Making sense of Lorentz symmetry violation and quantum anomalies in topological phases

Research in progress

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Outline:

1. Quantum Hall effect and topological insulators
2. Anomalous transport induced by axial anomaly in WSMs
3. Fermion sector of the SME in Weyls
4. Anomalous transport induced by parity anomaly in NLSM
5. Gravity + LV + TSCs

1945

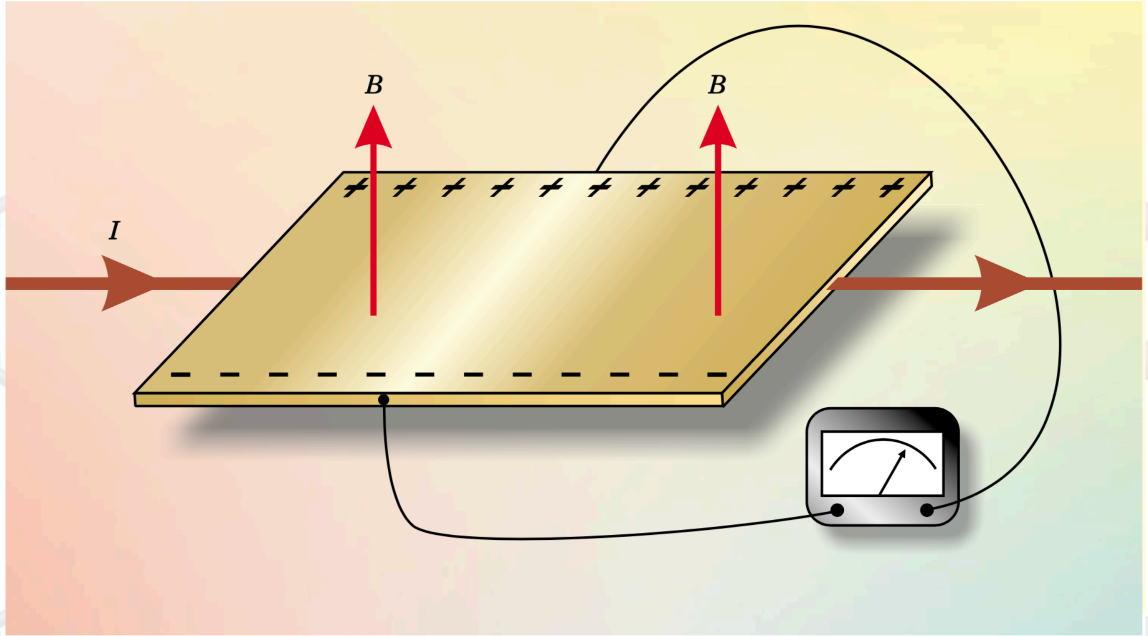
God made the bulk; surfaces were invented by the devil

Topological Matter School

Responses to Topological Matter. August 21-25 2023.

Optical and electronic responses of topological matter are fundamental to understand topological properties in real materials. The Berry curvature is behind numerous effects such as the anomalous Hall effect, the spin Hall effect or even heat currents as observed in the anomalous Nerst effect and the thermal Hall effect. Even more interestingly, the Berry curvature has been recently shown to determine novel and sizable non-linear optical effects, non-linear Hall responses without magnetic fields and universal responses of topological metals. Lastly, magnetotransport in topological metals is an exciting frontier to uncover exotic anomalous responses rooted in concepts from high-energy physics, such as the chiral anomaly. In this edition we will tackle all these phenomena, offering a pedagogical and broad picture of the main responses of topological matter.

Quantum Hall effect and topological insulators

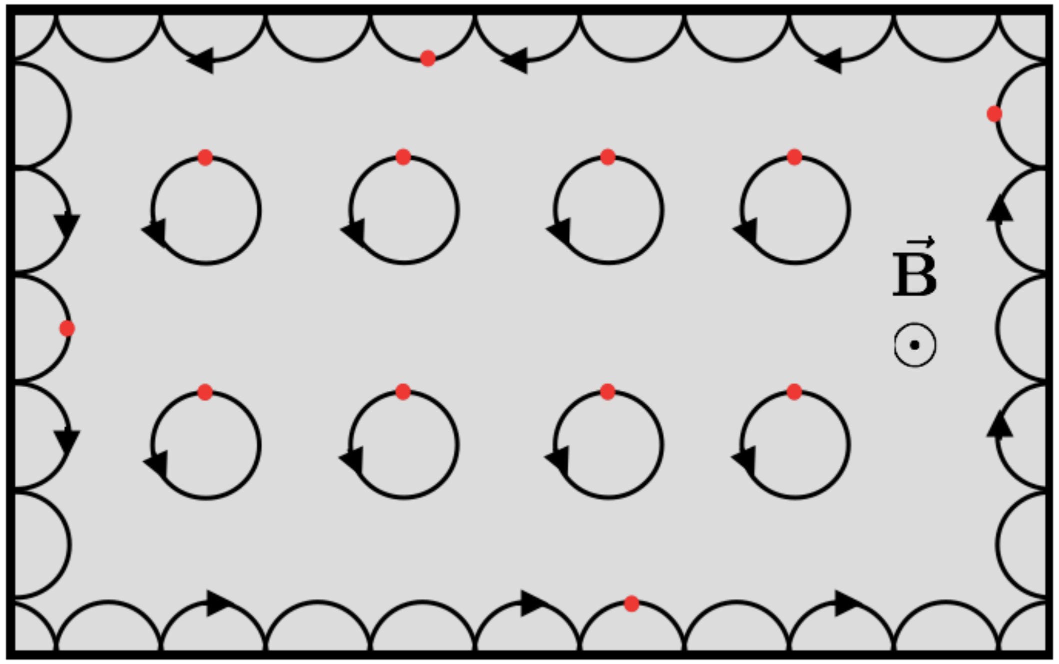


$$m\frac{d\mathbf{v}}{dt} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B} - \frac{m\mathbf{v}}{\tau}$$

Scattering time

$$J_i = \sigma_{ij} E_j$$

Conductivity



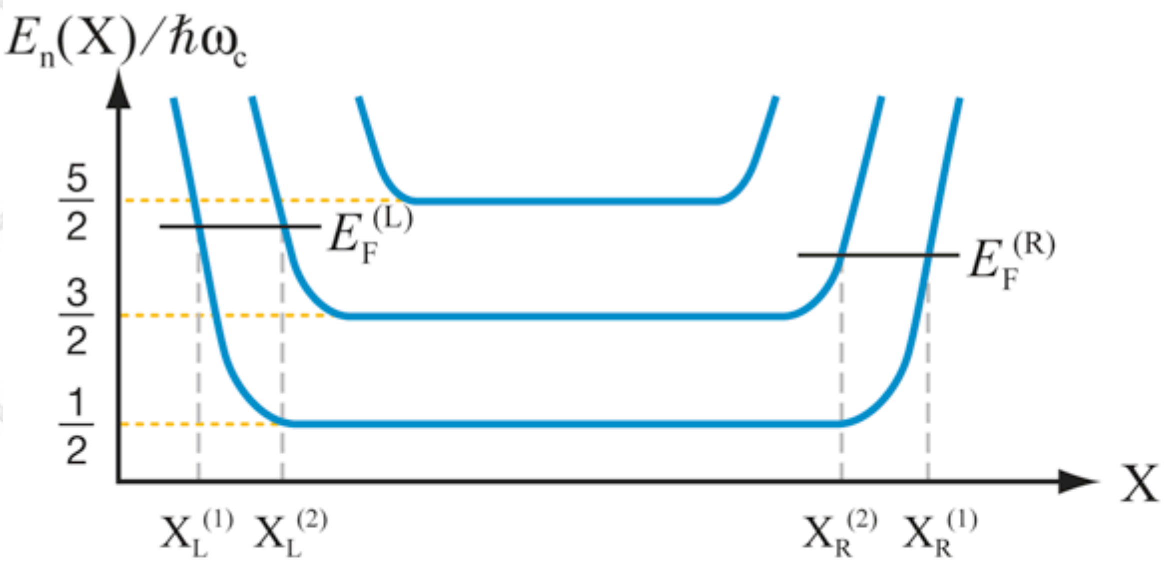
$$\rho_{xx} = \frac{m}{ne^2\tau} \qquad \rho_{xy} = \frac{B}{ne}$$

τ -independent

¡Protection against disorder!

E. H. Hall, Am. J. Math. **2**, 287 (1879)

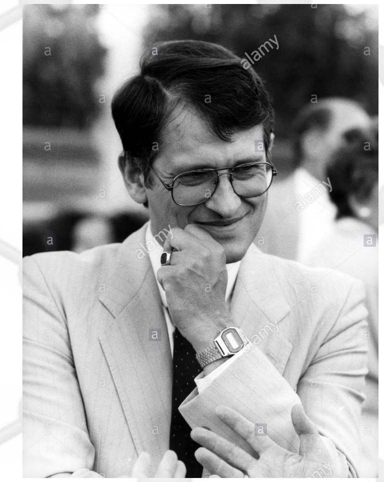
Quantum Hall effect and topological insulators



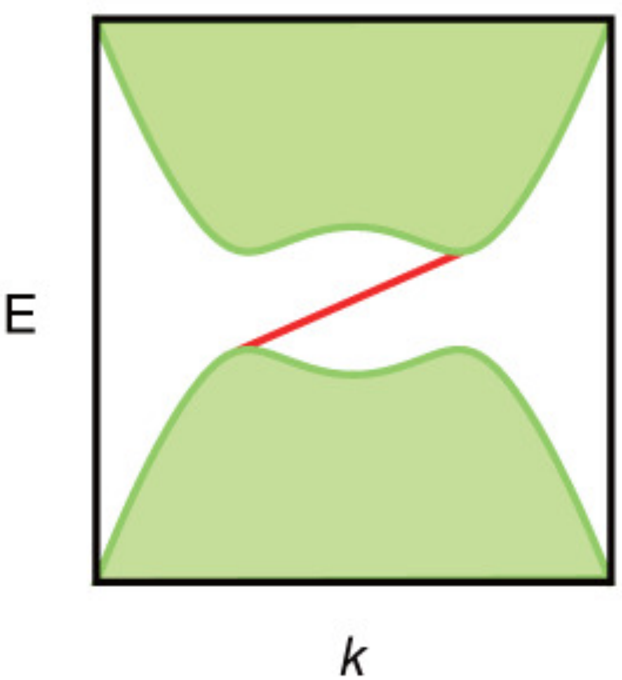
Landau levels

$$J_y = \frac{I_y}{A} = \nu \frac{e^2}{h} E_x$$

ν = # filled LLs

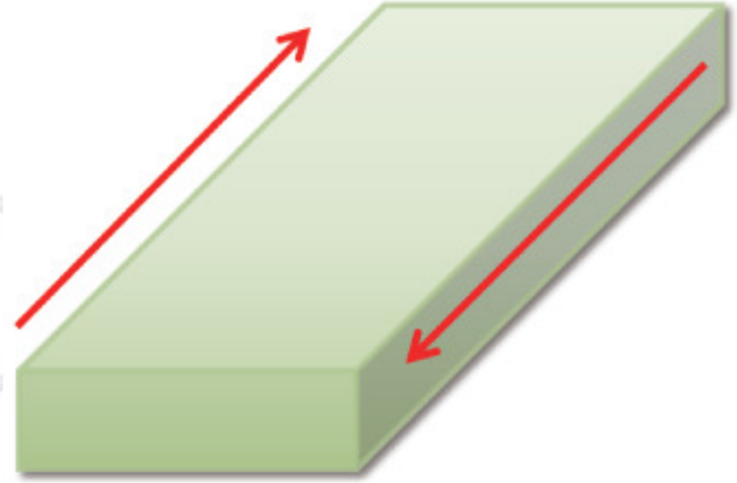


1985

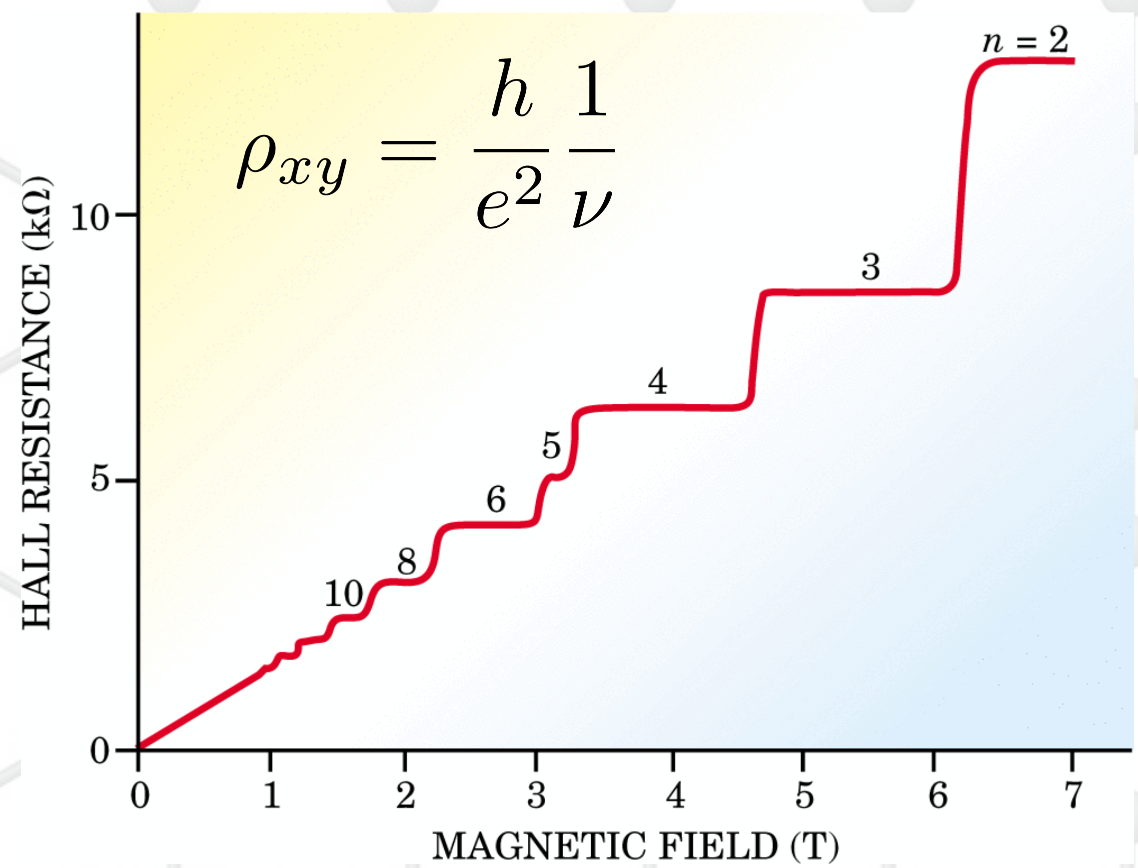


Bulk dispersion

Edge dispersion



Edge states



K. von Klitzing, Phys. Rev. Lett. **45**, 494 (1980)

Quantum Hall effect and topological insulators

3 routes towards the IQHE

2016

(1) TKNN invariant

$$\sigma_{xy} = \frac{e^2}{h} \sum_{\alpha} C_\alpha$$

Filled bands

$$C_\alpha = \frac{1}{2\pi} \int_{\text{B.Z.}} \mathbf{\Omega}_\alpha(\mathbf{k}) \cdot d\mathbf{S}_\mathbf{k}$$

Topological invariant

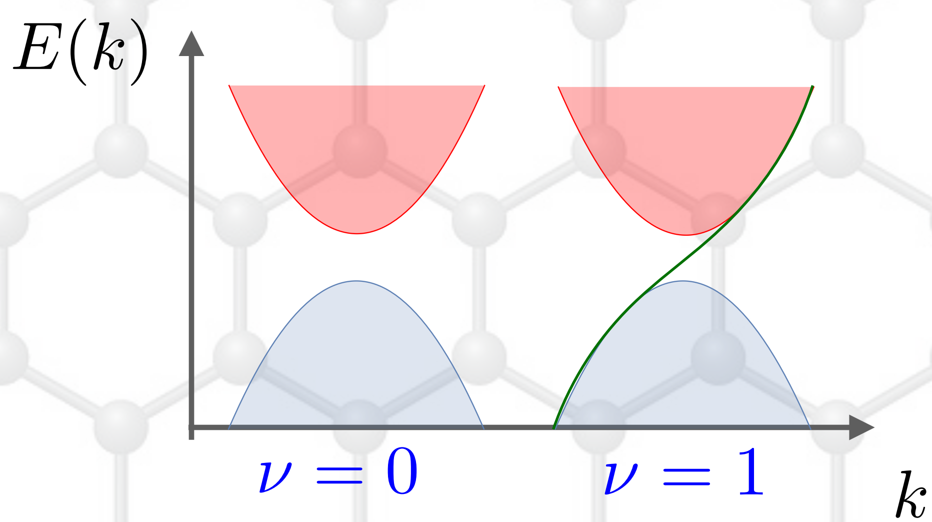
$$\nu = \sum_\alpha C_\alpha$$

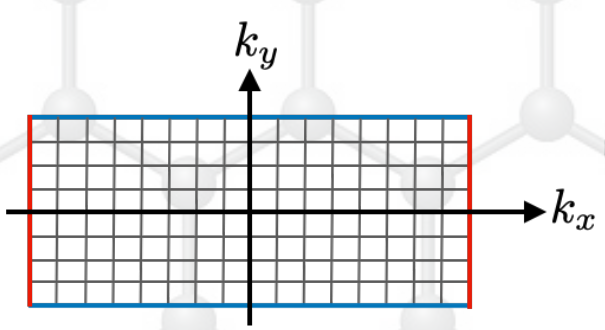
TKNN-invariant

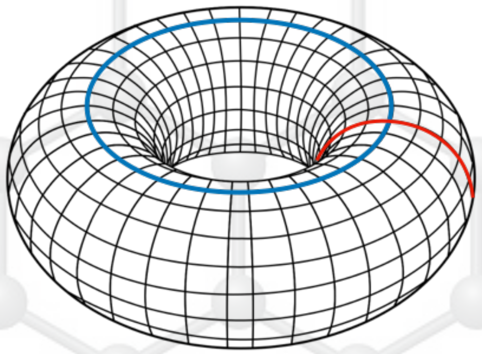
filled LLs

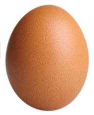
$$\mathbf{\Omega}_\alpha(\mathbf{k}) = \nabla_\mathbf{k} \times \langle u_\alpha(\mathbf{k}) | i \nabla_\mathbf{k} | u_\alpha(\mathbf{k}) \rangle$$

Berry curvature





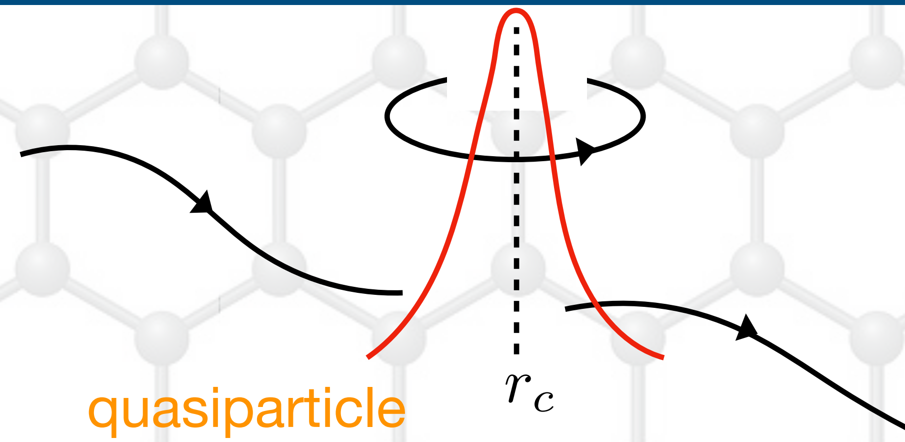


 $\neq$ 

TKNN, Phys. Rev. Lett. 49, 405 (1982)

Quantum Hall effect and topological insulators

(2) Kinetic theory



$$\mathbf{J} = -e \sum_{\alpha} \int_{\mathrm{B.Z.}} \dot{\mathbf{r}}_{\alpha}(\mathbf{k}) f_{\alpha}(\mathbf{k}) \frac{d^3\mathbf{k}}{(2\pi)^3}$$

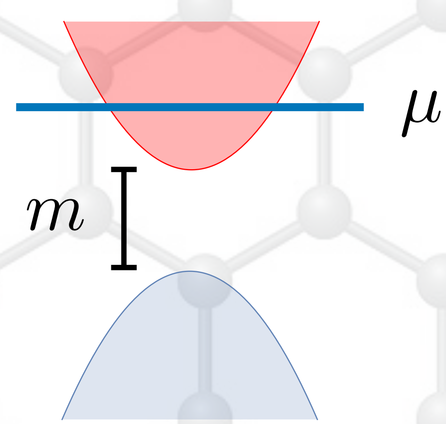
quasiparticle electric current

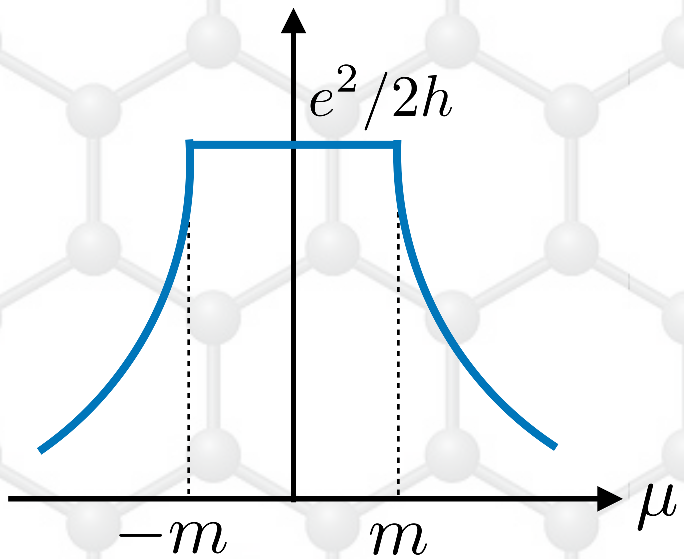
$$\dot{\mathbf{r}}_{\alpha}(\mathbf{k}) = \frac{1}{\hbar}\nabla_{\mathbf{k}} E_{\mathbf{k}} + \frac{e}{\hbar}\mathbf{E} \times \mathbf{\Omega}_{\alpha}(\mathbf{k})$$

Anomalous velocity

$$f_{\alpha}(\mathbf{k}) = \left[1 + e^{\frac{E_{\alpha}(\mathbf{k}) - \mu}{k_B T}}\right]^{-1}$$

Fermi-Dirac distribution





Sundaram, Niu, PR **53**, 7010 (1996)

D Xiao et al, Rev. Mod. Phys. **53**, 1959 (2010)

Quantum Hall effect and topological insulators

(3) Quantum anomalies

Fujikawa method

$$S[A_\mu] = \int \bar{\psi}(i\gamma^\mu \partial_\mu - eA_\mu \gamma^\mu - m)\psi \, d^4x$$

Fermions in a background U(1) gauge field

Integrating out fermions we get the effective action $S_\theta[A_\mu]$

$$e^{iS_\theta[A_\mu]} = \int \mathcal{D}\bar{\psi}\,\mathcal{D}\psi \; e^{iS[\bar{\psi},\psi,A_\mu]} \longrightarrow S_\theta[A_\mu] = \frac{\theta e^2}{32\pi^2} \int \underbrace{\epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}}_{\text{total derivative}} \, d^4x$$

Classical $\theta = 0$

QM: Contribution to partition function $e^{iS_\theta} = e^{i\theta C_2}$ ← second Chern-number quantized!!! (open manifold)

$$\mathcal{T}(\vec{E}\cdot\vec{B}) = -\vec{E}\cdot\vec{B} \longrightarrow e^{i\theta C_2} = e^{-i\theta C_2} \longrightarrow \theta = n\pi$$

Imposing TR

$$S_\theta = \frac{\alpha}{4\pi^2}\int \theta \vec{E}\cdot\vec{B} d^3x$$ $\mathbb{Z}_2$ classification

K. Fujikawa, Phys. Rev. D **29**, 285 (1984)

XL Qi et al, Phys. Rev. B **78**, 195424 (2008)

Quantum Hall effect and topological insulators

Consequences of the QHE

Image magnetic monopoles

XL Qi and SC Zhang, Science **323**, 1184 (2009)

A. Karch, Phys. Rev. Lett. **103**, 171601 (2009)

AMR, MCH and LFU, Phys. Rev. D **92**, 125015 (2015);

Phys. Rev. D **93**, 045022 (2016); Phys. Rev. A **97**, 022502 (2018)

Faraday and Kerr rotations

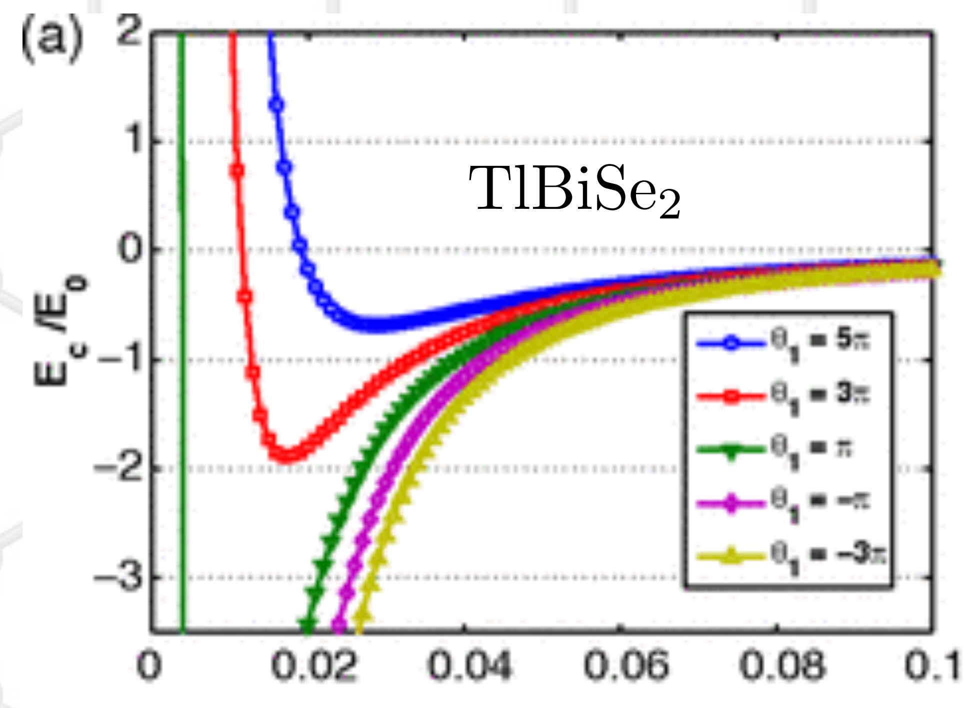
XL Qi et al, Phys. Rev. B 78, 195424 (2008)

J Maciejko et al, Phys. Rev. Lett. 105, 166803 (2010)

Casimir effect

Grushin & Cortijo, Phys. Rev. Lett. 106, 020403 (2011)

AMR, MCH and LFU, Europhys. Lett. 113, 60005 (2016)



Detected!

L. Ohnoutek et al., Sci. Rep. 6, 19087 (2016)

L. Wu, M. Salehi, et al, Science 354, 1124 (2

scientific reports

Article | Open Access | Published: 02 December 2022

Topological signatures in the entanglement of a topological insulator-quantum dot hybrid

L. A. Castro-Enríquez, A. Martín-Ruiz & Mauro Cambiaso

Scientific Reports **12**, Article number: 20856 (2022) | Cite this article

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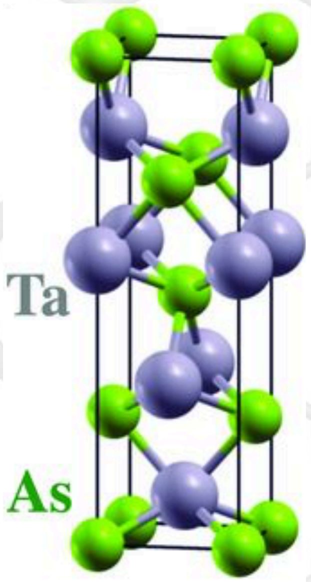
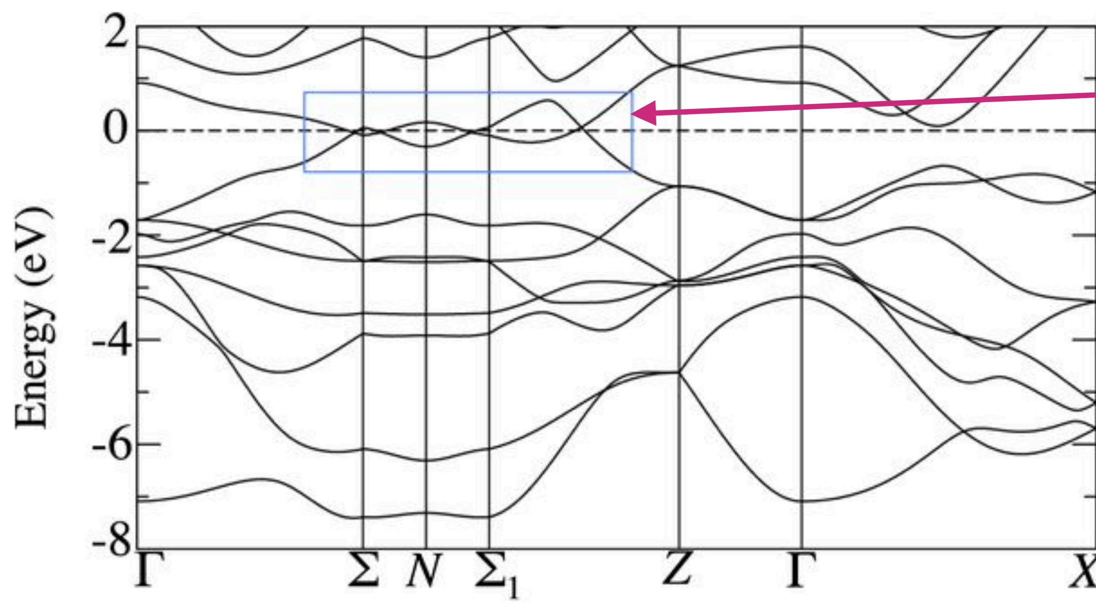
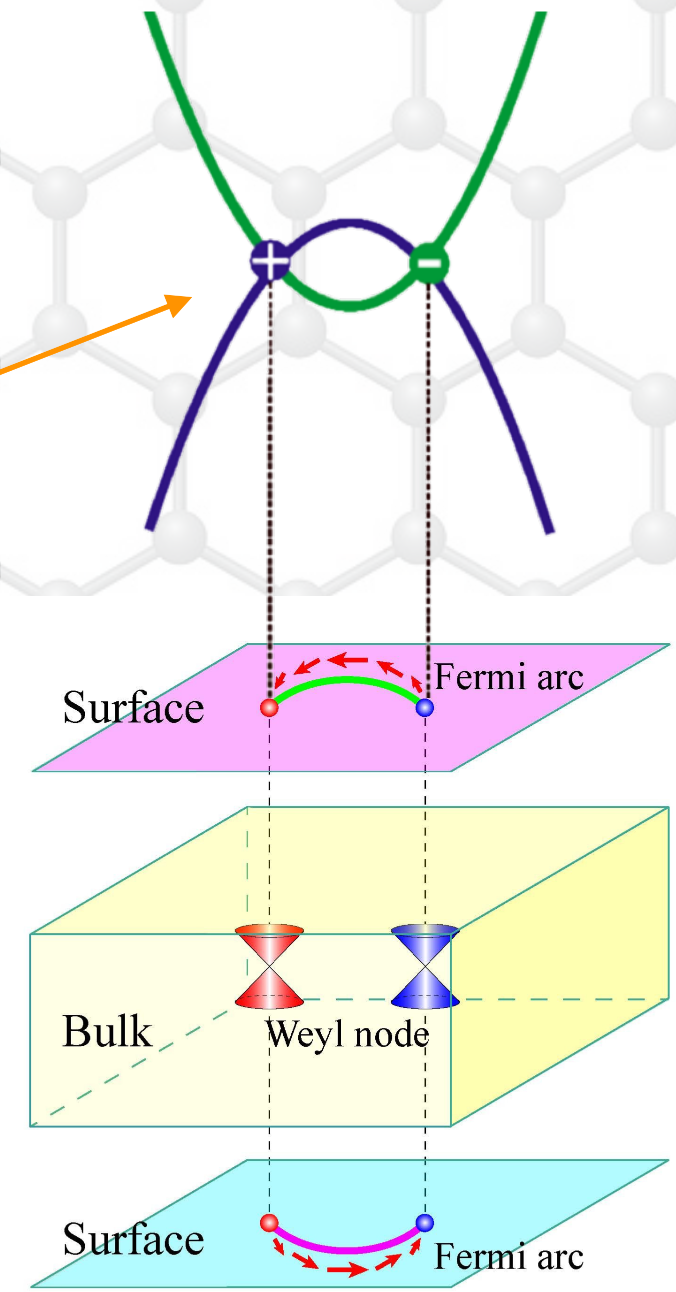
Anomalous transport induced by axial anomaly in WSMs

TaAs

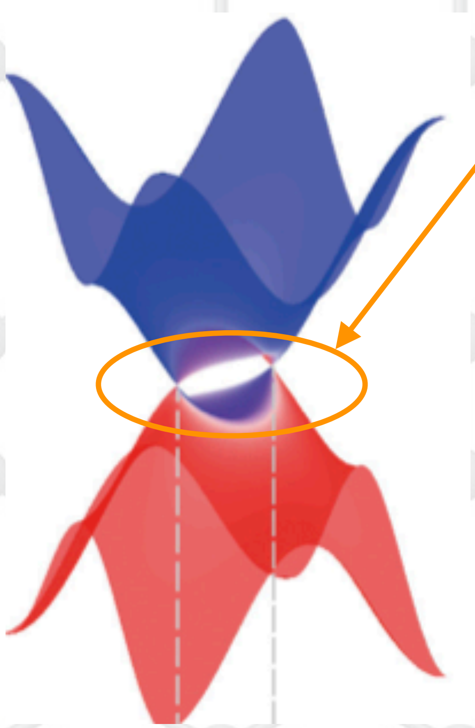




Band crossing points



two Dirac cones with different chiralities



$\mathrm{EuCd_2As_2}$

$$H = \chi \hbar v_F \, \boldsymbol{\sigma} \cdot (\mathbf{k} - \chi \mathbf{b})$$

broken TR

Nielsen-Ninomiya

$$\chi = \pm 1$$

Chirality

$$H\psi_\chi = \chi\psi_\chi$$

Weyl fermions

Anomalous transport induced by axial anomaly in WSMs

$$b_\mu = (b_0, \mathbf{b})$$

Weyl fermions coupled to U(1) gauge fields

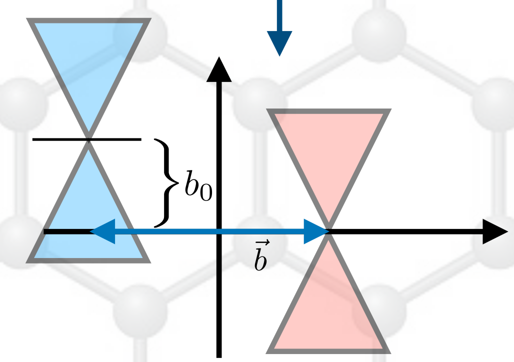
HEP

Lorentz-violating constant 4-vector

$$S = \int \psi^\dagger \left[\partial_\tau + ieA_0 + b_0\tau^z + \tau^z \vec{\sigma}\cdot(-i\vec{\nabla} + e\vec{A} + \vec{b}\tau^z)\right]\psi \, d^3x \, d\tau$$

chirality

CMP



$$S[A_\mu] = \int \bar{\psi}\, i\gamma^\mu(\partial_\mu + ieA_\mu + ib_\mu\gamma^5)\psi \, d^4x$$

eliminated by gauge transf

$$\psi \to e^{-i\theta(x)\gamma^5/2}\psi$$

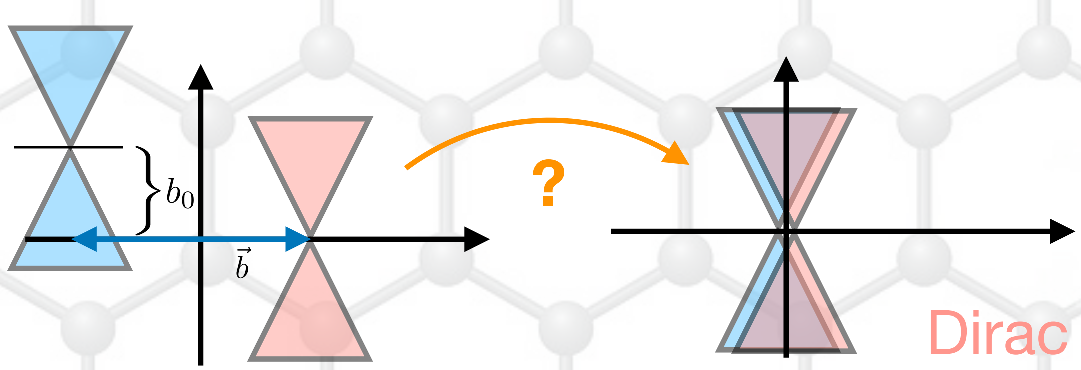
$$\bar{\psi} \to \bar{\psi}\, e^{-i\theta(x)\gamma^5/2}$$

$$\theta(x) = 2b_\mu x^\mu$$

Apparent conservation of # fermions L&R chirality

$$S[A_\mu] = \int \bar{\psi} i\gamma^\mu(\partial_\mu + ieA_\mu)\psi \, d^4x$$

2 Weyl nodes opposite chiralities



Wrong!

Axial/Chiral anomaly!!!

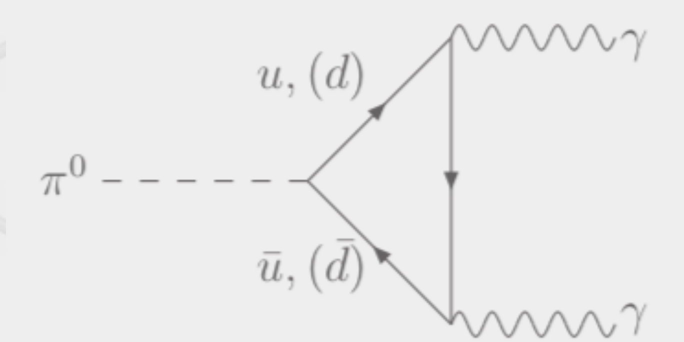
Anomalous transport induced by axial anomaly in WSMs

Recall HEP

$$J_5^\mu = \bar{\psi}\gamma_5\gamma^\mu\psi$$ axial current

$$\partial_\mu J_5^\mu = \frac{e^2}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

$$\pi^0 \rightarrow \gamma + \gamma$$



ABJ anomaly

Fujikawa method

$$e^{iS_\theta[A_\mu]} = \int \mathcal{D}\bar{\psi}\,\mathcal{D}\psi\; e^{iS[\bar{\psi},\psi,A_\mu]}$$

$$S_\theta = \frac{\alpha}{4\pi^2}\int \theta(\vec{r},t)\vec{E}\cdot\vec{B}\,d^3x$$

space-time dependent axion field

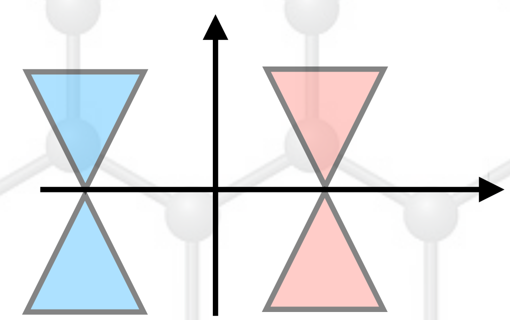
$$\theta(\vec{r},t) = 2(\vec{b}\cdot\vec{r} - b_0 t)$$

Physical consequences?

Symmetry considerations

TR-breaking

$$\mathbf{b} \neq 0$$

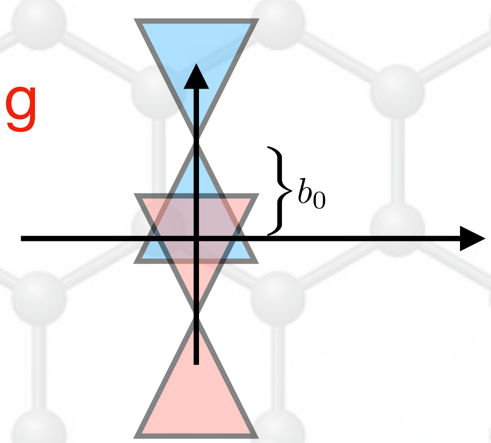


$$\mathbf{J} = \frac{\alpha}{2\pi^2}\mathbf{b}\times\mathbf{E}$$

anomalous Hall effect

parity-breaking

$$b_0 \neq 0$$



$$\mathbf{J} = -\frac{\alpha}{2\pi^2}b_0\mathbf{B}$$

S. L. Adler, Phys. Rev. **177**, 2426 (1969)

J. S. Bell and R. Jackiw, Nuovo Cim. **A60**, 47 (1969)

K. Fujikawa, Phys. Rev. D **29**, 285 (1984)

Anomalous transport induced by axial anomaly in WSMs

Chiral kinetic theory

quasiparticle electric current

$$\mathbf{J} = -e \sum_{s} \sum_{\chi=\pm 1} \int D_\alpha(\mathbf{k})\, \dot{\mathbf{r}}_\alpha(\mathbf{k})\, f_\alpha(\mathbf{k}) \frac{d^3\mathbf{k}}{(2\pi)^3}$$

Nonequilibrium distribution

$$D_\alpha(\mathbf{k}) = 1 + \frac{e}{\hbar}\mathbf{B}\cdot\mathbf{\Omega}_\alpha(\mathbf{k})$$

modified density of states

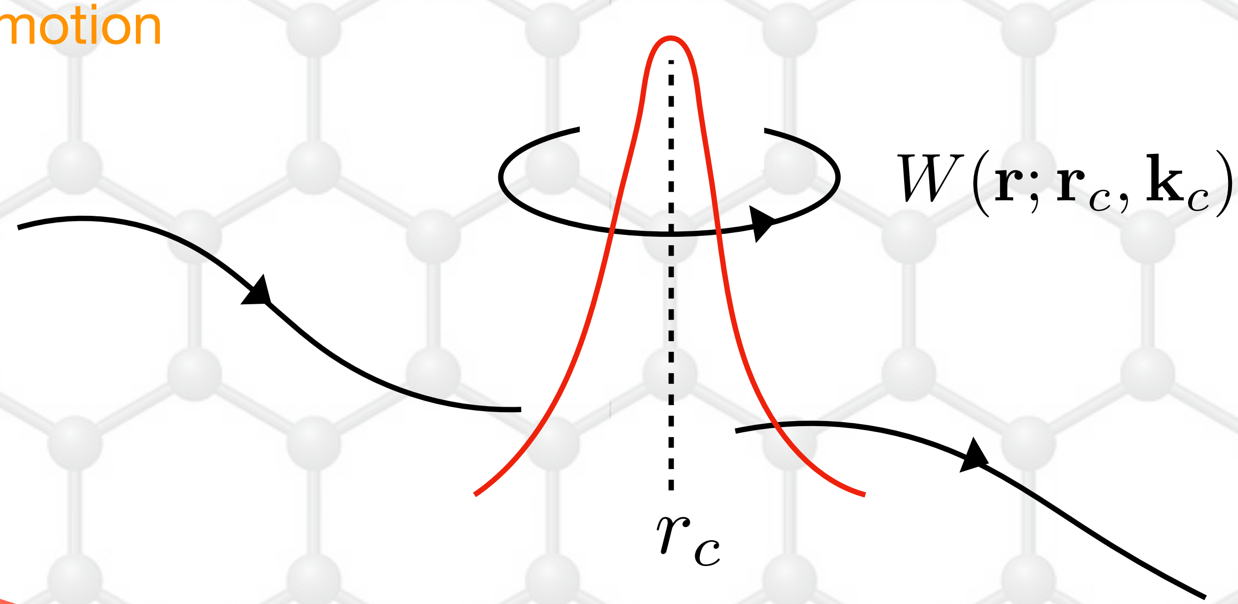
eqs. of motion

$$\dot{\mathbf{r}}_\alpha(\mathbf{k}) = \mathbf{v}_\alpha(\mathbf{k}) - \dot{\mathbf{k}}_\alpha(\mathbf{k}) \times \mathbf{\Omega}_\alpha(\mathbf{k})$$

$$\dot{\mathbf{k}}_\alpha(\mathbf{k}) = -e\mathbf{E} - e\dot{\mathbf{r}}_\alpha(\mathbf{k}) \times \mathbf{B}$$

$$\mathbf{v}_\alpha(\mathbf{k}) = \frac{1}{\hbar}\nabla_{\mathbf{k}}[E_\alpha(\mathbf{k}) - \mathbf{m}_\alpha(\mathbf{k})\cdot\mathbf{B}]$$

Zeeman splitting



$$\mathbf{m}_\alpha(\mathbf{k}) = -i\frac{e}{2\hbar}\langle\nabla_{\mathbf{k}} u_\alpha(\mathbf{k})| \times [H(\mathbf{k}) - E_\alpha(\mathbf{k})] |\nabla_{\mathbf{k}} u_\alpha(\mathbf{k})\rangle$$

Bloch

magnetic moment

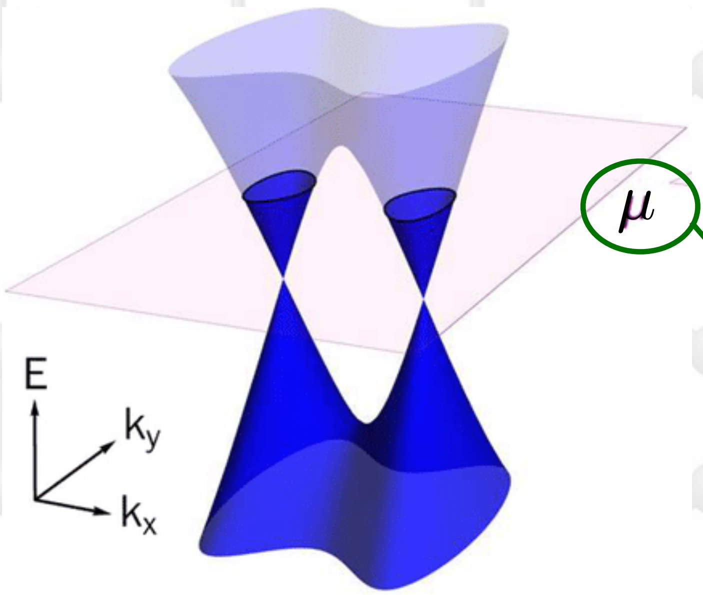
Sundaram, Niu, PR **53**, 7010 (1996)

D Xiao et al, Rev. Mod. Phys. **53**, 1959 (2010)

Anomalous transport induced by axial anomaly in WSMs

Anomalous Hall effect, Ohm law...

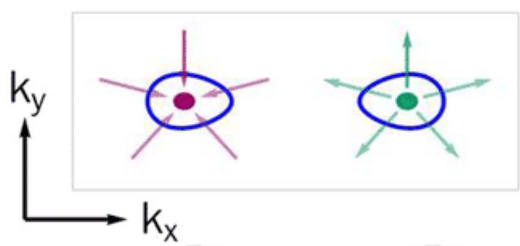
E-field $\mathbf{E}$ B-field $\mathbf{B} = 0$



topological part

$$\mathbf{J} = \frac{e^2}{\hbar}\mathbf{E} \times \int \mathbf{\Omega}_s(\mathbf{k})\, f_s(\mathbf{k}, t)\, \frac{d^3\mathbf{k}}{(2\pi)^3} = \frac{\alpha}{2\pi^2} c\mathbf{b} \times \mathbf{E}$$

anomalous Hall effect



non-topological part

$$\mathbf{J} = \sigma_D\, \mathbf{E} \qquad \sigma_D = \frac{\tau e^2 \mu^2}{3\pi^2 \hbar^3 v_F}$$

vanishing at neutrality

$EuCd_2As_2$

$$\mathbf{\Omega}_\alpha(\mathbf{k}) = -s\chi\frac{\mathbf{k}}{k^3}$$

monopoles

Anomalous transport induced by axial anomaly in WSMs

$$\partial_\mu J_5^\mu = \frac{e^2}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

E-field $\mathbf{E}$ B-field $\mathbf{B}$ $\mathbf{E}\cdot\mathbf{B}\neq 0$

Chiral anomaly

$$\mathbf{J} = \frac{e^2\tau}{\hbar}\sum_\alpha\sum_{\chi=\pm1}\int_{\text{B.Z.}} D_\alpha(\mathbf{k})\left[(\mathbf{v}_\alpha(\mathbf{k})\cdot\boldsymbol{\Omega}_\alpha(\mathbf{k}))\mathbf{B} - e(\mathbf{E}\cdot\mathbf{B})\boldsymbol{\Omega}_\alpha(\mathbf{k})\right]\frac{d^3\mathbf{k}}{(2\pi)^3}$$

$$\dot\rho_\chi = \frac{e^3}{8\pi^3}(\mathbf{E}\cdot\mathbf{B})\int(\nabla_\mathbf{k}\cdot\boldsymbol{\Omega}_\chi(\mathbf{k}))f_\chi(\mathbf{k})$$

cuasiparticle density

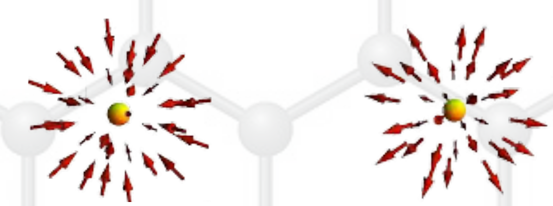
chiral charge

$$\nabla_\mathbf{k}\cdot\boldsymbol{\Omega}_\chi(\mathbf{k}) = \chi\,\delta(\mathbf{k})$$

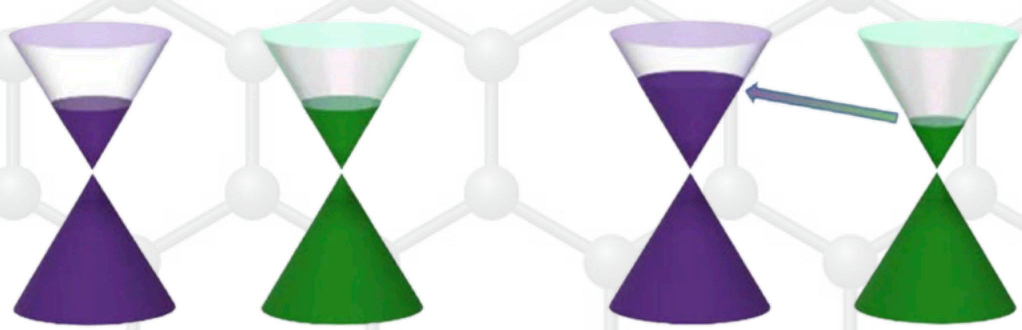
$$\dot\rho = \dot\rho_+ + \dot\rho_- = 0$$

total charge conserved

gauge invariance



$\mathbf{E}\cdot\mathbf{B}=0$ $\mathbf{E}\cdot\mathbf{B}\neq 0$



$$\dot\rho_5 = \dot\rho_+ - \dot\rho_- = \frac{e^3}{2\pi^2}(\mathbf{E}\cdot\mathbf{B})f_+(\mathbf{0})$$

nonconservation of chiral charge

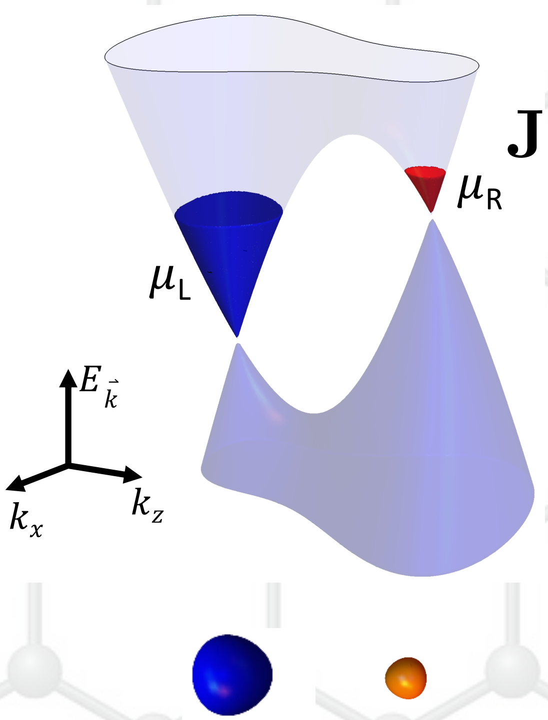
(if we impose gauge invariance)

AA Burkov, L Balents, Phys. Rev. Lett. **107**, 12720 (2012)

AA Zyuzin, AA Burkov, Phys. Rev. B (2012)

Anomalous transport induced by axial anomaly in WSMs

Statistical transport induced by axial anomaly



Fermi surface

Magnetization current

$$\mathbf{J} = -e \sum_{\alpha=\pm 1} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left(\mathbf{r}_\alpha + \frac{1}{e}\mathbf{m}_\alpha(\mathbf{k}) \times \nabla_\mathbf{r} \right) f_\alpha(\mathbf{k},\mathbf{r})$$

$\nabla \times \mathbf{M}$

$$\mathbf{J}_5 = \frac{2}{3}\sigma_0 \nabla \mu_5 \times \mathbf{E}$$

$\sigma_0 = e^2\tau/h^2$

conductance quantum

$\mu_5 = (\mu_R - \mu_L)/2$ chiral Fermi level

$$S[A_\mu] = \int \bar{\psi} i\gamma^\mu \left(\partial_\mu + ieA_\mu + ib_\mu\gamma^5 + \frac{4}{3} i(\tau/\hbar)\mu_5\gamma^5 \right)$$

$$\partial_\mu J_5^\mu = \frac{e^2}{2\pi h}\mathbf{E}\cdot\mathbf{B}$$

Chiral anomaly!!! Fujikawa method

Chiral anomaly induced statistical transport…

$$\mathcal{L}_\theta = \frac{\alpha}{4\pi^2}\theta\,\mathbf{E}\cdot\mathbf{B}$$

$$b_\mu = \partial_\mu\theta \quad \rightarrow \quad b_\mu + a_\mu = \partial_\mu\left(\theta + \frac{4}{3}(\tau/\hbar)\mu_5\right)$$

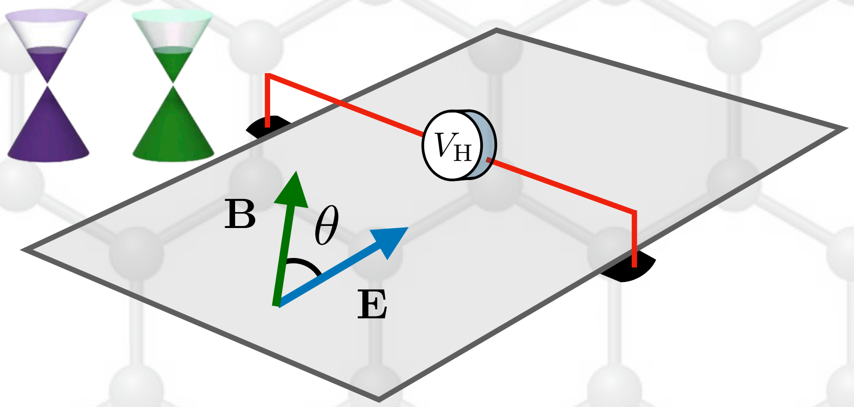
chiral gauge field!!!

- electroweak plasma in the early Universe
- quark-gluon plasmas created in heavy ion collisions
- supernova explosions

R. Flores & AMR Phys. Rev. B **103**, 035102 (2021)

Anomalous transport induced by axial anomaly in WSMs

Planar Hall effect



$$\sigma_{xx} = -e^2\tau \sum_\alpha \int D_\alpha \left[v_x + \frac{eB\cos\theta}{\hbar}(\mathbf{v}_\alpha \cdot \mathbf{\Omega}_\alpha) \right]^2 \frac{\partial f_\alpha}{\partial E_\alpha} \frac{d^3\mathbf{k}}{(2\pi)^3}$$

$\theta = 0$

longitudinal magnetoconductance

topological chiral anomaly

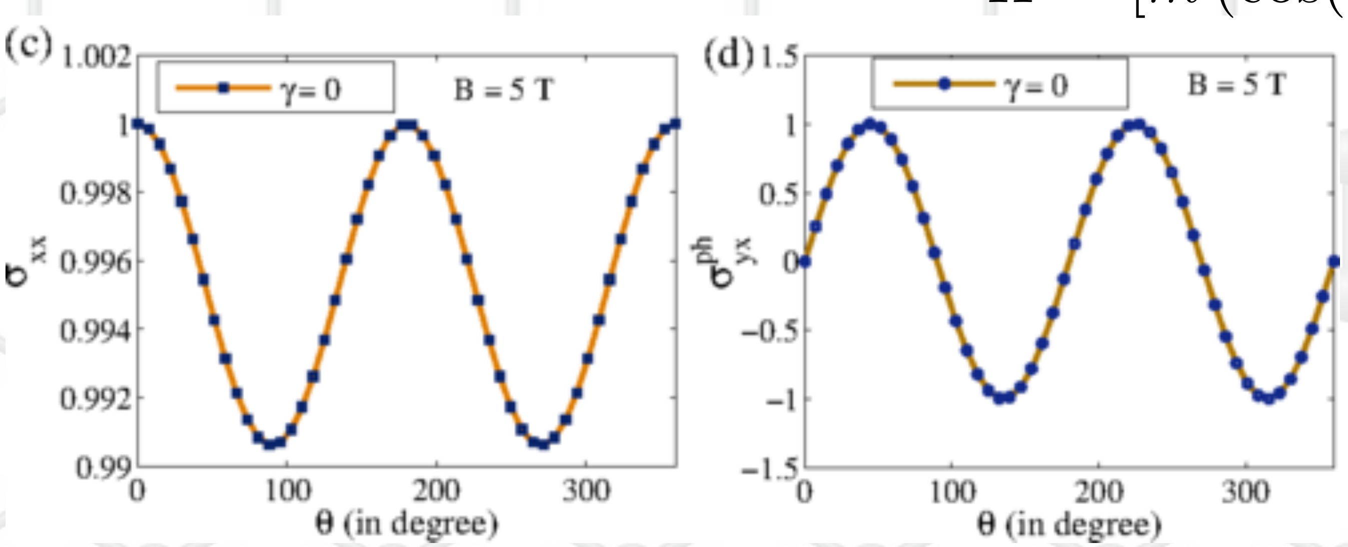
Absent for a regular Fermi liquid

$$\sigma_{yx} = -e^2\tau \sum_\alpha \int D_\alpha \left\{ \frac{eB\sin\theta}{\hbar}(\mathbf{v}_\alpha \cdot \mathbf{\Omega}_\alpha) \times \left[v_x + \frac{eB\cos\theta}{\hbar}(\mathbf{v}_\alpha \cdot \mathbf{\Omega}_\alpha) \right] \right\} \frac{\partial f_\alpha}{\partial E_\alpha} \frac{d^3\mathbf{k}}{(2\pi)^3}$$

$$H = [m(\cos(k_y b) + \cos(k_z c) - 2) + 2t(\cos(k_x a) - \cos k_0)]\sigma_x$$

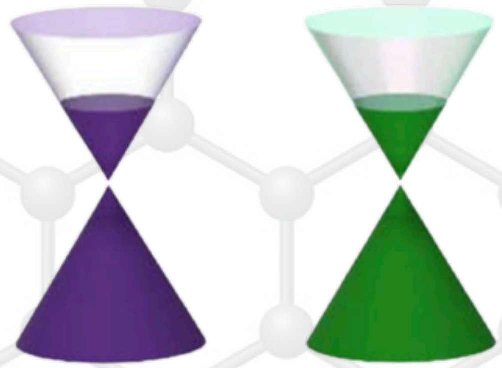
$$-2t\sin(k_y b)\sigma_y - 2t\sin(k_z c)\sigma_z$$

$$H_T = \gamma[\cos(k_x a) - \cos k_0]\sigma_0$$

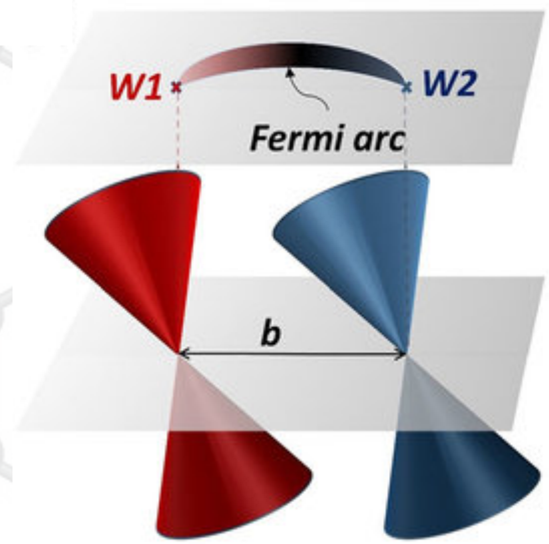


Under study: magnetic moment and spin-orbit coupling effects

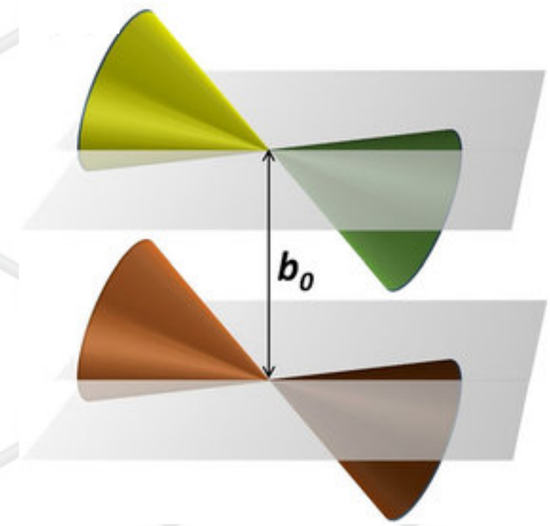
Fermion sector of the SME in Weyls



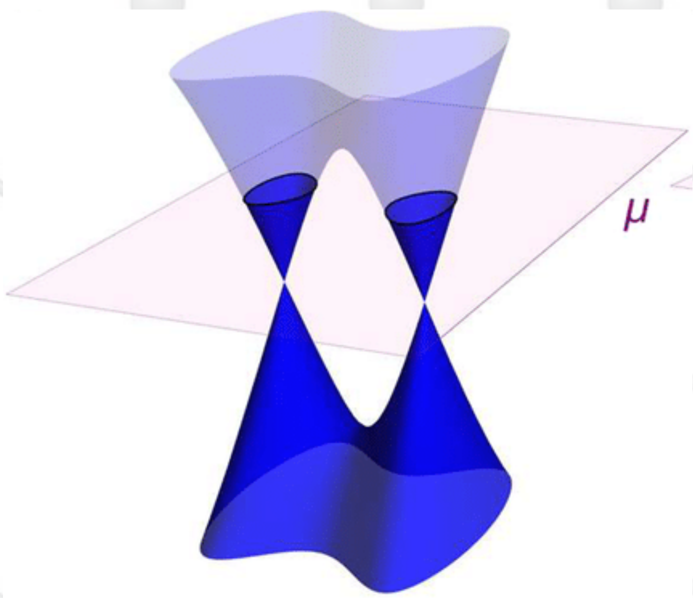
Weyl cones



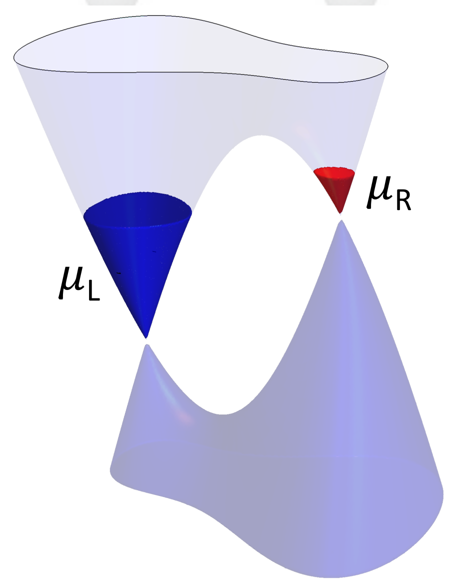
Tilting Type-I WSM



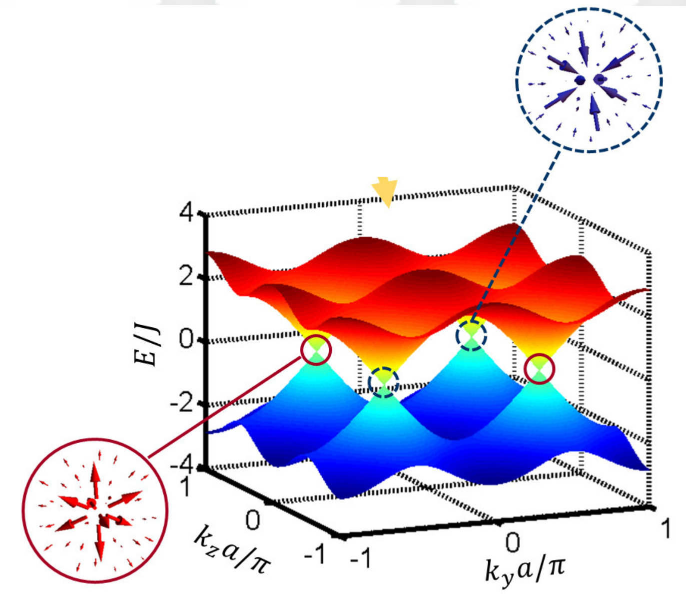
Type-I WSM



Band bending



Tilting + Band bending



Lattice

$$H\psi_\chi = \chi\psi_\chi$$ Weyl equation

Anomalous transport induced by axial anomaly in WSMs

Towards a complete field-theoretical description of Weyl semimetals

$$\mathcal{L}_{\mathrm{QED}} = \frac{1}{2}\bar{\psi}(i\Gamma^{\mu}D_{\mu} - M)\psi + \mathcal{L}_{\mathrm{ED}}$$

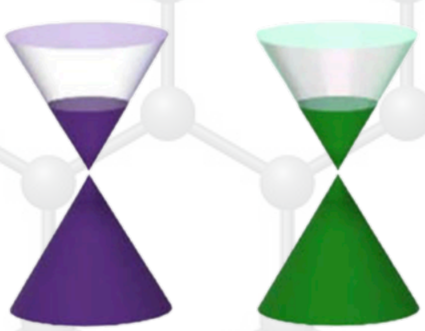
$$\Gamma^{\mu} = \gamma^{\mu} + \delta\Gamma^{\mu}$$

$$M = m + \delta M$$

$$\delta\Gamma^{\mu} = c^{\nu\mu}\gamma_{\mu} + d^{\nu\mu}\gamma_5\gamma_{\nu} + e^{\mu} + if^{\mu}\gamma_5 + \tfrac{1}{2}g^{\alpha\beta\mu}\sigma_{\alpha\beta}$$

$$\delta M = a^{\mu}\gamma_{\mu} + b^{\mu}\gamma_5\gamma_{\mu} + \tfrac{1}{2}H^{\mu\nu}\sigma_{\mu\nu}$$

axial coupling



Weyl semimetals

Anomalous transport induced by axial anomaly in WSMs

ELSEVIER

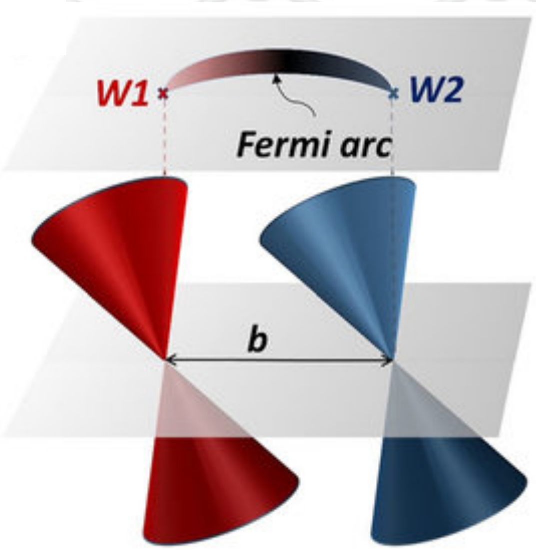
Physics Letters B
Volume 829, 10 June 2022, 137043

Effective electromagnetic actions for Lorentz violating theories exhibiting the axial anomaly

Andrés Gómez [a], A. Martín-Ruiz [b], Luis F. Urrutia [b]

$$\delta\Gamma^{\mu} = c^{\nu\mu}\gamma_{\mu} + d^{\nu\mu}\gamma_5\gamma_{\nu}$$

$$\delta M = a^{\mu}\gamma_{\mu} + b^{\mu}\gamma_5\gamma_{\mu}$$

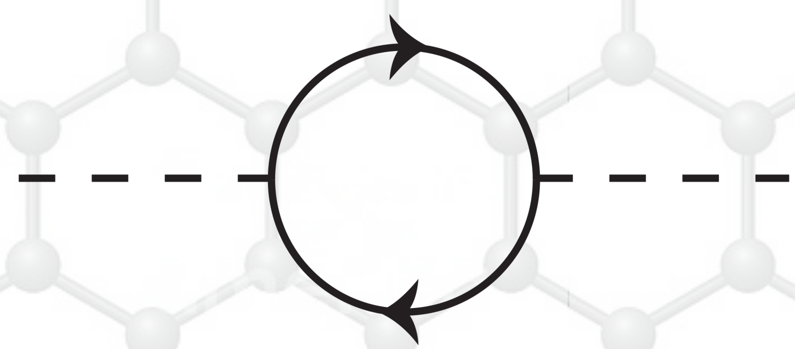


$$i\Pi^{\mu\nu}(p) = e^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left[S(k-p)\Gamma^{\mu}S(k)\Gamma^{\nu}\right]$$

$$S(k) = i/(\Gamma^{\mu}k_{\mu} - M)$$

Vacuum polarization

LV-fermion prop



Anomalous transport induced by axial anomaly in WSMs

$$\partial_\mu J_5^\mu = \frac{e^2}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

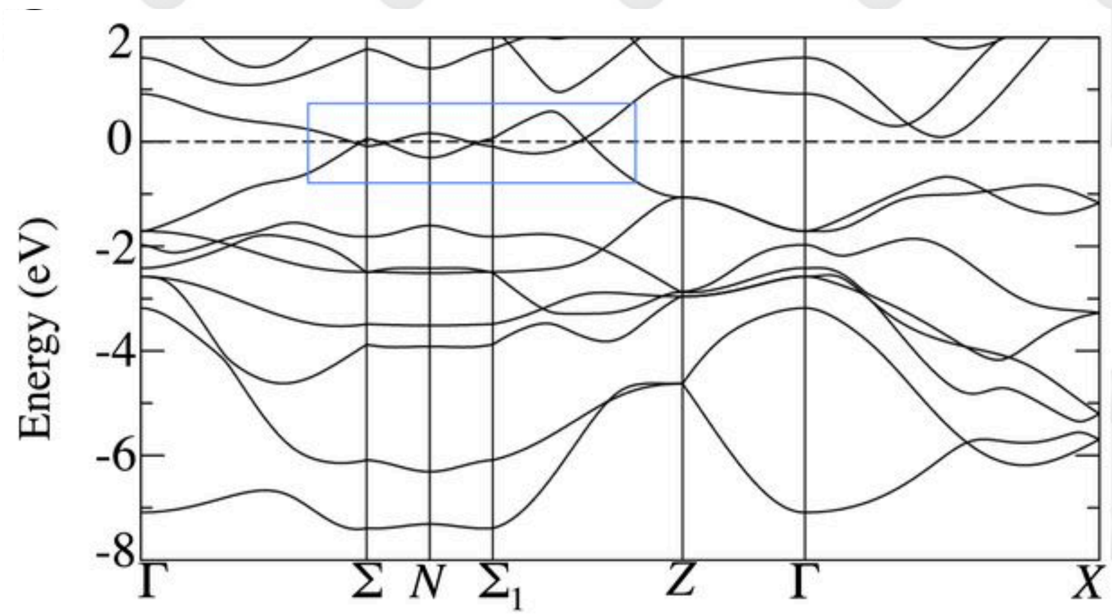
Axial anomaly is untouched by LV!!

Effective field theory?

Quantum corrections depend on the regularization procedure…

$$S_\theta = \frac{\alpha}{4\pi^2}\int \theta(\vec{r},t)\vec{E}\cdot\vec{B}\,d^3x$$

LV-coefficients here

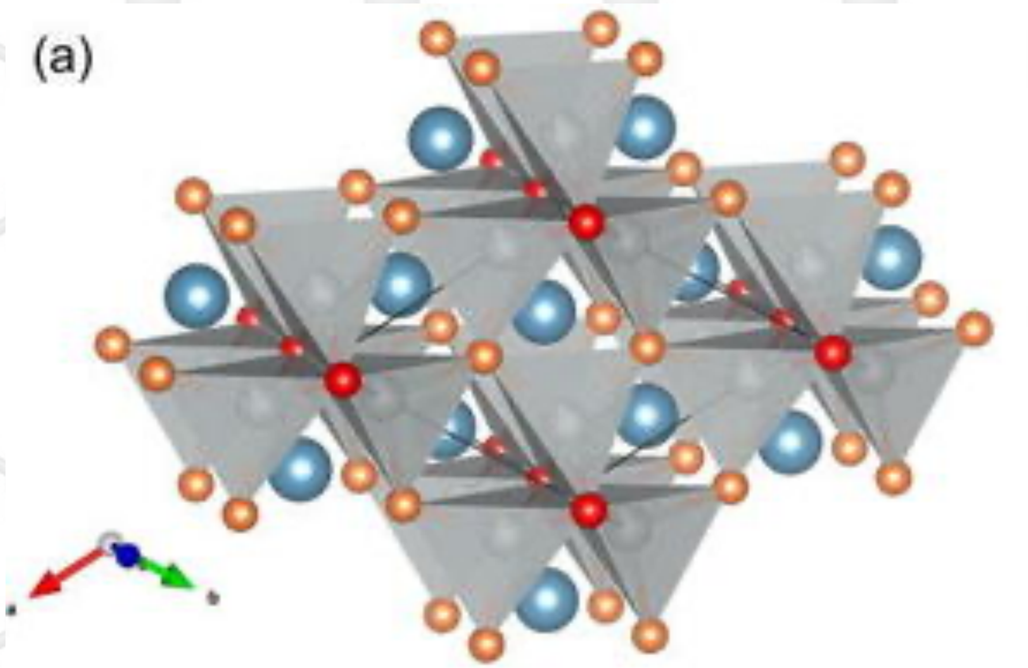


$$\theta = 2(\mathbf{B}\cdot\mathbf{r} - B_0 t)$$

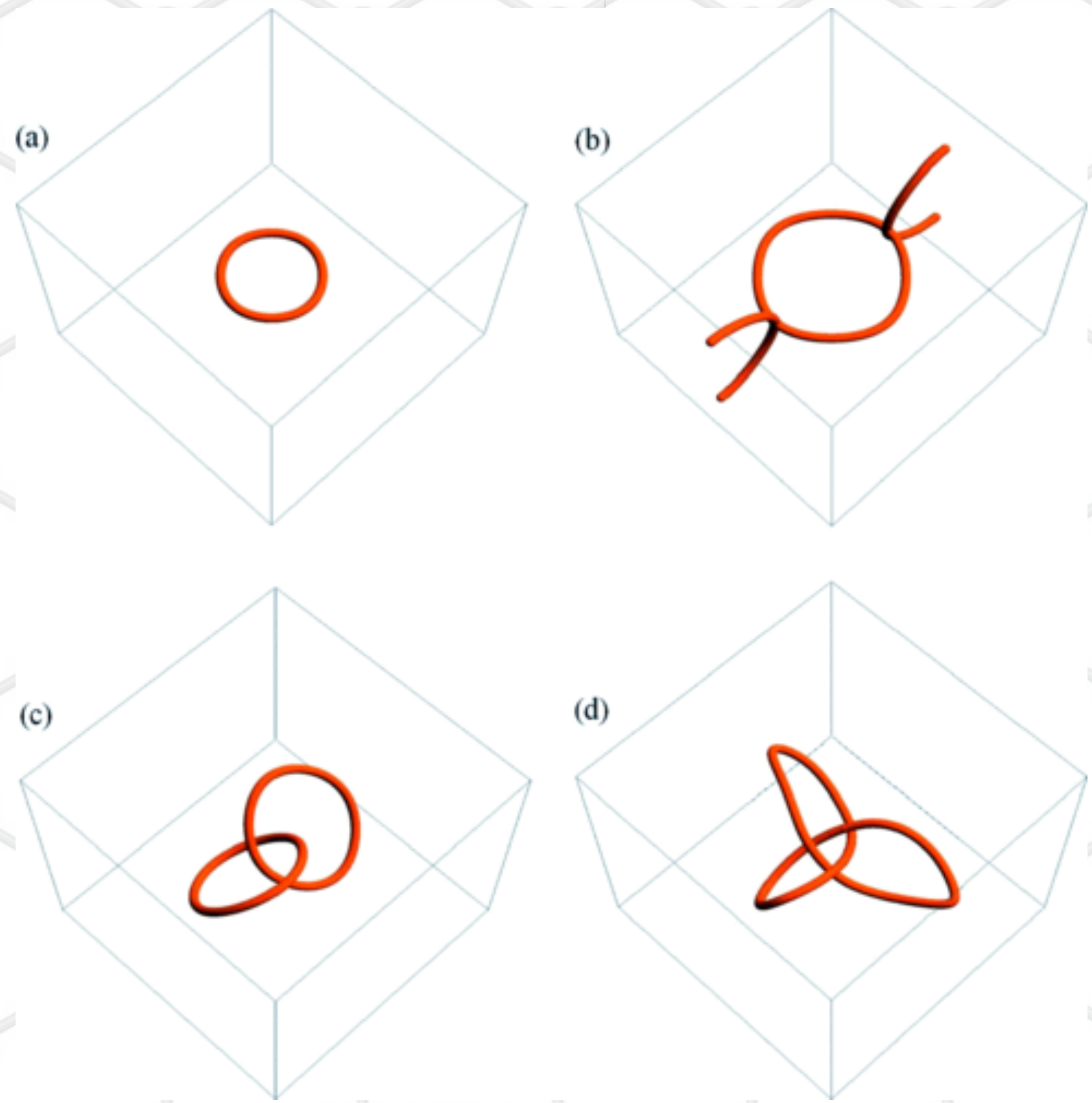
Anomalous transport induced by parity anomaly in NLSM

Nodal lines

Hexagonal pnictide CaAgP



Polyhedral representation



Nodal knot

Anomalous transport induced by parity anomaly in NLSM

$$H_{\mathrm{NL}} = \sum_{\boldsymbol{k}} \Psi^{\dagger}_{\boldsymbol{k}}\, h_{\mathrm{NL}}(\boldsymbol{k})\, \Psi_{\boldsymbol{k}} \qquad \Psi_{\boldsymbol{k}} = (c_{p\boldsymbol{k}}, c_{d\boldsymbol{k}})^T$$

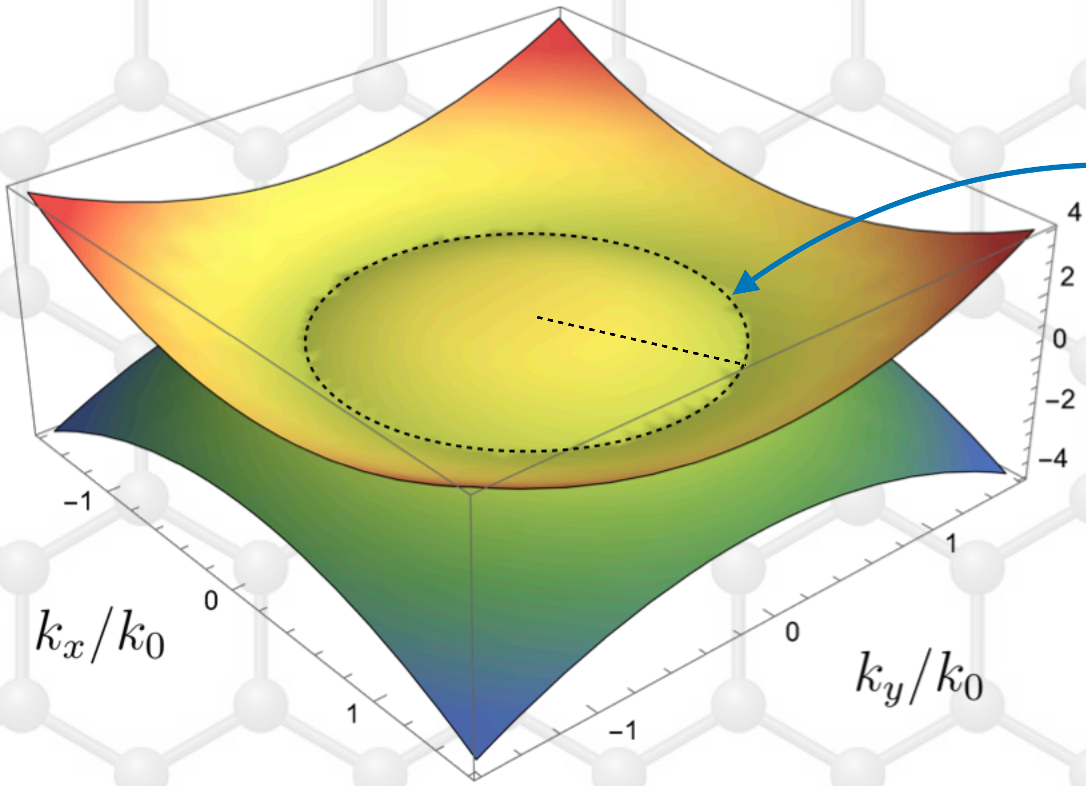
orbitals

$$h_{\mathrm{NL}}(\boldsymbol{k}) = \{\mu_z - 2t_{\parallel}[\cos(k_x a) + \cos(k_y a)] - 2t_{\perp}\cos(k_z a)\}\,\sigma_z - 2t'_{\perp}\sin(k_z a)\sigma_y$$

site energy (μ_z) · intra-orbital hopping ($t_{\parallel}$) · inter-orbital hopping ($t'_{\perp}$)

ab − initio $\quad \mu_z = 1.609\,\mathrm{eV} \quad t_{\parallel} = 0.638\,\mathrm{eV} \quad t'_{\perp} = 0.262\,\mathrm{eV} \quad t_{\perp} = -0.303\,\mathrm{eV}$

$$\mathcal{PT} = \sigma_z \mathcal{C}$$

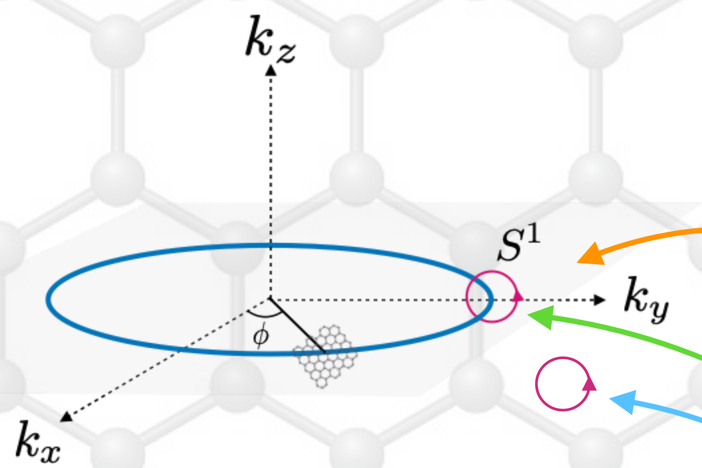


$$h_{\mathrm{NL}}(\boldsymbol{k}) = \frac{1}{\Lambda}\left(k_0^2 - k_x^2 - k_y^2 + b^2 k_z^2\right)\sigma_z + v_z k_z \sigma_y$$

$$k_0 = [\Lambda(\mu_z - 4t_{\parallel} - 2t_{\perp})]^{1/2} \qquad v_z = -2at'_{\perp}$$

$$\Lambda = -(a^2 t_{\parallel})^{-1} \qquad b = (t_{\perp}/t_{\parallel})^{1/2}$$

Anomalous transport induced by parity anomaly in NLSM



$$\nu[S^1] = \frac{1}{\pi}\int_{S^1} d\varphi\, \mathrm{tr}\mathcal{A}(\varphi) \mod 2$$

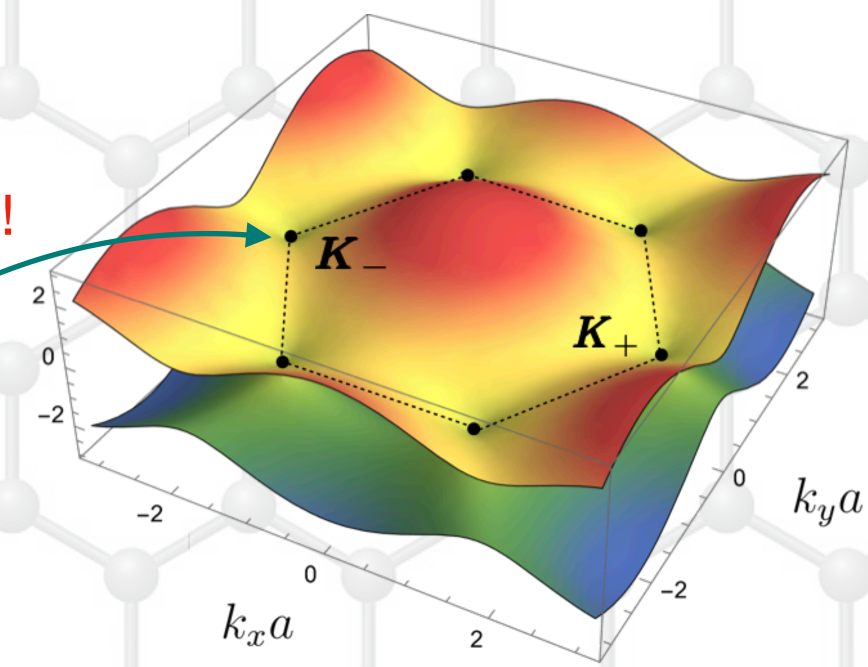
$$\mathcal{A}_{\alpha\beta,j} = \langle \alpha, \boldsymbol{k} | i\partial_{k_j} | \beta, \boldsymbol{k} \rangle$$

Berry connection

$\mathcal{PT} \longrightarrow \nu = 0, 1$ $\mathbb{Z}_2$ quantization

¡Stability of Dirac points in graphene!

Graphene $\longrightarrow$ $\mathcal{PT}$ $\longrightarrow$ Dirac points $\mathrm{cod} = 1$



NLSM: same topological classification of graphene

NLSM = Family of graphene sheets (2D QFTs)

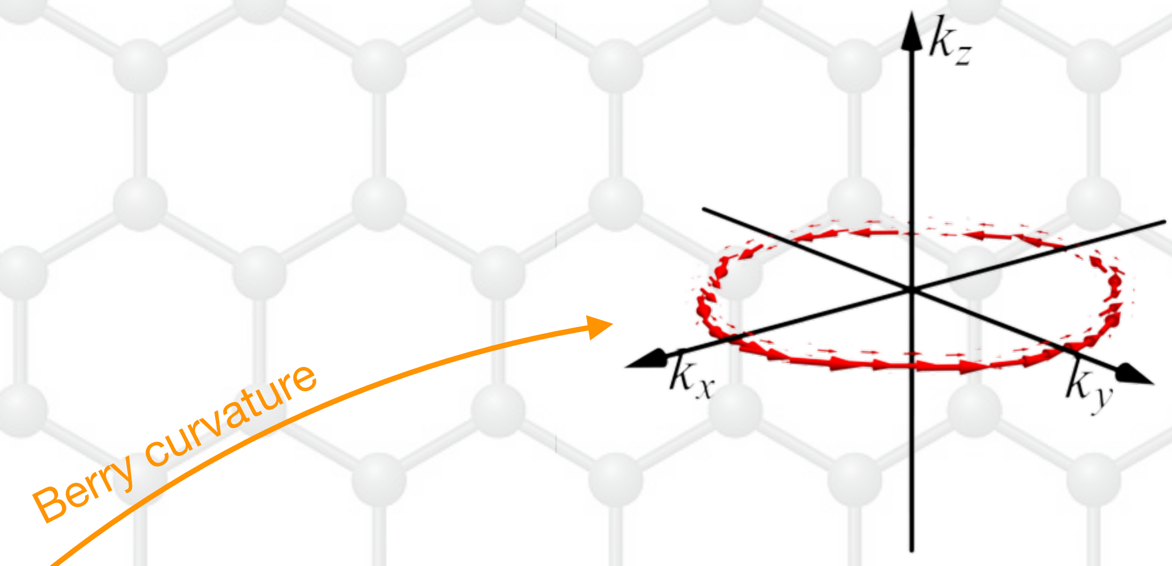
$$(3+1) = (2+1) + 1$$

Anomalous transport induced by parity anomaly in NLSM

Kinetic theory

$\boldsymbol{\Omega}_\nu(\boldsymbol{k}) = \nu\pi\delta(k_\rho - k_0)\delta(k_z)\hat{\boldsymbol{e}}_\phi$ singular at the NL

$\hat{\boldsymbol{e}}_\phi = (-\sin\phi, \cos\phi, 0)^T$



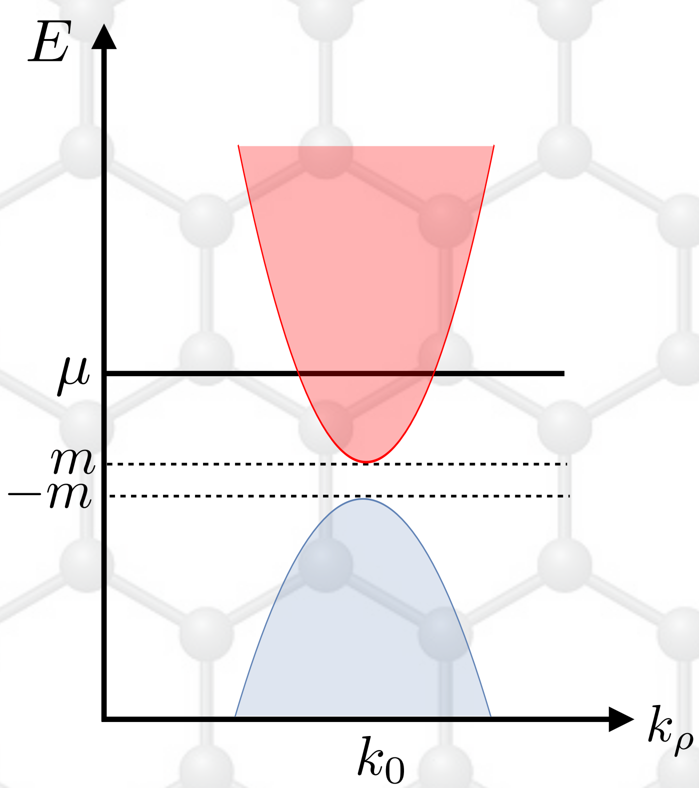
$\mathcal{PT}$ -breaking regularization $h_{\rm NL}(\boldsymbol{k}) + m\sigma_x$

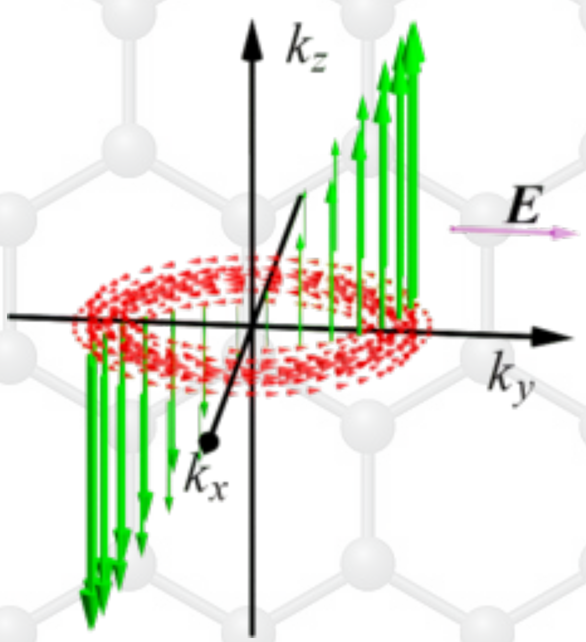
(as in graphene)

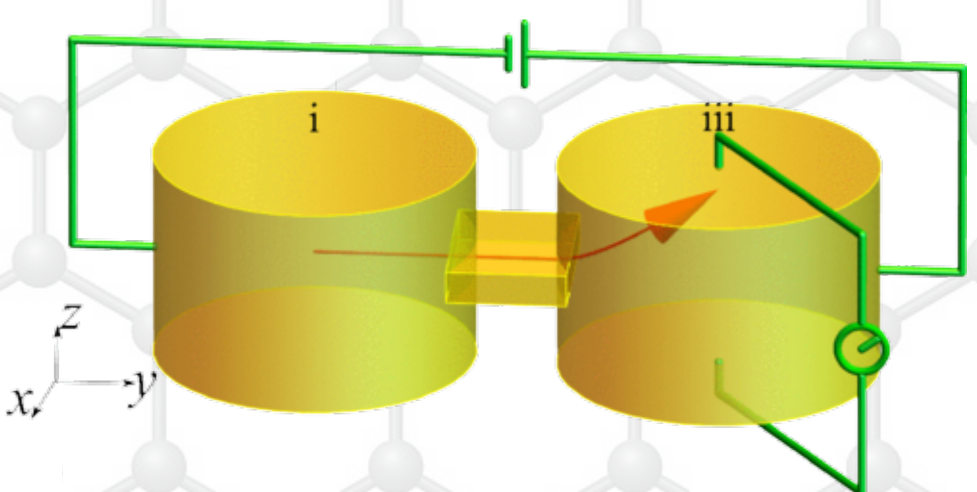
Recall valley HE

$$\sigma_\xi^{\rm Hall} = \frac{e^2}{\hbar}\lim_{m\to 0}\int_{\rm BZ}\frac{d^2\boldsymbol{q}}{(2\pi)^2}\Omega_{z,\xi}(\boldsymbol{q}) = \xi\frac{e^2}{\hbar}$$

$$\boldsymbol{J}(\phi) = \frac{e^2}{\hbar}\frac{k_0}{8\pi^2}\left(1 - \frac{m}{\mu}\right)\boldsymbol{E}\times\hat{\boldsymbol{e}}_\phi$$



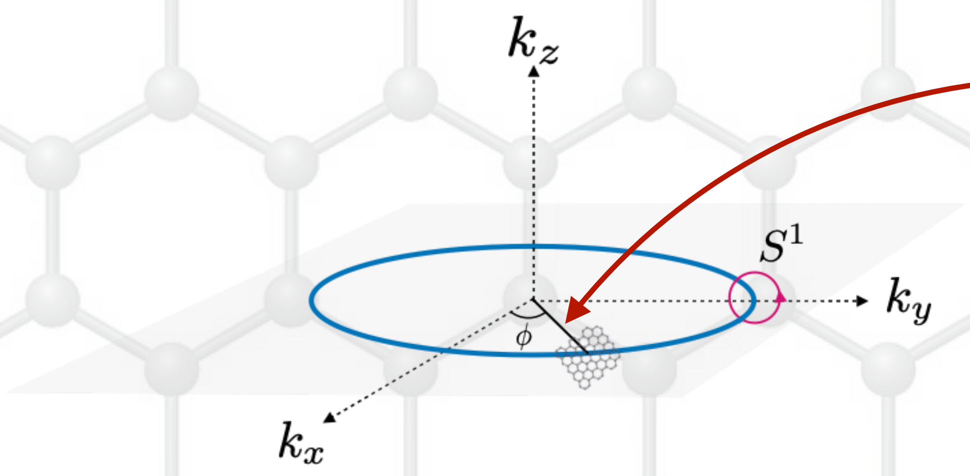




Dumbbell filter device

Anomalous transport induced by parity anomaly in NLSM

Parity anomaly



coupled to gauge field

$$S^\phi[A] = \int d^3x\, \bar\Psi(x)\,[i\gamma^\mu(\partial_\mu + ieA_\mu) + m]\,\Psi(x)$$ (2+1)D-QFT

$\mathcal{PT}$-breaking

$$\bar\Psi = \Psi^\dagger\gamma^0$$

$$\{\gamma^\mu,\gamma^\nu\} = 2\eta^{\mu\nu}$$

$$\eta^{\mu\nu} = \mathrm{diag}(1,-1,-1)$$

$$\mathcal{PT}\ \Psi \rightarrow \gamma^2\gamma^0\Psi$$

$$\mathcal{PT}\ \Psi^\dagger \rightarrow -\Psi^\dagger\gamma^0\gamma^2$$

Effective theory

$S_{\rm ef}[A,0]$ ← UV-divergent

Needs for a $\mathcal{PT}$-breaking for a consistent reg

Pauli-Villars reg $\quad S^{\rm reg}_{\rm ef}[A] = S_{\rm ef}[A,0] - \lim_{m\to\infty} S_{\rm ef}[A,m]$

preserves gauge invariance

$\mathcal{PT}$-breaking

$$S_{\rm CS}[A] = \xi\frac{e^2}{\hbar}\int d^3x\,\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$$

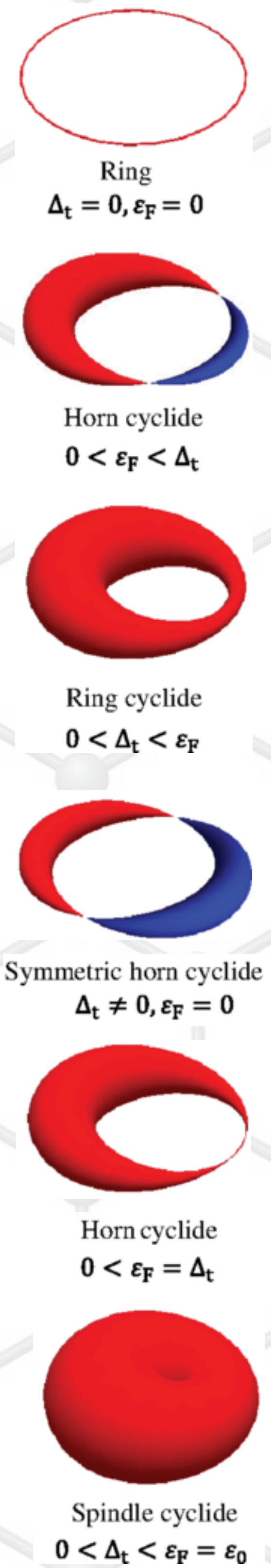
topological charge

valley index

$$J^\phi = \frac{\delta S_{\rm CS}[A]}{\delta A} = \xi\frac{e^2}{4\pi\hbar}\epsilon^{\mu\nu\alpha}\partial_\nu A_\alpha$$

Consequence of the parity anomaly

Anomalous transport induced by parity anomaly in NLSM



Nonlinear response induced by parity anomaly

$$H = \mathbf{v}\cdot\mathbf{k}\,\sigma_0 + \frac{1}{\Lambda}(k_0^2 - k_\rho^2)\sigma_x + m\sigma_y + v_z k_z \sigma_z$$

$$\mathbf{J} = \frac{e^2}{2\pi^2} k_0 \sqrt{1 - \frac{\mu^2}{v^2 k_0^2}}\,\mathbf{E}\times(\hat{\mathbf{v}}\times\hat{\mathbf{n}})$$

Linear Hall

Nonlinear Hall

$$\mathbf{J} = \frac{e^3}{2\pi^2 i\omega}\left(\frac{\mu}{v k_0}\right)\sqrt{1 - \frac{\mu^2}{v^2 k_0^2}}\,[\mathbf{E}\times\mathbf{E}^* - (\mathbf{S}\cdot\mathbf{E})(\mathbf{S}\times\mathbf{E}^*) - (\mathbf{n}\cdot\mathbf{E})(\mathbf{n}\times\mathbf{E}^*)]$$

$$\mathbf{S} = \mathbf{v} + \mathbf{n}\times\mathbf{v}$$

Injection + shift current

Interband effects

AMR & A. Cortijo, Phys. Rev. B **98**, 155125 (2018)

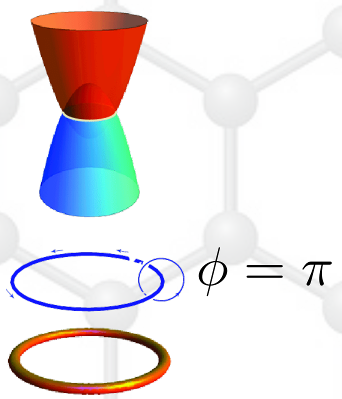
Anomalous transport induced by parity anomaly in NLSM

Planar Hall effect

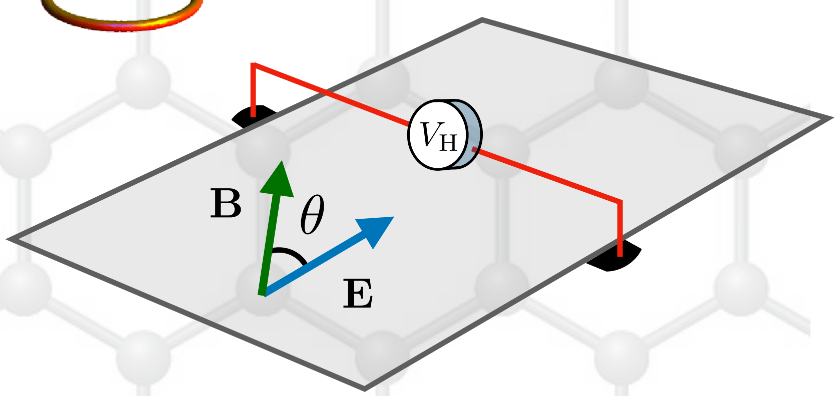
$$\tilde{\mathbf{v}}_\alpha(\mathbf{k}) = \mathbf{v}_\alpha(\mathbf{k}) - \frac{1}{\hbar}\nabla_{\mathbf{k}}(\mathbf{m}_{\alpha\mathbf{k}} \cdot \mathbf{B})$$

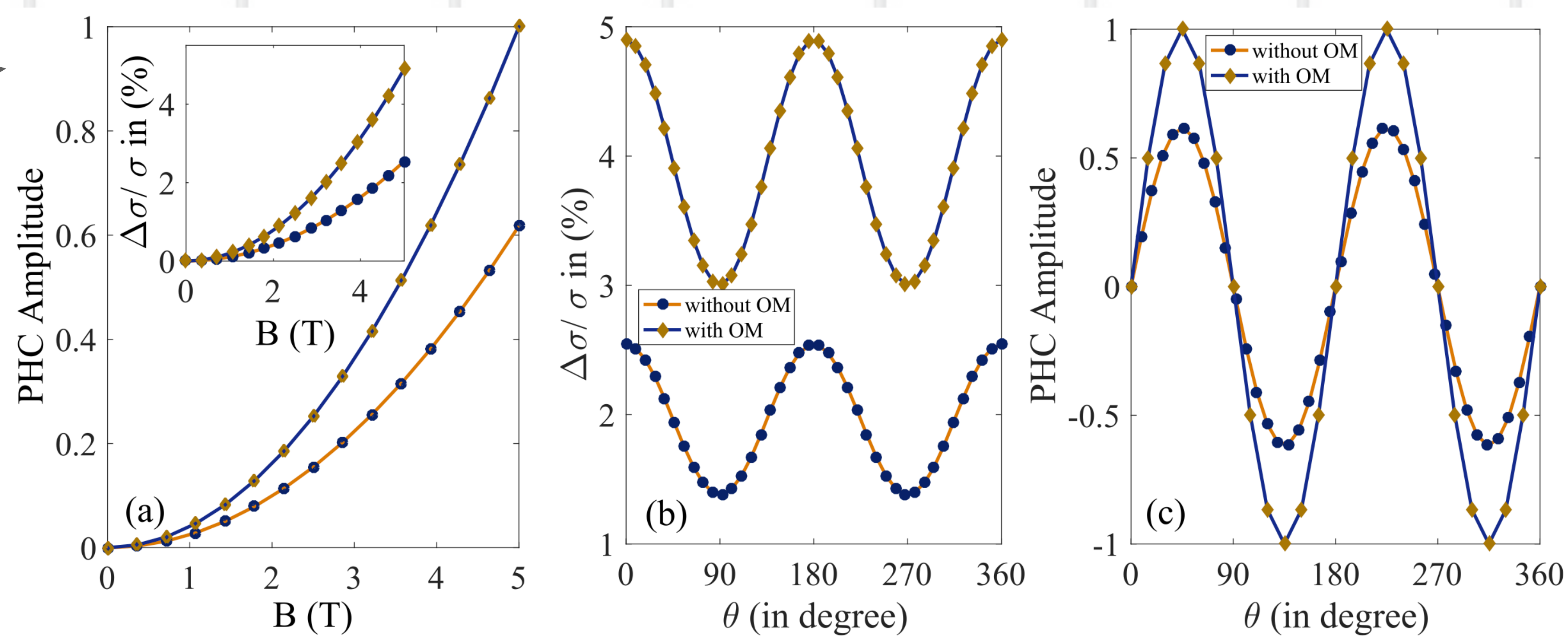
magnetic moment effects

$$\sigma_{ij} = \hbar\tau e \int \frac{d^3\mathbf{k}}{(2\pi)^3} D_\alpha(\mathbf{k})\,(\mathbf{B}, \boldsymbol{\Omega}_{s\mathbf{k}})[\frac{e}{\hbar}\tilde{v}_i\tilde{v}_j + \frac{e^2}{\hbar^2}(\tilde{\mathbf{v}}_{s\mathbf{k}} \cdot \boldsymbol{\Omega}_{s\mathbf{k}})(\tilde{v}_i B_j + B_i\tilde{v}_j) + \frac{e^3}{\hbar^3}(\tilde{\mathbf{v}}_{s\mathbf{k}} \cdot \boldsymbol{\Omega}_{s\mathbf{k}})^2 B_i B_j]\delta(E_{s\mathbf{k}} - \mathbf{m}_{s\mathbf{k}} \cdot \mathbf{B}$$



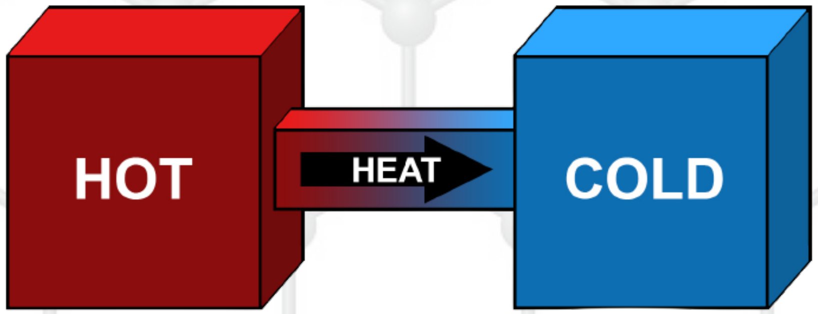
$$\frac{\Delta\sigma}{\sigma} = \frac{\sigma_{xx}(B) - \sigma_{xx}(B=0)}{\sigma_{xx}(B=0)}$$



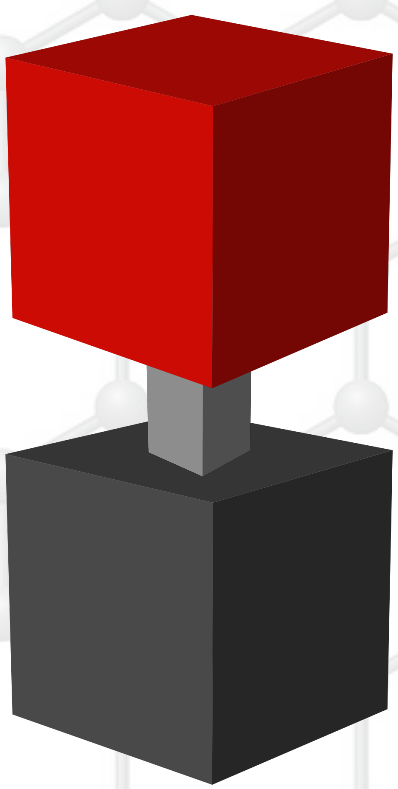


Gravity + LV + Topological SCs

Tolman-Ehrenfest: Thermal equilibrium in gravitational fields



$$\Delta S = \left(\frac{1}{T_1} - \frac{1}{T_2} \right) \Delta U$$



$$\Delta \phi \qquad \frac{\nabla T}{T} = -\frac{1}{c^2} \nabla \phi$$

Gradient of gravitational potential

Temperature gradient

$$\frac{1}{T}\frac{dT}{dr} = -\frac{g}{c^2} \sim 10^{-18} \mathrm{cm}^{-1}$$

At the Earth's surface

Gravity + LV + Topological SCs

Luttinger transport coefficient

$J^i = \sigma^{ij} E^j$ $E^j = i\omega A^j$ $\sigma^{ij} = \frac{1}{i\omega}\left\langle J^i J^j \right\rangle (\omega, k=0)$ Kubo

How do w describe statistical transport? $\nabla \mu$ ∇T

Einstein relation $\mathcal{E} = -\nabla \mu$ Electrochemical transport

(Donor an aceptor impurities)

What is the proper field to describe thermal transport?

Answer: **¡Gravity!** Luttinger-Tolman-Ehrenfest

Local source A_μ **coupled to current** J^μ**:** $S = -\int A_\mu J^\mu d^4x$

$$J^\mu = -\frac{\delta S}{\delta A_\mu}$$

J. M. Luttinger, Phys. Rev. 135 A1505 (1964)

Luttinger transport coefficient

Theory of Thermal Transport Coefficients*

J. M. LUTTINGER
Department of Physics, Columbia University, New York, New York
(Received 20 April 1964)

[7] These effects are actually extremely small, far too small to be observed in any ordinary experiment. They were first considered by A. Einstein, Ann. Physik 38, 443 (1912). See also R. C. Tolman, Phys. Rev. 35, 904 (1930) and R. C. Tolman and P. Ehrenfest, *ibid*. 36, 1791 (1930). (I am indebted to Professor G. Uhlenbeck for calling these interesting references to my attention.) Although the effect is very small, in practice we are only interested in questions of principle, and an arbitrarily small effect is just as good as a large one. In fact, if the gravitational field didn't exist, one could invent one for the purposes of this paper.

In analogy with electrical conductivity is proposed a **ficticius gravitational field**

$$H_{\text{int}}(t) = \int h(\mathbf{r}, t)\, \phi(\mathbf{r})\, d^3\mathbf{r}$$

Hamiltonian density

Gravity field

local

$$\mathbf{J} = \mathcal{L}^{(1)}\mathbf{E} - \mathcal{L}^{(2)}\nabla\phi$$

$$\mathbf{J}_{\mathcal{E}} = \mathcal{L}^{(3)}\mathbf{E} - \mathcal{L}^{(4)}\nabla\phi$$

$$\mathcal{L}^{(2)}_{ij} = \lim_{\omega\to 0}\frac{1}{i\omega}\left\langle J^{\mathcal{E}}_i J_j\right\rangle(\omega, k=0)$$

$$\mathcal{L}^{(4)}_{ij} = \lim_{\omega\to 0}\frac{1}{i\omega}\left\langle J^{\mathcal{E}}_i J^{\mathcal{E}}_j\right\rangle(\omega, k=0)$$

J. M. Luttinger, Phys. Rev. 135 A1505 (1964)

Gravity + LV + Topological SCs

$$\rho = \exp\left[-\frac{1}{kT}(u_\mu P^\mu - \mu Q)\right]$$

$$u_\mu = (1,0,0,0)$$

ensemble at rest

We look for the effects of variations of the metric and gauge field upon the ensemble…

$$\delta H = -\frac{1}{2}\delta g^{00} H + \delta A_0 Q$$

Can we mimic these effects with thermodynamical variations?

YES

Equilibrium conditions:

$$-\nabla A_0 = T\nabla\left(\frac{\mu}{T}\right) \qquad \nabla\phi = T\nabla\left(\frac{1}{T}\right)$$

$$\mathbf{J}_{\rm tr} = L^{(1)}\left[\mathbf{E} - T\nabla\left(\frac{\mu}{T}\right)\right] + L^{(2)}\left[-\nabla\phi + T\nabla\left(\frac{1}{T}\right)\right]$$

transport currents

$$\mathbf{J}^{\mathcal{E}}_{\rm tr} = L^{(3)}\left[\mathbf{E} - T\nabla\left(\frac{\mu}{T}\right)\right] + L^{(4)}\left[-\nabla\phi + T\nabla\left(\frac{1}{T}\right)\right]$$

Gravity + LV + Topological SCs

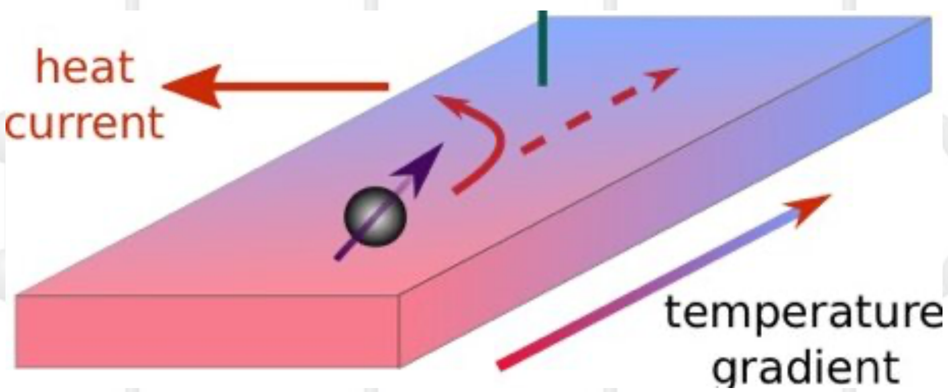
Effective field theory

$$S'_\theta \sim (k_B^2 T^2/24\hbar v) \int \theta\, \vec{E}_g \cdot \vec{B}_g\, d^4x$$

Similar to the electromagnetic axion coupling

Thermal Hall effect

$$\sigma_H = (\pi k_B^2 T/6\hbar)$$



$$J_i = -\frac{\delta S_\theta}{\delta A^i} = \sigma_{\rm H}\partial_i T$$

Is it consistent with the effective field theory?

Gravity + LV + Topological SCs

Example: $$H_{\rm BCS} = \sum_{\mathbf{k}\sigma} \zeta_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} - \sum_{\mathbf{k}} \Delta_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} - \sum_{\mathbf{k}} \Delta^*_{\mathbf{k}} c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow}$$

$\Delta = \mathrm{cte}$ s SC

$\Delta = \Delta(k_x + i k_y)$ $p + ip$ SC

B-phase $^3\mathrm{He}$ superfluid

$$\Psi^\dagger(\mathbf{k}) = (c^\dagger_{\uparrow,\mathbf{k}}, c^\dagger_{\downarrow,\mathbf{k}}, c_{\uparrow,-\mathbf{k}}, c_{\downarrow,-\mathbf{k}})$$ Nambu

$$\mathcal{H} = \int d^3\mathbf{k}\, \Psi^\dagger(\mathbf{k}) H(\mathbf{k}) \Psi(\mathbf{k})$$

Dirac-like

$$H = i\gamma^\mu k_\mu - m\gamma^0$$

How the TSC respond to the gravity field?

$$S[\bar\Psi, \Psi, e] = \int d^4x\, \sqrt{g}\, \left[\bar\Psi e_a{}^\mu \gamma^a \left(i\partial_\mu + \tfrac{1}{2}\omega_\mu{}^{cd}\Sigma_{cd}\right)\Psi + \mu\bar\Psi\Psi\right]$$

Fermions is curved spacetime

Gravity + LV + Topological SCs

$$S_{\text{eff}}[\mu, e] = -i \ln \int \mathcal{D}[\bar{\Psi}, \Psi] \exp\left(iS[\bar{\Psi}, \Psi, e]\right)$$

Integrating out fermions

Fujikawa method

$$S_\theta \sim -\frac{1}{1532\pi^2} \int \epsilon^{\mu\nu\alpha\beta} R^{\tau}{}_{\sigma\mu\nu} R^{\sigma}{}_{\tau\alpha\beta} \, d^4x$$

Gravitational anomaly

HEP: Nonconservation of Lepton current…

gauge broken with a boundary $\delta(S + S_\theta) = 0$ Anomaly induced restoring symmetry?

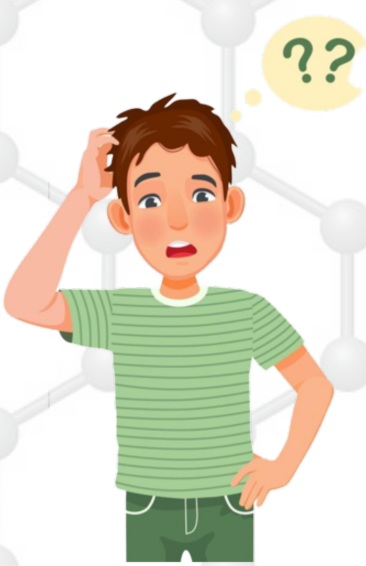
$$J_i = -\frac{\delta S_\theta}{\delta A^i}$$

$$S'_\theta \sim (k_B^2 T^2 / 24\hbar v) \int \theta \, \vec{E}_g \cdot \vec{B}_g \, d^4x$$

¡Nonlinear thermal Hall effect!

What's the correct result?

We do not have experiments yet…



AMR & LFU, IJMPD 26, 1750148 (2017)

Gravity + LV + Topological SCs

LV-gravity sector

Weyl conformal

$$\mathcal{L}_{\mathrm{Grav}} = \frac{1}{2\kappa}R + \frac{1}{2\kappa}(-uR + s^{\mu\nu}R^T_{\mu\nu} + t^{\mu\nu\alpha\beta}C_{\mu\nu\alpha\beta})$$

t-puzzle

Is there a Bogoliubov-de Genes microscopic Hamiltonian?

Some of the coefficients can generate the linear thermal Hall effect?

Luis Urrutia

Ricardo von Dossow

Eduardo Barrios

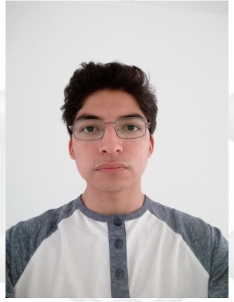
Leonardo Medel

Martín Ibarra

Edgar Briceño

Y. Sarahí García

Fernando López



Amilli Calatayud

Joseph Luna

¡Thanks!