Masses of Excited Mesons

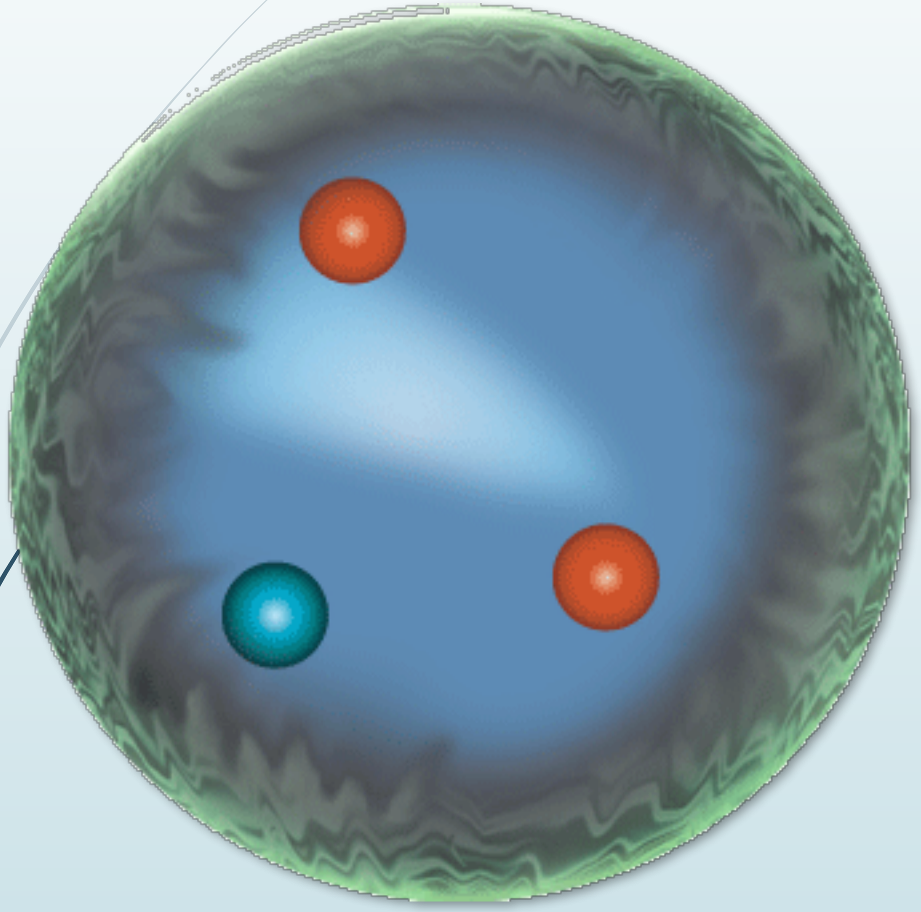
Gustavo Paredes Torres
Adnan Bashir
Laura Xiomara Gutiérrez Guerrero

INSTITUTO DE FÍSICA
Y MATEMÁTICAS
U M S N H

DPC-SMF
XVIII Mexican Workshop on Particles and Fields
21-25 November 2022
Edificio Carolino, BUAP
Mexico/General timezone

CONTENTS

QUARK'S MASS



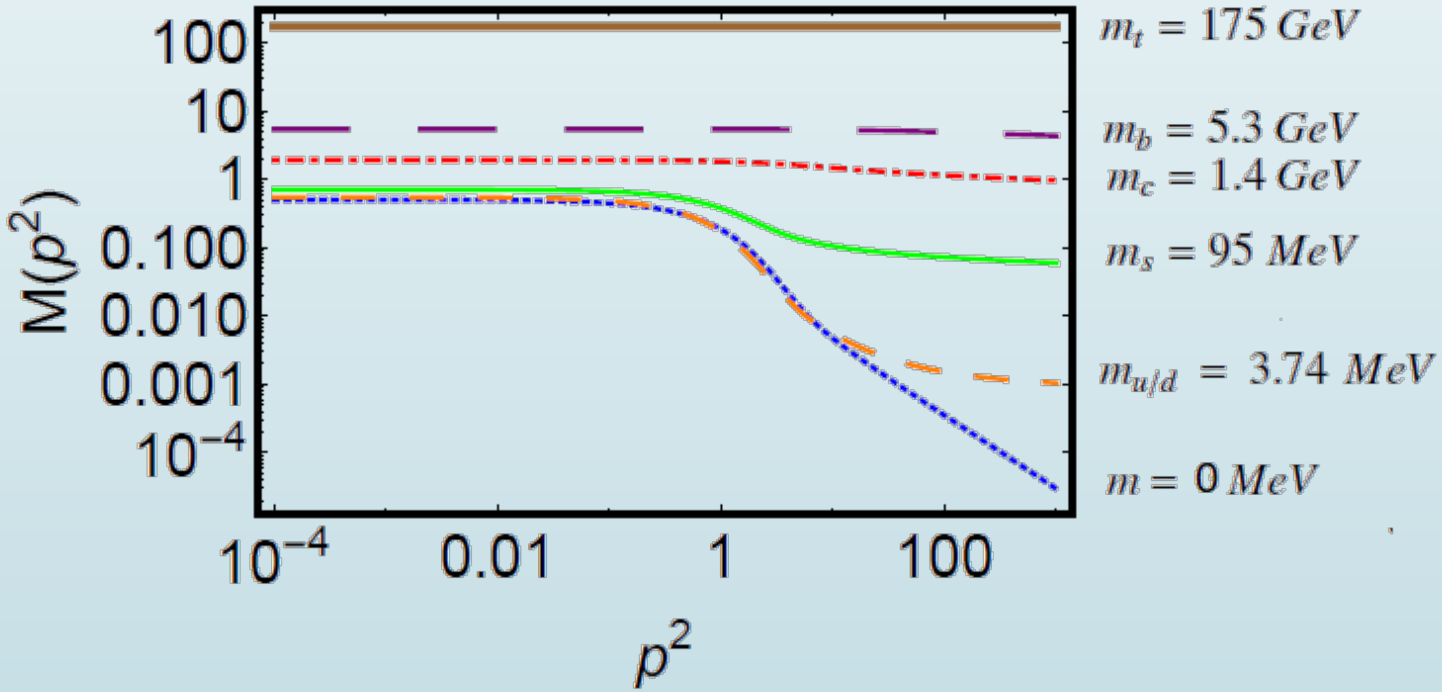
- We can distinguish among two different types of quark masses.
- The current mass that appears in the renormalized Lagrangian of QCD.
- Due to interactions, quarks and gluons acquire constituent/dressed mass dynamically within hadrons.

$$m_u + m_u + m_d \simeq 10\ MeV$$

$$m_{proton} \approx 1000\ MeV$$

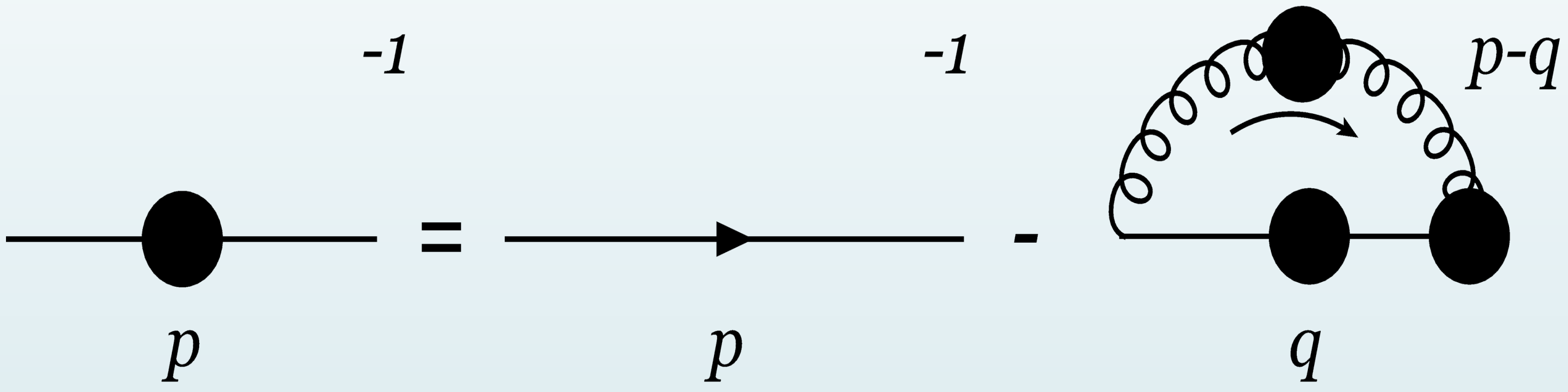
CURRENT VS CONSTITUENT MASS

- The breaking of chiral symmetry causes the difference between the current and constituent masses.
- About the same mass is generated for each quark (300 MeV); for the heavier quarks it is less noticeable because of the large value of their explicit mass.
- DCSB is the mechanism that generates 98% of the mass of visible matter.



SDE FOR THE QUARK PROPAGATOR

- **Ideal for studying non-perturbative phenomena as no assumptions are made about the value of the coupling.**



$$S^{-1}(p,\mu) = Z_{2F} S_0^{-1}(p) + Z_{1F} \int \frac{d^4 p}{(2\pi)^4} g^2 D_{\rho\nu}(p-q;\mu) \frac{\lambda^a}{2} \gamma_\rho S(q;\mu) \Gamma_\nu^a(q,p;\mu).$$

Complete Propagator, dressed

Bare propagator, tree level

Dressed gluon propagator

Dressed quark-gluon vertex

Gap equation

$\Sigma(\mathbf{p})$ **Self-energy**

CONTACT INTERACTION MODEL

- Simplified QCD model, where UV divergences are regularized to preserve QCD symmetries, compatible with confinement and DCSB.
- The gluon propagator is considered as a constant:

$$g^2 D_{\mu\nu}(k) = g^2 \left[g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2} \right] \frac{1}{k^2} \to \frac{4\pi\alpha_{IR}}{m_g^2}\delta_{\mu\nu} \equiv \delta_{\mu\nu}\frac{1}{m_G^2},$$

Interaction strength in infrared

Gluon mass scale

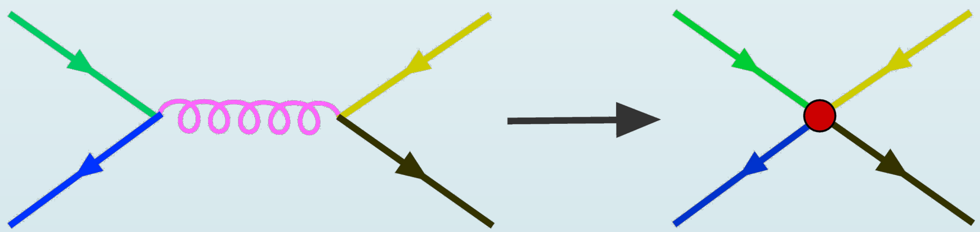
- It is one of the most drastic approximations of this model. However, according to our present knowledge, the gluon propagator does become constant in the infrared, at least to the Landau gauge. Therefore, in the Landau gauge it is a very good approximation for all those observables that only depend on infrared physics such as the hadron mass.

CONTACT INTERACTION MODEL

- **The vertex is assumed to be tree level, that is (bare vertex):**

$$\Gamma^a_\nu(p,q;\mu) \to \frac{\lambda^a}{2}\gamma_\nu.$$

- **Feynman diagram for $q\bar{q}$ scattering:**



- **It is the gauge for which the sensitivity to differences between the bare quark-gluon vertex and the full vertex is less noticeable. Furthermore, it is a gauge that is easily implemented in numerical simulations of regularized QCD lattice.**

CONTACT INTERACTION MODEL

- **With CI the equation of the quark propagator takes the form:**

$$S^{-1}(p) = i\gamma \cdot p + m_f + \frac{3}{4}\frac{1}{m_G^2}\int \frac{d^4q}{(2\pi)^4}\gamma_\mu S(q)\gamma_\mu,$$

- **It has quadratic and logarithmic divergences, that is why it must be regularized, for this, a proper time regularization scheme is used:**

$$M_f = m_f + \frac{M_f}{3\pi^2 m_G^2} C(M_f, \tau_{ir}, \tau_{uv})$$

$$\mathcal{C}(M_f, \tau_{ir}, \tau_{uv})/M_f = \overline{\mathcal{C}}(M_f, \tau_{ir}, \tau_{uv}) = \Gamma(-1, M_f\tau_{\text{uv}}^2) - \Gamma(-1, M_f\tau_{\text{ir}}^2)$$

QUAKR DRESSED MASS RESULTS

- **Using the following parameters:**

Quarks	Z_H	Λ_{uv} [GeV]
Light parameters (CI-LP)		
u, d, s, c, b	1	0.905
Heavy parameters (CI-HP)		
u, d, s	1	0.905
c, d, s	3.034	1.322
c	13.122	2.305
b, u, s	16.473	2.522
b, c	59.056	4.131
b	165.848	6.559

Light parameters (CI-LP)				
$m_0 = 0$ $M_0 = 0.357$	$m_u = 0.007$ $M_u = 0.367$	$m_s = 0.17$ $M_s = 0.53$	$m_c = 1.58$ $M_c = 1.60$	$m_b = 4.83$ $M_b = 4.83$
Heavy parameters (CI-HP)				
$m_0 = 0$ $M_0 = 0.357$	$m_u = 0.007$ $M_u = 0.367$	$m_s = 0.17$ $M_s = 0.53$	$m_c = 1.08$ $M_c = 1.52$	$m_b = 3.92$ $M_b = 4.68$

L. X. Gutiérrez-Guerrero, A. Bashir, I. C. Cloët, and C. D. Roberts, Phys. Rev. C, 81, 065202 (2010).

- **With these parameters we calculate the masses of bound states of quarks and its first radial excitation.**

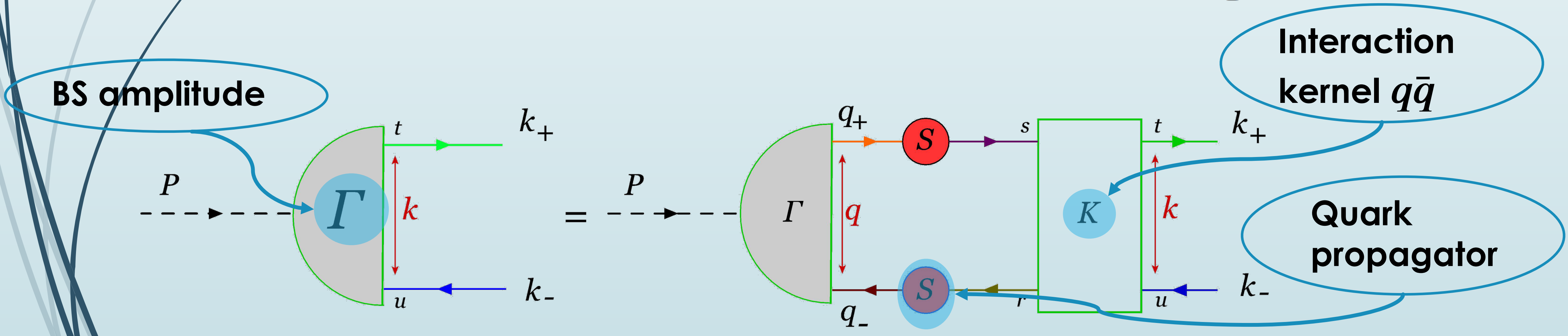
BETHE-SALPETER EQUATION

- Describes bound states of two particles such as mesons (flavored quarks f_1 and f_2).
- The mesons appear as poles of the Green function.
- BS equation:

$$\left[\Gamma_H^{f_1\bar{f}_2}(k;P)\right]_{tu} = \int \frac{d^4q}{(2\pi)^4} \left[\chi_H^{f_1\bar{f}_2}(q;P)\right]_{sr} K_{tu}^{rs}(q,k;P),$$

- BS wave function:

$$\chi_H^{f_1\bar{f}_2}(q;P) = S_{f_1}(q_+)\Gamma_H^{f_1\bar{f}_2}(q;P)S_{\bar{f}_2}(q_-),$$



E. E. Salpeter and H. A. Bethe, Phys. Rev. 84, 1232 (1951).

BS AMPLITUDES IN CI

- **Mesons types:**

Type	Form	Spin	L	Parity ($(-1)^{L+1}$)	J^{PC}
Scalar	$\bar{\psi}\psi$	0	1	+	0^{++}
Pseudoscalar	$\bar{\psi}\gamma^5\psi$	0	0	-	0^{-+}
Vector	$\bar{\psi}\gamma^\mu\psi$	1	0	-	1^{--}
Axial vector	$\bar{\psi}\gamma^\mu\gamma^5\psi$	1	1	+	1^{++}, 1^{+-}
Tensor	$\bar{\psi}\left(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\right)\psi$	2	1	+	2^{++}

- **BS amplitude:**

$$\Gamma_H^j(k;P) = \tau^j\gamma_5\left[iE_H(k;P) + \gamma\cdot P F_H(k;P) + \gamma\cdot k\, G_H(k;P) + \sigma_{\mu\nu}k_\nu P_\nu D_H(k;P)\right],$$

- **In the CI Model the BSA does not depend on k, the relative momentum:**

$$\begin{aligned}
\Gamma^{0^{++}}(P) &= \mathbb{1}E^{0^{++}}(P),\\
\Gamma^{0^{-+}}(P) &= \gamma_5\left[iE^{0^{-+}}(P) + \frac{\gamma\cdot P}{2M_R}F^{0^{-+}}(P)\right],\\
\Gamma_\mu^{1^{--}}(P) &= \gamma_\mu^T E^{1^{--}}(P) + \frac{1}{2M_R}\sigma_{\mu\nu}P_\nu F^{1^{--}}(P),\\
\Gamma_\mu^{1^{++}}(P) &= \gamma_5\left[\gamma_\mu^T E^{1^{++}}(P) + \frac{1}{2M_R}\sigma_{\mu\nu}P_\nu F^{1^{++}}(P)\right],
\end{aligned}$$

- **M_R is the reduced mass.**

MASSES OF MESONS

- **We solve the BS equation in the form of eigenequation:**

$$K_H(M_H) \cdot \Gamma_H(M_H) = \lambda_H(M_H)\Gamma_H(M_H),$$

- **We look for the solution with:** $\lambda_H(P^2 = -M_H^2) = 1$

$$\mathcal{K}_{PS}^{EE} = \frac{1}{3\pi^2 m_G^2} \int_0^1 dx \left[\left(M_{f_1} M_{\bar{f}_2} - x(1-x)P^2 - \mathfrak{M}^2\right)\overline{C}'(\mathfrak{M}, \tau_{ir}, \tau_{uv}) + C(\mathfrak{M}, \tau_{ir}, \tau_{uv})\right],$$

$$\mathcal{K}_{PS}^{EF} = \frac{1}{3\pi^2 m_G^2} \frac{P^2}{2M} \int_0^1 dx \left[M_{f_1} x + (1-x) M_{\bar{f}_2}\right]\overline{C}'(\mathfrak{M}, \tau_{ir}, \tau_{uv}),$$

$$\mathcal{K}_{PS}^{FE} = \frac{M}{3\pi^2 m_G^2} \int_0^1 dx \left[(1-x) M_{\bar{f}_2} + x M_{f_1}\right]\overline{C}'(\mathfrak{M}, \tau_{ir}, \tau_{uv}),$$

$$\mathcal{K}_{PS}^{FF} = -\frac{1}{3\pi^2 m_G^2} \frac{1}{2} \int_0^1 dx \left[M_{f_1} M_{\bar{f}_2} + M_{f_1}^2 x + M_{\bar{f}_2}^2 (1-x)\right]\overline{C}'(\mathfrak{M}, \tau_{ir}, \tau_{uv}),$$

Marco A. Bedolla, J. J. Cobos-Martínez, Adnan Bashir, Phys.Rev.D, 92, 054031 (2015).

FIRST RADIAL EXCITATION

- Estuding radial excited states shows a related zero, (Chebychev). This zero is seen from the relative momentum dependence.
- From the ABS independent of relative momentum, a zero cannot be obtained.
- A zero is inserted by hand in the Interaction Kernel expressions.

$$\Gamma_\pi(P) = -\frac{1}{3\pi^2}\frac{1}{m_g^2}\int_q^\Lambda \gamma_\mu\, S(q+P)\,\Gamma_\pi(P)\,S(q) \;\rightarrow\; \Gamma_\pi(P) = -\frac{1}{3\pi^2}\frac{1}{m_g^2}\int_q^\Lambda \gamma_\mu\, S(q+P)\,\Gamma_\pi(P)\,S(q)(1-q^2 d_F)$$

- Then $(1 - d_F\, q^2) \rightarrow 0$ when $q^2 = \frac{1}{d_F}$, in [1] they use $\frac{1}{d_F} = M^2(1 \pm 0.2)$,and in [2] they use $\frac{1}{d_F} = 2M^2(1 \pm 0.2)$.

[1] Hannes L. L. Roberts, Lei Chang, Ian C. Cloët , Craig D. Roberts Masses of ground and excited-state hadrons.
[2] Chen Chen, Lei Chang, Craig D. Roberts, Shaolong Wan, David J. Wilson Spectrum of hadrons with strangeness

FIRST RADIAL EXCITATION

- **Then by the regularization of the integrals the matrix elements change:**

$$\mathcal{K}^{\pi}_{EE} = \int_0^1 d\alpha \left[\mathcal{C}^{\rm iu}(\omega(M^2,\alpha,P^2)) - 2\alpha(1-\alpha)\,P^2\,\overline{\mathcal{C}}^{\rm iu}_1(\omega(M^2,\alpha,P^2))\right],$$
$$\mathcal{K}^{\pi}_{EF} = P^2 \int_0^1 d\alpha\,\overline{\mathcal{C}}^{\rm iu}_1(\omega(M^2,\alpha,P^2)),$$
$$\mathcal{K}^{\pi}_{FE} = \frac{1}{2}M^2 \int_0^1 d\alpha\,\overline{\mathcal{C}}^{\rm iu}_1(\omega(M^2,\alpha,P^2)),$$
$$\mathcal{K}^{\pi}_{FF} = -2\mathcal{K}_{FE}\,,$$

$$\mathcal{K}^{\pi^*}_{EE} = \int_0^1 d\alpha \left[\mathcal{F}^{\rm iu}(\omega(M^2,\alpha,P^2)) - 2\alpha(1-\alpha)\,P^2\,\overline{\mathcal{F}}^{\rm iu}_1(\omega(M^2,\alpha,P^2))\right],$$
$$\mathcal{K}^{\pi^*}_{EF} = P^2 \int_0^1 d\alpha\,\overline{\mathcal{F}}^{\rm iu}_1(\omega(M^2,\alpha,P^2)),$$
$$\mathcal{K}^{\pi^*}_{FE} = \frac{1}{2}M^2 \int_0^1 d\alpha\,\overline{\mathcal{F}}^{\rm iu}_1(\omega(M^2,\alpha,P^2)) - \frac{1}{2}M_0^2 \int_0^1 d\alpha\,\overline{\mathcal{F}}^{\rm iu}_1(\omega(M_0^2,\alpha,P^2)),$$
$$\mathcal{K}^{\pi^*}_{FF} = -2\mathcal{K}_{FE}\,.$$

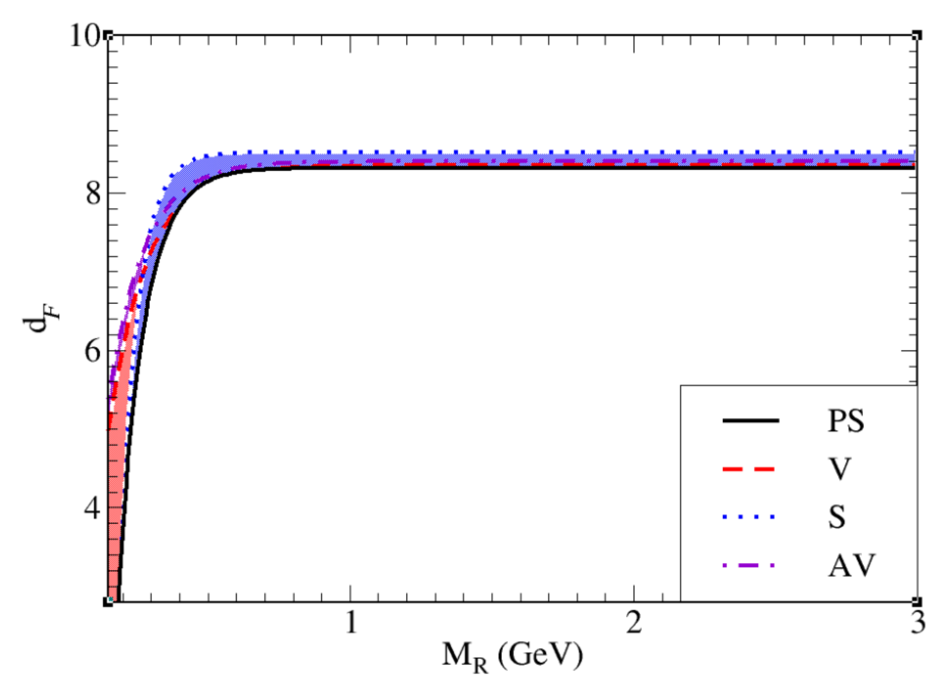
- **With:** $\mathcal{C}(M^2,\tau_{uv},\tau_{ir}) = \int_0^\infty ds\,\frac{s}{s+M^2}$ y $\mathcal{D}(M^2,\tau_{uv},\tau_{ir}) = \int_0^\infty ds\,\frac{s^2}{s+M^2}$

$$\mathcal{F}(M^2,\tau_{uv},\tau_{ir}) = \mathcal{C}(M^2,\tau_{uv},\tau_{ir}) - d_F\mathcal{D}(M^2,\tau_{uv},\tau_{ir})$$

- **Analyzing the phenomenology of the known experimental results we propose a functional form for $d_F(M_R)$:**

$$d_F(M_R) = a - be^{-cM_R} \quad (20)$$

Meson	a	b	c
PS	8.32	41.67	11.08
V	8.35	10.37	7.50
S	8.52	109.47	15.82
AV	8.44	10.00	8.00



RESULTS

- **Using the advocated parameters we obtain:**

S Mesons	Exp.	CI	E
$\sigma(u\bar{d})$	1.2	1.22	0.66
$K_0^*(u\bar{s})$	1.430	1.33	0.65
$f_0(s\bar{s})$	–	1.34	0.64
$D_0^*(c\bar{u})$	2.300	2.32	0.39
$D_{s0}^*(c\bar{s})$	2.317	2.43	0.37
$B_0^*(u\bar{b})$	–	5.50	0.21
$B_{s0}(s\bar{b})$	–	5.59	0.20
$B_{c0}(c\bar{b})$	–	6.45	0.08
$\chi_{c0}(c\bar{c})$	3.414	3.35	0.16
$\chi_{b0}(b\bar{b})$	9.859	9.50	0.04

PS Mesons	Exp.	CI	E	F
$\pi(u\bar{d})$	0.139	0.14	3.60	0.47
$K(u\bar{s})$	0.493	0.49	3.81	0.59
$h_s(s\bar{s})$	–	0.69	4.04	0.74
$D^0(c\bar{u})$	1.86	1.87	3.03	0.37
$D_s^+(c\bar{s})$	1.97	1.96	3.24	0.51
$B^+(u\bar{b})$	5.28	5.28	1.50	0.09
$B_s^0(s\bar{b})$	5.37	5.37	1.59	0.13
$B_c^+(c\bar{b})$	6.27	6.29	0.73	0.11
$\eta_c(c\bar{c})$	2.98	2.98	2.16	0.41
$\eta_b(b\bar{b})$	9.40	9.40	0.48	0.10

L. X. Gutiérrez-Guerrero,G. Paredes-Torres A. Bashir, Phys. Rev. D, 104, 094013 (2021).

- **Using the advocated parameters we obtain:**

V Mesons	Exp.	CI	E
$\rho\ (u\bar{d})$	0.78	0.93	1.53
$K_1\ (u\bar{s})$	0.89	1.03	1.62
$\phi\ (s\bar{s})$	1.02	1.12	1.73
$D^{*0}(c\bar{u})$	2.01	2.06	1.23
$D_s^*(c\bar{s})$	2.11	2.14	1.32
$B^{+*}(u\bar{b})$	5.33	5.33	0.65
$B_s^{0*}(s\bar{b})$	5.42	5.41	0.67
$B_c^*(c\bar{b})$	–	6.32	0.27
$J/\Psi\ (c\bar{c})$	3.10	3.15	0.61
$\Upsilon(b\bar{b})$	9.46	9.42	0.15

AV Mesons	Exp.	CI	E
$a_1(u\bar{d})$	1.260	1.37	0.32
$K_1(u\bar{s})$	1.34	1.48	0.32
$f_1(s\bar{s})$	1.43	1.58	0.32
$D_1(c\bar{u})$	2.420	2.41	0.20
$D_{s1}(c\bar{s})$	2.460	2.51	0.19
$B_1(u\bar{b})$	5.721	5.55	0.11
$B_{s1}(s\bar{b})$	5.830	5.64	0.10
$B_{cb}(c\bar{b})$	–	6.48	0.04
$\chi_{c1}(c\bar{c})$	3.510	3.40	0.08
$\chi_{b1}(b\bar{b})$	9.892	9.52	0.02

L. X. Gutiérrez-Guerrero, G. Paredes-Torres A. Bashir, Phys. Rev. D, 104, 094013 (2021).

MESONES RESULTS 1ST EXCITATION

- **Using the proposed parameters we obtain the following preliminary results:**

Pseudoscalar						Vector					
	n	Exp.	Others	CI	Diff. %		n	Exp.	Others	CI	Diff. %
$\pi(u\bar{d})$	0	0.139	-	0.14	0.72	$\rho(u\bar{d})$	0	0.78	-	0.93	19.23
	1	1.3	-	1.27	2.3		1	1.47	-	1.47	0
$K(s\bar{u})$	0	0.493	-	0.49	0.61	$K_1(s\bar{u})$	0	0.89	-	1.03	15.73
	1	1.46	-	1.51	3.43		1	1.68	-	1.63	2.98
$h_s(s\bar{s})$	0	-	-	0.69	-	$\phi(s\bar{s})$	0	1.02	-	1.12	9.80
	1	-	-	1.72	-		1	1.68	-	1.79	6.55
$D^0(c\bar{u})$	0	1.86	1.88 [75]	1.87	0.54	$D^{*0}(c\bar{u})$	0	2.01	2.04 [75]	2.06	2.49
	1	2.54 [24]	2.58 [75]	2.53	0.39		1	2.61 [24]	2.64 [75]	2.57	1.53
$D_s^+(c\bar{s})$	0	1.97	1.98 [75]	1.96	0.51	$D_s^*(c\bar{s})$	0	2.11	2.12 [75]	2.14	1.42
	1	-	2.67 [75]	2.78	-		1	2.70	2.73 [75]	2.78	2.96
$B^+(u\bar{b})$	0	5.28	5.28 [40]	5.28	0	$B^{+*}(u\bar{b})$	0	5.33	5.33 [40]	5.33	0
	1	5.86 [27]	5.91 [40]	5.68	3.07		1	5.97 [27]	5.94 [40]	5.68	4.86
$B_s^0(s\bar{b})$	0	5.37	5.36 [40]	5.37	0	$B_s^{0*}(s\bar{b})$	0	5.42	5.41 [40]	5.41	0.18
	1	-	5.98 [40]	5.94	-		1	-	6.0 [40]	5.91	-
$\eta_c(c\bar{c})$	0	2.98	2.93 [76]	2.98	0	$J/\Psi(c\bar{c})$	0	3.10	3.11 [76]	3.15	1.61
	1	3.64	3.68 [76]	3.66	0.55		1	3.686	3.7 [76]	3.92	6.35
$B_c^+(c\bar{b})$	0	6.27	6.27 [77]	6.28	0.16	$B_c^*(c\bar{b})$	0	6.27 [29]	6.33 [77]	6.32	-
	1	6.87	6.87 [77]	6.80	1.02		1	6.84 [29]	6.89 [77]	6.85	-
$\eta_b(b\bar{b})$	0	9.40	9.41 [76]	9.40	0	$\Upsilon(b\bar{b})$	0	9.46	9.49 [76]	9.42	0.42
	1	9.99	9.99 [76]	9.68	3.10		1	10.023	10.9 [76]	9.71	3.12

MESONES RESULTS 1ST EXCITATION

Scalar						Axial - Vector					
	n	Exp.	Others	CI	Diff. %		n	Exp.	Others	CI	Diff. %
$\sigma(u\bar{d})$	0	1.2	-	1.22	1.66	$a_1(u\bar{d})$	0	1.260	-	1.37	8.73
	1	-	1.358 [78]	1.34	-		1	1.65	-	1.58	4.01
$K_0^*(u\bar{s})$	0	1.430	-	1.33	6.99	$K_1(u\bar{s})$	0	1.34	-	1.48	10.44
	1	-	1.53 [4]	1.57	-		1	-	1.57 [4]	1.72	-
$f_0(s\bar{s})$	0	-	-	1.34	-	$f_1(s\bar{s})$	0	1.43	-	1.58	10.49
	1	-	-	1.82	-		1	-	1.67 [4]	1.88	-
$D_0^*(c\bar{u})$	0	2.30	2.45 [75]	2.32	0.87	$D_1(c\bar{u})$	0	2.420	2.5 [75]	2.41	0.41
	1	-	2.924 [75]	2.63	-		1	-	2.931 [75]	2.63	-
$D_{s0}^*(c\bar{s})$	0	2.317	2.55 [75]	2.43	4.88	$D_{s1}(c\bar{s})$	0	2.460	2.6 [75]	2.51	2.03
	1	3.044 [25]	3.018 [75]	3.27	7.42		1	-	3.005 [75]	2.90	-
$B_0^*(u\bar{b})$	0	-	5.72 [40]	5.50	-	$B_1(u\bar{b})$	0	5.721	5.77 [40]	5.55	2.99
	1	-	6.185 [40]	5.82	-		1	-	6.145 [40]	5.74	-
$B_{s0}(s\bar{b})$	0	-	5.80 [40]	5.59	-	$B_{s1}(s\bar{b})$	0	5.830	5.85 [40]	5.64	3.26
	1	-	6.241 [40]	6.57	-		1	-	6.2013 [40]	6.05	-
$\chi_{c0}(c\bar{c})$	0	3.414	3.32 [76]	3.35	1.87	$\chi_{c1}(c\bar{c})$	0	3.510	3.49 [76]	3.40	3.13
	1	-	3.83 [76]	4.7	-		1	-	3.67 [76]	4.19	-
$B_{c0}(c\bar{b})$	0	-	6.76 [77]	6.45	-	$B_{cb}(c\bar{b})$	0	-	6.71 [77]	6.48	-
	1	-	7.134 [77]	6.88	-		1	-	7.107 [77]	7.05	-
$\chi_{b0}(b\bar{b})$	0	9.859	9.815 [76]	9.50	3.64	$\chi_{b1}(b\bar{b})$	0	9.892	9.842 [76]	9.52	3.76
	1	10.232	10.254 [76]	10.234	0.02		1	10.255	10.120 [76]	9.53	7.07

- The Gell-Mann-Okubo formula provides a sum rule for the masses of the hadrons in a specific multiplet :

$$M = M_0 + aY + b\left[I(I+1) - \frac{Y^2}{4}\right],$$

$$Y = B + S + C + B',$$

$$B = (N_q - N_{\bar{q}})/3,$$

$$S = N_{\bar{s}} - N_s ,$$

$$C = N_c - N_{\bar{c}} ,$$

$$B' = N_{\bar{b}} - N_b$$

- Mass relation for pseudoscalar mesons is :

$$m_{D_s^+(c\bar{s})} - m_{D^0(c\bar{u})} + m_{B^+(u\bar{b})} - m_{B_s^0(s\bar{b})} = 0.$$

- Mass relation for vector mesons is :

$$m_{D_s^*(c\bar{s})} - m_{D^{0*}(c\bar{u})} + m_{B^{+*}(u\bar{b})} - m_{B_s^{0*}(s\bar{b})} = 0.$$

M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

S. Okubo, Prog. Theor. Phys. 27, 949 (1962).

RELATIONSHIPS BETWEEN THE MESONS

- **The Gell-Mann-Okubo relations for the excited states give the following results:**

$$1\,{}^1S_0 \;:\; m_{D_s^+(c\bar{s})} - m_{D^0(c\bar{u})} + m_{B^+(u\bar{b})} - m_{B_s^0(s\bar{b})} = 0,$$
$$m_{\eta_c(c\bar{c})} + m_{\eta_b(b\bar{b})} - 2m_{B_c^+(c\bar{b})} = 0,$$
$$1\,{}^1P_1 \;:\; m_{D_{s0}^*(c\bar{s})} - m_{D_0^*(c\bar{u})} + m_{B_0^*(u\bar{b})} - m_{B_{s0}(s\bar{b})} = 0.$$

$$1\,{}^3S_1 \;:\; m_{D_s^*(c\bar{s})} - m_{D^{0*}(c\bar{u})} + m_{B^{+*}(u\bar{b})} - m_{B_s^{0*}(s\bar{b})} = 0,$$
$$m_{J/\Psi(c\bar{c})} + m_{\Upsilon(bb)} - 2m_{B_c^*(c\bar{b})} = 0$$
$$1\,{}^3P_0 \;:\; m_{D_{s1}(c\bar{s})} - m_{D_1(c\bar{u})} + m_{B_1(u\bar{b})} - m_{B_{s1}(s\bar{b})} = 0.$$

$$m_{B_c^*(c\bar{b})} - m_{B_s^{0*}(s\bar{b})} - m_{B_c^+(c\bar{b})} + m_{B_s^0(s\bar{b})} \approx 0\,, \quad (20)$$
$$m_{B_s^{0*}(s\bar{b})} - m_{B^{+*}(u\bar{b})} - m_{B_s^0(s\bar{b})} + m_{B^+(u\bar{b})} = 0\,, \quad (21)$$
$$m_{B_s^{0*}(s\bar{b})} - m_{B^{+*}(u\bar{b})} - m_{D_s^+(c\bar{s})} + m_{D^0(c\bar{u})} = 0\,, \quad (22)$$
$$m_{\eta_b(b\bar{b})} - m_{\eta_c(c\bar{c})} - 2m_{B_s^{0*}(s\bar{b})} + 2m_{D_s^*(c\bar{s})} \approx 0\,, \quad (23)$$
$$m_{\eta_b(b\bar{b})} - m_{\eta_c(c\bar{c})} - 2m_{B_s^0(s\bar{b})} + 2m_{D_s^+(c\bar{s})} = 0\,, \quad (24)$$
$$m_{B_s^{0*}(s\bar{b})} - m_{D_s^*(c\bar{s})} - m_{B_s^0(s\bar{b})} + m_{D_s^+(c\bar{s})} = 0\,. \quad (25)$$
$$m_{\Upsilon(b\bar{b})} - m_{J/\Psi(c\bar{c})} - 2m_{B_s^0(s\bar{b})} + 2m_{D_s^+(c\bar{s})} = 0\,. \quad (26)$$
$$m_{\Upsilon(b\bar{b})} - m_{J/\Psi(c\bar{c})} - m_{\eta_b(b\bar{b})} + m_{\eta_c(c\bar{c})} \approx 0\,. \quad (27)$$
$$m_{\Upsilon(b\bar{b})} - m_{J/\Psi(c\bar{c})} - 2m_{B_s^{0*}(s\bar{b})} + 2m_{D_s^*(c\bar{s})} \approx 0\,. \quad (28)$$

$$n\;{}^{2S+1}L_J$$

Peudoscalar mesons	$\rightarrow$	n $^1S_0$
Vector mesons	$\rightarrow$	n $^3S_1$
Scalar mesons	$\rightarrow$	n $^1P_1$
Vector-Axial mesons	$\rightarrow$	(n $^3P_0$, n $^3P_1$, n $^3P_2$)

	n	Eq. (20)	Eq. (21)	Eq. (22)	Eq. (23)	Eq. (24)	Eq. (25)	Eq. (26)	Eq. (27)	Eq. (28)
CI	$n = 0$	−0.01	−0.01	−0.01	−0.12	−0.4	−0.14	−0.55	−0.15	−0.27
Exp.	$n = 0$	...	0.0	−0.02	−0.2	−0.38	−0.09	−0.44	−0.06	−0.26
CI	$n = 1$	0.08	−0.03	−0.02	−0.48	−0.54	−0.03	−0.53	0.01	−0.47

CONCLUSIONS

- The CI model is easy to implement and teaches us a lot about hadrons.

- The funtional form for df(MR) is interesting and we are trying to understand all its implications.

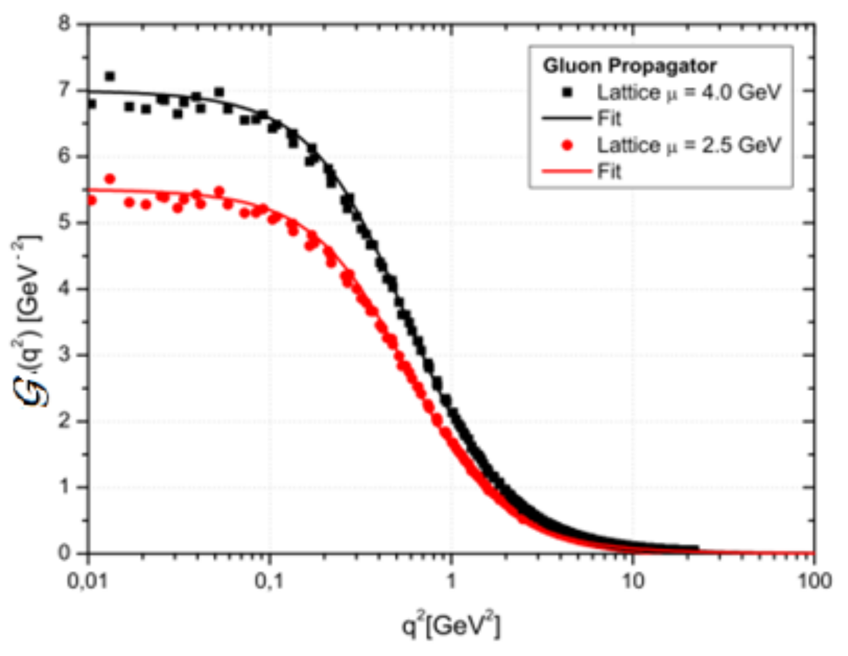
- The results obtained are in agreement with other theoretical approaches as well as with the experimental results that already exist.

- L. X. Gutiérrez-Guerrero, G. Paredes-Torres, and A. Bashir. Mesons and baryons: Parity partners. Phys. Rev. D, 104(9):094013, 2021.

Thank you

CONTACT INTERACTION MODEL

- It is one of the most drastic approximations of this model. However, according to our present knowledge, the gluon propagator does become constant in the infrared, at least to the Landau gauge. Therefore, in the Landau gauge it is a very good approximation for all those observables that only depend on infrared physics such as the hadron mass.



Función de vestimento del gluon

$$K(k,q,P)_{tu}^{rs} = -\frac{1}{m_G^2}\delta_{\mu\nu}\left[\frac{\lambda^a}{2}\gamma_\mu\right]\left[\frac{\lambda^a}{2}\gamma_\nu\right]_{ru}$$

- **Pseudoscalar meson radial excitations. A. Holl, A. Krassnigg, C.D. Roberts (Jun, 2004) Published in: Phys.Rev.C 70 (2004) 042203**

In Fig. 1 we depict the lowest Chebyshev moments of the pseudoscalar amplitude in Eq. (13), i.e., ${}^{0,2}E_{\pi_0}(k^2)$ and ${}^{0,2,4}E_{\pi_1}(k^2)$, where

$$ {}^{i}E_{\pi_{0,1}}(k^2) = \frac{2}{\pi}\int_0^\pi d\beta\, \sin^2\beta U_i(\cos\beta) E_{\pi_{0,1}}(k^2, k\cdot P; P^2), \tag{29}$$

with $U_i(x)$ a Chebyshev polynomial of the second kind and $k\cdot P := \cos\beta\sqrt{k^2P^2}$. The odd moments, $i=1,3,5,\ldots$, etc.,

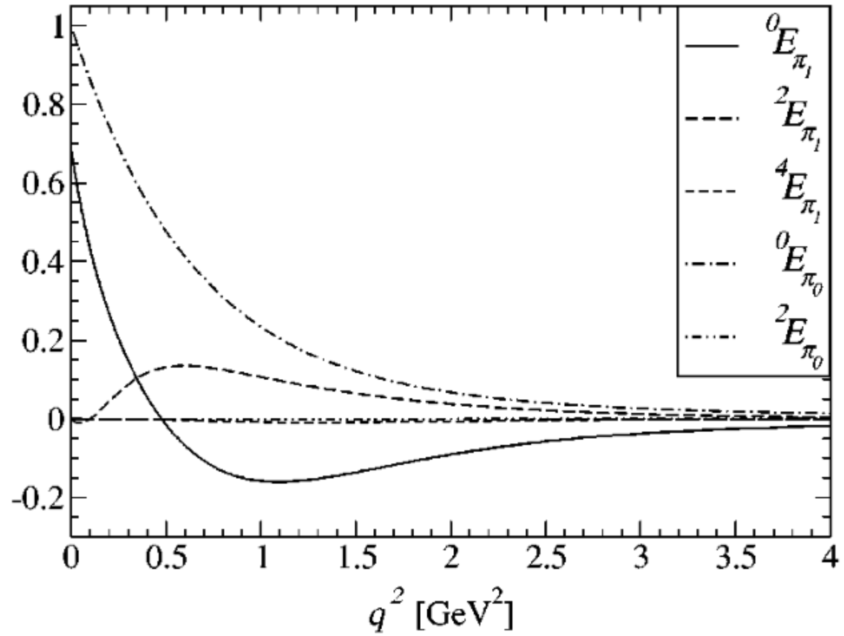
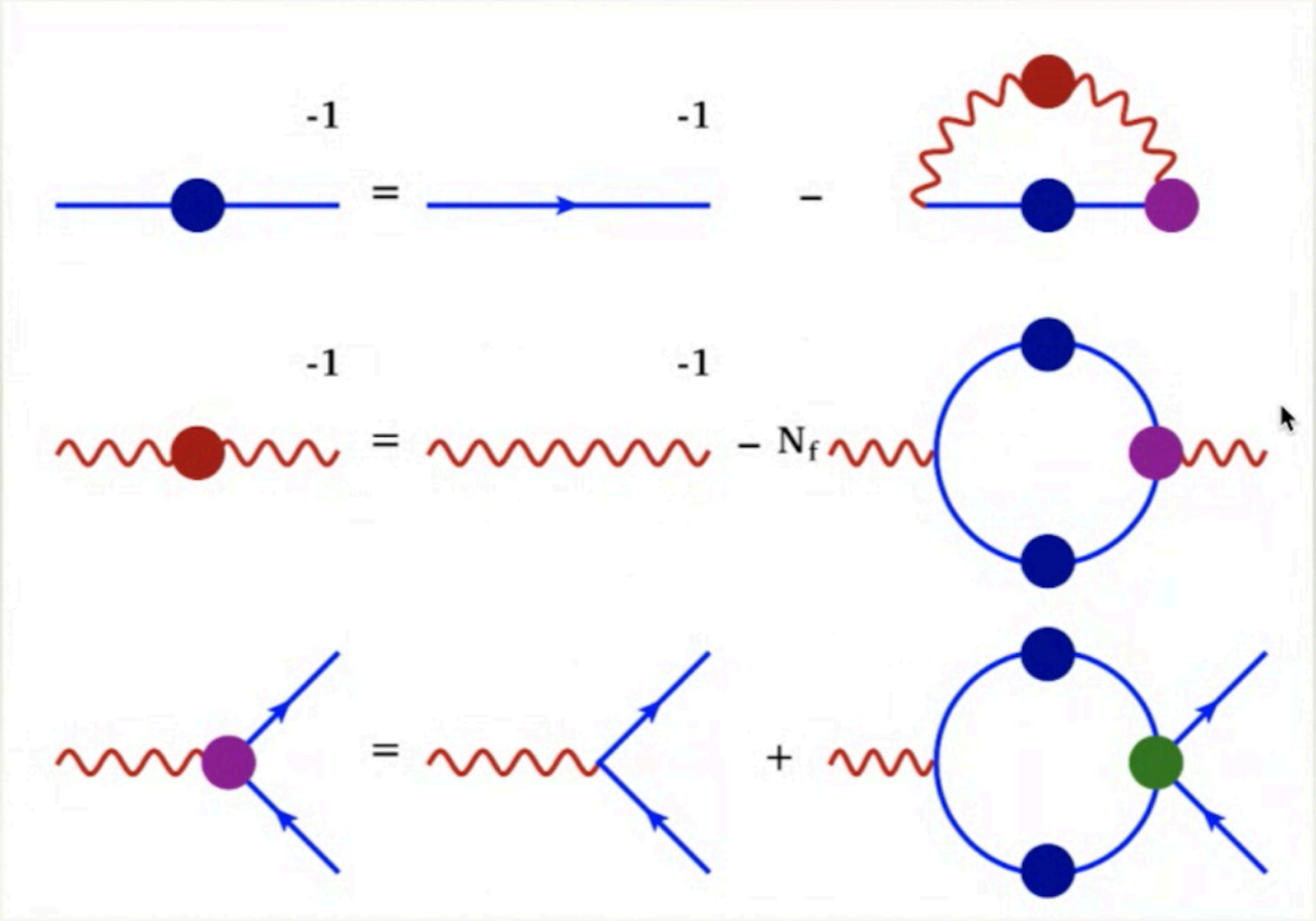


FIG. 1. Dimensionless low-order Chebyshev moments of the scalar function that characterizes the dominant amplitude in Eq. (13) for the ground (π_0) and first excited (π_1) states.

vanish in the case of equal mass constituents. The Chebyshev moments are obtained from the canonically normalized Bethe-Salpeter amplitudes but, for illustrative simplicity, the functions depicted are rescaled by the positive constant ${}^{0}E_{\pi_0}\,(k^2=0)$.

All Chebyshev moments of E_{π_1} possess a single zero, whereas those of E_{π_0} exhibit none. This similarity to the wave functions of radial excitations in quantum mechanics is not particular to manifestly covariant BSE studies [25,26]. It is evident that the zeroth Chebyshev moment almost completely determines the pseudoscalar amplitude in the ground state pseudoscalar meson. For the first excited state, however, the second moment is also required to obtain a good approximation to $E_{\pi_1}(k;P)$. The pseudovector and pseudotensor amplitudes are nonzero in the ground and first excited states, and they are materially important in the calculation of their properties. The bulk qualitative features of the scalar functions characterizing these amplitudes are the same as those described in connection with the pseudoscalar amplitude.


-1
=
-1
–
-1
=
-1
– N_f
=
+