Universidad Autónoma del Estado de Hidalgo

Neutrino Scale within a Universal Texture and Majorana Masses Model

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November 7, 2022

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Abstract

We give a prediction for the scale of the heavier neutrino using the concept of universal texture constraint and corrections from the Majorana mass contributions for neutrinos assuming normal ordering.

$$m_3 \in [2.488,\ 2.544] \times 10^{-3}$$
$$(\delta_1', \delta_2') \in [0.4766,\ 0.5157]$$

Introduction

Background

- The Standard Model groups particles with similar properties in families.
- Fermions in the Standard Model can have a Dirac or a Majorana Mass.

Gell-Mann Matrices

Although there are eight matrices, only those with a diagonal not composed out of only zeros are used.

$$\ell_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad , \quad \ell_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{1}$$

$$\underline{I} = \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \ell_0 \tag{2}$$

Mass Matrix & Exp. Measurements

Mass matrices are modified by multiplying them for the inverse of the heaviest fermion within each sector:

$$\underline{M_i} = \frac{1}{m_{3,i}}\underline{m_i} = \frac{1}{m_{3,i}} \begin{pmatrix} m_{1,i} & 0 & 0 \\ 0 & m_{2,i} & 0 \\ 0 & 0 & m_{3,i} \end{pmatrix} \tag{3}$$

$$= C_0 \cdot \underline{I} + C_3 \cdot \ell_3 + C_8 \cdot \ell_8 \quad ; \quad i = u, d, l \tag{4}$$

We want to reproduce the experimental data for the squared masses difference for neutrinos given by:

$$(\Delta m^2_{21})_{\mathrm{exp}} = 7.53 \pm 0.18 \times 10^{-5} eV^2 \tag{5}$$

$$(\Delta m^2_{32})_{\mathrm{exp}} = 2.453 \pm 0.034 \times 10^{-3} eV^2 \tag{6}$$

Parameters C_k & ΔC_k

- The universal texture permits taking the $C_{k,i}$ as invariant parameters in all sectors.

$$C_{k,i} = \frac{1}{2} Tr(\ell_k \cdot M_i) \tag{7}$$

$$\Delta C_{k,i} = \sum_{j=1}^{3} \frac{dC_{k,i}}{dm_j} \delta_{\mathrm{err}} m_j \tag{8}$$

C_0, C_3, C_8

Using equation (7) we can compute the three parameters for charged leptons:

$$C_{0,l} = \frac{m_e + m_\mu}{m_\tau} + 1 \quad , \quad C_{3,l} = \frac{m_e - m_\mu}{m_\tau} \, , \qquad (9)$$

$$C_{8,l} = \frac{1}{\sqrt{3}} \left(\frac{m_e + m_\mu}{m_\tau} - 2 \right)$$

Universal Texture Constraint - UTC

A bottom-up approach to distinguishing patterns.

Coeff.	Value	Error
$C_{0,l}$	0.4326	4.0365×10^{-6}
$C_{0,u}$	0.4112	1.0585×10^{-4}
$C_{0,d}$	0.4178	2.0038×10^{-3}

Table 1: Errors for coefficients $C_{0,i}$.

Coeff.	Value	Error
$C_{3,l}$	-2.9587×10^{-2}	3.9951×10^{-6}
$C_{3,u}$	-3.6708×10^{-3}	-1.7902×10^{-3}
$C_{3,d}$	-1.0613×10^{-2}	-1.0022×10^{-4}

Table 2: Errors for coefficients $C_{3,i}$.

Coeff.	Value	Error
$C_{8,l}$	-0.5601	-2.3290×10^{-6}
$C_{8,u}$	-0.5752	6.1115×10^{-5}
$C_{8,d}$	-0.5705	1.1569×10^{-3}

Table 3: Errors for coefficients $C_{8,i}$.

This suggest the existence of a common mechanism to describe every sector.

Dirac Masses Approach

Dirac's Approach

Using (7) we compute the parameters in the neutrino sector:

$$C_{0,\nu} = \frac{m_1 + m_2}{m_3} + 1 \quad , \quad C_{3,\nu} = \frac{m_1 - m_2}{m_3} \,,$$
$$C_{8,\nu} = \frac{1}{\sqrt{3}} \left(\frac{m_1 + m_2}{m_3} - 2 \right) \tag{10}$$

Solving for m_1 and m_2 and changing the neutrino sector parameters for those in (9), we obtain:

$$m_1 = \frac{m_3(C_{0,l} + C_{3,l})}{2} - \frac{m_3}{2}$$
$$m_2 = \frac{m_3(C_{0,l} - C_{3,l})}{2} - \frac{m_3}{2} \tag{11}$$

Dirac's Approach

Using a fixed value for m_3 such as $m_3 = 0.05\,eV$ we obtain:

$$\begin{aligned} m_1 &= \frac{(0.05eV)(C_{0,l} + C_{3,l})}{2} - \frac{(0.05eV)}{2} = 1.4379 \times 10^{-5}\,\mathrm{eV} \\ m_2 &= \frac{(0.05eV)(C_{0,l} - C_{3,l})}{2} - \frac{(0.05eV)}{2} = 2.9731 \times 10^{-3}\,\mathrm{eV} \end{aligned} \tag{12}$$

Therefore:

$$\begin{aligned} (\Delta m^2_{21})' &= m_2^2 - m_1^2 = 8.839 \times 10^{-6}\,\mathrm{eV}^2 \\ (\Delta m^2_{32})' &= m_3^2 - m_2^2 = 2.491 \times 10^{-3}\,\mathrm{eV}^2 \\ \Delta m^2_{21} &\neq (\Delta m^2_{21})' = 7.53 \times 10^{-5}\mathrm{eV}^2 \\ \Delta m^2_{32} &\approx (\Delta m^2_{32})' = 2.453 \times 10^{-3}\mathrm{eV}^2 \end{aligned} \tag{13}$$

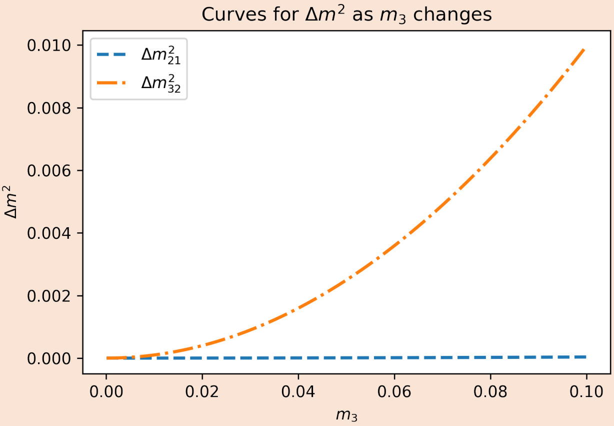

Figure 1: *Behaviour of Δm^2. For the case of Δm^2_{21} we see it doesn't grow within the range for m_3.*

Majorana Masses Approach

Majorana's Approach

We introduce a Majorana mass correction in two different ways; An addition to the mass and a scaling of the mass.

Case 1

$$\begin{aligned} m_1 &\to m_1 + \delta \\ m_2 &\to m_2 + \delta \end{aligned} \tag{14}$$

Case 2

$$\begin{aligned} m_1 &\to \frac{m_1}{\delta'_1} \\ m_2 &\to \frac{m_1}{\delta'_2} \end{aligned} \tag{15}$$

We want to accomplish:

$$f_1 = \Delta m^2_{21} - (\Delta m^2_{21})_{\mathrm{exp}} = 0 \tag{16}$$

$$f_2 = \Delta m^2_{32} - (\Delta m^2_{32})_{\mathrm{exp}} = 0 \tag{17}$$

Case 1

Substituting expressions (14) in (10) and solving m_1 and m_2 we obtain (18):

$$\begin{aligned} m_{1\mathscr{M}} &= \frac{m_3(C_{0,l} + C_{3,l})}{2} - \frac{m_3 + \delta}{2} \\ m_{2\mathscr{M}} &= \frac{m_3(C_{0,l} - C_{3,l})}{2} - \frac{m_3 - \delta}{2} \end{aligned} \tag{18}$$

Majorana's Approach Case 1

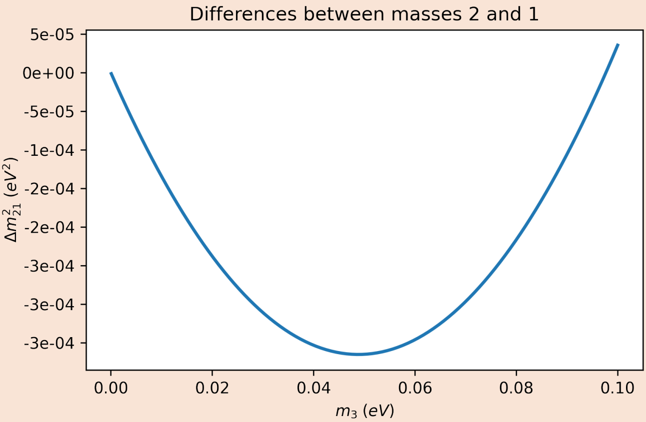

Figure 2: *Curve for Δm^2_{21} when; $m_3 \in (0.0, 0.1)$.*

Majorana's Approach Case 1

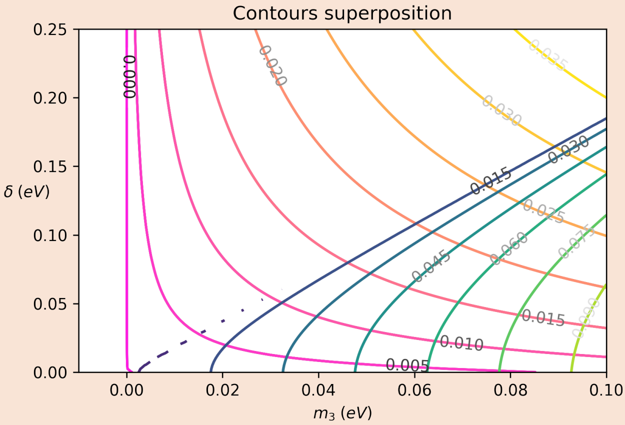

Figure 3: *Curves; f_1, f_2 using expressions in (16). Level zero curves do not intersect.*

Case 2

Now substituting (15) in (10) gives us the expressions in (19):

$$m'_{1\mathscr{M}} = \frac{m_3 \cdot \delta'_1 \cdot (C_{0,l} + C_{3,l})}{2} - \frac{m_3 \cdot \delta'_1}{2}$$
$$m'_{2\mathscr{M}} = \frac{m_3 \cdot \delta'_2 \cdot (C_{0,l} - C_{3,l})}{2} - \frac{m_3 \cdot \delta'_2}{2} \tag{19}$$

Majorana's Approach Case 2

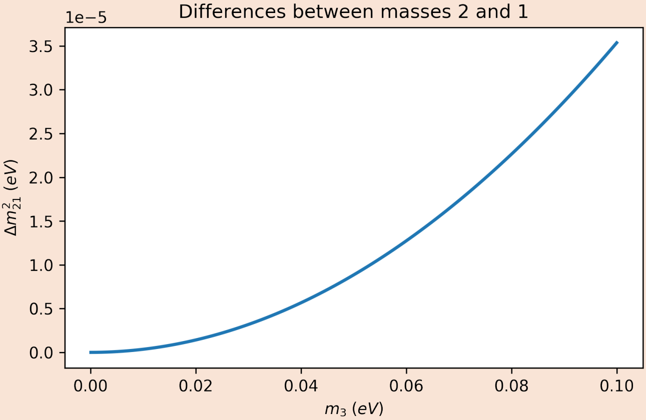

Figure 4: *Curve for Δm^2_{21} when; $m_3 \in (0.0, 0.1)$.*

Majorana's Approach Case 2

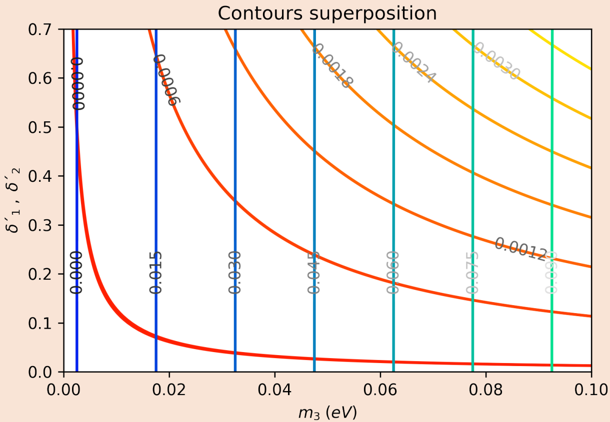

Figure 5: *Curve; f_1, f_2 using expressions in (17). Level zero curves do intersect.*

Prediction

We get the area in Figure 6 from where the ranges for m_3 , δ'_1 , δ'_2 can be defined as:

$$m_3 \in [2.488,\ 2.544] \times 10^{-3} \quad (20)$$

$$(\delta'_1, \delta'_2) \in [0.4766,\ 0.5157] \quad (21)$$

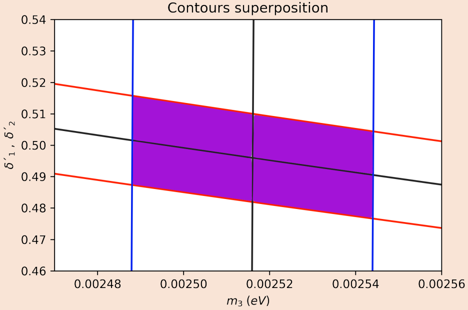

Figure 6: Prediction of sets of numerical values for $m_3, \delta'_1, \delta'_2$ so that; $\Delta m^2_{21} = (\Delta m^2_{21})_{\rm exp}$ y $\Delta m^2_{32} = (\Delta m^2_{32})_{\rm exp}$.

Conclusions

Closing Thoughts

- There is a set of values for m_3 , δ_1' , δ_2' such that the theoretical Δm_{21}^2 and Δm_{32}^2 agree with the experimental measurements and they are given by:

$$m_3 \in [2.488,\ 2.544] \times 10^{-3}$$
$$(\delta_1', \delta_2') \in [0.4766,\ 0.5157]$$

Perspectives

- Study of the relation between the Majorana correction and the electro weak Lagrangian.
- Study of the relation between three generation mixing mechanism for neutrinos and the Majorana correction.
- Repeat procedure using the C_8 parameters.

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