Amplitudes: a modern approach to QFT and Gravity



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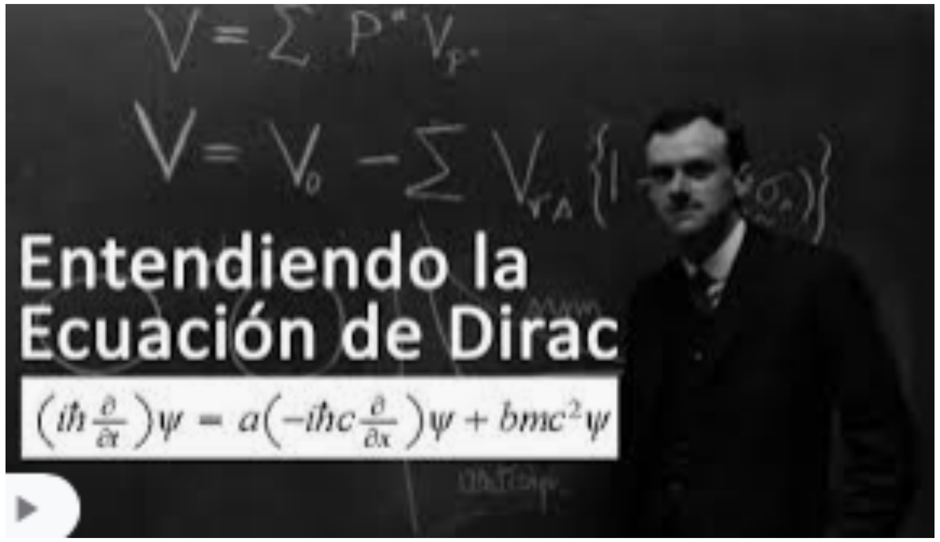
Outline:

- Part 1: Research

1. Introduction: Higgs - Scalars
2. pQG & Gravitons,
3. Higgs decay to gravitons,
4. Conclusions,

- Part 2: QFT & Amplitudes

1. What is QFT? What is a Particle?
2. Helicity Methods: “gluons almost for nothing and gravitons for free” (JJC),
3. Constructible QFT
4. Conclusions,



1. Introduction: Higgs - Scalars

- The first fundamental scalar has been detected at LHC (2012),

- Its mass (125 GeV) and its properties are in agreement with the SM, so far …

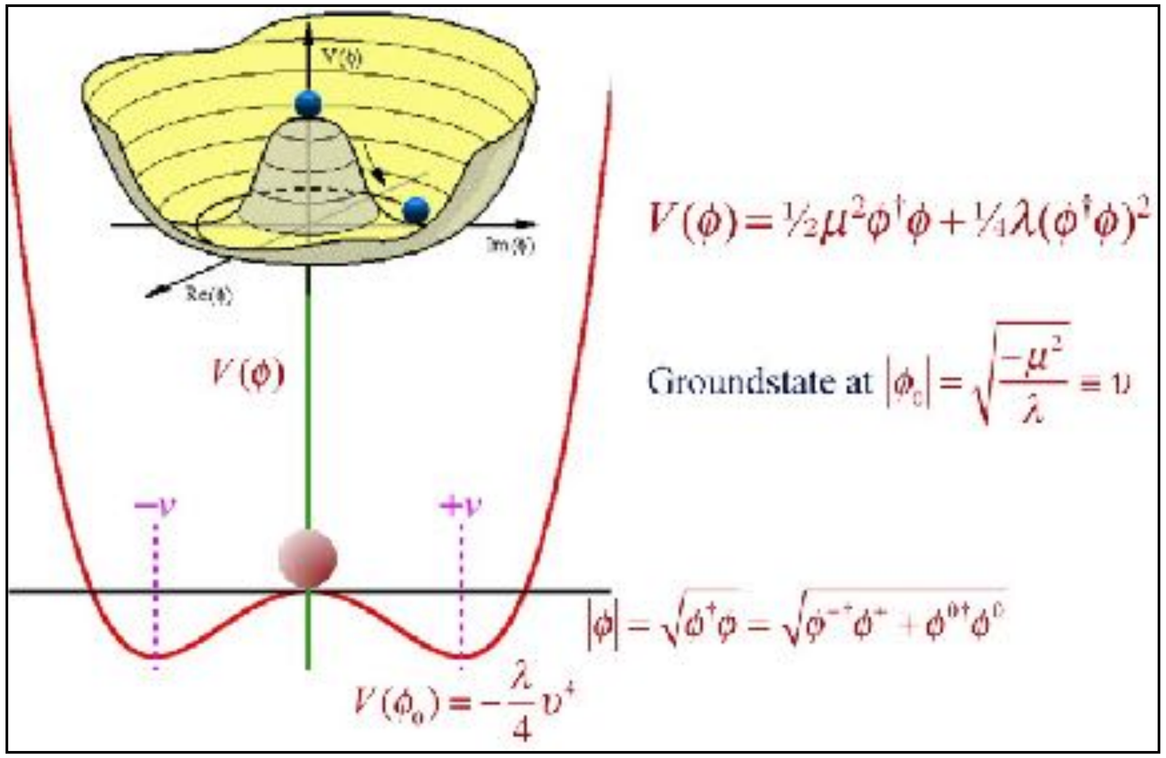
- But more properties have to be studied:

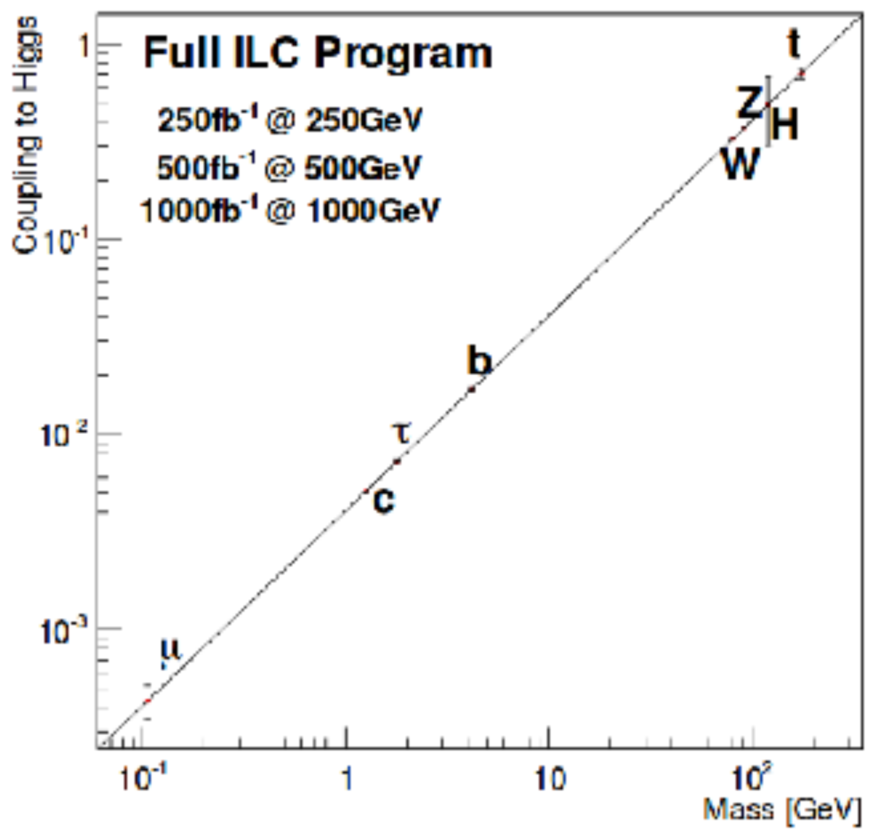
  - Higgs self-couplings (lambda),

  - Flavor changing couplings (FCNC),

  - New scalars: 2HDM, sfermions, axions, inflaton, modulus, etc,

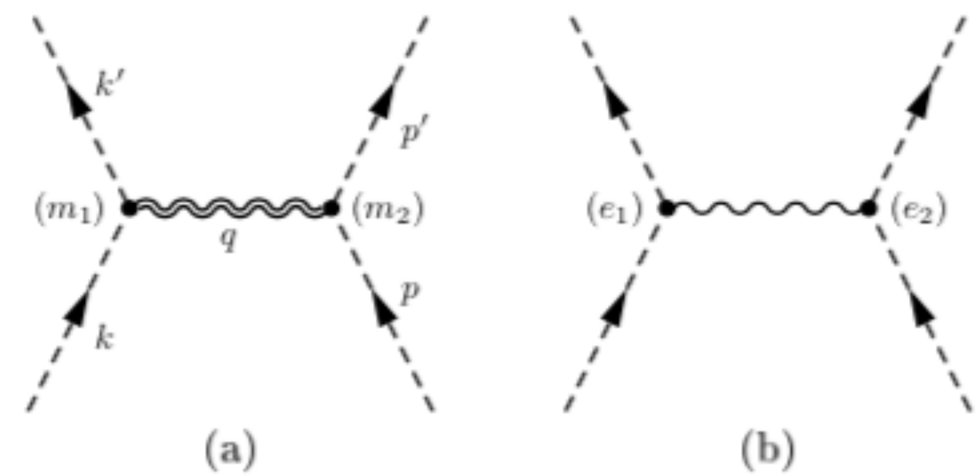
- SM and gravity: Higgs (scalar) couplings with gravitons,





2. pQuantum Gravity and the Higgs

- Here, we shall assume that pQG makes sense (at least in the IR),
- So, like all other fundamental forces, gravity is mediated by a particle
- The graviton is a fluctuation of the metric: massless, spin-2 particle,



$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

- Graviton couplings are extracted from Einstein-Hilbert action, considered as an effective filed theory (EFT) (J. Donoghue):

$$S_{eff} = \int d^4x \sqrt{-g}\mathcal{L}_{eff}$$

$$\begin{aligned}
\mathcal{L}_{eff} &= \mathcal{L}_{grav} + \mathcal{L}_{matter} \\
\mathcal{L}_{grav} &= \mathcal{L}_{g0} + \mathcal{L}_{g2} + \mathcal{L}_{g4} + \ldots \\
\mathcal{L}_{matter} &= \mathcal{L}_{m0} + \mathcal{L}_{m2} + \ldots
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{g0} &= \Lambda \\
\mathcal{L}_{g2} &= \frac{2}{\kappa^2} R \\
\mathcal{L}_{g4} &= c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu}
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{m0} &= \frac{1}{2}\left[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2\right] \\
\mathcal{L}_{m2} &= d_1 R^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + R\left(d_2\partial_\mu\phi\partial^\mu\phi + d_3 m^2\phi^2\right)
\end{aligned}$$

Gravitons arise from pQG, and deriving its interactions hurts …

- Need to consider Einstein-Hilbert action.

$$\boxed{S_{EH} = \int d^4x \sqrt{-g}\,\mathcal{R}} \qquad R \equiv g^{\mu\nu} R_{\mu\nu}$$

- Expansion of the metric defines graviton,

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} + \frac{\kappa^2}{2} h_{\mu\alpha} h^{\alpha}_{\nu} + \ldots$$

- Part of the 3-graviton vertex,

$$\begin{aligned}
&\frac{\delta S^3}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma\tau}\delta\varphi_{\rho\lambda}} \to 2\eta^{\mu\tau}\eta^{\nu\sigma}k_1{}^{\lambda}k_1{}^{\rho} + 2\eta^{\mu\sigma}\eta^{\nu\tau}k_1{}^{\lambda}k_1{}^{\rho} - 2\eta^{\mu\nu}\eta^{\sigma\tau}k_1{}^{\lambda}k_1{}^{\rho} + \\
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\end{aligned}$$

- Helicity method gives such a simple answer,

- There must be an alternative formulations to both QFT and pQG,

- More on-shell stuff …

3. Higgs couplings with gravitons

- We would like to derive the coupling of an scalar with two gravitons from the lagrangian:

$$\mathcal{L} = \sqrt{-g}\left(\kappa_1 \phi R + \kappa_2 \phi R^2 + \kappa_3 \phi R_{\mu\nu} R^{\mu\nu} + \kappa_4 \phi R_{\mu\rho\nu\sigma} R^{\mu\rho\nu\sigma} + \ldots\right)$$

- Couplings of graviton with pseudo-scalar:

$$\mathcal{L}_5 = \frac{1}{4\kappa^2} \kappa_5 \phi_5 \sqrt{-g}\, \epsilon_{\mu\nu\rho\sigma} R^{\mu\nu}{}_{\alpha\beta} R^{\rho\sigma\alpha\beta}.$$

- To first order one graviton can interact with two scalars, but no coupling of one scalar with two gravitons arise, I.e.

$$R^{(1)} = \partial_\mu \partial_\nu h^{\mu\nu} - \partial^2 h$$

- So, we need to consider the expansion of the action up to 2nd order,

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} + \frac{\kappa^2}{2} h_{\mu\alpha} h^{\alpha}_{\nu} + \ldots$$

4. Results:

A. Apparently, the scalar can couple with all geometric objects,

B. But when one evaluates the on-shell coupling, only the square of the Riemman tensor contributes,

C. Similarly for the coupling of the pseudo-scalar

13.1 R^2

$$(\sqrt{-g}R^2)^{(2)} = (R^2)^{(2)} = R^{(1)}R^{(1)} = (\partial_\mu\partial_\nu h^{\mu\nu} - \partial^2 h)(\partial_\alpha\partial_\beta h^{\alpha\beta} - \partial^2 h)$$

13.2 $R_{\mu\nu}R^{\mu\nu}$

$$(\sqrt{-g}R_{\mu\nu}R^{\mu\nu})^{(2)} = (R_{\mu\nu}R^{\mu\nu})^{(2)} = (R_{\mu\nu})^{(1)}(R^{\mu\nu})^{(1)} = \left[\frac{1}{2}(h^{\lambda}_{\nu,\mu,\lambda} - h^{\lambda}_{\lambda,\nu,\mu} + h^{\lambda}_{\mu,\nu,\lambda} - h_{\mu\nu}{}^{,\lambda}{}_{,\lambda})\right] \times \left[\frac{1}{2}(h^{\lambda\nu,\mu}{}_{,\lambda} - h^{\lambda,\nu,\mu}_{\lambda} + h^{\lambda\mu,\nu}{}_{,\lambda} - h^{\mu\nu,\lambda}{}_{,\lambda})\right]$$

13.3 $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$

$$(\sqrt{-g}R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta})^{(2)} = (R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta})^{(2)} = (R_{\mu\nu\alpha\beta})^{(1)}(R^{\mu\nu\alpha\beta})^{(1)} = \left[\frac{1}{2}(h_{\alpha\nu}{}^{,\beta}{}_{,\mu} - h^{\beta}_{\alpha,\nu,\mu} + h^{\beta}_{\mu,\nu,\alpha} - h_{\mu\nu}{}^{,\beta}{}_{,\alpha})\right] \times \left[\frac{1}{2}(h_{\alpha}^{\nu,\beta,\mu} - h_{\alpha}^{\beta,\nu,\mu} + h^{\beta\mu,\nu}{}_{,\alpha} - h^{\mu\nu,\beta}{}_{,\alpha})\right]$$

3.5 $\phi_5 hh$ interaction for a pseudoscalar

The expansion of Equation (6), gives the following result:

$$\left(\frac{\sqrt{-g}}{4}\epsilon_{\mu\nu\rho\sigma}R^{\mu\nu}{}_{\alpha\beta}R^{\rho\sigma\alpha\beta}\right)^{(2)} = -\frac{1}{2}\epsilon_{\mu\rho\sigma\alpha}\partial^\alpha\partial_\beta h^\sigma_\nu\partial^\rho\partial^\nu h^{\beta\mu} + \frac{1}{2}\epsilon_{\mu\rho\sigma\alpha}\partial^\alpha\partial_\nu h^\sigma_\beta\partial^\rho\partial^\nu h^{\beta\mu} + \frac{1}{4}\epsilon_{\nu\rho\sigma\alpha}\partial^\alpha\partial^\sigma h_{\beta\mu}\partial^\rho\partial^\nu h^{\beta\mu}.$$

4.3 The decay width for $\phi \to hh$

In order to obtain the decay width for the two-body mode $\phi \to h\,h$, one has to square the amplitudes (20) and (21) and add them. Thus, the squared amplitude, summed over polarizations ($h_1 = \pm 2$ and $h_2 = \pm 2$), takes the form:

$$\langle |\mathcal{M}(\phi \to hh)|^2 \rangle = \sum_{h_i} \kappa_4^2 |A_3(\phi h^{h_i} h^{h_i})|^2 = \frac{\kappa_4^2}{2} m_\phi^8. \tag{30}$$

The expression for the decay width of the Higgs (in the rest frame) is given as follows:

$$\Gamma(\phi \to hh) = \frac{S|\vec{p}|}{8\pi m_\phi^2} \langle |\mathcal{M}(\phi \to hh)|^2 \rangle, \tag{31}$$

10

4. More Results:

A. We have considered scalar 3-body decays, and again only squared Riemann contributes,

B. Similar result holds when one consideres scalar contribution to graviton loops,

the decay $\phi(p) \to h(p_1) + X(p_3)X(p_3)$ is given by:

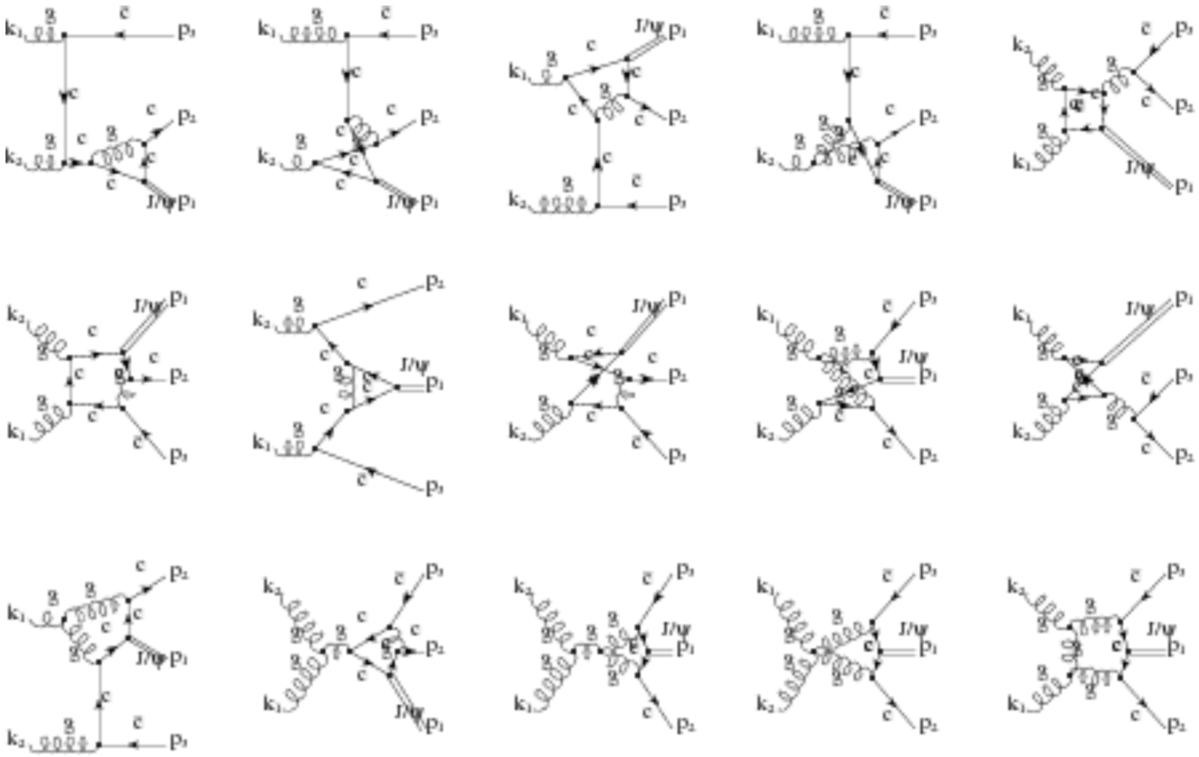
$$M^{h_i} = \epsilon^{\mu\nu}(p_1) V_{\mu\nu\rho\sigma}(p, p_1, q) \frac{iP^{\rho\sigma\alpha\beta}}{q^2} T_{\alpha\beta}(XX).$$

- Alonzo, Avilez, Diaz-Cruz, Larios, arXive: 2105.11684 [hep-th]

Part II

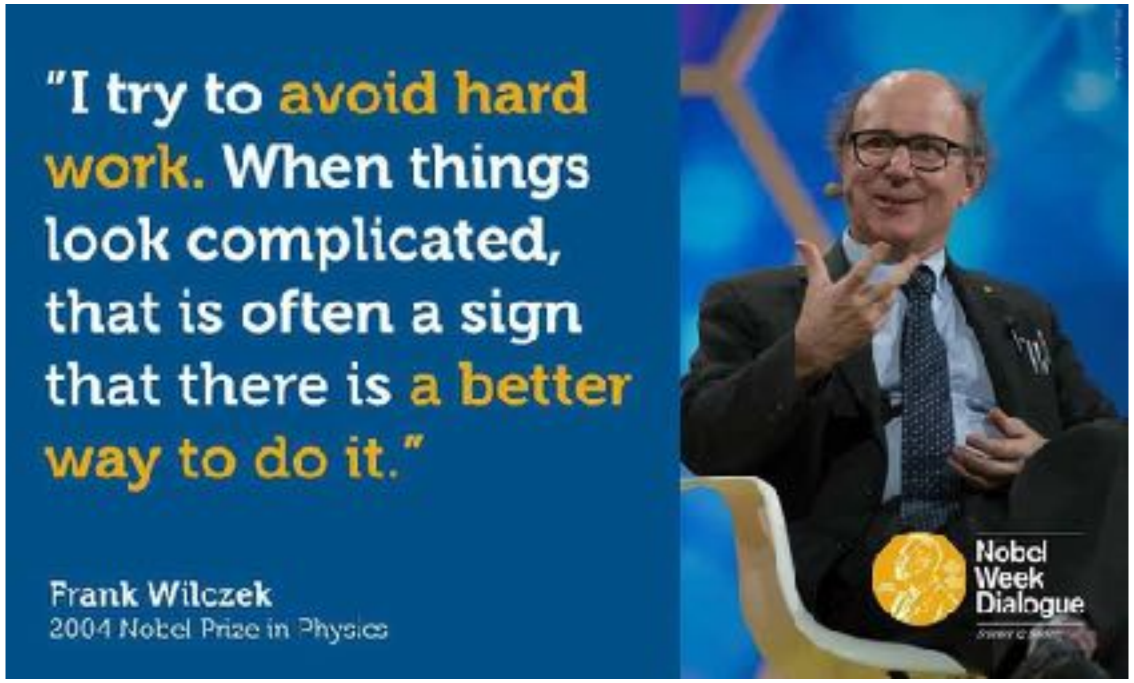
QFT theory and experiments agree, but heroic efforts needed …

- Shut up and calculate!



- But the method has its limitations, ex. In QCD the number of diagrams grows fast with the number of final states!

$$
\begin{array}{llll}
g+g & \rightarrow & g+g & \text{4 diagrams} \\
g+g & \rightarrow & g+g+g & \text{25 diagrams} \\
g+g & \rightarrow & g+g+g+g & \text{220 diagrams}
\end{array}
$$



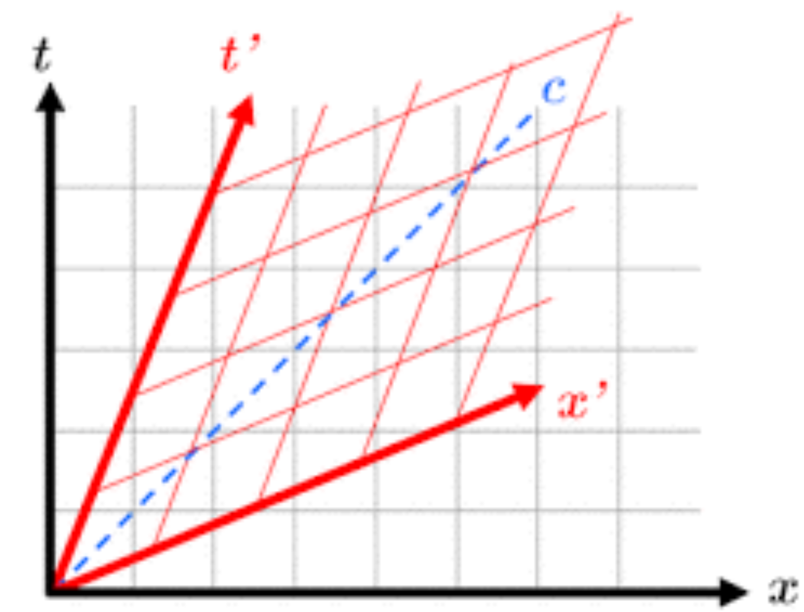
1) What is a QFT? RQT

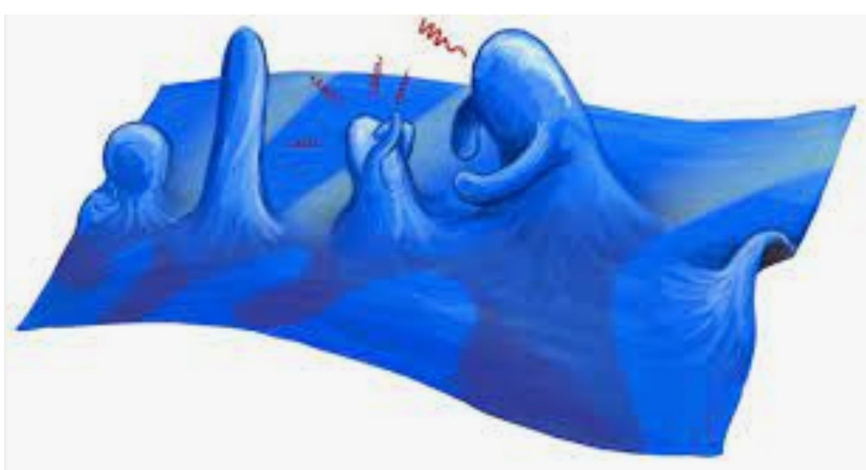
- Something that is Quantum and Relativistic!

- So, need to consider how Quantum states transform under Poincare Group (Lorentz & Translations),

(What is a Particle? Particle or Field? Or *Parfield?*).

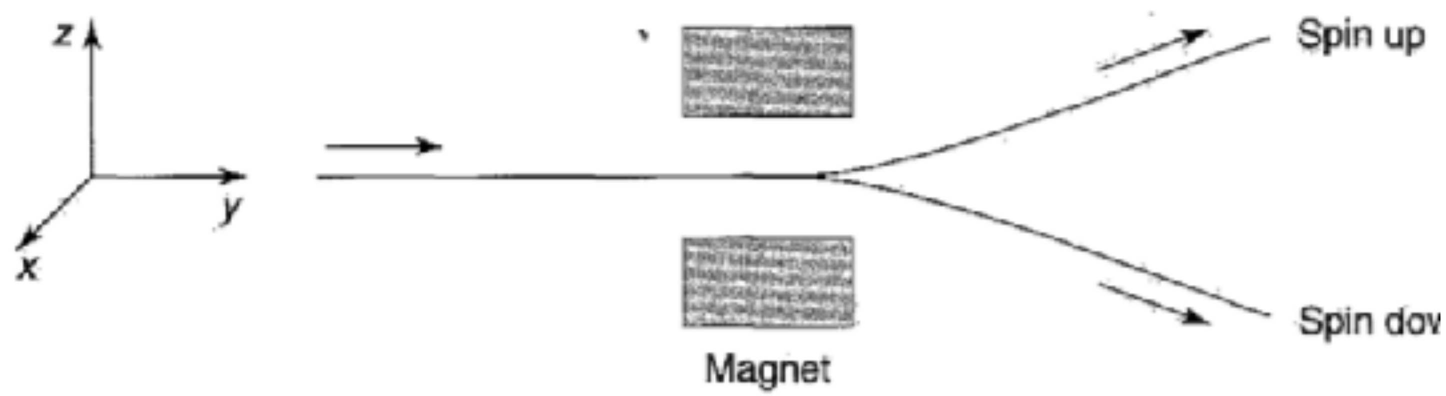
$$|\psi > \to U(\Lambda)|\psi >$$





- Here, we will discuss that in order to define and work with particles in QFT, all we need is ~~love~~ spinors!

1.x Spinors and Spin States in QM



$$\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \left(\text{eigenvalue} + \frac{\hbar}{2}\right); \qquad \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \left(\text{eigenvalue} \; - \frac{\hbar}{2}\right).$$

- Those functions are eigen-states of spin-z operador, and are known as spinors.

$$\mathbf{S}_z \chi_+ = \frac{\hbar}{2} \chi_+, \qquad \mathbf{S}_z \chi_- = -\frac{\hbar}{2} \chi_-,$$

$$S_i = \frac{h}{2} \sigma_i$$

$$\boxed{\sigma_x \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.}$$

1.2) Quantum states and Relativity

- Space-time, energy-momentum, 4-potential are all 4-vectors,
- Lorentz Transformation (boosts & 3-dim rotations) are such that: x^2 = x'^2,

$$x^\mu = (x^0, x^i) = (ct, x^i)$$

$$p^\mu = (E, p^i c)$$

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\mu$$

$$\Lambda^\mu{}_\rho \Lambda^\nu{}_\sigma g_{\mu\nu} = g_{\rho\sigma}$$

- In Quantum Mechanics we work with: states, operators, symmetries, etc.
- States must transform under Lorentz Transformation (and translations),

$$|\psi> \to U(\Lambda)|\psi>$$

$$U(\Lambda_1 * \Lambda_2) = U(\Lambda_1)U(\Lambda_2)$$

QFT, Wigner & Little group

- States depend on momentum and "other" degrees of freedom,
- Wigner Classification - what are those d. of f.
- Look for the subgroup of Lorentz Transf. that leave invariant the momentum,
- Little Group: L P = P (Weinberg, QFT, V1)

$$|\Psi>=|P_0,j>$$

$$|\psi>\rightarrow U(\Lambda)|\psi>$$

$$U(L)|\Psi>=\Sigma_i C_{ij}|P_0,j>$$

- Massive particles are defined by its mass (m) and spin (s)
- Little Group = SO(3)
- Known cases:

  s=0 (Higgs), 1/2 (q and l), 1 (W, Z)

- Massless particles are defined by its Helicity (h)
- Little Group = U(1) x T_2
- Known cases: h = +-1 (Photon, gluon), +- 2 (graviton)

1.3 Lorentz Group and Spinors

- SU(2) Reprs. given by J^2 and J_z; one can have: J = 0, 1/2, 1, 3/2, 2.
- Spinors are objects that transform under the SU(2) group (set of 2x2 unitary matrices, with det=1), with S= 1/2

- Lorentz transformations include rotations and boosts,
- Lorentz transformation form a group, it is:
- Two types of Fund. Repr.= Spinors (Weyl): LH (1/2, 0) and RH (0,1/2)

$$\Lambda = \exp(i\theta_i J_i + i\beta_i K_i)$$

$$J_\pm = \frac{1}{2}[J_i \pm iK_i]$$

$$SU(2)_L \times SU(2)_R$$

$$\psi_\alpha \ \text{ and } \ \tilde{\psi}_{\dot{\alpha}} \qquad \alpha(\dot{\alpha}) = 1,2(\dot{1},\dot{2})$$

2. Helicity Methods and Amplitudes

- We can use a matrix to represent the momentum, i.e.

- For massless case, det(p)=0 (Rank=1),

- So, matrix can be written as:

$$p_{\alpha,\dot{\alpha}} = \sigma^{\mu}_{\alpha,\dot{\alpha}} p_{\mu}$$

$$\sigma^{\mu} = (1, \sigma^{i})$$

$$p_{\alpha\dot{\alpha}} = p_{\mu}\sigma^{\mu}_{\alpha\dot{\alpha}} = \begin{pmatrix} p_0 + p_3 & p_1 - ip_2 \\ p_1 + ip_2 & p_0 - p_3 \end{pmatrix},$$

$$p_{\alpha,\dot{\alpha}} = \chi_{\alpha}\tilde{\chi}_{\dot{\alpha}}$$

$$\alpha(\dot{\alpha}) = 1, 2(\dot{1}, \dot{2})$$

- For massless case, in the frame where: $k_{\mu} = (E, 0, 0, E)$

$$P_{\alpha,\dot{\alpha}} = \begin{pmatrix} 2E & 0 \\ 0 & 0 \end{pmatrix} \qquad \chi_{\alpha} = \sqrt{2E}(1,0)^{T} \text{ and } \tilde{\chi}_{\dot{\alpha}} = \sqrt{2E}(1,0).$$

- For arbitrary momentum apply rotations/Boosts,

For Photons (QED), we also need Polarization Vectors

- Start from the usual definition of Pol. Vectors: e(k)
- Contract with sigma matrices:
- Since det(e)=0, we can write it as the product of two spinors,
- For 2-dims spinors, choose one of them to be the momentum spinor, while the other one is an auxiliary spinor
- Same for the other polarization:

$$\epsilon_\mu^{\pm} = \frac{1}{\sqrt{2}}(0, 1, \pm i, 0)$$

$$\epsilon^{+}_{\alpha\dot{\alpha}} = \sigma^{\mu}_{\alpha\dot{\alpha}}\epsilon^{+}_{\mu} \qquad e^{+}_{\alpha,\dot{\alpha}} = \begin{pmatrix} 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix}$$

$$e^{+}_{\alpha,\dot{\alpha}} = \frac{\lambda_\alpha \tilde{\eta}_{\dot{\alpha}}}{< \lambda\eta >} \qquad e^{-}_{\alpha,\dot{\alpha}} = \frac{\eta_\alpha \tilde{\lambda}_{\dot{\alpha}}}{[\tilde{\lambda}\tilde{\eta}]}$$

$$\boxed{p_{\alpha,\dot{\alpha}} = \chi_\alpha \tilde{\chi}_{\dot{\alpha}}}$$

- Polarization vectors/matrix satisfy:

- Eta = auxiliary spinors (massless),

- They satisfy transversality and "orthogonality",

$$\boxed{\epsilon^{\pm} \cdot p = 0.}$$

$$\boxed{\epsilon^{+} \cdot \epsilon^{+} = \epsilon^{-} \cdot \epsilon^{-} = 0}$$

$$\epsilon_p^-(r) \cdot \epsilon_p^+(r) = \frac{1}{2}[\epsilon_p^-(r)]_{\dot{\alpha}\alpha}[\epsilon_p^+(r)]^{\alpha\dot{\alpha}} = \frac{1}{2}\frac{2}{[pr]\langle rp\rangle}\mathrm{tr}\Big\{r]\langle p\, r\rangle[p\Big\} = -1$$

- Polarization products are easy to handle:

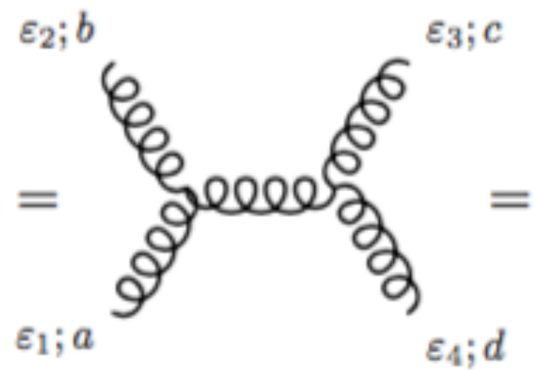
$$\epsilon_1^-(i) \cdot \epsilon_2^+(j) = \frac{\langle 1j\rangle[2i]}{[1i]\langle j2\rangle}, \quad \epsilon_1^+(i) \cdot \epsilon_2^+(j) = \frac{\langle ij\rangle[21]}{\langle i1\rangle\langle j2\rangle} \qquad \epsilon_1^-(i) \cdot \epsilon_2^-(j) = \frac{1}{2}\mathrm{tr}\left(2\frac{i]\langle 1}{[1i]}\frac{2\rangle[j}{[2j]}\right) = \frac{\langle 12\rangle[ji]}{[1i][2j]}.$$

$$\epsilon_1^-(i) \cdot p_3 = \frac{1}{\sqrt{2}}\frac{\langle 13\rangle[3i]}{[1i]}, \qquad \epsilon_1^+(i) \cdot p_3 = \frac{1}{\sqrt{2}}\frac{[13]\langle 3i\rangle}{\langle i1\rangle},$$

- So, we have achieved a "unification" of kinematics and dynamics!

Hybrid Method: Gluon Scattering: gg -> gg

We start by working out $\mathcal{M}(1^-2^-3^+4^+)$. We choose the reference momentum for ϵ_1 and ϵ_2 to be $r = p_4$ and the reference momentum for ϵ_3 and ϵ_4 to be p_1. Then the only polarization contraction that does not vanish is $\epsilon_2 \cdot \epsilon_3$. Also, we now have $\epsilon_1 \cdot p_4 = \epsilon_2 \cdot p_4 = \epsilon_3 \cdot p_1 = \epsilon_4 \cdot p_1 = 0$ as well as $\epsilon_i \cdot p_i = 0$. All of these constraints vastly simplify the

$i\mathcal{M}_s =$ 

$$= \frac{-ig_s^2}{s} f^{abe} f^{cde} \times [(\epsilon_1 \cdot \epsilon_2)(p_1 - p_2)^\mu + 2\epsilon_2^\mu (p_2 \cdot \epsilon_1) - 2\epsilon_1^\mu (p_1 \cdot \epsilon_2)] \times [(\epsilon_3 \cdot \epsilon_4)(p_3 - p_4)^\mu + 2\epsilon_4^\mu (p_4 \cdot \epsilon_3) - 2\epsilon_3^\mu (p_3 \cdot \epsilon_4)] \, .$$

$$\mathcal{M}_s(1^-2^-3^+4^+) = \frac{4g_s^2}{s} f^{abe} f^{cde} (\epsilon_2^- \cdot \epsilon_3^+)(p_2 \cdot \epsilon_1^-)(p_3 \cdot \epsilon_4^+) \, . \qquad = -2g_s^2 f^{abe} f^{cde} \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle},$$

- Similarly, can show that 4-point contribution vanishes, as well as t-channel, while u-channel contribution is:

$$\mathcal{M}_t(1^-2^-3^+4^+) = 0. \qquad \mathcal{M}_u(1^-2^-3^+4^+) = -2g_s^2 f^{ace} f^{bde} \left(\frac{\langle 21 \rangle^2 [34]^2}{[13] \langle 13 \rangle \langle 41 \rangle [14]} \right)$$

$$\mathcal{M}(1^-2^-3^+4^+) = -2g_s^2 \left[f^{abe} f^{cde} \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} + f^{ace} f^{bde} \frac{\langle 21 \rangle^4}{\langle 14 \rangle \langle 42 \rangle \langle 23 \rangle \langle 31 \rangle} \right].$$

- The squaring of the amplitude gives:

$$\sum_{\text{colors}} \left|\mathcal{M}(1^-2^-3^+4^+)\right|^2 = 4g_s^4 N^2(N^2-1)\left(\frac{s^4}{t^2u^2} - \frac{s^2}{tu}\right),$$

- Other combinations can be obtained by crossing, i.e. $\mathcal{M}(1^-2^+3^-4^+)$ is given by $\mathcal{M}(1^-2^-3^+4^+)$ with $s \leftrightarrow u$.

- Also: $\mathcal{M}_s(1^+,2^+,3^+,4^+) = 0 = \mathcal{M}_s(1^-,2^+,3^+,4^+)$

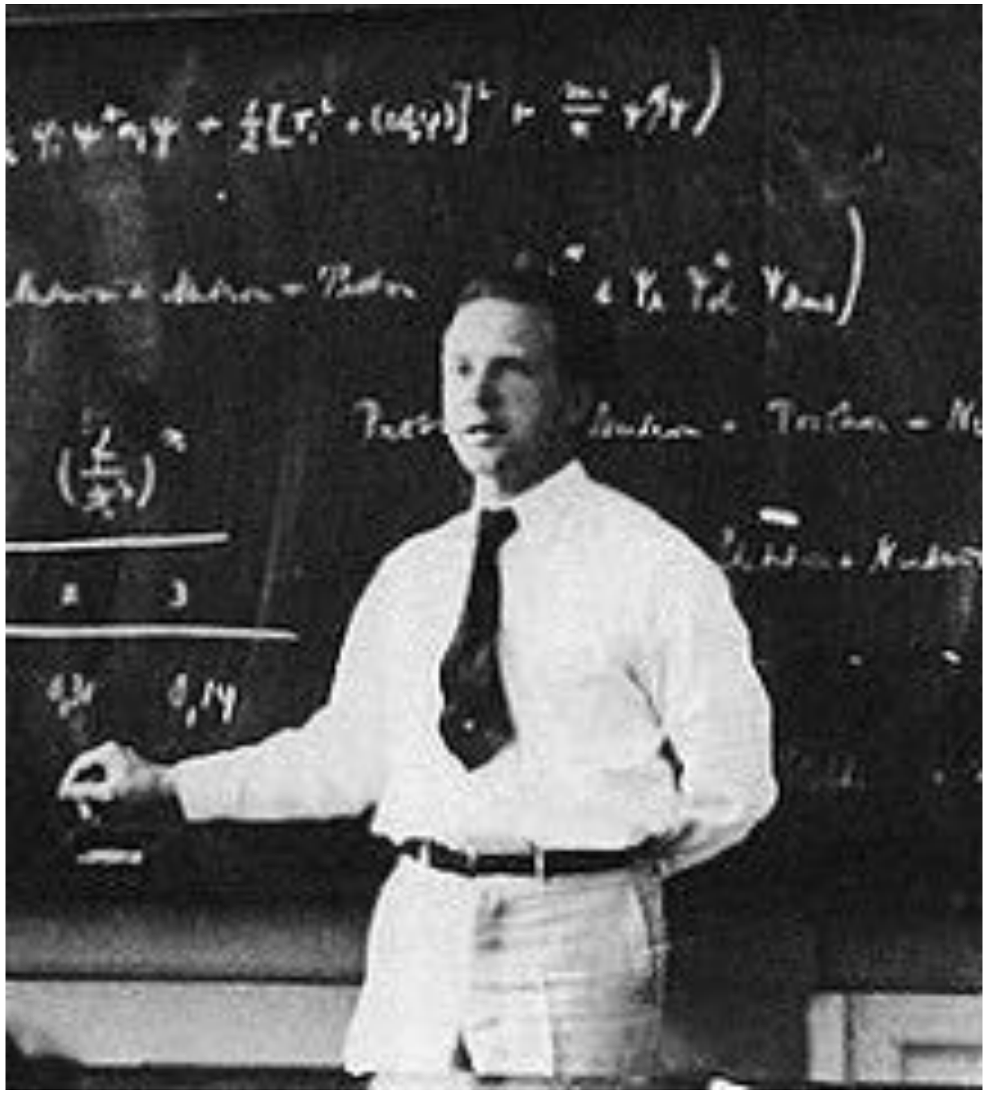
- Then, total squared amplitude is simply:

$$\frac{1}{256}\sum_{\substack{\text{pols.}\\ \text{colors}}} |\mathcal{M}|^2 = \frac{9}{2}g_s^4\left(3 - \frac{su}{t^2} - \frac{ut}{s^2} - \frac{st}{u^2}\right).$$

3. Constructible QFT (A la Heisenberg)

- Since the very early work in QM, Heisenberg wanted a formalism based on physical observables,
- He was against using the concept of "trayectory" for an electron in the atom,
- Einstein questioned him, but he resisted,
- A memorable debate opened between Schrodinger (Wave-functions) and Heisenberg (Matrix mechanics) -

  - Both were right, but …



Could we work in QFT with Physical Amplitudes, building them from scratch? (without ghosts, etc)

Constructible QFT: Little Group Scaling

- Under Little group momentum is invariant (p -> p) , and spinors transform as:

$$\lambda_i \to t_i \lambda_i \qquad \text{and} \qquad \tilde{\lambda}_i \to t_i^{-1} \tilde{\lambda}_i.$$

$$p_{\alpha,\dot{\alpha}} = \chi_\alpha \tilde{\chi}_{\dot{\alpha}}$$

- Polarization vectors and Amplitudes transform as:

$$e^+_{\alpha\dot{\alpha}} \to t^{-2} e^+_{\alpha\dot{\alpha}} \qquad \text{and} \qquad e^-_{\alpha\dot{\alpha}} \to t^2 e^-_{\alpha\dot{\alpha}},$$

$$A(1^{h_1} \cdots n^{h_n}) = e^{h_1}_{\mu_1} \ldots e^{h_n}_{\mu_n} A^{\mu_1 \cdots \mu_n},$$

$$\to \prod_i t_i^{-2h_i} A(1^{h_1} \cdots n^{h_n}).$$

- From momentum conservation for 3-pt interaction: p1+p2+p3=0

$$\langle 12 \rangle = \langle 23 \rangle = \langle 31 \rangle = 0 \qquad \Rightarrow \qquad \lambda_1 \propto \lambda_2 \propto \lambda_3.$$

$$[12] = [23] = [31] = 0 \qquad \Rightarrow \qquad \tilde{\lambda}_1 \propto \tilde{\lambda}_2 \propto \tilde{\lambda}_3,$$

Master formula for 3-point interactions

- The 3-point Amplitudes must be: $\boxed{A(1^{h_1}2^{h_2}3^{h_3}) = \langle 12\rangle^{n_3}\langle 23\rangle^{n_1}\langle 31\rangle^{n_2}.}$

- Then, by looking at scaling of both sides, one can solve:

$$\begin{aligned} -2h_1 &= n_2 + n_3 \\ -2h_2 &= n_3 + n_1 \\ -2h_3 &= n_1 + n_2 \end{aligned} \qquad \Rightarrow \qquad \begin{aligned} n_1 &= h_1 - h_2 - h_3 \\ n_2 &= h_2 - h_3 - h_1 \\ n_3 &= h_3 - h_1 - h_2 \end{aligned}$$

- The master-equation for the 3-point amplitude:

$$\boxed{A(1^{h_1}2^{h_2}3^{h_3}) = \begin{cases} \langle 12\rangle^{h_3-h_1-h_2}\langle 23\rangle^{h_1-h_2-h_3}\langle 31\rangle^{h_2-h_3-h_1}, & h \leq 0 \\ [12]^{h_1+h_2-h_3}[23]^{h_2+h_3-h_1}[31]^{h_3+h_1-h_2}, & h \geq 0 \end{cases}}$$

- For vector particles (gluons, with h=+-1) we obtain :

$$A(1_a^- 2_b^- 3_c^+) = f_{abc} \frac{\langle 12\rangle^3}{\langle 13\rangle\langle 32\rangle} \quad \text{and} \quad A(1_a^+ 2_b^+ 3_c^-) = f_{abc} \frac{[12]^3}{[13][32]}.$$

$$A(1_a^- 2_b^- 3_c^-) = f_{abc}\langle 12\rangle\langle 23\rangle\langle 31\rangle, \qquad A(1_a^+ 2_b^+ 3_c^+) = f_{abc}[12][23][31],$$

- For tensor particles (h=+-2), such as the graviton, we find:

$$A(1^{--}2^{--}3^{--}) = \langle 12\rangle^2\langle 23\rangle^2\langle 31\rangle^2, \qquad A(1^{++}2^{++}3^{++}) = [12]^2[23]^2[31]^2.$$

$$A(1^{--}2^{--}3^{++}) = \frac{\langle 12\rangle^6}{\langle 13\rangle^2\langle 32\rangle^2}, \qquad A(1^{++}2^{++}3^{--}) = \frac{[12]^6}{[13]^2[32]^2},$$

- Seems that GR= YM x YM !!!
- “I could bow and feel proud in the light of this power …”

Amplitudes &Perturbative Jewels

- Parke-Taylor for MHV Amplitudes,
- KLT relations (first derived from String theory),
- Double-copy,
- Unitarity cuts for Tree-loop calculations,
- Color-kinematics duality,
- Simplest QFT - N=4 SYM

$$\widetilde{\mathcal{M}}(1^+2^+\cdots j^-\cdots k^-\cdots n^+) = \frac{\langle jk\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\cdots\langle n1\rangle},$$

$$\begin{aligned}
M_4^{\rm tree}(1234) &= -s_{12}\, A_4^{\rm tree}[1234]\, A_4^{\rm tree}[1243]\,,\\
M_5^{\rm tree}(12345) &= s_{23}s_{45}\, A_5^{\rm tree}[12345]\, A_5^{\rm tree}[13254] + (3\leftrightarrow 4)\,,\\
M_6^{\rm tree}(123456) &= -s_{12}s_{45}A_6^{\rm tree}[123456]\Big(s_{35}A_6^{\rm tree}[153462] + (s_{34}+s_{35})A_6^{\rm tree}[154362]\Big)\\
&\quad + \mathcal{P}(2,3,4)\,.
\end{aligned}$$

$$\left\langle \frac{\delta S^3}{\delta A_\mu^{-a}\delta A_\sigma^{-b}\delta A_\rho^{+c}}\right\rangle_{\text{on-shell}} \to -2if^{abc}\left(k_1{}^\sigma\eta^{\mu\rho} - k_2{}^\mu\eta^{\rho\sigma}\right)\,. \qquad (20)$$

$$\left\langle \frac{\delta S^3}{\delta\varphi_{\mu\nu}^-\delta\varphi_{\sigma\tau}^-\delta\varphi_{\rho\lambda}^+}\right\rangle_{\text{on-shell}} \to 4\left(k_1{}^\sigma\eta^{\mu\rho} - k_2{}^\mu\eta^{\rho\sigma}\right)\left(k_1{}^\tau\eta^{\nu\lambda} - k_2{}^\nu\eta^{\lambda\tau}\right)\,. \qquad (21)$$

4.1 Higgs-graviton couplings: Where is the action?
(A. Alonzo, A. Avilez, B. Larios, LDC)

- We can use master formulae to find coupling of an scalar (h=0) with two gravitons (h=+-2),

$$A(1^0, 2^{-2}, 3^{-2}) = c < 23 >^4 \qquad A(1^0, 2^{+2}, 3^{+2}) = c[23]^4$$

- BTW, the coupling of the Higgs with two gluons is:

$$A(1^0, 2^-, 3^-) = c' < 23 >^2$$

- Scalar-graviton couplings are the same as the ones derived from the Effective Lagrangian:

$$\mathcal{L} = \sqrt{-g}\left(\kappa_1 \phi R + \kappa_2 \phi R^2 + \kappa_3 \phi R_{\mu\nu} R^{\mu\nu} + \kappa_4 \phi R_{\mu\rho\nu\sigma} R^{\mu\rho\nu\sigma} + \ldots\right)$$

- Graviton (h) corresponds to fluctuations of the metric:

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} + \frac{\kappa^2}{2} h_{\mu\alpha} h^{\alpha}_{\nu} + \ldots$$

4.2 New results for the Massive case

- For massive case we have that det(P)= m^2, and need more spinors to express the momentum matrix,
- For instance in the frame where: p=(m,0,0,0),
- Then, the corresponding spinors are:
- Many applications have appeared recently, ex. SMEFFT approach,

$$p_{\alpha\dot{\alpha}} = p_\mu \sigma^\mu_{\alpha\dot{\alpha}} = \begin{pmatrix} p_0 + p_3 & p_1 - ip_2 \\ p_1 + ip_2 & p_0 - p_3 \end{pmatrix},$$

$$P_{\alpha,\dot{\alpha}} = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \qquad det(P) = m^2$$

$$P_{\alpha,\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}} + \chi_\alpha \tilde{\chi}_{\dot{\alpha}}$$

$$\lambda_\alpha = \sqrt{m}(0,1)^T, \ \tilde{\lambda}_{\dot{\alpha}} = \sqrt{m}(0,1)$$

$$\chi_\alpha = \sqrt{m}(1,0)^T, \ \tilde{\chi}_{\dot{\alpha}} = \sqrt{m}(1,0)$$

- We have used a similar method (Light-cone decomposition) to study massive gravitino amplitudes (B. Larios, PhD thesis),

Recently, Arkani-Hamed et al proposed a method that take into account the Little Group properties for the massive case, (I=1,2)

$$P_{\alpha,\dot{\alpha}} = \eta^I_\alpha \eta_{I\dot{\alpha}}$$

4. Conclusions: The language of QFT has evolved with time

In the following we will limit ourselves to a world without strange particles, and with exact conservation of isospin **. Suppose there exists a triplet of vector-boson fields, $W^a_\mu(x)$, $a = 1, 2, 3$, coupled to a triplet of hadrons $J^a_\mu(x)$ such that

$$\partial_\mu J^a_\mu = -g\,\epsilon_{abc}\, W^b_\nu J^c_\nu \,. \tag{3}$$

Having that in mind we propose to use for the gauge invariant infrared regularization of the Yang-Mills theory the following Lagrangian

$$L = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} - m^{-2}(D^2\tilde{\phi})^*(D^2\tilde{\phi}) + (D_\mu e)^*(D_\mu b) + (D_\mu b)^*(D_\mu e)$$

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We first consider the massive fermion-fermion-vector amplitude, with equal-mass fermions. Since this is our first example, we will cover it in detail. It is not difficult to show that the $\mathcal{M}(\mathbf{1}_{\psi^c}, \mathbf{2}_\psi, \mathbf{3}_Z)$ amplitude is fully characterized by the following spinorial structures:

$$\mathcal{M}(1_{\psi^c}, \mathbf{2}_\psi, \mathbf{3}_Z) = \frac{c^{RRR}_{\psi^c\psi Z}}{\Lambda}[\mathbf{13}][\mathbf{23}] + \frac{c^{LR0}_{\psi^c\psi Z}}{m_Z}\langle\mathbf{13}\rangle[\mathbf{23}] + \frac{c^{RL0}_{\psi^c\psi Z}}{m_Z}[\mathbf{13}]\langle\mathbf{23}\rangle + \frac{c^{LLL}_{\psi^c\psi Z}}{\Lambda}\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle\,. \tag{3.1}$$

5. Conclusions:

1. Higgs-graviton couplings are an interesting arena to learn and play …
2. QFT reigns in all over many physics domains

 (from LHC to stars,   superconductors, early universe, and many more …)

3. Amplitudes and Helicity methods are like a “super-power”
4. Perturbative Jewels decorate with their beauty our QFT kingdom,
5. It is possible to build QFT from Little Group (Constructible)
6. The massive case & Little Group Contraction are under study ..

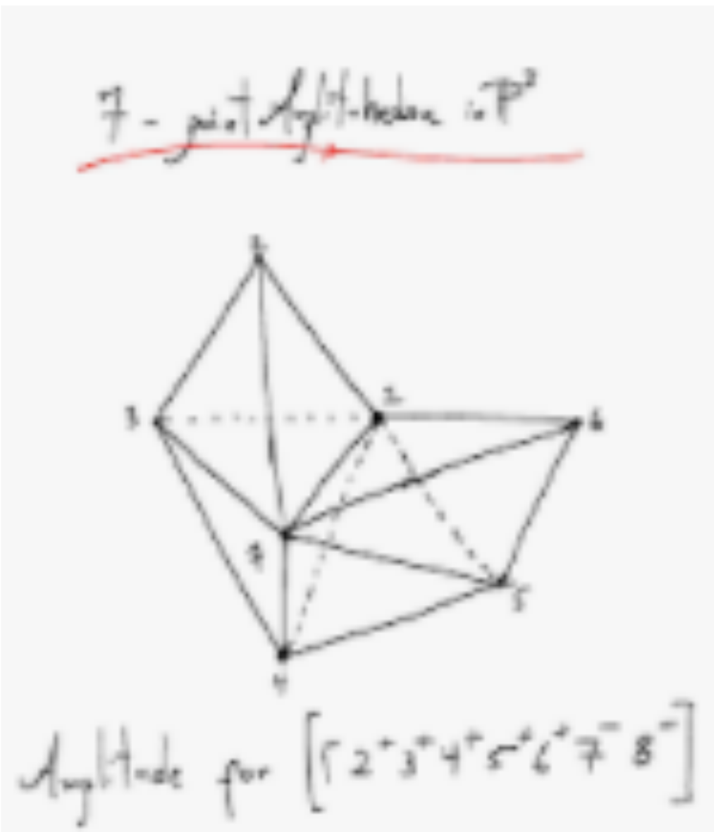
Recollections and reflections

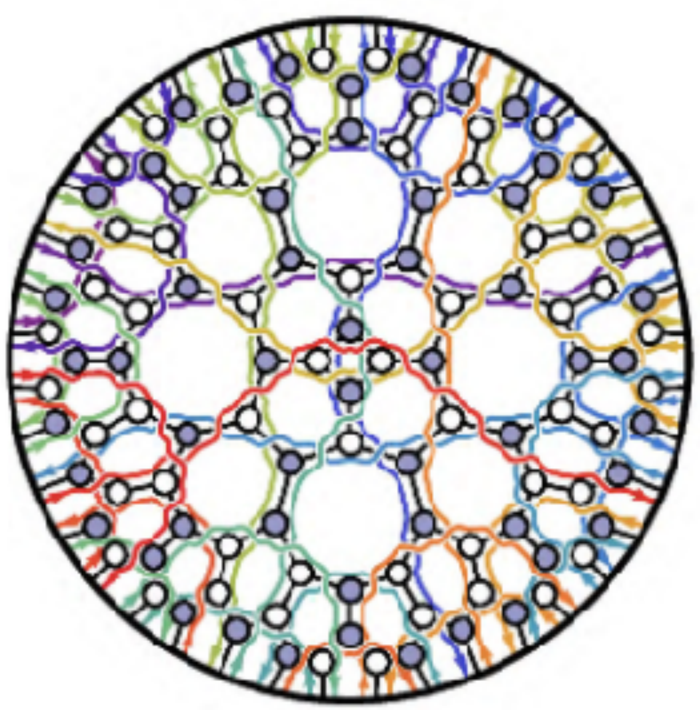
Theme:

- Nature is more beautiful than we think
- Nature is smarter than we are

(A quote from G. ’t Hooft)

Geometry and Amplitudes





Embedding Feynman Integral (Calabi-Yau) Geometries in Weighted Projective Space

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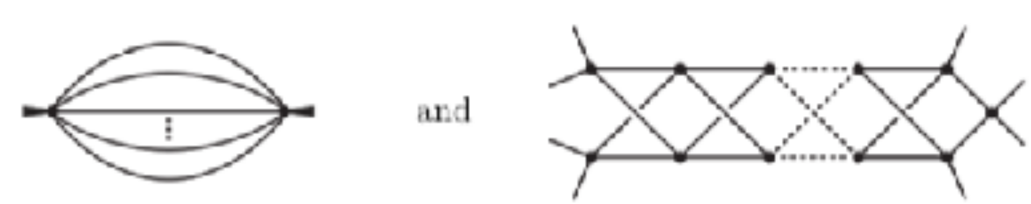
Abstract: It has recently been demonstrated that Feynman integrals relevant to a wide range of perturbative quantum field theories involve periods of Calabi-Yau manifolds of arbitrarily large dimension. While the number of Calabi-Yau manifolds of dimension three or higher is considerable (if not infinite), those relevant to most known examples come from a very simple class: degree-$2k$ hypersurfaces in k-dimensional weighted projective space $\mathbb{WP}^{1,\ldots,1,k}$. In this work, we describe some of the basic properties of these spaces and identify additional examples of Feynman integrals that give rise to hypersurfaces of this type. Details of these examples at three loops and of illustrations of open questions at four loops are included as ancillary files to this work.



Color-Kinematic Duality

M-
. Duff

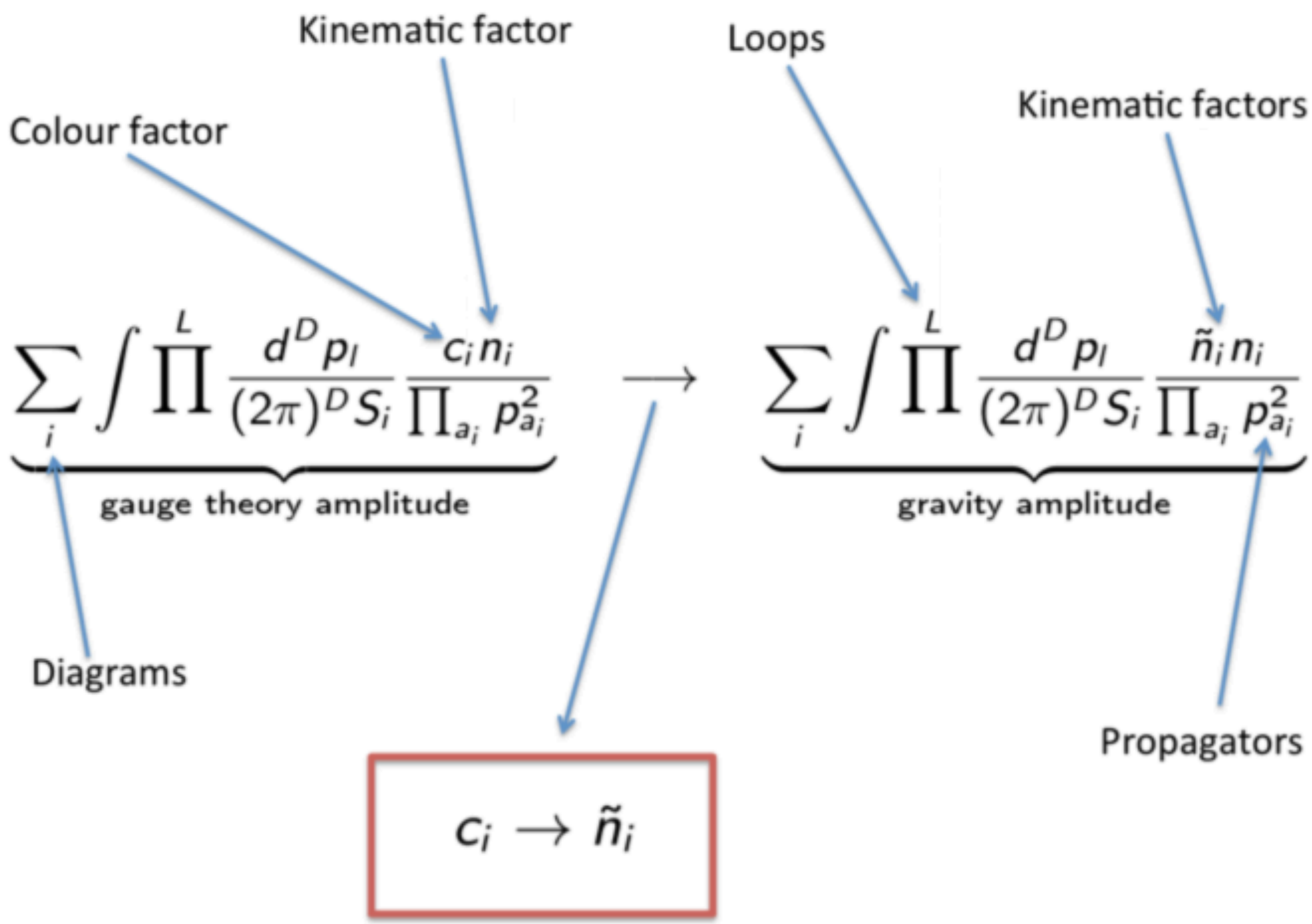
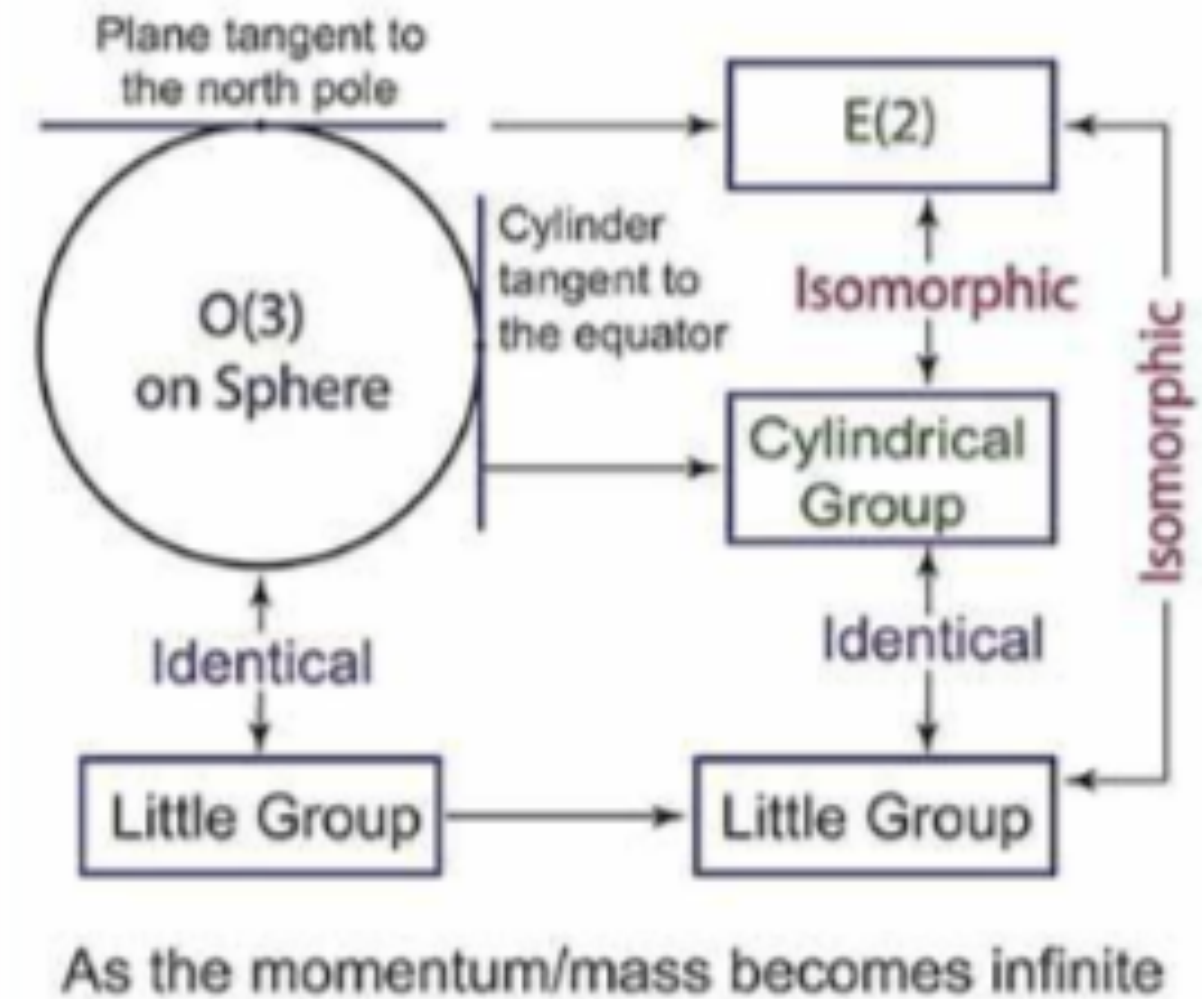


Figure 1: The double-copy procedure. Assuming the gauge theory amplitude on the left has been arranged to display colour-kinematic duality then the gravity amplitude on the right is straight-forwardly obtained by replacing the colour factors with a second copy of the kinematic factors. Note, the second factor does not have correspond to the same Yang-Mills theory. The (supressed) Yang-Mills coupling constants must be replaced by the gravitational coupling constant $g \to \kappa/2$, where $\kappa^2 = 16\pi G_N$.

Little Group Contraction and the massless limit in QFT (Kim & Wigner)



The O(3) group can be contracted to the Euclidian or to the cylindrical group.

The cylindrical group tells physics.

Spinor Notation: Dirac Bra-Kets

$$\chi_i^{\alpha} \to |i> \quad \text{and} \quad \tilde{\chi}_{\mathrm{j}\alpha} \to |\mathrm{j}]$$

$$\chi_{j\alpha} \to <j| \text{ and } \tilde{\chi}_{\mathrm{j}}^{\dot{\alpha}} \to [\mathrm{j}|$$

- Spinor products are defined as:

$$<ij>= \chi_{i\alpha}\chi_{j\beta}\epsilon^{\alpha\beta}$$

$$[ij] = \tilde{\chi}_{i\dot{\alpha}}\tilde{\chi}_{j\dot{\beta}}\epsilon^{\dot{\alpha}\dot{\beta}}$$

- Also for massless spinors:

< ii > = 0 = [jj] ,

$$<ij>[ij] = 2p_i \cdot p_j$$

$$q \cdot p = q^{\mu}p_{\mu} = \frac{1}{2}q_{\dot{\alpha}\alpha}p^{\alpha\dot{\alpha}} \equiv \frac{1}{2}\mathrm{tr}\Big\{q]\langle q\, p\rangle[p\Big\} = \frac{1}{2}\langle qp\rangle[pq]$$

The Birth of QFT

- In one of the fundamental papers on Quantum Mechanics (“3 man paper”= BHJ), the seeds of QFT were planted, from the relations: $[p,q]=ih$,

- P. Jordan also wrote similar relations for the EM field:

  $[a(k), a^+(k')] = i h\, d(k-k')$ —> Field

- Heisenberg and Pauli first wrote the complete program of QFT,

- Dirac proposed a relativistic equation for the electron, it predicted “Antimatter”

- Big problem: 2nd order calculations -> infinities,

Renormalization of QFT

- Quantum Electrodynamics (QED): the theory of electrons, positrons and photons,
- After 2nd WW, new data (electron AMM & Lambf shift) suggested needed for 2nd order QED,
- Killing the Hydra head I: Feynman, Schwinger and Tomonaga found that infinities can be absorbed into charge, mass and wave-function of the electron,
- F. Dyson found that P.I.´s (Feynman) are equivalent to Canonical Quantization (Schwinger, Tomonaga)

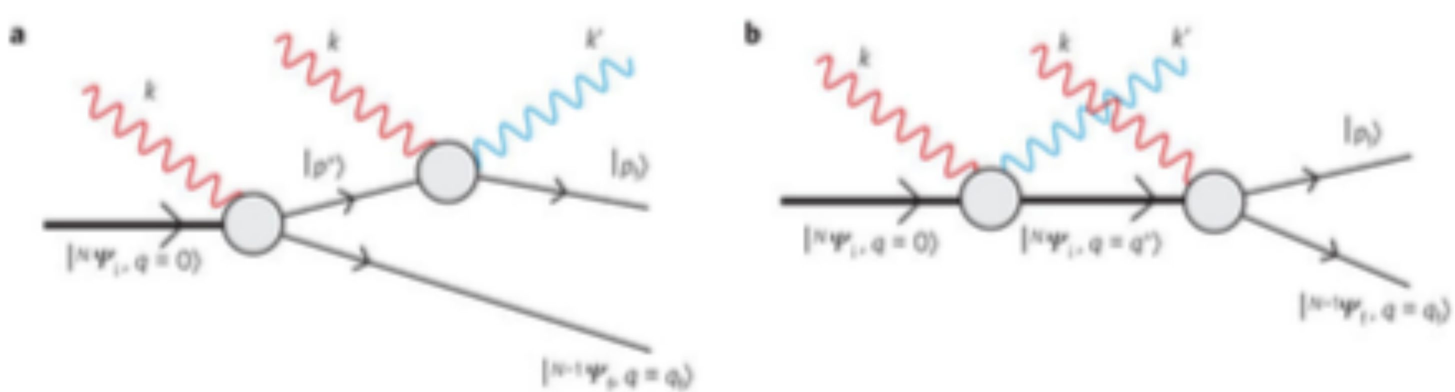
Sin-Itiro Tomonaga

Julian Schwinger

Standard treatment of QFT (as in textbooks)

$$\begin{tikzcd}[column sep=huge]
\textbf{Classical Mechanics} \arrow[r, "{\text{(i): Quantization}\ (x, p \to \hat{x}, \hat{p})}"] \arrow[d, "N\to\infty"] & \textbf{Quantum Mechanics} \arrow[d, "\text{(ii): } N\to\infty"] \\
\begin{array}{c}\text{Non-Relativistic}\\ \text{Classical Field Theory}\end{array} \arrow[r] \arrow[d, "\text{\& SRT}"] & \begin{array}{c}\text{Non-Relativistic}\\ \text{Quantum Field Theory}\end{array} \arrow[d, "\text{(iii): \& SRT}"] \\
\begin{array}{c}\textbf{Relativistic}\\ \textbf{Classical Field Theory}\\ \textbf{(e.g. Electrodynamics)}\end{array} \arrow[r, "{\text{Quantization}\ (\Phi, \Pi \to \hat{\Phi}, \hat{\Pi})}"] & \begin{array}{c}\textbf{Relativistic}\\ \textbf{Quantum Field Theory}\end{array}
\end{tikzcd}$$

- We use a perturbative language to identify particles,
- S-Matrix (LSZ) $\to$ Feynman rules $\to$ Physical Process,
- Amplitud $= \sum$ (Feynman Diagrams)
- Diagramm $=$ Ext. Lines $+$ Int. Lines (Propagators) $+$ Vertices



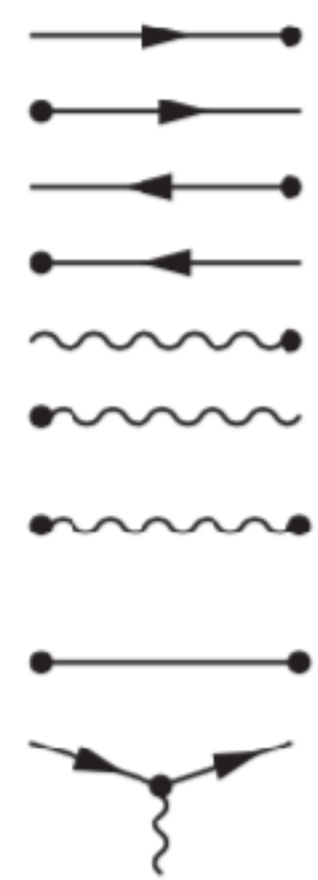
$$|in> \to |out> = S|in> \quad (\mathrm{S-Matrix})$$

$$< e^-(p_3)e^+(p_4)|S|e^-(p_1)e^+(p_2) >$$

QED was first successful QFT

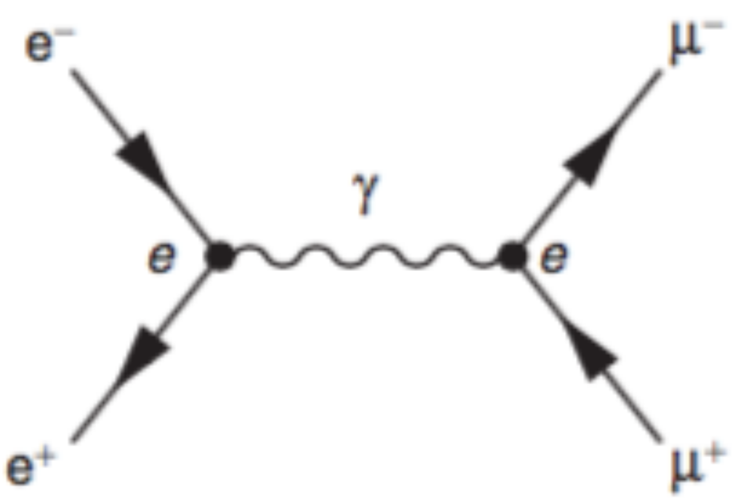
- Pert.-QFT is defined by a set of Feynman rules,
- External lines,
- Internal lines (Propagators),
- Vertices

initial-state particle:	$u(p)$	
final-state particle:	$\bar{u}(p)$	
initial-state antiparticle:	$\bar{v}(p)$	
final-state antiparticle:	$v(p)$	
initial-state photon:	$\varepsilon_\mu(p)$	
final-state photon:	$\varepsilon^*_\mu(p)$	
photon propagator:	$-\frac{ig_{\mu\nu}}{q^2}$	
fermion propagator:	$-\frac{i(\gamma^\mu q_\mu + m)}{q^2 - m^2}$	
QED vertex:	$-iQe\gamma^\mu$	



- Ex. e+ e- -> mu+ mu-

$$-i\mathcal{M} = [\bar{u}(p_3)\{ie\gamma^\mu\}u(p_1)]\frac{-ig_{\mu\nu}}{q^2}[\bar{u}(p_4)\{ie\gamma^\nu\}u(p_2)],$$

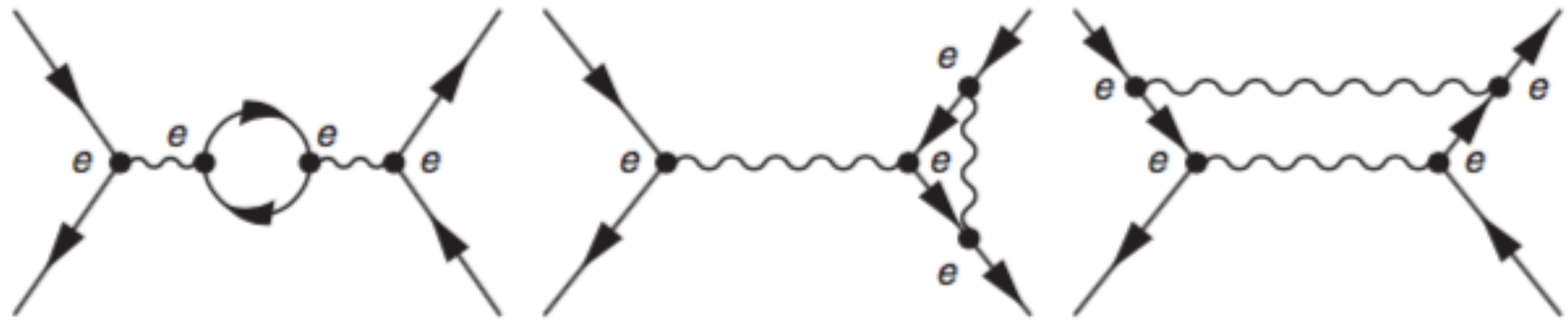


$$\sum_{\text{spins}} |\mathcal{M}_{fi}|^2 = \frac{Q_q^2 e^4}{q^4} \mathrm{Tr}\left([\not{p}_3 + m_e]\gamma^\mu[\not{p}_1 + m_e]\gamma^\nu\right) \mathrm{Tr}\left([\not{p}_4 + m_q]\gamma_\mu[\not{p}_2 + m_q]\gamma_\nu\right)$$

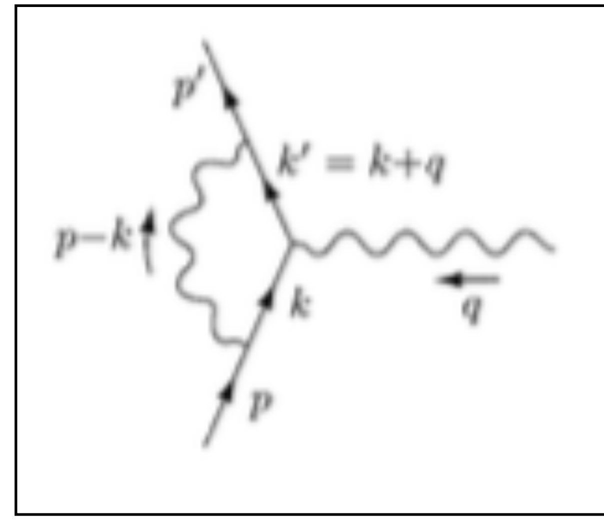
$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} e^4(1 + \cos^2\theta),$$

$$\boxed{\sigma = \frac{4\pi\alpha^2}{3s}.}$$

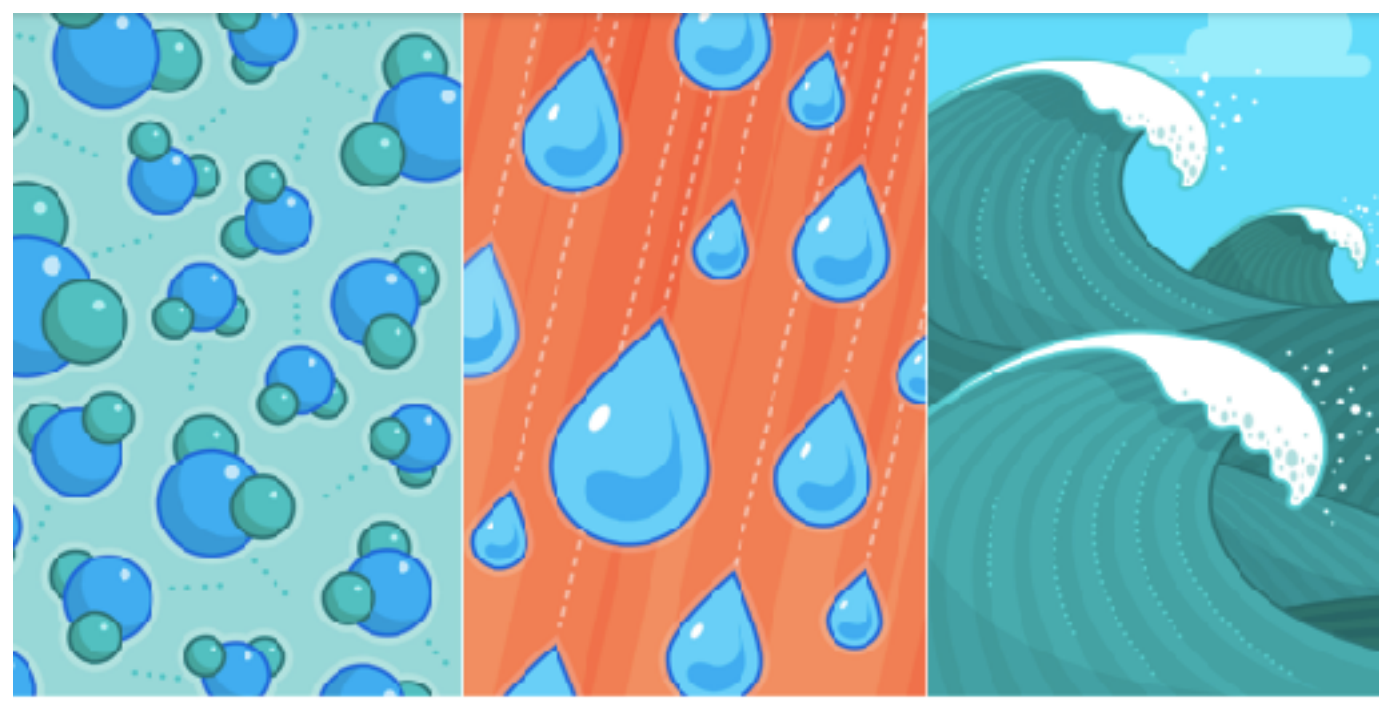
QED-Loops and Renormalization



- Loop diagrams (with infinites) - Renormalization.
- Finite loop diagrams:
  QED (and QFT) has been proved with a high precision, e.g.
  $a_\mu^{th(exp.)} = (1\,159\,652\,157 \pm 28) \times 10^{-12} [(1\,159\,652\,188 \pm 4) \times 10^{-12}],$



- First, Renormalization was considered ugly (Dirac),
- May be that made QFT to start late in Mexico,
- But, now it is better understood: relevant d.of f. are associated with the scale,



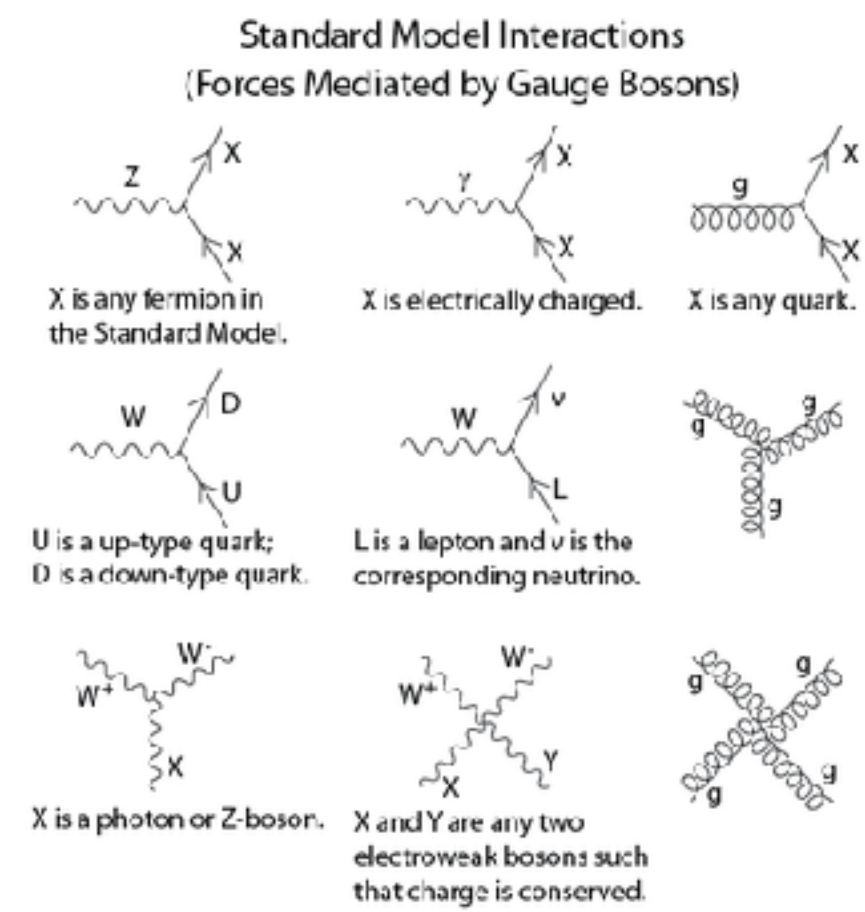
You don't have to analyze individual water molecules to understand the behavior of droplets, or droplets to study a wave. This ability to shift focus across various scales is the essence of renormalization.

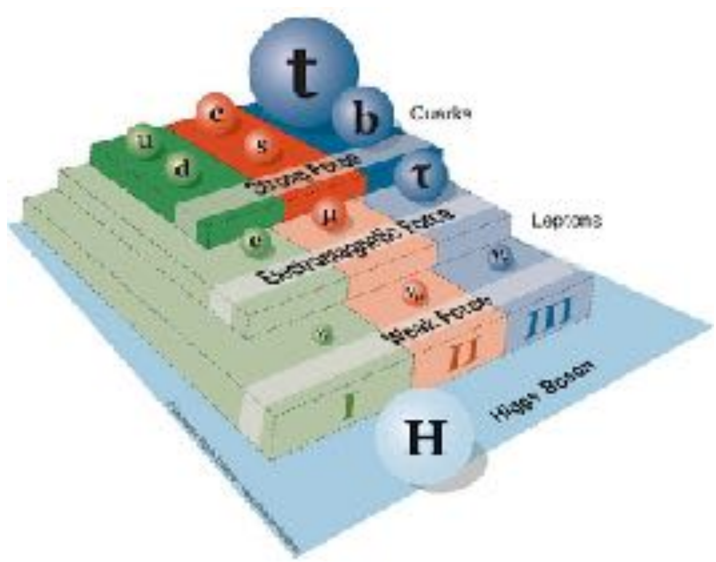
Samuel Velasco/Quanta Magazine

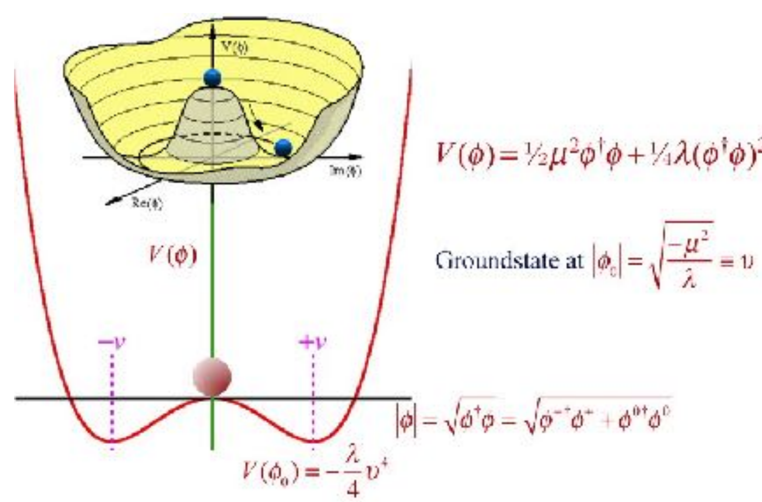
QFT: from QED to the Standard Model

- Quarks and Leptons are the fundamental blocks of nature (3 families),
- Forces obey the gauge principle (Weak,Strong, E.M.),
- Mass comes from the interaction of particles with the vacuum,
- The Higgs boson was found at LHC (m=125 GeV),

- SM (pert.) is defined by a set of Feynman rules

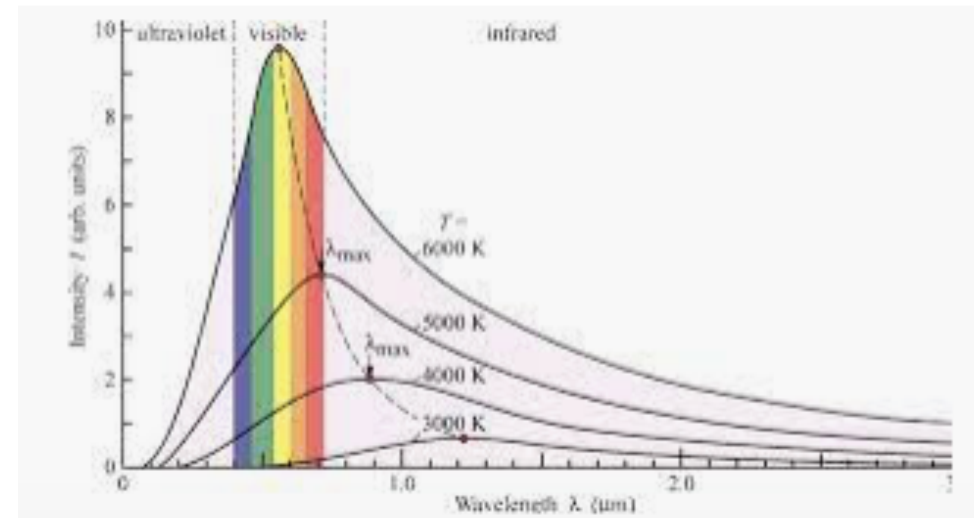


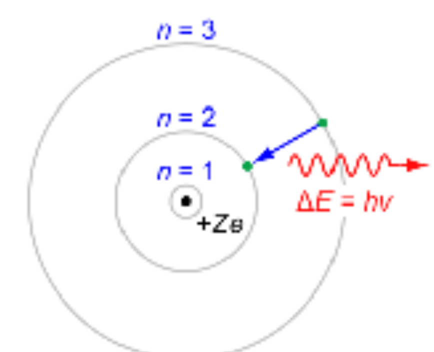




I) Introduction: The Birth of Quantum Mechanics

- Planck and the Black-body radiation -> Quantum (E=hν),
- Einstein and the photoelectric effect,
- Bohr and the atomic spectrum,





- Heisenberg, Born, Jordan,

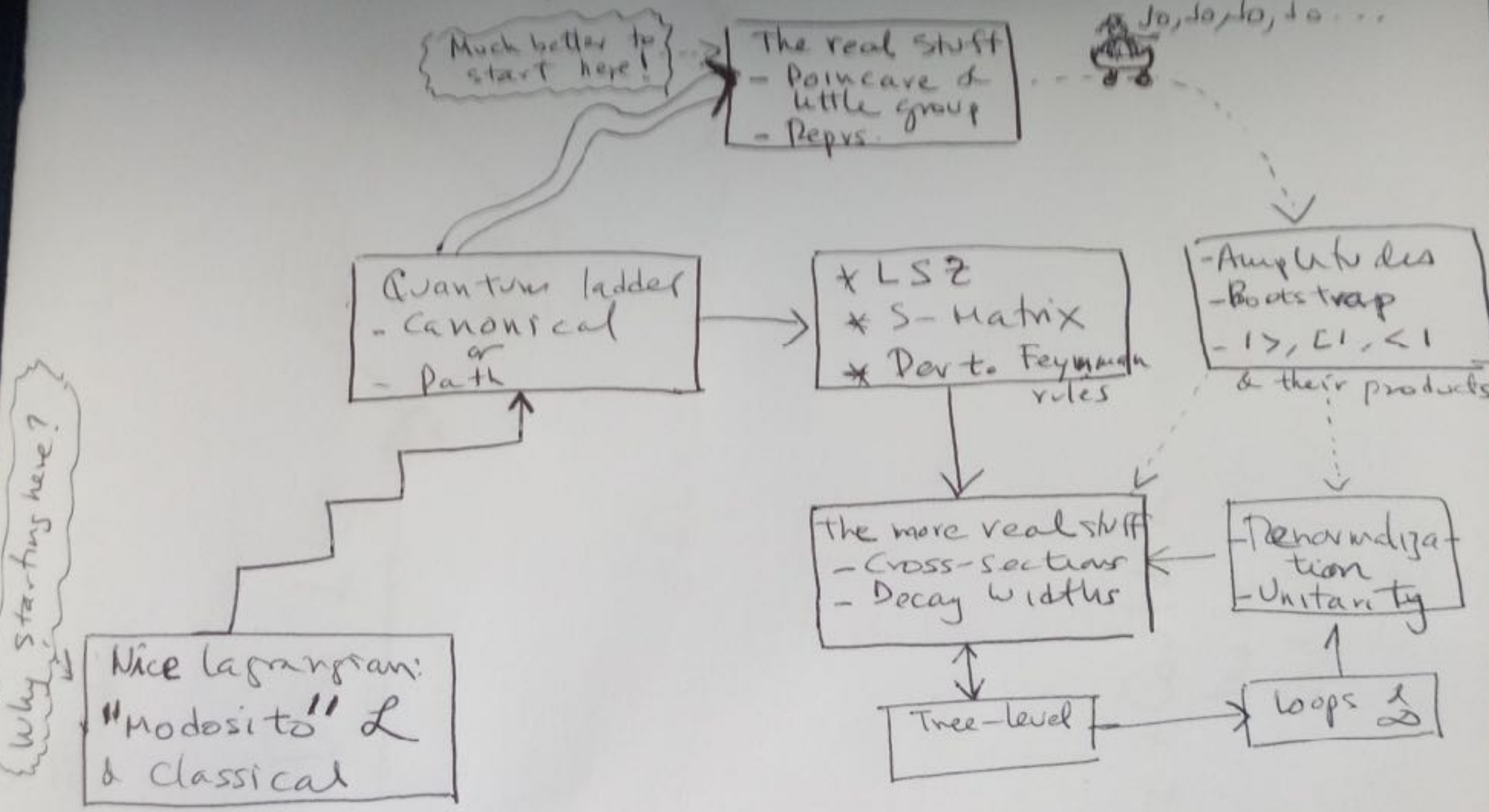
 Matrix Mechanics -> p & q are matrices
(-> Uncertaintity Principle)

- Schrodinger, Bohr, Born,

 Wave-functions -> Probability

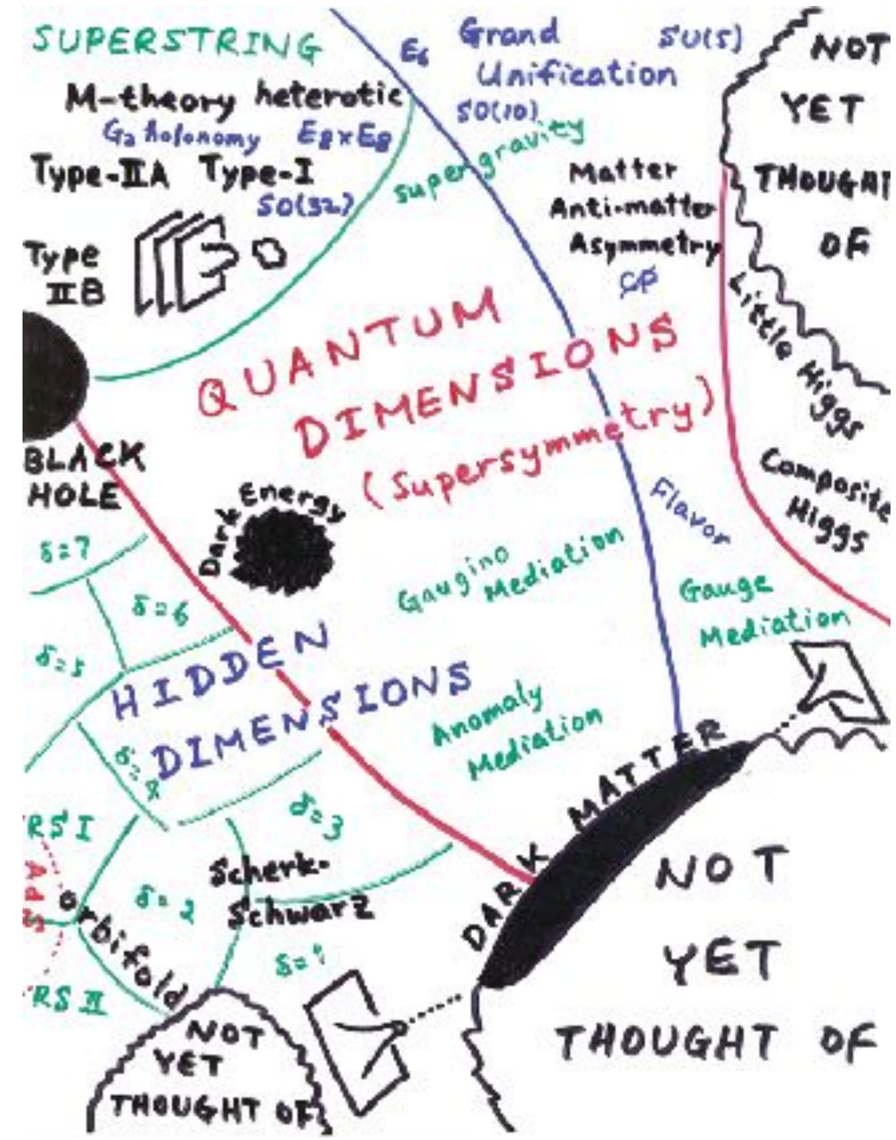
- Pauli, Dirac - bras and kets

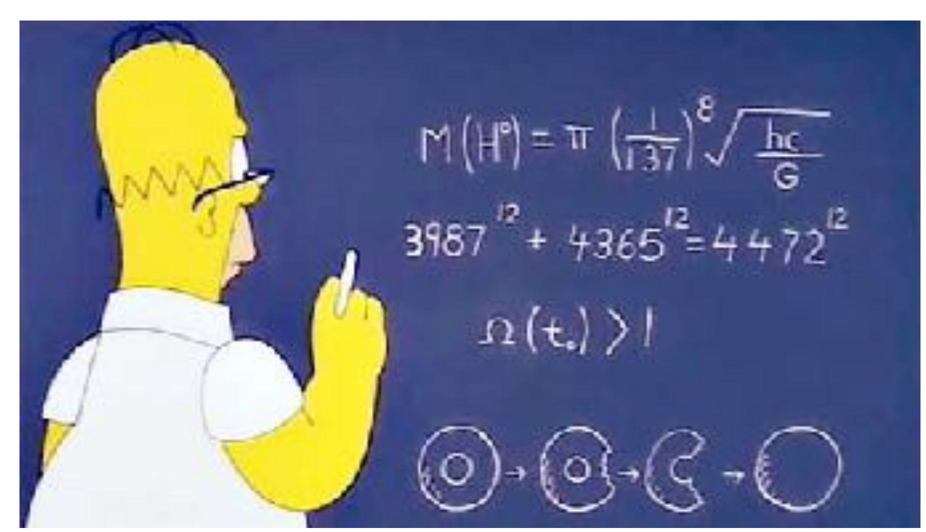
QFT for thinkers & Rebels



In summary QFT/SM is great, but …

- Great success, from QED to the SM,
- We want more: Unification,
  - Gran Unfied Theories (GUTs)
- Solve the Dark Matter and Dark Energy Enigmas,
- The Higgs generates the inertial mass, what about the gravitational mass?
- Find a quantum theory of gravity (QG)!
- String theory (A TOE?) or LQG,
- But is this all? Are there other formalisms for QFT? Is it a theory of particles or fields?





Results up to 2nd order in kappa:

- Expand the metric first,
- Evaluate Christoffel symbols:
- Then, can calculate Riemann tensor:

$$\Gamma_{\alpha\beta}{}^{\gamma} = \frac{1}{2} g^{\gamma\lambda}(g_{\lambda\alpha,\beta} + g_{\lambda\beta,\alpha} - g_{\alpha\beta,\lambda})$$

$$\Gamma_{\alpha\beta}{}^{\gamma} = \Gamma^{(0)\gamma}_{\alpha\beta} + \kappa\Gamma^{(1)\gamma}_{\alpha\beta} + \kappa^2\Gamma^{(2)\gamma}_{\alpha\beta} + \dots$$

$$R_{\mu\alpha\nu}{}^{\beta} = \Gamma_{\mu\nu}{}^{\beta}{}_{,\alpha} - \Gamma_{\alpha\nu}{}^{\beta}{}_{,\mu} + \Gamma_{\mu\nu}{}^{\sigma}\Gamma_{\sigma\nu}{}^{\beta} - \Gamma_{\alpha\nu}{}^{\sigma}\Gamma_{\sigma\mu}{}^{\beta}$$

$$R_{\mu\alpha\nu}{}^{\beta} = R^{(0)}_{\mu\alpha\nu}{}^{\beta} + \kappa R^{(1)}_{\mu\alpha\nu}{}^{\beta} + \dots$$

- Ricci Tensor:
- Scalar curvature:

$$R_{\mu\nu} \equiv R_{\mu\lambda\nu}{}^{\lambda} \qquad R_{\mu\nu} = R^{(0)}_{\mu\nu} + \kappa R^{(1)}_{\mu\nu} + \kappa^2 R^{(2)}_{\mu\nu} + \dots$$

$$R \equiv g^{\mu\nu} R_{\mu\nu} \qquad R = R^{(0)} + \kappa R^{(1)} + \kappa^2 R^{(2)} + \dots$$

- And do not forget to include expansion of Sqrt(-g):

$$\boxed{\begin{aligned}\sqrt{-g} &= 1 + \frac{1}{2}\kappa h^{\lambda}_{\lambda} + \frac{1}{8}\kappa^2 h^{\alpha}_{\alpha} h^{\beta}_{\beta} \\ &= 1 + \frac{1}{2}\kappa h + \frac{1}{8}\kappa^2 h^2 + \dots\end{aligned}}$$

Results for amplitudes on-shell:

- For squared Riemann tensor:

1.2 $\mathcal{L} = \sqrt{-g}\phi R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$

1.2.1 Caso I: $\lambda_1 = \lambda_2 = +2$

$$\mathcal{A}_3^{G.B.}(\phi h^+ h^+) = \frac{\kappa^2}{2}[kk']^4$$

Caso II: $\lambda_1 = +2, \lambda_2 = -2$

$$\mathcal{A}_3^{G.B.}(\phi h^+ h^-) = 0$$

2 Interacción con campo pseudoescalar ϕ_5

2.1 $\mathcal{L} = \sqrt{-g}\phi_5 \epsilon_{\mu\nu\rho\sigma} R^{\mu\nu}{}_{\alpha\beta} R^{\rho\sigma\alpha\beta}/4$

2.1.1 Caso I: $\lambda_1 = \lambda_2 = +2$

$$\mathcal{A}_3^{P.S.}(\phi_5 h^+ h^+) = \kappa^2 \frac{[kk']^2}{4\langle kk'\rangle^2} \epsilon_{\beta\delta\nu\sigma} k^\beta k'^\delta \langle k'|\gamma^\nu|k]\langle k|\gamma^\sigma|k']$$

$$|\mathcal{A}_3^{P.S.}(\phi_5 h^+ h^+)|^2 = \frac{\kappa^4}{16}\langle kk'\rangle^2[kk']^2\langle k'k\rangle^2[kk']^2 = \frac{\kappa^4}{16}\langle kk'\rangle^4[kk']^4$$

Caso II: $\lambda_1 = +2, \lambda_2 = -2$

$$\mathcal{A}_3^{P.S.}(\phi_5 h^+ h^-) = 0$$

… and more!

What is next?

- Evaluate the loop function,
- Result known for 3-point vertex at one-loop
- We are interested in applying Higgs Low-Energy Theorems I.e. From graviton self-energy -> Higgs-graviton interactions
- Work in progress: D. Amaya, J.L. Diaz-Cruz, B. Larios, Mario Aldair Perez,