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Michel Parameters in the presence of massive Dirac and Majorana neutrinos

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Outline

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Lorentz Structure of the Charged Current

To describe $l \to l' \nu \bar{\nu}'$, we use the most general, derivative-free, four-lepton interaction Hamiltonian, consistent with Lorentz invariance:

$$\mathcal{H} = 4\frac{G_{ll'}}{\sqrt{2}} \sum_{n,\epsilon,\omega} g^n_{\epsilon\omega} \left[\bar{l'}_\epsilon \Gamma^n (\nu_{l'})_\sigma\right] \left[(\bar{\nu}_l)_\lambda \Gamma_n l_\omega\right] + h.c. \qquad (1)$$

Where $\epsilon, \omega, \sigma, \lambda$ label the chiralities (L, R) of fermions, and $n = S, V, T$ the type of interaction: scalar $(\Gamma^S = I)$,vector $(\Gamma^V = \gamma^\mu)$ and tensor $(\Gamma^T = \sigma^{\mu\nu}/\sqrt{2})$.
The global factor G_{ll} ,determined from the total decay rate, leads to the following normalization of the coupling constants

$$\begin{aligned} 1 = \frac{1}{4}(|g^S_{RR}|^2 + |g^S_{RL}|^2 + |g^S_{LR}|^2 + |g^S_{LL}|^2) + 3(|g^T_{RL}|^2 + |g^T_{RL}|^2) \\ + (|g^V_{RR}|^2 + |g^V_{RL}|^2 + |g^V_{LR}|^2 + |g^V_{LL}|^2). \end{aligned} \qquad (2)$$

Thus, $|g^S_{\epsilon\omega}| \leq 2, |g^V_{\epsilon\omega}| \leq 1$ and $|g^T_{\epsilon\omega}| \leq 1/3$.

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Lorentz Structure of the Charged Current

The Standard Model predicts $|g_{LL}^V| = 1$ and all others couplings vanishing. In terms of chirality operators:

$$
\begin{aligned}
\mathcal{H} = \frac{G_{ll'}}{\sqrt{2}} \Big\{ & g_{LL}^S \left[\bar{l}' (1+\gamma^5) \nu_{l'} \right] \left[\bar{\nu}_l (1-\gamma^5) l \right] + g_{LL}^V \left[\bar{l}' \gamma^\mu (1-\gamma^5) \nu_{l'} \right] \left[\bar{\nu}_l \gamma_\mu (1-\gamma^5) l \right] \\
& + g_{RR}^S \left[\bar{l}' (1-\gamma^5) \nu_{l'} \right] \left[\bar{\nu}_l (1+\gamma^5) l \right] + g_{RR}^V \left[\bar{l}' \gamma^\mu (1+\gamma^5) \nu_{l'} \right] \left[\bar{\nu}_l \gamma_\mu (1+\gamma^5) l \right] \\
& + g_{LR}^S \left[\bar{l}' (1+\gamma^5) \nu_{l'} \right] \left[\bar{\nu}_l (1+\gamma^5) l \right] + g_{LR}^V \left[\bar{l}' \gamma^\mu (1-\gamma^5) \nu_{l'} \right] \left[\bar{\nu}_l \gamma_\mu (1+\gamma^5) l \right] \\
& + g_{LR}^T \left[\bar{l}' \frac{\sigma^{\mu\nu}}{\sqrt{2}} (1+\gamma^5) \nu_{l'} \right] \left[\bar{\nu}_l \frac{\sigma_{\mu\nu}}{\sqrt{2}} (1+\gamma^5) l \right] + g_{RL}^S \left[\bar{l}' (1-\gamma^5) \nu_{l'} \right] \left[\bar{\nu}_l (1-\gamma^5) l \right] \\
& + g_{RL}^V \left[\bar{l}' \gamma^\mu (1+\gamma^5) \nu_{l'} \right] \left[\bar{\nu}_l \gamma_\mu (1-\gamma^5) l \right] + g_{RL}^T \left[\bar{l}' \frac{\sigma^{\mu\nu}}{\sqrt{2}} (1-\gamma^5) \nu_{l'} \right] \left[\bar{\nu}_l \frac{\sigma_{\mu\nu}}{\sqrt{2}} (1-\gamma^5) l \right] \Big\}
\end{aligned} \tag{3}
$$

The lepton polarization is accounted via the spin projector $l \longrightarrow \frac{1}{2}(1+\gamma^5 \not{s}) l$.

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Lorentz Structure of the Charged Current

For the case of massless neutrinos, the differential decay rate is:

$$\frac{d\Gamma}{dxd\cos\theta} = \frac{m_1}{4\pi^3}\omega^4 G_{ll'}^2 \sqrt{x^2 - x_0^2}\left(F(x) - \frac{\xi}{3}\mathcal{P}\sqrt{x^2 - x_0^2}\cos\theta A(x)\right) \times \left[1 + \hat{\zeta}\cdot\vec{\mathcal{P}}_{l'}(x,\theta)\right], \tag{4}$$

where $\mathcal{P}$ is the degree of the initial lepton polarization, θ is the angle between the l^- spin and the final charged-lepton momenta, $\omega \equiv (m_1^2 + m_4^2)/2m_1$, $x \equiv E_4/\omega$ is the reduced energy and $x_0 \equiv m_4/\omega$, $\hat{\zeta}$ is an arbitrary direction parallel to the final charged-lepton spin and the polarization vector $\vec{\mathcal{P}}_{l'}$ is:

$$\vec{\mathcal{P}}_{l'} = P_{T_1}\cdot\hat{x} + P_{T_2}\cdot\hat{y} + P_L\cdot\hat{z}. \tag{5}$$

The components of $\vec{\mathcal{P}}_{l'}$ are, respectively:

$$\begin{aligned} P_{T_1} &= \mathcal{P}\sin\theta\cdot F_{T_1}(x) \;/\; \left\{F(x) - \frac{\xi}{3}\mathcal{P}\sqrt{x^2 - x_0^2}\cos\theta A(x)\right\}, \\ P_{T_2} &= \mathcal{P}\sin\theta\cdot F_{T_2}(x) \;/\; \left\{F(x) - \frac{\xi}{3}\mathcal{P}\sqrt{x^2 - x_0^2}\cos\theta A(x)\right\}, \\ P_L &= \frac{-F_{IP}(x) + \mathcal{P}\cos\theta\cdot F_{AP}(x)}{F(x) - \frac{\xi}{3}\mathcal{P}\sqrt{x^2 - x_0^2}\cos\theta A(x)}. \end{aligned} \tag{6}$$

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Lorentz Structure of the Charged Current

These functions are written in terms of the well-known **Michel Parameters** $(\rho, \eta, \delta, \xi, \eta'', \xi', \xi'', \alpha', \beta')$:

$$\begin{aligned}
F(x) &= x(1-x) + \frac{2}{9}\rho\left(4x^2 - 3x - x_0^2\right) + \eta x_0(1-x), \\
A(x) &= 1 - x + \frac{2}{3}\delta\left(4x - 4 + \sqrt{1-x_0^2}\right), \\
F_{T_1}(x) &= \frac{1}{12}\left[-2\left(\xi'' + 12\left(\rho - \frac{3}{4}\right)\right)(1-x)x_0 - 3\eta(x^2 - x_0^2) + \eta''(-3x^2 + 4x - x_0^2)\right], \\
F_{T_2}(x) &= \frac{1}{3}\sqrt{x^2 - x_0^2}\left[3\frac{\alpha'}{\mathcal{A}}(1-x) + 2\frac{\beta'}{\mathcal{A}}\sqrt{1-x_0^2}\right], \\
F_{IP}(x) &= \frac{1}{54}\sqrt{x^2 - x_0^2}\left[9\xi'\left(-2x + 2 + \sqrt{1-x_0^2}\right) + 4\xi\left(\delta - \frac{3}{4}\right)\left(4x - 4 + \sqrt{1-x_0^2}\right)\right], \\
F_{AP}(x) &= \frac{1}{6}\left[\xi''(2x^2 - x - x_0^2) + 4\left(\rho - \frac{3}{4}\right)(4x^2 - 3x - x_0^2) + 2\eta''(1-x)x_0\right].
\end{aligned} \tag{7}$$

As an example:

$$\eta = \frac{1}{2}\mathrm{Re}[g^V_{LL}g^{S*}_{RR} + g^V_{RR}g^{S*}_{LL} + g^V_{LR}(g^{S*}_{RL} + 6g^{T*}_{RL}) + g^V_{RL}(g^{S*}_{LR} + 6g^{T*}_{LR})]. \tag{8}$$

Lorentz Structure of the Charged Current

In the SM, $\rho = \delta = 3/4$, $\eta = \eta^{''} = \alpha^{'} = \beta^{'} = 0$ and $\xi = \xi^{'} = \xi^{''} = 1$.

	$\mu^- \to e^- \nu_\mu \bar{\nu}_e$	$\tau^- \to e^- \nu_\tau \bar{\nu}_e$	$\tau^- \to \mu^- \nu_\tau \bar{\nu}_\mu$
ρ	0.74979 ± 0.00026	0.747 ± 0.010	0.763 ± 0.020
η	0.057 ± 0.034	—	0.094 ± 0.073
ξ	$1.0009^{+0.0016}_{-0.0007}$	0.994 ± 0.040	1.030 ± 0.059
$\xi\delta$	$0.7511^{+0.0012}_{-0.0006}$	0.734 ± 0.028	0.778 ± 0.037
$\xi^{'}$	1.00 ± 0.04	—	—
$\xi^{''}$	0.65 ± 0.36	—	—

Effective Hamiltonian for Massive Neutrinos

The current neutrino ($\nu_{L,R}$) is assumed to be the superposition of the mass-eigenstate neutrinos (N_j) with the mass m_j, that is,

$$\nu_{lL} = \sum_j U_{lj} N_{jL}, \quad \nu_{lR} = \sum_j V_{lj} N_{jR}, \tag{9}$$

where $j = \{1, 2, ..., n\}$ with n the number of mass-eigenstate neutrinos. Thus, we can write the effective Hamiltonian in the mass basis, for the process $l^- \longrightarrow l'^- \overline{N}_j N_k$.

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Effective Hamiltonian for Massive Neutrinos

$$
\begin{aligned}
\mathcal{H} = 4\frac{G_{ll'}}{\sqrt{2}}\sum_{j,k}\Big\{ & g^S_{LL}\left[\bar{l}'_L V_{l'j} N_{jR}\right]\left[\overline{N}_{kR} V^*_{lk} l_L\right] + g^V_{LL}\left[\bar{l}'_L \gamma^\mu U_{l'j} N_{jL}\right]\left[\overline{N}_{kL} U^*_{lk}\gamma_\mu l_L\right] \\
& + g^S_{RR}\left[\bar{l}'_R U_{l'j} N_{jL}\right]\left[\overline{N}_{kL} U^*_{lk} l_R\right] + g^V_{RR}\left[\bar{l}'_R \gamma^\mu V_{l'j} N_{jR}\right]\left[\overline{N}_{kR} V^*_{lk}\gamma_\mu l_R\right] \\
& + g^S_{LR}\left[\bar{l}'_L V_{l'j} N_{jR}\right]\left[\overline{N}_{kL} U^*_{lk} l_R\right] + g^V_{LR}\left[\bar{l}'_L \gamma^\mu U_{l'j} N_{jL}\right]\left[\overline{N}_{kR} V^*_{lk}\gamma_\mu l_R\right] \\
& + g^T_{LR}\left[\bar{l}'_L \frac{\sigma^{\mu\nu}}{\sqrt{2}} V_{l'j} N_{jR}\right]\left[\overline{N}_{kL} U^*_{lk}\frac{\sigma_{\mu\nu}}{\sqrt{2}} l_R\right] + g^S_{RL}\left[\bar{l}'_R U_{l'j} N_{jL}\right]\left[\overline{N}_{kR} V^*_{lk} l_L\right] \\
& + g^V_{RL}\left[\bar{l}'_R \gamma^\mu V_{l'j} N_{jR}\right]\left[\overline{N}_{kL} U^*_{lk}\gamma_\mu l_L\right] + g^T_{RL}\left[\bar{l}'_R \frac{\sigma^{\mu\nu}}{\sqrt{2}} U_{l'j} N_{jL}\right]\left[\overline{N}_{kR} V^*_{lk}\frac{\sigma_{\mu\nu}}{\sqrt{2}} l_L\right]\Big\}.
\end{aligned}
\tag{10}
$$

Note that $\overline{N}$ represents an antineutrino for the Dirac neutrino case, but should be identified with N for the Majorana neutrino case ($N{=}N^c{=}C\overline{N}^T$).

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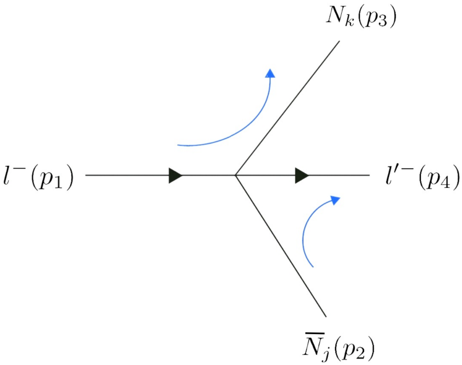
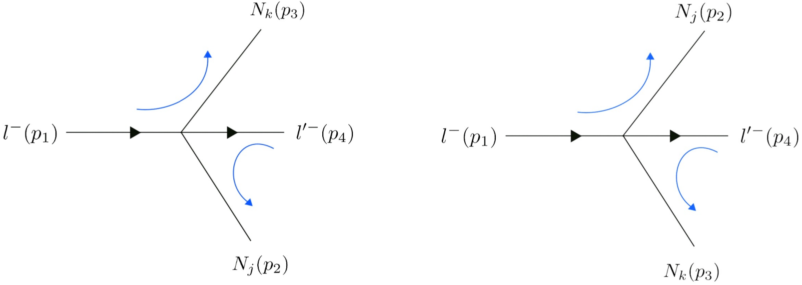
Dirac Neutrinos



- Neutrino $\neq$ Antineutrino.
- One possible first-order Feynman diagram.
- Well defined fermionic flux.

Majorana Neutrinos

The possible first order Feynman diagrams for the $l^- \longrightarrow l'^- N_j N_k$ decay are:



The first diagram leads to the same matrix element as the Dirac case, while the second diagram is only possible in the Majorana neutrino case and we already defined the orientation for each fermion chain.

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Majorana Neutrinos

Then, after integrating over the neutrinos momenta, the decay rate will have the following dependence on the amplitude:

$$\begin{aligned} d\Gamma &\propto \frac{1}{2}\sum_{j,k} |\mathcal{M}^D_{jk} - \mathcal{M}^M_{jk}|^2 \\ &= \frac{1}{2}\sum_{j,k} \left\{ |\mathcal{M}^D_{jk}|^2 + |\mathcal{M}^M_{jk}|^2 - 2\,\mathrm{Re}(\mathcal{M}^D_{jk}\mathcal{M}^{M*}_{jk}) \right\} \\ &= \sum_{j,k} |\mathcal{M}^D_{jk}|^2 - \sum_{j,k} \mathrm{Re}(\mathcal{M}^D_{jk}\mathcal{M}^{M*}_{jk}). \end{aligned} \tag{11}$$

The interference term distinguishes between Dirac and Majorana cases, which is sometimes called the Majorana term.

Differential Decay Rate

The differential decay rate taking into account finite Dirac or Majorana neutrino masses is:

$$\begin{aligned}\frac{d\Gamma}{dxd\cos\theta} &= \sum_{j,k}\frac{m_1}{4\pi^3}\omega^4 G_{ll'}^2\sqrt{x^2-x_0^2} \\ &\times\Big(\big(F_{IS}(x)+F'_{IS}(x)+F''_{IS}(x)\big)-\mathcal{P}\cos\theta\big(F_{AS}(x)+F'_{AS}(x)+F''_{AS}(x)\big)\Big) \\ &\times\left[1+\hat{\zeta}\cdot\vec{\mathcal{P}}_{l'}(x,\theta)\right],\end{aligned} \tag{12}$$

where

$$\vec{\mathcal{P}}_{l'} = P_{T_1}\cdot\hat{x}+P_{T_2}\cdot\hat{y}+P_L\cdot\hat{z}. \tag{13}$$

and the components of $\vec{\mathcal{P}}_{l'}$ are, respectively,

$$\begin{aligned}P_{T_1} &= \mathcal{P}\sin\theta\cdot\left(F_{T_1}(x)+F'_{T_1}(x)+F''_{T_1}(x)\right)\ /\ N, \\ P_{T_2} &= \mathcal{P}\sin\theta\cdot\left(F_{T_2}(x)+F'_{T_2}(x)+F''_{T_2}(x)\right)\ /\ N, \\ P_L &= \Big(-\left(F_{IP}(x)+F'_{IP}(x)+F''_{IP}(x)\right)+\mathcal{P}\cos\theta\cdot\left(F_{AP}(x)+F'_{AP}(x)+F''_{AP}(x)\right)\Big)\ /\ N.\end{aligned} \tag{14}$$

with N the normalization factor:
$N=\left(F_{IS}(x)+F'_{IS}(x)+F''_{IS}(x)\right)-\mathcal{P}\cos\theta\big(F_{AS}(x)+F'_{AS}(x)+F''_{AS}(x)\big)$.

Differential Decay Rate

For example:

$$F'_{T_1}(x) = \frac{1}{4}\frac{m_j}{m_1}\,\mathrm{Re}\left[(\lambda_L^+)_{jk}\left(x_0(1-x) + x_0\sqrt{1-x_0^2}\right) - (\lambda_R^+)_{kj}\left(x\left(1+\sqrt{1-x_0^2}\right) - x_0^2\right)\right],$$
$$F''_{T_1}(x) = \frac{1}{2}\frac{m_j m_k}{m_1^2}\left(1+\sqrt{1-x_0^2}\right)\mathrm{Re}\left(x_0(C^{'+})_{jk} - 2x(J^+)_{jk}\right), \tag{15}$$

with ($\epsilon = 0(1)$ for Dirac (Majorana)):

$$(\lambda_N^{\pm})_{jk} = -(f_{NN}^S)_{jk}(f_{LR}^V)^*_{jk} + (f_{NN}^V)_{jk}((f_{LR}^S)^*_{jk} + 2(f_{LR}^T)^*_{jk}) + 2(f_{NN}^S)_{kj}(f_{LR}^T)^*_{kj} - 2(f_{NN}^V)_{kj}(f_{LR}^V)^*_{kj}$$
$$\pm (R \leftrightarrow L) + \epsilon\Big[-2(f_{NN}^V)_{kj}(f_{LR}^V)^*_{jk} + \frac{1}{2}(f_{NN}^S)_{kj}(f_{LR}^S)^*_{jk} + (f_{NN}^S)_{kj}(f_{LR}^T)^*_{jk} + 4(f_{NN}^V)_{jk}$$
$$(f_{LR}^T)^*_{kj} - (f_{NN}^S)_{jk}(f_{LR}^V)^*_{kj} \pm (L \leftrightarrow R)\Big],$$
$$(C^{'\pm})_{jk} = (f_{LL}^S)_{jk}(f_{LL}^V)^*_{jk} - (f_{RL}^V)^*_{jk}((f_{RL}^S)_{jk} + 6(f_{RL}^T)_{jk}) \pm (R \leftrightarrow L) + \epsilon\Big[\frac{1}{4}(f_{LL}^S)_{jk}(f_{LL}^S)^*_{kj}$$
$$+ (f_{LL}^V)_{jk}(f_{LL}^V)^*_{kj} - (f_{RL}^S)_{jk}(f_{RL}^V)^*_{kj} - 6(f_{RL}^V)_{jk}(f_{RL}^T)^*_{kj} \pm (L \leftrightarrow R)\Big],$$
$$(J^+)_{jk} = (f_{LR}^S)_{jk}(f_{RL}^T)^*_{jk} + (f_{LR}^T)_{jk}(f_{RL}^S)^*_{jk} + 2(f_{LR}^V)_{jk}(f_{RL}^V)^*_{jk} + 4(f_{LR}^T)_{jk}(f_{RL}^T)^*_{jk} + \epsilon\Big[\frac{1}{4}(f_{LR}^S)_{kj}$$
$$(f_{RL}^S)^*_{jk} + \frac{1}{2}(f_{LR}^S)_{kj}(f_{RL}^T)^*_{jk} + \frac{1}{2}(f_{LR}^T)_{kj}(f_{RL}^S)^*_{jk} + 5(f_{LR}^T)_{kj}(f_{RL}^T)^*_{jk} + 2(f_{LR}^V)_{jk}(f_{RL}^V)^*_{kj}\Big] \tag{16}$$

Dirac vs Majorana

Considering the constraints on an invisible heavy neutrino[1], we can estimate the suppression of the neutrino mass dependent terms compared with the ones without this dependence (standard Michel distribution).

Neutrino	Mass (MeV)	Mixing $\|U_{l4}\|^2$	Process
Heavy ($l = e$)	0.001 - 0.45	10^{-3}	$n \rightarrow p + e + \nu_4$
	10 - 55	10^{-8}	$\pi \rightarrow e\nu_4$
	135 - 350	10^{-6}	$k \rightarrow e\nu_4$
Heavy ($l = \mu$)	10 - 30	10^{-4}	$\pi \rightarrow \mu\nu_4$
	70 - 300	10^{-5}	$k \rightarrow \mu\nu_4$
	175 - 300	10^{-8}	$k \rightarrow \mu\nu_4$
Heavy ($l = \tau$)	100 - 1.2×10^3	$10^{-7} - 10^{-3}$	$\tau \rightarrow \nu_4 + 3\pi$
	1×10^3-60×10^3	$10^{-5} - 10^{-3}$	$Z \rightarrow \nu\nu_4$

[1] A. de Gouvea and A. Kobach, Phys.Rev.D 93 (2016).

Dirac vs Majorana

Neutrino	Mass (MeV)	Mixing Suppression	Linear Term Suppression (m_ν)	Quadratic Term Suppression (m_ν^2)
Light (2)	1×10^{-6}	—	10^{-9}	10^{-18}
Heavy (1) ($l=e$)	0.001 - 0.45 10 - 55 135 - 350	10^{-3} 10^{-8} 10^{-6}	$10^{-9} - 10^{-7}$ 10^{-10} 10^{-7}	$10^{-18} - 10^{-16}$ 10^{-19} 10^{-16}
Heavy (1) ($l=\mu$)	10 - 30 70 - 300 175 - 300	10^{-4} 10^{-5} 10^{-8}	10^{-6} $10^{-7} - 10^{-6}$ 10^{-9}	10^{-15} $10^{-16} - 10^{-15}$ 10^{-18}
Heavy (1) ($l=\tau$)	100 - 1.2×10^{3} $1\times10^{3} - 60\times10^{3}$	$10^{-7} - 10^{-3}$ $10^{-5} - 10^{-3}$	$10^{-8} - 10^{-3}$ $10^{-5} - 10^{-3}$	$10^{-18} - 10^{-12}$ $10^{-14} - 10^{-12}$
Heavy (2) ($\mu \to eNN$)	10 - 30 175 - 300	10^{-12} $10^{-14} - 10^{-11}$	10^{-14} $10^{-15} - 10^{-12}$	10^{-16} $10^{-16} - 10^{-13}$
Heavy (2) ($\tau \to eNN$)	135 - 350	$10^{-13} - 10^{-9}$	$10^{-14} - 10^{-10}$	$10^{-14} - 10^{-10}$
Heavy (2) ($\tau \to \mu NN$)	100 - 300 175 - 350	$10^{-12} - 10^{-8}$ $10^{-15} - 10^{-11}$	$10^{-13} - 10^{-9}$ $10^{-16} - 10^{-12}$	$10^{-14} - 10^{-10}$ $10^{-16} - 10^{-12}$

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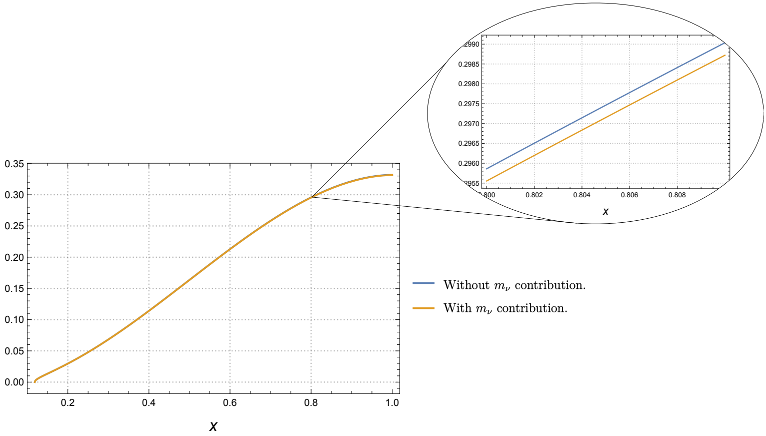
Dirac vs Majorana

For a realistic scenario ($g^V_{LL} = 0.96$, $g^S_{RR} = 0.25$ and $g^S_{LR} = 0.5$) a suppression of order 10^{-4} was estimated. For Dirac neutrinos, the energy spectrum is:



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Dirac vs Majorana

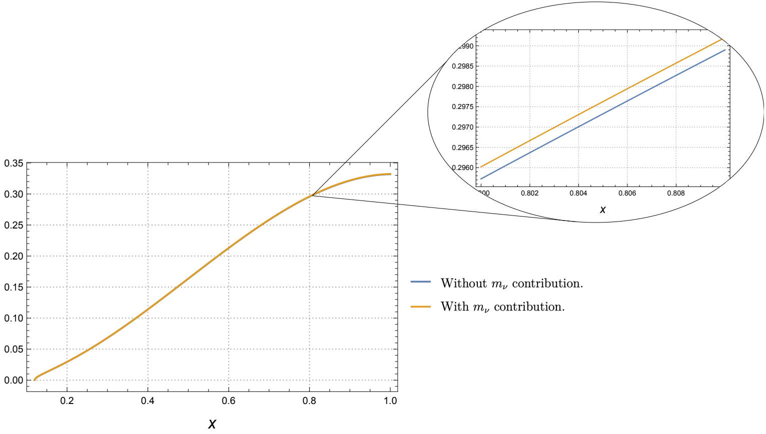

Figure: Majorana neutrinos.

Dirac vs Majorana

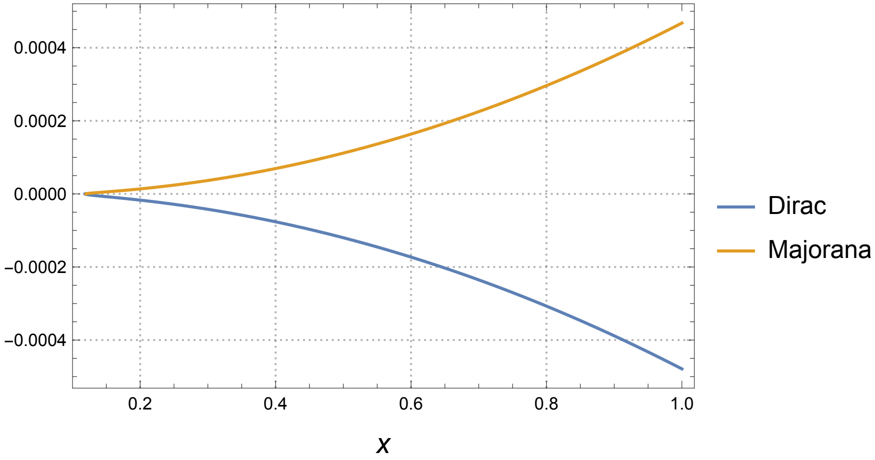

Figure: Neutrino mass contribution to Dirac and Majorana distributions.

Summary and Conclusions

- In this work we have studied the leptonic decay $l^- \longrightarrow l^{'-} N_j N_k$, where N_j and N_k are mass-eigenstate neutrinos.
- We have constructed its matrix element by using the most general four-lepton effective interaction Hamiltonian and obtained the specific energy and angular distribution of the final charged lepton, complemented with the decaying and final charged-lepton polarization and the effects of Dirac and Majorana neutrino masses.
- We have introduced generalized Michel parameters, that arise due to considering finite neutrino masses and a specific neutrino nature.
- Specifically, for the case of τ-decay with one heavy final-state neutrino with a mass around $10^2 - 10^3 MeV$ the linear term suppression could be of order 10^{-4}, low enough to be measured in current and forthcoming experiments.
- Finally, it would also be interesting to analyze other type of leptonic decays, such as radiative muon and tau decay with Dirac and Majorana neutrinos, where new information could be obtained.

What is new:

- We write our expressions in the PDG parametrization form, in a way that complements all previous results, facilitating their application to model-dependent scenarios.
- We classify the Dirac and Majorana contributions with the help of a flag parameter $\epsilon = 0, 1$, making easier to distinguish between Dirac and Majorana nature of neutrinos.
- We discuss their main differences, together with some examples of its application to model-dependent theories.
- We also introduced and discussed the leading W-boson propagator correction to the differential decay rate including the final charged-lepton polarization.

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BACKUP

For massless neutrinos the total decay rate is:

$$\Gamma_{l\to l'} = \frac{\hat{G}_{ll'}^2 m_1^5}{192\pi^3} f(m_4^2/m_1^2)\left(1+\delta_{RC}^{ll'}\right), \tag{17}$$

where

$$\hat{G}_{ll'} \equiv G_{ll'}\sqrt{1+4\eta\frac{m_4}{m_1}\frac{g(m_4^2/m_1^2)}{f(m_4^2/m_1^2)}} \tag{18}$$

$f(x) = 1-8x-12x^2\log(x)+8x^3-x^4$,$g(x) = 1+9x-9x^2-x^3+6x(1+x)log(x)$ and the SM radiative correction $\delta_{RC}^{ll'}$ has been included.

$$\delta_{RC}^{ll'} = \frac{\alpha}{2\pi}\left[\frac{25}{4}-\pi^2+\mathcal{O}\left(\frac{m_4^2}{m_1^2}\right)\right]+\ldots \tag{19}$$

$$G_{ll'}^2 = \left[\frac{g^2}{4\sqrt{2}M_W^2}(1+\Delta r)\right]^2\left[1+\frac{3}{5}\frac{m_1^2}{M_W^2}+\frac{9}{5}\frac{m_4^2}{M_W^2}+\mathcal{O}\left(\frac{m_4^4}{m_1^2M_W^2}\right)\right] \tag{20}$$

For massive neutrinos the total decay rate is:

$$\Gamma_{l\to l'} = \sum_{j,k} \frac{\hat{G}^2_{ll'} m_1^5}{192\pi^3} f(m_4^2/m_1^2)\left(1+\delta_{RC}^{ll'}\right), \tag{21}$$

where

$$\hat{G}_{ll'} \equiv G_{ll'} \left\{ (I)_{jk} + 4(\eta)_{jk} \frac{m_4}{m_1} \frac{g(m_4^2/m_1^2)}{f(m_4^2/m_1^2)} - 2\frac{m_j}{m_1}\left[(\kappa_L^+)_{jk} \frac{f'(m_4^2/m_1^2)}{f(m_4^2/m_1^2)} + (\kappa_R^+)_{kj} \frac{m_4}{m_1} \frac{g'(m_4^2/m_1^2)}{f(m_4^2/m_1^2)}\right] \right.$$
$$\left. - 4\frac{m_j m_k}{m_1^2}\left[(C^+)_{jk} \frac{f''(m_4^2/m_1^2)}{f(m_4^2/m_1^2)} + 3(H^+)_{jk} \frac{m_4}{m_1} \frac{g''(m_4^2/m_1^2)}{f(m_4^2/m_1^2)}\right] \right\}^{1/2}, \tag{22}$$

with the functions defined as:

$$\begin{aligned}
f'(x) &= -1 + 6x - 2x^3 + 3x^2\left(4 \operatorname{arctanh}\left(\frac{x-1}{x+1}\right) - 1\right), \\
f''(x) &= 1 - 3x + 3x^2 - x^3, \\
g'(x) &= 2 - 6x^2 + x^3 + 3x\left(4 \operatorname{arctanh}\left(\frac{x-1}{x+1}\right) + 1\right), \\
g''(x) &= 1 - x^2 + 2x\log(x).
\end{aligned} \tag{23}$$

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The main contributions are:

Radiative Corrections and Mass Effects	Numerical Effect (μ-decay)	Numerical Effect (τ-decay)
Electroweak	$(3/5)(m_\mu^2/M_W^2) \sim 1.0 \times 10^{-6}$	$(3/5)(m_\tau^2/M_W^2) \sim 2.9 \times 10^{-4}$
QED	$\mathcal{O}(\alpha) \sim 10^{-3}$	$\mathcal{O}(\alpha) \sim 10^{-3}$
Hadronic	$\mathcal{O}(\alpha^2/\pi^2) \sim 10^{-5}$	$\mathcal{O}(\alpha^2/\pi^2) \sim 10^{-5}$

The sub-leading contributions will be of order $\mathcal{O}(10^{-11} - 10^{-7})$.

At the level of differential decay rate, the W boson propagator corrections look like:

$$\frac{d^2\Gamma}{dxd\cos\theta} = \frac{G_{ll'}^2 M^5}{192\pi^3} x \left\{ 3x - 2x^2 + r_W^2 \left[2x^2 - x^3\right] - \cos\theta \; x\left[2x - 1 + r_W^2 \; x^2\right] \right\}, \tag{24}$$

where $r_W = M/M_W$. Then the Michel parameter, defined by the following energy spectrum:

$$\frac{d\Gamma_{l\to l'}}{dxd\cos\theta} = \frac{G_{ll'}^2 M^5}{192\pi^3} x \left\{ 6x(1-x) + \frac{4}{3}\rho\left(4x^2 - 3x\right) - 2\xi x\cos\theta\left(1 - x + \frac{2}{3}\delta(4x-3)\right)\right\}, \tag{25}$$

is given by:

$$\bullet\; \rho \to \rho_{eff} = \frac{3}{4} + \frac{3}{2}\left(\frac{M}{M_W}\right)^2. \tag{26}$$

For the final charged-lepton polarization dependence:

$$\frac{d\Gamma_{l\to l'}}{dx d\cos\theta} = \frac{G_{ll'}^2 M^5}{64\pi^3} x \left\{ -\left[\frac{1}{6}x(-2x+3) + r_W^2 \frac{1}{6}x(2x-x^2)\right]\cos\phi + \frac{1}{6}\left[(2x^2-x) + r_W^2 x^3\right]\cos\theta\cos\phi\right\} \tag{27}$$

Then the Michel parameters, defined by the following energy spectrum:

$$\begin{aligned}\frac{d\Gamma_{l\to l'}}{dx d\cos\theta} &= \frac{G_{ll'}^2 M^5}{64\pi^3} x\left\{-F_{IP}\cos\phi + F_{AP}\cos\theta\cos\phi\right\}\\ &= \frac{G_{ll'}^2 M^5}{64\pi^3} x \left\{ -\frac{1}{54}x\left[9\xi^{'}(3-2x) + 4\xi(\delta-\frac{3}{4})(4x-3)\right]\cos\phi + \frac{1}{6}\left[\xi^{''}(2x^2-x)\right.\right.\\ &\left.\left. + 4(\rho-\frac{3}{4})(4x^2-3x)\right]\cos\theta\cos\phi\right\},\end{aligned} \tag{28}$$

are given by:

$$\begin{aligned}&\bullet\ \xi' \to \xi'_{eff} = 1 + \left(\frac{M}{M_W}\right)^2\\ &\bullet\ \xi\left(\delta - \frac{3}{4}\right) \to \xi\left(\delta-\frac{3}{4}\right)_{eff} = 0 + \frac{9}{4}\left(\frac{M}{M_W}\right)^2\end{aligned} \tag{29}$$

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The Majorana term for each parameter is:

Term	Coupling Dependence
No Neutrino Mass Dependence	
$(\mathrm{I})^M_{jk}$	$\frac{1}{8}\Big[12(f^T_{LR})_{jk}(f^S_{LR})^*_{kj} + 12(f^T_{LR})_{jk}(f^T_{LR})^*_{kj} + 8(f^V_{RL})_{jk}(f^V_{RL})^*_{kj} - (f^S_{LR})_{jk}(f^S_{LR})^*_{kj}$ $+8(f^S_{LL})_{jk}(f^V_{LL})^*_{kj} + (L \leftrightarrow R)\Big]$
$(\rho)^M_{jk}$	$\frac{3}{16}\Big[-(f^S_{LR})_{jk}(f^S_{LR})^*_{kj} + 4(f^S_{LR})_{jk}(f^T_{LR})^*_{kj} + 4(f^S_{LL})_{jk}(f^V_{LL})^*_{kj} - 4(f^T_{LR})_{jk}(f^T_{LR})^*_{kj}$ $+(L \leftrightarrow R)\Big]$
$(\xi)^M_{jk}$	$-(f^S_{RR})_{jk}(f^V_{RR})^*_{kj} + \frac{17}{2}(f^T_{LR})_{jk}(f^T_{LR})^*_{kj} + \frac{1}{2}(f^S_{LR})_{jk}(f^T_{LR})^*_{kj} + 3(f^V_{LR})_{jk}(f^V_{LR})^*_{kj}$ $+\frac{5}{8}(f^S_{LR})_{jk}(f^S_{LR})^*_{kj} - (L \leftrightarrow R)$
$(\xi\delta)^M_{jk}$	$\frac{3}{4}\Big[-(f^S_{RR})_{jk}(f^V_{RR})^*_{kj} + (f^T_{LR})_{jk}(f^T_{LR})^*_{kj} - (f^S_{LR})_{jk}(f^T_{LR})^*_{kj} + \frac{1}{4}(f^S_{LR})_{jk}(f^S_{LR})^*_{kj} - (L \leftrightarrow R)\Big]$
$(\eta)^M_{jk}$	$\frac{1}{8}\Big[4(f^S_{LR})_{jk}(f^V_{RL})^*_{kj} + 24(f^T_{LR})_{jk}(f^V_{RL})^*_{kj} + (f^S_{LL})_{jk}(f^S_{RR})^*_{kj} + 4(f^V_{LL})_{jk}(f^V_{RR})^*_{kj} + (L \leftrightarrow R)\Big]$
$(\xi^{'})^M_{jk}$	$(f^S_{LL})_{jk}(f^V_{LL})^*_{kj} + \frac{3}{2}(f^S_{LR})_{jk}(f^T_{LR})^*_{kj} + \frac{3}{2}(f^T_{LR})_{jk}(f^T_{LR})^*_{kj} + (f^V_{LR})_{jk}(f^V_{LR})^*_{kj}$ $-\frac{1}{8}(f^S_{LR})_{jk}(f^S_{LR})^*_{kj} - (L \leftrightarrow R)$
$(\xi^{''})^M_{jk}$	$\frac{1}{2}(f^S_{LR})_{jk}(f^T_{LR})^*_{kj} + \frac{17}{2}(f^T_{LR})_{jk}(f^T_{LR})^*_{kj} + (f^S_{LL})_{jk}(f^V_{LL})^*_{kj} + \frac{5}{8}(f^S_{RL})_{jk}(f^S_{RL})^*_{kj}$ $+3(f^V_{RL})_{jk}(f^V_{RL})^*_{kj} + (L \leftrightarrow R)$
$(\eta^{''})^M_{jk}$	$\frac{1}{2}\Big[3(f^S_{LR})_{jk}(f^V_{RL})^*_{kj} + 18(f^T_{LR})_{jk}(f^V_{RL})^*_{kj} - \frac{1}{4}(f^S_{LL})_{jk}(f^S_{RR})^*_{kj} - (f^V_{LL})_{jk}(f^V_{RR})^*_{kj} + (L \leftrightarrow R)\Big]$
$(\frac{\alpha^{'}}{\mathcal{A}})^M_{jk}$	$\frac{1}{2}(f^V_{RL})^*_{jk}((f^S_{LR})_{kj} + 6(f^T_{LR})_{kj}) - (L \leftrightarrow R)$
$(\frac{\beta^{'}}{\mathcal{A}})^M_{jk}$	$-\frac{1}{2}(f^V_{LL})_{jk}(f^V_{RR})^*_{kj} - \frac{1}{8}(f^S_{LL})_{jk}(f^S_{RR})^*_{kj}$

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Linear Neutrino Mass Dependence	
$(\kappa_L^{\pm})^M_{jk}$	$-2(f^V_{LL})_{kj}(f^V_{LR})^*_{jk} - \frac{1}{2}(f^S_{LL})_{kj}(f^S_{LR})^*_{jk} + 3(f^S_{LL})_{kj}(f^T_{LR})^*_{jk} + 2(f^V_{LL})_{jk}(f^S_{LR})^*_{kj}$ $-(f^S_{LL})_{jk}(f^V_{LR})^*_{kj} \pm (L \leftrightarrow R)$
$(\kappa_R^{\pm})^M_{jk}$	$-2(f^V_{RR})_{kj}(f^V_{LR})^*_{jk} - \frac{1}{2}(f^S_{RR})_{kj}(f^S_{LR})^*_{jk} + 3(f^S_{RR})_{kj}(f^T_{LR})^*_{jk} + 2(f^V_{RR})_{jk}(f^S_{LR})^*_{kj}$ $-(f^S_{RR})_{jk}(f^V_{LR})^*_{kj} \pm (L \leftrightarrow R)$
$(\lambda_L^{\pm})^M_{jk}$	$-2(f^V_{LL})_{kj}(f^V_{LR})^*_{jk} + \frac{1}{2}(f^S_{LL})_{kj}(f^S_{LR})^*_{jk} + (f^S_{LL})_{kj}(f^T_{LR})^*_{jk} + 4(f^V_{LL})_{jk}(f^T_{LR})^*_{kj}$ $-(f^S_{LL})_{jk}(f^V_{LR})^*_{kj} \pm (L \leftrightarrow R)$
$(\lambda_R^{\pm})^M_{jk}$	$-2(f^V_{RR})_{kj}(f^V_{LR})^*_{jk} + \frac{1}{2}(f^S_{RR})_{kj}(f^S_{LR})^*_{jk} + (f^S_{RR})_{kj}(f^T_{LR})^*_{jk} + 4(f^V_{RR})_{jk}(f^T_{LR})^*_{kj}$ $-(f^S_{RR})_{jk}(f^V_{LR})^*_{kj} \pm (L \leftrightarrow R)$
Quadratic Neutrino Mass Dependence	
$(\mathrm{C}^{\pm})^M_{jk}$	$(f^V_{LL})_{jk}(f^V_{LL})^*_{kj} + \frac{1}{4}(f^S_{LL})_{jk}(f^S_{LL})^*_{kj} + (f^S_{RL})_{jk}(f^V_{RL})^*_{kj} + 6(f^V_{RL})_{jk}(f^T_{RL})^*_{kj} \pm (L \leftrightarrow R)$
$(\mathrm{C}^{'\pm})^M_{jk}$	$\frac{1}{4}(f^S_{LL})_{jk}(f^S_{LL})^*_{kj} + (f^V_{LL})_{jk}(f^V_{LL})^*_{kj} - (f^S_{RL})_{jk}(f^V_{RL})^*_{kj} - 6(f^V_{RL})_{jk}(f^T_{RL})^*_{kj} \pm (L \leftrightarrow R)$
$(\mathrm{J}^{+})^M_{jk}$	$\frac{1}{4}(f^S_{LR})_{kj}(f^S_{RL})^*_{jk} + \frac{1}{2}(f^S_{LR})_{kj}(f^T_{RL})^*_{jk} + \frac{1}{2}(f^T_{LR})_{kj}(f^S_{RL})^*_{jk} + 5(f^T_{LR})_{kj}(f^T_{RL})^*_{jk}$ $+2(f^V_{LR})_{jk}(f^V_{RL})^*_{kj}$
$(\mathrm{H}^{+})^M_{jk}$	$2(f^V_{LL})_{jk}(f^S_{RR})^*_{kj} - \frac{1}{4}(f^S_{LR})_{jk}(f^S_{RL})^*_{kj} + 3(f^T_{LR})_{jk}(f^S_{RL})^*_{kj} + 3(f^T_{LR})_{jk}(f^T_{RL})^*_{kj}$ $+2(f^V_{LR})_{jk}(f^V_{RL})^*_{kj} + (L \leftrightarrow R)$

Easier implementation in specific model-dependent theories:

Term	Coupling Dependence	Term	Coupling Dependence
No Neutrino Mass Dependence		**Linear Neutrino Mass Dependence**	
$(\mathrm{I})^M_{jk}$	0	$(\kappa^{\pm}_L)^M_{jk}$	0
$(\rho)^M_{jk}$	0	$(\kappa^{\pm}_R)^M_{jk}$	0
$(\xi)^M_{jk}$	0	$(\lambda^{\pm}_L)^M_{jk}$	0
$(\xi\delta)^M_{jk}$	0	$(\lambda^{\pm}_R)^M_{jk}$	0
$(\eta)^M_{jk}$	0	**Quadratic Neutrino Mass Dependence**	
$(\xi^{'})^M_{jk}$	0	$(\mathrm{C}^{\pm})^M_{jk}$	$(f^V_{LL})_{jk}(f^V_{LL})^*_{kj}$
$(\xi^{''})^M_{jk}$	0	$(\mathrm{C}^{'\pm})^M_{jk}$	$(f^V_{LL})_{jk}(f^V_{LL})^*_{kj}$
$(\eta^{''})^M_{jk}$	0	$(\mathrm{J}^{+})^M_{jk}$	0
$(\frac{\alpha}{A}^{'})^M_{jk}$	0	$(\mathrm{H}^{+})^M_{jk}$	0
$(\frac{\beta}{A}^{'})^M_{jk}$	0		

Table: SM case $|f^V_{LL}| = 1$.

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Reproduced well-known results:

Term	Coupling Dependence	Term	Coupling Dependence
No Neutrino Mass Dependence		**Linear Neutrino Mass Dependence**	
$(\mathrm{I})^M_{jk}$	0	$(\kappa^{\pm}_L)^M_{jk}$	$-2(f^V_{LL})_{kj}(f^V_{LR})^*_{jk}$
$(\rho)^M_{jk}$	0	$(\kappa^{\pm}_R)^M_{jk}$	$\mp 2(f^V_{LL})_{kj}(f^V_{RL})^*_{jk}$
$(\xi)^M_{jk}$	0	$(\lambda^{\pm}_L)^M_{jk}$	$-2(f^V_{LL})_{kj}(f^V_{LR})^*_{jk}$
$(\xi\delta)^M_{jk}$	0	$(\lambda^{\pm}_R)^M_{jk}$	$\mp 2(f^V_{LL})_{kj}(f^V_{RL})^*_{jk}$
$(\eta)^M_{jk}$	$(f^V_{LL})_{jk}(f^V_{RR})^*_{kj}$	**Quadratic Neutrino Mass Dependence**	
$(\xi^{'})^M_{jk}$	0	$(\mathrm{C}^{\pm})^M_{jk}$	$(f^V_{LL})_{jk}(f^V_{LL})^*_{kj}$
$(\xi^{''})^M_{jk}$	0	$(\mathrm{C}^{'\pm})^M_{jk}$	$(f^V_{LL})_{jk}(f^V_{LL})^*_{kj}$
$(\eta^{''})^M_{jk}$	$-(f^V_{LL})_{jk}(f^V_{RR})^*_{kj}$	$(\mathrm{J}^{+})^M_{jk}$	0
$(\frac{\alpha^{'}}{\mathcal{A}})^M_{jk}$	0	$(\mathrm{H}^{+})^M_{jk}$	0
$(\frac{\beta^{'}}{\mathcal{A}})^M_{jk}$	$-\frac{1}{2}(f^V_{LL})_{jk}(f^V_{RR})^*_{kj}$		

Table: $V \pm A$ Currents.

As an example, considering only transverse polarization component in the $V \pm A$ theory, the energy distribution takes the form:

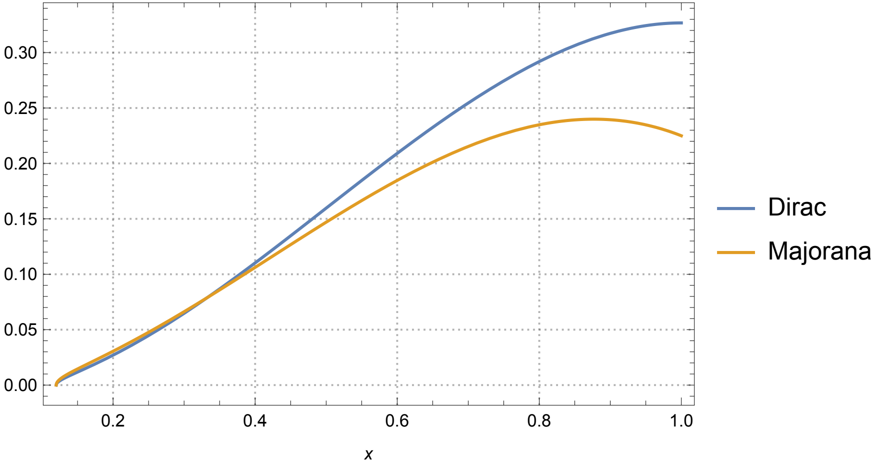

Figure: Energy spectrum for the $V \pm A$ theory with P_{T_2} polarization.

Prospects for τ-lepton physics (Michel Parameters):

- BELLE II (Expected statistical uncertainty of the order of 10^{-4})
- Super Charm-Tau Factory
  - First measure of $\xi', \xi'', \alpha', \beta'$
  - 2.1×10^{10} tau pairs.
- Radiative decays, new methods for measure of the Michel parameters, etc.
- Many others...

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