field charges are computable
- field couplings arise from string interactions
  $\rightarrow$ coupling strengths are computable modular forms

Heterotic Orbifolds



Dixon, Harvey, Vafa, Witten (1985-86)

Ibáñez, Nilles, Quevedo (1987)

Font, Ibáñez, Quevedo, Sierra (1990)
Katsuki, Kawamura, Kobayashi, Ohtsubo, Ono, Tanioka (1990)

Kobayashi, Raby, Zhang (2004)

Buchmüller, Hamaguchi, Lebedev, Ratz (2004-06)

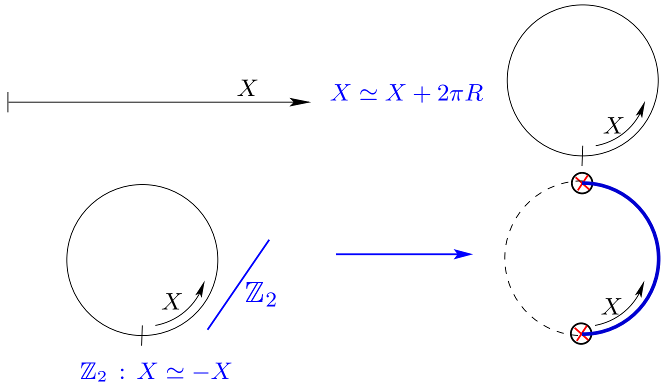
Kobayashi, Nilles, Plöger, Raby, Ratz (2006)
Lebedev, Nilles, Ratz, SRS, Vaudrevange, Wingerter (2006-08)
⋮

Mütter, Parr, Vaudrevange + Biermann, Ratz (2018-19)

Baur, Nilles, Trautner, Vaudrevange (2018-19)

...

1D $S^1/\mathbb{Z}_2$ orbifold



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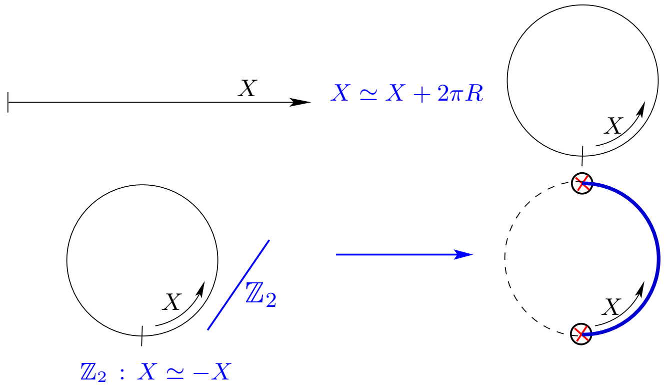
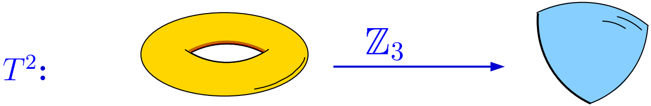


In general, an orbifold $\mathcal{O} := X/S$

2D $\mathbb{T}^2/\mathbb{Z}_N$ orbifolds and G_{flavor}

- $\mathbb{T}^2/\mathbb{Z}_3$



triangular pillow $\rightarrow$ symmetry of a triangle ($S_3 \rightarrow \Delta(27)$)

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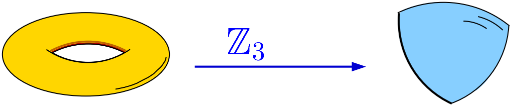
T^2:



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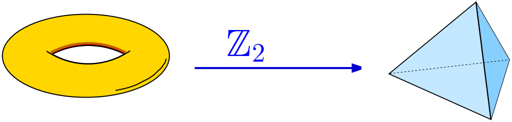

tetrahedron $\rightarrow$ symmetry of a tetrahedron ($A_4 \rightarrow (D_8 \times D_8)/\mathbb{Z}_2$)

In Abelian, toroidal heterotic orbifolds

- Orbifold $\mathcal{O} = \mathbb{R}^6/S \leftarrow$ space group: rotations, reflexions and shifts
- Localized states are subject to 2 kinds of symmetries

  A: geometric symmetries G_{flavor}

  B: stringy modular symmetries $\rightarrow \Gamma_N, \Gamma'_N, \ldots$

(Technically, both arise as outer automorphisms of S in Narain formalism)

Baur, Nilles, Trautner, Vaudrevange (1901.03251, 1908.00805)

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A & B combine to provide an *eclectic* picture of flavor

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Advantage vs pure modular symmetries:

kinetic terms (Kähler potential) under full control! ☺

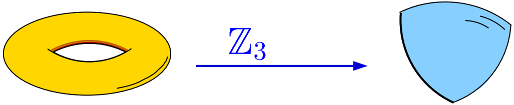
Flavor in

semi-realistic orbifold models

Explicit string model $\mathbb{T}^6/\mathbb{Z}_3 \times \mathbb{Z}_3$

- Contains a sector $\mathbb{T}^2/\mathbb{Z}_3$

T^2:



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$$\Delta(27) \cup \{\mathrm{S}^2\} \;\cong\; \Delta(54)$$

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	quarks and leptons						Higgs fields	
label	q	$\bar{u}$	$\bar{d}$	ℓ	$\bar{e}$	$\bar{\nu}$	H_u	H_d
$\mathrm{SU}(3)_c$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathrm{SU}(2)_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$
$\mathrm{U}(1)_Y$	$1/6$	$-2/3$	$1/3$	$-1/2$	1	0	$1/2$	$-1/2$
$\Delta(54)$	$\mathbf{3}_2$	$\mathbf{3}_2$	$\mathbf{3}_2$	$\mathbf{3}_2$	$\mathbf{3}_2$	$\mathbf{3}_2$	$\mathbf{1}$	$\mathbf{1}$
T'	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$

Baur, Nilles, SRS, Trautner, Vaudrevange (2207.10677)

Explicit string model $\mathbb{T}^6/\mathbb{Z}_3 \times \mathbb{Z}_3$

After

- writing the corresponding action,
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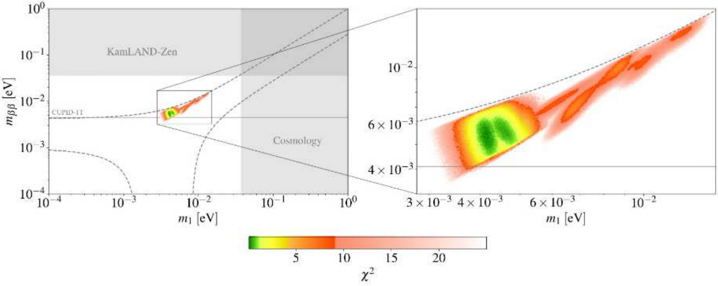
Predictions:



Explicit string model $\mathbb{T}^6/\mathbb{Z}_3 \times \mathbb{Z}_3$

Predictions:

	observable	model best fit	exp. best fit	exp. 1σ interval
quark sector	m_u/m_c	0.00193	0.00193	$0.00133 \to 0.00253$
	m_c/m_t	0.00280	0.00282	$0.00270 \to 0.00294$
	m_d/m_s	0.0505	0.0505	$0.0443 \to 0.0567$
	m_s/m_b	0.0182	0.0182	$0.0172 \to 0.0192$
	ϑ_{12} [deg]	13.03	13.03	$12.98 \to 13.07$
	ϑ_{13} [deg]	0.200	0.200	$0.193 \to 0.207$
	ϑ_{23} [deg]	2.30	2.30	$2.26 \to 2.34$
	$\delta^q_{\mathcal{CP}}$ [deg]	69.2	69.2	$66.1 \to 72.3$
lepton sector	m_e/m_μ	0.00473	0.00474	$0.00470 \to 0.00478$
	m_μ/m_τ	0.0586	0.0586	$0.0581 \to 0.0590$
	$\sin^2\theta_{12}$	0.303	0.304	$0.292 \to 0.316$
	$\sin^2\theta_{13}$	0.0225	0.0225	$0.0218 \to 0.0231$
	$\sin^2\theta_{23}$	0.449	0.450	$0.434 \to 0.469$
	$\delta^\ell_{\mathcal{CP}}/\pi$	1.28	1.28	$1.14 \to 1.48$
	η_1/π	0.029	-	-
	η_2/π	0.994	-	-
	$J_{\mathcal{CP}}$	-0.026	-0.026	$-0.033 \to -0.016$
	$J^{\max}_{\mathcal{CP}}$	0.0335	0.0336	$0.0329 \to 0.0341$
	$\Delta m^2_{21}/10^{-5}$ [eV^2]	7.39	7.42	$7.22 \to 7.63$
	$\Delta m^2_{31}/10^{-3}$ [eV^2]	2.521	2.510	$2.483 \to 2.537$
	m_1 [eV]	0.0042	< 0.037	-
	m_2 [eV]	0.0095	-	-
	m_3 [eV]	0.0504	-	-
	$\sum_i m_i$ [eV]	0.0641	< 0.120	-
	$m_{\beta\beta}$ [eV]	0.0055	< 0.036	-
	m_β [eV]	0.0099	< 0.8	-
	χ^2	0.11		

Baur, Nilles, SRS, Trautner, Vaudrevange (2207.10677)

Siegel modular flavor group

from string theory

Baur, Kade, Nilles, SRS, Vaudrevange: 2008.07534, 2012.09586, 2104.03981

Siegel modular symmetries from $\mathbb{T}^2/\mathbb{Z}_2$

- Recall that $\mathbb{T}^2/\mathbb{Z}_2$ yields $G_{flavor} = (D_8 \times D_8)/\mathbb{Z}_2$

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bottom-up and top-down phenomenology unexplored !!

Quasi-Eclectic realization

of a simple lepton model

Quasi-eclectic picture $A_4 \times \Gamma_3 \to A_4$

Chen, Knapp-Pérez, Ramos-Hamud, SRS, Ratz, Shukla (2021)

	$(E_1^{\mathcal{C}}, E_2^{\mathcal{C}}, E_3^{\mathcal{C}})$	L	H_d	H_u	χ	φ	S_χ	S_φ	Y
$A_4^{\text{traditional}}$	$(\mathbf{1}_0, \mathbf{1_2}, \mathbf{1_1})$	$\mathbf{3}$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$
Γ_3	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{3}$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{3}$
modular weights	$(1,1,1)$	-1	0	0	0	0	0	0	2

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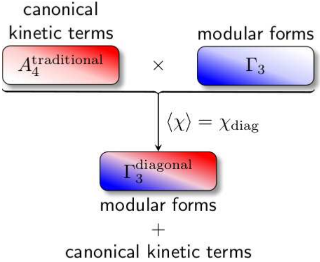

$$m_\nu = \frac{v_u^2 \varepsilon_1}{\sqrt{3}\Lambda} \begin{pmatrix} 2Y_1(\tau) & -Y_3(\tau) & -Y_2(\tau) \\ -Y_3(\tau) & 2Y_2(\tau) & -Y_1(\tau) \\ -Y_2(\tau) & -Y_1(\tau) & 2Y_3(\tau) \end{pmatrix}$$

phenomenology like Feruglio's first model

In summary

Concluding remarks

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To work on

- flavor with $\mathbb{T}^2/\mathbb{Z}_4$ & $\mathbb{T}^2/\mathbb{Z}_6$?

  Baur, Nilles, SRS, Trautner, Vaudrevange (2023)
- $\mathcal{CP}$ and $\mathcal{CP}$ violation ?

  Nilles, Ratz, Trautner, Vaudrevange (2018)
- bottom-up pheno with $\Gamma_N, N > 6$?

  Arriaga, SRS,... (2023)
- dynamic moduli stabilization & de Sitter ?

  Knapp, Liu, Nilles, SRS, Ratz (2023)
- more pheno in these models ?
- non-supersymmetric constructions ?

  Pérez-Martínez, SRS, Vaudrevange (2105.03460)
- already testable predictions ?

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And beyond...

- DM + inflation with KK ALPs?

  Gordillo, Morales, SRS (2023)
- DM in multi-Higgs non-SUSY models ?

  Cervantes, Pérez-Figueroa, Pérez-Martínez, SRS (2302.08520)
- Spectral distortions in inflation?

  Baur, Henríquez, García, SRS (2023)
- non-Abelian orbifolds & flavor ?

  Hernández-Segura, SRS (2023)
- Machine learning for better models?

  Escalante-Notario, Portillo-Castillo, SRS (2212.00821,23xx.xxxx)
- ...


THANK
YOU FOR
SUPPORTING
MY SMALL
BUSINESS

Backup slides

Modular symmetries as flavor symmetries

Congruence modular subgroups: $\Gamma(N) \subset \mathrm{SL}(2,\mathbb{Z})$

$$\Gamma(N) = \{\gamma \in \mathrm{SL}(2,\mathbb{Z}) \,|\, \gamma = \mathbb{1} \mod N\}$$

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$$\Gamma'_N = \left\langle \mathrm{S},\mathrm{T} \,|\, \mathrm{S}^4 = (\mathrm{ST})^3 = T^N = \mathbb{1}, \quad \mathrm{S}^2\mathrm{T} = \mathrm{TS}^2, \qquad N = 2,3,4,5 \right\rangle$$

$$\Gamma'_2 \cong S_3,\ \Gamma'_3 \cong T',\ \Gamma_4 \cong \mathrm{SL}(2,4),\ \Gamma_5 \cong \mathrm{SL}(2,5), \ldots$$

e.g. Liu, Ding (2019)

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$$\Gamma_2 \cong S_3,\ \Gamma_3 \cong A_4,\ \Gamma_4 \cong S_4,\ \Gamma_5 \cong A_5, \ldots, \Gamma_7 \cong \Sigma(168), \ldots$$

e.g. de Adelhaart, Feruglio, Hagedorn (2011)

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Thus far, models with modular flavor symmetries are supersymmetric

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Admissible iff

$$W(\Phi_{n_1},\ldots) \xrightarrow{\gamma} (cT+d)^{-1}\,\mathbb{1}\,W(\Phi_{n_1},\ldots), \qquad \text{i.e. } {\scriptstyle n_Y+\sum n_i=-1}, \quad {\scriptstyle \prod\rho(\gamma)=\mathbb{1}}$$

Note the nontrivial $automorphy\ factor$ ${\scriptstyle (cT+d)^{-1}} \rightarrow W$ covariant

How to proceed with *modular* flavor symmetries

- Take your favorite symmetry: $G_{mod} = \Gamma_N \in \{S_3, A_4, S_4, A_5, \ldots\}$
- Choose your favorite representations $\rho(\gamma)$ for quark and lepton fields

  e.g. quark doublets Q as $\mathbf{3}$ or $\mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}''$ of $\Gamma_3 \cong A_4, \ldots$

- Pick your favorite modular weights n_i and n_Y
- Write your G_{mod}-covariant superpotential W

$$\text{e.g. } W \supset \hat{Y}^u H_u Q \bar{u} + \hat{Y}^d H_d Q \bar{d} + \hat{Y}^e H_d L \bar{e} + \frac{\hat{Y}}{\Lambda} L H_u \, L H_u$$

- Take your favorite inv. Kähler potential K; typical choice $K = \sum |\Phi_{n_i}|^2$

  MANY other modular invariant K possible! - Chen, SR-S, Ratz (1909.06910)

- Choose a $\langle T \rangle \neq 0$ $\rightarrow$ nontrivial rep. of $\hat{Y}(\langle T \rangle)$ breaks G_{mod}
- EW breakdown with $\langle H_u \rangle, \langle H_d \rangle \neq 0$
- Diagonalize quark and lepton matrices to compute V_{CKM} and U_{PMNS} and adjust only $\langle T \rangle$ to data

From top-down to bottom-up

eclectic flavor symmetries

Eclectic flavor groups

Key observation: T' is an outer automorphism group of $\Delta(54)$ ☺

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- Verify whether there is a third (class-inverting) outer automorphism that act as a $\mathbb{Z}_2$ CP-like transformation to further enhance the eclectic flavor symmetry

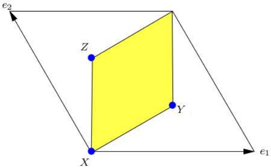
Eclectic flavor groups

flavor group $\mathcal{G}_{\text{fl}}$	GAP ID	$\text{Aut}(\mathcal{G}_{\text{fl}})$	finite modular groups		eclectic flavor group
Q_8	[8, 4]	S_4	without $\mathcal{CP}$	S_3	$\text{GL}(2,3)$
			with $\mathcal{CP}$	–	–
$\mathbb{Z}_3 \times \mathbb{Z}_3$	[9, 2]	$\text{GL}(2,3)$	without $\mathcal{CP}$	S_3	$\Delta(54)$
			with $\mathcal{CP}$	$S_3 \times \mathbb{Z}_2$	[108, 17]
A_4	[12, 3]	S_4	without $\mathcal{CP}$	S_3 S_4	S_4 S_4
			with $\mathcal{CP}$	–	–
T'	[24, 3]	S_4	without $\mathcal{CP}$	S_3	$\text{GL}(2,3)$
			with $\mathcal{CP}$	–	–
$\Delta(27)$	[27, 3]	[432, 734]	without $\mathcal{CP}$	S_3 T'	$\Delta(54)$ $\Omega(1)$
			with $\mathcal{CP}$	$S_3 \times \mathbb{Z}_2$ $\text{GL}(2,3)$	[108, 17] [1296, 2891]
$\Delta(54)$	[54, 8]	[432, 734]	without $\mathcal{CP}$	T'	$\Omega(1)$
			with $\mathcal{CP}$	$\text{GL}(2,3)$	[1296, 2891]

Nilles, SR-S, Vaudrevange (2001.01736)

Back in the $\mathbb{T}^2/\mathbb{Z}_3$ example

Restricted superpotential



$$\Rightarrow \mathcal{W} \quad \supset \quad c\Big[\hat{Y}_2(T)\big(X_1\,X_2\,X_3 + Y_1\,Y_2\,Y_3 + Z_1\,Z_2\,Z_3\big) - \frac{\hat{Y}_1(T)}{\sqrt{2}}\big(X_1\,Y_2\,Z_3 + X_1\,Y_3\,Z_2 + X_2\,Y_1\,Z_3 + X_3\,Y_1\,Z_2 + X_2\,Y_3\,Z_1 + X_3\,Y_2\,Z_1\big)\Big]\,,$$

with $\quad \Phi^i_{-2/3} := (X_i, Y_i, Z_i)^T, \quad c \in \mathbb{R}$

Back in the $\mathbb{T}^2/\mathbb{Z}_3$ example

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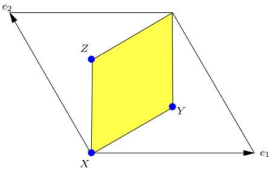

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with $\Phi^{i}_{-2/3} := (X_i, Y_i, Z_i)^T, \quad c \in \mathbb{R}$

More interestingly

$$K = -\log(-\mathrm{i}T + \mathrm{i}T) + \sum_i (-\mathrm{i}T + \mathrm{i}T)^{-2/3} |\Phi^{i}_{-2/3}|^2$$

Only canonical terms are allowed

$\rightarrow$ predictability of bottom-up models with Γ'_N recovered! ☺

Nilles, SRS, Vaudrevange (2004.05200)

Towards the *eclectic* flavor picture

Use Narain formalism: split string in independent components

$$X(\tau,\sigma) = X_R(\sigma-\tau) + X_L(\sigma+\tau)$$

Groot-Nibbelink, Vaudrevange (2017)

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Perform $\mathbb{T}^2/\Theta$ (e.g. $\Theta = \mathbb{Z}_3$) on each 2D independent string component

$$\mathcal{O}_{Narain} = (\mathbb{R}^2_R \otimes \mathbb{R}^2_L)/S_{Narain}$$

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Inspiration: C, P, T in SM are outer automorphisms of the Poincaré symmetry group

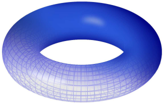
What are the outer automorphisms of $S_{Narain} = \{g\}$?

$$Out(S_{Narain}) = \left\{h = (\Sigma, t) \notin S_{Narain} \mid hgh^{-1} \in S_{Narain}\right\}$$

Rotations: $h_\Sigma = (\Sigma, 0) \to O(2,2;\mathbb{Z})$, Translations: $h_t = (\mathbb{1}_4, t)$

Towards the *eclectic* picture: what $Out(S_{Narain})$ is

String 2D toroidal compactifications have two moduli: T, U



$$G = \frac{\operatorname{Im} T}{\operatorname{Im} U} \begin{pmatrix} 1 & \operatorname{Re} U \\ \operatorname{Re} U & |U|^2 \end{pmatrix}, \qquad B = \operatorname{Re} T \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

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$h_\Sigma =$	S_U	T_U	S_T	T_T	M	K_*
$U \xrightarrow{h_\Sigma}$	$-1/U$	$U+1$	U	U	T	$-\bar{U}$
$T \xrightarrow{h_\Sigma}$	T	T	$-1/T$	$T+1$	U	$-\bar{T}$

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Recall: in $\mathrm{SL}(2,\mathbb{Z})$ $\qquad T \xrightarrow{\mathrm{S}} -\frac{1}{T}, \qquad T \xrightarrow{\mathrm{T}} T+1$

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$$\mathrm{SL}(2,Z)_T = \langle \mathrm{S}_T, \mathrm{T}_T \rangle, \quad \mathrm{SL}(2,Z)_U = \langle \mathrm{S}_U, \mathrm{T}_U \rangle$$ ☺

M: mirror symmetry, K$_*$: $\mathcal{CP}$-like transformation ☺

Nilles, Ratz, Trautner, Vaudrevange (2018); Novichkov, Penedo, Petcov, Titov (2019)

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Nilles, Ratz, Trautner, Vaudrevange (2018); Novichkov, Penedo, Petcov, Titov (2019)

Further, $\{h_t\}$ don't change T, U, but do transform fields
$\rightarrow$ traditional flavor symmetry ☺

Common origin of modular and traditional flavor

Modular weights n_i, representations and couplings of Φ_{n_i} not *ad hoc*! ☺

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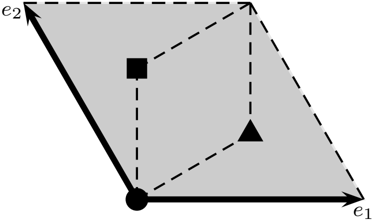
Example $\mathbb{T}^2/\mathbb{Z}_3$: must fix U to $\langle U\rangle = \omega = e^{2\pi \mathrm{i}/3} \to$ broken $\mathrm{SL}(2,\mathbb{Z})_U$



Lauer, Mas, Nilles (1989)

By using CFT formalism, inspect $\mathrm{SL}(2,\mathbb{Z})_T$ on the triplet of matter fields:

$$h_\Sigma : \rho(\mathrm{S}_T) = \frac{\mathrm{i}}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}, \quad \rho(\mathrm{T}_T) = \begin{pmatrix} \omega^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\rho(\mathrm{S}_T)$ and $\rho(\mathrm{S}_T)$ build the reps. $\mathbf{2}' \oplus \mathbf{1}$ of modular group $\Gamma_3' = T'$ ☺

$$\Phi_{n=-2/3,-5/3} \xrightarrow{\mathrm{S}_T} (-T)^n \rho(\mathrm{S}_T)\Phi_n, \qquad \Phi_n \xrightarrow{\mathrm{T}_T} \rho(\mathrm{T}_T)\Phi_n$$

Ibáñez, Lüst (1992)

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Modular weights n_i, representations and couplings of Φ_{n_i} not *ad hoc*! ☺

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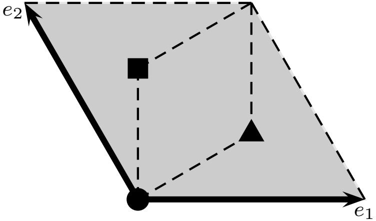

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$$h_t : \rho(\mathrm{A}) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \ \rho(\mathrm{B}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \ \rho(\mathrm{C}) = \rho(\mathrm{S}_T^2)$$

$\rho(\mathrm{A})$, $\rho(\mathrm{B})$ and $\rho(\mathrm{C})$ build the reps $\mathbf{3}_{2(1)}$ and $\mathbf{3}_{1(1)}$ of traditional flavor group $\Delta(54)$ for $\Phi_{-2/3}$ and $\Phi_{-5/3}$

cf. also in Kobayashi, Plöger, Nilles, Raby, Ratz (2006)

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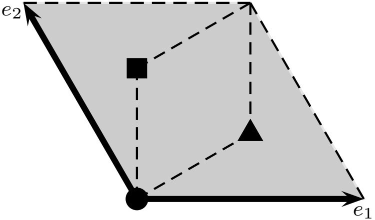

first eclectic flavor symmetry: modular + traditional flavor

$$\Delta(54) \cup T' \cong \Omega(1) = SG[648, 533]$$

with $\mathcal{CP}$: $\Delta(54) \cup T' \cup \mathbb{Z}_2^{\mathcal{CP}} \cong SG[1296, 2891]$

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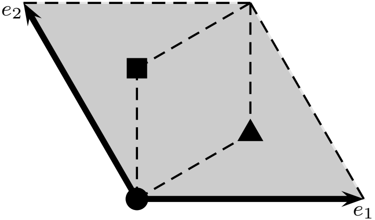

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Can we generalize this in a bottom-up fashion ?