Exclusive hadronic tau decays as probes of non-SM interactions High energy physics seminar, ICN-UNAM, IF-UNAM

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¹J. R., P. Roig and G. Toledo Sánchez, Phys. Rev. D **99**, no.9, 093005 (2019).

²S. González-Solís, A. Miranda, J. R. and P. Roig, Phys. Rev. D **101**, no.3, 034010 (2020).

³S. González-Solís, A. Miranda, J. R. and P. Roig, Phys. Lett. B **804**, 135371 (2020).

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Effective Field Theories

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Tau Physics

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The SM of particle physics



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Some problems with the SM

- The SM does not explain the mass of the neutrinos
- The SM does not explain dark matter
- It does not explain dark energy
- It does not explain the matter-antimatter asymmetry in the universe

The Tau lepton

The Tau is the heaviest lepton. Due to its large mass, it is the only lepton capable of decaying into hadrons (mesons).



Semileptonic decays of the Tau lepton

$$\begin{split} \tau^- &\to (\pi/K)^- \nu_\tau \\ \tau^- &\to \pi^- \pi^0 \nu_\tau, \\ \tau^- &\to \pi^- \eta^{(\prime)} \nu_\tau, \\ \tau^- &\to \pi^- K_{\mathsf{S}} \nu_\tau, \\ \tau^- &\to \pi^- \ell^+ \ell^- \nu_\tau, \\ \mathsf{etc.} \end{split}$$

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Semileptonic decays of the Tau lepton

$$\tau^{-} \rightarrow (\pi/K)^{-}\nu_{\tau}, \leftarrow S. Aoki \text{ et al.}^{4}$$

$$\tau^{-} \rightarrow \pi^{-}\pi^{0}\nu_{\tau}, \leftarrow Miranda, Roig^{5}$$

$$\tau^{-} \rightarrow \pi^{-}\eta^{(\prime)}\nu_{\tau}, \leftarrow E. Garc\acute{es}, Villanueva, L\acute{o}pez - Castro, Roig,^{6}$$

$$\tau^{-} \rightarrow \pi^{-}K_{S}\nu_{\tau}, \leftarrow Rend\acute{o}n, Roig, Toledo^{7}$$

$$\tau^{-} \rightarrow \pi^{-}\ell^{+}\ell^{-}\nu_{\tau} \leftarrow A. Guevara, L\acute{o}pez - Castro, Roig,^{8}$$

etc.

⁴S. Aoki et al., Http://flag.unibe.ch/ (2019 version).

⁵J. A. Miranda and P. Roig, JHEP **1811**, 038 (2018).

⁶E. A. Garcés, M. Hernández Villanueva, G. López Castro and P. Roig, JHEP **1712**, 027 (2017).

⁷J. R., P. Roig and G. Toledo Sánchez, Phys. Rev. D **99**, no. 9, 093005 (2019).

⁸A. Guevara, G. López Castro and P. Roig, Phys. Rev. D **88**, no.3, 033007 (2013).

Tau Physics

- Provides a clean environment to study low energy QCD effects
- It is important in the searches for lepton flavor violation
- Important in the determination of V_{us} to complement K_{ℓ3} decays and in the determination of α_S
- Important in the studies of lepton universality
- Important in the searches for new physics
- Important for providing an independent evaluation of $a_{\mu}^{HVP,LO}$

Effective Field Theories

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Effective Field Theories

$$\mathcal{L}^{(\textit{eff})} = \mathcal{L}_{\textit{SM}} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \frac{1}{\Lambda^3}\mathcal{L}_7 + \cdots$$

Fermi Theory for the Weak Interaction

$${\cal H}={G_F\over\sqrt{2}}J^\dagger_\mu J^\mu$$

Where,

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

$$J^{\mu} = \overline{e}\gamma^{\mu}(1-\gamma_{5})\nu_{e} + \overline{\mu}\gamma^{\mu}(1-\gamma_{5})\nu_{\mu} + V^{*}_{ud}\overline{d}\gamma^{\mu}(1-\gamma_{5})u + h.c + \dots$$

The scale of new physics in this case is the mass of the W boson.

Motivation for the work

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Motivation to do this study

► There is a 2.8 σ discrepancy between the $\tau^+ \rightarrow \pi^+ K_s \bar{\nu}_\tau$ rate asymmetry measured by BaBar $(A_{CP}^{\tau,exp} = -3.6(2.3)(1.1) \times 10^{-3})^{-9}$ and the one coming from the expected value due to $K^0 - \bar{K}^0$ mixing $(A_{CP}^{\tau,SM} = 3.6(1) \times 10^{-3})^{-10}$, where

$$A_{CP}^{\tau} = \frac{\Gamma(\tau^+ \to \pi^+ K_S \bar{\nu_{\tau}}) - \Gamma(\tau^- \to \pi^- K_S \nu_{\tau})}{\Gamma(\tau^+ \to \pi^+ K_S \bar{\nu_{\tau}}) + \Gamma(\tau^- \to \pi^- K_S \nu_{\tau})}$$

The $K_s \pi^-$ spectrum, particularly the first few Belle data points cannot be explained within the SM.

⁹J. P. Lees *et al.* [BaBar Collaboration], Phys. Rev. D **85**, 031102 (2012) Erratum: [Phys. Rev. D **85**, 099904 (2012)].

¹⁰I. I. Bigi and A. I. Sanda, Phys. Lett. B **625**, 47 (2005).

$au^- ightarrow K_S \pi^- u_{ au}$ spectrum



Figure: Distribution of $\tau^- \to K_S \pi^- \nu_{\tau}$ events measured by Belle¹¹

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¹¹Belle Collaboration, Phys. Lett. B 654 (2007) 65.

¹²A. Pich, Prog. Part. Nucl. Phys. **75**, 41 (2014)

Motivation to do this study

- The main motivation to do this work was to probe that semileptonic tau decays are really important in complementing traditional low-energy probes such as nuclear β decays, semileptonic pion and kaon decays, and also high energy measurements at the LHC.
- Show the importance of semileptonic tau decays as golden modes at Belle-II.
- We wanted to check if a purely exclusive global analysis is consistent with an inclusive+exclusive global analysis
- Do the first global analysis for $|\Delta S| = 1$ in the tau sector

EFT analysis of the $au^- ightarrow (K\pi)^- u_ au$ decays

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Effective theory analysis of $au^-
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The effective lagrangian density constructed with dimension six operators and invariant under the $SU(2)_L \otimes U(1)$ group has the following form,

$$\mathcal{L}^{(eff)} = \mathcal{L}_{SM} + \frac{1}{\Lambda^2} \sum_i \alpha_i O_i$$

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Effective theory analysis of $\tau^- \rightarrow \nu_\tau \bar{u}s$

We can explicitly construct the low-scale O(1 GeV) effective lagrangian for semi-leptonic transitions as follows:

$$\begin{split} \mathcal{L}_{cc} &= -\frac{G_F V_{us}}{\sqrt{2}} (1 + \epsilon_L + \epsilon_R) \\ &\times \left[\bar{\tau} \gamma_\mu (1 - \gamma_5) \nu_\tau \cdot \bar{u} [\gamma^\mu - (1 - 2\hat{\epsilon}_R) \gamma^\mu \gamma^5] s \right. \\ &+ \bar{\tau} (1 - \gamma_5) \nu_\tau \cdot \bar{u} [\hat{\epsilon}_s - \hat{\epsilon}_p \gamma_5] s \\ &+ 2\hat{\epsilon}_T \bar{\tau} \sigma_{\mu\nu} (1 - \gamma_5) \nu_\tau \cdot \bar{u} \sigma^{\mu\nu} s \right] + h.c \,, \end{split}$$

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where $\hat{\epsilon}_i = \epsilon_i / (1 + \epsilon_L + \epsilon_R)$ for i = R, S, P, T.

Amplitude

Due to the parity of pseudoscalar mesons, only the vector, scalar and tensor currents give a non-zero contribution to the decay amplitude $^{\rm 13}$

$$egin{aligned} \mathcal{M} =& \mathcal{M}_V + \mathcal{M}_S + \mathcal{M}_T \ &= rac{G_F V_{us} \sqrt{S_{EW}}}{\sqrt{2}} (1 + \epsilon_L + \epsilon_R) \ & imes [L_\mu H^\mu + \hat{\epsilon}_S L H + 2 \hat{\epsilon_T} L_{\mu
u} H^{\mu
u}] \,, \end{aligned}$$

where the leptonic currents have the following structure,

$$egin{aligned} & L_{\mu} = ar{u}(p')\gamma_{\mu}(1-\gamma_5)u(p)\,, \ & L = ar{u}(p')(1+\gamma_5)u(p)\,, \ & L_{\mu
u} = ar{u}(p')\sigma_{\mu
u}(1+\gamma_5)u(p)\,, \end{aligned}$$

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¹³E. A. Garcés, M. Hernández Villanueva, G. López Castro and P. Roig, JHEP **1712**, 027 (2017).

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Amplitude

The Hadronic matrix elements for $\tau^- \rightarrow \bar{K}^0 \pi^- \nu_{\tau}$ are given as follows,

$$\begin{split} H^{\mu} &= \langle \pi^{-} \bar{K}^{0} | \bar{s} \gamma^{\mu} u | 0 \rangle = Q^{\mu} F_{+}(s) + \frac{\Delta_{K\pi}}{s} q^{\mu} F_{0}(s) , \\ H &= \langle \pi^{-} \bar{K}^{0} | \bar{s} u | 0 \rangle = F_{s}(s) , \\ H^{\mu\nu} &= \langle \pi^{-} \bar{K}^{0} | \bar{s} \sigma^{\mu\nu} u | 0 \rangle = i F_{T}(s) (p^{\mu}_{K} p^{\nu}_{\pi} - p^{\mu}_{\pi} p^{\nu}_{K}) , \end{split}$$

where $q^{\mu} = (p_{\pi} + p_{K})^{\mu}$, $Q^{\mu} = (p_{K} - p_{\pi})^{\mu} - \frac{\Delta_{K\pi}}{s}q^{\mu}$, $s = q^{2}$, and $\Delta_{ij} = m_{i}^{2} - m_{j}^{2}$.

Amplitude

Similarly for the
$$\tau^- \rightarrow K^- \pi^0 \nu_{\tau}$$
 decay we have,
 $\sqrt{2}F^{K^-\pi^0}_{0,+,T}(s) = F^{\bar{K}^0\pi^-}_{0,+,T}(s).$

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The unpolarized spin-averaged squared amplitude is given by:

$$\overline{|\mathcal{M}|^2} = \frac{G_F^2 |V_{us}|^2 S_{EW}}{2} (1 + \epsilon_L + \epsilon_R)^2 \times (M_{0+} + M_{T+} + M_{T0} + M_{00} + M_{++} + M_{TT})$$

Vector and Scalar Form Factors

Here we benefit from previous works for the VFF and SFF cases. The VFF is taken from ref. 14 and the SFF is taken from ref. 15 .

¹⁴D. R. Boito, R. Escribano and M. Jamin, Eur. Phys. J. C **59**, 821 (2009).
 ¹⁵M. Jamin, J. A. Oller and A. Pich, Phys. Rev. D **74**, 074009 (2006).

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Tensor Form Factor

For the TFF we obtain its normalization at zero momentum transfer using χ PT with tensor sources ¹⁶ and Lattice data ¹⁷ and its energy dependence using a dispersion relation¹⁸.

$$egin{aligned} &i\langle \pi^-ar{K^0}|rac{\delta\mathcal{L}}{\deltaar{t}_{lphaeta}}|0
angle &=rac{\Lambda_2}{F^2}\left(p_K^lpha p_0^eta-p_0^lpha p_K^eta
ight)\,.\ &rac{F_T(s)}{F_T(0)} = exp\left[rac{s}{\pi}\int_{s_{\pi K}}^{s_{cut}}ds^{'}rac{\delta_T(s^{'})}{s^{'}(s^{'}-s-i\epsilon)}
ight]\,,\ &\kappa=(m_{ar{k}^0}+m_{\pi^-})^2. \end{aligned}$$

where $s_{\pi K} = (m_{ar{K}^0} + m_{\pi^-})^2$

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¹⁸J. A. Miranda and P. Roig, JHEP **1811**, 038 (2018).

¹⁶O. Cata and V. Mateu, JHEP **0709**, 078 (2007).

¹⁷I. Baum, V. Lubicz, G. Martinelli, L. Orifici and S. Simula, Phys. Rev. D 84, 074503 (2011).

Tensor Form Factor



Figure: Modulus and phase, $|F_T(s)|$ (left) and $\delta_T(s) = \delta_+(s)$ (right), of the tensor form factor, $F_T(s)$. On the left plot, the dotted line corresponds to $s_{cut} = 9$ GeV², the dashed one to $s_{cut} = 4$ GeV², and the solid one to $s_{cut} = M_{\tau}^2$.

Dalitz Plots



Figure: Dalitz plot distribution $\overline{|\mathcal{M}|^2}_{00}$ in the SM: Differential decay distribution for $\tau^- \to K_S \pi^- \nu_\tau$ in the (s, t) variables (left), and in the $(s, \cos\theta)$ variables (right)

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Dalitz Plots



Figure: Dalitz plot distribution $\tilde{\Delta}(\hat{\epsilon}_S, \hat{\epsilon}_T)$ in the $\tau^- \to K_S \pi^- \nu_{\tau}$ decays: left-hand side corresponds to the (s, t) variables and the right-hand side corresponds to the differential decay distribution in the $(s, \cos\theta)$ variables, both with $(\hat{\epsilon}_S = 0, \hat{\epsilon}_T = 0.6)$.

In the previous plot we have defined,

$$\tilde{\Delta}(\hat{\epsilon}_{\mathcal{S}},\hat{\epsilon}_{\mathcal{T}}) = \frac{\left|\overline{|\mathcal{M}(\hat{\epsilon}_{\mathcal{S}},\hat{\epsilon}_{\mathcal{T}})|^2} - \overline{|\mathcal{M}(0,0)|^2}\right|}{\overline{|\mathcal{M}(0,0)|^2}}$$

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Decay rate

$$\begin{aligned} \frac{d\Gamma}{ds} = & \frac{G_F^2 |V_{us}|^2 M_\tau^3 S_{EW}}{384 \pi^3 s} (1 + \epsilon_L + \epsilon_R)^2 \left(1 - \frac{s}{M_\tau^2}\right)^2 \lambda^{1/2} (s, m_\pi^2, m_K^2) \\ & \times \left[X_{V\!A} + \hat{\epsilon}_S X_S + \hat{\epsilon}_T X_T + \hat{\epsilon}_S^2 X_{S^2} + \hat{\epsilon}_T^2 X_{T^2}\right], \end{aligned}$$

$$\begin{split} X_{VA} &= \frac{1}{2s^2} \left[3|F_0(s)|^2 \Delta_{K\pi}^2 + |F_+(s)|^2 \left(1 + \frac{2s}{M_\tau^2} \right) \lambda(s, m_\pi^2, m_K^2) \right] \,, \\ X_S &= \frac{3}{sM_\tau} |F_0(s)|^2 \frac{\Delta_{K\pi}^2}{m_s - m_d} \,, \\ X_T &= \frac{6}{sM_\tau} Re[F_T(s)F_+^*(s)]\lambda(s, m_\pi^2, m_K^2) \,, \\ X_{S^2} &= \frac{3}{2M_\tau^2} |F_0(s)|^2 \frac{\Delta_{K\pi}^2}{(m_s - m_u)^2} \,, \\ X_{T^2} &= \frac{4}{s} |F_T(s)|^2 \left(1 + \frac{s}{2M_\tau^2} \right) \lambda(s, m_\pi^2, m_K^2) \,. \end{split}$$

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Decay Rate



Figure: The $\bar{K}^0\pi^-$ hadronic invariant mass distribution for the SM (solid line) and $\hat{\epsilon}_S = -0.5$, $\hat{\epsilon}_T = 0$ (dashed line) and $\hat{\epsilon}_S = 0$, $\hat{\epsilon}_T = 0.6$ (dotted line). The decay distributions are normalized to the tau decay width.

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Forward-backward asymmetries

$$\mathcal{A}_{K\pi}(s) = \frac{\int_{0}^{1} d\cos\theta \frac{d^{2}\Gamma}{dsd\cos\theta} - \int_{-1}^{0} d\cos\theta \frac{d^{2}\Gamma}{dsd\cos\theta}}{\int_{0}^{1} d\cos\theta \frac{d^{2}\Gamma}{dsd\cos\theta} + \int_{-1}^{0} d\cos\theta \frac{d^{2}\Gamma}{dsd\cos\theta}} \,.$$
$$\mathcal{A}_{K\pi} = \frac{3\sqrt{\lambda(s), m_{\pi}^{2}, m_{K}^{2}}}{2s^{2}[X_{VA} + \hat{\epsilon}_{S}X_{S} + \hat{\epsilon}_{T}X_{T} + \hat{\epsilon}_{S}^{2}X_{S^{2}} + \hat{\epsilon}_{T}^{2}X_{T^{2}}]} \\ \left(1 + \frac{s\hat{\epsilon}_{S}}{M_{\tau}(m_{s} - m_{u})}\right) \Delta_{\pi K} \left[-Re[F_{0}(s)F_{+}^{*}(s)] + \frac{2s\hat{\epsilon}_{T}}{M_{\tau}}Re[F_{T}(s)F_{0}^{*}(s)]\right]$$

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Forward-backward asymmetries



Figure: FB asymmetries: SM (solid line), $\hat{\epsilon}_S = -0.5$, $\hat{\epsilon}_T = 0$ (dashed line) and $\hat{\epsilon}_S = 0$, $\hat{\epsilon}_T = 0.6$ (dotted line).

Limits on $\hat{\epsilon}_S$ and $\hat{\epsilon}_T$

$$\Delta \equiv \frac{\Gamma - \Gamma^0}{\Gamma^0} = \alpha \hat{\epsilon}_S + \beta \hat{\epsilon}_T + \gamma \hat{\epsilon}_S^2 + \delta \hat{\epsilon}_T^2 \,,$$

where we obtained the following results for the coefficients: $\alpha \in [0.30, 0.34]$, $\beta \in [-2.92, -2.35]$, $\gamma \in [0.95, 1.13]$ and $\delta \in [3.57, 5.45]$.

Limits on $\hat{\epsilon}_S$ and $\hat{\epsilon}_T$



Figure: Δ as a function of $\hat{\epsilon}_S$ for $\hat{\epsilon}_T = 0$ (left hand) and of $\hat{\epsilon}_T$ for $\hat{\epsilon}_S = 0$ (right hand) for $\tau^- \rightarrow K_S \pi^- \nu_\tau$ decays. Horizontal lines represent the values of Δ according to the current measurement and theory errors (at three standard deviations) of the branching ratio (dashed line) and in the hypothetical case where the measured branching ratio at Belle-II has a three times reduced uncertainty (dotted line).

Limits on $\hat{\epsilon}_S$ and $\hat{\epsilon}_T$



Figure: Constraints on the scalar and tensor couplings obtained from $\Delta(\tau^- \rightarrow K_S \pi^- \nu_\tau)$ using theory and the measured value reported in the PDG, with their corresponding uncertainties at three standard deviations (solid line). The dashed line ellipse corresponds to the case where the measurements error was reduced to a third of the current uncertainty.

Δ limits	$\hat{\epsilon}_{S}(\hat{\epsilon}_{T}=0)$	$\hat{\epsilon}_{\mathcal{T}}(\hat{\epsilon}_{\mathcal{S}}=0)$	ês	êτ
Current bounds	[-0.57, 0.27]	$[-0.059, 0.052] \cup [0.60, 0.72]$	[-0.89, 0.58]	[-0.07, 0.72]
Future bounds	[-0.52, 0.22]	$[-0.047, 0.036] \cup [0.62, 0.71]$	[-0.87, 0.56]	[-0.06, 0.71]

Table: Constraints on the scalar and tensor couplings obtained through the limits on the current branching ratio at three standard deviations using the current theory and experimental errors and assuming the latter be reduced to a third ('Future bounds').
Limits on $\hat{\epsilon}_S$ and $\hat{\epsilon}_T$

When we make the fit to the whole spectrum we obtain,

Best fit values	$\hat{\epsilon}_{S}$	$\hat{\epsilon}_T$	χ^2	χ^2 in the SM
No $i = 5, 6, 7$ bins	$(1.3\pm 0.9) imes 10^{-2}$	$(0.7 \pm 1.0) imes 10^{-2}$	[72, 73]	[74, 77]
i = 5, 6, 7 bins	$(0.9 \pm 1.0) imes 10^{-2}$	$(1.7\pm 1.7) imes 10^{-2}$	[83, 86]	[91, 95]

Table: Best fit values to the Belle spectrum and branching ratio of the $\tau^- \to K_S \pi^- \nu_\tau$ decays

 19 The previous limits translate into bounds of the corresponding NP scale $\Lambda\approx[2,5]\text{TeV}$, while Kaon physics may reach $\mathcal{O}(500)\text{TeV}_{20}$

$$\Lambda \sim v(V_{us}\hat{\epsilon}_{S,T})^{-1/2}$$

¹⁹D. Epifanov *et al.* [Belle Collaboration], Phys. Lett. B **654**, 65 (2007).
 ²⁰M. González-Alonso and J. Martin Camalich, JHEP **1612**, 052 (2016).

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What about CP violation?

$$egin{aligned} & {\cal A}_{CP}^{BSM} \, = \, rac{2{
m sin}\delta_T^W |\hat{\epsilon}_T| G_F^2 |V_{us}|^2 S_{EW}}{256\pi^3 M_ au^2 \Gamma(au o K_S \pi
u_ au)} imes \ & \int_{s_{\pi K}}^{M_ au^2} {
m d}s |f_+(s)| |F_T(s)| {
m sin} \left(\delta_+(s) - \delta_T(s)
ight) rac{\lambda^{3/2}(s,m_\pi^2,m_K^2) (M_ au^2 - s)^2}{s^2} \,, \end{aligned}$$

In agreement with Ref. ²¹ we confirmed that it is not possible to understand the BaBar A_{CP} measurement.

$$A_{CP}^{BSM} \lesssim 8 \cdot 10^{-7}$$
,

which is a slightly weaker bound than the one reported in the previous reference ($A_{CP}^{BSM} \lesssim 3 \cdot 10^{-7}$)

²¹V. Cirigliano, A. Crivellin and M. Hoferichter, Phys. Rev. Lett. **120**, no. 14, 141803 (2018).

Global analysis

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Effective theory analysis of $au^- ightarrow u_{ au} \overline{u} D$ (D = d, s)

We can explicitly construct the low-scale O(1GeV) effective lagrangian for semi-leptonic transitions as follows: 22 23 :

$$\begin{split} \mathcal{L}_{CC} &= -\frac{G_F V_{uD}}{\sqrt{2}} \bigg[(1 + \epsilon_L^\tau) \bar{\tau} \gamma_\mu (1 - \gamma^5) \nu_\tau \cdot \bar{u} \gamma^\mu (1 - \gamma^5) D \\ &+ \epsilon_R^\tau \bar{\tau} \gamma_\mu (1 - \gamma^5) \nu_\tau \cdot \bar{u} \gamma^\mu (1 + \gamma^5) D \\ &+ \bar{\tau} (1 - \gamma^5) \nu_\tau \cdot \bar{u} (\epsilon_S^\tau - \epsilon_P^\tau \gamma^5) D \\ &+ \epsilon_T^\tau \bar{\tau} \sigma_{\mu\nu} (1 - \gamma^5) \nu_\tau \, \bar{u} \sigma^{\mu\nu} (1 - \gamma^5) D \bigg] + h.c. \,, \end{split}$$

²²T. Bhattacharya, V. Cirigliano, S. D. Cohen, A. Filipuzzi,

M. Gonzalez-Alonso, M. L. Graesser, R. Gupta and H. W. Lin, Phys. Rev. D 85, 054512 (2012).

²³S. González-Solís, A. Miranda, J. Rendón and P. Roig, Phys. Lett. B **804**, 135371 (2020).

One-meson decay modes $\tau^- \rightarrow P^- \nu_{\tau}$ ($P = \pi, K$).

$$\begin{split} \Gamma(\tau^- \to \pi^- \nu_\tau) &= \frac{G_F^2 |\tilde{V}_{ud}^e|^2 f_\pi^2 m_\tau^3}{16\pi} \left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^2 \\ &\times \left(1 + \delta_{\rm em}^{\tau\pi} + 2\Delta^{\tau\pi} + \mathcal{O}(\epsilon_i^\tau)^2 + \mathcal{O}(\delta_{\rm em}^{\tau\pi} \epsilon_i^\tau)\right), \end{split}$$

where f_{π} is the pion decay constant, the quantity $\delta_{\rm em}^{\tau\pi}$ accounts for the EM radiative corrections and the term $\Delta^{\tau\pi}$ contains the tree-level NP corrections that arise from \mathcal{L}_{eff} that are not absorbed in \tilde{V}_{ud}^{e} .

One-meson decay modes $\tau^- \rightarrow P^- \nu_\tau \ (P = \pi, K)$.

The product $G_F V_{uD}$ in \mathcal{L}_{eff} denotes that its determination from the superallowed nuclear Fermi β decays carries implicitly a dependence on ϵ_L^e and ϵ_R^e that is given by ²⁴

$$G_F \tilde{V}^e_{uD} = G_F \left(1 + \epsilon^e_L + \epsilon^e_R\right) V_{uD},$$

For the channel $\tau^- \to K^- \nu_{\tau}$, the decay rate is that of Eq. (1) but replacing $\tilde{V}^e_{ud} \to \tilde{V}^e_{us}$, $f_{\pi} \to f_K$, $m_{\pi} \to m_K$, and $\delta^{\tau\pi}_{em}$ and $\Delta^{\tau\pi}$ by $\delta^{\tau K}_{em}$ and $\Delta^{\tau K}$, respectively.

²⁴M. González-Alonso and J. Martin Camalich, JHEP 12, 052 (2016)

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Two-meson decay modes $\tau^- \rightarrow (PP')^- \nu_{\tau}$.

$$\begin{aligned} \frac{d\Gamma}{ds} &= \frac{G_F^2 |\tilde{V}_{uD}^e|^2 m_\tau^3 S_{EW}}{384 \pi^3 s} \left(1 - \frac{s}{m_\tau^2}\right)^2 \lambda^{1/2} (s, m_P^2, m_{P'}^2) \\ &\times \left[\left(1 + 2(\epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e)\right) X_{VA} \right. \\ &\left. + \epsilon_S^\tau X_S + \epsilon_T^\tau X_T + (\epsilon_S^\tau)^2 X_{S^2} + (\epsilon_T^\tau)^2 X_{T^2} \right], \end{aligned}$$

 Two-meson decay modes $\tau^- \rightarrow (PP')^- \nu_{\tau}$.

$$\begin{split} X_{VA} &= \frac{1}{2s^2} \left\{ 3 \left(C_{PP'}^{S} \right)^2 |F_0^{PP'}(s)|^2 \Delta_{PP'}^2 \\ &+ \left(C_{PP'}^{V} \right)^2 |F_+^{PP'}(s)|^2 \left(1 + \frac{2s}{m_\tau^2} \right) \lambda(s, m_P^2, m_{P'}^2) \right\}, \\ X_S &= \frac{3}{s \, m_\tau} \left(C_{PP'}^{S} \right)^2 |F_0^{PP'}(s)|^2 \frac{\Delta_{PP'}^2}{m_d - m_u}, \\ X_T &= \frac{6}{s \, m_\tau} C_{PP'}^{V} \operatorname{Re} \left[F_T^{PP'}(s) \left(F_+^{PP'}(s) \right)^* \right] \lambda(s, m_P^2, m_{P'}^2), \\ X_{S^2} &= \frac{3}{2 \, m_\tau^2} \left(C_{PP'}^{S} \right)^2 |F_0^{PP'}(s)|^2 \frac{\Delta_{PP'}^2}{(m_d - m_u)^2}, \\ X_{T^2} &= \frac{4}{s} |F_T^{PP'}(s)|^2 \left(1 + \frac{s}{2 \, m_\tau^2} \right) \lambda(s, m_P^2, m_{P'}^2), \end{split}$$

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Tensor form factors

For the tensor form factors $F_T^{PP'}(s)$ we have^{25 26},

$$F_T^{PP'}(s) = F_T^{PP'}(0) \exp\left[\frac{s}{\pi} \int_{s_{\rm th}}^{s_{\rm cut}} \frac{ds'}{s'} \frac{\delta_T^{PP'}(s')}{(s'-s-i0)}\right] \,,$$

where $s_{\rm th}$ is the two-meson production threshold for the lightest pair of mesons with the same quantum numbers as the given pair PP'.

 $F_T^{PP'}(0)$: ChPT with Tensor Sources+Lattice

²⁵O. Cata and V. Mateu, JHEP **0709**, 078 (2007).

²⁶I. Baum, V. Lubicz, G. Martinelli, L. Orifici and S. Simula, Phys. Rev. D 84, 074503 (2011).

New Physics bounds from $\Delta S = 0$ decays

from the $\tau^- \to \pi^- \nu_\tau$ decay rate alone, we obtain the following constraint,

$$\epsilon_L^{\tau} - \epsilon_L^e - \epsilon_R^{\tau} - \epsilon_R^e - \frac{m_{\pi}^2}{m_{\tau}(m_u + m_d)} \epsilon_P^{\tau} = (-0.12 \pm 0.68) \times 10^{-2} \,,$$

Input:
$$f_{\pi} = 130.2(8)$$
 (FLAG)²⁷; $\delta_{em}^{\tau\pi} = 1.92(24)\%^{28}$ ²⁹³⁰;
 $|\tilde{V}_{ud}^{e}| = 0.97420(21)$ (β decays, PDG).

²⁷S. Aoki *et al.* [Flavour Lattice Averaging Group], Eur. Phys. J. C **80**, no.2, 113 (2020).

²⁸R. Decker and M. Finkemeier, Nucl. Phys. B **438**, 17-53 (1995).

²⁹V. Cirigliano and I. Rosell, JHEP **10**, 005 (2007).

³⁰J. L. Rosner, S. Stone and R. S. Van de Water, [arXiv:1509.02220 [hep-ph]].

Global analysis for $\Delta S = 0$ decays

- ► the high-statistics τ⁻ → π⁻π⁰ν_τ experimental data reported by the Belle collaboration, including both the normalized unfolded spectrum and the branching ratio.
- the branching ratio for the decay $\tau^- \rightarrow K^- K^0 \nu_{\tau}$.
- the branching ratio for $\tau^- \rightarrow \pi^- \nu_{\tau}$.

The χ^2 function that is minimized in our fits is

$$\begin{split} \chi^2 &= \sum_k \left(\frac{\bar{N}_k^{\text{th}} - \bar{N}_k^{\text{exp}}}{\sigma_{\bar{N}_k^{\text{exp}}}} \right)^2 + \left(\frac{BR_{\pi\pi}^{\text{th}} - BR_{\pi\pi}^{\text{exp}}}{\sigma_{BR_{\pi\pi}^{\text{exp}}}} \right)^2 \\ &+ \left(\frac{BR_{KK}^{\text{th}} - BR_{KK}^{\text{exp}}}{\sigma_{BR_{KK}^{\text{exp}}}} \right)^2 + \left(\frac{BR_{\tau\pi}^{\text{th}} - BR_{\tau\pi}^{\text{exp}}}{\sigma_{BR_{\tau\pi}^{\text{exp}}}} \right)^2 \,, \end{split}$$

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New Physics bounds from $\Delta S = 0$ decays

The bounds for the non-SM effective couplings resulting from the global fit are found to be (in the \overline{MS} scheme at scale $\mu = 2 \text{ GeV}$)

$$\begin{pmatrix} \epsilon_L^{\tau} - \epsilon_L^{e} + \epsilon_R^{\tau} - \epsilon_R^{e} \\ \epsilon_R^{\tau} + \frac{m_{\pi}^2}{2m_{\tau}(m_u + m_d)} \epsilon_P^{\tau} \\ \epsilon_S^{\tau} \\ \epsilon_T^{\tau} \end{pmatrix} = \begin{pmatrix} 0.5 \pm 0.6^{+2.3}_{-1.8} \stackrel{+0.2}{_{-0.1}} \pm 0.4 \\ 0.3 \pm 0.5^{+1.1}_{-1.8} \stackrel{+0.1}{_{-0.0}} \pm 0.2 \\ 9.7^{+0.5}_{-0.6} \pm 21.5^{+0.0}_{-0.1} \pm 0.2 \\ -0.1 \pm 0.2^{+1.1}_{-1.4} \stackrel{+0.0}{_{-0.1}} \pm 0.2 \end{pmatrix} \times 10^{-2},$$

31 1.stat. fit uncertainty, 2.pion VFF, 3.quark masses, 4.TFF

³¹ if we use the $\pi\eta$ channel we reproduce the limits in E. A. Garcés, M. Hernández Villanueva, G. López Castro and P. Roig, JHEP **12**, 027 (2017).

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New Physics bounds from $|\Delta S| = 1$ decays

from the $\tau^- \to K^- \nu_\tau$ decay rate alone, we obtain the following constraint,

$$\epsilon_L^ au - \epsilon_L^e - \epsilon_R^ au - \epsilon_R^e - rac{m_K^2}{m_ au(m_u+m_s)}\epsilon_P^ au = (-0.41\pm0.93) imes10^{-2}\,.$$

Input: $f_{\mathcal{K}} = 155.7(7) \text{ (FLAG)}^{32}; \ \delta_{em}^{\tau \mathcal{K}} = 1.98(31)\%^{33} \ ^{3435};$ $|\tilde{V}_{us}^{e}| = 0.2231(7) \text{ (PDG)}.$

³²S. Aoki *et al.* [Flavour Lattice Averaging Group], Eur. Phys. J. C **80**, no.2, 113 (2020).

³³R. Decker and M. Finkemeier, Nucl. Phys. B **438**, 17-53 (1995).

³⁴V. Cirigliano and I. Rosell, JHEP **10**, 005 (2007).

³⁵J. L. Rosner, S. Stone and R. S. Van de Water, [arXiv:1509.02220 [hep-ph]].

Global analysis for $\Delta S = 1$ decays

- ▶ the $\tau^- \rightarrow K_S \pi^- \nu_\tau$ Belle spectrum together with the measured branching ratio, $BR_{K\pi}^{exp} = 0.404(2)(13)\%$.
- the branching ratio of the decay $\tau^- \to K^- \eta \nu_{\tau}$ $(BR_{K\eta}^{exp} = 1.55(8) \times 10^{-4})$.

• the branching ratio of the decay $\tau^- \rightarrow K^- \nu_{\tau}$ $(BR_{\tau K}^{exp} = 6.96(10) \times 10^{-3}).$

$$\begin{split} \chi^2 &= \sum_k \left(\frac{\bar{N}_k^{\text{th}} - \bar{N}_k^{\text{exp}}}{\sigma_{\bar{N}_k^{\text{exp}}}} \right)^2 + \left(\frac{BR_{K\pi}^{\text{th}} - BR_{K\pi}^{\text{exp}}}{\sigma_{BR_{K\pi}^{\text{exp}}}} \right)^2 \\ &+ \left(\frac{BR_{K\eta}^{\text{th}} - BR_{K\eta}^{\text{exp}}}{\sigma_{BR_{K\eta}^{\text{exp}}}} \right)^2 + \left(\frac{BR_{\tau K}^{\text{th}} - BR_{\tau K}^{\text{exp}}}{\sigma_{BR_{\tau K}^{\text{exp}}}} \right)^2 \,, \end{split}$$

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New Physics bounds from $|\Delta S| = 1$ decays

In this case, the limits for the NP effective couplings are found to be (in the $\overline{\rm MS}$ scheme at scale $\mu = 2$ GeV)

$$\begin{pmatrix} \epsilon_L^{\tau} - \epsilon_L^{e} + \epsilon_R^{\tau} - \epsilon_R^{e} \\ \epsilon_R^{\tau} + \frac{m_K^2}{2m_{\tau}(m_u + m_s)} \epsilon_P^{\tau} \\ \epsilon_S^{\tau} \\ \epsilon_T^{\tau} \end{pmatrix} = \begin{pmatrix} 0.5 \pm 1.5 \pm 0.3 \\ 0.4 \pm 0.9 \pm 0.2 \\ 0.8^{+0.8}_{-0.9} \pm 0.3 \\ 0.9 \pm 0.7 \pm 0.4 \end{pmatrix} \times 10^{-2},$$

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1.stat. fit uncertainty, 2.TFF

New Physics bounds from a global fit to both $\Delta S = 0$ and $|\Delta S| = 1$

$$\begin{pmatrix} \epsilon_L^{\tau} - \epsilon_L^e + \epsilon_R^{\tau} - \epsilon_R^e \\ \epsilon_R^{\tau} \\ \epsilon_P^{\tau} \\ \epsilon_S^{\tau} \\ \epsilon_T^{\tau} \end{pmatrix} = \\ \begin{pmatrix} 2.9 \pm 0.6 & ^{+1.0}_{-0.9} \pm 0.6 \pm 0.0 \pm 0.4 & ^{+0.2}_{-0.3} \\ 7.1 \pm 4.9 & ^{+0.5}_{-0.4} & ^{+1.3}_{-1.5} & ^{+1.2}_{-1.3} \pm 0.2 & ^{+40.9}_{-14.1} \\ -7.6 \pm 6.3 \pm 0.0 & ^{+1.9}_{-1.6} & ^{+1.7}_{-1.6} \pm 0.0 & ^{+19.0}_{-53.6} \\ 5.0 & ^{+0.7}_{-0.8} & ^{+0.8}_{-1.3} & ^{+0.2}_{-0.1} \pm 0.0 \pm 0.2 & ^{+1.1}_{-0.6} \\ -0.5 \pm 0.2 & ^{+0.8}_{-1.0} \pm 0.0 \pm 0.0 \pm 0.6 \pm 0.1 \end{pmatrix} \times 10^{-2},$$

1.stat. fit uncertainty, 2.pion VFF, 3.CKM parameters, 4.rad. cor., 5.TFF, 6.quark masses

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Comparison between different probes for the ϵ 's



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Conclusions

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Conclusions

- Both the BaBar A_{CP} anomaly and the i = 5, 6, 7 Belle data points cannot be explained by heavy NP contributions.
- This work highlights that hadronic tau lepton decays remain to be not only a privileged tool for the investigation of the hadronization of QCD currents but also offer an interesting scenario as New Physics probes.

▶ In general, our bounds on the NP couplings, are competitive. This is specially the case for the combination of couplings $\epsilon_L^{\tau} - \epsilon_L^e + \epsilon_R^{\tau} - \epsilon_R^e$, which is found to be in accord with ³⁷ and ϵ_T^{τ} , that can even compete with the constraints set by the theoretically cleaner $K_{\ell 3}$ decays.

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A. Rodríguez-Sánchez, Phys. Rev. Lett. 122, no.22, 221801 (2019).

³⁷V. Cirigliano, A. Falkowski, M. González-Alonso and

Conclusions

- As for ε^τ_S, it is impossible to compete with the limits coming from K_{ℓ3} decays (the decay τ⁻ → π⁻ην_τ has not been taken into account because the lack of data).
- $\tau \rightarrow \pi \eta \nu_{\tau}$ is good constraining ϵ_S but is limited due to lack of data.
- $\tau \to \pi \pi \nu_{\tau}$ and $\tau \to \pi K \nu_{\tau}$ are good constraining ϵ_{T} .
- We have performed the first global analysis for $|\Delta S| = 1$ in the tau sector.

Gracias

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Parameters of the SM

where we have a total of 19 independent parameters:

- ▶ 3 gauge couplings (g, g', g_s)
- 6 quark masses
- 3 masses for the charged leptons
- 3 mixing angles from the CKM matrix
- 1 CP phase also from the CKM matrix
- 2 parameters from the scalar sector (μ^2 and λ)

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► θ_{QCD}

Dimension Six Operators (Backup)

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2 \varphi^3$	
\mathcal{O}_G	$f^{ABC}G^{A\nu}_{\mu}G^{B\rho}_{\nu}G^{C\mu}_{\rho}$	\mathcal{O}_{φ}	$(\varphi^{\dagger}\varphi)^{3}$	$\mathcal{O}_{e\varphi}$	$(\varphi^{\dagger}\varphi)(\bar{l}_{p}e_{r}\varphi)$
$\mathcal{O}_{\widetilde{G}}$	$f^{ABC} {\widetilde G}^{A\nu}_\mu G^{B\rho}_\nu G^{C\mu}_\rho$	$\mathcal{O}_{\varphi \Box}$	$(\varphi^{\dagger}\varphi)\Box(\varphi^{\dagger}\varphi)$	$\mathcal{O}_{u\varphi}$	$(\varphi^{\dagger}\varphi)(\bar{q}_{p}u_{r}\widetilde{\varphi})$
\mathcal{O}_W	$\varepsilon^{IJK}W^{I\nu}_{\mu}W^{J\rho}_{\nu}W^{K\mu}_{\rho}$	$\mathcal{O}_{\varphi D}$	$\left(\varphi^{\dagger}D^{\mu}\varphi\right)^{\star}\left(\varphi^{\dagger}D_{\mu}\varphi\right)$	$\mathcal{O}_{d\varphi}$	$(\varphi^{\dagger}\varphi)(\bar{q}_{p}d_{r}\varphi)$
$\mathcal{O}_{\widetilde{W}}$	$\varepsilon^{IJK}\widetilde{W}^{I\nu}_{\mu}W^{J\rho}_{\nu}W^{K\mu}_{\rho}$				
$X^2 \varphi^2$		$\psi^2 X \varphi$		$\psi^2 \varphi^2 D$	
$\mathcal{O}_{\varphi G}$	$\varphi^{\dagger}\varphiG^{A}_{\mu\nu}G^{A\mu\nu}$	\mathcal{O}_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I \varphi W^I_{\mu\nu}$	$\mathcal{O}_{\varphi l}^{(1)}$	$(\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\varphi)(\overline{l}_{p}\gamma^{\mu}l_{r})$
$\mathcal{O}_{\varphi \widetilde{G}}$	$\varphi^{\dagger}\varphi\widetilde{G}^{A}_{\mu\nu}G^{A\mu\nu}$	\mathcal{O}_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$\mathcal{O}_{\varphi l}^{(3)}$	$(\varphi^{\dagger}i \overset{\leftrightarrow}{D}{}^{I}_{\mu} \varphi)(\bar{l}_{p} \tau^{I} \gamma^{\mu} l_{r})$
$\mathcal{O}_{\varphi W}$	$\varphi^{\dagger}\varphiW^{I}_{\mu\nu}W^{I\mu\nu}$	\mathcal{O}_{uG}	$(\bar{q}_p \sigma^{\mu u} T^A u_r) \widetilde{\varphi} G^A_{\mu u}$	$\mathcal{O}_{\varphi e}$	$(\varphi^{\dagger}i \stackrel{\leftrightarrow}{D}_{\mu} \varphi)(\bar{e}_p \gamma^{\mu} e_r)$
$\mathcal{O}_{\varphi \widetilde{W}}$	$\varphi^{\dagger}\varphi\widetilde{W}^{I}_{\mu\nu}W^{I\mu\nu}$	\mathcal{O}_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \widetilde{\varphi} W^I_{\mu\nu}$	$\mathcal{O}_{\varphi q}^{(1)}$	$(\varphi^{\dagger}i \overset{\leftrightarrow}{D}_{\mu} \varphi)(\bar{q}_p \gamma^{\mu} q_r)$
$\mathcal{O}_{\varphi B}$	$\varphi^{\dagger}\varphi B_{\mu\nu}B^{\mu\nu}$	\mathcal{O}_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \widetilde{\varphi} B_{\mu\nu}$	$\mathcal{O}_{\varphi q}^{(3)}$	$(\varphi^{\dagger}i \overleftrightarrow{D}_{\mu}^{I} \varphi)(\bar{q}_{p} \tau^{I} \gamma^{\mu} q_{r})$
$\mathcal{O}_{\varphi \widetilde{B}}$	$\varphi^{\dagger}\varphi\widetilde{B}_{\mu\nu}B^{\mu\nu}$	\mathcal{O}_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) \varphi G^A_{\mu\nu}$	$\mathcal{O}_{\varphi u}$	$(\varphi^{\dagger}i \overset{\leftrightarrow}{D}_{\mu} \varphi)(\bar{u}_p \gamma^{\mu} u_r)$
$\mathcal{O}_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi W^I_{\mu\nu} B^{\mu\nu}$	\mathcal{O}_{dW}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I\varphiW^I_{\mu\nu}$	$\mathcal{O}_{\varphi d}$	$(\varphi^{\dagger}i \overleftrightarrow{D}_{\mu} \varphi)(\bar{d}_p \gamma^{\mu} d_r)$
$\mathcal{O}_{\varphi \widetilde{W}B}$	$\varphi^\dagger \tau^I \varphi \widetilde{W}^I_{\mu\nu} B^{\mu\nu}$	\mathcal{O}_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \varphi B_{\mu\nu}$	$\mathcal{O}_{\varphi ud}$	$i(\widetilde{\varphi}^{\dagger}D_{\mu}\varphi)(\bar{u}_{p}\gamma^{\mu}d_{r})$

Figure: Dimension-six operators other than the four-fermion ones, ³⁸.

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³⁸JHEP 10, 085 (2010).

Dimension Six Operators

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$		
\mathcal{O}_{ll}	$(\bar{l}_p \gamma_\mu l_r) (\bar{l}_s \gamma^\mu l_t)$	\mathcal{O}_{ee}	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$	\mathcal{O}_{le}	$(\bar{l}_p \gamma_\mu l_r) (\bar{e}_s \gamma^\mu e_t)$	
$\mathcal{O}_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$	\mathcal{O}_{uu}	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$	\mathcal{O}_{lu}	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$	
$\mathcal{O}_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r) (\bar{q}_s \gamma^\mu \tau^I q_t)$	\mathcal{O}_{dd}	$(\bar{d}_p \gamma_\mu d_r) (\bar{d}_s \gamma^\mu d_t)$	\mathcal{O}_{ld}	$(\bar{l}_p \gamma_\mu l_r) (\bar{d}_s \gamma^\mu d_t)$	
$\mathcal{O}_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$	\mathcal{O}_{eu}	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	\mathcal{O}_{qe}	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$	
$\mathcal{O}_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r) (\bar{q}_s \gamma^\mu \tau^I q_t)$	\mathcal{O}_{ed}	$(\bar{e}_p \gamma_\mu e_r) (\bar{d}_s \gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$	
		$\mathcal{O}_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r) (\bar{d}_s \gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$	
		$\mathcal{O}_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r) (\bar{d}_s \gamma^\mu T^A d_t)$	$\mathcal{O}_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r) (\bar{d}_s \gamma^\mu d_t)$	
				$\mathcal{O}_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r) (\bar{d}_s \gamma^\mu T^A d_t)$	
$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$						
\mathcal{O}_{ledq}	$(ar{l}_p^j e_r)(ar{d}_s q_t^j)$]				
$\mathcal{O}_{quqd}^{(1)}$	$(\bar{q}_p^j u_r) \varepsilon_{jk} (\bar{q}_s^k d_t)$					
$\mathcal{O}_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r) \varepsilon_{jk} (\bar{q}_s^k T^A d_t)$	59 operators				
$\mathcal{O}_{lequ}^{(1)}$	$(\bar{l}_p^j e_r) \varepsilon_{jk} (\bar{q}_s^k u_t)$	(1 family & B conservation)				
$\mathcal{O}_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r) \varepsilon_{jk} (\bar{q}_s^k \sigma^{\mu\nu} u_t)$. /		,	

Figure: Four-fermion operators ³⁹.

³⁹JHEP **10**, 085 (2010).

Effective Lagrangian

$$\begin{split} \mathcal{L}_{cc} = & \frac{-4G_F}{\sqrt{2}} V_{us} \Big[(1 + [v_L]_{\ell\ell}) \bar{\ell}_L \gamma_\mu \nu_{\ell L} \bar{u}_L \gamma^\mu s_L + [v_R]_{\ell\ell} \bar{\ell}_L \gamma_\mu \nu_{\ell L} \bar{u}_R \gamma^\mu s_R \\ &+ [s_L]_{\ell\ell} \bar{\ell}_R \nu_{\ell L} \bar{u}_R s_L + [s_R]_{\ell\ell} \bar{\ell}_R \nu_{\ell L} \bar{u}_L s_R \\ &+ [t_L]_{\ell\ell} \bar{\ell}_R \sigma_{\mu\nu} \nu_{\ell L} \bar{u}_R \sigma^{\mu\nu} s_L \Big] + \text{h.c.} \,, \end{split}$$

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Operators

$$O_{lq}^{(3)} = (\bar{l}\gamma^{\mu}\sigma^{a}l)(\bar{q}\gamma_{\mu}\sigma^{a}q)$$

$$O_{qde} = (\bar{\ell}e)(\bar{d}q) + \text{h.c.}$$

$$O_{lq} = (\bar{l}_{a}e)\epsilon^{ab}(\bar{q}_{b}u) + \text{h.c.}$$

$$O_{lq}^{t} = (\bar{l}_{a}\sigma^{\mu\nu}e)\epsilon^{ab}(\bar{q}_{b}\sigma_{\mu\nu}u) + \text{h.c.}$$

and vertex corrections

$$\begin{array}{lll} O_{\varphi\varphi} &=& i(\varphi^T \epsilon D_\mu \varphi)(\overline{u} \gamma^\mu d) + {\rm h.c.} \ , \\ O^{(3)}_{\varphi q} &=& i(\varphi^\dagger D^\mu \sigma^a \varphi)(\overline{q} \gamma_\mu \sigma^a q) + {\rm h.c.}. \end{array}$$

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Operators

$$O_{ll}^{(3)} = \frac{1}{2} (\bar{l}\gamma^{\mu} \sigma^{a} l) (\bar{l}\gamma_{\mu} \sigma^{a} l)$$
(60a)

$$O_{\varphi l}^{(3)} = i(\varphi^{\dagger} D^{\mu} \sigma^{a} \varphi)(\bar{l} \gamma_{\mu} \sigma^{a} l) + \text{h.c.} .$$
(60b)

In terms of the coefficients of the above operators, the low-energy effective couplings appearing in \mathcal{L}_{CC} (see Eq. 2) are given by

$$V_{ij} \cdot [v_L]_{\ell\ell ij} = 2 V_{ij} \left[\hat{\alpha}_{\varphi l}^{(3)} \right]_{\ell\ell} + 2 V_{im} \left[\hat{\alpha}_{\varphi q}^{(3)} \right]_{jm}^* - 2 V_{im} \left[\hat{\alpha}_{lq}^{(3)} \right]_{\ell\ell mj}$$
(61a)

$$V_{ij} \cdot [v_R]_{\ell\ell ij} = -[\hat{\alpha}_{\varphi\varphi}]_{ij} \tag{61b}$$

$$V_{ij} \cdot [s_L]_{\ell \ell ij} = - [\hat{\alpha}_{lq}]^*_{\ell \ell ji}$$
(61c)

$$V_{ij} \cdot [s_R]_{\ell \ell ij} = -V_{im} \left[\hat{\alpha}_{qde} \right]_{\ell \ell jm}^* \tag{61d}$$

$$V_{ij} \cdot [t_L]_{\ell\ell ij} = - \left[\hat{\alpha}_{lq}^t\right]_{\ell\ell ji}^* .$$
(61e)

ChPT with tensor sources (Backup)

$$\mathcal{L}_4 = \Lambda_1 \langle t_+^{\mu\nu} f_{+\mu\nu} \rangle - i\Lambda_2 \langle t_+^{\mu\nu} u_\mu u_\nu \rangle + \Lambda_3 \langle t_+^{\mu\nu} t_{\mu\nu}^+ \rangle + \Lambda_4 \langle t_+^{\mu\nu} \rangle^2 \,.$$

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Interpretation of subtractions

$$F_V^{\pi}(s) = 1 + \frac{1}{6} \langle r^2 \rangle_V^{\pi} s + c_V^{\pi} s^2 + d_V^{\pi} s^3 + \dots$$
$$\langle r^2 \rangle_V^{\pi} = 6\alpha_1, \quad c_V^{\pi} = \frac{1}{2} (\alpha_2 + \alpha_1^2)$$

Amplitude ($K\eta^{(\prime)}$ case)

The hadronic matrix elements are given by,

$$H = \langle K^{-} \eta^{(\prime)} | \bar{s} u | 0 \rangle \equiv F_{S}^{K^{-} \eta^{(\prime)}}(s), \qquad (1)$$

$$H^{\mu} = \langle K^{-} \eta^{(\prime)} | \bar{s} \gamma^{\mu} u | 0 \rangle = C_{K^{-} \eta^{(\prime)}}^{V} Q^{\mu} F_{+}^{K^{-} \eta^{(\prime)}}(s) + C_{K^{-} \eta^{(\prime)}}^{S} \left(\frac{\Delta_{K\pi}}{s} \right) q^{\mu} F_{0}^{K^{-} \eta^{(\prime)}}(s), \qquad (2)$$

$$H^{\mu\nu} = \langle K^{-} \eta^{(\prime)} | \bar{s} \sigma^{\mu\nu} u | 0 \rangle = i F_{T}^{K^{-} \eta^{(\prime)}}(s) (p_{\eta^{(\prime)}}^{\mu} p_{K}^{\nu} - p_{K}^{\mu} p_{\eta^{(\prime)}}^{\nu}), \qquad (3)$$

where $q^{\mu} = (p_{K} + p_{\eta^{(\prime)}})^{\mu}$, $Q^{\mu} = (p_{\eta^{(\prime)}} - p_{K})^{\mu} + (\Delta_{K\eta^{(\prime)}}/s)q^{\mu}$, $s = q^{2}$ and $\Delta_{ij} = m_{i}^{2} - m_{j}^{2}$, and with the Clebsch-Gordan coefficients: $C_{K\eta^{(\prime)}}^{V} = -\sqrt{\frac{3}{2}}$, $C_{K\eta}^{S} = -\frac{1}{\sqrt{6}}$ and $C_{K\eta^{\prime}}^{S} = \frac{2}{\sqrt{3}}$. The process is analogous for $K\bar{K}$.

Two-meson Form Factors

For the $K\eta^{(\prime)}$ case we have ⁴⁰,

$$F_{+}^{K\eta^{(\prime)}}(s) = \cos\theta_{P}(\sin\theta_{P})F_{+}^{K\pi}(s), \qquad (4)$$

where, $heta_P = (-13.3 \pm 0.5)^\circ$ 41 and 42 ,

$$F_{+}^{K\pi}(s) = F_{+}^{K\pi}(0) \exp\left[\alpha_{1}\frac{s}{m_{\pi}^{2}} + \frac{1}{2}\alpha_{2}\frac{s^{2}}{m_{\pi}^{4}} + \frac{s^{3}}{\pi}\int_{s_{K\pi}}^{s_{\text{cut}}} ds' \frac{\delta_{+}^{K\pi}(s')}{(s')^{3}(s'-s-i0)}\right], \quad (5)$$

⁴⁰R. Escribano, S. Gonzalez-Solis and P. Roig, JHEP 10, 039 (2013).

⁴¹F. Ambrosino et al. [KLOE], Phys. Lett. B 648, 267-273 (2007).

⁴²D. R. Boito, R. Escribano and M. Jamin, Eur. Phys. J. C **59**, 821-829 (2009).

Two-meson form factors

For the $K\bar{K}$ case we have ⁴³

$$F_{+}^{KK}(s) = \exp\left[\tilde{\alpha}_{1}s + \frac{\tilde{\alpha}_{2}}{2}s^{2} + \frac{s^{3}}{\pi}\int_{4m_{\pi}^{2}}^{s_{\text{cut}}} ds' \frac{\delta_{+}^{KK}(s)}{(s')^{3}(s'-s-i0)}\right],$$
(6)

⁴³S. Gonzàlez-Solís and P. Roig, Eur. Phys. J. C **79**, no.5, 436 (2019),

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For the scalar form factors we also take advantage from previous literature, for the $F_0^{\pi\pi}(s)$ we use ⁴⁴, for the $F_0^{KK}(s)$ we use ⁴⁵ ⁴⁶ ⁴⁷ and for $F^{K\pi}(s)$ and $F^{K\eta^{(\prime)}}(s)$ we use ⁴⁸

⁴⁴S. Descotes-Genon and B. Moussallam, Eur. Phys. J. C 74, 2946 (2014).
 ⁴⁵Z. H. Guo and J. A. Oller, Phys. Rev. D 84, 034005 (2011).

⁴⁶Z. H. Guo, J. A. Oller and J. Ruiz de Elvira, Phys. Rev. D **86**, 054006 (2012).

⁴⁷Z. H. Guo, L. Liu, U. G. Meißner, J. A. Oller and A. Rusetsky, Phys. Rev. D **95**, no.5, 054004 (2017).

⁴⁸M. Jamin, J. A. Oller and A. Pich, Nucl. Phys. B **622**, 279-308 (2002).

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Tensor form factors

For the tensor form factors $F_T^{PP'}(s)$ we have^{49 50},

$$F_{T}^{PP'}(s) = F_{T}^{PP'}(0) \exp\left[\frac{s}{\pi} \int_{s_{\rm th}}^{s_{\rm cut}} \frac{ds'}{s'} \frac{\delta_{T}^{PP'}(s')}{(s'-s-i0)}\right], \quad (7)$$

where $s_{\rm th}$ is the two-meson production threshold for the lightest pair of mesons with the same quantum numbers as the given pair PP' and we have the normalizations,

$$F_T^{K^-\eta}(0) = \left(\frac{C_q}{\sqrt{2}} + C_s\right) \frac{\Lambda_2}{F_\pi^2}, \qquad (8)$$

$$F_T^{K^-\eta'}(0) = \left(\frac{C'_q}{\sqrt{2}} - C'_s\right) \frac{\Lambda_2}{F_\pi^2},$$
 (9)

$$F_T^{K^-K^0}(0) = \frac{\Lambda_2}{F_\pi^2}.$$
 (10)

⁴⁹O. Cata and V. Mateu, JHEP **0709**, 078 (2007).

⁵⁰I. Baum, V. Lubicz, G. Martinelli, L. Orifici and S. Simula, Phys. Rev. D 84, 074503 (2011).

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Tensor form factors



Figure: Normalized absolute value of the tensor form factor $F_T^{K\eta^{(\prime)}}(s)$ (left), for $s_{\text{cut}} = 4 \text{ GeV}^2$ (dotted line), 9 GeV² (dashed line) and $s_{\text{cut}} \to \infty$ (solid line), and tensor form factor phase $\delta_T^{K\eta^{(\prime)}}(s)$ (right).

Effective theory analysis of $au^-
ightarrow
u_{ au} \overline{u} D$ (D = d, s)

The effective lagrangian density constructed with dimension six operators and invariant under the $SU(2)_L \otimes U(1)$ group has the following form,

$$\mathcal{L}^{(eff)} = \mathcal{L}_{SM} + \frac{1}{\Lambda^2} \sum_{i} \alpha_i O_i$$
(11)

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Effective theory analysis of $au^- o
u_{ au} ar{u} D$ (D = d, s)

The low-scale O(1 GeV) effective lagrangian for semi-leptonic transitions is given by, ⁵¹⁵²⁵³:

$$\mathcal{L}_{CC} = -\frac{G_F}{\sqrt{2}} V_{uD} (1 + \epsilon_L + \epsilon_R) [\bar{\tau} \gamma_\mu (1 - \gamma^5) \nu_\tau \\ \cdot \bar{u} [\gamma^\mu - (1 - 2\hat{\epsilon}_R) \gamma^\mu \gamma^5] D \\ + \bar{\tau} (1 - \gamma^5) \nu_\tau \, \bar{u} (\hat{\epsilon}_S - \hat{\epsilon}_P \gamma^5) D \\ + 2\hat{\epsilon}_T \bar{\tau} \sigma_{\mu\nu} (1 - \gamma^5) \nu_\tau \, \bar{u} \sigma^{\mu\nu} D] + h.c. , \qquad (12)$$

where $\hat{\epsilon}_i = \epsilon_i / (1 + \epsilon_L + \epsilon_R)$ for i = R, S, P, T.

⁵¹V. Cirigliano, J. Jenkins and M. Gonzalez-Alonso, Nucl. Phys. B **830**, 95-115 (2010).

⁵²S. González-Solís, A. Miranda, J. R. and P. Roig, Phys. Rev. D **101**, no.3, 034010 (2020).

⁵³E. A. Garcés, M. Hernández Villanueva, G. López Castro and P. Roig, JHEP **12**, 027 (2017).

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Amplitude for two-meson decay modes $au^- o (PP')^-
u_ au$

$$\mathcal{M} = \mathcal{M}_{V} + \mathcal{M}_{S} + \mathcal{M}_{T}$$
$$= \frac{G_{F} V_{\mu D} \sqrt{S_{EW}}}{\sqrt{2}} (1 + \epsilon_{L} + \epsilon_{R})$$
$$\times \left[L_{\mu} H^{\mu} + \hat{\epsilon}_{S} L H + 2 \hat{\epsilon}_{T} L_{\mu \nu} H^{\mu \nu} \right], \qquad (13)$$

where the leptonic currents are defined by:

$$L_{\mu} = \bar{u}(P')\gamma_{\mu}(1-\gamma^{5})u(P), \qquad (14)$$

$$L = \bar{u}(P')(1+\gamma^{5})u(P), \qquad (15)$$

$$L_{\mu\nu} = \bar{u}(P')\sigma_{\mu\nu}(1+\gamma^5)u(P).$$
 (16)

Amplitude for two-meson decay modes $au^-
ightarrow (PP')^-
u_{ au}$

and where the hadronic matrix elements are given by

$$H = \langle K^{-}K^{0}|\bar{d}u|0\rangle = F_{S}^{K^{-}K^{0}}(s), \qquad (17)$$

$$H^{\mu} = \langle K^{-}K^{0}|\bar{d}\gamma^{\mu}u|0\rangle = C_{K^{-}K^{0}}^{V}Q^{\mu}F_{+}^{K^{-}K^{0}}(s)$$

$$+ C_{K^{-}K^{0}}^{S}\left(\frac{\Delta_{KK}}{s}\right)q^{\mu}F_{0}^{K^{-}K^{0}}(s), \qquad (18)$$

$$H^{\mu\nu} = \langle K^{-}K^{0} | \bar{d}\sigma^{\mu\nu} u | 0 \rangle = i F_{T}^{K^{-}K^{0}}(s) (p_{K^{0}}^{\mu} p_{K}^{\nu} - p_{K}^{\mu} p_{K^{0}}^{\nu}),$$
(19)

Two-meson Form Factors

$$F_{+}^{\pi\pi}(s) = \exp\left[\alpha_{1}s + \frac{\alpha_{2}}{2}s^{2} + \frac{s^{3}}{\pi}\int_{4m_{\pi}^{2}}^{\infty} ds' \frac{\delta_{1}^{1}(s')}{(s')^{3}(s'-s-i0)}\right],$$
(20)

⁵⁴S. Gonzàlez-Solís and P. Roig, Eur. Phys. J. C **79** (2019) no.5, 436.
 ⁵⁵A. Pich and J. Portolés, Phys. Rev. D **63** (2001), 093005.

⁵⁶D. Gómez Dumm and P. Roig, Eur. Phys. J. C **73** (2013) no.8, 2528.

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Two-meson Form Factors

$$F_{+}^{K\pi}(s) = \exp\left[\alpha_{1}s + \frac{\alpha_{2}}{2}s^{2} + \int_{s_{\pi K}}^{\infty} ds' \frac{\delta_{1}^{1/2}(s)}{(s')^{3}(s' - s - i\epsilon)}\right], \quad (21)$$

57 58 59

- ⁵⁷D. R. Boito, R. Escribano and M. Jamin, Eur. Phys. J. C **59** (2009) 821.
- $^{58}\text{D.}$ R. Boito, R. Escribano and M. Jamin, JHEP 1009 (2010) 031.
- ⁵⁹R. Escribano, S. González-Solís, M. Jamin and P. Roig, JHEP **1409** (2014) 042.

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For the scalar form factors we also take advantage from previous literature, for the $F_0^{\pi\pi}(s)$ we use ⁶⁰, for the $F_0^{KK}(s)$ we use ⁶¹ 62 63 and for $F_0^{K\pi}(s)$ and $F_0^{K\eta^{(\prime)}}(s)$ we use ⁶⁴

- ⁶⁰S. Descotes-Genon and B. Moussallam, Eur. Phys. J. C 74, 2946 (2014).
 ⁶¹Z. H. Guo and J. A. Oller, Phys. Rev. D 84, 034005 (2011).
- ⁶²Z. H. Guo, J. A. Oller and J. Ruiz de Elvira, Phys. Rev. D **86**, 054006 (2012).

⁶³Z. H. Guo, L. Liu, U. G. Meißner, J. A. Oller and A. Rusetsky, Phys. Rev. D **95**, no.5, 054004 (2017).

⁶⁴M. Jamin, J. A. Oller and A. Pich, Nucl. Phys. B **622**, 279-308 (2002).

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Tensor form factors

For the tensor form factors $F_T^{PP'}(s)$ we have⁶⁵ ⁶⁶,

$$F_{T}^{PP'}(s) = F_{T}^{PP'}(0) \exp\left[\frac{s}{\pi} \int_{s_{\rm th}}^{s_{\rm cut}} \frac{ds'}{s'} \frac{\delta_{T}^{PP'}(s')}{(s'-s-i0)}\right], \quad (22)$$

where $s_{\rm th}$ is the two-meson production threshold for the lightest pair of mesons with the same quantum numbers as the given pair PP'.

 $F_T^{PP'}(0)$: ChPT with Tensor Sources+Lattice

⁶⁵O. Cata and V. Mateu, JHEP **0709**, 078 (2007).

⁶⁶I. Baum, V. Lubicz, G. Martinelli, L. Orifici and S. Simula, Phys. Rev. D 84, 074503 (2011).

Tensor form factors



Figure: Normalized absolute value of the tensor form factor $F_T^{KK}(s)$ (left), for $s_{\text{cut}} = 4 \text{ GeV}^2$ (dotted line), 9 GeV² (dashed line) and $s_{\text{cut}} \to \infty$ GeV² (solid line), and tensor form factor phase $\delta_T^{KK}(s)$ (right).

Limits on $\hat{\epsilon}_S$ and $\hat{\epsilon}_T$

The upper part of the table are the new results and the lower part are our previous results

Decay channel	$\hat{\epsilon}_{S}(\hat{\epsilon}_{T}=0)$	$\hat{\epsilon}_T (\hat{\epsilon}_S = 0)$	$\hat{\epsilon}_{S}$	$\hat{\epsilon}_T$
$\tau^- \rightarrow K^- \eta \nu_{\tau}$	[-0.38, 0.16]	$[-1.4, -0.7] \cup [-4.7, 8.5] \cdot 10^{-2}$	[-0.7, 0.5]	[-1.5, 0.1]
$\tau^- \rightarrow K^- \eta' \nu_\tau$	[-0.20, 0.05]	[-7.6, 14.9]	[-0.21, 0.05]	[-10.4, 17.7]
$\tau^- ightarrow K^- K^0 u_{ au}$	$[-0.12, -0.08] \cup [0.08, 0.12]$	$[-0.12, -0.06] \cup [0.92, 0.99]$	[-0.2, 0.2]	[-0.12, 0.98]
$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$	[-1.33, 1.31]	$[-0.79, -0.57] \cup [-1.4, 1.3] \cdot 10^{-2}$	[-5.2, 5.2]	[-0.79, 0.013]
$\tau^- \rightarrow (K\pi)^- \nu_\tau$	[-0.57, 0.27]	$[-0.059, 0.052] \cup [0.60, 0.72]$	[-0.89, 0.58]	[-0.07, 0.72]
$\tau^- \rightarrow \pi^- \eta \nu_\tau$	$[-8.3, 3.9] \cdot 10^{-3}$	[-0.43, 0.39]	$[-0.83, 0.37] \cdot 10^{-2}$	[-0.55, 0.50]
$\tau^- \rightarrow \pi^- \eta' \nu_\tau$	$[-1.13, 0.68] \cdot 10^{-2}$	$ \hat{\epsilon}_{\mathcal{T}} < 11.4$	$[-1.13, 0.67] \cdot 10^{-2}$	[-11.9, 11.9]

Table: Constraints on the scalar and tensor couplings obtained (at three standard deviations) through the limits on the current branching ratio measurements. Theory errors are included.

Important Data (Backup)



Figure: Belle measurement of the modulus squared of the pion vector form factor as compared to our fits .

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⁶⁷M. Fujikawa *et al.* [Belle], Phys. Rev. D **78**, 072006 (2008)

⁶⁸S. González-Solís and P. Roig, Eur. Phys. J₆C **79** no.5 436 (2019) $_{\odot}$

Important Data



Figure: Belle $\tau^- \rightarrow K_S \pi^- \nu_\tau$ (red circles) and $\tau^- \rightarrow K^- \eta \nu_\tau$ (green squares) measurements as compared to our best fit results in (solid black and blue lines) obtained from a combined fit to both data sets. The small scalar contributions are represented by black and blue dashed lines.

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⁶⁹D. Epifanov et al. [Belle], Phys. Lett. B **654**, 65-73 (2007)

⁷⁰K. Inami *et al.* [Belle], Phys. Lett. B **672**, 209-218 (2009)

Comparison between different probes for the ϵ 's



It is important to remerber the result G. Aad et al. [ATLAS], [arXiv:2007.14040 [hep-ex].

Límites para $\hat{\epsilon}_S$ y $\hat{\epsilon}_T$



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Leptoquarks

	F	Spin	${\rm SU(3)}_{\rm C}$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	Dimension-6 operators
S_1	-2	0	$\overline{3}$	1	2/3	$\mathcal{O}_{la}^{(1)}, \mathcal{O}_{Nd}, \mathcal{O}_{lNqd}, \mathcal{O}_{lNqd}',$
						$\mathcal{O}_{eluq}, \mathcal{O}_{eNud}$
S'_1	-2	0	$\overline{3}$	1	8/3	1
S_1''	$^{-2}$	0	$\overline{3}$	1	-4/3	\mathcal{O}_{Nu}
S_3	$^{-2}$	0	$\overline{3}$	3	2/3	$\mathcal{O}_{lq}^{(3)}$
V_2	-2	1	$\overline{3}$	2	5/3	$\mathcal{O}_{ld}, \mathcal{O}_{elqd}$
V'_2	$^{-2}$	1	$\overline{3}$	2	-1/3	$\mathcal{O}_{Nq}, \mathcal{O}_{lu}, \mathcal{O}_{lNuq}$
R_2	0	0	3	2	7/3	$\mathcal{O}_{lu}, \mathcal{O}_{eluq}$
R'_2	0	0	3	2	1/3	$\mathcal{O}_{ld}, \mathcal{O}_{Nq}, \mathcal{O}_{lNqd}, \mathcal{O}'_{lNqd}$
U_1	0	1	3	1	4/3	$\mathcal{O}_{lq}^{(1)}, \mathcal{O}_{Nu}, \mathcal{O}_{elqd}, \mathcal{O}_{lNuq}, \mathcal{O}_{eNud}$
U'_1	0	1	3	1	10/3	
U_1''	0	1	3	1	-2/3	\mathcal{O}_{Nd}
U_3	0	1	3	3	4/3	$\mathcal{O}_{lq}^{(3)}$

Table 10: Coupling constants and operators appearing in generic neutral-current (2) and charged-current Lagrangians (3).

$a_{\mu}^{HVP,LO}$ anomaly?



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Inelasticidades

Estimación de inelasticidades en la fase del TFF⁷³



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 $^{^{73}}V.$ Cirigliano, A. Crivellin and M. Hoferichter, Phys. Rev. Lett. 120, no. 14, 141803 (2018)

Dalitz plots



Figure: Dalitz plot distribution of $\widetilde{\Delta}(\hat{\epsilon}_{S}, \hat{\epsilon}_{T})$ for $\tau^{-} \rightarrow K^{-}K^{0}\nu_{\tau}$ with $(\hat{\epsilon}_{S} = 0.10, \hat{\epsilon}_{T} = 0)$ (left panels) and $(\hat{\epsilon}_{S} = 0, \hat{\epsilon}_{T} = 0.9)$ (right panels). The lower row show the differential decay distribution in the $(s, \cos \theta)$ variables.

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Forward-Backward asymmetries



Figure: Forward-backward asymmetry for the decay $\tau^- \rightarrow K^- K^0 \nu_{\tau}$ in the SM (solid line), and for $\hat{\epsilon}_S = 0.1$, $\hat{\epsilon}_T = 0$ (dashed line), and $\hat{\epsilon}_T = 0.9$, $\hat{\epsilon}_S = 0$ (dotted line).

Spectrum



Figure: Invariant mass distribution for the decay $\tau^- \rightarrow K^- K^0 \nu_{\tau}$ in the SM (solid line), and for $\hat{\epsilon}_S = 0.1$, $\hat{\epsilon}_T = 0$ (dashed line) and $\hat{\epsilon}_S = 0$, $\hat{\epsilon}_T = 0.9$ (dotted line). The decay distribution is normalized to the tau decay width.