Lorentz violation in electromagnetic moments of fermions

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Field theories

Two main ingredients:

Dynamic variables

Symmetries

SM fields

- EW gauge bosons
- QCD gauge bosons
- Dirac spinors
- Higgs scalar

SM symmetries

- Gauge symmetry
- Lorentz symmetry
  - $\Rightarrow$ CPT symmetry
- Other global symmetries

Beyond the SM

Electrodynamics: $\mathcal{L}_{\mathrm{U}(1)_e} = \frac{1}{4}(F_{\mu\nu})^2 + \overline{\psi_i}(i\not{D} - m_j)\psi_j$ + electroweak stuff

EW symmetry breaking

Standard Model:

$$\mathcal{L}_{\mathrm{SM}} = -\frac{1}{4}(F_{\mu\nu})^2 + i\overline{\psi}\not{D}\psi$$
$$-\overline{\psi_i}Y_{ij}\psi_j + \mathrm{H.c.}$$
$$+|D_\mu\phi|^2 - V(\phi)$$

+ LV stuff, perhaps?

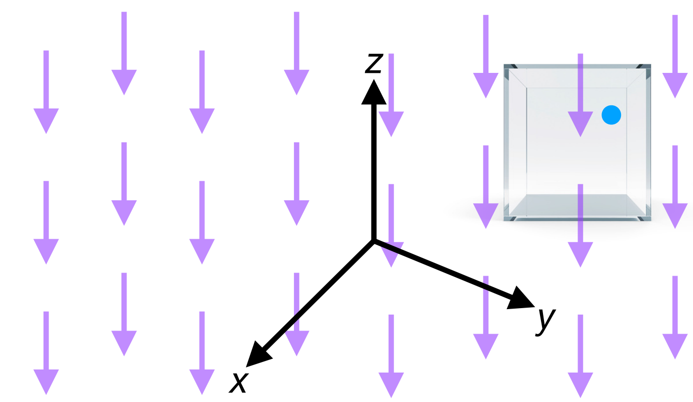
Not perfect, but our best theory so far!!!

Fundamental description:
- Planck Scale
- Dynamic variables and symmetries
- Lorentz violation? $\Longrightarrow$ Deviations from SM observables

Observer VS particle

The gravitational linear potential:*

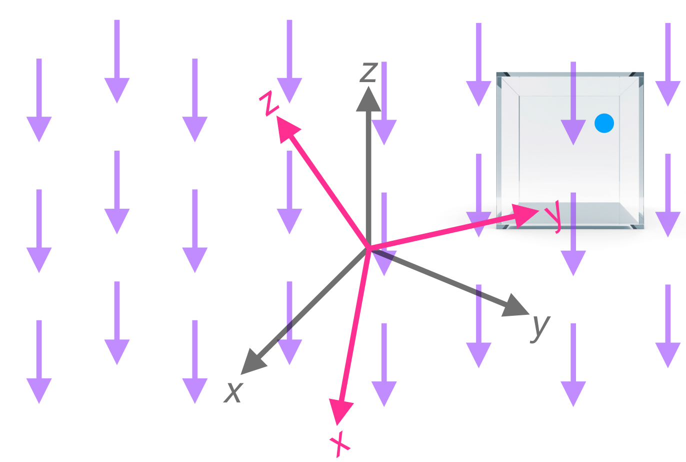
$$L = \frac{m\dot{\boldsymbol{r}}^2}{2} - m\boldsymbol{g}\cdot\boldsymbol{r}$$



Observer LT

$$L \xrightarrow{\text{obs.}} L' = \frac{m\dot{\boldsymbol{r}}^2}{2} - m\boldsymbol{g}\cdot\boldsymbol{r} = L$$

Invariant

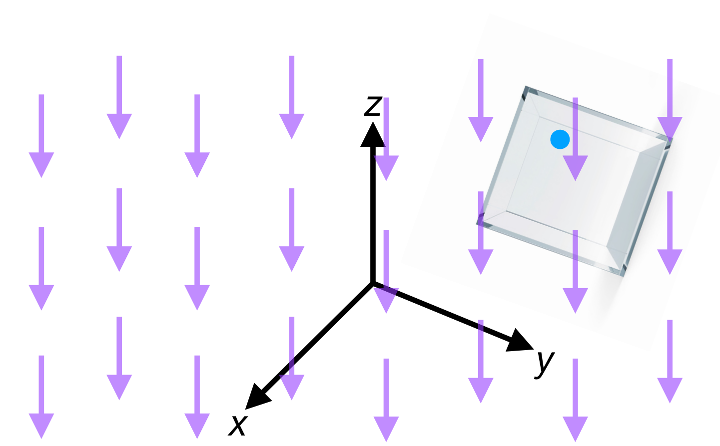


New frame

Particle LT

$$L \xrightarrow{\text{part.}} L' = \frac{m\dot{\boldsymbol{r}}^2}{2} - m\,g_k\,(R_{kj}rj) \neq L$$

Non invariant



Same frame

*Jay D. Tasson, What do we know about Lorentz invariance?, *Rep. Prog. Phys.* **77** 062901 (2014).

The Lorentz-violating SME

Effective Lagrangian:

- SM symmetries
- SM dynamic variables
- Effects from high energies $\Rightarrow$ Novel phenomena; Deviations in SM observables

$$\mathcal{L}_{\text{SME}} = \underline{\mathcal{L}_{\text{mSME}}} + \underline{\mathcal{L}_{\text{nrSME}}}$$

- Renormalizable or minimal SME: mass units $= 4$
- Non-renormalizable SME: mass units > 4

$$\mathcal{L}_{\text{mSME}} = \underline{\mathcal{L}_{\text{SM}}} + \underline{\mathcal{L}_{\text{LV}}}$$

- The SM Lagrangian
- Lorentz-violating terms: $\underline{T_{\mu_1 \ldots \mu_n}}\,\underline{\mathcal{O}^{\mu_1 \cdots \mu_n}}$
  - Background fields
  - Field content
  - Transformations: OT: $T_{\mu_1 \mu_2 \ldots \mu_n} \mathcal{O}^{\mu_1 \mu_2 \cdots \mu_n} \xrightarrow{\text{obs.}} T_{\mu_1 \mu_2 \ldots \mu_n} \mathcal{O}^{\mu_1 \mu_2 \cdots \mu_n}$ **Invariant!**

    PT: $T_{\mu_1 \ldots \mu_n} \mathcal{O}^{\mu_1 \cdots \mu_n} \xrightarrow{\text{part.}} T_{\mu_1 \ldots \mu_n} \Lambda^{\mu_1}{}_{\nu_1} \ldots \Lambda^{\mu_n}{}_{\nu_n} \mathcal{O}^{\nu_1 \cdots \nu_n}$ **Non-invariant!**

The Yukawa sector*

- Leptons
- Quarks
- Antisymmetric (spacetime)

$$\mathcal{L}_{\mathrm{Y}}^{\mathrm{SME}} = -\frac{1}{2}(H_L)_{\mu\nu}^{AB}\overline{L_A}\phi\sigma^{\mu\nu}R_B - \frac{1}{2}(H_U)_{\mu\nu}^{AB}\overline{Q_A}\tilde{\phi}\sigma^{\mu\nu}U_B - \frac{1}{2}(H_D)_{\mu\nu}^{AB}\overline{Q_A}\phi\sigma^{\mu\nu}D_B + \mathrm{H.c.}$$

SSB + unitary gauge $\Longrightarrow$

$$\mathcal{L}_{\mathrm{Y}}^{\mathrm{SME}} = -\frac{1}{2}(v+H)\sum_{f=l,u,d}\overline{f_A}\left[(Y_f)_{\mu\nu}^{AB}P_L + (Y_f)_{\mu\nu}^{BA*}P_R\right]\sigma^{\mu\nu}f_B$$

- Biunitary transformation: $(Y_f)_{\mu\nu} = U_L^{f\dagger}(H_f)_{\mu\nu}U_R^f$
- Mass eigenspinors
- Higgs field, *H*

* D. Colladay and V. A. Kostelecky, *Lorentz-violating extension of the standard model*, Phys. Rev. D **58**, 116002 (1998).

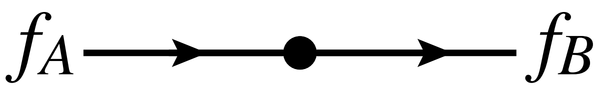
$$\mathcal{L}_{\mathrm{Y}}^{\mathrm{SME}} = -\frac{1}{2}(v+H) \sum_{f=l,u,d} \overline{f_A}\big[(Y_f)_{\mu\nu}^{AB} P_L + (Y_f)_{\mu\nu}^{BA*} P_R\big]\sigma^{\mu\nu} f_B$$

■ Parameters:

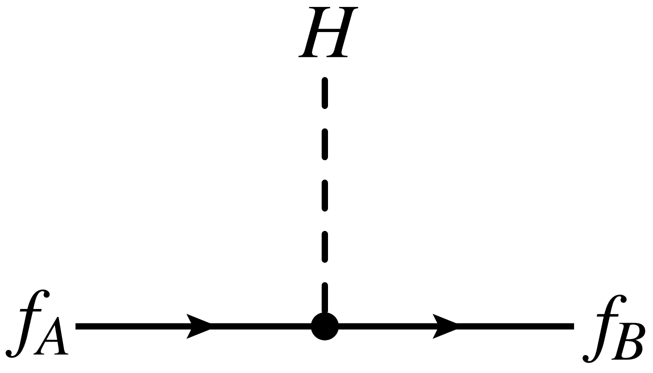
$$Y_{0i}^{AB} = e_i^{AB}, \qquad Y_{ij}^{AB} = \epsilon_{ijk} b^{ABk}, \qquad \Longrightarrow \qquad \mathbf{e}^{AB},\ \mathbf{b}^{AB}$$

$\mathcal{L}_{\mathrm{Y}}^{\mathrm{SME}} \Longrightarrow$ Vertices:

■ Two-point insertion

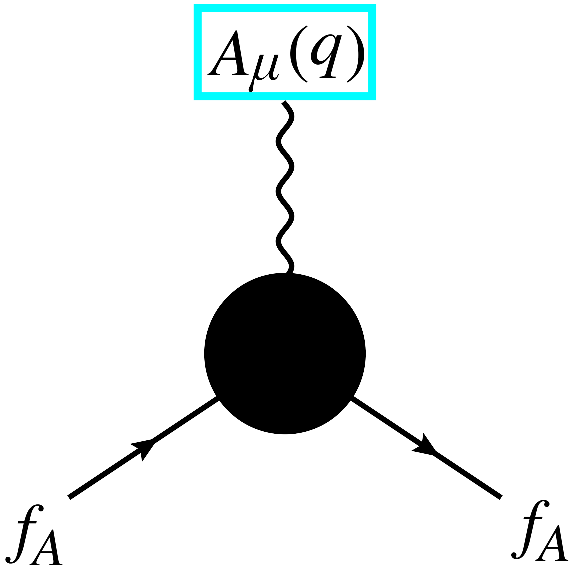


■ Three-point vertex



$\Longrightarrow$ **Contributing Feynman diagrams!**

Contributions to EMMs



$$= \overline{\mathcal{U}_A}\Big(f_A^{\mathrm{M}}(q^2)\sigma_{\mu\nu}q^\nu + f_A^{\mathrm{E}}(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 \Big)\mathcal{U}_A + \cdots$$

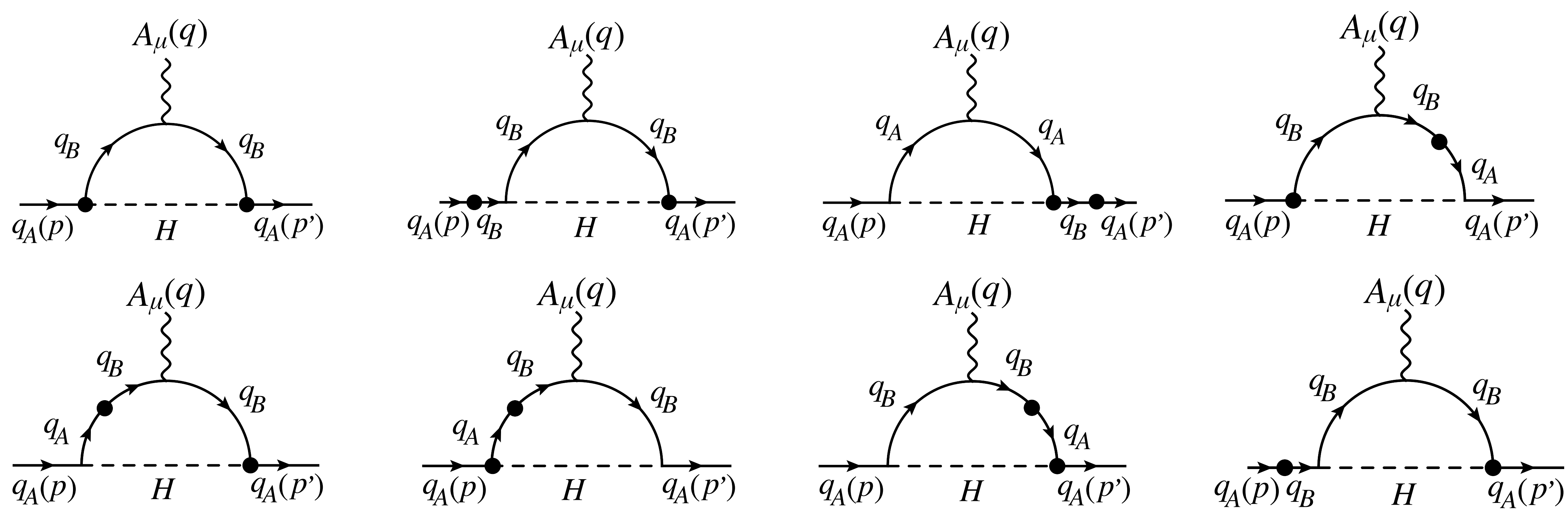
- Off-shell photon
- Lorentz invariant contributions
- Several more terms with Lorentz violation
- Electromagnetic moments:

AMM $a_A^{\mathrm{SME}} = f_A^{\mathrm{M}}(q^2 = 0)$

EDM $d_A^{\mathrm{SME}} = f_A^{\mathrm{E}}(q^2 = 0)$

Contributing diagrams

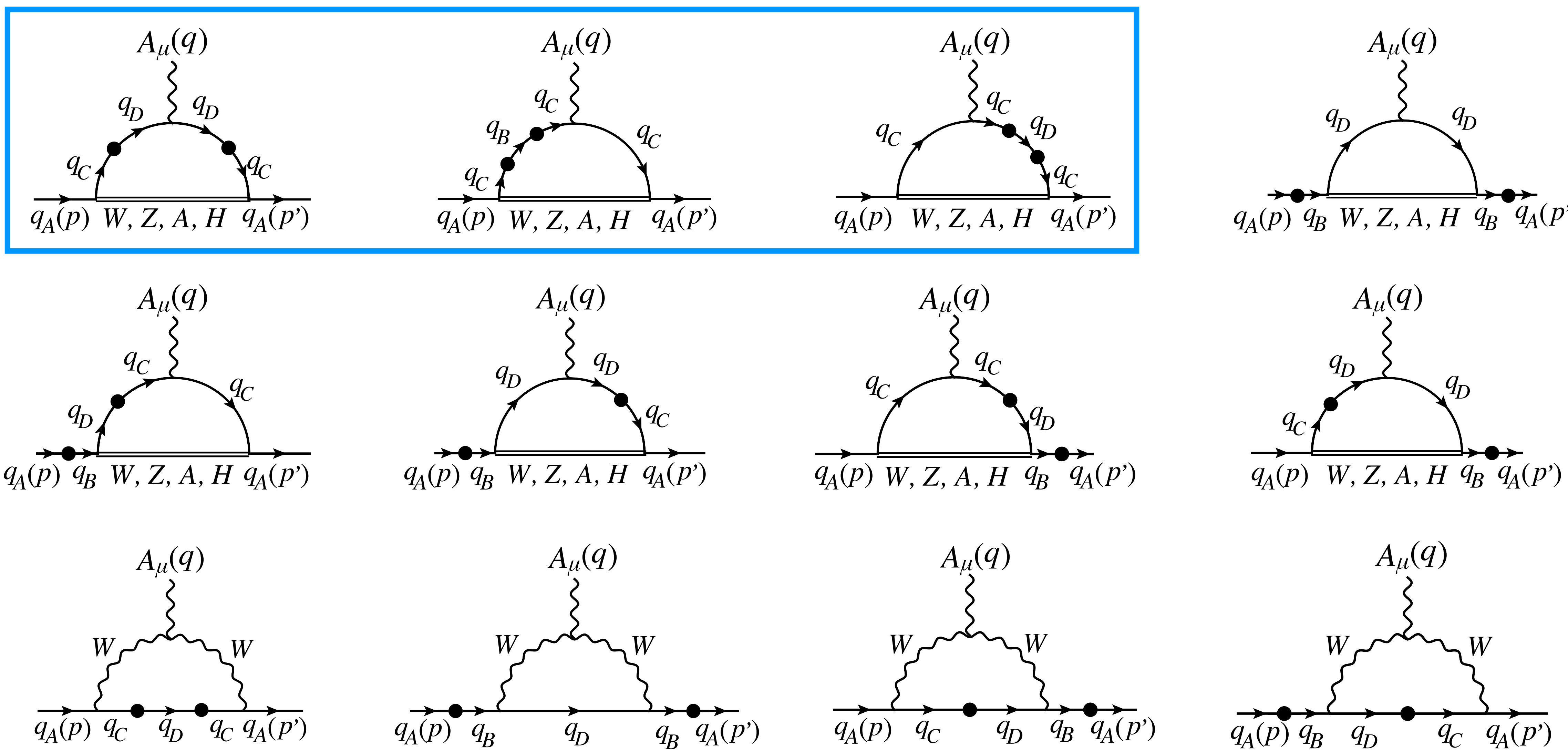
Three-point vertices and two-point insertions



Always a virtual Higgs-boson line

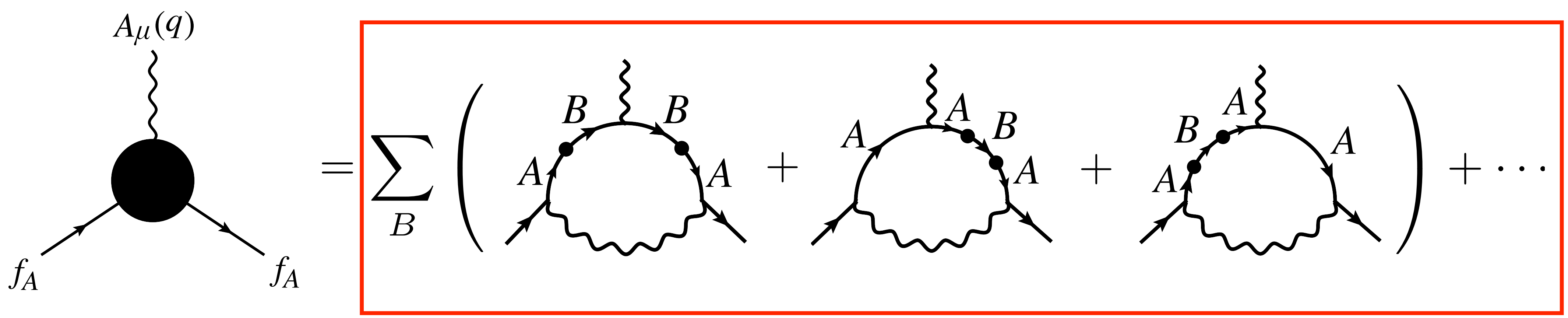
Contributing diagrams

Only two-point insertions



■ Dominant contributions from virtual-photon diagrams

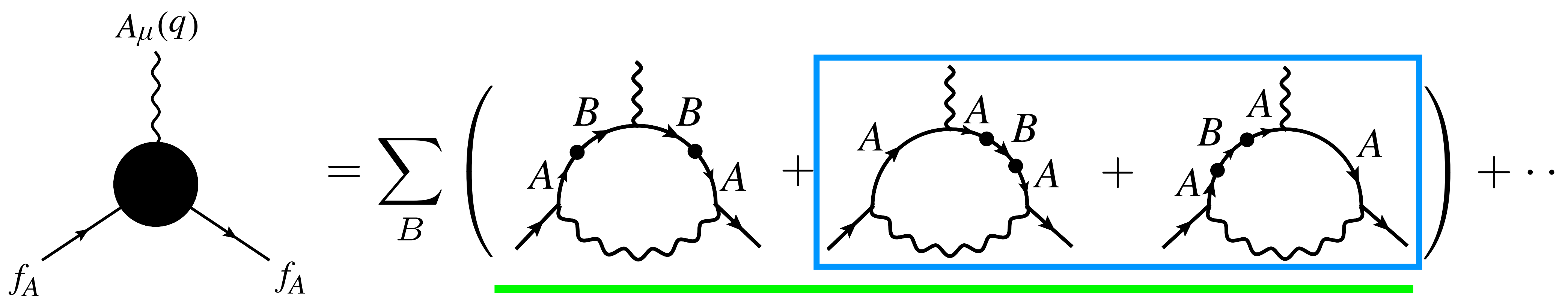
The calculation



■ Full contribution:

- Diagonal electromagnetic form factors
- Unitary gauge ⟶ No pseudo-Goldstones
- Passarino-Veltman tensor-reduction method
- Mathematica, Feyncalc, and Package-X
- Ward identity
- AMMs a_A^{SME} and EDMs d_A^{SME}
- UV-finite electromagnetic moments

The calculation



■ Dominant contributions

■ IR divergences remain → To be canceled from cross section

The contributions:

$$a_A^{\mathrm{SME}} = \sum_B \left[\tilde{a}_{AB}\left(|\mathrm{Re}\,\mathbf{e}^{AB}|^2 + |\mathrm{Re}\,\mathbf{b}^{AB}|^2\right) + \hat{a}_{AB}\left(|\mathrm{Im}\,\mathbf{e}^{AB}|^2 + |\mathrm{Im}\,\mathbf{b}^{AB}|^2\right)\right]$$

$$d_A^{\mathrm{SME}} = \sum_B \tilde{d}_{AB}\left(|\mathrm{Re}\,\mathbf{e}^{AB}||\mathrm{Im}\,\mathbf{b}^{AB}| + |\mathrm{Re}\,\mathbf{b}^{AB}||\mathrm{Im}\,\mathbf{e}^{AB}|\right)$$

IR divergences

$$a_A^{\text{SME}} = \frac{e^3 v^2}{2\pi^2 m_A^2}\left(\Delta_{\text{IR}} + \log\frac{\mu^2}{m_A^2}\right)\left(|\text{Re}\,\mathbf{e}^{AA}|^2 + |\text{Re}\,\mathbf{b}^{AA}|^2\right) + \text{IR finite}$$

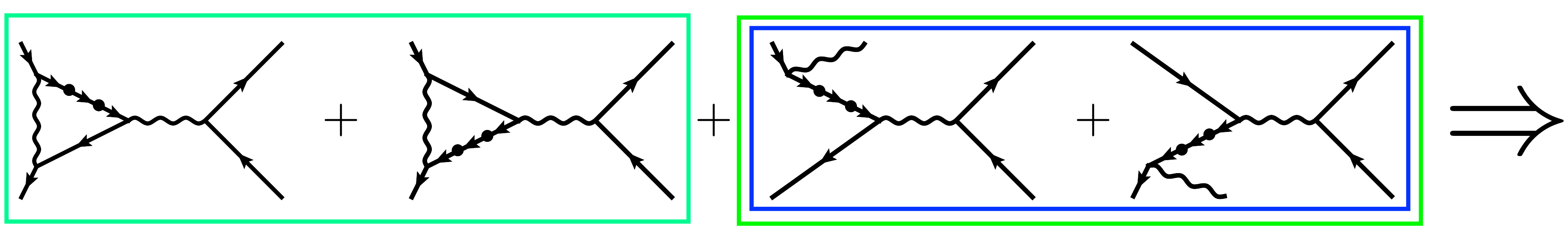
$$d_A^{\text{SME}} = \frac{e^3 v^2}{4\pi^2 m_A^3}\left(\Delta_{\text{IR}} + \log\frac{\mu^2}{m_A^2}\right)|\text{Im}\,\mathbf{e}^{AA}||\text{Re}\,\mathbf{b}^{AA}| + \text{IR finite}$$

- IR divergences
- Contributions are not observables

Soft final-state photon

IR divergent

Forthcoming work: cancelation of IR divergences at cross-section level



$\Rightarrow$ **IR finite cross section**

- Soft final-state photon
- IR divergent

SM quarks

- Proton magnetic moment:

$$a_p = \frac{4}{3}\boxed{a_u} - \frac{1}{3}\boxed{a_d}$$

- Up quark AMM
- Down quark AMM

- Current measurement*:

$$\mu_p = 2.7928473446\boxed{(8)}\,\mu_N$$

- Error $\Longrightarrow$ Bounds on SME

- Most restrictive bounds $\sim 10^{-11}$

- Access to 2nd and 3rd quark families

LVP	**Bounds**
$\lvert\mathrm{Re}\{\mathbf{e}^{uu},\mathbf{b}^{uu}\}\rvert$	6.828×10^{-10}
$\lvert\mathrm{Im}\{\mathbf{e}^{uu},\mathbf{b}^{uu}\}\rvert$	1.526×10^{-9}
$\lvert\mathrm{Re}\{\mathbf{e}^{uc},\mathbf{b}^{uc}\}\rvert$	1.921×10^{-8}
$\lvert\mathrm{Im}\{\mathbf{e}^{uc},\mathbf{b}^{uc}\}\rvert$	1.925×10^{-8}
$\lvert\mathrm{Re}\{\mathbf{e}^{ut},\mathbf{b}^{ut}\}\rvert$	7.156×10^{-11}
$\lvert\mathrm{Im}\{\mathbf{e}^{ut},\mathbf{b}^{ut}\}\rvert$	7.156×10^{-11}
$\lvert\mathrm{Re}\{\mathbf{e}^{dd},\mathbf{b}^{dd}\}\rvert$	6.828×10^{-9}
$\lvert\mathrm{Im}\{\mathbf{e}^{dd},\mathbf{b}^{dd}\}\rvert$	1.526×10^{-8}
$\lvert\mathrm{Re}\{\mathbf{e}^{ds},\mathbf{b}^{ds}\}\rvert$	3.267×10^{-8}
$\lvert\mathrm{Im}\{\mathbf{e}^{ds},\mathbf{b}^{ds}\}\rvert$	3.441×10^{-8}
$\lvert\mathrm{Re}\{\mathbf{e}^{db},\mathbf{b}^{db}\}\rvert$	2.194×10^{-7}
$\lvert\mathrm{Im}\{\mathbf{e}^{db},\mathbf{b}^{db}\}\rvert$	2.220×10^{-7}

*G. Schneider *et al.*, *Double-trap measurement of the proton magnetic moment at 0.3 parts per billion precision*, Science **358**, 1081 (2017).

SM quarks

- Neutron EDM:

$$d_n = \frac{4}{3} d_d - \frac{1}{3} d_u$$

- Down quark EDM
- Up quark EDM

- Current bound*:

$$|d_n| < 1.8 \times 10^{-26}\, e \cdot \mathrm{cm}$$

$\Longrightarrow$ Bounds on SME

LVP	Bounds
$\lvert \mathbf{e}^{uu}, \mathbf{b}^{uu} \rvert$	4.308×10^{-12}
$\lvert \mathbf{e}^{uc}, \mathbf{b}^{uc} \rvert$	9.401×10^{-11}
$\lvert \mathbf{e}^{ut}, \mathbf{b}^{ut} \rvert$	1.096×10^{-9}
$\lvert \mathbf{e}^{dd}, \mathbf{b}^{dd} \rvert$	1.703×10^{-11}
$\lvert \mathbf{e}^{ds}, \mathbf{b}^{ds} \rvert$	6.449×10^{-11}
$\lvert \mathbf{e}^{db}, \mathbf{b}^{db} \rvert$	4.264×10^{-10}

- Most restrictive bound $\sim 10^{-12}$

- Access to 2nd and 3rd quark families

*P. J. Mohr, D. B. Newell, and B. N. Taylor, *CODATA recommended values of the fundamental physical constants: 2014*, Rev. Mod. Phys. **88**, 035009 (2016).

SM charged leptons

- Electron AMM*

$$\Delta a_e = -1.06(082) \times 10^{-12}$$

- Muon AMM**

$$\Delta a_\mu = 249(87) \times 10^{-11}$$

- Tau AMM $\Longrightarrow$ Poor bounds

- Most restrictive bounds $\sim 10^{-11}$

LVP	Bounds
$\lvert\mathrm{Re}\{\mathbf{e}^{ee},\mathbf{b}^{ee}\}\rvert$	1.619×10^{-11}
$\lvert\mathrm{Im}\{\mathbf{e}^{ee},\mathbf{b}^{ee}\}\rvert$	3.620×10^{-11}
$\lvert\mathrm{Re}\{\mathbf{e}^{e\mu},\mathbf{b}^{e\mu}\}\rvert$	2.596×10^{-10}
$\lvert\mathrm{Im}\{\mathbf{e}^{e\mu},\mathbf{b}^{e\mu}\}\rvert$	2.609×10^{-10}
$\lvert\mathrm{Re}\{\mathbf{e}^{e\tau},\mathbf{b}^{e\tau}\}\rvert$	1.067×10^{-9}
$\lvert\mathrm{Im}\{\mathbf{e}^{e\tau},\mathbf{b}^{e\tau}\}\rvert$	1.067×10^{-9}
$\lvert\mathrm{Re}\{\mathbf{e}^{\mu e},\mathbf{b}^{\mu e}\}\rvert$	1.436×10^{-7}
$\lvert\mathrm{Im}\{\mathbf{e}^{\mu e},\mathbf{b}^{\mu e}\}\rvert$	1.451×10^{-7}
$\lvert\mathrm{Re}\{\mathbf{e}^{\mu\mu},\mathbf{b}^{\mu\mu}\}\rvert$	1.622×10^{-7}
$\lvert\mathrm{Im}\{\mathbf{e}^{\mu\mu},\mathbf{b}^{\mu\mu}\}\rvert$	3.628×10^{-7}
$\lvert\mathrm{Re}\{\mathbf{e}^{\mu\tau},\mathbf{b}^{\mu\tau}\}\rvert$	7.243×10^{-7}
$\lvert\mathrm{Im}\{\mathbf{e}^{\mu\tau},\mathbf{b}^{\mu\tau}\}\rvert$	7.686×10^{-7}

*T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, *Tenth-Order QED Contribution to the Electron g–2 and an Improved Value of the Fine Structure Constant*, Phys. Rev. Lett. **109**, 111807 (2012).

T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, *Complete Tenth-Order QED Contribution to the Muon g–2*, Phys. Rev. Lett. **109, 111808 (2012).

SM charged leptons

- Electron EDM*

$$|d_e| < 8.7 \times 10^{-29}\, e \cdot \text{cm}$$

- Muon EDM**

$$|d_\mu| < 1.8 \times 10^{-19}\, e \cdot \text{cm}$$

- Tau EDM $\Longrightarrow$ Poor bounds

- Most restrictive bounds $\sim 10^{-14}$

LVP	Bounds
$\|\mathbf{e}^{ee}, \mathbf{b}^{ee}\|$	5.282×10^{-14}
$\|\mathbf{e}^{e\mu}, \mathbf{b}^{e\mu}\|$	6.577×10^{-13}
$\|\mathbf{e}^{e\tau}, \mathbf{b}^{e\tau}\|$	2.697×10^{-12}
$\|\mathbf{e}^{\mu e}, \mathbf{b}^{\mu e}\|$	5.516×10^{-3}
$\|\mathbf{e}^{\mu\mu}, \mathbf{b}^{\mu\mu}\|$	7.143×10^{-6}
$\|\mathbf{e}^{\mu\tau}, \mathbf{b}^{\mu\tau}\|$	2.530×10^{-5}

J. Baron *et al.* (ACME Collaboration), *Order of magnitude smaller limit on the electric dipole moment of the electron*, Science **343**, 269 (2014).

G. W. Bennett *et al.*, *Improved limit on the muon electric dipole moment*, Phys. Rev. D **80**, 052008 (2009).

Thank you!!!