Conceptual difficulties in quantum theory and their impact in black hole physics, and cosmology

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Seminar of the High Energy Department

March 21, 2023

PLAN OF THIS TALK:

1) The Quantum Measurement Problem in Gravitational Contexts and examples of confusion .

2) Exploring the Gravity/ Quantum Interface.

3) The inflationary account for the emergence of the seeds of cosmic structure and the CMB

4) The question of tensor modes (primordial gravity waves).

5) The Black Hole Information Puzzle.

6) Final considerations.

It is hard to think about physics without a space-time framework. Adopting a clear view about QT makes it harder.

Failure to do so frequently drives to confusion:

A classic one connects to the notion of *“fluctuations"*: We should distinguish at least 3 uses of the word:

i) Variations through space-time of well defined attributes of an “extended" entity (e.g. the water level on the ocean).

ii) Variations of well defined quantities within an ensemble of systems (e.g. the energy of a classical canonical ensemble of similar gas filled boxes).

iii) Quantum indeterminacies or uncertainties in a single system (fluctuations in the position of a harmonic oscillator in its ground state).

(Failures to distinguish between proper and improper mixtures).

Moreover, we should try to be clear on whether the fluctuations one is talking about are ontological (conditions in our universe) or epistemic (the quantum state of our universe taken as selected from an imaginary ensemble).

Confusion is frequent and problematic : Example

Stochastic Gravity It is based on

$$G_{ab} = 8\pi G(\langle\psi|\hat{T}_{ab}(x)|\psi\rangle + \xi_{ab})$$

where ξ_{ab} is a stochastic tensor introduced to characterize *the quantum fluctuations*. One problem is that while $\overline{\nabla^a \xi_{ab}} = 0$ it need not satisfy $\nabla^a \xi_{ab} = 0$ for individual elements of the stochastic ensemble. The problem is often not appreciated.

These problems can often be traced to the tendency to ignore (or to adopt an unclear posture about) :

THE “ M" PROBLEM : (the measurement, or reality, problem in QT).

THE “ M" PROBLEM: (or part of it)

2 rules determining the change in the quantum state: *U* and *R*. No satisfactory rule specifying which one applies.

The following 3 premises can not be held simultaneously in a self consistent manner. [Tim Maudlin (*Topoi* **14**, 1995)].

i) The characterization of a system by its wave function is complete. Its negation leads, for instance to hidden variable theories.

ii) The evolution of the wave function is always according to Schrödinger's equation. Its negation leads, for instance to spontaneous collapse theories.

iii) The results of experiments lead to definite results. Its negation leads for instance to Many World/ Minds Interpretations, Consistent histories approach, etc.

It is NOT solved by decoherence! .

THE “M" PROBLEM: Becomes exacerbated in situations in which trying to hold on to a Copenhagen type Interpretation, becomes even harder, due to the lack of an identifiable observer that might be used to “ justify" application of the R rule.

We approach the exploration of the GR/ QT regime in a top - bottom approach.

Usual bottom -up approach: postulates FT (String Theory , LQG, Causal sets, dynamical triangulations, etc.) and attempts to connect to regimes of interest of the "world out there" : Cosmology, Black Holes, etc.

The top - bottom approach, pushes existing, well tested and developed theories, to address open issues that seem to lie beyond their domain. Possible modifications can serve as clues about the nature of the more fundamental theory .

The idea is to push GR together with QFT (i.e. semi-classical gravity) into realms/questions usually not explored.

We will try to stick to a concrete and fixed view based on a *provisional* ontological posture:

Describe space-time in classical terms expected to be a good approximation in regimes where curvatures $<< (1/l_{Planck})^2$ and quantum uncertainties *"are not too large"*. Matter to be treated quantum mechanically (QFT in CS). It is natural to say that the **local beable** at x is what restricts its curvature, i.e. the quantity appearing in the RHS of Einstein's equations.

Difficulty: In a recent work, (T Maudlin & E. Okón) we offered strong arguments indicating that all *reaasonnable* approaches to the M-Problem seem to lead to violations of $\nabla^a T_{ab} = 0$.

Whatever we do we need to deal with this!

It is *provisional* in that, we expect something dramatically different could be required when dealing with a theory in which space-time itself is described quantum mechanically (regarded as necessary), and perhaps emergent.

Regarding the M problem we will focus on spontaneous collapse theories

Collapse Theories: Large amount of work: GRW, Pearle, Diosi, Penrose, Bassi (recent advances to make it compatible with relativity Tumulka, Bedningham, Pearle).

The basic idea is to unify U and R. The changes are small when a few DOF are involved and become large when something like 10^{23} are entangled (and delocalized).

These address the problem successfully and are empirically viable (at least in the Non Relativistic regimes).

Our ontology will thus be centered on:

$$\langle\psi|\hat{T}_{ab}(x)|\psi\rangle$$

where $|\psi\rangle$ is the quantum state (on the past light cone of x ?) .

It is NOT to be regarded as an average, but as the general relativistic version of the mass density in m-GRW.

Continuous Spontaneous Localization (CSL) P. Pearle . The theory is defined by two equations:
i) A modified Schrödinger equation, whose solution is:

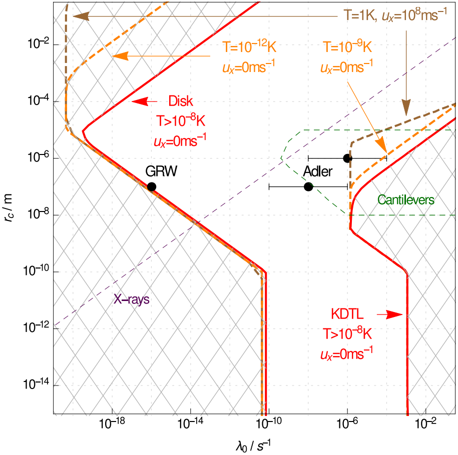
$$|\psi, t\rangle_w = \hat{\mathcal{T}} e^{-\int_0^t dt' \left[i\hat{H} + \frac{1}{4\lambda}[w(t') - 2\lambda\hat{A}]^2\right]} |\psi, 0\rangle. \tag{1}$$

($\hat{\mathcal{T}}$ is the time-ordering operator). $w(t)$ is a random classical function of time, of white noise type, whose probability is given by the second equation, ii) the Probability Rule:

$$PDw(t) \equiv {}_w\langle\psi, t|\psi, t\rangle_w \prod_{t_i=0}^{t} \frac{dw(t_i)}{\sqrt{2\pi\lambda/dt}}. \tag{2}$$

The processes U and R (corresponding to the observable $\hat{A}$) are unified. For non-relativistc QM the proposal assumes :
$\hat{A} = \hat{\vec{X}}$ (smeared with scale $r_c \sim 10^{-5} cm$).
Here λ must be small enough not to conflict with tests of QM in the domain of subatomic physics and big enough to result in rapid localization of “macroscopic objects". GRW suggested range: $\lambda \sim 10^{-16} sec^{-1}$. (Likely depends on particle mass).

The theory is being experimentally tested.



How to deal with our problem?

Regard semi-classical GR as an approximated description with limited applicability. During the collapse the equations can not be valid.

At the formal level we rely on the notion of *Semi-classical Self-consistent Configuration* (SSC).

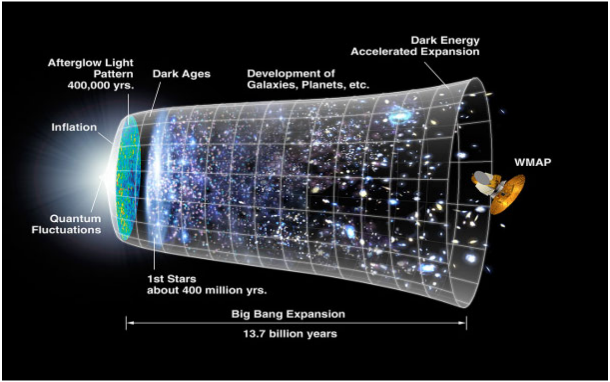
DEFINITION: The set $g_{\mu\nu}(x), \hat{\varphi}(x), \hat{\pi}(x), \mathcal{H}, |\xi\rangle \mathrm{in} \mathcal{H}$ represents a SSC iff $\hat{\varphi}(x)$, $\hat{\pi}(x)$ y $\mathcal{H}$ corresponds to QFT in CS over the space-time with metric $g_{\mu\nu}(x)$, and MOREOVER the state $|\xi\rangle$ in $\mathcal{H}$ is such that:

$$G_{\mu\nu}[g(x)] = 8\pi G \langle \xi | \hat{T}_{\mu\nu}[g(x), \hat{\varphi}(x), \hat{\pi}(x)] | \xi \rangle.$$

Note that this is a kind of GR version of the Schödinger -Newton system (and, as non-linear !).

Collapse: a transition for one complete SSC to another one. That is, we do not have simple jumps in states but jumps of the form $....SSC1.... \to ....SSC2....$

COSMIC INFLATION Contemporary cosmology includes inflation as one of its most attractive components.



Its biggest success: the account for emergence of the seeds of cosmic structure as a result of “quantum fluctuations" with the ‘ right" spectrum.

However at the theoretical/conceptual level the account is not truly satisfactory.

The starting point of the analysis is a cosmological space-time (in a specific gauge)

$$ds^2 = a^2(\eta)\{-(1+2\Psi)d\eta^2 + [(1-2\Psi)\delta_{ij} + h_{ij}]dx^i dx^j\}$$

with matter represented by an inflaton field written as $\phi = \phi_0(\eta) + \delta\phi$ with $\delta\phi, \Psi, ..., \delta h_{ij}$ small perturbations containing the spatial dependencies. The background (a, ϕ_0) is treated classically and assumed to be dominated by the inflaton potential (slow roll regime) so ϕ_0 changes slowly and a is approximately given by:

$$a(\eta) = \frac{-1}{\eta H_I} .$$

We set $a = 1$ at the “present cosmological time", assume that inflationary regime corresponds to η in $(-\mathcal{T}, \eta_0), \eta_0 < 0$.

The "perturbations": $\delta\phi, \Psi, ..., \delta h_{ij}$, treated quantum mechanically & assumed to be characterized by the “ vacuum state" (essentially the BD vacuum) $|0\rangle$.

Inflation dilutes all preexisting features and drives all space dependent fields towards their vacuum states.

The state of the quantum field is *"also characterized"* by the so called "quantum fluctuations" or "uncertainties".

Now we face an instance of the first confusion we discussed:

In the usual treatments, those quantum indeterminacies are **unjustifiably** identified as the primordial inhomogeneities (fluctuations of type i)) which eventually evolved into all the structure in our Universe: galaxies, stars planets, etc...

However note that, according to the inflationary picture: The Universe was H&I, (both in the part that could be described at the “classical level", and the quantum level) as a result of inflation (up to e^{-N}).

[A displacement of the state by $\vec{D}$ is $e^{i\hat{\vec{P}}.\vec{D}}|0\rangle = |0\rangle$ so, it is completely homogeneous.]

The end situation (with galaxies, stars, planets and us) is not.

How does this happen if the dynamics of the closed system does not break those symmetries.?

Our approach can deal with that problem:

Space-time is treated classically. The scalar field must be treated using QFT in curved space-time.

Thus, Quantum-Mechanically, the zero mode of the field $\hat{\phi}_0$) is taken to be in a highly excited (and sharply peaked) state, while the space dependent modes are in the vacuum state.
The quantum state of the scalar field and the space-time metric satisfy Einstein's semi-classical eq.

$$G_{\mu\nu} = 8\pi G \langle \xi | \hat{T}_{\mu\nu} | \xi \rangle.$$

under those conditions one obtains almost essentially the standard behavior for the background

$a(\eta) = \frac{-1}{\eta H_I}$ and slow roll for $\langle \hat{\phi}_0 \rangle$ in $(-\mathcal{T}, \eta_0), \eta_0 < 0$.

We will concentrate next on the $\vec{k} \neq 0$ modes.

Rely on a *practical procedure* which we have checked to give equivalent results as the SSC formalism.

Early stages of inflation $\eta = -\mathcal{T}$, the state is $|0\rangle$, the operators $\delta\hat{\phi}_k$ $\hat{\pi}_k$ are characterized by gaussian wave functions centered on 0 with uncertainties $\Delta\delta\phi_k$ and $\Delta\pi_k$, and $\Psi(\eta, x) = 0, h_{ij}(\eta, x) = 0$.

The collapse modifies the quantum state, and the expectation values of $\widehat{\delta\phi_k}(\eta)$ and $\hat{\pi}_k(\eta)$.

Assume the collapse occurs mode by mode and is described by an adapted version of collapse theories.

Our universe would correspond to one specific realization of the stochastic functions (one for each $\vec{k}$).

First consider the scalar metric perturbations $\Psi(\eta, x)$. which characterize the CMB temperature fluctuations (and seeds of structure).
The Fourier decomposition of the semi classical Einstein's Equations give:

$$-k^2\Psi(\eta, \vec{k}) = \frac{4\pi G\phi_0'(\eta)}{a}\langle\hat{\pi}(\vec{k}, \eta)\rangle \tag{3}$$

With reasonable choices in the details of the collapse theory, agreement with observations can be achieved:

In CSL version: Collapse in the field operator or the momentum conjugate operators with $\lambda = \tilde{\lambda} k^{\pm 1}$ fixed by dimensional considerations (or collapse in the operators $(-\nabla^2)^{-1/4}\hat{\pi}(\vec{x})$ or $(-\nabla^2)^{1/4}\hat{\phi}(\vec{x})$) . Why is this the right thing?.

The resulting prediction for the power spectrum is:

$$P_S(k) \sim (1/k^3)(1/\epsilon)(V/M_{Pl}^4)\tilde{\lambda}\mathcal{T} \tag{4}$$

Taking GUT scale for the inflation potential, and standard values for the slow-roll, leads to agreement with observation for: $\tilde{\lambda} \sim 10^{-5} MpC^{-1} \approx 10^{-19} sec^{-1}$.

Not very different from GRW suggestion ! .

[*PRD*, **87**, 104024 (2013)] .Other treatments with similar spirit by J. Martin, V. Vennin & P. Peter, [*PRD*, **86** , 103524 (2012)], and S. Das, K. Lochan, S. Sahu & T. P. Singh [*PRD*, **88**, 085020 (2013)]

TENSOR MODES

Similarly, the equation of motion for the tensor perturbations is:

$$(\partial_0^2 - \nabla^2)h_{ij} + 2(\dot{a}/a)\dot{h}_{ij} = 16\pi G \langle(\partial_i\delta\phi)(\partial_j\delta\phi)\rangle_{Ren}^{tr-tr} \qquad (5)$$

$tr - tr$ stands for the transverse trace-less part of the expression (retaining only dominant terms).

Note that it is quadratic in the collapsing quantities !!

Passing to a Fourier decomposition, we solve the eq.

$$\ddot{\tilde{h}}_{ij}(\vec{k},\eta) + 2(\dot{a}/a)\dot{\tilde{h}}_{ij}(\vec{k},\eta) + k^2\tilde{h}_{ij}(\vec{k},\eta) = S_{ij}(\vec{k},\eta), \qquad (6)$$

with zero initial data, and source term:

$$S_{ij}(\vec{k},\eta) = 16\pi G \int \frac{d^3x}{\sqrt{(2\pi)^3}} e^{i\vec{k}\vec{x}} \langle(\partial_i\delta\phi)(\partial_j\delta\phi)\rangle_{Ren}^{tr-tr}(\eta,\vec{x}). \qquad (7)$$

The result is formally divergent. However, we must introduce a cut-off *(the scale of diffusion (Silk) dumping with* $p_{UV} \approx 0.078 MpC^{-1}$ *)*.

The prediction for the power spectrum of tensor perturbations is:

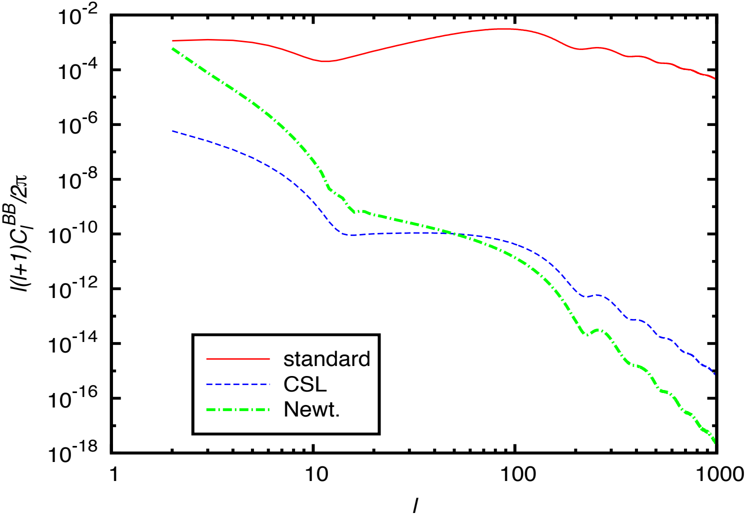
$$P_h(k) \sim (1/k^3)(V/M_{Pl}^4)^2(\tilde{\lambda}^2 \mathcal{T}^4 p_{UV}^5/k^3) \tag{8}$$

($\mathcal{T}$ the conformal time at the start of inflation taken for standard inflationary parameters as 10^4 MpC) while the power spectrum for the scalar perturbations is

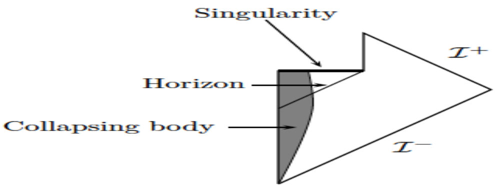
$$P_S(k) \sim (1/k^3)(1/\epsilon)(V/M_{Pl}^4)\tilde{\lambda}\mathcal{T} \tag{9}$$

That is a very different relation between them than usual. Tensor modes are not expected at the level they are being looked for!!
PRD **96**, 101301(R) (2017); *PRD* **98** 023512 (2018) .

We also considered a simpler collapse model, and again obtained **reduced tensor mode amplitude** but with a different shape.



The BH Information Issue.



S. Hawking: QFT effects cause BH to radiate. It should loose mass and, essentially disappears leaving just thermal radiation.

QT requires a unitary relation between the final state and the initial one (on Cauchy hypersurfaces!).

People often modify the demand requiring unitary relation between states in $\mathcal{I}-$ and $\mathcal{I}+$

That seems both unwarranted, but also very hard to account for.

Beware: The real BH information paradox only arises when we assume that QG cures the singularity, (and the need for an additional boundary to space-time). (Otherwise see postures by R. Wald or T. Maudlin).

One then faces that lack of unitarity indicated by the Hawking evaporation of the BH (assume no remnants), and the conflict with QM.

But wait!!... QM involves some departures from unitarity: in connection to measurements.
Brings us back to the M problem.

In theories of spontaneous collapse departure from unitary evolution is present in general. [FoP 44, 114-143, (2014), FoP 45, 461-470 (2015)].

We consider a picture where the two kinds of departures from unitarity are unified.

We have studied this explicitly only in a 2-D Model (CGHS). [PRD 91, 12, 124009].

Quantum Fields In a BH space-time

QFT in CS treatment for the matter fields ϕ. First in the *in* region, before the black hole forms.

Use the Heisenberg picture: The state remains fixed, but the field operators depend on time (and space) $\hat{\phi}(x)$.

The initial state can be written schematically as

$$|\Psi_{in}\rangle = |0_{in}\rangle \otimes |Matter - Pulse\rangle \tag{10}$$

The matter pulse undergoes gravitational collapse and the space-time develops a Black Hole region.

One describes the state of a quantum field at late times in terms of degrees of freedom *inside* and *outside* the Black Hole.

The vacuum state described in this form is:

$$|0_{in}\rangle = \sum_{F_\alpha} C_{F_\alpha} |F_\alpha\rangle^{ext} \otimes |F_\alpha\rangle^{int} \quad (11)$$

where a particle state F_α consists of an arbitrary, but *finite*, number of particles (or individual mode excitations).

Tracing over the interior DOF, would lead to an improper thermal state corresponding to the Hawking flux.

The complete initial state can be written schematically as

$$|\Psi_{in}\rangle = \sum_{F_\alpha} C_{F_\alpha} |F_\alpha\rangle^{ext} \otimes |F_\alpha\rangle^{int} \otimes |MatterPulse\rangle \quad (12)$$

We consider the evolution using a modified theory involving spontaneous collapse. For instance a CSL type theory. Introduce the folliation (and time the parameter τ) corresponding to $W^2 = \text{const.}$ in the inside, (and “almost arbitrary") outside.

The CSL collapse operator

The CSL equations can be generalized to drive collapse into a state of a joint eigen-basis of a set of commuting operators $\hat{A}^I$, $[\hat{A}^I, \hat{A}^J] = 0$. For each $\hat{A}^I$ there will be one $w^I(t)$. In this case,we have

$$|\psi, t\rangle_w = \hat{\mathcal{T}} e^{-\int_0^t dt' \left[i\hat{H} + \frac{1}{4\lambda} \sum_I [w^I(t') - 2\lambda \hat{A}^I]^2 \right]} |\psi, 0\rangle. \qquad (13)$$

We call $\hat{A}^I$ the *set of collapse operators*. In this work we make simplifying choices

i) States will collapse to a state of definite number of particles in the inside region.

ii) We are working in the interaction picture, so $\hat{H} \to 0$ in the above equation.

The curvature dependent coupling λ in modified CSL

Assume that the CSL collapse mechanism is amplified by the curvature of space-time: i .e. that the rate of collapse λ, will depend,on the Weyl tensor scalar:

$$\lambda(R) = \lambda_0 \left[1 + \left(\frac{W^2}{\mu} \right)^{\gamma} \right] \quad (14)$$

where $W^2 = W_{abcd} W^{abcd}$ space-time and $\gamma > 1/2$ is a constant, μ provides an appropriate scale (R^2 in 2D).

In the region of interest, we will have $\lambda = \lambda(\tau)$.

This evolution achieves, in the finite time to the singularity, what ordinary CSL achieves in infinite time, i.e. drives the state to one of the eigenstates of the collapse operators.

Thus, the effect of CSL on the initial state:

$$|\Psi_{in}\rangle = |0_{in}\rangle \otimes |MPulse\rangle = N \sum_{F_\alpha} C_{F_\alpha} |F_\alpha\rangle^{ext} \otimes |F_\alpha\rangle^{int} \otimes |MPulse\rangle \tag{15}$$

is to drive it to one of the eigenstates of the joint number operators.

Thus at the hypersurfaces $\tau = Constant$ very close to the singularity the state will be

$$\left|\Psi_{in,\tau}\right\rangle = NC_{F_\alpha} |F_\alpha\rangle^{ext} \otimes |F_\alpha\rangle^{int} \otimes |MPulse\rangle \tag{16}$$

There is no summation. It is a pure state. We do not know which one!

The role of quantum gravity: Assume that QG :

a) : resolves the singularity and leads, on the other side, to a reasonable space-time.
b) : does not lead to large violations of the basic space-time conservation laws.

Thus, the effects of QG can be represented by the curing of the singularity and the transformation of the state:

$$\left|\Psi_{in,\tau}\right\rangle = NC_{F_\alpha} \left|F_\alpha\right\rangle^{ext} \otimes \left|F_\alpha\right\rangle^{int} \otimes \left|MPulse\right\rangle$$

$$\to NC_{F_\alpha} \left|F_\alpha\right\rangle^{ext} \otimes \left|0^{post-singularity}\right\rangle \tag{17}$$

Where $\left|0^{post-singularity}\right\rangle$ represents a zero energy momentum state corresponding to a trivial region of space-time. (We ignored possible small remnants).

ENTER THE ENSEMBLES

We ended up with a pure quantum state, but we do not know which one. That depends on the particular realization of the functions w^α.

Consider now an ensemble of systems prepared in the same initial state:

$$|\Psi_{in}\rangle = |0_{in}\rangle_R \otimes |Pulse\rangle_L \tag{18}$$

We describe this ensemble, by the pure density matrix:

$$\rho(\tau_0) = |\Psi_{in}\rangle \langle\Psi_{in}| \tag{19}$$

Consider the CSL evolution of this density matrix up to the hypersurface just before the singularity.

Finally, add the matter pulse and use what was assumed about QG. The density matrix characterizing the ensemble after the would-be-singularity is then :

$$\rho^{Final} = N^2 \sum_F e^{-\frac{E_F}{T}} \, |F\rangle^{\text{out}} \otimes \left|0^{post-sing}\right\rangle \langle F|^{\text{out}} \otimes \left\langle 0^{post-sing}\right|$$

$$= \left|0^{post-sing}\right\rangle \left\langle 0^{post-sing}\right| \otimes \rho^{out}_{Thermal} \qquad (20)$$

Start: a pure state of $\hat{f}$, and space-time initial data on past null infinity. End: a “proper " thermal state on future null infinity followed by an empty region!

Information was lost as a result of general quantum evolution (in a slightly modified theory). !!

And there is nothing paradoxical.

CONCERNS:

Energy violation: Early concerns by Banks-Susskind-Peskin, but further analysis by Unruh and Wald indicated these were exaggerated. Dynamical collapse theories have been constructed to ensure compatibility with experimental bounds. [In fact, recent work indicates that "cumulative effects" of $\nabla^a \langle \hat{T}_{ab} \rangle \neq 0$ might account for the Dark Energy (using Unimodular Gravity). See *PRL* **122**, 221302 (2019)].

Foliation dependence: When using the non-relativistic CSL version this is an issue. Should be eliminated by passing to the relativistic versions of collapse dynamics.

Relativistic Covariance: In : PRD, 94, 045009 (2016) we carried out a similar analysis as the one performed here using a relativistic version of Dynamical collapse theory recently developed by D. Bedingham. (Again in 2-D models).

The programs has more potential: Eternal inflation, Anomalous, Low power at low l, Problem of time in CQG, ... Lots of work ahead!!.

This whole approach could, in the future, be shown to be non-viable. However, as noted by Sir Francis Bacon when considering the scientific enterprise in general: “Truth emerges more readily from error than from confusion”.

We believe that ignoring “ the measurement problem" in the application of QT to macro problems can be a serious source of confusion, particularly when referring to situations beyond the Lab, as the ones considered here.

THANKS

COMMON CONFUSIONS :

1) Decoherence solve the measurement problem. Counter-Examples EPR, (Mini Mott), Our theorem.

2) The early universe is not 100% H& I. Yes, but if we rely on those highly suppressed) departures from H& I we must give up the predictions of Inflation that rely on the BD vacuum.

3) In de Sitter space-time there is a cosmological horizon and so the state is thermal, and the *thermal fluctuations* break the H& I. (example of the mistake we noted at the start). Furthermore there is a different cosmological horizon associated with each co-moving observer, just as there is a Rindler Horizon associated with each accelerated observer in Minkowski space-time. The Unruh effect shows that each such observer sees the Minkowski vacuum as thermal. That does not mean the Minkowski vacuum fail to be H& I .

Consideraciones Globales sobre el espacio de de-Sitter

Este se puede considerar como una subvariaedad de el espacio tiempo de Minkowski en 5 dimensiones:

La variedad R^5 con métrica

$dS^2 = -dT^2 + (dX^2 + dY^2 + dZ^2 + dW^2)$

Consideramos la subvariedad S_4^- (conocida como la pseudoesfera de 4 dimensiones) correspondinete a

$(X^2 + Y^2 + Z^2 + W^2) - T^2 = A^2$

En esta la podemos poner coordenadas radiales :

$X = rSen\theta Sen\varphi$, $Y = rSen\theta Cos\varphi$, $Z = rCos\theta$.

y cuando $r < A$

$W = A\sqrt{1 - r^2/A^2} Cosh(t/A)$,

$T = A\sqrt{1 - r^2/A^2} Senh(t/A)$

mientras que cuando $r > A$

$W = A\sqrt{r^2/A^2 - 1} Senh(t/A)$,

$T = A\sqrt{r^2/A^2 - 1} Cosh(t/A)$

Es fácil ver que la métrica inducida en S_4^- es una sección estática del espacio-tiempo de de-Sitter, específicamente:

$$dS^2 = -(1 - r^2/A^2)dt^2 + (1 - r^2/A^2)^{-1}dr^2 + r^2(d\theta^2 + Sen^2\theta d\varphi^2)$$

Hay un Horizonte de Killing en $r = A$, pero es como el de Rindler.... asociado con la elección de coordenadas!

No es una distancia sino un valor de la coordenada. Su area es $4\pi A^2$.

Notar: Los observadores estacionarios no sigen geodésicas. Tienen acceleración propia constante $a = \frac{Ar}{\sqrt{1-Ar^2}}$.

En el contexto cosmológico ponemos coordenadas de manera distinta.

$X = e^{(\tau/A)} RSen\theta Sen\varphi$, $Y = e^{(\tau/A)} RSen\theta Cos\varphi$,
$Z = e^{(\tau/A)} RCos\theta$,

y

$$W = ASenh(\tau/A) - (R^2/2A)e^{(\tau/A)},$$
$$T = ACosh(\tau/A) + (R^2/2A)e^{(\tau/A)}$$

En este caso la métrica resultante se ve

$$dS^2 = -d\tau^2 + e^{2(\tau/A)}[dR^2 + R^2(d\theta^2 + Sen^2\theta d\varphi^2)]$$

Estas coordenadas cubren la región

$W + T = Ae^{(\tau/A)} > 0$, o sea la parte de S_4^- por encima del plano $T = -W$.

Las regiones de $t = Constante$ corresponden a la intersección de S_4^- con los planos $W + T = constante$ que son regiones tipo “hiperbólicas".

Notar que tenemos : $Re^{\tau/A} = r$.

Horizonte de Eventos Cosmológico Asociado con un observador comóvil con linea de mundo γ el Horizonte de Evntos C. de esta es $\partial J^{-}(\gamma)$: los eventos que nunca serán visibles por γ.

Consideremos, sin perder gereralidad el observaador en el origen. El H.C. corresponde a la geodésica radial nula que llega a $R = 0$ en $\tau = \infty$.
Esta satisface $d\tau = -e^{\tau/A} dR$. Demos a sus eventos coordenadas τ_i, R_i , entonces se tiene:
$-\int_{R_i}^{0} dr = \int_{\tau_i}^{\infty} e^{-\tau'/A} d\tau'$ o $R_i = A e^{-\tau_i/A}$, ($r_i = A$) que en la hipersuperficie $\tau = \tau_i$ está a una distancia del orígen igual $D(\tau_i) = A$ (independiente de τ_i !).

Sin embargo no es un lugar físico epecial...solo tiene sentido en referencia al observador comóvil en $R = 0$, que depende de la elección de coordenadas y es equivalente a cualquier otro observador comóvil.

Horizonte de Partícula:

Consideremos la distancia máxima desde donde un observador co-móvil (en $r = 0, t = t_f$) puede haber recibido señales luminosas.

$$\int_0^{t_f} \frac{1}{a(t)} dt = \int_0^{r_e} \frac{1}{\sqrt{1-kr^2}} dr = \Psi_k(r_e).$$

La distancia espacial (en $t = t_f$) es :

$$D = a(t_f)\Psi_k(r_e) = a(t_f) \int_0^{t_f} a(t)^{-1} dt \; .$$

Si la integral

$$\int_0^{t_f} a(t)^{-1} dt$$

converge en su límite inferior, entonces D será una distancia finita que denotaremos $D_{HP}(t_f)$.

Siempre y cuando el límite inferior de integración, tomado aca como (t=0) sea el inicio del Universo !! habrá regiones del Universo que en $t = t_f$ no han tenido contacto causal.

1) En el caso $a(t) = Ct^{\alpha}, \alpha < 1$, tenemos:

$D_{HP}(t_f) = t_f^{\alpha}[\frac{1}{(1-\alpha)} t_f^{1-\alpha}] = \frac{1}{(1-\alpha)} t_f$.

Y la taza de expnsión ($H(t) = \dot{a}/a$) en t_f $H(t_f) = \alpha t_f^{-1}$ así que el radio de Hubble es $R_H(t_f) \equiv H(t_f)^{-1} = \alpha^{-1} t_f$ que es en general del mismo orden de magnitud que $D_{HP}(t_f)$. No confundir!!.

2) Pero en el caso $a(t) = Ce^{Ht}$, tenemos:

$D_{HP}(t_f) = e^{Ht_f}[\frac{-1}{H} e^{-Ht}|_{t_i}^{t_f}] = e^{Ht_f}(\frac{-1}{H})[e^{-Ht_f} - e^{-Ht_i}] = \frac{1}{H}[e^{H(t_f - t_i)} - 1]$.

En ese caso $H(t_f) = H$ es constante y el radio de Hubble es $R_H(t_f) = H(t_f)^{-1} = \frac{1}{H}$, muy diferente a $D_{HP}(t_f)$.

En general el radio de Hubble $R_H(t_f) = H(t_f)^{-1}$ nos da *una idea de la escala eficiente de interacciones causales* pero NO es un Horizonte de Partícula!!