The role of the rho(1450) in low energy observables

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Based on hep-ph
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Outline

- The importance of the rho(1450) state
- Description of low energy processes
- Global analysis. From decay modes to cross section data
- Results and Conclusions

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The importance of the rho(1450) state

Citation: R.L. Workman *et al.* (Particle Data Group), Prog.Theor.Exp.Phys. **2022**, 083C01 (2022)

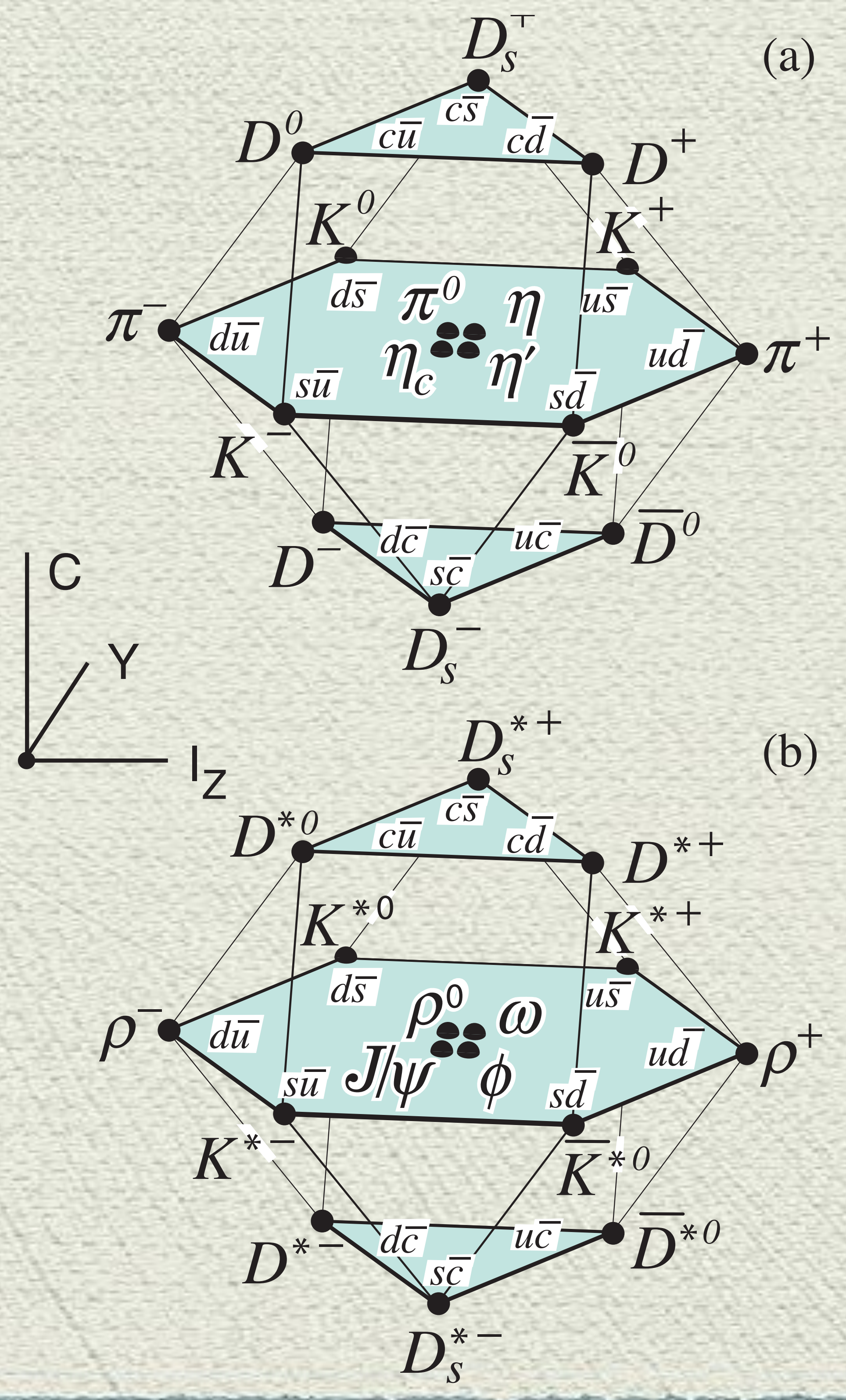
$\rho(1450)$

$I^G(J^{PC}) = 1^+(1^{--})$

ρ(1450) MASS

ρ(1450) MASS

VALUE (MeV)	*DOCUMENT ID*
1465±25 OUR ESTIMATE This is only an educated guess; the error given is larger than the error on the average of the published values.	



Rho(770) excited state

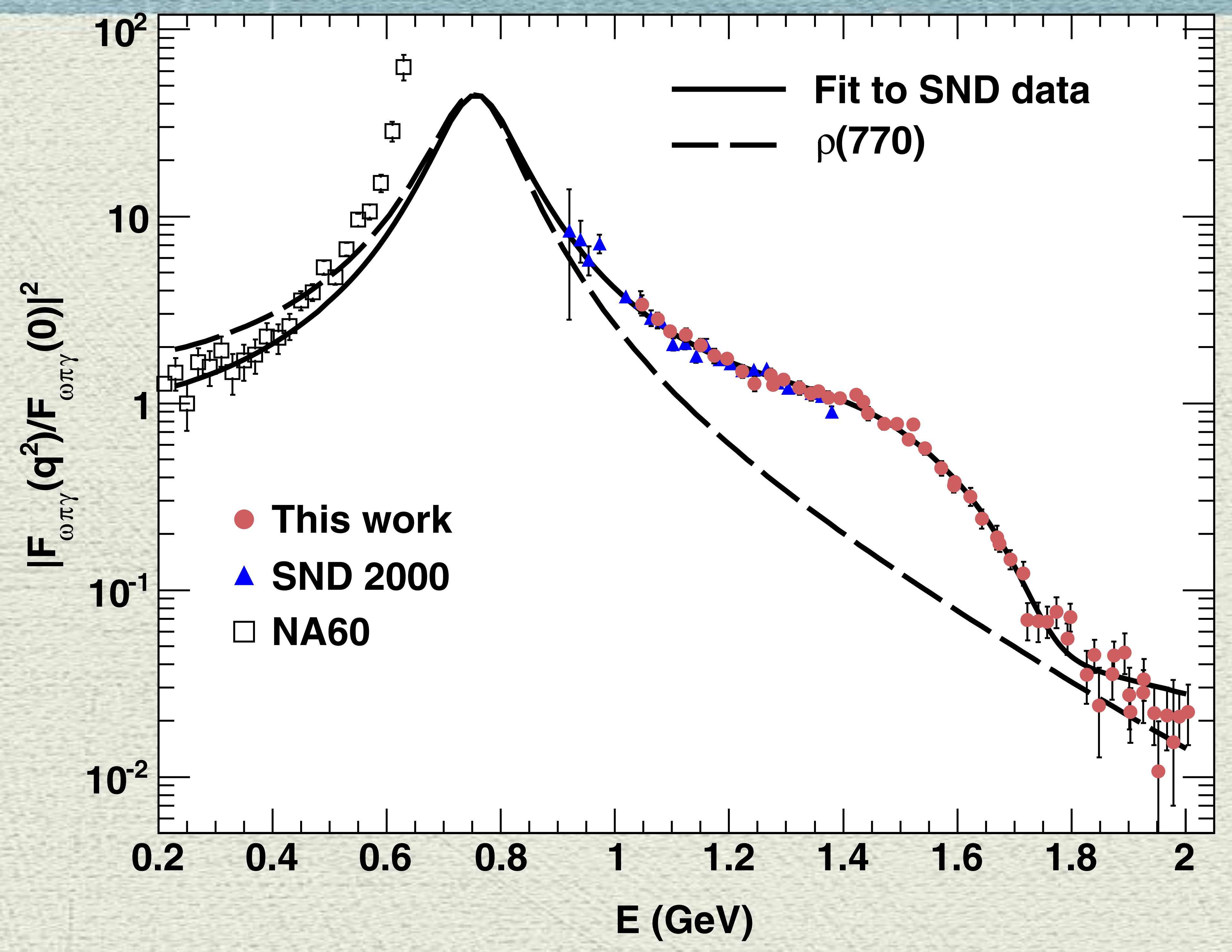
Parameters are not settled
PDG

ρ(1450) DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi$	seen
Γ_2	$\pi^+\pi^-$	seen
Γ_3	4π	seen
Γ_4	$\omega\pi$	
Γ_5	$a_1(1260)\pi$	
Γ_6	$h_1(1170)\pi$	
Γ_7	$\pi(1300)\pi$	
Γ_8	$\rho\rho$	
Γ_9	$\rho(\pi\pi)_{S\text{-wave}}$	
Γ_{10}	e^+e^-	seen
Γ_{11}	$\eta\rho$	seen
Γ_{12}	$a_2(1320)\pi$	not seen
Γ_{13}	$K\bar{K}$	seen
Γ_{14}	K^+K^-	seen
Γ_{15}	$K\bar{K}^*(892)$ + c.c.	possibly seen
Γ_{16}	$\pi^0\gamma$	
Γ_{17}	$\eta\gamma$	seen
Γ_{18}	$f_0(500)\gamma$	not seen
Γ_{19}	$f_0(980)\gamma$	not seen
Γ_{20}	$f_0(1370)\gamma$	not seen
Γ_{21}	$f_2(1270)\gamma$	not seen

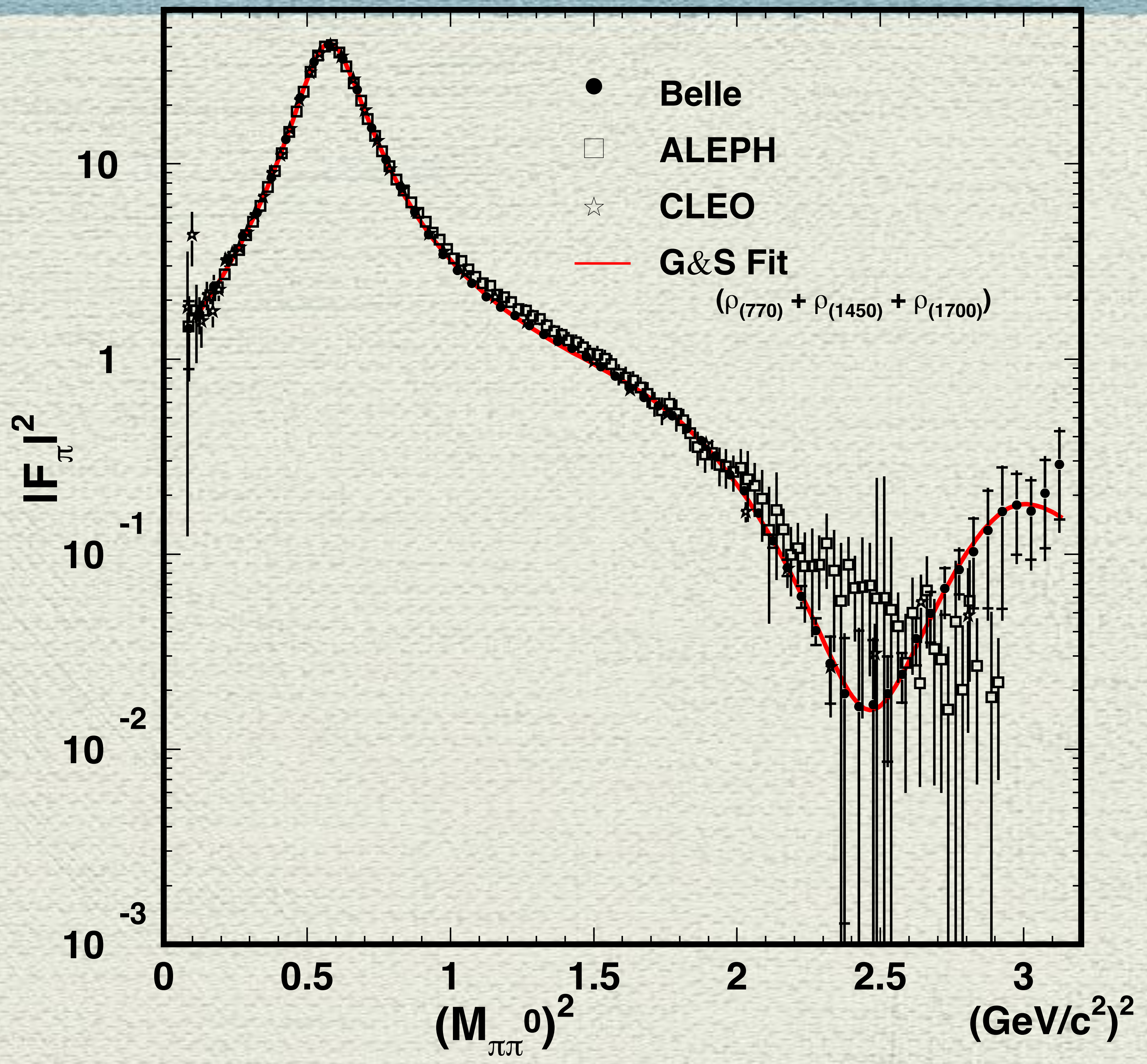
The importance …

Observed in tau decays and e+e- -> hadrons cross sections



e+e- -> pi pi gamma cross section

Phys.Rev.D 94 (2016) 11, 112001



tau -> pi pi nu decay

Phys.Rev.D 78 (2008) 072006

Contribution to muon g-2, rho MDM from e+e- -> 4 pi, pion form factor, etc

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Description of low energy processes

Vector meson dominance approach

$$\mathcal{L} = \sum_{V=\rho,\,\rho'} g_{V\pi\pi}\,\epsilon_{abc}\,V_\mu^a\,\pi^b\,\partial^\mu\,\pi^c + \sum_{V=\rho,\,\rho'} g_{\omega V\pi}\,\delta_{ab}\,\epsilon^{\mu\nu\lambda\sigma}\,\partial_\mu\,\omega_\nu\,\partial_\lambda\,V_\sigma^a\,\pi^b$$

$$+\; g_{3\pi}\,\epsilon_{abc}\,\epsilon^{\mu\nu\lambda\sigma}\,\omega_\mu\,\partial_\nu\,\pi^a\,\partial_\lambda\,\pi^b\,\partial_\sigma\,\pi^c + \sum_{V=\rho,\,\rho',\,\omega} \frac{e\,m_V^2}{g_V}\,V_\mu\,A^\mu.$$

Consider hadrons as the relevant degrees of freedom at low energies.
Couplings are free parameters to be determined from experiment.

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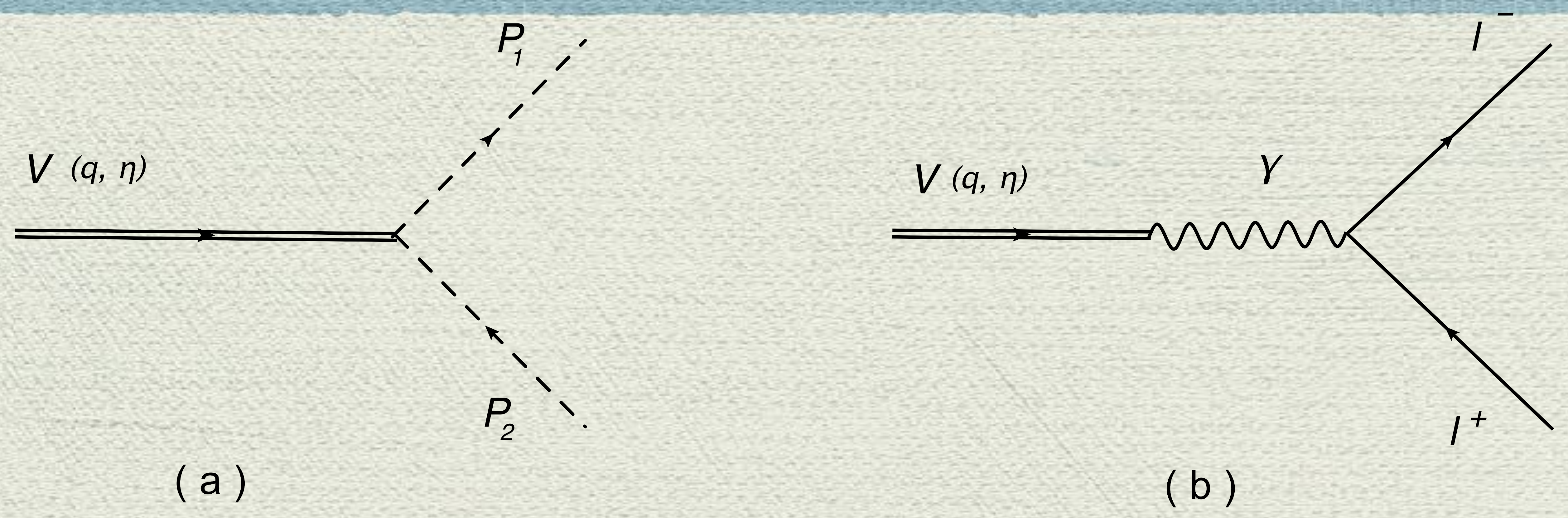
V -> PP and V -> l^+ l^- decays



Corresponding amplitudes and decay widths

$$\mathcal{M} = i\, g_{V P_1 P_2}\, (p_1 - p_2)^\mu\, \eta_\mu(q)$$

$$\Gamma_{V P_1 P_2} = \frac{g^2_{V P_1 P_2}\, \lambda^{3/2}(m_V^2, m_{P_1}^2, m_{P_2}^2)}{48\, \pi\, m_V^5}$$

g_{VPP}

$$\mathcal{M} = -i\, \frac{e^2}{g_V}\, \bar{u}(l_1)\, \gamma^\nu\, v(l_2)\, \eta_\nu(q)$$

$$\Gamma_{V\ell\ell} = \frac{4\, \pi\, \alpha^2 (2\, m_{L_1}^2 + m_V^2)\, (m_V^2 - 4\, m_{L_1}^2)^{1/2}}{3\, m_V^2\, g_V^2}.$$

g_V

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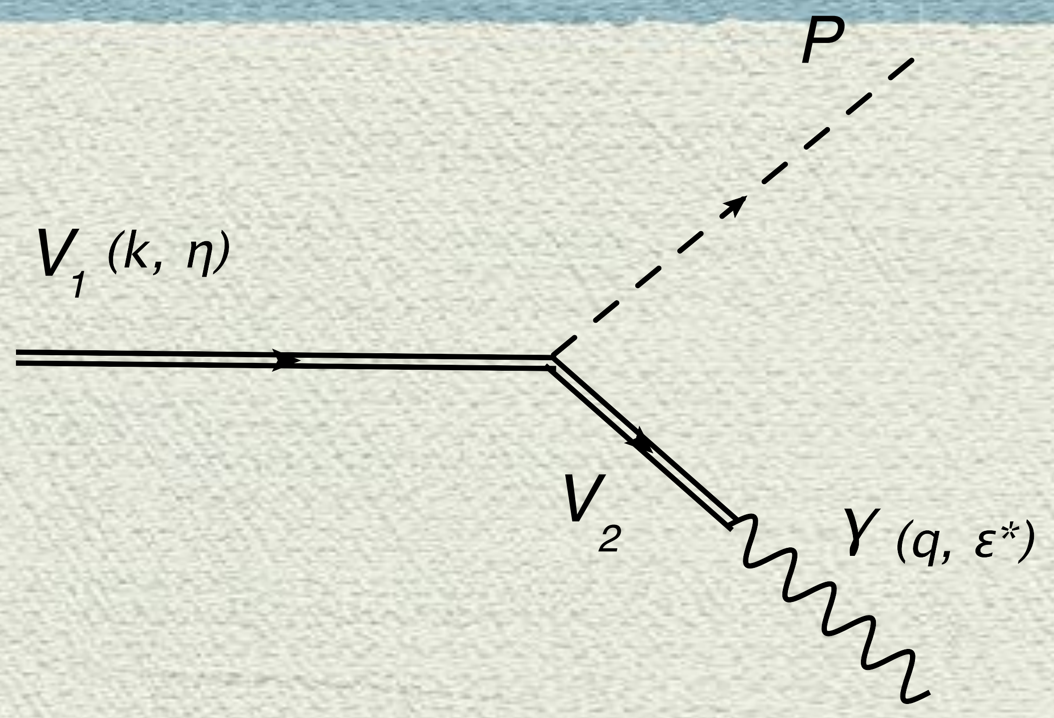
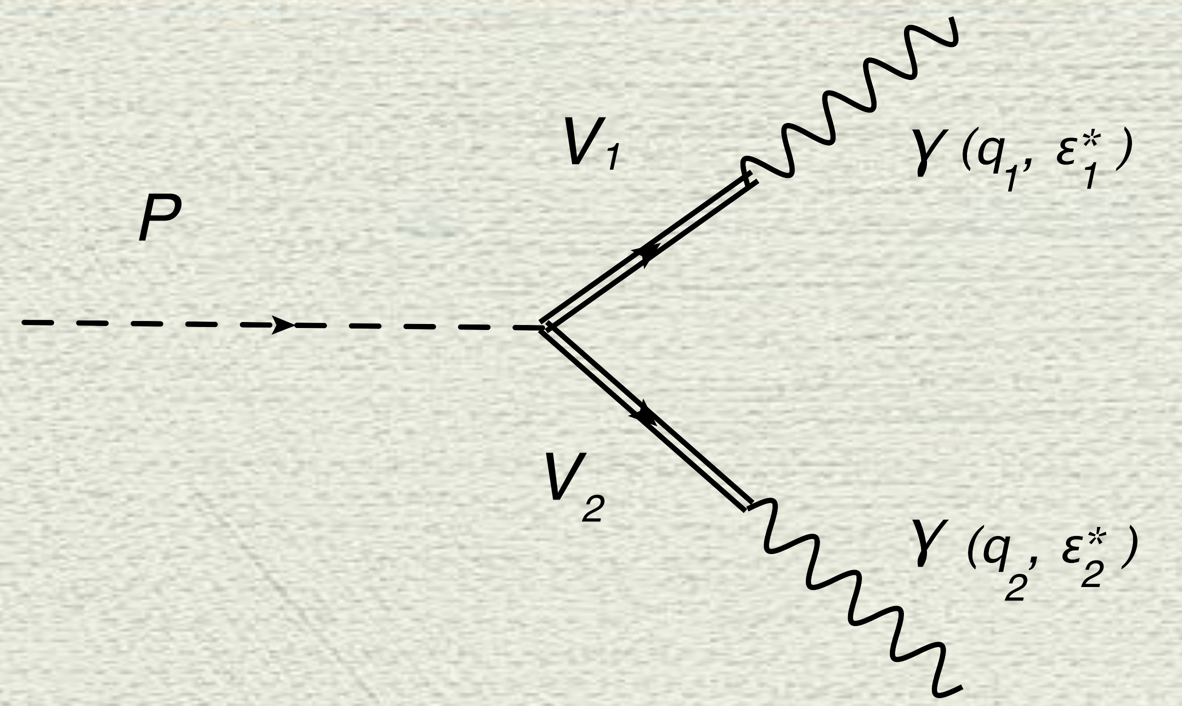
Radiative decays



(a)



(b)

Corresponding amplitudes and decay widths

$$\mathcal{M} = i\, g_{V_1 P\gamma}\, \epsilon^{\beta\nu\alpha\mu}\, k_\beta\, q_\alpha\, \eta_\mu\, \epsilon^*_\nu \qquad \Gamma_{V_1 P\gamma} = g^2_{V_1 P\gamma}\left[\frac{(m^2_{V_1} - m^2_P)^3}{96\,\pi\, m^3_P}\right].$$

$\mathbf{g_{VPV}}$

$$\mathcal{M} = i\, g_{P\gamma\gamma}\, \epsilon^{\alpha\mu\beta\nu}\, q_{1\beta}\, q_{2\alpha}\, \epsilon^*_{1\mu}\, \epsilon^*_{2\nu} \qquad \Gamma_{P\gamma\gamma} = \left[\frac{g^2_{P\gamma\gamma}\, m^3_P}{32\,\pi}\right].$$

$\mathbf{g_{PVV}}$

Couplings from individual decays

Process	Coupling
$\rho^0(770) \to \pi^+\pi^-$	5.944 ± 0.018
$\rho^+(770) \to \pi^+\pi^0$	5.978 ± 0.048
Weighted Average	5.953 ± 0.017

Process	$g_{\rho\omega\pi}$ (GeV^{-1})
$\omega(782) \to \pi^0\,\gamma$	11.489 ± 0.039
$\rho^0(770) \to \pi^0\,\gamma$	14.224 ± 2.227
$\rho^+(770) \to \pi^+\,\gamma$	12.358 ± 1.806
$\pi^0 \to \gamma\,\gamma$	15.631 ± 1.6121

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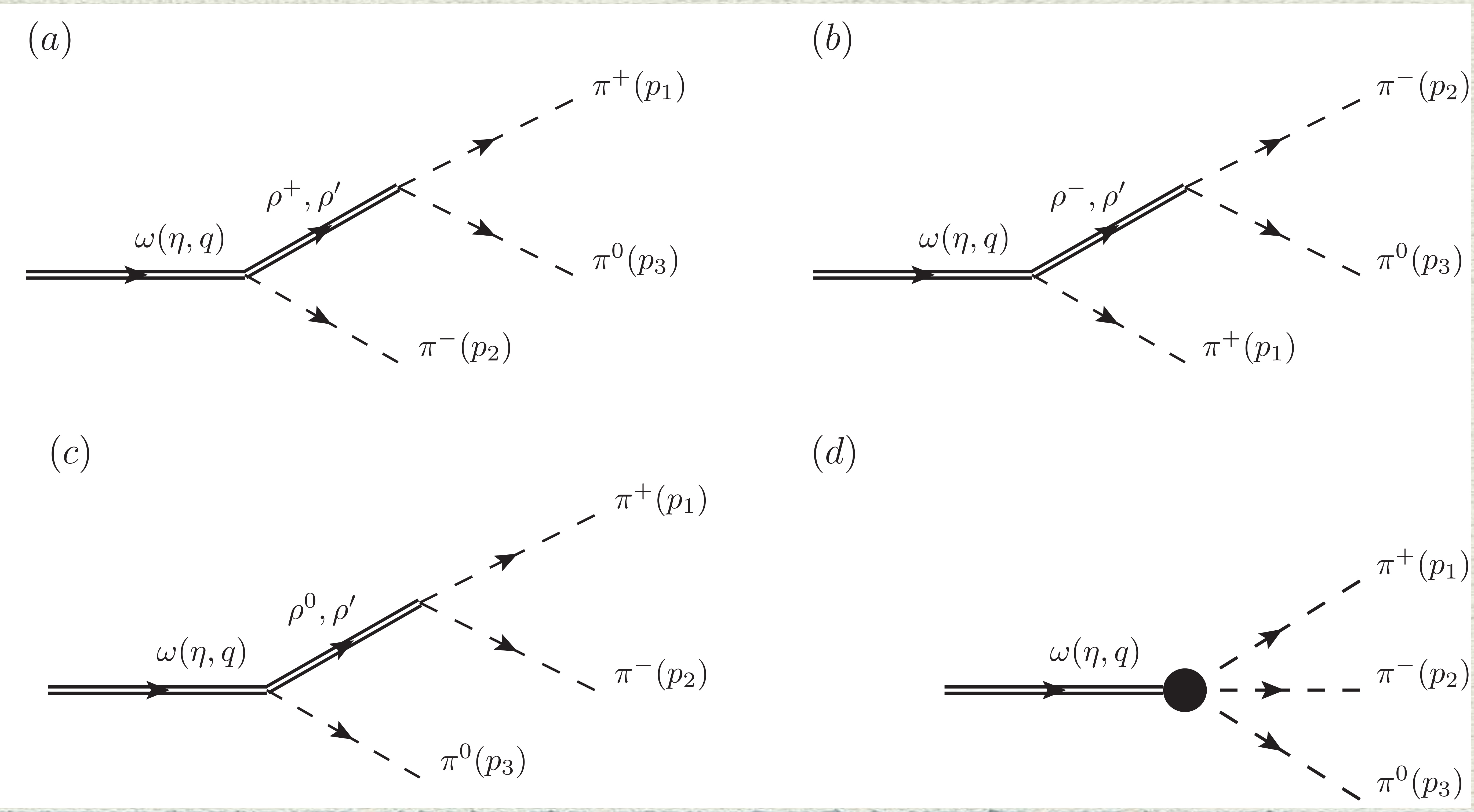
Omega -> 3pi
rho(770), rho(1450), contact

rho(1450) + contact contributions needed to describe the process

$$\mathcal{L} = \sum_{V=\rho,\,\rho'} g_{V\pi\pi}\,\epsilon_{abc}\,V^a_\mu\,\pi^b\,\partial^\mu\,\pi^c + \sum_{V=\rho,\,\rho'} g_{\omega V\pi}\,\delta_{ab}\,\epsilon^{\mu\nu\lambda\sigma}\,\partial_\mu\,\omega_\nu\,\partial_\lambda\,V^a_\sigma\,\pi^b$$

$$+\; g_{3\pi}\,\epsilon_{abc}\,\epsilon^{\mu\nu\lambda\sigma}\,\omega_\mu\,\partial_\nu\,\pi^a\,\partial_\lambda\,\pi^b\,\partial_\sigma\,\pi^c + \sum_{V=\rho,\,\rho',\,\omega} \frac{e\,m_V^2}{g_V}\,V_\mu\,A^\mu.$$



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Omega -> 3 pi decay width

The amplitude $\mathcal{M}_{\omega\to 3\pi} = i\,\epsilon_{\mu\alpha\beta\gamma}\,\eta^\mu\, {p_1}^\alpha\, {p_2}^\beta\, {p_3}^\gamma\, \mathcal{A}(m_\omega^2),$

$$\mathcal{A}(m_\omega^2) \;=\; 6\,g_{3\pi} + 2\,g_{\omega\rho\pi}\,g_{\rho\pi\pi}\,\left(D_{\rho^0}[s_{12}] + D_{\rho^+}[s_{13}] + D_{\rho^-}[s_{23}]\right)$$
$$+2\,g_{\omega\rho'\pi}\,g_{\rho'\pi\pi}\,\left(D_{\rho'}[s_{12}] + D_{\rho'}[s_{13}] + D_{\rho'}[s_{23}]\right),$$

g_{3Pi}

$g_{VV'P}$

$$s_{ij} = p_i + p_j, \qquad D_V[p] = 1/(p^2 - m_V^2 + \imath\, m_V\,\Gamma_V)$$

Decay width as a function of all the couplings involved

$$\Gamma_{\omega 3\pi} \;=\; A_1\,g_{3\pi}^2 + A_2\,g_{\omega\rho\pi}^2\,g_{\rho\pi\pi}^2 + A_3\,g_{3\pi}\,g_{\omega\rho\pi}\,g_{\rho\pi\pi} + A_4\,g_{\omega\rho'\pi}^2\,g_{\rho'\pi\pi}^2$$
$$+A_5\,g_{\omega\rho'\pi}\,g_{3\pi}\,g_{\rho'\pi\pi} + A_6\,g_{\omega\rho'\pi}\,g_{\omega\rho\pi}\,g_{\rho\pi\pi}\,g_{\rho'\pi\pi},$$

The Ai coefficients are computed at the omega mass energy

e+e- -> omega -> 3 pi

Similar to the previous contribution but adding the omega production process from e+e-

Amplitude

$$\mathcal{M}_{e^+\,e^-\to 3\,\pi} = \frac{e}{q^2}\frac{m_\omega^2}{g_\omega}\,D_\omega(q)\,\mathcal{A}(q^2)\,\epsilon_{\mu\alpha\beta\gamma}\,{p_1}^\alpha {p_2}^\beta {p_3}^\gamma\, l^\mu$$

Cross section as a function of all the couplings involved

$$\sigma(e^+e^- \to \omega \to 3\,\pi) \;=\; \frac{1}{g_\omega^2}\Big(B_1\, g_{3\pi}^2 + B_2\, g_{\omega\rho\pi}^2\, g_{\rho\pi\pi}^2 + B_3\, g_{3\pi}\, g_{\omega\rho\pi}\, g_{\rho\pi\pi} + B_4\, g_{\omega\rho'\pi}^2\, g_{\rho'\pi\pi}^2 + B_5\, g_{\omega\rho'\pi}\, g_{3\pi}\, g_{\rho'\pi\pi} + B_6\, g_{\omega\rho'\pi}\, g_{\omega\rho\pi}\, g_{\rho\pi\pi}\, g_{\rho'\pi\pi}\Big),$$

The Bi coefficients are computed at each energy data of the experimental cross section

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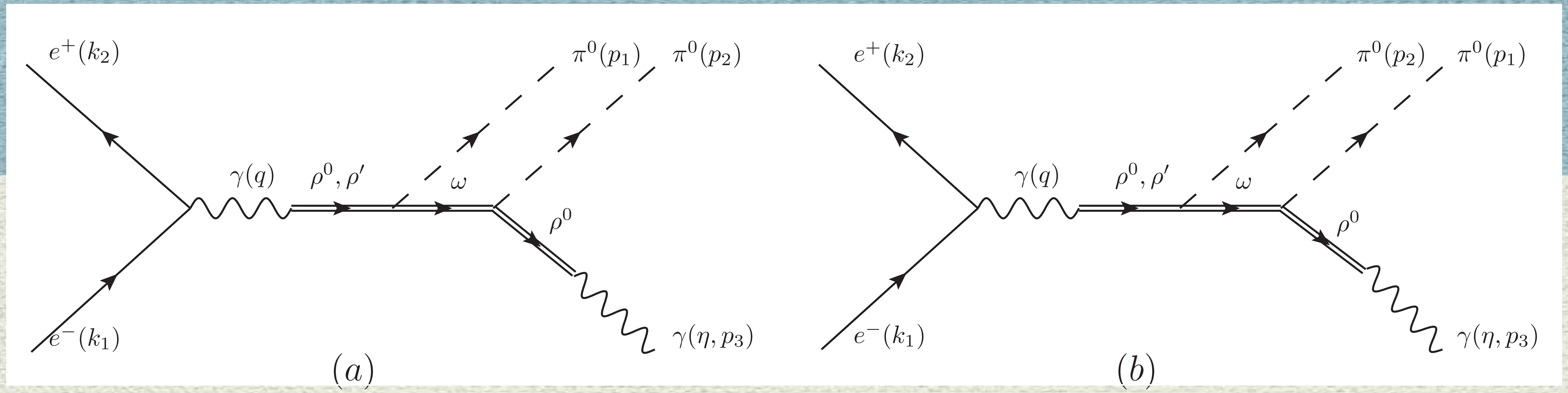
e+e- -> 2pi gamma



Amplitude $$\mathcal{M}_{(a)} = \frac{e^2}{q^2}\left(C_{\rho^0} + e^{i\theta}C_{\rho'}\right) D_\omega(q-p_1)\,\epsilon_{\mu\sigma\epsilon\lambda}\, q^\sigma\,(q-p_1)^\epsilon\,\epsilon_{\alpha\lambda\beta\nu}\,(q-p_1)^\alpha\, {p_3}^\beta\,\eta^{*\nu}\, l^\mu,$$

$$C_{\rho^0} = \left(\frac{g_{\omega\rho\pi}}{g_\rho}\right)^2 m^2_{\rho^0}\, D_{\rho^0}(q), \qquad C_{\rho'} = \frac{g_{\omega\rho'\pi}\, g_{\omega\rho\pi}}{g_\rho\, g_{\rho'}}\, m^2_{\rho'}\, D_{\rho'}(q),$$

Cross section as a function of all the couplings involved

$$\sigma(e^+e^- \to 2\pi^0\gamma) = \left(\frac{g_{\omega\rho\pi}}{g_\rho}\right)^4 C_1 + \left(\frac{g_{\omega\rho\pi}}{g_\rho}\,\frac{g_{\omega\rho'\pi}}{g_{\rho'}}\right)^2 C_2 + \left(\frac{g^3_{\omega\rho\pi}}{g^3_\rho}\,\frac{g_{\omega\rho'\pi}}{g_{\rho'}}\right)\left(Cos(\theta)\,C_3 - Sin(\theta)\,C4\right).$$

The Ci coefficients are computed at each energy data of the experimental cross section

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See poster by Leonardo Esparza for more physical insights

Global analysis. From decay modes to cross section data

We minimize the function

$$\chi^2(\theta) = \sum_{i=1}^{N} \frac{(y_i - \mu(x_i;\theta))^2}{E_i^2},$$

considering the couplings as free parameters, for the following data:

(a) 10 decay modes: $\rho \to \pi\pi \quad \rho^0 \to e^+e^-,\ \mu^+\mu^- \quad \omega \to e^+e^-,\ \mu^+\mu^-$

$\rho \to \pi\gamma \quad \omega \to \pi^0\gamma. \quad \pi^0 \to \gamma\gamma.$

11 decay modes: (a) + $\omega \to 3\pi$

(b)11 decay modes + $e^+e^- \to \pi^0\pi^0\gamma$ SND (00), (13), (16), CMD2

(c)11 decay modes + $e^+e^- \to 3\pi$ SND, BABAR, CMD2, BES 3

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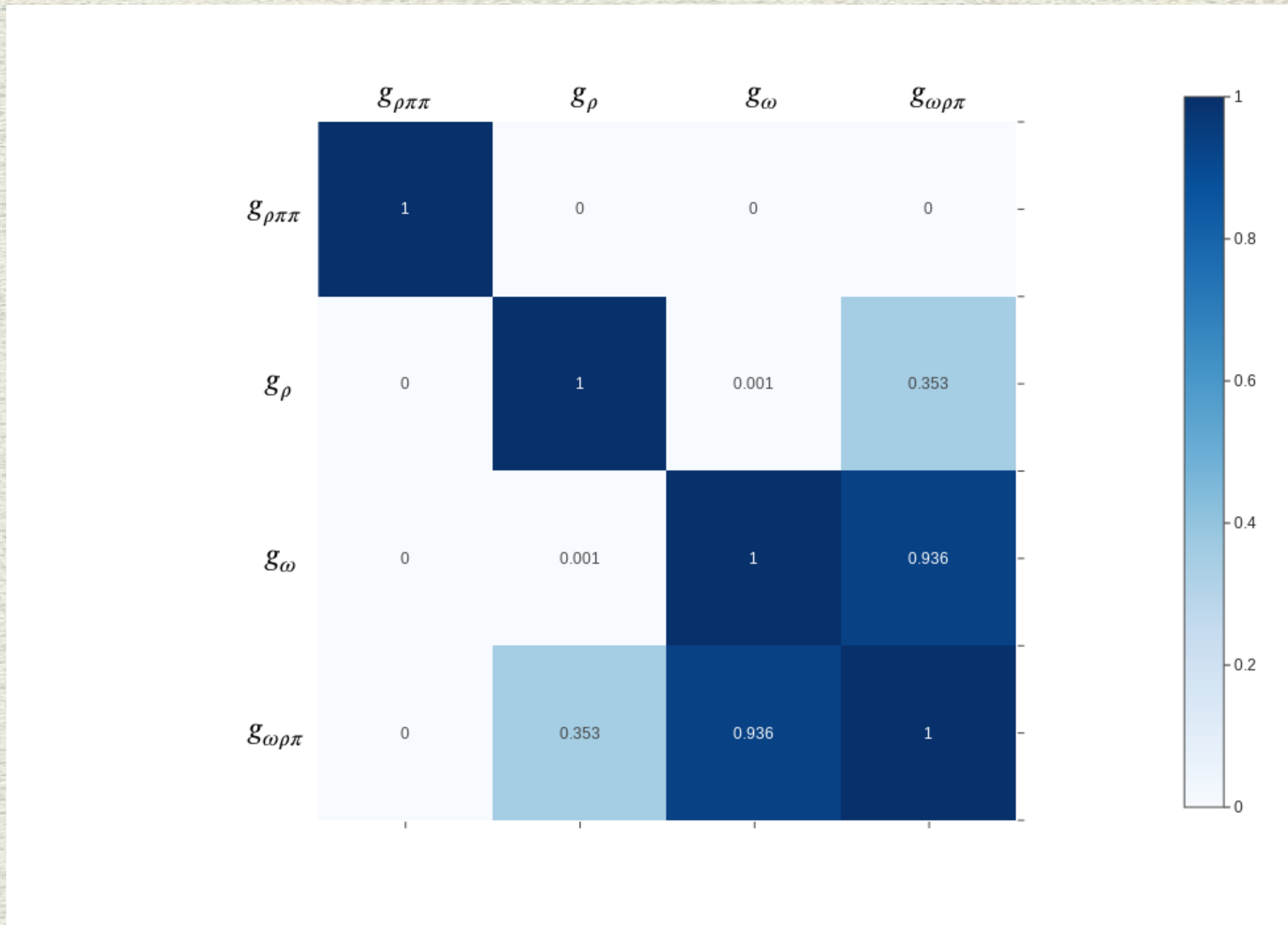
10 decay modes (4 parameters)

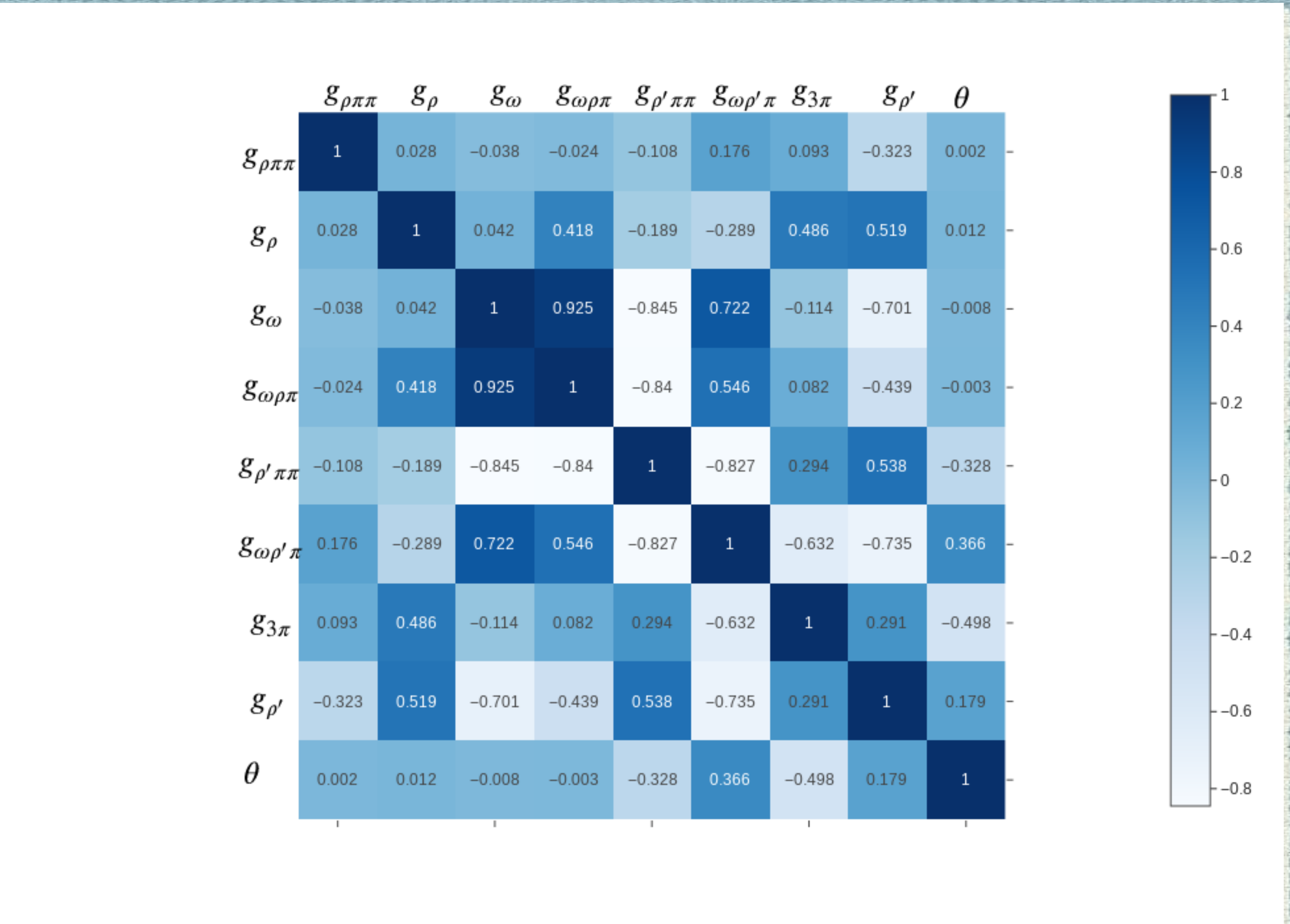
Parameter	Central value	Error
$g_{\rho\pi\pi}$	5.9485	0.0536
g_{ρ}	4.9619	0.0661
g_{ω}	17.038	0.603
$g_{\omega\rho\pi}$ (GeV^{-1})	11.575	0.438



Correlation matrix

11 decay modes + $e^+e^- \to \pi^0\pi^0\gamma$

Parameter	Central value	Error
$g_{\rho\pi\pi}$	5.9484	0.0668
g_{ρ}	4.9618	0.0819
g_{ω}	16.907	0.6625
$g_{\omega\rho\pi}$ (GeV^{-1})	11.486	0.4951
$g_{\rho'\pi\pi}$	4.5103	1.0371
$g_{\omega\rho'\pi}$ (GeV^{-1})	3.1363	1.7702
$g_{3\pi}$ (GeV^{-3})	-53.612	6.8932
$g_{\rho'}$	12.472	1.2437
θ (in π units)	0.8697	0.0452



Correlation matrix

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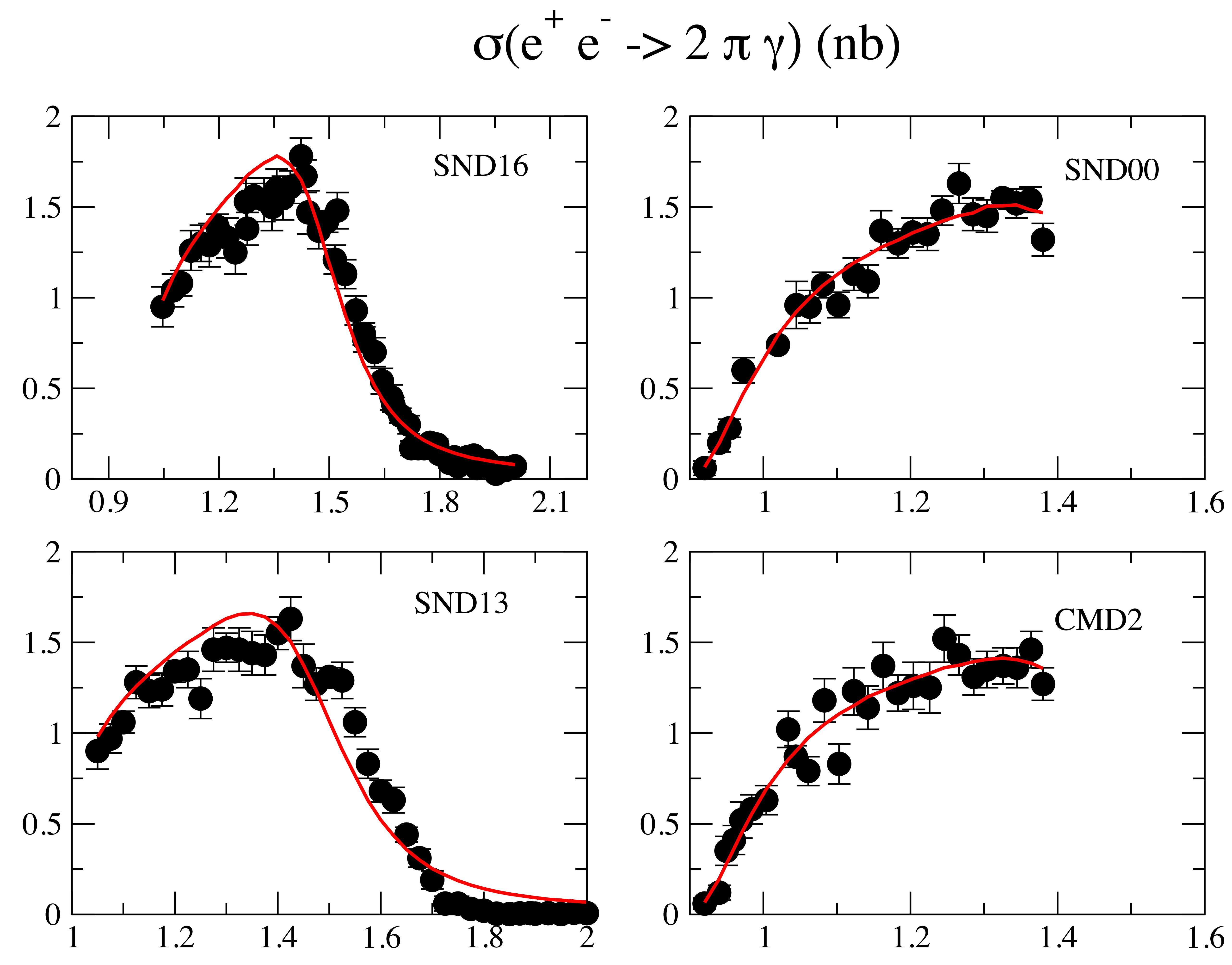
11 decay modes + $e^+e^- \to \pi^0\pi^0\gamma$



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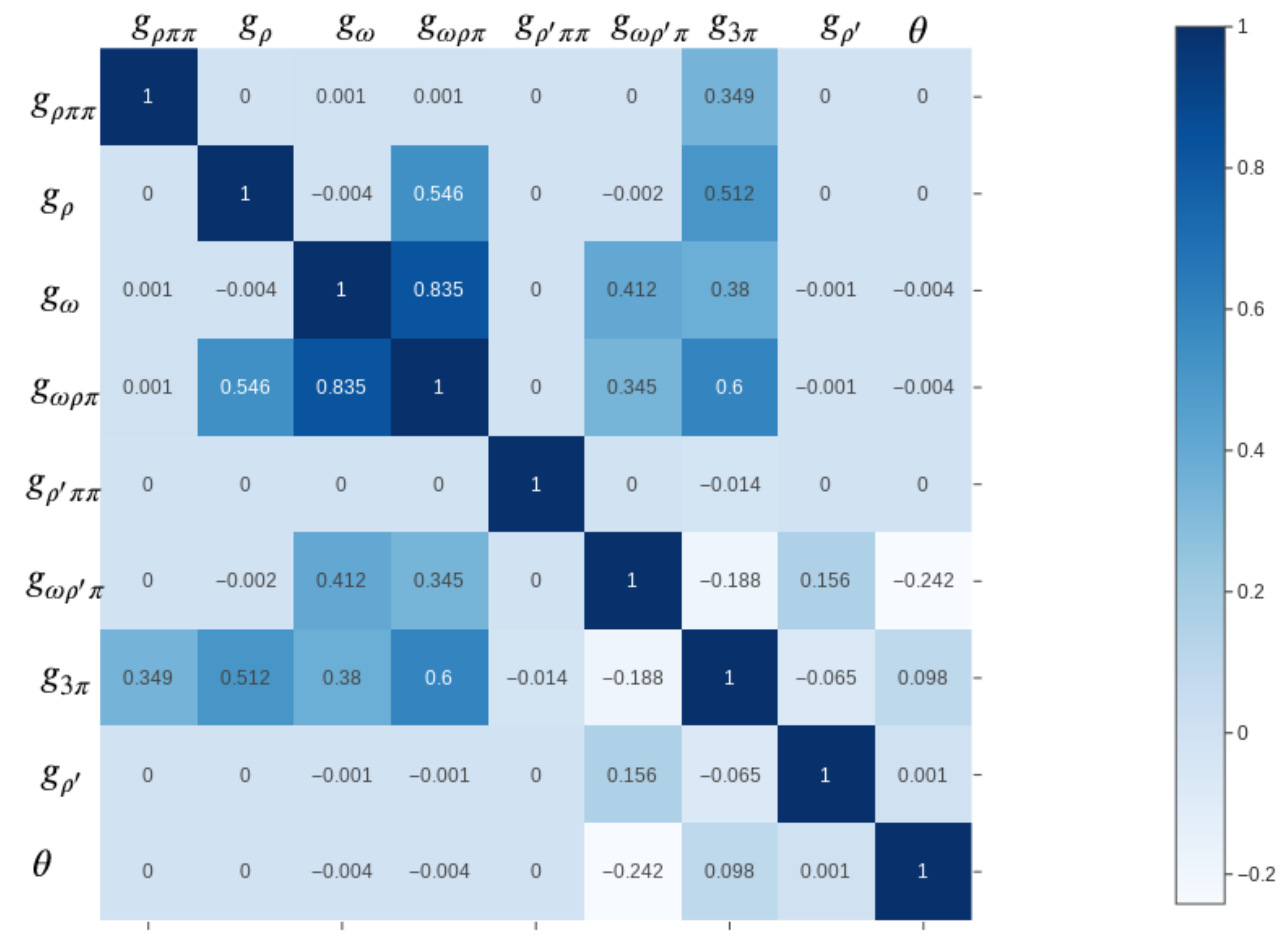
11 decay modes + $e^+e^- \to 3\pi$

Parameter	Central value	Error
$g_{\rho\pi\pi}$	5.9486	0.0755
g_ρ	4.9622	0.0928
g_ω	16.652	0.4726
$g_{\omega\rho\pi}$ (GeV^{-1})	11.314	0.383
$g_{\rho'\pi\pi}$	5.4999	1.0597
$g_{\omega\rho'\pi}$ (GeV^{-1})	3.4774	0.96262
$g_{3\pi}$ (GeV^{-3})	-54.338	6.6739
$g_{\rho'}$	12.918	1.1907
θ (in π units)	0.8715	0.0512



Correlation matrix

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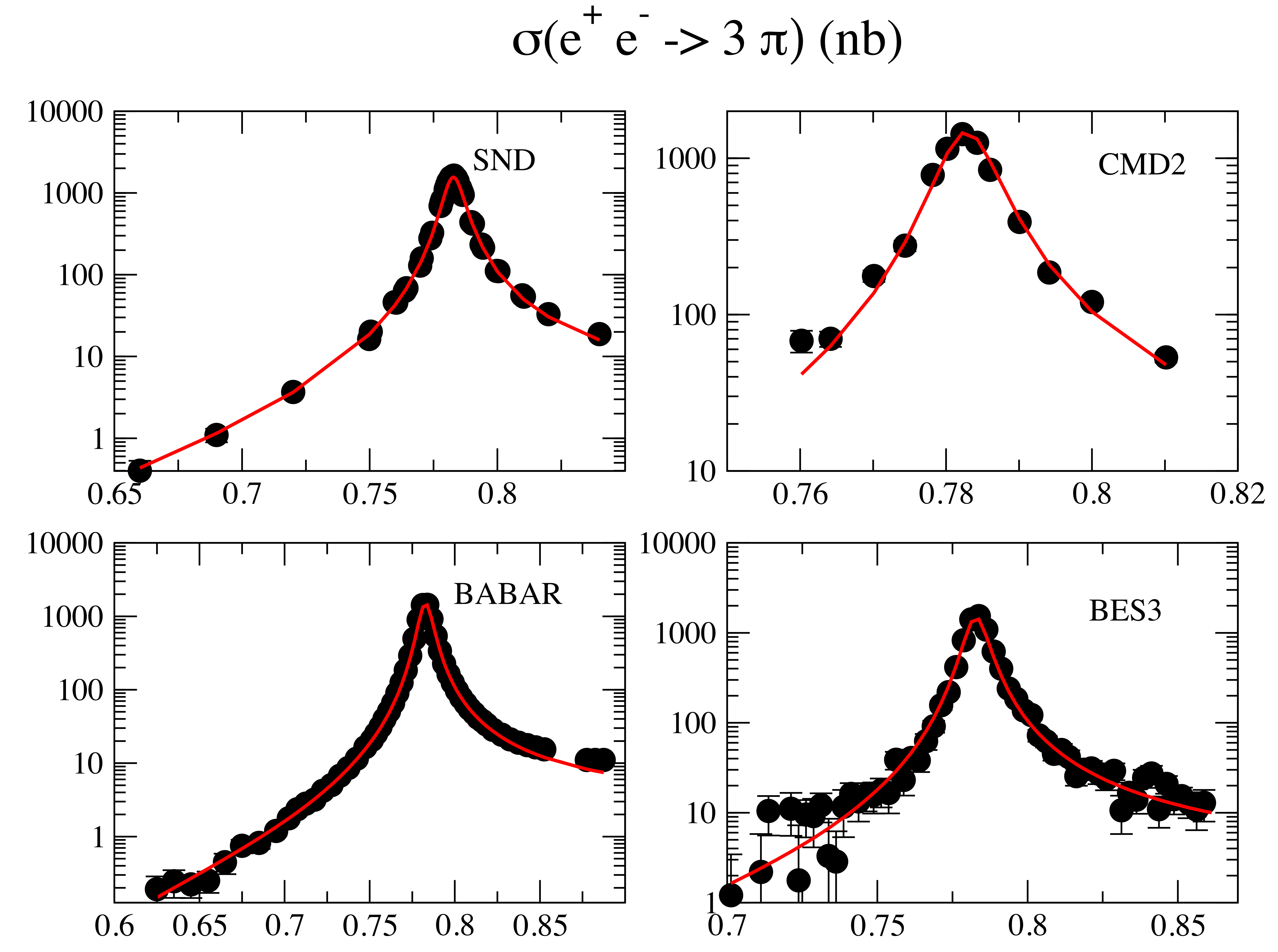
G. Toledo, MWPF22

11 decay modes + $e^+e^- \to 3\pi$



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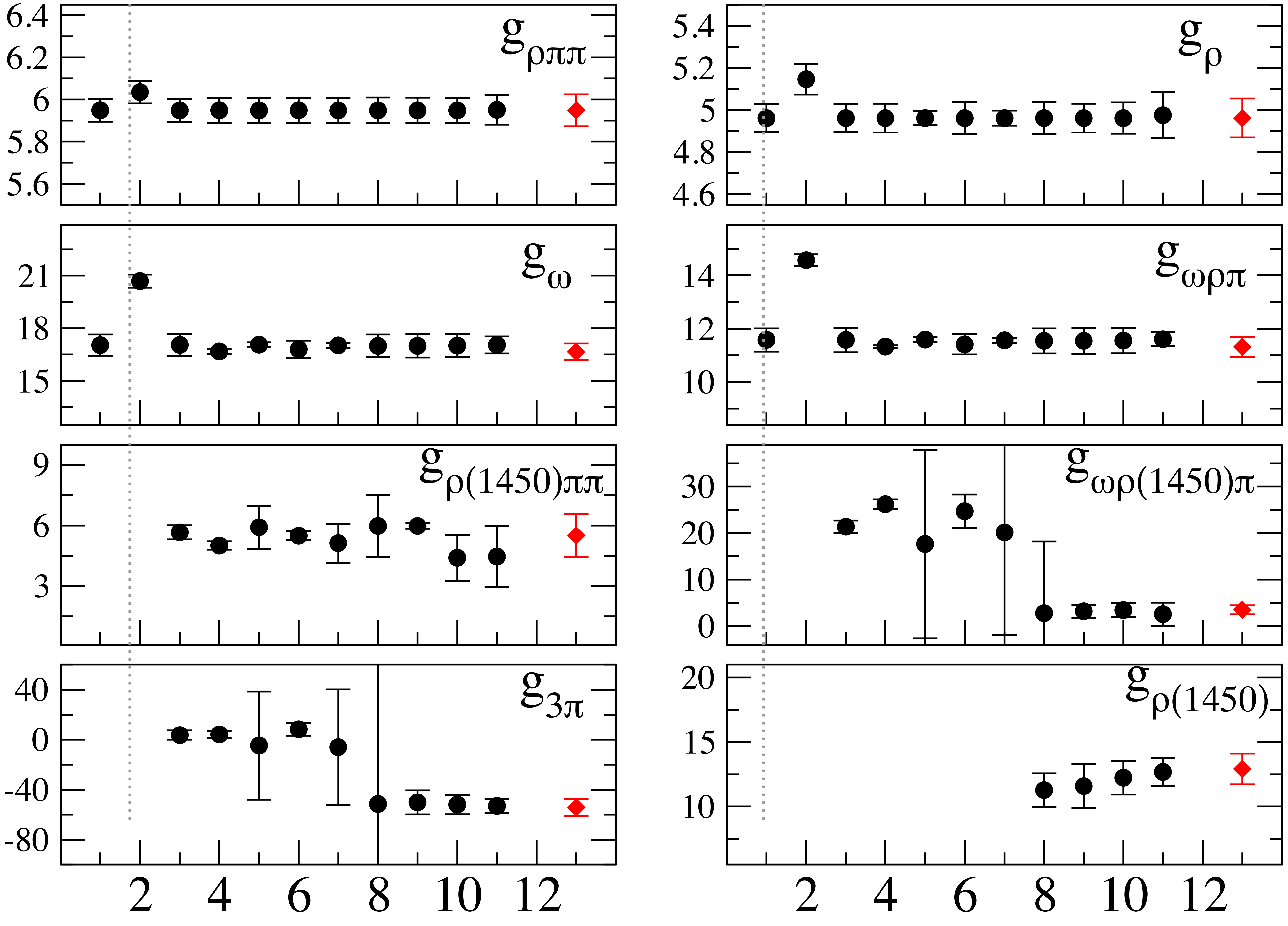
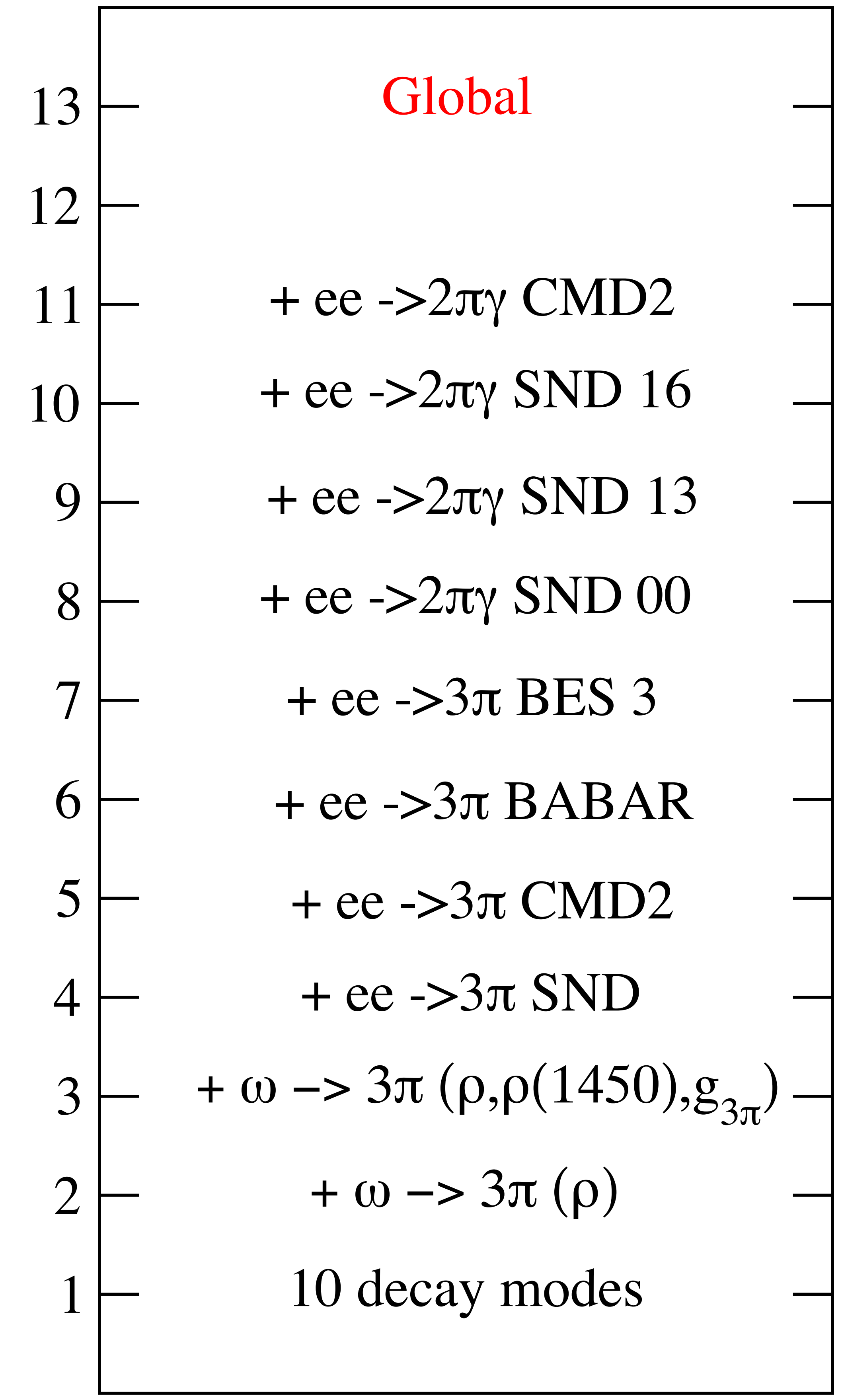
Couplings behavior for individual data



X axis labels



The $e^+e^- \to \pi\,\omega \to 4\,\pi$ cross section

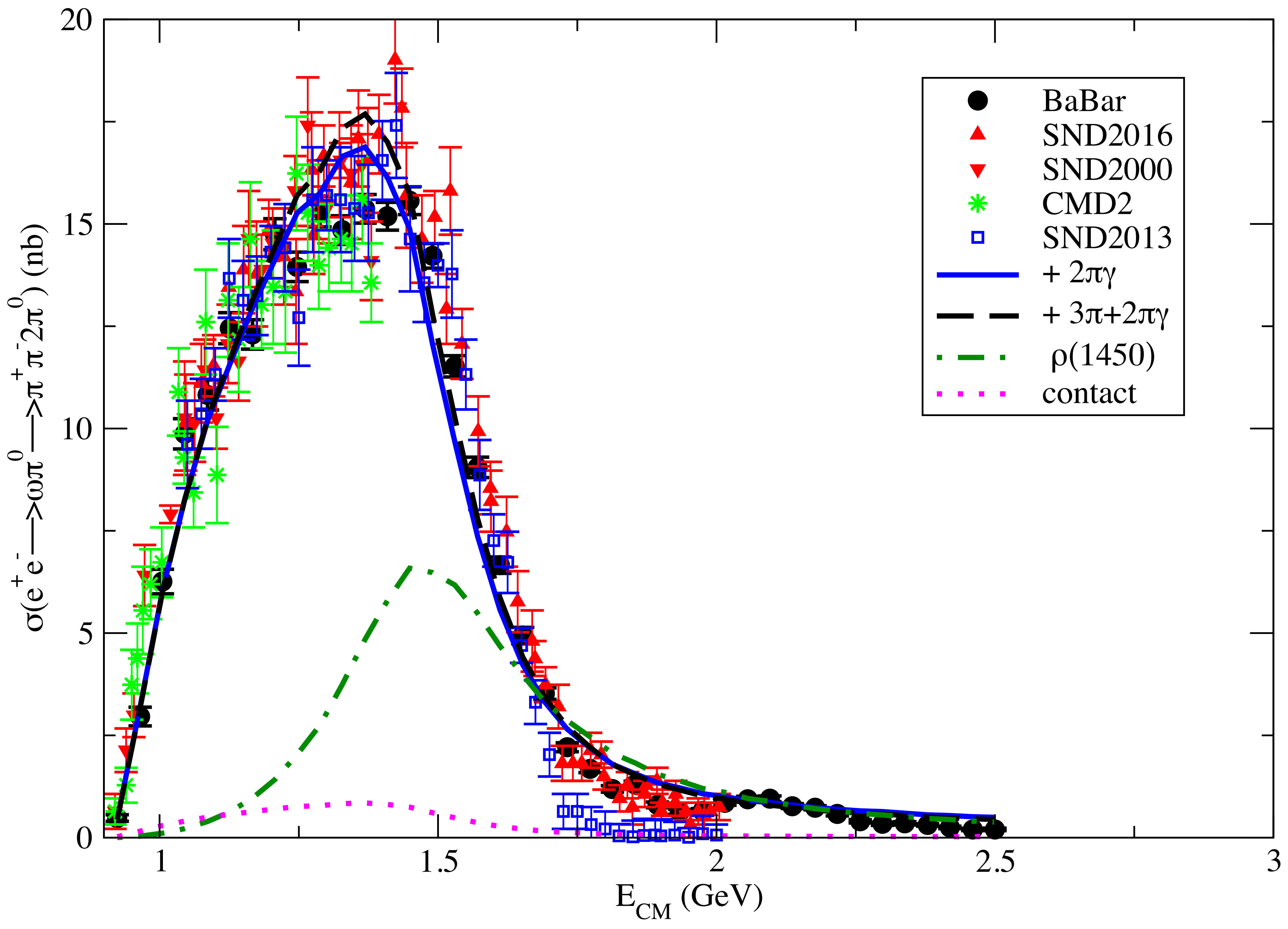
Using the previous results, NO fit to these data.

See poster by Antonio Rojas for more details

Amplitude $\mathcal{M}_{e^+e^-\to 4\,\pi} = \frac{e}{q^2}\left(G_\rho + e^{i\theta}G_{\rho'}\right) D_\omega(q-p_4)\,\mathcal{A}((q-p_4)^2)\,\epsilon_{\sigma\alpha\eta\beta}\,\epsilon_{\mu\gamma\chi\sigma}\,q^\gamma\,{p_1}^\alpha\,{p_2}^\eta\,{p_3}^\beta\,{p_4}^\chi\,l^\mu$

$$G_\rho = \frac{g_{\omega\rho\pi}}{g_\rho} m^2_{\rho^0} D_{\rho^0}(q),$$

$$G_{\rho'} = \frac{g_{\omega\rho'\pi}}{g_{\rho'}} m^2_{\rho'} D_{\rho'}(q).$$



G. Toledo, MWPF22

Conclusions

We performed a global analysis of a set of decay modes and cross sections in the context of the vector meson dominance model.

In a first step we determined the parameters of the model involving the light mesons, from 10 decay modes which are insensitive to the ϱ'. Then, we considered the $\omega \to 3\pi$ decay, and exhibit the need of the ϱ' and a contact term as prescribed by the WZW anomaly.

In a second step, we incorporated the data from the $e^+e^- \to 3\pi$ cross section (as measured by SND, CMD2, BABAR and BES III), and then the $e^+e^- \to \pi^0\pi^0\gamma$ data (as measured by SND and CDM2) to further restrict the ϱ' parameters validity region.

As an application, we computed the $e^+e^- \to 4\pi$ cross section for the so-called omega channel, measured by BABAR and find a good description of the data considering the parameters found.

As a byproduct, the coupling $g_{\varrho\omega\pi} = 11.314 \pm 0.383$ GeV^{-1} is found to be consistent with all the relevant observables.

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THANK YOU

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