The Quark Propagator in the Light Cone Gauge

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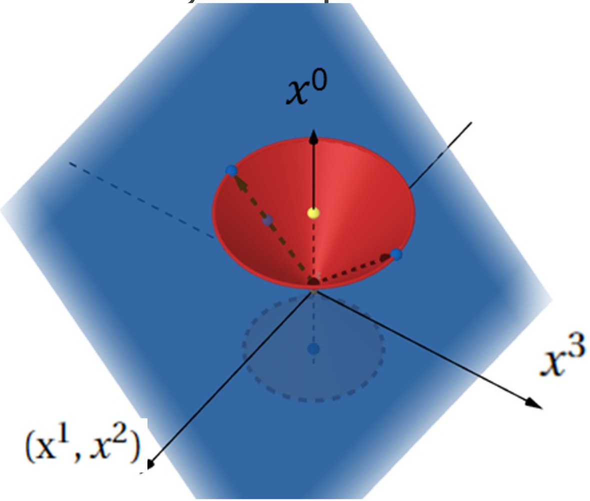
Motivation

- The Schwinger-Dyson Equations (SDE) are the equations of motion corresponding to the Green's function in QCD.
- SDE have been solved in covariant gauge.
- The main adventage of the light-cone approach is that it automatically gets rid of all ghosts and non-physical degrees of freedom.

Light cone coordinates

For an arbitrary four-vector in Minkowski, we perform the following transformation:

$$(a^0, a^1, a^2, a^3) \rightarrow (a^+, a^1, a^2, a^-)$$



Where we have defined:

$$a^{\pm} = a^0 \pm a^3, \quad \mathbf{a}_{\perp} = (a^1, a^2)$$

The scalar product:

$$a \cdot b = g_{\mu\nu} a^{\mu} b^{\nu} = \frac{1}{2} a^+ b^- + \frac{1}{2} a^- b^+ - a^i b^i, \quad i = 1,2$$

$$n_{\mu} = \eta(1,0,0,1)$$
$$n^*_{\mu} = \eta(1,0,0,-1)$$

Metric convention $g_{\mu\nu}$ =(+,-,-,-,) $\longrightarrow$ $\delta_{\mu\nu}$=(+,+,+,+)

QCD Lagrangian in the light cone gauge

$$\mathscr{L}_{LCG} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a - \frac{1}{2\xi}(n_\mu A^\mu_a)^2 + \sum_{j=1}^{N_f} \bar{\psi}^j_l (i\gamma^\mu D_\mu - m_j)\psi^j_l, \quad \text{with } n^2 = 0$$

A represents the gluonic field and **ψ** is the fermionic field

With the covariant derivative $D_\mu \psi^j_l = \partial_\mu \psi^j_l - igR^a_{lk}\psi^j_k A^a_\mu.$

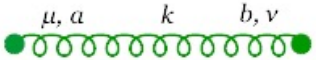
And the gluonic field tensor $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf_{abc}A^b_\mu A^c_\nu$

Feynman rules

Quark propagator:

$$S^{(0)}(p) = \frac{1}{\not{p} - m_j}\delta_{\alpha\beta}\delta_{jj'}$$

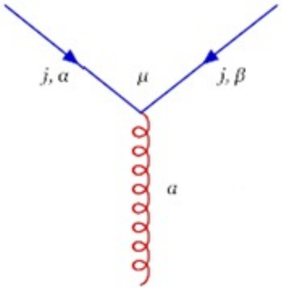
Gluon propagator:



$$\Delta^{(0)}_{\mu\nu}(k) = \frac{1}{k^2}\left[\delta_{\mu\nu} - \frac{k_\mu n_\nu + k_\nu n_\mu}{n\cdot k} + \xi\frac{k^2 k_\mu k_\nu}{(n\cdot k)^2}\right]$$

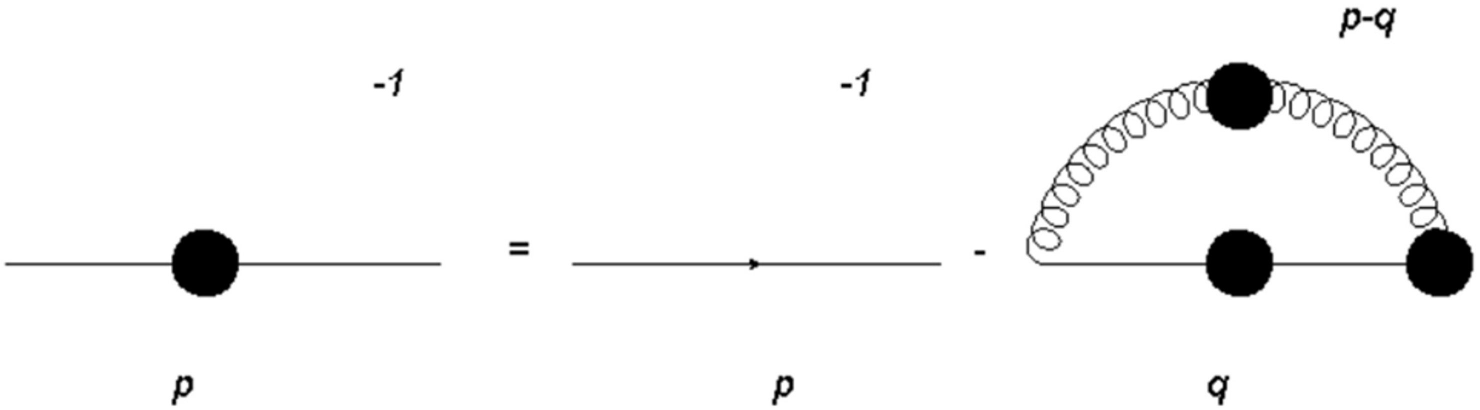
Non-transverse i.e. $k_\mu \Delta^0_{\mu\nu} \neq 0$

Quark-gluon vertex:



$$\Gamma^{(0)a}_{\mu}(p) = g\left(\frac{\lambda_a}{2}\right)_{\beta\alpha}\gamma^\mu$$

Schwinger-Dyson Equations



- Poincare invariance
- They form a set of infinitely many functional differential equations.
- They work very well for studying non-perturbative phenoma

SDE for the quark propagator

For a fully dressed quark of flavor *f*, the SDE in Euclidean space is given by:

$$S_f^{-1}(p;\Lambda) = i\gamma\cdot p + m_f(\Lambda) + \Sigma_f(p;\Lambda)\,,$$

$$\Sigma_f(p;\Lambda) = C_F\int^\Lambda \frac{d^4q}{(2\pi)^4} g^2(\Lambda)\Delta_{\mu\nu}(k;\Lambda)\gamma_\mu S_f(q;\Lambda)\Gamma_\nu(q,p;\Lambda)\,,$$

with $k=p-q$

The most general decomposition for the quark self-energy can be written as

These structures do not appear in covariant gauge!

$$\Sigma_f(p;\Lambda) = i\hat{A}(p^2;\Lambda)\gamma\cdot p + \hat{B}(p^2;\Lambda) + i\hat{C}(p^2;\Lambda)\gamma\cdot n + i\hat{D}(p^2,\Lambda)\gamma\cdot n^*$$

Renormalized SDE

$$Z_2 \equiv \text{Quark propagator renormalization constant.}$$
$$Z_\pi \equiv \text{Self-energy renormalization constant}$$
$$Z_m \equiv \text{Renormalization constant for the mass of the quark}$$

$$S_f^{-1}(p) = Z_2[i\gamma \cdot p + Z_m m_f(\mu)] + Z_\pi Z_2^2 C_F \int^\Lambda \frac{d^4q}{(2\pi)^4} g^2 \Delta_{\mu\nu}(k)\gamma_\mu S_f(q)\Gamma_\nu(q,p)$$

Where the fully dressed quark propagator can be expressed quite generally as:

$$S_f^{-1}(p) = iA(p^2)\gamma \cdot p + B(p^2) + iC(p^2)\gamma \cdot n + iD(p^2)\gamma \cdot n^*,$$

$$S_f(p) = \frac{-iA(p^2)\gamma \cdot p + B(p^2) - iC(p^2)\gamma \cdot n - iD(p^2)\gamma \cdot n^*}{\beta(p)},$$

$$\begin{aligned}\beta(p) \;=\; & p^2 A^2(p^2) + B^2(p^2) + 2A(p^2)C(p^2)(n \cdot p) \\ & + 2A(p^2)D(p^2)(n^* \cdot p) + 2C(p^2)D(p^2)(n^* \cdot n),\end{aligned}$$

$$\begin{aligned}A(p^2) &= \hat{A}(p^2) + 1 \\ B(p^2) &= \hat{B}(p^2) + m_f(\mu), \\ C(p^2) &= \hat{C}(p^2), \\ D(p^2) &= \hat{D}(p^2),\end{aligned}$$

Renormalization conditions

–

$$A^{(1)}(\mu^2) = 1$$
$$B^{(1)}(\mu^2) = m_q$$

We would also expect

$$C^{(1)}(\mu^2) = 0$$
$$D^{(1)}(\mu^2) = 0$$

One-loop calculations suggest that in LCG we need to choose a "renormalization direction"!

at tree level $S^{-1}_{(0)}(p) = \gamma \cdot p + m$

thus at the renormalization point $p^2=\mu^2$ one would expect this behavior

One loop solutions

In the leading log. approximation:

$$
\begin{aligned}
A(p^2) &= Z_2(1 + Z_\pi I_A(p^2)) \\
B(p^2) &= Z_2 Z_m m(\mu) + Z_\pi I_B(p^2) \\
C(p^2) &= \frac{1}{2} Z_2 Z_\pi \frac{n^* \cdot p}{n^* \cdot n}(I_C(p^2) - I_D(p^2)) \\
D(p^2) &= \frac{1}{2} Z_2 Z_\pi \frac{n \cdot p}{n^* \cdot n}(I_D(p^2) - I_C(p^2))
\end{aligned}
$$

$$\kappa = \frac{C_F \alpha(\mu)}{2\pi^3}$$

$$
\begin{aligned}
I_A^{(1)}(p^2) &= \kappa I(p^2) \left[\frac{p_{||}^2}{p^2} \left(1 - \frac{I(p_{||}^2)}{I(p^2)} \right) - \frac{1}{2} \right] \\
I_B^{(1)}(p^2) &= \kappa I(p^2) \\
I_C^{(1)}(p^2) &= \frac{\kappa}{2} I(p^2) \left(1 - 4 \frac{I(p_{||}^2)}{I(p^2)} \right) \\
I_D^{(1)}(p^2) &= \frac{\kappa}{2} I(p^2) \\
I(p^2) &= \pi^2 \log(\Lambda^2 / p^2)
\end{aligned}
$$

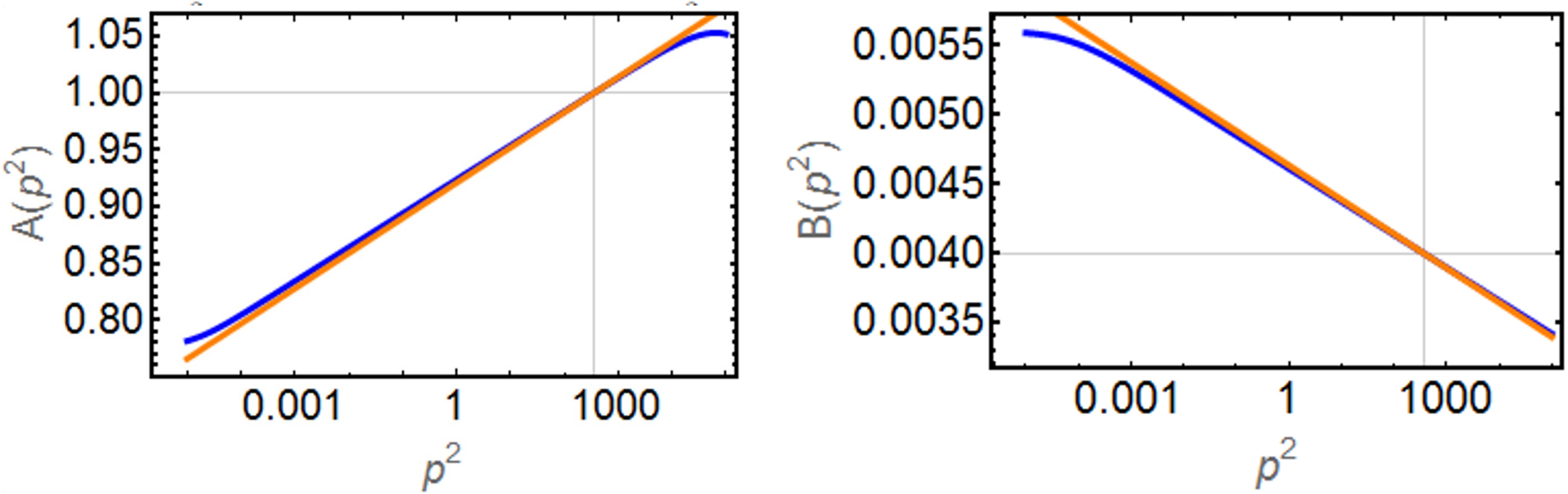
Recall that $p_\mu = p_n n_\mu + p_{n^*} n_\mu^* + \mathbf{p}_\perp \implies p^2 = \mathbf{p}_{||}^2 + \mathbf{p}_\perp^2$

Numerical results (for $p_\perp$=0)

$\alpha = 0.118, \quad m_q = 0.004 GeV, \quad \mu = 19.0 GeV$
$Z_2 = 1.07, \quad Z_m = 0.78, \quad Z_\pi = 1.0$

$A(p^2) = Z_2(1 + Z_\pi I_A(p^2))$

$B(p^2) = Z_2 Z_m m(\mu) + Z_\pi I_B(p^2)$



Analytical results (orange), numerical results(blue)

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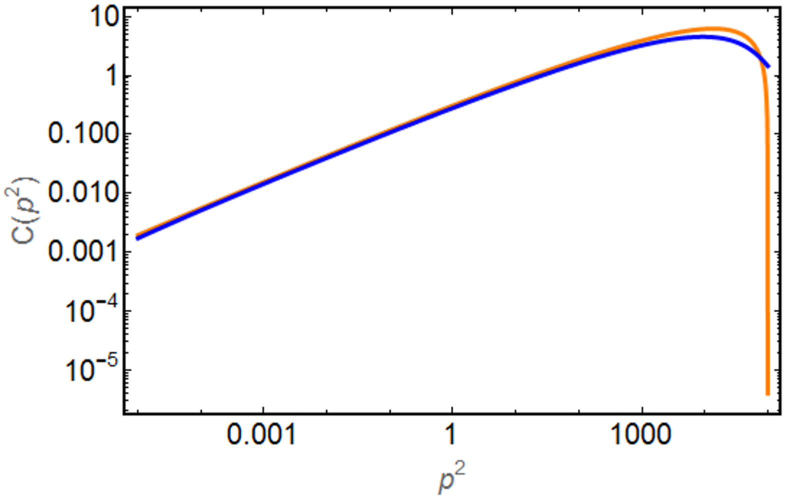
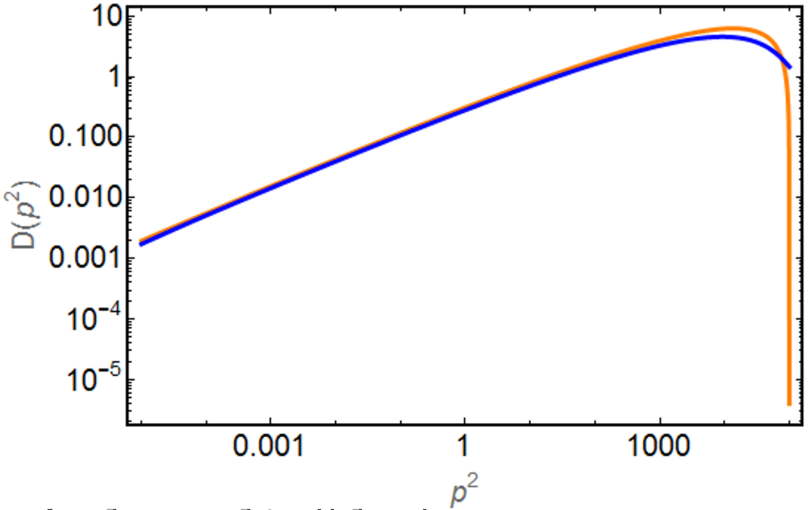
$Z_2 = 1.07 \quad, Z_m = 0.78 \quad, Z_\pi \;=\; 1.0$

$$C(p^2) \;=\; \frac{1}{2} Z_2 Z_\pi \frac{n^* \cdot p}{n^* \cdot n}(I_C(p^2) - I_D(p^2)) \qquad D(p^2) \;=\; \frac{1}{2} Z_2 Z_\pi \frac{n \cdot p}{n^* \cdot n}(I_D(p^2) - I_C(p^2))$$





Analytical results (orange), numerical results(blue)

Conclusions and work for the future

- $A(p_1, p_2, p_3, p_4)$, the same feature stands for C & D.
- Lorentz symmetry hides.
- Tree level gluon propagator in this gauge is not transverse.
- In order to compute non-perturbative solutions, we must propose an interaction model taking into account the above features.

Thank you!

QCD Lagrangian

$$\mathcal{L}_B = -\frac{1}{4}Z_3(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)(\partial^\mu A_a^\nu - \partial^\nu A_a^\mu) - \frac{1}{2}Z_1 g f_{abc}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)A_b^\mu A_c^\nu$$
$$- \frac{1}{4}Z_5 g^2 f_{abc} f_{ade} A_\mu^b A_\nu^c A_d^\mu A_e^\nu - \frac{1}{2\xi}Z_6(n^\mu A_\mu^a)^2$$
$$+ \sum_{j=1}^{Nf}(iZ_{2Fj}\bar{\psi}_l^j \gamma^\mu \partial_\mu \psi_l^j + Z_{1Fj} g \bar{\psi}_l^j R_{lk}^a \gamma^\mu \psi_k^j A_\mu^a - Z_{4j} m_j \bar{\psi}_l^j \psi_l^j), \quad n^2 = 0$$

Z_i renormalization constants

Rainbow-ladder aproximation

The quark-gluon vertex obeys its own SDE, so we must truncate the infinite system of equations.

$$g^2\Delta_{\mu\nu}(k)\Gamma_\nu(q,p) \equiv Z_2 D_{\mu\nu}(k)\gamma_\nu\,,$$

Where

$$D_{\mu\nu}(k) = 4\pi\frac{\alpha(k^2)}{k^2}\left[\delta_{\mu\nu} - \frac{k_\mu n_\nu + n_\mu k_\nu}{n\cdot k}\right],$$

We choose ξ=0

SDE after rainbow-ladder aproximation

$$S_f^{-1}(p) = Z_2[i\gamma \cdot p + Z_m m_f(\mu)] + Z_2^2 C_F \int^\Lambda \frac{d^4 q}{(2\pi)^4} D_{\mu\nu}(k)\gamma_\mu S_f(q)\gamma_\nu \,,$$

In order to get a coupled system of equations for *A*, *B*, *C* and *D*, *we apply the following projectors*

$$P_A = \frac{-i\gamma \cdot p}{4p^2}, \quad P_B = \frac{1}{4}\mathbf{I}, \quad P_C = \frac{-i\gamma \cdot n}{4(n \cdot p)}, \quad P_D = \frac{-i\gamma \cdot n^*}{4(n^* \cdot p)},$$

SDE

$$B_p = Z_2 Z_m m_f(\mu) + \frac{Z_2^2 C_F}{4} \int \frac{d^4 q}{(2\pi)^4} D_{\mu\nu}(k)\, \mathrm{Tr}\left[\gamma_\mu S_f(q) \gamma_\nu\right],$$

$$A_p + \frac{(n\cdot p)}{p^2} C_p + \frac{(n^*\cdot p)}{p^2} D_p = Z_2 - \frac{i Z_2^2 C_F}{4p^2} \int \frac{d^4 q}{(2\pi)^4} D_{\mu\nu}(k)\, \mathrm{Tr}\left[(\gamma\cdot p)\gamma_\mu S_f(q) \gamma_\nu\right],$$

$$A_p + \frac{n^*\cdot n}{(n\cdot p)} D_p = Z_2 - \frac{i Z_2^2 C_F}{4(n\cdot p)} \int \frac{d^4 q}{(2\pi)^4} D_{\mu\nu}(k)\, \mathrm{Tr}\left[(\gamma\cdot n)\gamma_\mu S_f(q) \gamma_\nu\right],$$

$$A_p + \frac{n^*\cdot n}{(n^*\cdot p)} C_p = Z_2 - \frac{i Z_2^2 C_F}{4(n^*\cdot p)} \int \frac{d^4 q}{(2\pi)^4} D_{\mu\nu}(k)\, \mathrm{Tr}\left[(\gamma\cdot n^*)\gamma_\mu S_f(q) \gamma_\nu\right],$$