Scalar and gauge sectors in the 3-Higgs Doublet Model under the S_3 symmetry

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Outline

Motivation

- Studies have been started in the 70's, hoping to find global symmetry that explains the mass and mixing patterns.

  Pakvasa et al (1978); Derman and Tsao (1979);Yahalom (1984); Wyler (1979); A. Mondragón et al (1999); Kubo et al (2004); etc

- S_3 is the smallest flavour symmetry suggested by data.
- Previous works in the quarks and neutrinos sector.

  Kubo et al (2004); A. Mondragón et al (2007); F. Gonzalez (2012); etc

- Extending the concept of flavour to the Higgs sector by adding two more EW doublets.
- Without the symmetry $\rightarrow$ 54 real parameters in the potential.
- Low energy model
- Testable model

The potential of three-Higgs-doublets under the symmetry S_3

- The lagrangian of the Higgs sector under the symmetry S_3 is given as:

$$\mathcal{L}_\phi = (D_\mu H_s)^2 + (D_\mu H_1)^2 + (D_\mu H_2)^2 - V(H_1, H_2, H_s). \qquad (1)$$

- The potential $V(H_1, H_2.H_s)$ more general for the three higgs doublet model invariant under $SU(3)_c \times SU(2)_L \times U(1)_Y \times S_3$ is the following:

$$\begin{aligned} V &= \mu_1^2\left(H_1^\dagger H_1 + H_2^\dagger H_2\right) + \mu_0^2\left(H_s^\dagger H_s\right) + \frac{a}{2}\left(H_s^\dagger H_s\right)^2 + b\left(H_s^\dagger H_s\right)\left(H_1^\dagger H_1 + H_2^\dagger H_2\right) \\ &+ \frac{c}{2}\left(H_1^\dagger H_1 + H_2^\dagger H_2\right)^2 + \frac{d}{2}\left(H_1^\dagger H_2 - H_2^\dagger H_1\right)^2 + ef_{ijk}\left(\left(H_s^\dagger H_i\right)\left(H_j^\dagger H_k\right) + h.c.\right) \\ &+ f\left\{\left(H_s^\dagger H_1\right)\left(H_1^\dagger H_s\right) + \left(H_s^\dagger H_2\right)\left(H_2^\dagger H_s\right)\right\} + \frac{g}{2}\left\{\left(H_1^\dagger H_1 - H_2^\dagger H_2\right)^2 + \left(H_1^\dagger H_2 + H_2^\dagger H_1\right)^2\right\} \\ &+ \frac{h}{2}\left\{\left(H_s^\dagger H_1\right)\left(H_s^\dagger H_1\right) + \left(H_s^\dagger H_2\right)\left(H_s^\dagger H_2\right) + \left(H_1^\dagger H_s\right)\left(H_1^\dagger H_s\right) + \left(H_2^\dagger H_s\right)\left(H_2^\dagger H_s\right)\right\}. \qquad (2) \end{aligned}$$

- The three Higgs doubles of $SU(2)$: H_1, H_2 and H_s can be writing in the following way:

$$H_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 + i\phi_4 \\ \phi_7 + i\phi_{10} \end{pmatrix}, \quad H_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_2 + i\phi_5 \\ \phi_8 + i\phi_{11} \end{pmatrix} \qquad (3)$$

$$H_s = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_3 + i\phi_6 \\ \phi_9 + i\phi_{12} \end{pmatrix}.$$

Kubo et al (2004); Felix-Beltrán, Rodríguez-Jáuregui, M.M (2009), Das and Dey (2014), Barradas et al (2014), Costa, Ogreid, Osland and Rebelo (2016), etc

- We worked in the normal minimum, where the configurations of the fields are:

$$\phi_7 = v_1, \phi_8 = v_2, \phi_9 = v_3, \phi_i = 0, \quad i \neq 7, 8, 9, \tag{4}$$

they must satisfy the condition $v = \sqrt{v_1^2 + v_2^2 + v_3^2} = 246\ GeV$.

- The condition of the minimum fixes :

$$v_1^2 = 3v_2^2, \quad \wedge \quad e = 0. \tag{5}$$

Felix-Beltrán, Rodríguez-Jáuregui, M.M; Costa et al

- The minimum of potential can be parameterized in spherical coordinates, two angles and v.

$$v_1 = v\cos\varphi\sin\theta, \quad v_2 = v\sin\varphi\sin\theta \quad v_3 = v\cos\theta. \tag{6}$$

$\tan\varphi = \pm\frac{1}{\sqrt{3}}$, which means $\varphi = \pi/6$, therefore:

$$\tan\varphi = 1/\sqrt{3} \quad \Rightarrow \quad \sin\varphi = \frac{1}{2} \quad \& \quad \cos\varphi = \frac{\sqrt{3}}{2} \tag{7}$$

$$\tan\theta = \frac{2v_2}{v_3} \quad \Rightarrow \quad \sin\theta = \frac{2v_2}{v} \quad \& \quad \cos\theta = \frac{v_3}{v} \tag{8}$$

- The properties Higgs and Goldstone bosons have been found after diagonalize the 12×12 matrix $(\mathcal{M}^2_H)_{ij} = \frac{1}{2} \left. \frac{\partial^2 V}{\partial\phi_i \partial\phi_j} \right|_{\text{min}}$. We took the next convention:

$$[\mathcal{M}^2_{diag}]_I = R^T_I \mathcal{M}^2_I R_I \quad I = S, A, C. \tag{9}$$

- The rotation matrix is the product of two rotations, i.e., $R_i = A * B_i$. The rotation matrix R which diagonalize $\mathcal{M}^2_A$ and $\mathcal{M}^2_C$ is:

$$R_{a,c} = \begin{pmatrix} \sin\theta\cos\varphi & -\sin\varphi & -\cos\theta\cos\varphi \\ \sin\theta\sin\varphi & \cos\varphi & -\cos\theta\sin\varphi \\ \cos\theta & 0 & \sin\theta \end{pmatrix}. \tag{10}$$

The rotation matrix R which diagonalize $\mathcal{M}^2_S$ is:

$$R_s = \begin{pmatrix} \sin\alpha\cos\varphi & -\sin\varphi & -\cos\alpha\cos\varphi \\ \sin\alpha\sin\varphi & \cos\varphi & -\cos\alpha\sin\varphi \\ \cos\alpha & 0 & \sin\alpha \end{pmatrix}. \tag{11}$$

we define the angle $\tan(2\alpha) = -\frac{M_b^2}{M_a^2 - M_c^2}$.

Higgs masses

- The Higgs masses can be rewritten in the following way:

$$m^2_{h_0} = -\frac{9}{2} e v^2 \sin\theta\cos\theta \tag{12}$$

$$m^2_{H_1,H_2} = \frac{1}{2}\left[(M_a^2 + M_c^2) \pm \sqrt{(M_a^2 - M_c^2)^2 + (M_b^2)^2}\right], \tag{13}$$

$$M_a^2 = \left[(c+g)v^2\sin^2\theta + \frac{3}{2} e v^2 \sin\theta\cos\theta\right],$$

$$M_b^2 = \left[3ev^2\sin^2\theta + 2(b+f+h)v^2\sin\theta\cos\theta\right],$$

$$M_c^2 = av^2\cos^2\theta - \frac{ev^2\tan\theta\sin^2\theta}{2}.$$

$$m^2_{A_1} = -v^2\left[(d+g)\sin^2\theta + \frac{5}{2} e\cos\theta\sin\theta + h\cos^2\theta\right] \tag{14}$$

$$m^2_{A_2} = -v^2\left(\frac{e}{2}\tan\theta + h\right). \tag{15}$$

$$m^2_{H_1^\pm} = -v^2\left[\frac{5}{2} e\sin\theta\cos\theta + \frac{(f+h)}{2}\cos^2\theta + g\sin^2\theta\right] \tag{16}$$

$$m^2_{H_2^\pm} = -\frac{v^2}{2}\left[e\tan\theta + (f+h)\right]. \tag{17}$$

Higgs Basis

- This is the basis in which one of the Higgs' doublets has the complete vacuum expectation value, ϕ_{vev}, and the other doubles are perpendicular to the first one,ψ_1, ψ_2.
- The Higgs basis is define in the following form:

$$\begin{pmatrix} \phi_{vev} \\ \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \cos\varphi \sin\theta & \sin\varphi \sin\theta & \cos\theta \\ -\sin\varphi & \cos\varphi & 0 \\ -\cos\varphi\cos\theta & -\sin\varphi\cos\theta & \sin\theta \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix}. \quad (18)$$

- The doublets in the Higgs basis are given as:

$$\phi_{vev} = \begin{pmatrix} G^{\pm} \\ \frac{1}{\sqrt{2}}(v + \widetilde{h} + iG_0) \end{pmatrix}, \quad \psi_1 = \begin{pmatrix} H_1^{\pm} \\ \frac{1}{\sqrt{2}}(\widetilde{H_1} + iA_1) \end{pmatrix}, \quad \psi_2 = \begin{pmatrix} H_2^{\pm} \\ \frac{1}{\sqrt{2}}(\widetilde{H_2} + iA_2) \end{pmatrix}.$$

where

$$\begin{pmatrix} \widetilde{H} \\ \widetilde{H_a} \\ \widetilde{H_b} \end{pmatrix} = \begin{pmatrix} \cos(\alpha-\theta) & 0 & \sin(\alpha-\theta) \\ 0 & 1 & 0 \\ -\sin(\alpha-\theta) & 0 & \cos(\alpha-\theta) \end{pmatrix} \begin{pmatrix} H_1 \\ h_0 \\ H_2 \end{pmatrix}. \quad (19)$$

Das and Dey (2014)

- After the EWSB it remains a residual symmetry $\mathcal{Z}_2$, that is going to have different changes asociated with the particles from the model. We summarize in the next table:

Das and Dey (2014)

Neutral scalar		Pseudoscalars		Charged scalars	
h_0	Odd	A_1	Odd	$H_1^\pm$	Odd
$\tilde{H}$	Even	A_2	Even	$H_2^\pm$	Even
$\tilde{H}_b$	Even				

Table 1: The $\mathcal{Z}_2$ assignment for the physical states $h_0, A_{1,2}$ and $H^\pm_{1,2}$, and the intermediate-basis states $\tilde{H}$, and $\tilde{H}_b$. In the alignment limit the last two will correspond also to the physical states.

- h_0 decoupled from gauge bosons.
- There is an "alignment" limit, where H_2 is the SM Higgs boson $\rightarrow H_1$ also decoupled from the gauge bosons.

- On the other hand the kinetic term in this basis is given as follow:

$$\mathcal{L}_{kin} = (\mathcal{D}_\mu H_1)^\dagger (\mathcal{D}_\mu H_1) + (\mathcal{D}_\mu H_2)^\dagger (\mathcal{D}_\mu H_2) + (\mathcal{D}_\mu H_s)^\dagger (\mathcal{D}_\mu H_s) \ . \qquad (20)$$

- A summary of the couplings h_0, H_1 and H_2, with two vector bosons and with vector bosons and charged scalars.

$\cos(\alpha-\theta)$	$\sin(\alpha-\theta)$
$g_{H_1 W^+ W^-}$	$g_{H_2 W^+ W^-}$
$g_{H_1 ZZ}$	$g_{H_2 ZZ}$
$g_{ZA_2 H_2}$	$g_{ZA_2 H_1}$
$g_{W^\pm H_2^\mp H_2}$	$g_{W^\pm H_2^\mp H_1}$
$g_{ZW^\pm H_2^\mp H_2}$	$g_{ZW^\pm H_2^\mp H_1}$
$g_{\gamma W^\pm H_2^\mp H_2}$	$g_{\gamma W^\pm H_2^\mp H_1}$

- h0 has no trilinear gauge couplings, only:

$$g_{ZA_1 h_0},\ g_{ZW^\pm H_1^\pm h_0},\ g_{W^\pm H_1^\pm h_0} \text{ and } g_{\gamma W^\pm H_1^\pm h_0}. \qquad (21)$$

Higgs couplings

$$g_{h_0 W^\pm W^\mp} = 0, \quad g_{h_0 ZZ} = 0; \tag{22}$$

$$g_{H_1 W^\pm W^\mp} = \frac{2M_W^2 \cos(\alpha - \theta) g^{\mu\nu}}{v}, \quad g_{H_2 W^\pm W^\mp} = \frac{2M_W^2 \sin(\alpha - \theta) g^{\mu\nu}}{v}; \tag{23}$$

$$g_{H_1 ZZ} = \frac{M_Z^2 \cos(\alpha - \theta) g^{\mu\nu}}{v}, \quad g_{H_2 ZZ} = \frac{M_Z^2 \sin(\alpha - \theta) g^{\mu\nu}}{v}; \tag{24}$$

$$g_{h_0 h_0 W^\pm W^\mp} = \frac{M_W^2 g^{\mu\nu}}{v^2}, \quad g_{h_0 h_0 ZZ} = \frac{M_Z^2 g^{\mu\nu}}{2v^2}; \tag{25}$$

$$g_{H_1 H_1 W^\pm W^\mp} = \frac{M_W^2 g^{\mu\nu}}{v^2}, \quad g_{H_2 H_2 W^\pm W^\mp} = \frac{M_W^2 g^{\mu\nu}}{v^2}; \tag{26}$$

$$g_{H_1 H_1 ZZ} = \frac{M_Z^2 g^{\mu\nu}}{2v^2}, \quad g_{H_2 H_2 ZZ} = \frac{M_Z^2 g^{\mu\nu}}{2v^2}. \tag{27}$$

Higgs self-couplings

- The Higgs trilinear self-couplings are defining as:

$$\lambda_{ijk} = \frac{-i\partial^3 V}{\partial H_i \partial H_j \partial H_k}. \quad (28)$$

- Some of the trilinear self-couplings.

$$g_{h_0 h_0 h_0} = 0,$$

$$g_{H_1 H_1 H_1} = \frac{1}{v s_{2\theta}} \left(m_{h_0}^2 \left(\frac{s_{\alpha-\theta}^3}{9 c_\theta^2} \right) - m_{H_1}^2 \left(c_\alpha^2 s_{\alpha-\theta} - s_\alpha c_\theta \right) \right),$$

$$g_{A_1 A_1 A_1} = 0,$$

$$g_{h_0 h_0 H_1} = \frac{1}{2 v s_\theta} (m_{h_0}^2 \frac{s_{\alpha+\theta}}{c_\theta} + s_\alpha m_{H_1}^2),$$

$$g_{H_2 H_2 H_2} = -\frac{1}{v s_{2\theta}} \left(m_{h_0}^2 \left(\frac{c_{\alpha-\theta}^3}{9 c_\theta^2} \right) + m_{H_2}^2 \left(c_\alpha^2 c_{\alpha-\theta} - s_\alpha s_\theta \right) \right),$$

$$g_{h_0 h_0 H_2} = -\frac{1}{2 v s_\theta} (m_{h_0}^2 \frac{c_{\alpha+\theta}}{c_\theta} + c_\alpha m_{H_2}^2),$$

Per Osland et al (2008); John F. Gunion and Howard E. Haber (2003); Barradas-Guevara et al. (2014)

Higgs self-couplings

$$g_{H_2 H_2 H_1 H_1} = \frac{1}{8v^2 s_{2\theta}^2}\left(\frac{4m_{h_0}^2 s_{2(\alpha-\theta)}}{3c_\theta^2}\left(2s_{2\alpha} + s_{2(\alpha-\theta)}\right) - 2m_{H_1}^2 s_{2\alpha}(3c_{2\alpha} s_{2(\alpha-\theta)} - 3s_{2\alpha} + s_{2\theta}) + 2m_{H_2}^2 s_{2\alpha}(3c_{2\alpha} s_{2(\alpha-\theta)} + 3s_{2\alpha} + s_{2\theta})\right), \quad (29)$$

$$g_{H_2 H_2 H_2 H_2} = \frac{1}{v^2 s_{2\theta}^2}\left(m_{h_0}^2 c_{\alpha-\theta}^3 \frac{(c_{\alpha-\theta} + 2c_{\alpha+\theta})}{9c_\theta^2} + m_{H_1}^2 \frac{s_{2\alpha}^2 c_{\alpha-\theta}^2}{4} + m_{H_2}^2 (c_\alpha^2 c_{\alpha-\theta} - s_\alpha s_\theta)^2\right), \quad (30)$$

Couplings in scenario A

The scalar couplings are reduced in the alignment limit of scenario A as:

$$\sin\alpha = \cos\theta; \quad \cos\alpha = -\sin\theta \tag{31}$$

In scenario A in the alignment limit, the Higgs boson H_2, trilinear coupling coincides exactly with the trilinear coupling of the SM Higgs boson λ_{SM}.

$$g_{H_2H_2H_2} = \frac{1}{v\, s_{2\theta}}\left[m_{H_2}^2 s_\alpha s_\theta\right] = \frac{1}{2v}\frac{s_\alpha}{c_\theta} m_{H_2}^2 = \frac{m_{H_2}^2}{2v}. \tag{32}$$

$$g_{H_1H_1H_1} = \frac{1}{v\, s_{2\theta}}\left[\frac{1}{9c_\theta^2} m_{h_0}^2 - s_\theta^2 m_{H_1}^2\right] = \frac{1}{v\, s_{2\theta} c_\theta^2}\left[\frac{1}{9} m_{h_0}^2 - \frac{1}{2} s_{2\theta} m_{H_1}^2\right]. \tag{33}$$

$$g_{H_2H_2H_2H_2} = \frac{1}{2v^2 s_{2\theta}^2} m_{H_2}^2 (-s_\theta^3 c_\theta - c_\theta^3 s_\theta)^2 = \frac{m_{H_2}^2}{8v^2}.$$

Some of the reduced scalar couplings for scenario A depend only on the masses involved, and are given as:

$$g_{H_2h_0h_0} = \frac{1}{2v}(m_{H_2}^2 + 2m_{h_0}^2), \quad g_{H_2A_1A_1} = \frac{1}{2v}(m_{H_2}^2 + 2m_{A_1}^2), \quad g_{H_2A_2A_2} = \frac{1}{2v}(m_{H_2}^2 + 2m_{A_2}^2),$$
$$g_{H_2H_1^\pm H_1^\mp} = \frac{1}{v}(m_{H_2}^2 + 2m_{H_1^\pm}^2), \quad g_{H_2H_2^\pm H_2^\mp} = \frac{1}{v}(m_{H_2}^2 + 2m_{H_2^\pm}^2), \quad g_{H_2H_2H_2H_1} = 0. \tag{34}$$

If we take $\cos(\alpha - \theta) = \cos\left(\frac{\pi}{2} - \epsilon\right) = \sin\epsilon \equiv \delta$, we get

$$g_{H_2H_2H_2} \equiv \lambda_{SM}\kappa_\lambda = \frac{m_{H_2}^2}{2v}\left[(1+2\delta^2)\sqrt{1-\delta^2} + \delta^3(\tan\theta - \cot\theta) - \frac{m_{h_0}^2}{m_{H_2}^2}\frac{\delta^3}{9 s_\theta c_\theta^3}\right],$$

Higgs one loop self energy

$$\Sigma^{\phi}(s) = \begin{pmatrix} \Sigma^{\phi}_{h_0}(s) & 0 & 0 \\ 0 & \Sigma^{\phi}_{H_1}(s) & \Sigma^{\phi}_{H_1 H_2}(s) \\ 0 & \Sigma^{\phi}_{H_2 H_1}(s) & \Sigma^{\phi}_{H_2}(s) \end{pmatrix}. \qquad (35)$$

$$\Sigma^{\phi}_{H_n} = \sum_i \frac{g_{H_n H_n \phi^0_i \phi^0_i}}{16\pi^2} A_0(m^2_{\phi^0_i}) + \sum_{i,j} \frac{g^2_{H_n \phi^0_i \phi^0_j}}{8\pi^2} B_0(p^2, m^2_{\phi^0_i}, m^2_{\phi^0_j}) + \sum_k \frac{g^2_{H_n \phi^{\pm}_k \phi^{\mp}_k}}{8\pi^2} B_0(p^2, m^2_{\phi^{\pm}_k}, m^2_{\phi^{\pm}_k})$$
$$+ \sum_i \frac{g_{H_n H_n V_i V_i}}{16\pi^2} A_0(m^2_{V_i}) + \sum_i \frac{g^2_{H_n V_i V_i}}{8\pi^2} B_0(p^2, m^2_{V_i}, m^2_{V_i}), \qquad (36)$$

	Masses (GeV)	$\tan\theta$
light spectrum	$m_{h_0} = 80$, $m_{H_1} = 200$, $m_{A_{1,2}} = 80$, $m_{H^{\pm}_{1,2}} = 100$	1
heavy spectrum	$m_{h_0} = 800$, $m_{H_1} = 800$, $m_{A_{1,2}} = 800$, $m_{H^{\pm}_{1,2}} = 800$	2.119

Table 2: Parameter values in scenario A that make the one-loop mixing parameter vanish, $\Sigma^{\phi}_{H_1 H_2} = 0$, taking into account only the scalar and gauge contributions.

Numerical analysis in the Higgs potential

We are going to have two scenarios
Scenario A

$$\sin(\alpha - \theta) = 1 \quad \text{so} \quad \cos(\alpha - \theta) = 0. \tag{37}$$

Scenario B

$$\sin(\alpha - \theta) = 0 \quad \text{y} \quad \cos(\alpha - \theta) = 1. \tag{38}$$

Proceeding

- We made a scan in the parameter space, we imposed the stability and unitarity conditions.

  Das and Dey (2014), Barradas et al (2014)
- We imposed the alignment limit, which means $(\alpha - \theta) = \frac{\pi}{2}$. From each scenario we imposed by hand that H_2 or H_1 takes values between [120-130] GeV.
- We take a precision of the 10%. i.e., ± 0.1, on the $(\alpha - \theta)$ values.

Scenario A

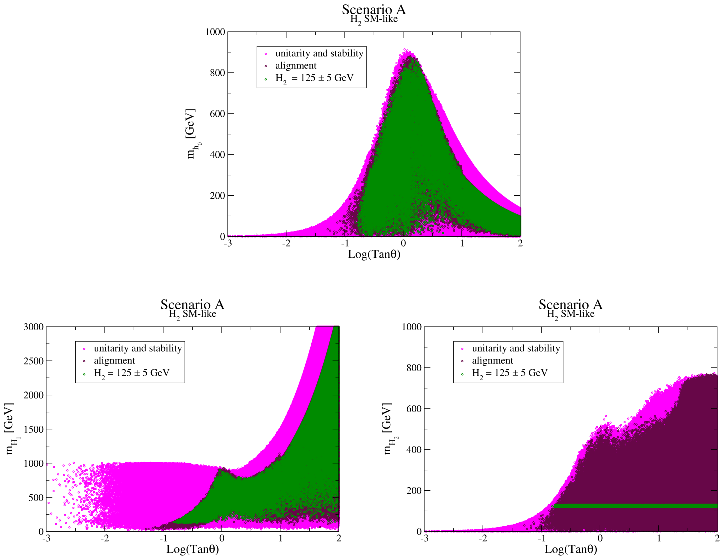

Figure 1: Dependence of the neutral scalar masses, h_0, H_1, H_2, on $\log(\tan\theta) \in (-3, 2)$, for scenario A, when $(\alpha - \theta) = \pi/2$. The first graph shows the values of h_0, the second graph shows the values of H_1 and the last one shows the values of H_2.

Scenario B

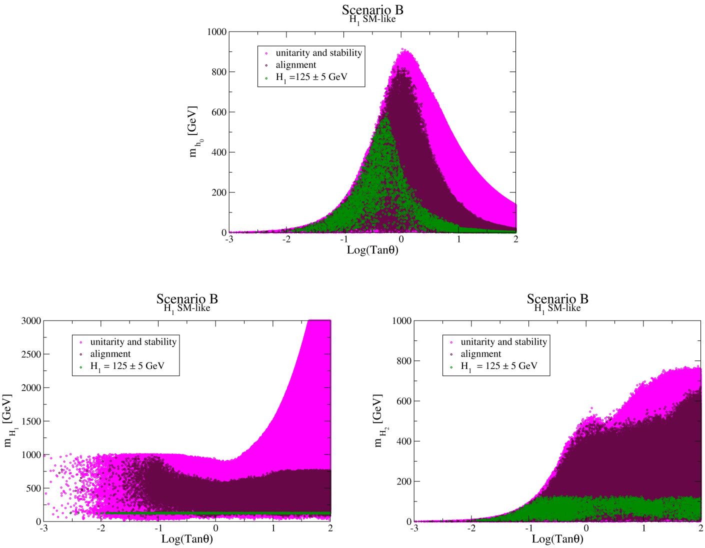

Figure 2: Dependence of the neutral scalar masses, h_0, H_1, H_2, on $\log(\tan\theta) \in (-3, 2)$, for scenario B, when $(\alpha - \theta) = \pi/2$. The first graph shows the values of h_0, the second graph shows the values of H_1 and the last one shows the values of H_2.

Precision on $(\alpha - \theta)$

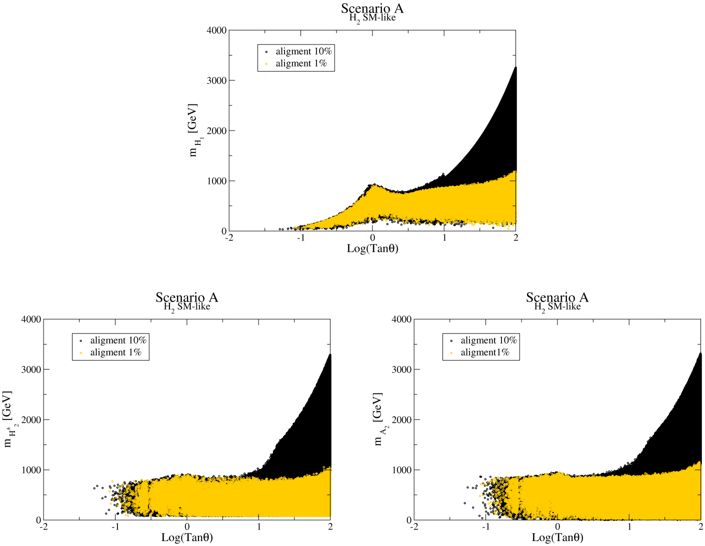

Figure 3: Dependence of the neutral scalar masses, H_1, $H_2^{\pm}$, A_2 for scenario A. Applied with a 10% uncertainty ((black points) and 1% uncertainty (yellow points) on $(\alpha - \theta)$. 2 The points shown comply with the unitarity and stability conditions, and the restriction of H_2 as the SM Higgs.

Charged and Pseudoscalars Higgs

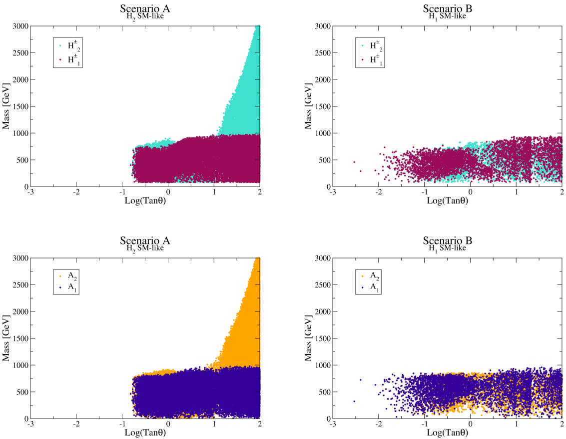

Figure 4: The points shown comply with the constraints of previous figures plus the bounds on the SM-like Higgs boson mass for each scenario.

Summary

- We compute the Higgs self-couplings and the Higgs couplings with the vector bosons.
- We found values for the parameters of the model and the angle θ, that pass all Higgs bounds and satisfy the alignment limit.
- We are able to compute different process in order to make boundaries to the parameters of the model.
- There is a residual symmetry $\mathcal{Z}_2$ as was reported. Which decouples one of the neutral scalars h_0 from the gauge bosons, this raises the possibility of treating this decoupled scalar as a dark matter candidate.
- We obtained the analytical expressions for the one-loop corrections to the SM-like Higgs, due to scalar and gauge bosons in the loop, and found that the decoupling of h_0 remains at one-loop level.
- The small deviation δ we considered of the alignment limit at tree level, is compatible with the latest experimental results on Higgs-gauge boson couplings
- doi: 10.1140/epjc/s10052-021-09731-3. arXiv: 2102.02800 [hep-ph].

Thank you so much !