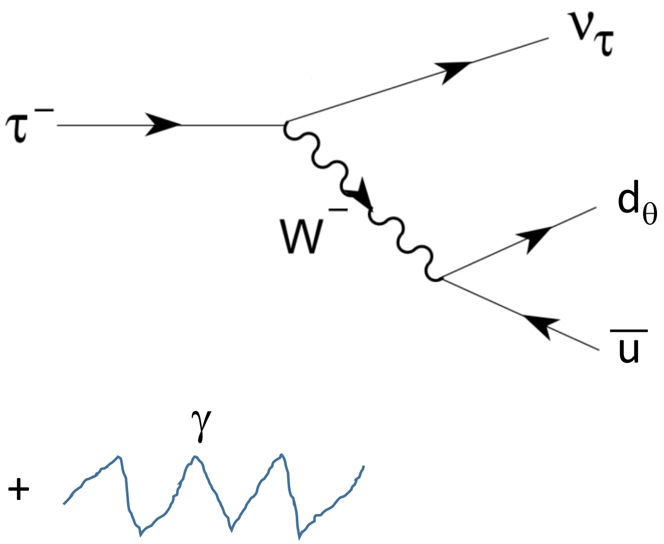
XVIII MWPF
Ed. Carolino, BUAP, Puebla, Mexico, Nov. 21-25, 2022



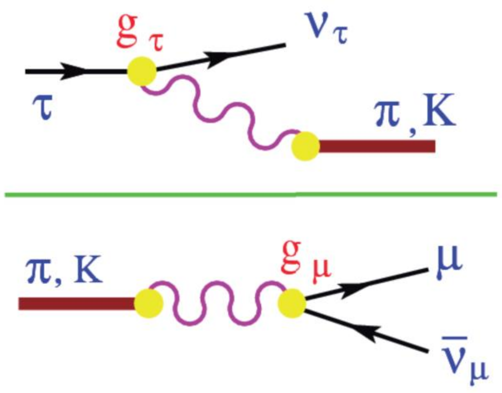
Radiative corrections in semileptonic tau decays and reliable new physics tests

Pablo Roig
Cinvestav (Mexico)

Cinvestav

XVIIIth Mexican Workshop on Particles and Fields

XVIII MWPF
Ed. Carolino, BUAP, Puebla, Mexico, Nov. 21-25, 2022



Radiative corrections in **one-meson** tau decays and reliable new physics tests

Pablo Roig
Cinvestav (Mexico)

Cinvestav

In collaboration with:
M.A. Arroyo-Ureña (BUAP & Cinvestav, Mexico)
G. Hernández-Tomé (IF-UNAM, Mexico)
G. López-Castro (Cinvestav, Mexico)
I. Rosell (UCH-CEU,Valencia,Spain)

JHEP 02 (2022) 173 [arXiv:2112.01859]
PRD 104 (2021) 9, L091502 [arXiv:2107.04603]

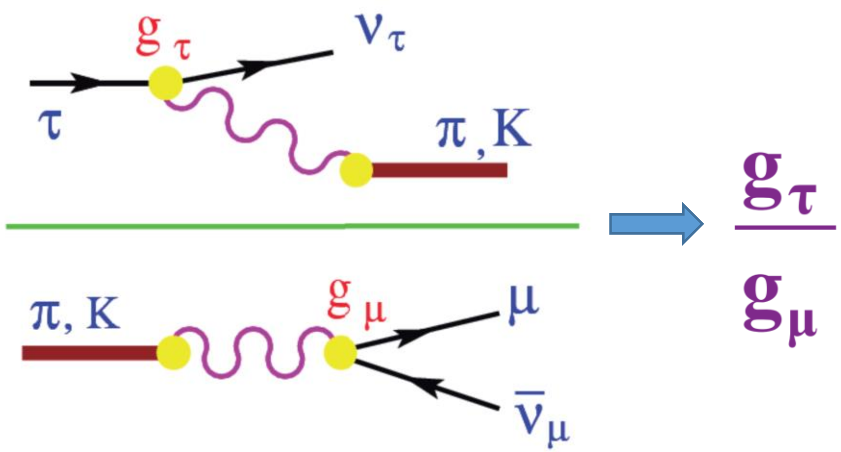
OUTLINE

1) Motivation

2) $P \to \mu\, \nu_\mu\, [\gamma]$ (P=π,K)

3) $\tau \to P\, \nu_\tau\, [\gamma]$ (P=π,K)

4) Calculation of $R_{\tau/P} \equiv \dfrac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])}$



5) Results

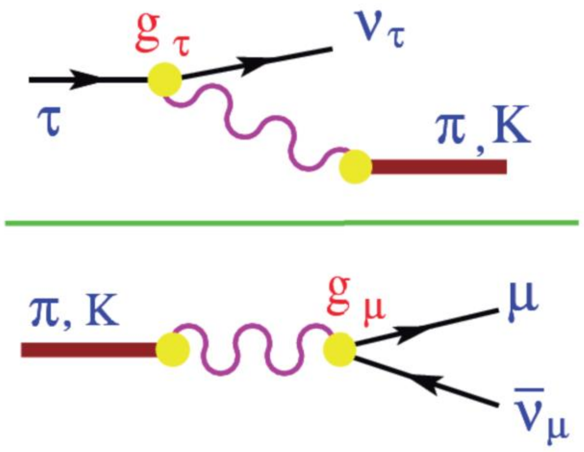
6) Applications

7) Conclusions

1. Motivation

- ✓ Lepton Universality (LU) as a basic tenet of the Standard Model (SM).
  - ✓ A few anomalies observed in semileptonic B meson decays*. (See talks by Irina and E. Rojas)
  - ✓ Lower energy observables currently provide the most precise test of LU**.
- ✓ We aim to test muon-tau lepton universality through the ratio (P = π, K)***:

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 R^{(0)}_{\tau/P}\left(1+\delta R_{\tau/P}\right)$$



  - ✓ $g_\tau = g_\mu$ according to LU.
  - ✓ $R_{\tau/P}^{(0)}$ is the LO result $R^{(0)}_{\tau/P} = \frac{1}{2}\frac{M_\tau^3}{m_\mu^2 m_P}\frac{(1-m_P^2/M_\tau^2)^2}{(1-m_\mu^2/m_P^2)^2}$.
  - ✓ $\delta R_{\tau/P}$ encodes the radiative corrections.
- ✓ $\delta R_{\tau/P}$ was calculated by Decker & Finkemeier (DF'95) ^ :
  - ✓ $\delta R_{\tau/\pi}$ = (0.16 ± 0.14)% and $\delta R_{\tau/K}$ = (0.90 ± 0.22)%.
- ✓ Important phenomenological and theoretical reasons to address the analysis again.

* Albrecht et al.'21
** Bryman et al.'21

*** Marciano & Sirlin'93
^ Decker & Finkemeier'95

1. Motivation

✓ Phenomenological disagreement in LU tests:

✓ Using $\frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])}$ and DF'95*, HFLAV** reported:

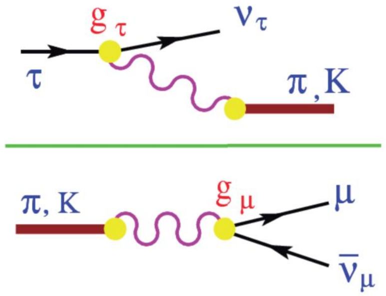
✓ $|g_\tau/g_\mu|_\pi = 0.9958 \pm 0.0026$ (at 1.6σ of LU)
✓ $|g_\tau/g_\mu|_K = 0.9879 \pm 0.0063$ (at 1.9σ of LU)

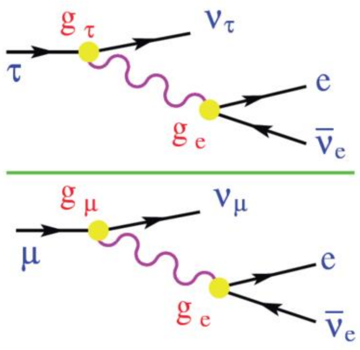
✓ Using $\frac{\Gamma(\tau \to e\bar{\nu}_e\nu_\tau[\gamma])}{\Gamma(\mu \to e\bar{\nu}_e\nu_\mu[\gamma])}$, HFLAV** reported:

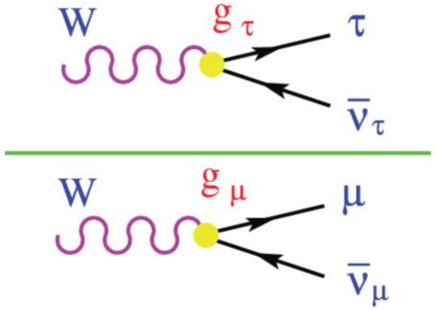
✓ $|g_\tau/g_\mu| = 1.0010 \pm 0.0014$ (at 0.7σ of LU)

✓ Using $\frac{\Gamma(W \to \tau\nu_\tau)}{\Gamma(W \to \mu\nu_\mu)}$, CMS and ATLAS*** and reported:

✓ $|g_\tau/g_\mu| = 0.995 \pm 0.006$ (at 0.8σ of LU)







* Decker & Finkemeier'95
** HFLAV'21
*** CMS'21, ATLAS'21

1. Motivation

✓ Phenomenological disagreement in LU tests:

✓ Using $\frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])}$ and DF'95*, HFLAV** reported:

✓ $|g_\tau/g_\mu|_\pi = 0.9958 \pm 0.0026$ (at 1.6σ of LU)

✓ $|g_\tau/g_\mu|_K = 0.9879 \pm 0.0063$ (at 1.9σ of LU)

✓ Using $\frac{\Gamma(\tau \to e\bar{\nu}_e\nu_\tau[\gamma])}{\Gamma(\mu \to e\bar{\nu}_e\nu_\mu[\gamma])}$, HFLAV** reported:

✓ $|g_\tau/g_\mu| = 1.0010 \pm 0.0014$ (at 0.7σ of LU)

✓ Using $\frac{\Gamma(W \to \tau\nu_\tau)}{\Gamma(W \to \mu\nu_\mu)}$, CMS and ATLAS*** and reported:

✓ $|g_\tau/g_\mu| = 0.995 \pm 0.006$ (at 0.8σ of LU)

✓ Theoretical issues within DF'95*:

✓ Hadronic form factors are different for real- and virtual-photon corrections, do not satisfy the correct QCD short-distance behavior, violate unitarity, analicity and the chiral limit at leading non-trivial orders.

✓ A cutoff to regulate the loop integrals (separating long- and short-distance corrections)

✓ Unrealistic uncertainties (purely $O(e^2p^2)$ ChPT size).

* Decker & Finkemeier'95
** HFLAV'21
*** CMS'21, ATLAS'21

1. Motivation

- ✓ Phenomenological disagreement in LU tests:
  - ✓ Using $\frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])}$ and DF'95*, HFLAV** reported:
    - ✓ $|g_\tau/g_\mu|_\pi = 0.9958 \pm 0.0026$ (at 1.6σ of LU)
    - ✓ $|g_\tau/g_\mu|_K = 0.9879 \pm 0.0063$ (at 1.9σ of LU)
  - ✓ Using $\frac{\Gamma(\tau \to e\bar{\nu}_e\nu_\tau[\gamma])}{\Gamma(\mu \to e\bar{\nu}_e\nu_\mu[\gamma])}$, HFLAV** reported:
    - ✓ $|g_\tau/g_\mu| = 1.0010 \pm 0.0014$ (at 0.7σ of LU)
  - ✓ Using $\frac{\Gamma(W \to \tau\nu_\tau)}{\Gamma(W \to \mu\nu_\mu)}$, CMS and ATLAS*** and reported:
    - ✓ $|g_\tau/g_\mu| = 0.995 \pm 0.006$ (at 0.8σ of LU)
- ✓ By-products of the project:
  - ✓ Radiative corrections in $\Gamma(\tau \to P\nu_\tau[\gamma])$.
  - ✓ CKM unitarity test via $\Gamma(\tau \to K\nu_\tau[\gamma])$ or via the ratio $\Gamma(\tau \to K\nu_\tau[\gamma]) / \Gamma(\tau \to \pi\nu_\tau[\gamma])$.
  - ✓ Constraints on possible non-standard interactions in $\Gamma(\tau \to P\nu_\tau[\gamma])$^.

- ✓ Theoretical issues within DF'95*:
  - ✓ Hadronic form factors are different for real- and virtual-photon corrections, do not satisfy the correct QCD short-distance behavior, violate unitarity, analicity and the chiral limit at leading non-trivial orders.
  - ✓ A cutoff to regulate the loop integrals (separating long- and short-distance corrections)
  - ✓ Unrealistic uncertainties (purely $O(e^2p^2)$ ChPT size).

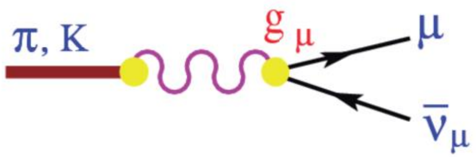
* Decker & Finkemeier'95
** HFLAV'21
*** CMS'21, ATLAS'21

^ Cirigliano et al.'10 '19, '21
^ González-Alonso & Martín-Camalich '16
^ Gonzàlez-Solís et al. '20

2. P → $\mu\, \nu_\mu$ [γ]   (P=π,K)

✓ Calculated unambigously within the Standard Model (Chiral Perturbation Theory, ChPT*).

✓ Notation by Marciano & Sirlin** and numbers by Cirigliano & Rosell*** (D=d,s for π,K and $F_\pi \approx 92.2$ MeV):



LO result

short-distance EW correction ≈ 1.0232**

structure independent (SI) contributions (point-like approximation)^

$$\Gamma(P\to\mu\nu_\mu[\gamma]) = \frac{G_F^2|V_{uD}|^2F_P^2}{4\pi}\, m_P\, m_\mu^2 \left(1-\frac{m_\mu^2}{m_P^2}\right)^2 S_{\rm EW}\left\{1+\frac{\alpha}{\pi}F(m_\mu^2/m_P^2)\right\}\times$$

$$\left\{1-\frac{\alpha}{\pi}\left[\frac{3}{2}\log\frac{m_\rho}{m_P}+c_1^{(P)}+\frac{m_\mu^2}{m_\rho^2}\left(c_2^{(P)}\log\frac{m_\rho^2}{m_\mu^2}+c_3^{(P)}+c_4^{(P)}(m_\mu/m_P)\right)-\frac{m_P^2}{m_\rho^2}\tilde{c}_2^{(P)}\log\frac{m_\rho^2}{m_\mu^2}\right]\right\}$$

structure-dependent (SD) contributions
[coefficents reported in Cirigliano & IR'07]

✓ The only model-dependence is the determination of the counterterms in $c_1^{(P)}$ and $c_3^{(P)}$:

- ✓ Large-N_C expansion of QCD: ChPT is enlarged by including the lightest multiplets of spin-one resonances such that the relevant Green functions are well-behaved at high energies†.

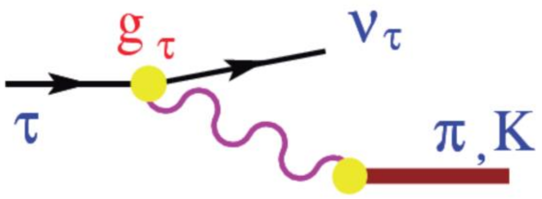
* Weinberg'79
* Gasser & Leutwyler'84 '85
** Marciano & Sirlin'93

*** Cirigliano & IR'07
^ Kinoshita'59

† Ecker et al.'89
† Cirigliano et al.'06

3. $\tau \to P\,\nu_\tau\,[\gamma]$ (P=π,K)

- ✓ Calculated within an effective approach encoding the hadronization:
  - ✓ Large-N_C expansion of QCD: ChPT is enlarged by including the lightest multiplets of spin-one resonances such that the relevant Green functions are well-behaved at high energies*.
- ✓ We follow a similar notation to $P\to\mu\nu_\mu[\gamma]$ (D=d,s for π,K and $F_\pi \approx$ 92.2 MeV):



LO result

short-distance EW correction ≈ 1.0232**

structure independent (SI) contributions (point-like approximation)***

$$\Gamma(\tau\to P\nu_\tau[\gamma]) = \frac{G_F^2|V_{uD}|^2F_P^2}{8\pi}M_\tau^3\left(1-\frac{m_P^2}{M_\tau^2}\right)^2 S_{\rm EW}\left\{1+\frac{\alpha}{\pi}G(m_P^2/M_\tau^2)\right\}\times$$
$$\left\{1-\frac{3\alpha}{2\pi}\log\frac{m_\rho}{M_\tau}+\delta_{\tau P}\big|_{\rm rSD}+\delta_{\tau P}\big|_{\rm vSD}\right\}$$

real-photon structure-dependent (rSD) contributions

virtual-photon structure-dependent (vSD) contributions

  - ✓ Real-photon structure-dependent (rSD) contributions from Guo & Roig'10^.
  - ✓ Virtual-photon structure-dependent (vSD) contributions not calculated in the literature.

* Ecker et al.'89
* Cirigliano et al.'06
** Erler'02

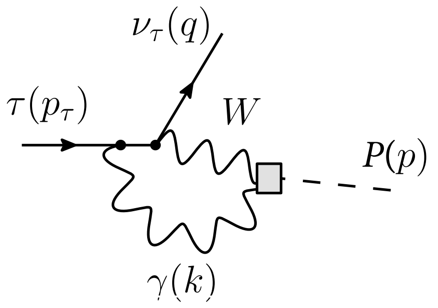
*** Kinoshita'59
^ Guo & Roig'10

3. $\tau \to P\, \nu_\tau\, [\gamma]$ (P=π,K)

✓ Virtual-photon structure-dependent contribution (vSD):

$$i\mathcal{M}[\tau\to P\nu_\tau]|_{\rm SD}=G_F V_{uD} e^2\int\frac{d^dk}{(2\pi)^d}\frac{\ell^{\mu\nu}}{k^2[(p_\tau+k)^2-M_\tau^2]}\left[i\epsilon_{\mu\nu\lambda\rho}k^\lambda p^\rho F_V^P(W^2,k^2)+F_A^P(W^2,k^2)\lambda_{1\mu\nu}+2B(k^2)\lambda_{2\mu\nu}\right]$$

$$\begin{aligned}\ell^{\mu\nu} &= \bar u(q)\gamma^\mu(1-\gamma_5)[(\not p_\tau+\not k)+M_\tau]\gamma^\nu u(p_\tau)\\ \lambda_{1\mu\nu} &= \left[(p+k)^2+k^2-m_P^2\right]g_{\mu\nu}-2k_\mu p_\nu\\ \lambda_{2\mu\nu} &= k^2 g_{\mu\nu}-\frac{k^2(p+k)_\mu p_\nu}{(p+k)^2-m_P^2}\end{aligned}$$



✓ Form factors from Guo & Roig'10 and Guevara et al.'13,'21*:

$$\begin{aligned}F_V^P(W^2,k^2) &= \frac{-N_C M_V^4}{24\pi^2 F_P(k^2-M_V^2)(W^2-M_V^2)}\\ F_A^P(W^2,k^2) &= \frac{F_P}{2}\frac{M_A^2-2M_V^2-k^2}{(M_V^2-k^2)(M_A^2-W^2)}\\ B(k^2) &= \frac{F_P}{M_V^2-k^2}\end{aligned}$$

✓ Well-behaved two- and three-point Green functions.

✓ Chiral and U(3) limits.

✓ M_V and M_A vector- and axial-vector resonance mass: $M_V=M_\rho$ and $M_A=M_{a1}$ (π case); $M_V=M_{K*}$ and $M_A\approx M_{f1}$ (K case).

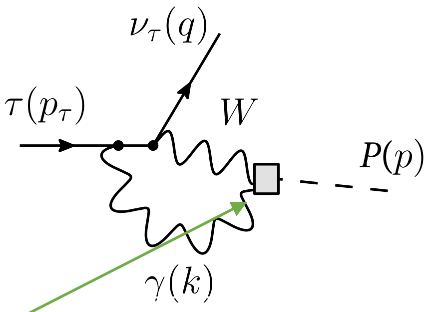
* Guo & Roig'10
* Guevara et al.'13,'21

3. $\tau \to P\, \nu_\tau\, [\gamma]$ (P=π,K)

✓ Virtual-photon structure-dependent contribution (vSD):

$$i\mathcal{M}[\tau\to P\nu_\tau]|_{\rm SD} = G_F V_{uD} e^2 \int \frac{d^dk}{(2\pi)^d} \frac{\ell^{\mu\nu}}{k^2[(p_\tau+k)^2-M_\tau^2]} \left[i\epsilon_{\mu\nu\lambda\rho}k^\lambda p^\rho F_V^P(W^2,k^2) + F_A^P(W^2,k^2)\lambda_{1\mu\nu} + 2B(k^2)\lambda_{2\mu\nu}\right]$$

$$\begin{aligned}
\ell^{\mu\nu} &= \bar{u}(q)\gamma^\mu(1-\gamma_5)[(\not{p}_\tau + \not{k}) + M_\tau]\gamma^\nu u(p_\tau) \\
\lambda_{1\mu\nu} &= \left[(p+k)^2 + k^2 - m_P^2\right] g_{\mu\nu} - 2k_\mu p_\nu \\
\lambda_{2\mu\nu} &= k^2 g_{\mu\nu} - \frac{k^2(p+k)_\mu p_\nu}{(p+k)^2 - m_P^2}
\end{aligned}$$



✓ Form factors from Guo & Roig'10 and Guevara et al.'13,'21*:

$$\begin{aligned}
F_V^P(W^2,k^2) &= \frac{-N_C M_V^4}{24\pi^2 F_P (k^2 - M_V^2)(W^2 - M_V^2)} \\
F_A^P(W^2,k^2) &= \frac{F_P}{2}\, \frac{M_A^2 - 2M_V^2 - k^2}{(M_V^2 - k^2)(M_A^2 - W^2)} \\
B(k^2) &= \frac{F_P}{M_V^2 - k^2}
\end{aligned}$$

✓ Well-behaved two- and three-point Green functions.

✓ Chiral and U(3) limits.

✓ M_V and M_A vector- and axial-vector resonance mass: $M_V=M_\rho$ and $M_A=M_{a1}$ (π case); $M_V=M_{K^*}$ and $M_A \approx M_{f1}$ (K case).

* Guo & Roig'10
* Guevara et al.'13,'21

4. Calculation of $R_{\tau/P} = R_{\tau/P}^{(0)} (1 + \delta R_{\tau/P}) = R_{\tau/P}^{(0)} (1 + \delta_{\tau P} - \delta_{P\mu})$

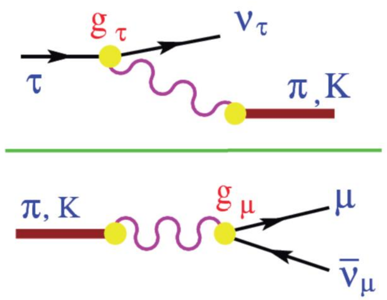
1. Structure-independent contribution (point-like approximation): SI.

✓ We confirm the results by DF'95*.

$$\delta R_{\tau/P}\big|_{\rm SI} = \frac{\alpha}{2\pi}\left\{\frac{3}{2}\log\frac{M_\tau^2 m_P^2}{m_\mu^4} + \frac{3}{2} + g\left(\frac{m_P^2}{M_\tau^2}\right) - f\left(\frac{m_\mu^2}{m_P^2}\right)\right\}$$

$$f(x) = 2\left(\frac{1+x}{1-x}\log x - 2\right)\log(1-x) - \frac{x(8-5x)}{2(1-x)^2}\log x + 4\frac{1+x}{1-x}{\rm Li}_2(x) - \frac{x}{1-x}\left(\frac{3}{2} + \frac{4}{3}\pi^2\right)$$

$$g(x) = 2\left(\frac{1+x}{1-x}\log x - 2\right)\log(1-x) - \frac{x(2-5x)}{2(1-x)^2}\log x + 4\frac{1+x}{1-x}{\rm Li}_2(x) + \frac{x}{1-x}\left(\frac{3}{2} - \frac{4}{3}\pi^2\right)$$



$\delta R_{\tau/\pi}|_{SI} = 1.05\%$ and $\delta R_{\tau/K}|_{SI} = 1.67\%$

Real-photon structure-dependent contribution: rSD.

✓ $\delta_{P\mu}|_{rSD}$ from Cirigliano & IR'07**: $\delta_{\pi\mu}|_{rSD} = -1.3\cdot10^{-8}$ and $\delta_{K\mu}|_{rSD} = -1.7\cdot10^{-5}$.

✓ $\delta_{\tau P}|_{rSD}$ from Guo & Roig'10***: $\delta_{\tau\pi}|_{rSD} = 0.15\%$ and $\delta_{\tau K}|_{rSD} = (0.18 \pm 0.05)\%$.

$\delta R_{\tau/\pi}|_{rSD} = 0.15\%$ and $\delta R_{\tau/K}|_{rSD} = (0.18 \pm 0.15)\%$

* Decker & Finkemeier'95
** Cirigliano & Rosell'07

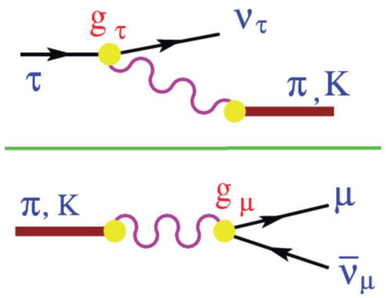
*** Guo & Roig'10

4. Calculation of $R_{\tau/P} = R_{\tau/P}^{(0)} (1 + \delta R_{\tau/P}) = R_{\tau/P}^{(0)} (1 + \delta_{\tau P} - \delta_{P\mu})$

3. Virtual-photon structure-dependent contribution: vSD.

- ✓ $\delta_{P\mu}|_{vSD}$ from Cirigliano & IR'07*: $\delta_{\pi\mu}|_{vSD} = (0.54 \pm 0.12)\%$ and $\delta_{K\mu}|_{vSD} = (0.43 \pm 0.12)\%$.
- ✓ $\delta_{\tau P}|_{vSD}$, **new calculation**: $\delta_{\tau\pi}|_{vSD} = (-0.48 \pm 0.56)\%$ and $\delta_{\tau K}|_{vSD} = (-0.45 \pm 0.57)\%$.

$$\delta R_{\tau/\pi}|_{vSD} = (-1.02 \pm 0.57)\% \text{ and } \delta R_{\tau/K}|_{vSD} = (-0.88 \pm 0.58)\%$$



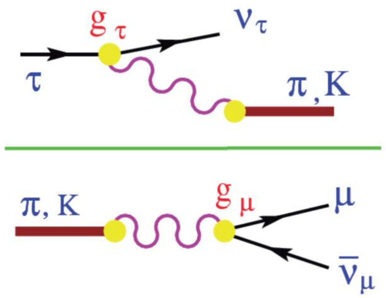
* Cirigliano & IR'07

4. Calculation of $R_{\tau/P} = R_{\tau/P}^{(0)} (1 + \delta R_{\tau/P}) = R_{\tau/P}^{(0)} (1 + \delta_{\tau P} - \delta_{P\mu})$

3. Virtual-photon structure-dependent contribution: vSD.

- ✓ $\delta_{P\mu}|_{vSD}$ from Cirigliano & IR'07*: $\delta_{\pi\mu}|_{vSD} = (0.54 \pm 0.12)\%$ and $\delta_{K\mu}|_{vSD} = (0.43 \pm 0.12)\%$.
- ✓ $\delta_{\tau P}|_{vSD}$, **new calculation**: $\delta_{\tau\pi}|_{vSD} = (-0.48 \pm 0.56)\%$ and $\delta_{\tau K}|_{vSD} = (-0.45 \pm 0.57)\%$.

$$\delta R_{\tau/\pi}|_{vSD} = (-1.02 \pm 0.57)\% \text{ and } \delta R_{\tau/K}|_{vSD} = (-0.88 \pm 0.58)\%$$



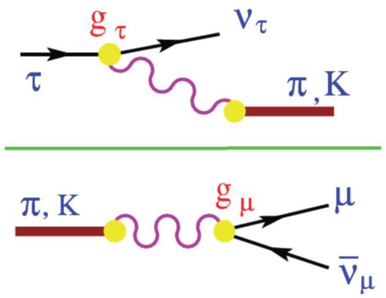
- ✓ Uncertainties dominated by $\delta_{\tau P}|_{vSD}$:
  - ✓ P decays within ChPT [counterterms can be determined by matching ChPT with the resonance effective approach at higher energies], whereas τ decays within resonance effective approach [no matching to determine the counterterms].
  - ✓ Estimation of the model-dependence by comparing our results with a less general scenario where only well-behaved two-point Green functions and a reduced resonance Lagrangian is used: ±0.22% and ±0.24% for the pion and the kaon case.
  - ✓ Estimation of the counterterms by considering the running between 0.5 and 1.0 GeV: ±0.52% (similar procedure in Marciano & Sirlin'93). **Conservative estimate**, since vSD counterterms affecting in P decays imply similar corrections to our estimation of the vSD counterterms in τ decays.

* Cirigliano & Rosell'07

5. Results

Contribution	$\delta R_{\tau/\pi}$	$\delta R_{\tau/K}$	Ref.
SI	$+1.05\%$	$+1.67\%$	*
rSD	$+0.15\%$	$+(0.18 \pm 0.05)\%$	**
vSD	$-(1.02 \pm 0.57)\%$	$-(0.88 \pm 0.58)\%$	new
Total	$+(0.18 \pm 0.57)\%$	$+(0.97 \pm 0.58)\%$	new

Errors are not reported if they are lower than 0.01%.

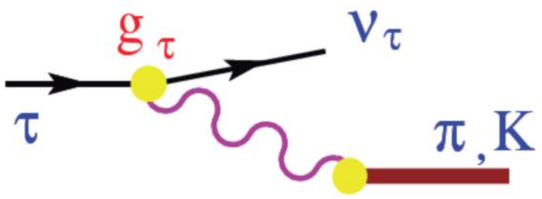


Central values agree remarkably with DF'95, merely a coincidence: $\delta R_{\tau/\pi} = (0.16 \pm 0.14)\%$ and $\delta R_{\tau/K} = (0.90 \pm 0.22)\%$, **but** in that work:

- ✓ problematic hadronization: form factors are different for real- and virtual-photon corrections, do not satisfy the correct QCD short-distance behavior, violate unitarity, analicity and the chiral limit at leading non-trivial orders.
- ✓ a cutoff to regulate the loop integrals, splitting unphysically long- and short-distance regimes.
- ✓ unrealistic uncertainties (purely $O(e^2p^2)$ ChPT size).

* Decker & Finkemeier'95
** Cirigliano & Rosell'07
** Guo & Roig'10

6. Application I: Radiative corrections in $\Gamma(\tau \to P\nu_\tau[\gamma])$



short-distance EW correction ≈ 1.0201*

$$\Gamma(\tau \to P\nu_\tau[\gamma]) = \frac{G_F^2|V_{uD}|^2F_P^2}{8\pi}M_\tau^3\left(1-\frac{m_P^2}{M_\tau^2}\right)^2 S_{\mathrm{EW}}(1+\delta_{\tau P})$$

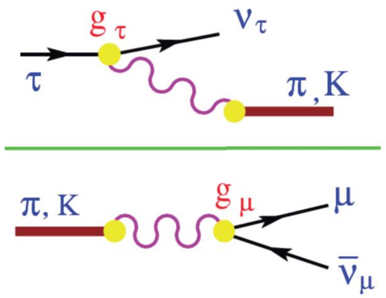
✓ $\delta_{\tau P}$ includes SI and SD radiative corrections.

$$\delta_{\tau P} = \frac{\alpha}{2\pi}\left(g\left(\frac{m_P^2}{M_\tau^2}\right)+\frac{19}{4}-\frac{2\pi^2}{3}-3\log\frac{m_\rho}{M_\tau}\right)+\delta_{\tau P}\big|_{\mathrm{rSD}}+\delta_{\tau P}\big|_{\mathrm{vSD}} = \begin{cases}\delta_{\tau\pi} = (-0.24\pm 0.56)\% \\ \delta_{\tau K} = (-0.15\pm 0.57)\%\end{cases}$$

* Erler'02

6. Application II: lepton universality test

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 \frac{1}{2}\frac{M_\tau^3}{m_\mu^2 m_P}\frac{(1-m_P^2/M_\tau^2)^2}{(1-m_\mu^2/m_P^2)^2}\left(1+\delta R_{\tau/P}\right)$$



$$\left|\frac{g_\tau}{g_\mu}\right|_\pi = 0.9964 \pm 0.0028_{\text{th}} \pm 0.0025_{\text{exp}} = 0.9964 \pm 0.0038$$

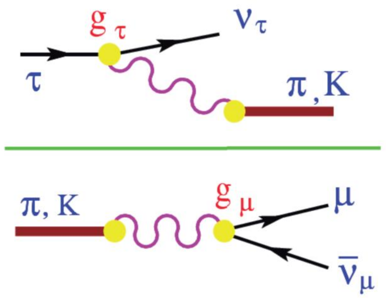
$$\left|\frac{g_\tau}{g_\mu}\right|_K = 0.9857 \pm 0.0028_{\text{th}} \pm 0.0072_{\text{exp}} = 0.9857 \pm 0.0078$$

6. Application II: lepton universality test

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 \frac{1}{2}\frac{M_\tau^3}{m_\mu^2 m_P}\frac{(1-m_P^2/M_\tau^2)^2}{(1-m_\mu^2/m_P^2)^2}\left(1+\delta R_{\tau/P}\right)$$

PDG

$\delta R_{\tau/\pi}$ = (0.18 ± 0.57)%
$\delta R_{\tau/K}$ = (0.97 ± 0.58)%



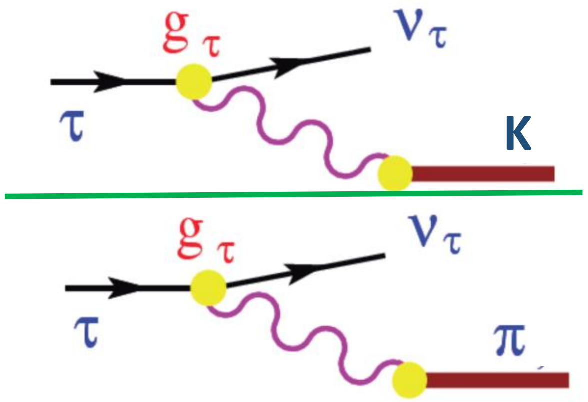
$$\left|\frac{g_\tau}{g_\mu}\right|_\pi = 0.9964 \pm 0.0028_{\rm th} \pm 0.0025_{\rm exp} = 0.9964 \pm 0.0038$$

$$\left|\frac{g_\tau}{g_\mu}\right|_K = 0.9857 \pm 0.0028_{\rm th} \pm 0.0072_{\rm exp} = 0.9857 \pm 0.0078$$

- ✓ π case: at 0.9σ of LU vs. 1.6σ of LU in HFLAV'21* using DF'95**
- ✓ K case: at 1.8σ of LU vs. 1.9σ of LU in HFLAV'21* using DF'95**

* HFLAV'21
** Decker & Finkemeier'95

6. Application III: CKM unitarity test in the ratio $\Gamma(\tau \to K\nu_\tau[\gamma]) / \Gamma(\tau \to \pi\nu_\tau[\gamma])$



$$\frac{\Gamma(\tau \to K\nu_\tau[\gamma])}{\Gamma(\tau \to \pi\nu_\tau[\gamma])} = \frac{|V_{us}|^2}{|V_{ud}|^2}\frac{F_K^2}{F_\pi^2}\frac{(1-m_K^2/M_\tau^2)^2}{(1-m_\pi^2/M_\tau^2)^2}(1+\delta)$$

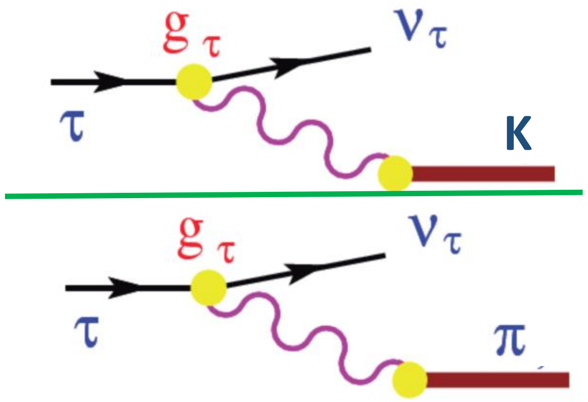
$$\left|\frac{V_{us}}{V_{ud}}\right| = 0.2288 \pm 0.0010_{\rm th} \pm 0.0017_{\rm exp} = 0.2288 \pm 0.0020$$

6. Application III: CKM unitarity test in the ratio $\Gamma(\tau \to K\nu_\tau[\gamma]) / \Gamma(\tau \to \pi\nu_\tau[\gamma])$

FLAG'20*:
$F_K/F_\pi = 1.1932 \pm 0.0019$

PDG

$$\delta = \delta_{\tau K} - \delta_{\tau\pi} = +(0.10 \pm 0.80)\%$$



$$\frac{\Gamma(\tau \to K\nu_\tau[\gamma])}{\Gamma(\tau \to \pi\nu_\tau[\gamma])} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{F_K^2}{F_\pi^2} \frac{(1-m_K^2/M_\tau^2)^2}{(1-m_\pi^2/M_\tau^2)^2} (1+\delta)$$

$$\left|\frac{V_{us}}{V_{ud}}\right| = 0.2288 \pm 0.0010_{\rm th} \pm 0.0017_{\rm exp} = 0.2288 \pm 0.0020$$

- ✓ 2.1σ away from CKM unitarity, considering $|V_{ud}|=0.97373\pm0.00031$**.
- ✓ To be compared with $|V_{us}/V_{ud}|=0.2291\pm0.0009$***, obtained with kaon semileptonic decays. Our error does not reach this level due to lack of statistics in τ decays.

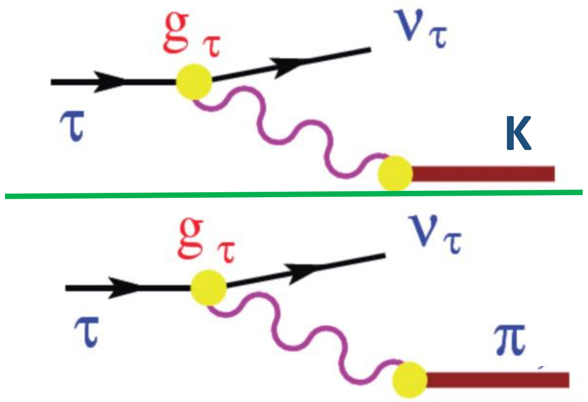
* FLAG'20
** Hardy & Towner'20
*** Seng et al.'21

6. Application III: CKM unitarity test in the ratio $\Gamma(\tau \to K\nu_\tau[\gamma]) / \Gamma(\tau \to \pi\nu_\tau[\gamma])$

FLAG'20*:
$F_K/F_\pi = 1.1932 \pm 0.0019$

$$\delta = \delta_{\tau K} - \delta_{\tau\pi} = +(0.10 \pm 0.80)\%$$

PDG



$$\frac{\Gamma(\tau \to K\nu_\tau[\gamma])}{\Gamma(\tau \to \pi\nu_\tau[\gamma])} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{F_K^2}{F_\pi^2} \frac{(1-m_K^2/M_\tau^2)^2}{(1-m_\pi^2/M_\tau^2)^2} (1+\delta)$$

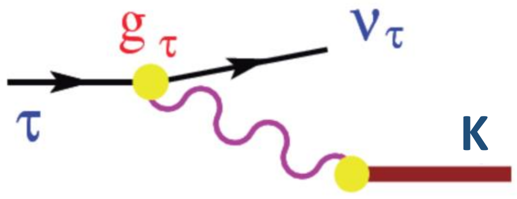
Conservative estimation of the errors in δ, since we have directly propagated the uncertainties of $\delta_{\tau K}$ and $\delta_{\tau\pi}$.

$$\left|\frac{V_{us}}{V_{ud}}\right| = 0.2288 \pm 0.0010_{\rm th} \pm 0.0017_{\rm exp} = 0.2288 \pm 0.0020$$

- ✓ 2.1σ away from CKM unitarity, considering $|V_{ud}|=0.97373\pm0.00031$**.
- ✓ To be compared with $|V_{us}/V_{ud}|=0.2291\pm0.0009$***, obtained with kaon semileptonic decays. Our error does not reach this level due to lack of statistics in τ decays.

* FLAG'20
** Hardy & Towner'20
*** Seng et al.'21

6. Application IV: CKM unitarity test in $\Gamma(\tau \to K\nu_\tau[\gamma])$



$$\Gamma(\tau \to K\nu_\tau[\gamma]) = \frac{G_F^2 F_K^2}{8\pi}|V_{us}|^2 M_\tau^3 \left(1 - \frac{m_K^2}{M_\tau^2}\right)^2 S_{\mathrm{EW}} \; (1 + \delta_{\tau K})$$

$$|V_{us}| = 0.2220 \pm 0.0008_{\mathrm{th}} \pm 0.0016_{\mathrm{exp}} = 0.2220 \pm 0.0018$$

6. Application IV: CKM unitarity test in $\Gamma(\tau \to K\nu_\tau[\gamma])$

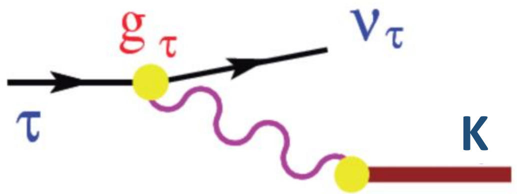
FLAG'20*: $\sqrt{2}F_K = (155.7 \pm 0.3)$ MeV

short-distance EW correction ≈ 1.0201**

$\delta_{\tau K} = (-0.15 \pm 0.57)\%$

PDG

$$\Gamma(\tau \to K\nu_\tau[\gamma]) = \frac{G_F^2 F_K^2}{8\pi}|V_{us}|^2 M_\tau^3 \left(1 - \frac{m_K^2}{M_\tau^2}\right)^2 S_{\rm EW}(1 + \delta_{\tau K})$$

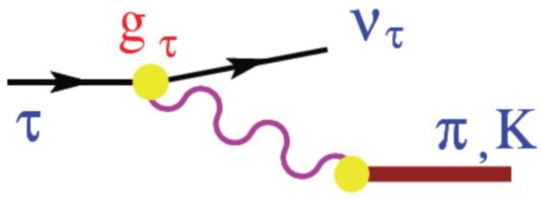


$$|V_{us}| = 0.2220 \pm 0.0008_{\rm th} \pm 0.0016_{\rm exp} = 0.2220 \pm 0.0018$$

- ✓ 2.6σ away from CKM unitarity, considering $|V_{ud}|$=0.97373±0.00031***.
- ✓ To be compared with $|V_{us}|$=0.2234±0.0015^ or $|V_{us}|$=0.2231±0.0006†, obtained this last one with kaon semileptonic decays. Our error does not reach this level due to lack of statistics in τ decays.

* FLAG'20
** Erler'02
*** Hardy & Towner'20
^ HFLAV'21
† Seng et al.'21

6. Application V: constraining non-standard interactions in $\Gamma(\tau \to P\nu_\tau[\gamma])$



$$\Gamma(\tau \to P\nu_\tau[\gamma]) = \frac{G_F^2|\widetilde{V}_{uD}|^2 F_P^2}{8\pi} M_\tau^3 \left(1 - \frac{m_P^2}{M_\tau^2}\right)^2 S_{\rm EW}\ \left(1 + \delta_{\tau P} + 2\Delta^{\tau P}\right)$$

Values of $\Delta^{\tau P}$ reported in the $\overline{\text{MS}}$-scheme and at a scale of μ=2 GeV.

$$\Delta^{\tau P} = \epsilon_L^\tau - \epsilon_L^e - \epsilon_R^\tau - \epsilon_R^e - \frac{m_P^2}{M_\tau(m_u + m_D)}\epsilon_P^\tau = \begin{cases} \Delta^{\tau\pi} = -(0.15 \pm 0.72)\cdot 10^{-2} \\ \Delta^{\tau K} = -(0.36 \pm 1.18)\cdot 10^{-2} \end{cases}$$

6. Application V: constraining non-standard interactions in $\Gamma(\tau \to P\nu_\tau[\gamma])$

$|V_{ud}|$ = 0.97373 ± 0.00031*
$|V_{us}/V_{ud}|$ = 0.2288 ± 0.0020

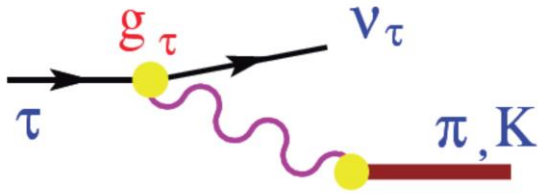
FLAG'20*:
$\sqrt{2}F_\pi$ = (130.2 ± 0.8) MeV
$\sqrt{2}F_K$ = (155.7 ± 0.3) MeV

short-distance EW correction ≈ 1.0201**

$$\delta_{\tau\pi} = (-0.24 \pm 0.56)\%$$
$$\delta_{\tau K} = (-0.15 \pm 0.57)\%$$

PDG

$$\Gamma(\tau \to P\nu_\tau[\gamma]) = \frac{G_F^2 |\widetilde{V}_{uD}|^2 F_P^2}{8\pi} M_\tau^3 \left(1 - \frac{m_P^2}{M_\tau^2}\right)^2 S_{\rm EW} \left(1 + \delta_{\tau P} + 2\Delta^{\tau P}\right)$$



Values of $\Delta^{\tau P}$ reported in the MS-scheme and at a scale of μ=2 GeV.

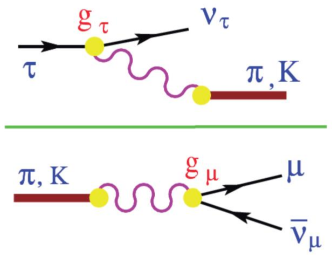
$$\Delta^{\tau P} = \epsilon_L^\tau - \epsilon_L^e - \epsilon_R^\tau - \epsilon_R^e - \frac{m_P^2}{M_\tau(m_u + m_D)}\epsilon_P^\tau = \begin{cases} \Delta^{\tau\pi} = -(0.15 \pm 0.72) \cdot 10^{-2} \\ \Delta^{\tau K} = -(0.36 \pm 1.18) \cdot 10^{-2} \end{cases}$$

- ✓ To be compared with $\Delta^{\tau\pi} = -(0.15 \pm 0.67)\cdot 10^{-2}$ of Cirigliano et al.'19^.
- ✓ To be compared with $\Delta^{\tau\pi} = -(0.12 \pm 0.68)\cdot 10^{-2}$ and $\Delta^{\tau K} = (-0.41 \pm 0.93)\cdot 10^{-2}$ of González-Solís et al.'20†.

* Hardy & Towner'20
** FLAG'20
*** Erler'02

^ Cirigliano et al.'19
† Gonzàlez-Solís et al. '20

7. Conclusions



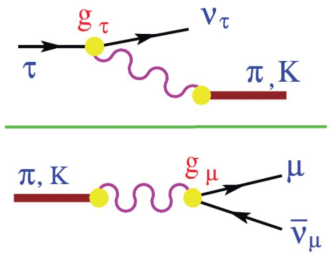
- ✓ The observable and **our result**:

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 R^{(0)}_{\tau/P}\left(1+\delta R_{\tau/P}\right) \quad \longrightarrow \quad \begin{cases} \delta R_{\tau/\pi} = (0.18 \pm 0.57)\% \\ \delta R_{\tau/K} = (0.97 \pm 0.58)\% \end{cases}$$

- ✓ Framework: ChPT for π decays and a resonance extension of ChPT for τ decays.
- ✓ Consistent with DF'95*, but with more robust assumptions and yielding a reliable uncertainty.
- ✓ Applications:
  - ✓ Theoretical determination of radiative corrections in $\Gamma(\tau \to P\nu_\tau[\gamma])$.
  - ✓ $|g_\tau/g_\mu|_P$ at 0.9σ (π) and 1.8σ (K) of LU, reducing HFLAV'21** disagreement with LU.
  - ✓ CKM unitarity in $\Gamma(\tau\to K\nu_\tau[\gamma])/\Gamma(\tau\to\pi\nu_\tau[\gamma])$: $|V_{us}/V_{ud}| = 0.2288 \pm 0.0020$, at 2.1σ from unitarity.
  - ✓ CKM unitarity in $\Gamma(\tau\to K\nu_\tau[\gamma])$: $|V_{us}| = 0.2220 \pm 0.0018$, at 2.6σ from unitarity.
  - ✓ Constraining non-standard interactions in $\Gamma(\tau \to P\nu_\tau[\gamma])$: update of $\Delta^{\tau P}$.
- ✓ Our results have been incorporated in the very recent HFLAV'22.

* Decker & Finkemeier'95
** HFLAV'21

Reliable NP tests for present & future exps.



- ✓ The observable and **our result**:

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 R^{(0)}_{\tau/P}\left(1+\delta R_{\tau/P}\right) \longrightarrow \begin{cases} \delta R_{\tau/\pi} = (0.18 \pm 0.57)\% \\ \delta R_{\tau/K} = (0.97 \pm 0.58)\% \end{cases}$$

- ✓ Framework: ChPT for π decays and a resonance extension of ChPT for τ decays.
- ✓ Consistent with DF'95*, but with more robust assumptions and yielding a reliable uncertainty.
- ✓ Applications:
  - ✓ Theoretical determination of radiative corrections in $\Gamma(\tau \to P\nu_\tau[\gamma])$.
  - ✓ $|g_\tau/g_\mu|_P$ at 0.9σ (π) and 1.8σ (K) of LU, reducing HFLAV'21** disagreement with LU.
  - ✓ CKM unitarity in $\Gamma(\tau\to K\nu_\tau[\gamma])/\Gamma(\tau\to\pi\nu_\tau[\gamma])$: $|V_{us}/V_{ud}| = 0.2288 \pm 0.0020$, at 2.1σ from unitarity.
  - ✓ CKM unitarity in $\Gamma(\tau\to K\nu_\tau[\gamma])$: $|V_{us}| = 0.2220 \pm 0.0018$, at 2.6σ from unitarity.
  - ✓ Constraining non-standard interactions in $\Gamma(\tau \to P\nu_\tau[\gamma])$: update of $\Delta^{\tau P}$.
- ✓ Our results have been incorporated in the very recent HFLAV'22.

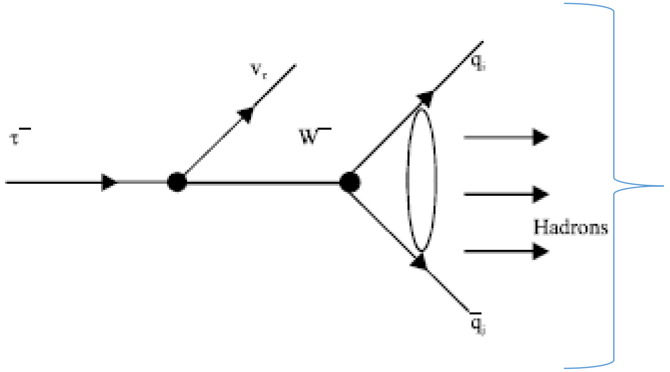
* Decker & Finkemeier'95
** HFLAV'21

Comparison with Decker & Finkemeier'95 (DF'95) in the π case

Contribution	$\delta R_{\tau\pi}$ by DF'95 [μ_{cut} =1.5 GeV]	our $\delta R_{\tau\pi}$
SI	$+0.84\%^*$	$+1.05\%$
rSD	$+0.05\%$	$+0.15\%$
vSD	$-0.49\%^*$	$-(1.02 \pm 0.57)\%$
short-distance	$-0.25\%^*$	0
Total	$+(0.16 \pm 0.14)\%^*$	$+(0.18 \pm 0.57)\%$

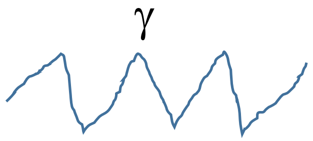
- ✓ Virtual corrections by DF'95 are μ_{cut}-dependent, since long- and short-distance photonic contributions were separated unphysically: figures with an asterisk are cutoff-dependent.
- ✓ The quoted error in the radiative correction of DF'95 arises from uncertainties in hadronic parameters of SD contributions and from variations in the cutoff parameter, μ_{cut}.
- ✓ For the SI contribution in DF'95 we have added to the result obtained in the point-like approximation (1.05%) the term coming from cutting off the loops at μ_{cut} (−0.21%).
- ✓ Different contributions of $\delta R_{\tau/K}$ are not provided in DF'95, which prevents a comparison.
- ✓ Although central values for the sum of all the corrections agree remarkably, this is a coincidence, since central values for the SD corrections are largely different within both approaches.

XVIII MWPF
Ed. Carolino, BUAP, Puebla, Mexico, Nov. 21-25, 2022



$\pi\pi$,
πK,
…

Radiative corrections in **two-meson** tau decays and reliable new physics tests

\+ 

Pablo Roig
Cinvestav (Mexico)

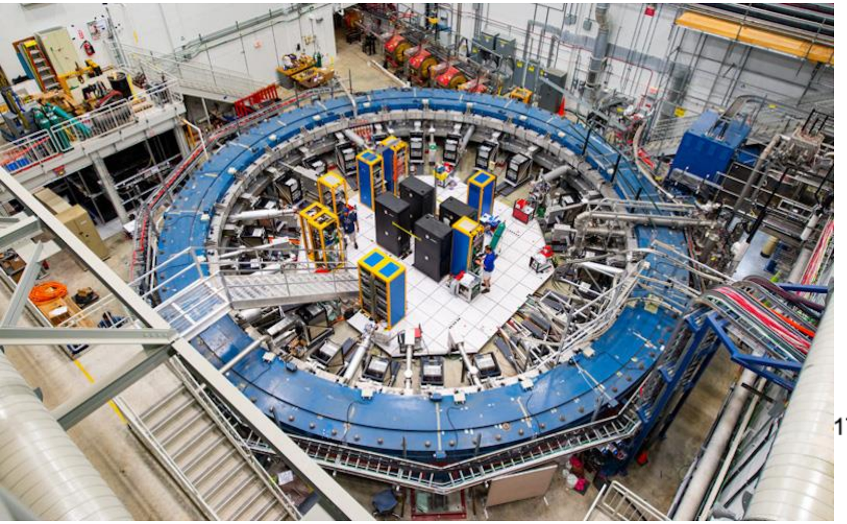
Cinvestav

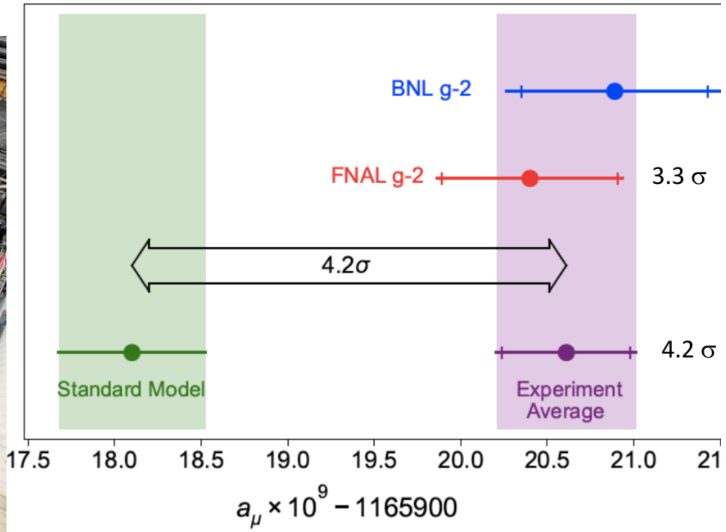
In collaboration with:
J.A. Miranda (Cinvestav, Mexico & IFAE, Barcelona)
and also with R. Escribano (IFAE, Barcelona)

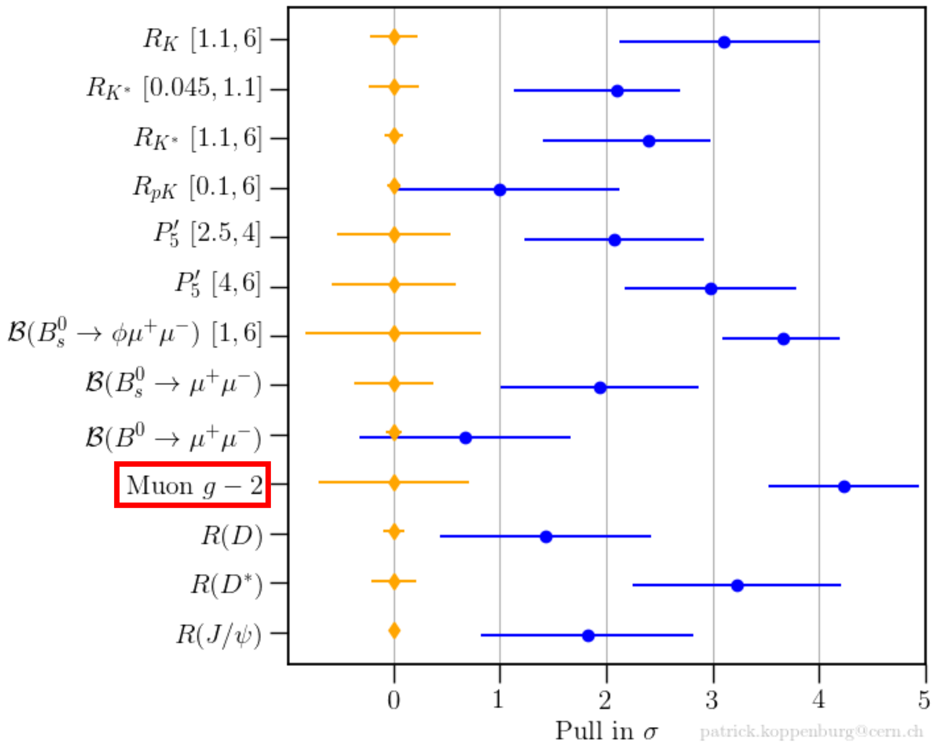
PRD 102 (2020), 114117 [arXiv:2007.11019] → $\pi\pi$
& to appear soon… → All other modes

THE MUON ANOMALOUS MAGNETIC MOMENT & POSSIBLE NEW PHYSICS AFTER THE FNAL 1st MEASUREMENT

Support from Cátedra Marcos Moshinsky is acknowledged



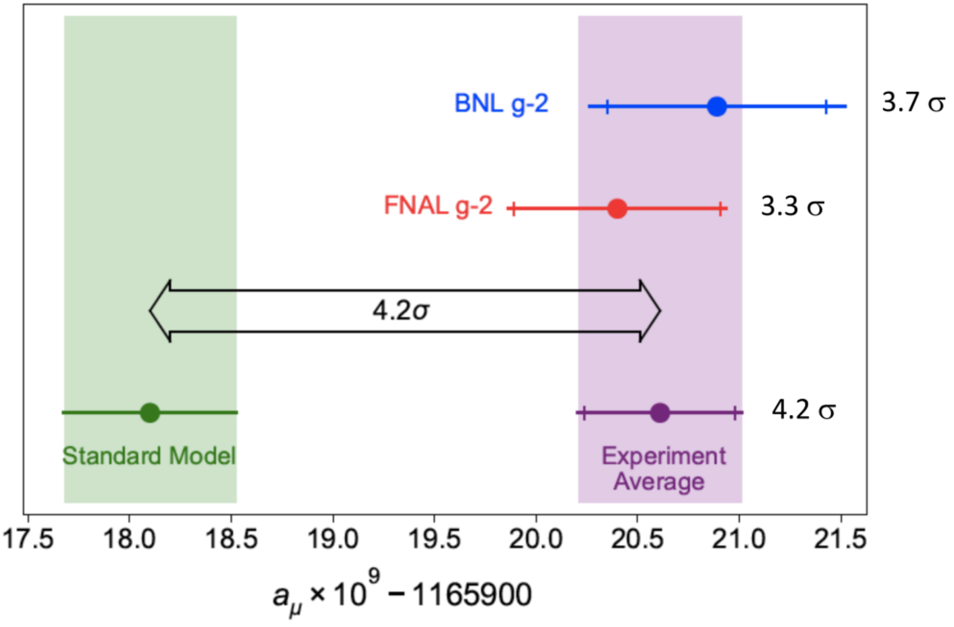


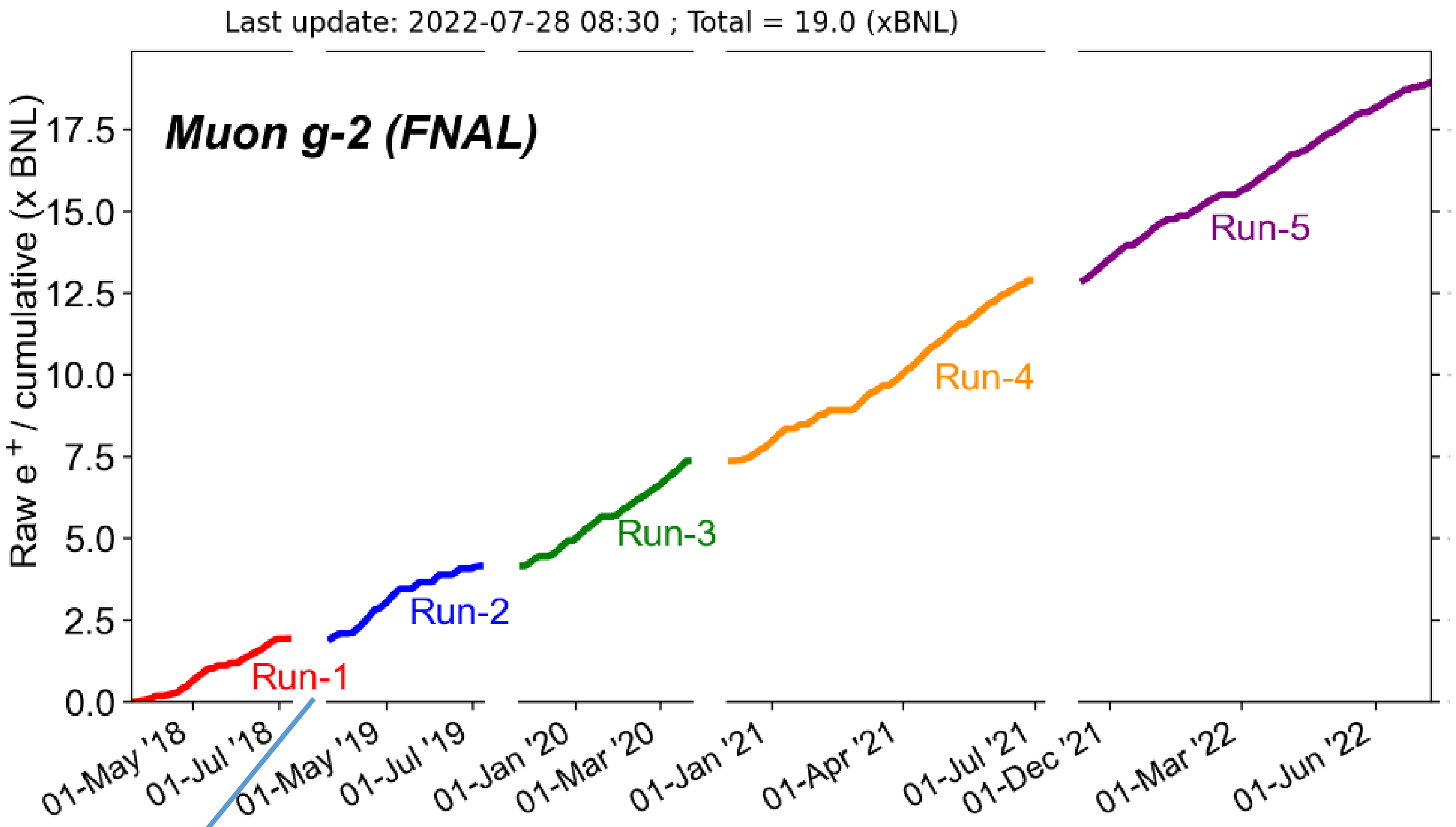


Pablo Roig

Cinvestav (CDMX)

Waiting eagerly for next FNAL measurement in 2023…





Only 6% of final data set!!

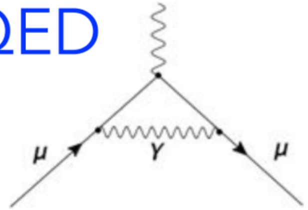
Muon g-2 Theory Initiative

$$a_\mu(\mathrm{SM}) = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{Weak}) + a_\mu(\mathrm{Hadronic})$$

https://doi.org/10.1016/j.physrep.2020.07.006 (SM prediction, '**White Paper**')

https://muon-gm2-theory.illinois.edu

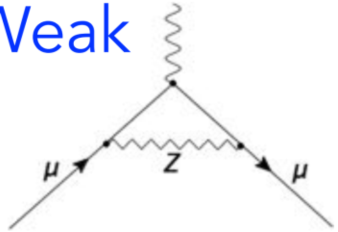
QED



+…

$116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

Weak



+…

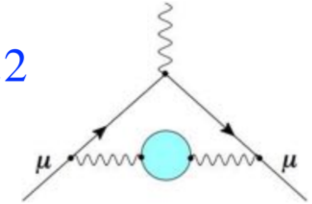
$153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

Hadronic…

Saturate the error

…Vacuum Polarization (HVP)

α^2

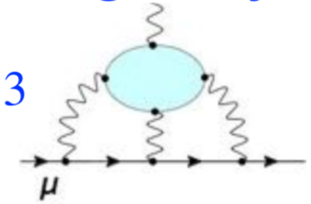


+…

$6845\,(40) \times 10^{-11}$ [0.6%] 0.37 ppm

…Light-by-Light (HLbL)

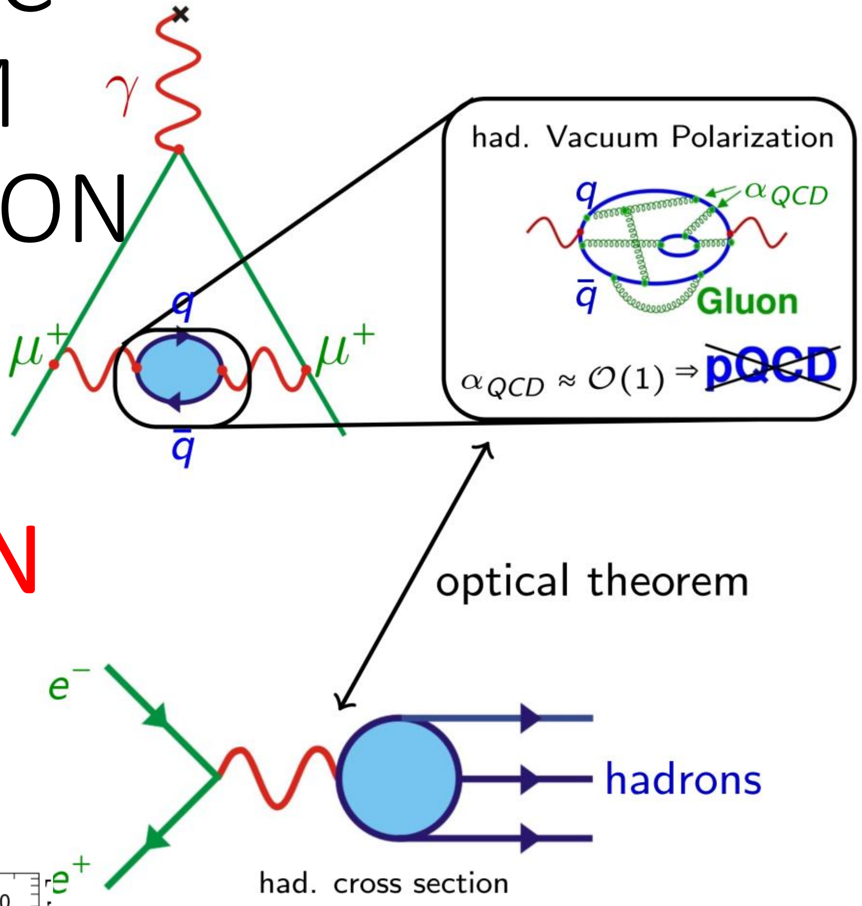
α^3



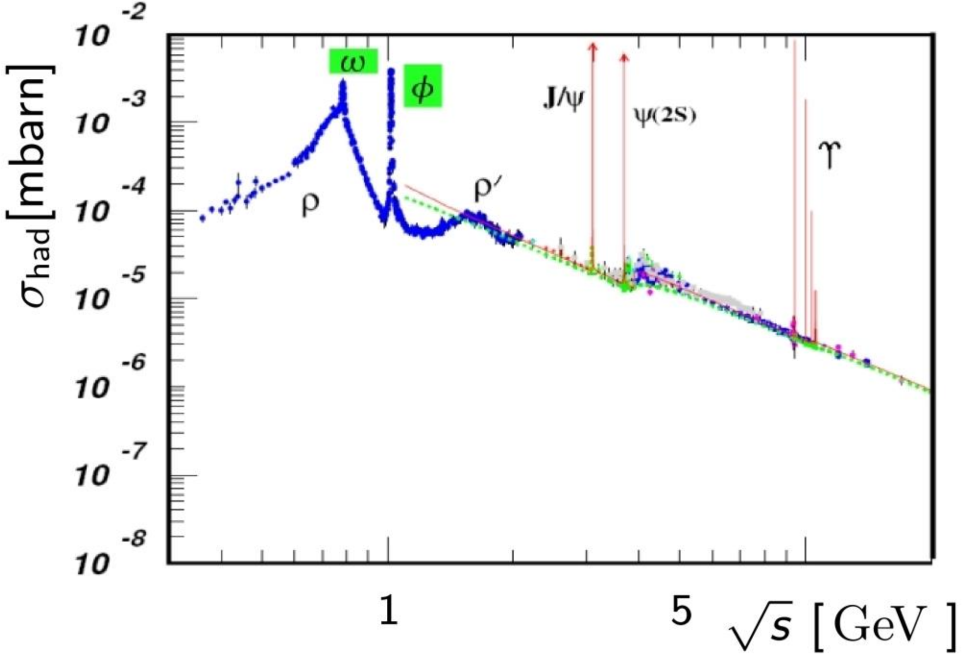
+…

$92\,(18) \times 10^{-11}$ [20%] 0.15 ppm

HADRONIC VACUUM POLARIZATION



DATA-DRIVEN METHOD



$$a_{\mu,LO}^{had} = \frac{1}{4\pi^3} \int_{m_{\pi^0}^2}^{\infty} ds\, K(s)\, \sigma_{had}(s)$$

$\sigma_{had}(s) \sim 1/s$ & $K(s) \sim 1/s$

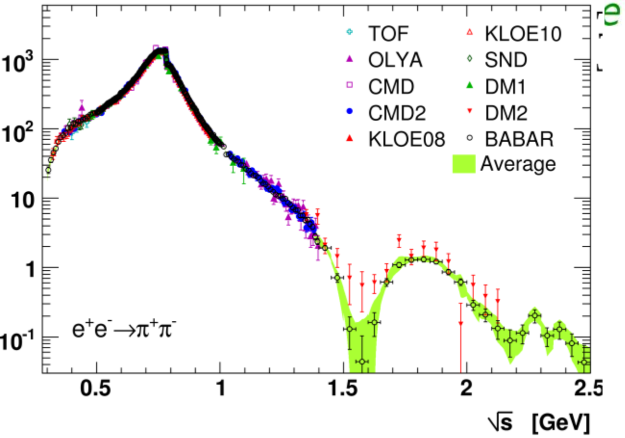
$\Downarrow$

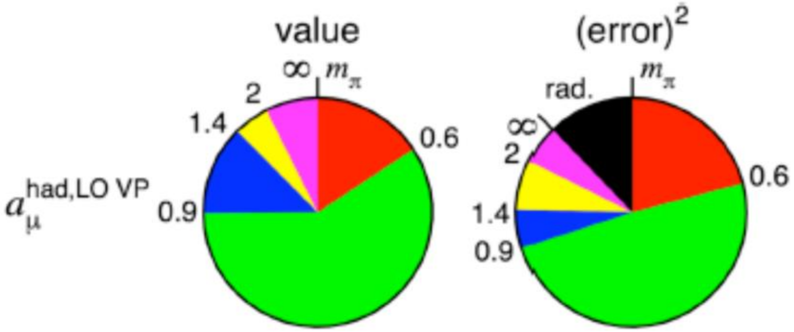
Low energy region important! $\sim 1/s^2$

Sum of exclusive σ_{had}

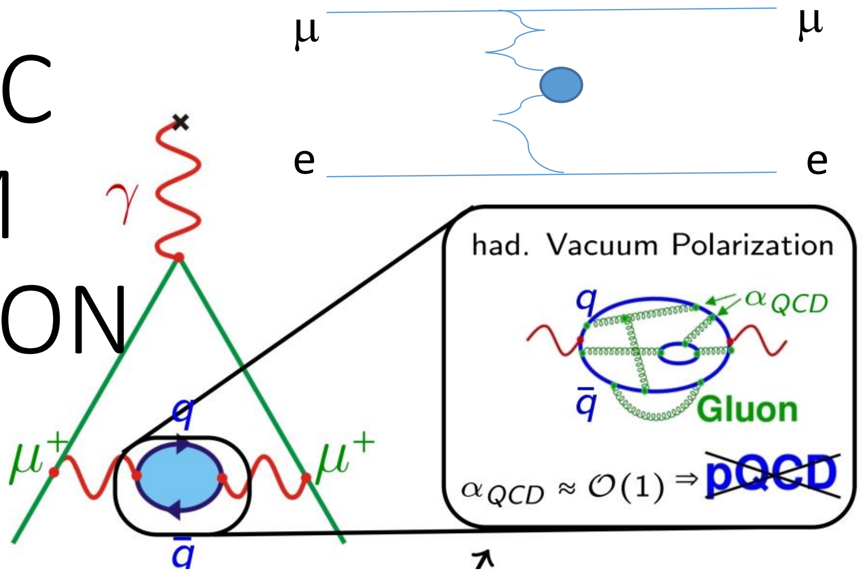
$\Downarrow$

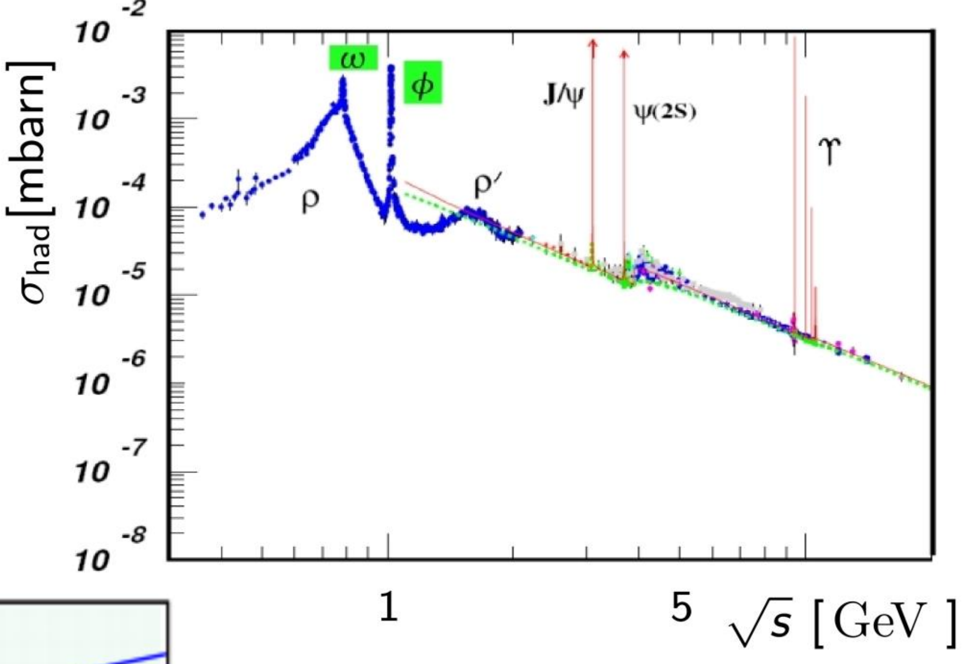
Hadronic contribution of a_μ





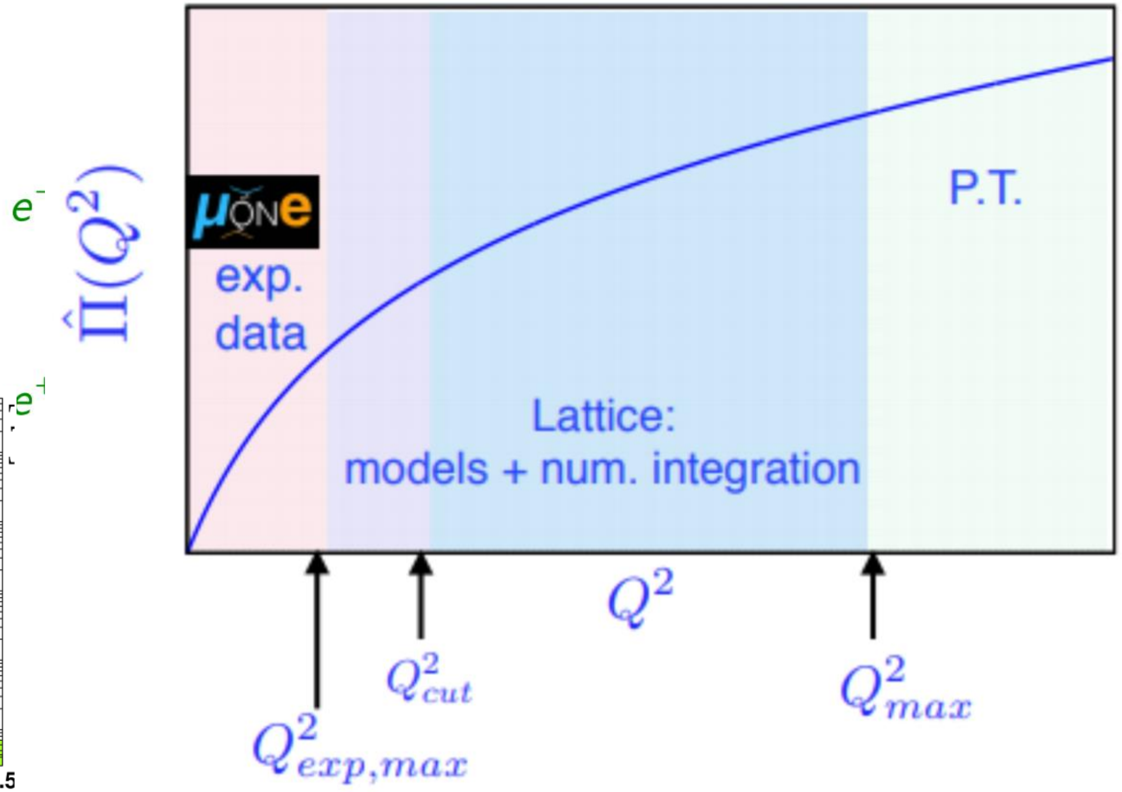
HADRONIC VACUUM POLARIZATION

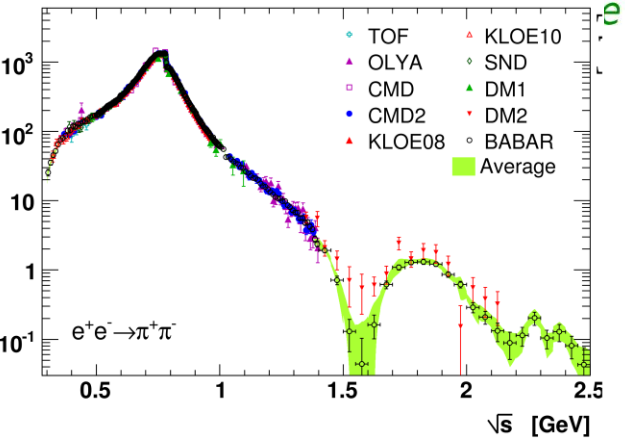




DATA-DRIVEN METHOD

(In future also MUonE)





$$: \frac{1}{4\pi^3} \int_{m^2_{\pi^0}}^{\infty} ds\, K(s)\, \sigma_{had}(s)$$

$_d(s) \sim 1/s$ & $K(s) \sim 1/s$

$\Downarrow$

ıergy region important! $\sim 1/s^2$

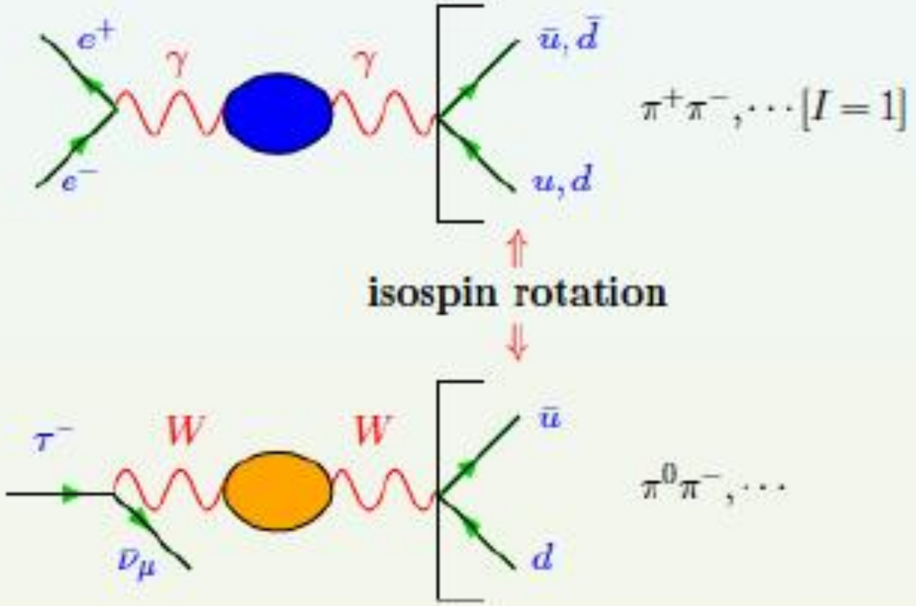
Sum of exclusive σ_{had}

$\Downarrow$

adronic contribution of a_μ

Need precise data:

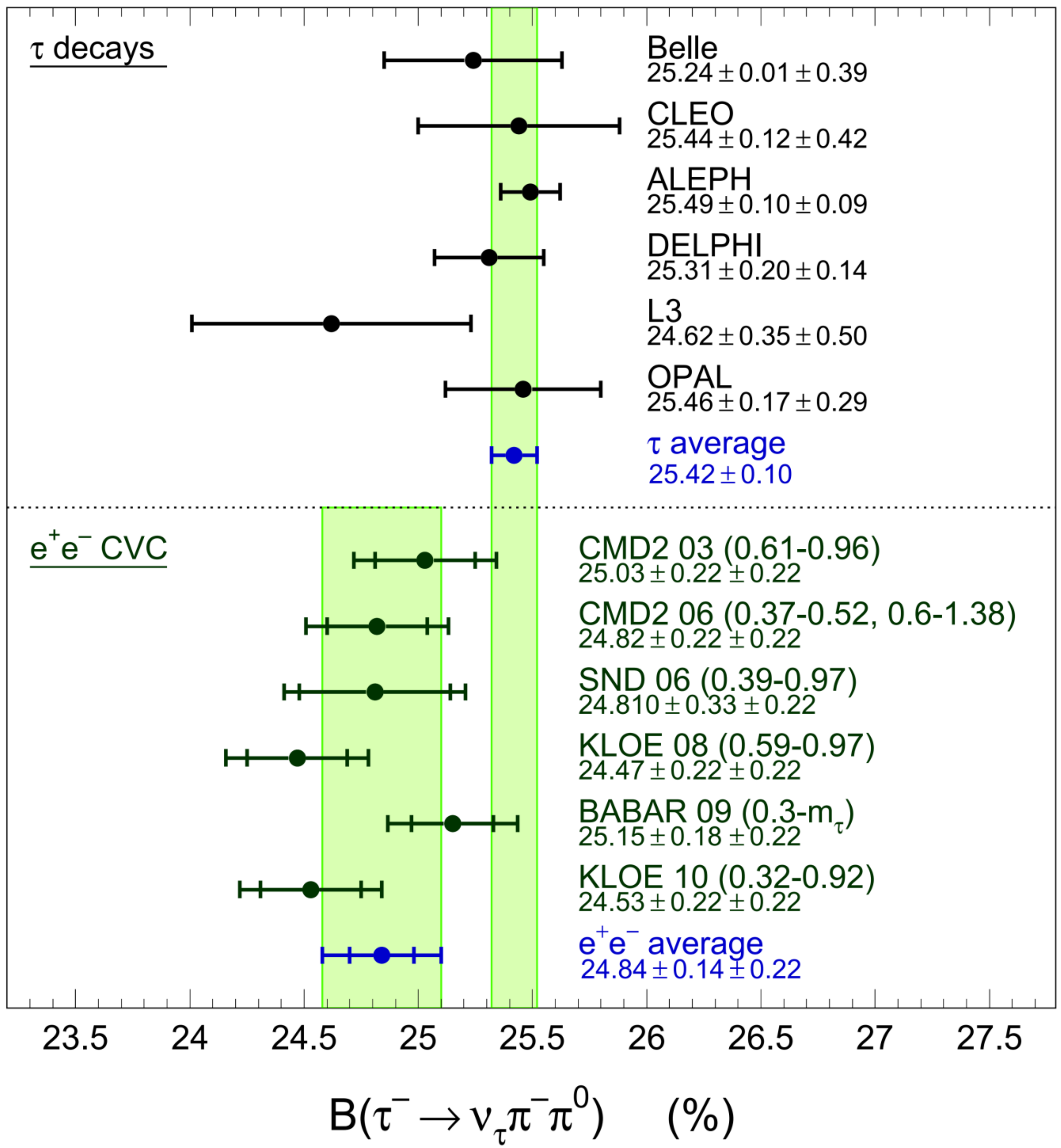
- Good old idea: use isospin symmetry to include existing high quality τ–data (including isospin corrections)

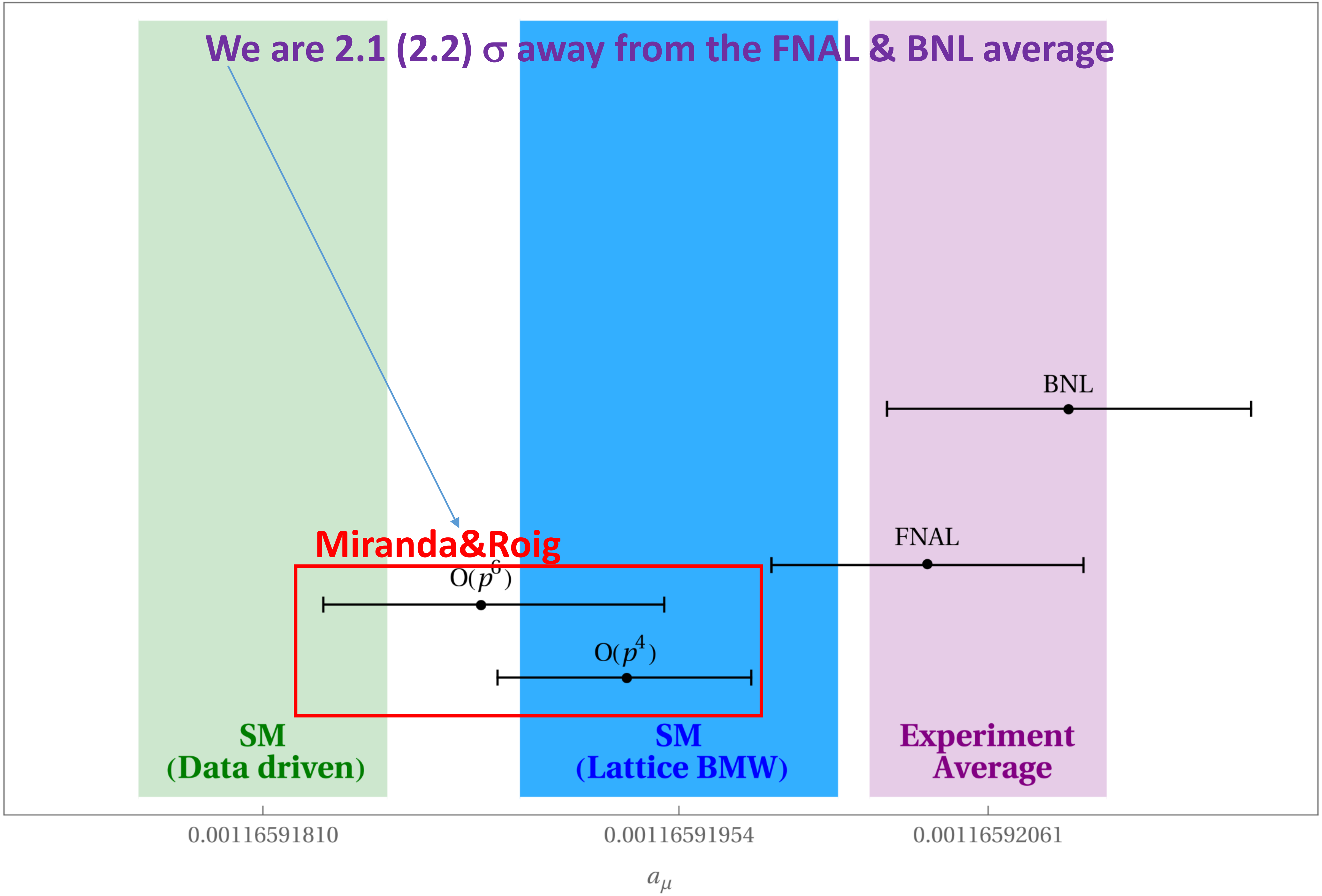


(Francisco Flores-Baez, Alain Flores-Tlalpa, Gabriel López Castro & Genaro Toledo'06&'07)

(Gabriel López Castro, Genaro Toledo & Orsay, CERN & IHEP collaborators'10)

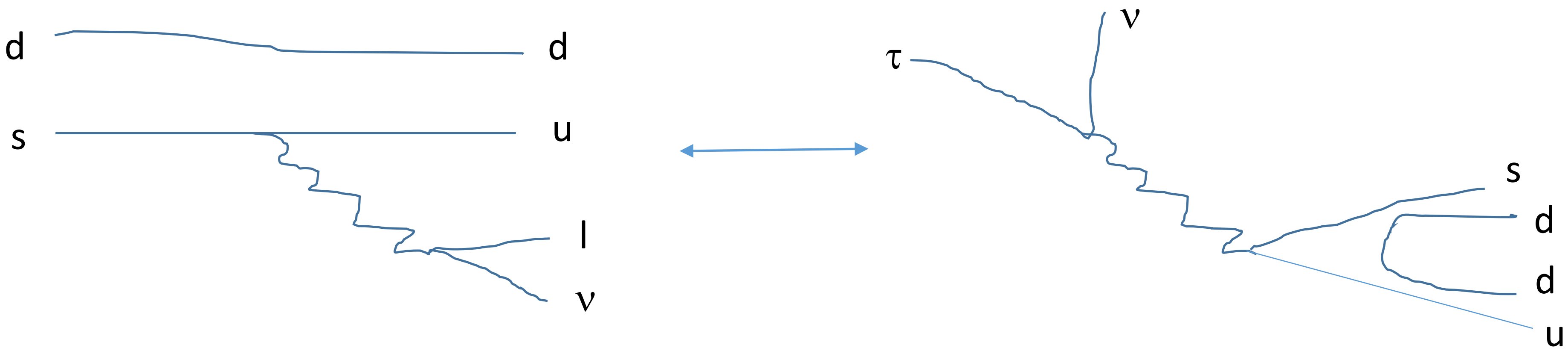
Corrected data: large discrepancy [~ 10%] persists! τ vs. e^+e^- problem! [manifest since 2002]


τ decays
Belle
25.24 ± 0.01 ± 0.39
CLEO
25.44 ± 0.12 ± 0.42
ALEPH
25.49 ± 0.10 ± 0.09
DELPHI
25.31 ± 0.20 ± 0.14
L3
24.62 ± 0.35 ± 0.50
OPAL
25.46 ± 0.17 ± 0.29
τ average
25.42 ± 0.10
e+e− CVC
CMD2 03 (0.61-0.96)
25.03 ± 0.22 ± 0.22
CMD2 06 (0.37-0.52, 0.6-1.38)
24.82 ± 0.22 ± 0.22
SND 06 (0.39-0.97)
24.810 ± 0.33 ± 0.22
KLOE 08 (0.59-0.97)
24.47 ± 0.22 ± 0.22
BABAR 09 (0.3-mτ)
25.15 ± 0.18 ± 0.22
KLOE 10 (0.32-0.92)
24.53 ± 0.22 ± 0.22
e+e− average
24.84 ± 0.14 ± 0.22
23.5
24
24.5
25
25.5
26
26.5
27
27.5
B(τ− → ντπ−π0) (%)


We are 2.1 (2.2) σ away from the FNAL & BNL average
BNL
FNAL
Miranda&Roig
O(p^6)
O(p^4)
SM
(Data driven)
SM
(Lattice BMW)
Experiment
Average
0.00116591810
0.00116591954
0.00116592061
a_μ

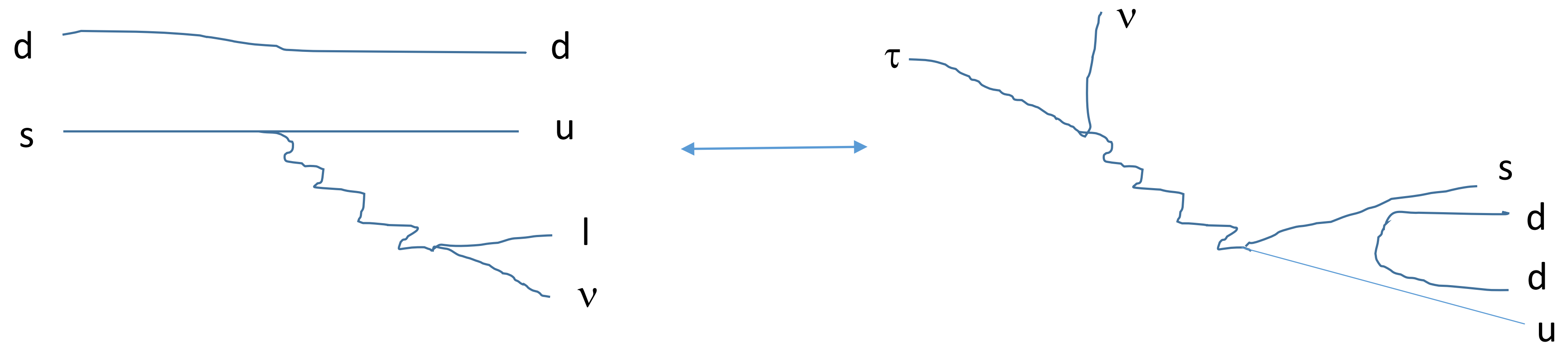
Radiative corrections to other two-meson channels

Antonelli-Cirigliano-Lusiani-Passemar'13 reduced the CKM unitarity violation by using hadron input from Kaon semileptonic decays in strangeness-changing tau decays



Radiative corrections to other two-meson channels

Antonelli-Cirigliano-Lusiani-Passemar'13 reduced the CKM unitarity violation by using hadron input from Kaon semileptonic decays in strangeness-changing tau decays



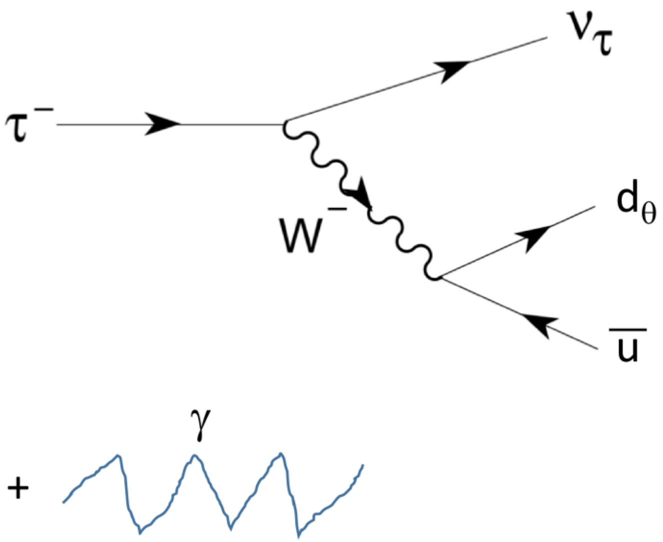
In the one-meson tau decays they used the old Decker-Finkemeier RadCors & for the two-meson channels they computed only the SI part and estimated the size of the model-dependent corrections, **which we have calculated now**.

$$\delta^{K^-\pi^0} = \left(-0.009^{+0.008}_{-0.118}\right)\%\,, \qquad \delta^{\bar{K}^0\pi^-} = \left(-0.166^{+0.010}_{-0.122}\right)\%$$

$$\delta^{K^-K^0} = \left(-0.030^{+0.026}_{-0.179}\right)\%\,, \qquad \delta^{\pi^-\pi^0} = \left(-0.186^{+0.024}_{-0.169}\right)\%$$

The first two were -0.20(20) and -0.15(20) in Antonelli et al. Our RadCors (also for one-meson case) will enable improved NP tests: most notably CKM unitarity.

XVIII MWPF
Ed. Carolino, BUAP, Puebla, Mexico, Nov. 21-25, 2022



Radiative corrections in semileptonic tau decays and reliable new physics tests

Pablo Roig
Cinvestav (Mexico)