Geometrical causality: casting Feynman integrals into quantum algorithms.

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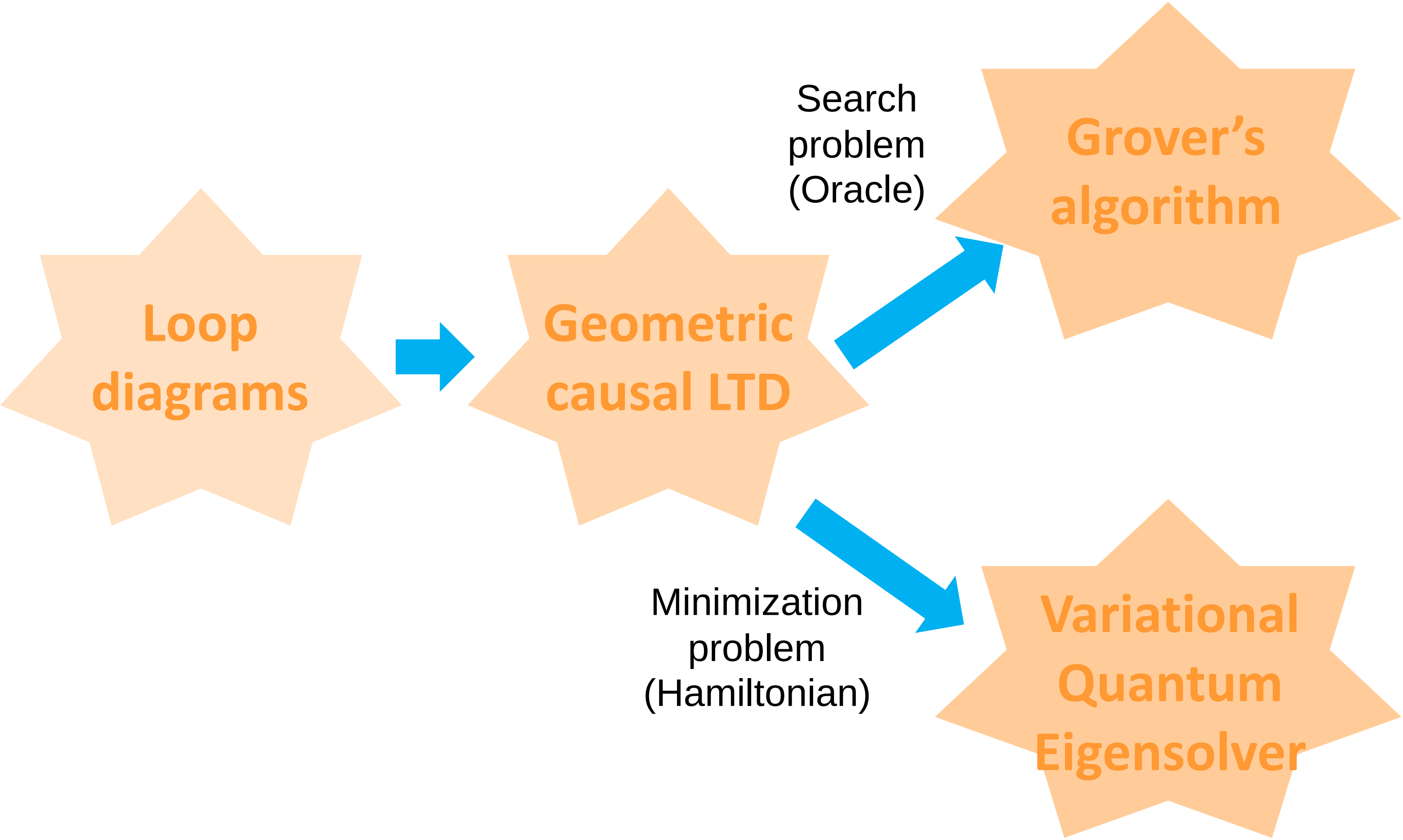


Motivation

- **Data extracted from particle colliders must be confronted with theoretical models**

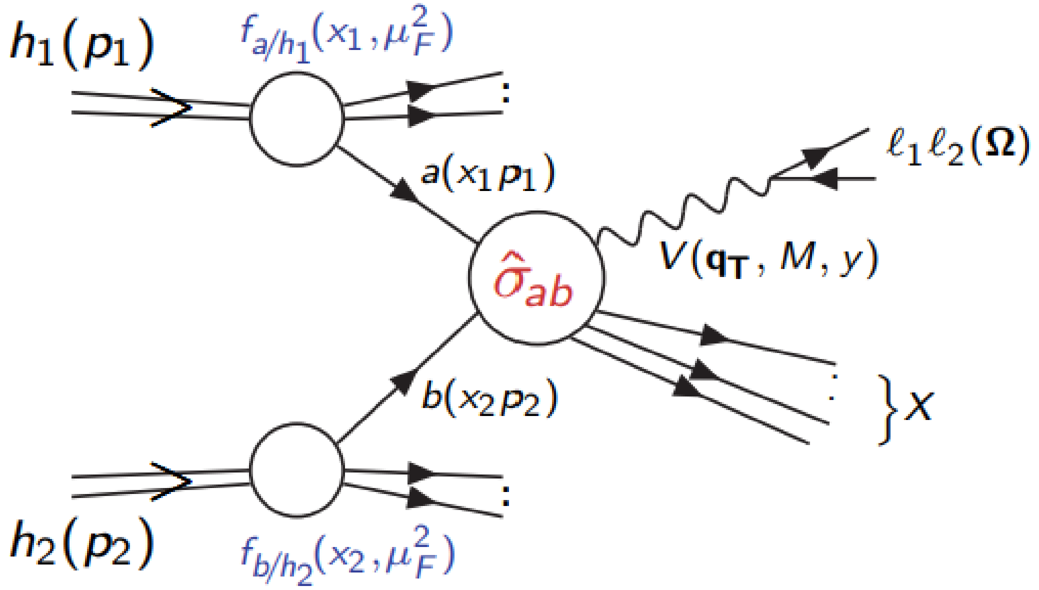
Small effects can be discovered only if theoretical predictions match experimental accuracy…

- **What we need to calculate? Cross-sections and production/decay rates at colliders**
- How to calculate? Use the parton model and SM (or other QFT…)

$$\frac{d\sigma}{d^2\vec{q}_T\, dM^2 d\Omega dy} = \sum_{a,b} \int dx_1 dx_2\, f_a^{h_1}(x_1)\, f_b^{h_2}(x_2) \frac{d\hat{\sigma}_{ab\to V+X}}{d^2\vec{q}_T\, dM^2 d\Omega dy}$$

PDFs (non-perturbative)

Partonic cross-section (perturbative)

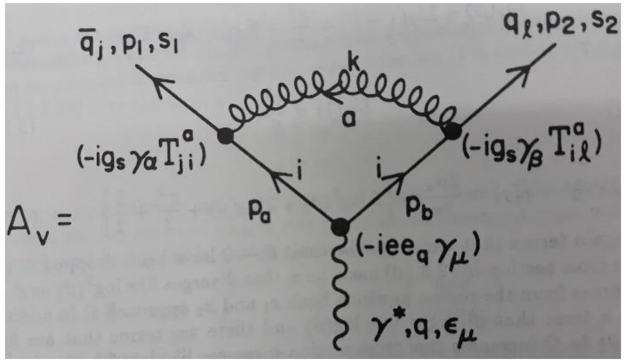


- Intermediate steps contain mathematical issues
- Need for regularization ⇨ **DREG**
- It changes the number of **space-time dimensions** in order to **achieve integrability**

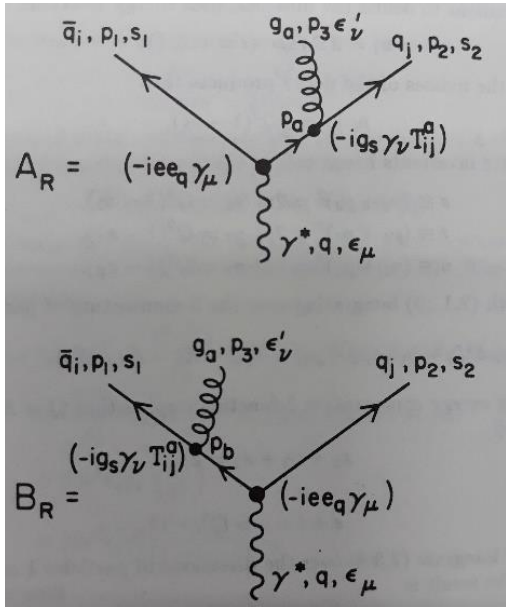
$$\mathcal{O}_d[F] = \int d^d \mathrm{x}\, F(\mathrm{x}) \qquad d = 4 - 2\varepsilon$$

Motivation

- **Parton Distribution Functions:**
  - Extracted from data (fits, neural networks, etc)
  - Scale dependence determined by DGLAP equations (perturbative kernels)
  - Several PDFs sets available in the market (different datasets, models, approximations, etc)
- **Partonic Cross Sections:**
  - Directly obtained from QFT (applying perturbative methods)



Loop contributions (quantum fluctuations of vacuum)



Real corrections (additional particles)

*Appears **after** integration*

$$\frac{C_r}{\epsilon} \times d\sigma^{(0)}$$

Counter-terms (fix the problems of the other two)

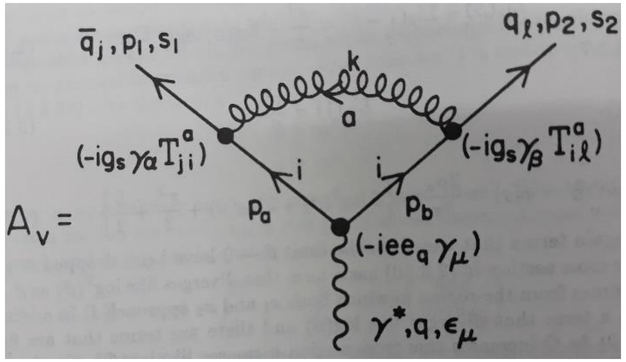
= FINITE NUMBER (compare to experiments)

CANCELLATION *AFTER* INTEGRATION

- **Parton Distribution Functions:**
  - Extracted from data (fits, neural networks, etc)
  - Scale dependence determined by DGLAP equations (perturbative kernels)
  - Several PDFs sets available in the market (different datasets, models, approximations, etc)

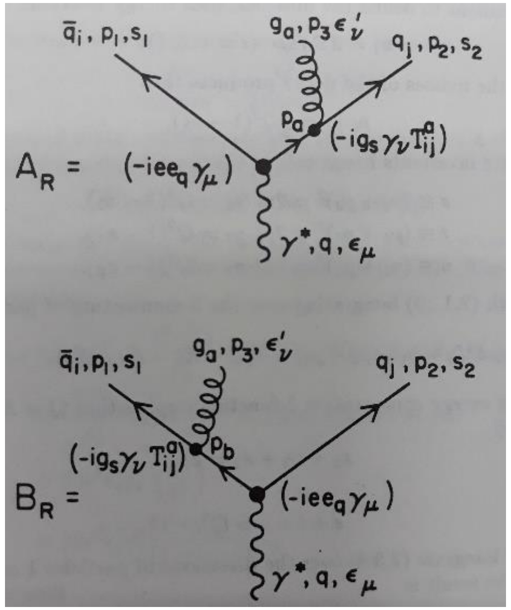
Several bottlenecks make it difficult to increase the precision (phase-space integrals, loop integrals, singularities)

- **Partonic Cross Sections:**
  - Directly obtained from QFT (applying perturbative methods)



Loop contributions (quantum fluctuations of vacuum)

+



Real corrections (additional particles)

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*Appears **after** integration*

$$\frac{C_r}{\epsilon} \times d\sigma^{(0)}$$

Counter-terms (fix the problems of the other two)

=

FINITE NUMBER (compare to experiments)

CANCELLATION *AFTER* INTEGRATION

PROBLEM

Loop integrals are hard to compute (specially, in closed analytical form) and they live in a space different than real corrections.

QUESTION

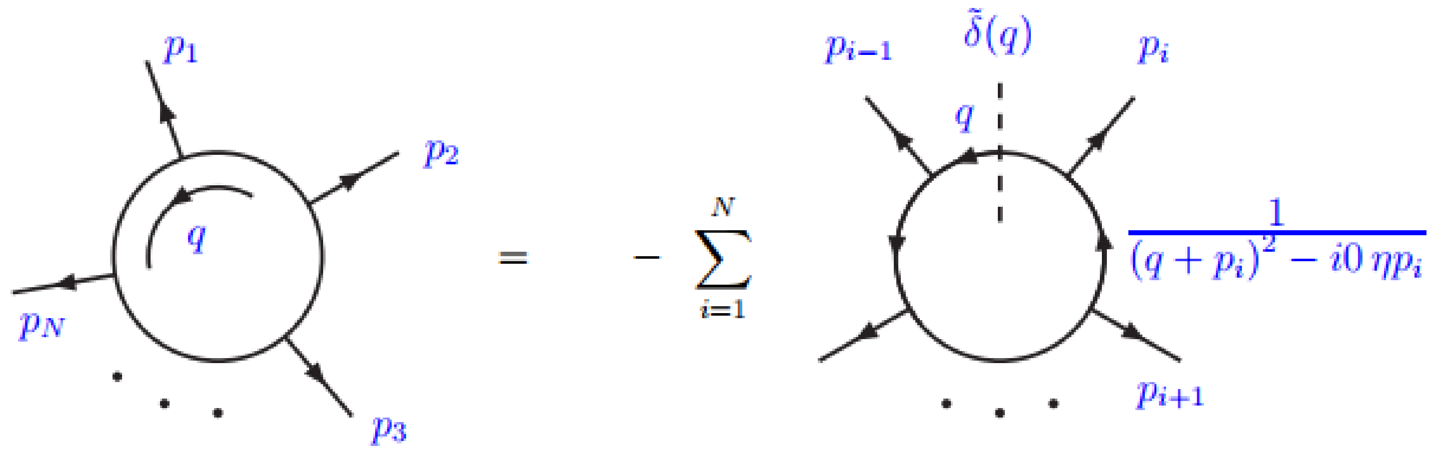
Can we do something to combine them?

ANSWER

We can use…

Loop-Tree Duality!

Tackling the loops: Loop-Tree Duality



Graphical representation of one-loop opening into trees *(original idea by Catani et al '08)*

LOOP AMPLITUDES

- ***Virtual internal momenta***
- ***Defined in Minkowski space-time***

⇨

Loop-Tree Duality

⇨

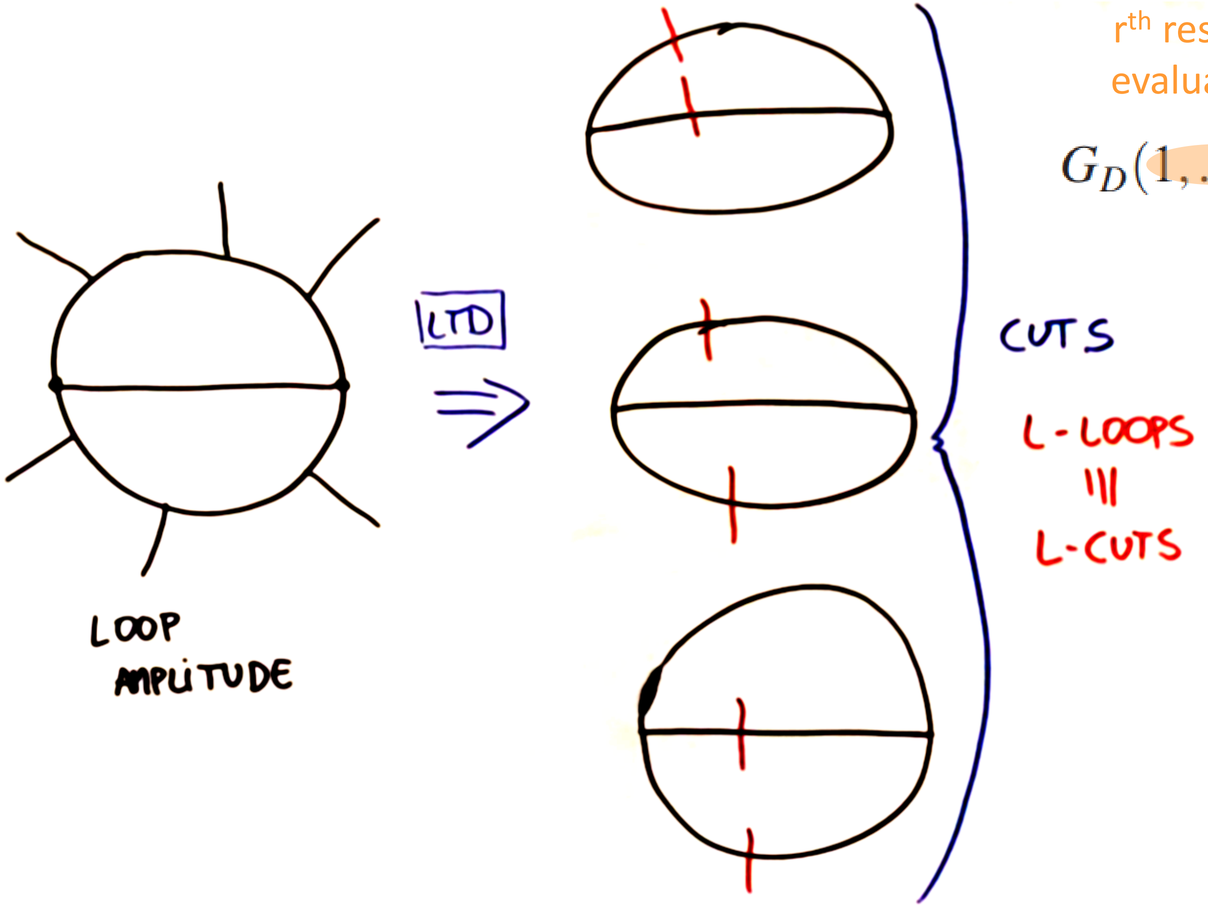
DUAL AMPLITUDES

- ***On-shell cut momenta***
- ***Defined in Euclidean space-time***

Advantages

- **Real-radiation** contributions are defined in **Euclidean space** (i.e. phase-space integrals)
- Finite loop integrals **numerically integrable** (adding local counter-terms), **just like phase-space ones**
- **We can combine real and virtual (plus local counter-terms) in a single finite and integrable expression!!**

- New strategy: iterate Cauchy's theorem to *open loops into trees*
- Energy component is removed by using **Cauchy's residue theorem**
- **Multiloop require to iterate ("nest") the procedure (remove all the energy components)**



r^{th} residue evaluation

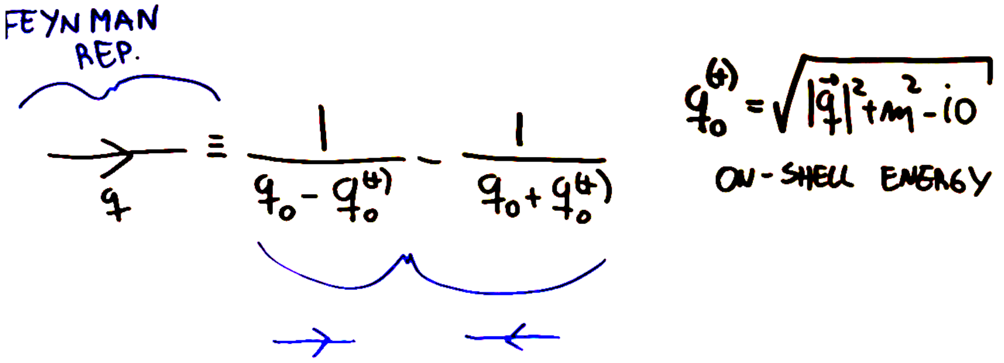
Remaining sets (no residue evaluation)

$$G_D(1,\dots,r;n) = -2\pi\imath \sum_{i_r\in r} \mathrm{Res}(G_D(1,\dots,r-1;r,n), \mathrm{Im}(\eta\cdot q_{i_r})<0)$$

Sum over all the elements of the r^{th} set

$(r-1)^{th}$ dual function

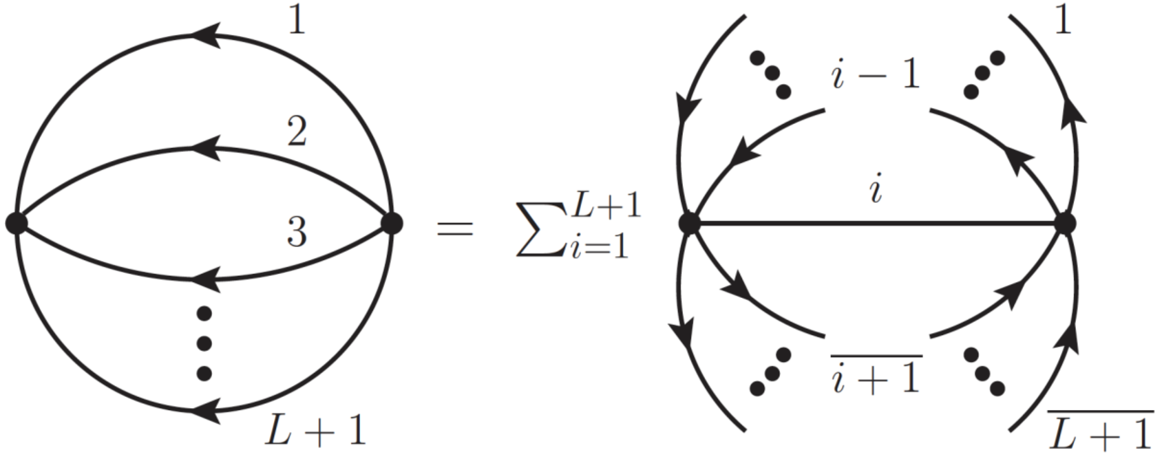
Depends on integration variables (q_i)



Causality and nested residues

- **Explicit calculation of nested residues** ⇨ **Very compact formulae!**

Maximal Loop Topology (2 vertices, L+1 lines)



REMARK: External particles can be attached to each momenta set

On-shell lines with reversed momenta

Lines = sets of propagators

$$\mathcal{A}^{(L)}_{\rm MLT}(1,2,\ldots,L+1) = \int_{\vec\ell_1,\ldots,\vec\ell_L} \sum_{i=1}^{L+1} \mathcal{A}_D(1,\ldots,i-1,\overline{i+1},\ldots,\overline{L+1};i)$$

Defined in Minkowski space

Defined in Euclidean space

On-shell lines

1 off-shell line

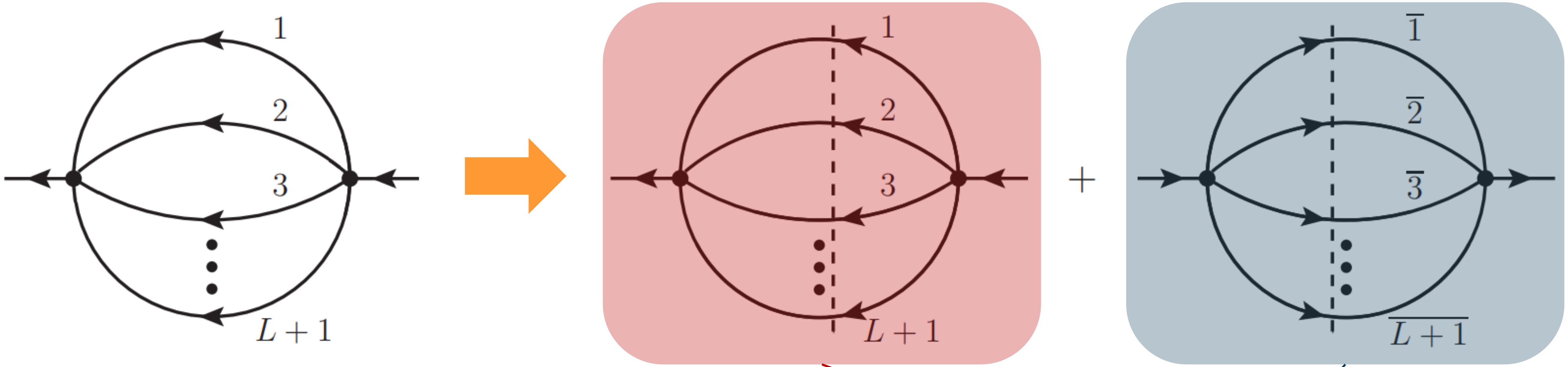
- We define the Maximal Loop Topology (MLT) as a building block to describe multi-loop amplitudes
- **Important:** *"Any one and two-loop amplitude can be described by MLT topologies"*

Inductive proofs of these formulae to all-loop orders available in JHEP 02 (2021) 112

Aguilera-Verdugo et al (2020) Phys. Rev. Lett. 124, 211602 & JHEP 01 (2021) 069

Causality and nested residues

- <u>**Summing all the terms in LTD representation leads to noticeable simplifications**</u>
- There is a strict connection between **aligned contributions** and **causal terms!!!**
- *MLT example*: If we **sum over all the possible cuts**, we get **this extremely compact** result:



$$\mathcal{A}^{(L)}_{\mathrm{MLT}}\left(1,2,\ldots,(L+1)_{-p_1}\right) = -\int_{\vec{\ell}_1,\cdots,\vec{\ell}_L} \frac{1}{x_{L+1}}\left(\frac{1}{\lambda_1^-}+\frac{1}{\lambda_1^+}\right)$$

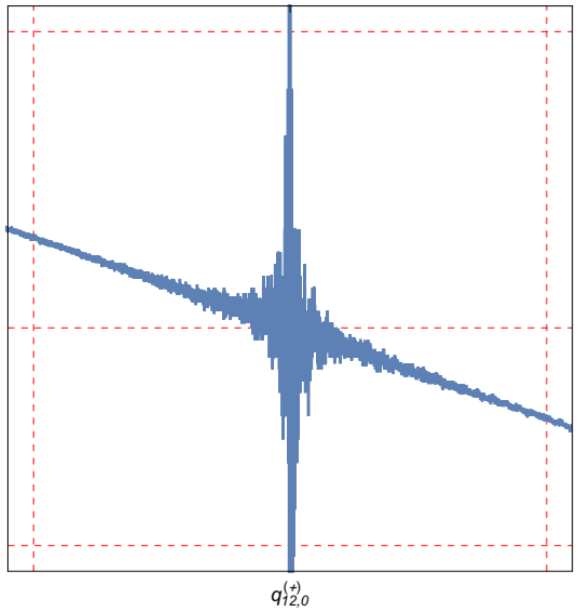
with $\lambda_1^{\pm} = \sum_{i=1}^{L+1} q_{i,0}^{(+)} \pm p_{1,0}$ and $x_{L+k} = 2^{L+k}\prod_{i=1}^{L+k} q_{i,0}^{(+)}$

CAUSAL PROPAGATORS

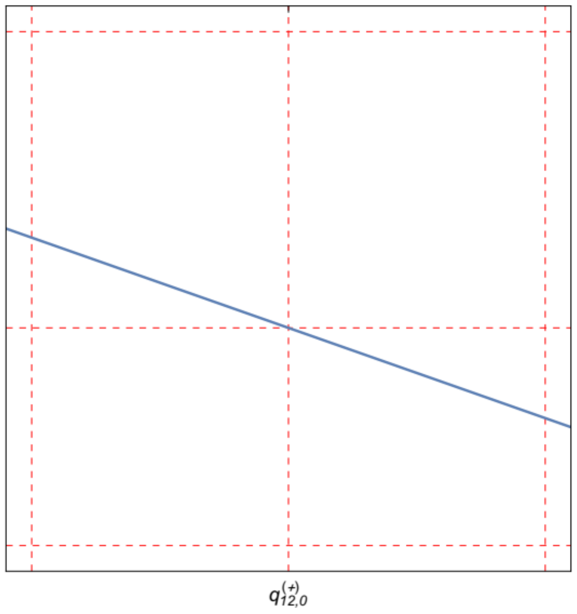
Aguilera-Verdugo et al, JHEP 01 (2021) 069; JHEP 02 (2021) 112

- **Causal Representation exists for any QFT amplitude!**
- **Advantages**

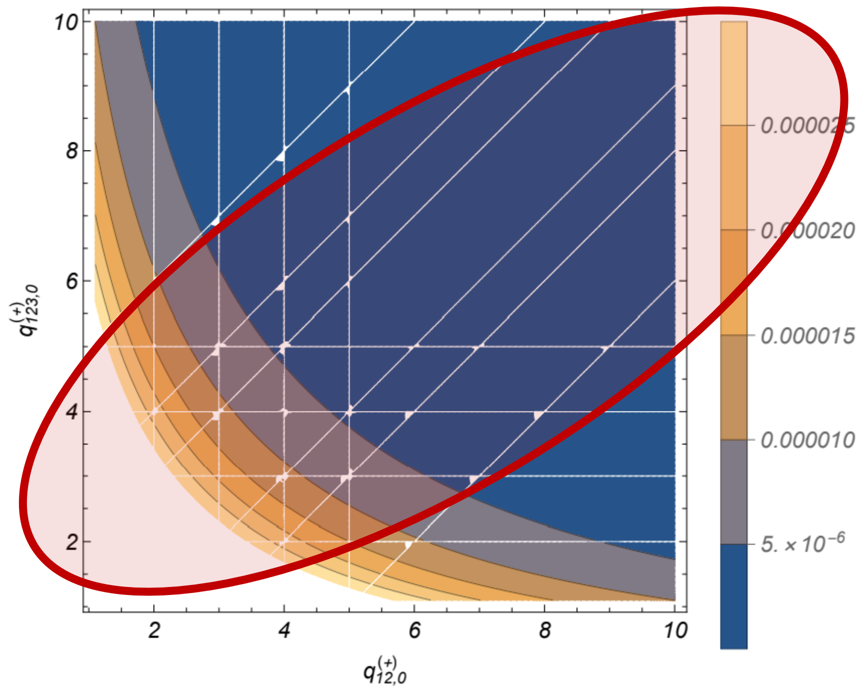
  1. Causal denominators have **same-sign combinations of on-shell energies** (positive numbers), thus are **more stable numerically!**
  2. **Only physical thresholds remain**; spurious un-physical instabilities are removed!



Without causal representation



With causal representation



White lines = Numerical instabilities

Aguilera-Verdugo et al, JHEP 01 (2021) 069
Ramírez Uribe et al, JHEP 04 (2021) 129

Causal Reconstruction from graphs

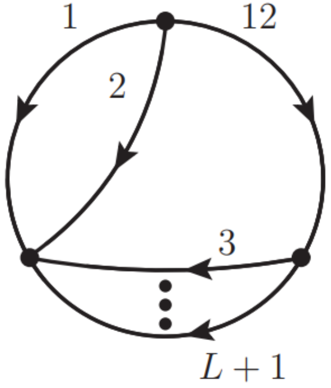
- Further studies were performed with several topological families

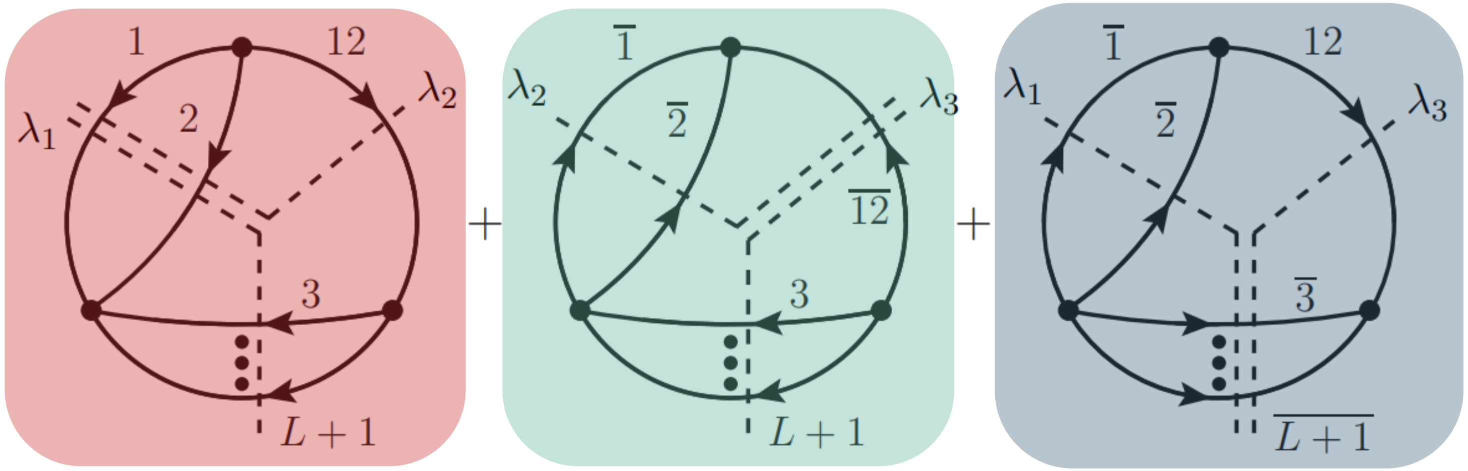
  JHEP 01 (2021) 069; JHEP 04 (2021) 129; JHEP 04 (2021) 183; Eur.Phys.J.C 81 (2021) 6, 514

- <u>**Graphical interpretation in terms of entangled thresholds**</u>

  1. **Each causal** propagator represents a **threshold** of the diagram
  2. **Each diagram** contains **several thresholds**
  3. **The causal representation involves products of (*compatible*) thresholds**

Causal denominators (λ) are associated to *cut lines* in the diagrams: momenta flow must be adjusted to be compatible





$$\mathcal{A}^{(L)}_{\rm NMLT}(1,2,\ldots,L+2) = \int_{\vec{\ell}_1,\ldots,\vec{\ell}_L} \frac{2}{x_{L+2}} \left(\frac{1}{\lambda_1\lambda_2} + \frac{1}{\lambda_2\lambda_3} + \frac{1}{\lambda_3\lambda_1} \right)$$

Aguilera-Verdugo et al, JHEP 01 (2021) 069
Ramírez Uribe et al, JHEP 04 (2021) 129

Causal Reconstruction from graphs

- **Causal representation** obtained directly after **summing over all the nested residues**

Master formula

$$\mathcal{A}_N^{(L)}(1,\dots,L+k)=\sum_{\sigma\in\Sigma}\int_{\vec{\ell}_1,\cdots,\vec{\ell}_L}\frac{\mathcal{N}_\sigma(\{q_{r,0}^{(+)}\},\{p_{j,0}\})}{x_{L+k}}\times\prod_{i=1}^{k}\frac{1}{-\lambda_{\sigma(i)}}+(\sigma\leftrightarrow\bar{\sigma})$$

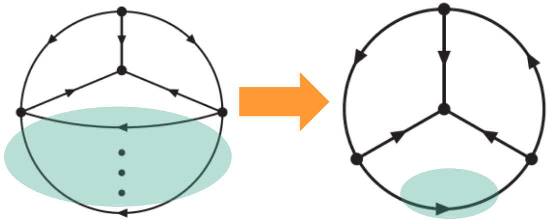
Set of entangled thresholds

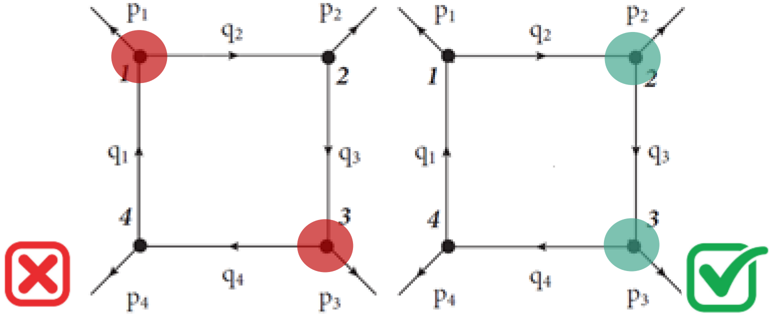
Products of k causal propagators

- *Is it possible to do it in other way?* ➡
  - **Geometrical reconstruction** — **Sborlini '21**
  - **Algebraic reconstruction (Lotty)** — **Torres Bobadilla '21**

- **Previous concepts**

  1. **Diagrams** are made of **vertices** and **multi-edges** *(bunches of propagators, connecting two given vertices)*
  2. **Multi-edges** define a **basis of momenta**, that lead to the **"vertex matrix"** ➡ **Defines the casual structure!**
  3. **Binary partitions** are given by **subsets of vertices** that **splits in two** the original diagram ➡ **Connected partitions!**



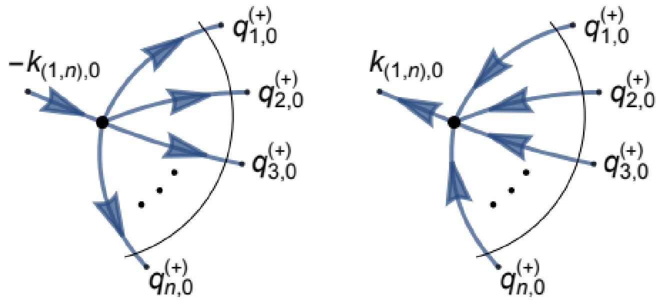


Sborlini, Phys.Rev.D 104 (2021) 3, 036014

Causal Reconstruction from graphs

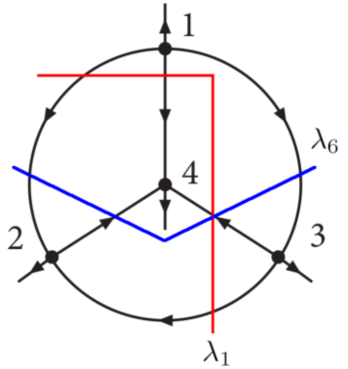
More details in arXiv:2102.05062 [hep-ph]

1. **Generate causal propagators**
   - Causal propagators are associated to **binary connected partitions** of the diagram, namely *"connected sub-blocks of the diagram"*
   - They encode the possible **physical thresholds**
   - Involve a **consistent (aligned) energy flow** through the cut lines

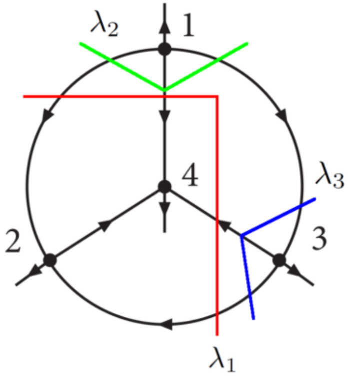


2. **Order of a diagram:** it quantifies the complexity of a given topology
   - *k=1 for MLT, k=2 for NMLT and so on* → ***k = vertices - 1***
   - A diagram of **order k** involves **products of k causal propagators**

3. **Geometric compatibility rules:** determine the entangled thresholds
   a) **All the multi-edges are cut at least once**
   b) **Causal propagators do no intersect;** i.e. they are associated to disjoint or extended partitions of the diagram
   c) **All the multi-edges** involved in a causal threshold must carry **momenta flowing in the same direction** → Distinction λ^{+} / λ^{-}



Presence of intersections



Incompatible causal flux

PROBLEM

Complex topologies have many causal configurations, it takes a lot of time to test all the possibilities.

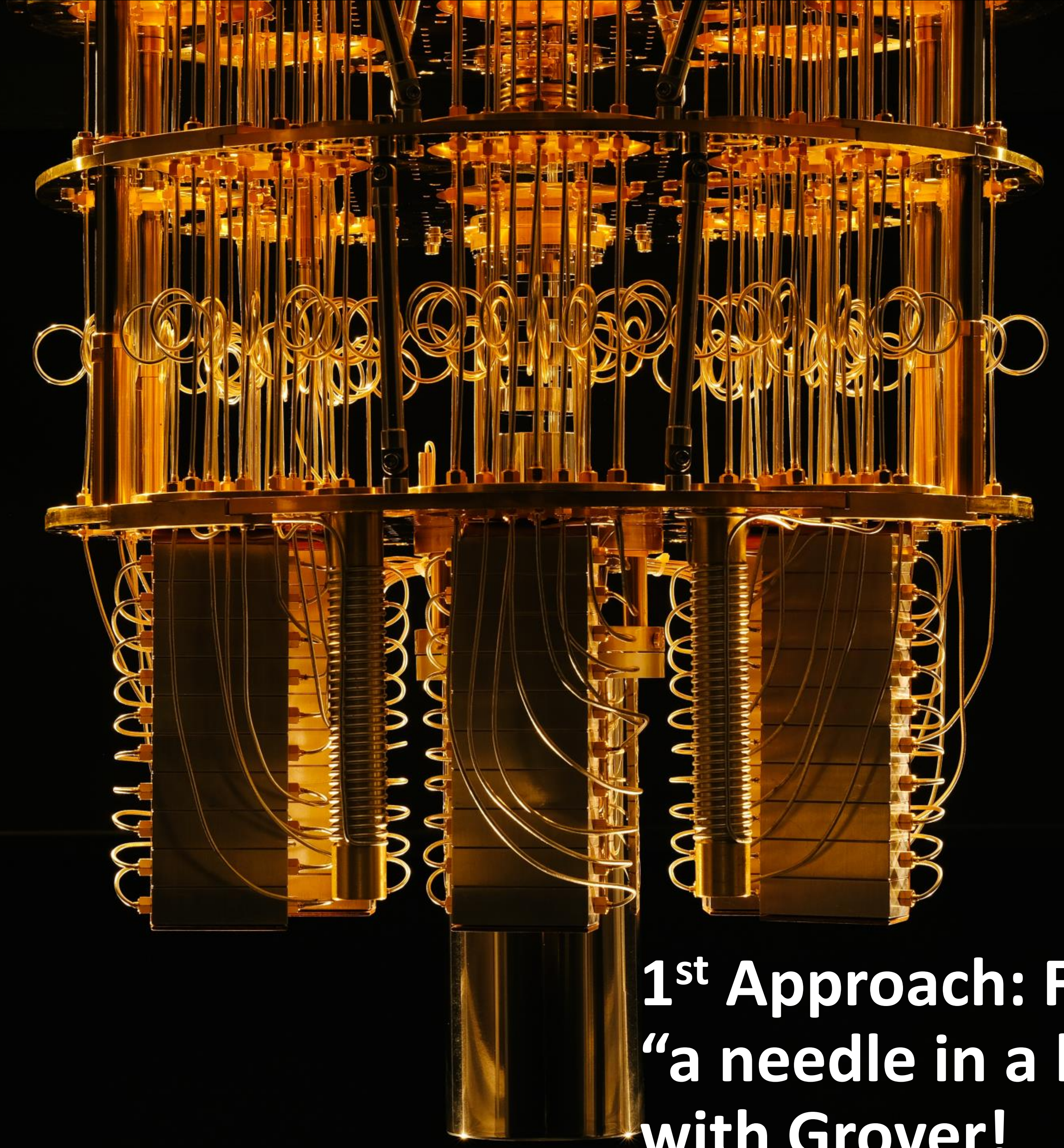
QUESTION

Can we use other techniques to identify the causal terms?

ANSWER

We can explore …

Quantum Search Algorithms!



1st Approach: Finding “a needle in a haystack” with Grover!

- **Purpose:** Search "selected" states from a bunch of possible configurations
- **Idea:** Build a quantum uniform superposition of N states and paralellize a selection condition
- **Aim:** Achieve an speed-up compared to the classical search algorithms

Strategy: Preparation

- From the N total states, there are **r** "winning" states and **N-r** orthogonal ones

$$|q\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle \quad \text{with} \quad N = 2^n$$

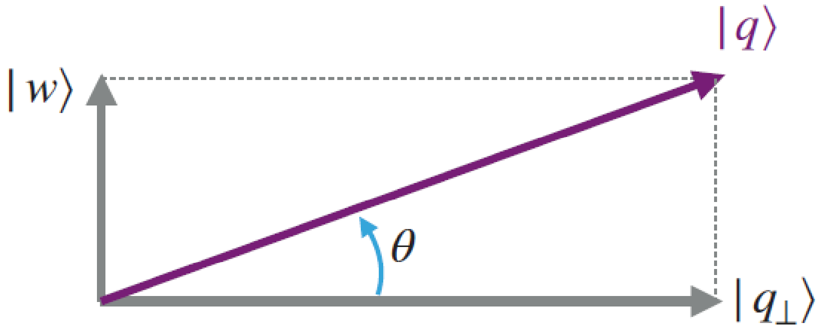
Projection over orthogonal subspaces

$$|q\rangle = \sin\theta |w\rangle + \cos\theta |q_\perp\rangle$$

$$|w\rangle = \frac{1}{\sqrt{r}} \sum_{x \in w} |x\rangle \qquad |q_\perp\rangle = \frac{1}{\sqrt{N-r}} \sum_{x \notin w} |x\rangle$$

Winning states

Non-winning states



$$\theta = \arcsin\sqrt{r/N}$$

Mixing angle

Grover (1997) arXiv: quant-ph/9712011

G. Rodrigo (DESY Seminar 21)

Strategy: Amplitude amplification

- We define the oracle operator to mark the "winning" states

$$U_w = \boldsymbol{I} - 2\,|w\rangle\langle w|$$

- It flips the phase of winning states, and left unaltered the others

$$U_w |x\rangle = -\,|x\rangle \qquad U_w |x\rangle = |x\rangle$$

Action over winning subspace — Action over orthogonal subspace

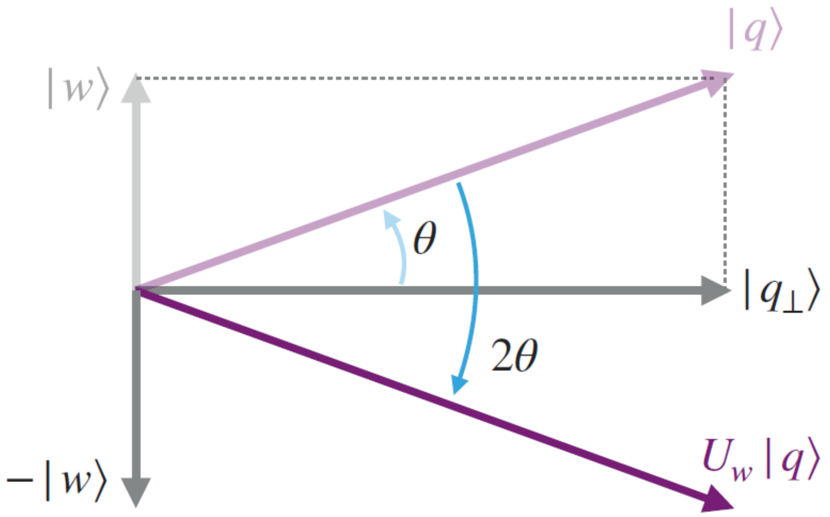
- Then, the diffusion operator reflect over the initial state:

$$U_q = 2\,|q\rangle\langle q| - \boldsymbol{I}$$

- Iterate the procedure to achieve an amplification:

$$(U_q U_w)^t |q\rangle = \sin\theta_t |w\rangle + \cos\theta_t |q_\perp\rangle$$

with $\theta_t = (2t+1)\,\theta$



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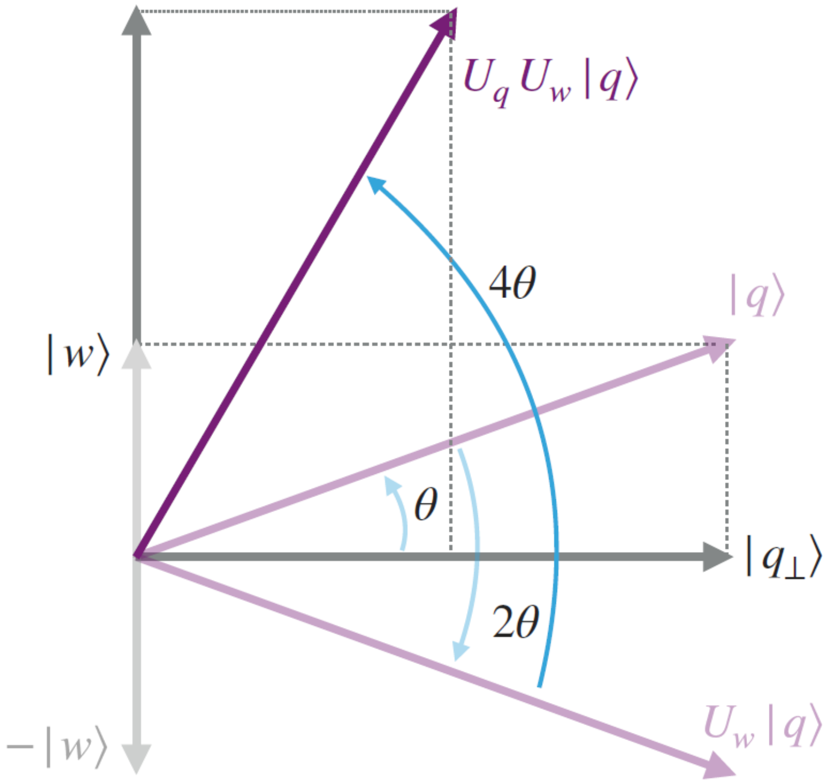

G. Rodrigo (DESY Seminar 21)

Grover (1997) arXiv: quant-ph/9712011

More detailed explanation arXiv:2102.05062 [hep-ph] & arXiv:2105.08703 [hep-ph]

1. Generate causal propagators
   - Causal propagators are associated to **binary connected partitions** of the diagram, namely *"connected sub-blocks of the diagram"*
   - **Non-cyclical configurations = Causal flux**
   - Involve a **consistent (aligned) energy flow** through the cut lines
2. Order of a diagram: it quantifies the complexity of a given topology
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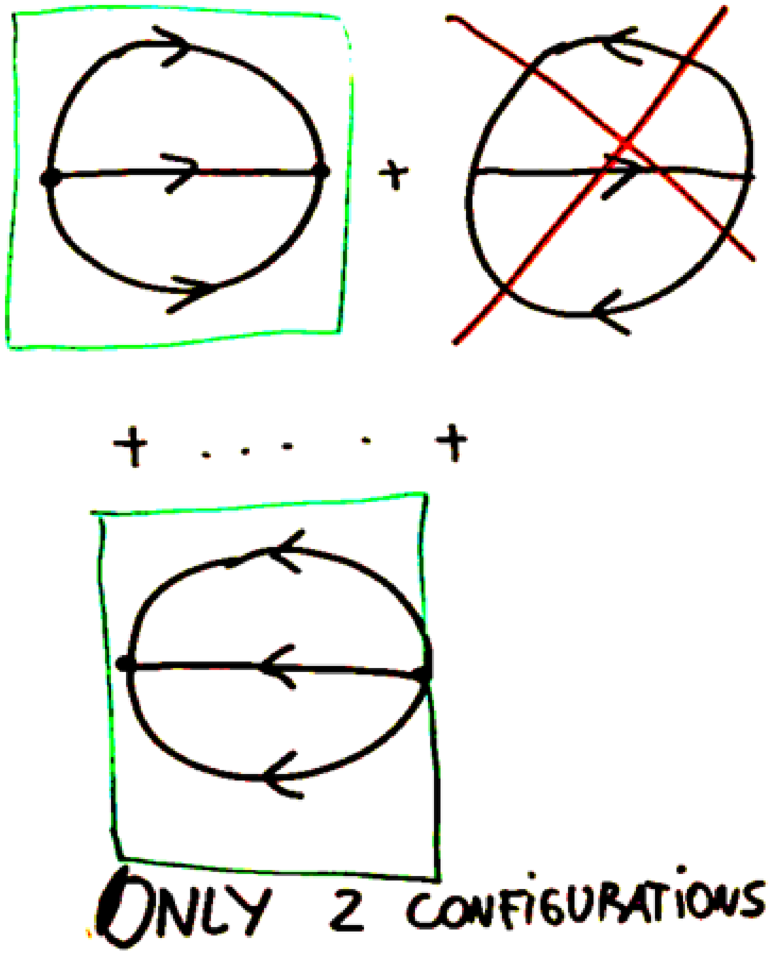
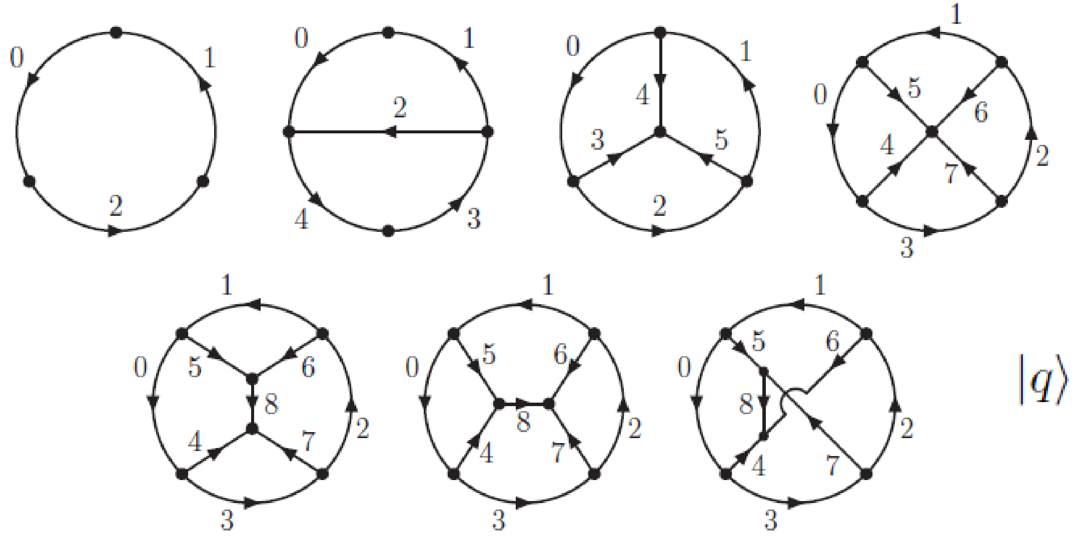


- **Identify momentum-orderings compatible with causality using Grover's search algorithm!**
- We assign **1 qubit to each edge**, and impose logical conditions to select configurations without closed cycles → **Non-cyclical configurations = Causal flux**
- **Important:** **"loop"** refers to **integration variables; "eloop"** to loops in the **graph**



Total number of orderings (n = nº of edges)

$$N = 2^n \rightarrow |q\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle$$

Quantum superposition of N flux configurations

$$|q\rangle = \cos\theta\, |q_\perp\rangle + \sin\theta\, |w\rangle$$

$$|w\rangle = \frac{1}{\sqrt{r}} \sum_{x \in w} |x\rangle$$

States with causal flow = "Winning states"

$$|q_\perp\rangle = \frac{1}{\sqrt{N-r}} \sum_{x \notin w} |x\rangle$$

States with non-causal flow = "Orthogonal states"

Ramírez-Uribe et al, JHEP 05 (2022) 100

- We use Grover's algorithm to **enhances** the probability of the **causal states**:

$$U_w = \boldsymbol{I} - 2|w\rangle\langle w|$$

Oracle operator (changes sign of causal states)

$$U_q = 2|q\rangle\langle q| - \boldsymbol{I}$$

Diffusion operator (reflects with respect to initial state)

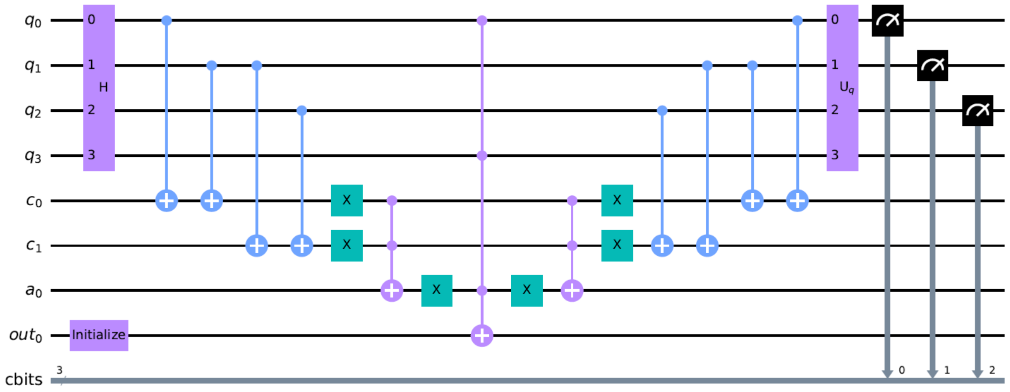
$$\rightarrow (U_q U_w)^t |q\rangle = \cos\theta_t\, |q_\perp\rangle + \sin\theta_t\, |w\rangle$$

with $\sin^2\theta_t \sim 1$

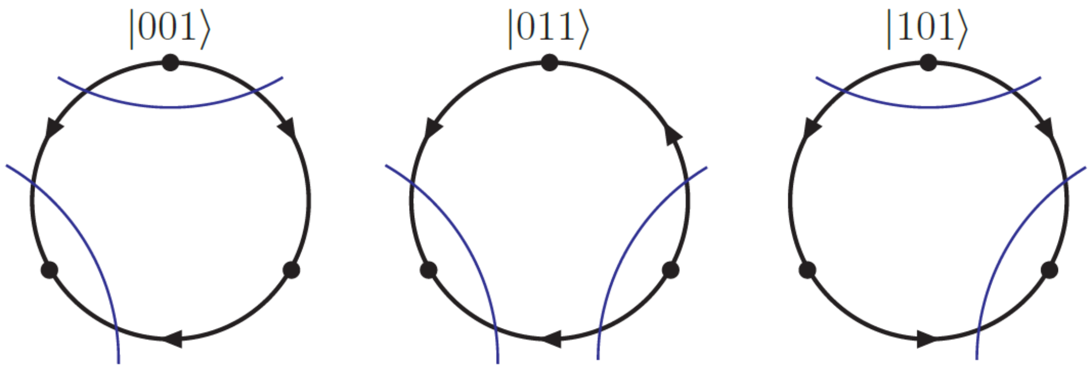
Grover's algorithm for Causal Reconstruction

- Implemented with Qiskit and run in **IBM Q** (simulator & *real QC*)
- Several topologies studied!! **Enhanced performance** with extra-qubits

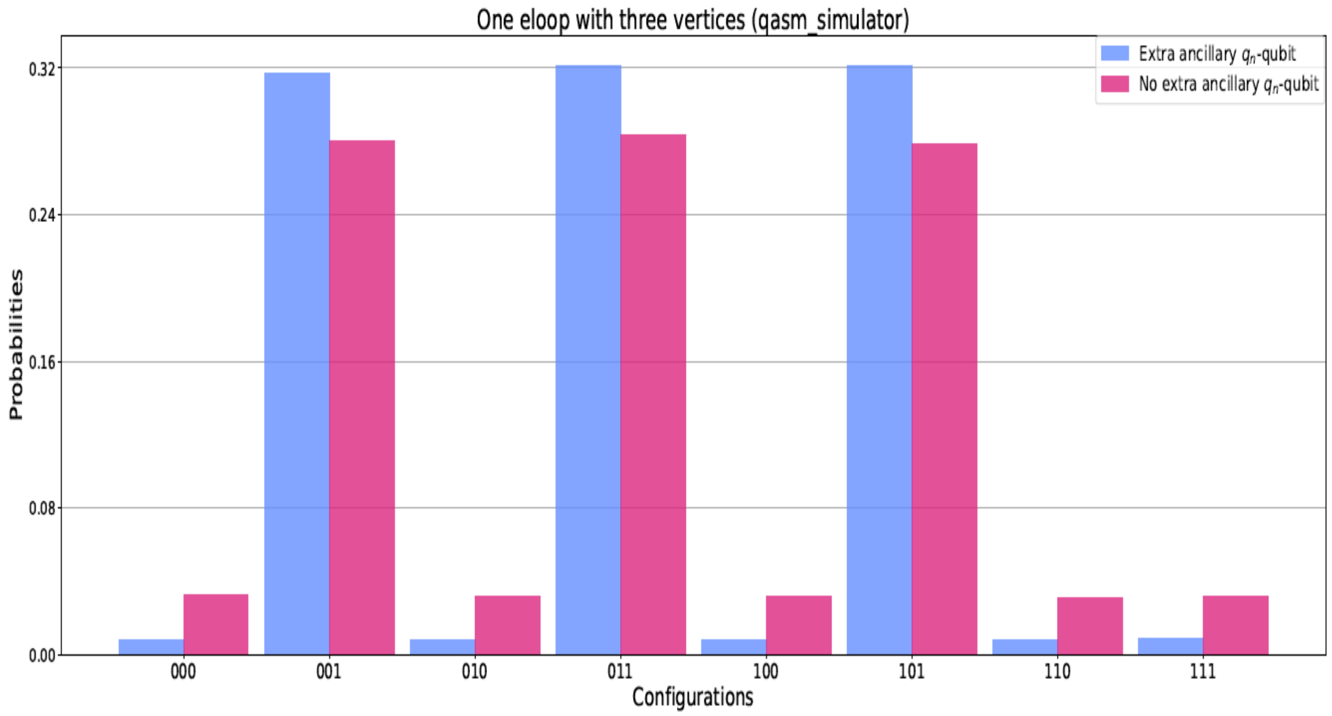
JHEP 05 (2022) 100
arXiv:2105.08703 [hep-ph]



Quantum circuit



Causal configurations



The selected configurations are exactly |001>, |011>, |101>

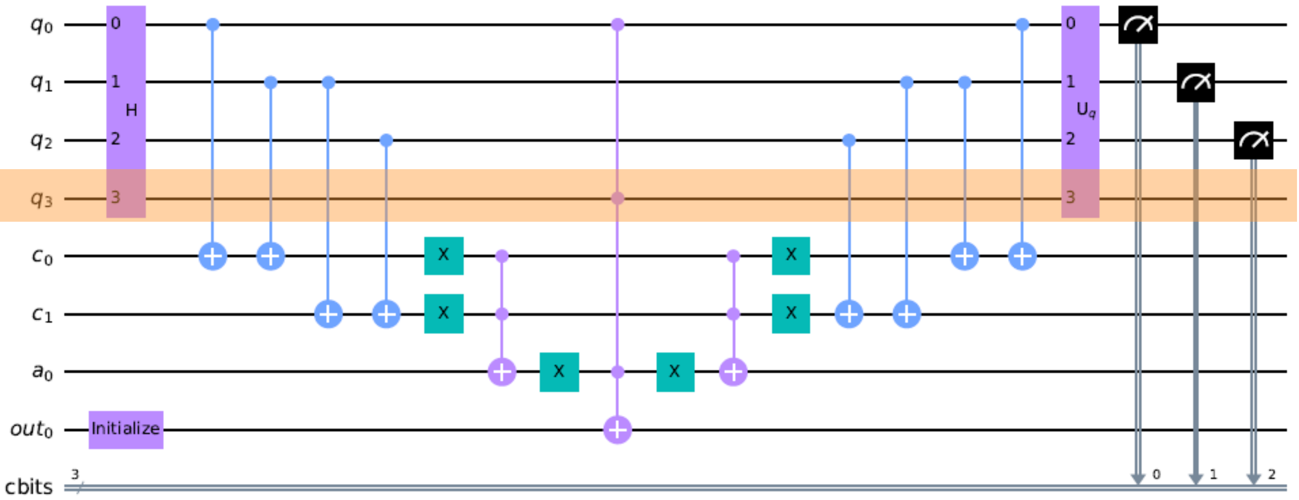
The algorithm identifies the causal flux, relying on geometrical concepts!

Ramírez-Uribe et al, JHEP 05 (2022) 100

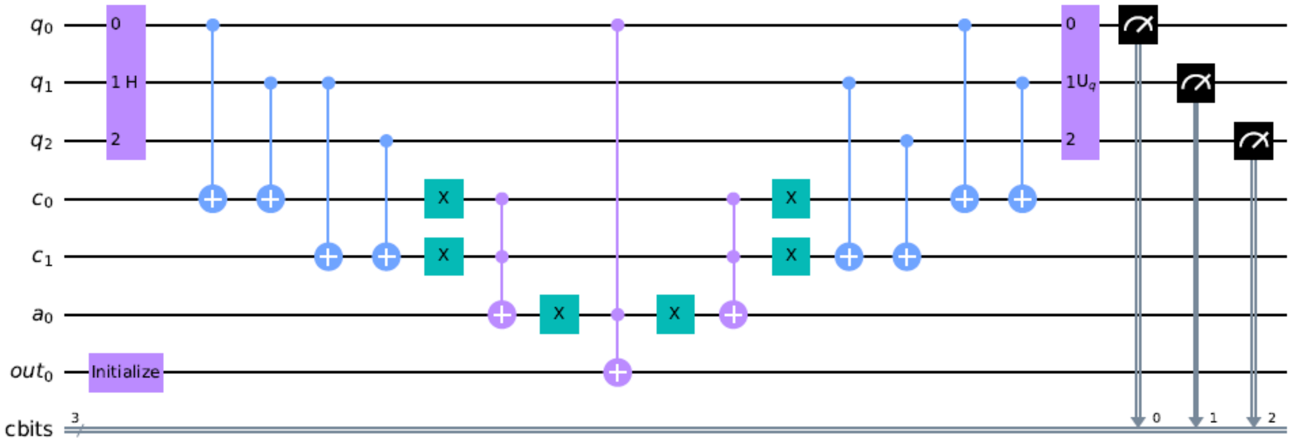
Grover's algorithm for Causal Reconstruction

- **<u>Details about the circuit:</u>** one eloop with three vertices

Extra qubit



No-extra qubit



- *Adding an additional qubit increases the total configurations, without increasing the winning states*

- Grover's algorithm could reach a quadratic speed-up (subtleties related to the number of shots)

JHEP 05 (2022) 100
arXiv:2105.08703 [hep-ph]

Ramírez-Uribe et al, JHEP 05 (2022) 100

Grover's algorithm for Causal Reconstruction

- **<u>Details about the circuit:</u>** one eloop with three vertices (no extra-qubit)

JHEP 05 (2022) 100
arXiv:2105.08703 [hep-ph]

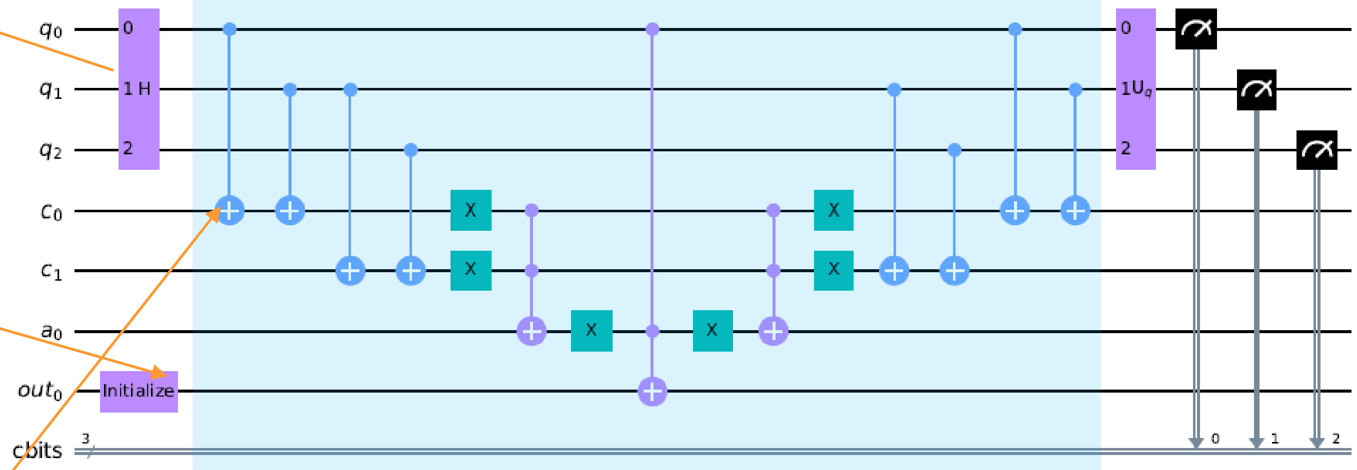
Hadamard

$$|q\rangle = H^{\otimes n}|0\rangle$$

$$U_w|q\rangle|c\rangle|a\rangle|out_0\rangle = (-1)^{f(a,q)}\,|q\rangle|c\rangle|a\rangle|out_0\rangle$$

Oracle operator

$$|out_0\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} \equiv |-\rangle$$



Diffuser (defined in Qiskit)

n=2 in this example

Ramírez-Uribe et al, JHEP 05 (2022) 100

$$c_{ij} \equiv (q_i = q_j)\,,$$
$$\bar{c}_{ij} \equiv (q_i \neq q_j)\,, \qquad i,j \in \{0,\ldots,n-1\}$$

Binary clauses

$$f^{(1)}(a,q) = a_0 \wedge q_0 \wedge q_n$$

$$a_0(\{c_{ij}\}) \equiv \neg\,(c_{01} \wedge c_{12} \wedge \cdots \wedge c_{n-2,n-1})$$

Implementation of the marker f

$$(U_q U_w)^t|q\rangle = \cos\theta_t\,|q_\perp\rangle + \sin\theta_t\,|w\rangle$$

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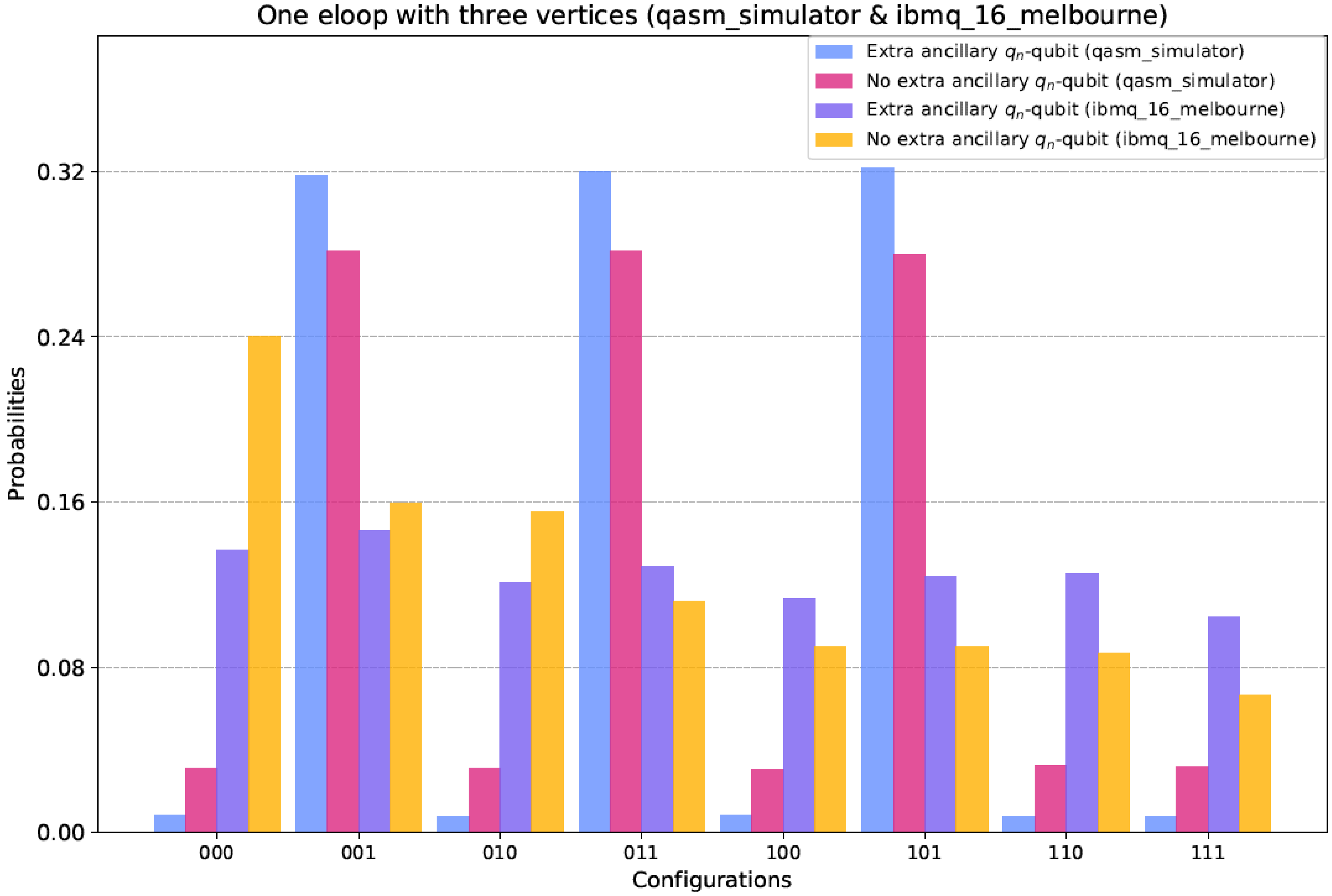


- **Simulator:**
  - Very good performance
  - Extra-qubit enhances the amplification
- **Real devices:**
  - **Several limitations due to large quantum depth**
  - Efficient error mitigation required (noisy output)
  - *Could be improved in future devices*

Ramírez-Uribe et al, JHEP 05 (2022) 100

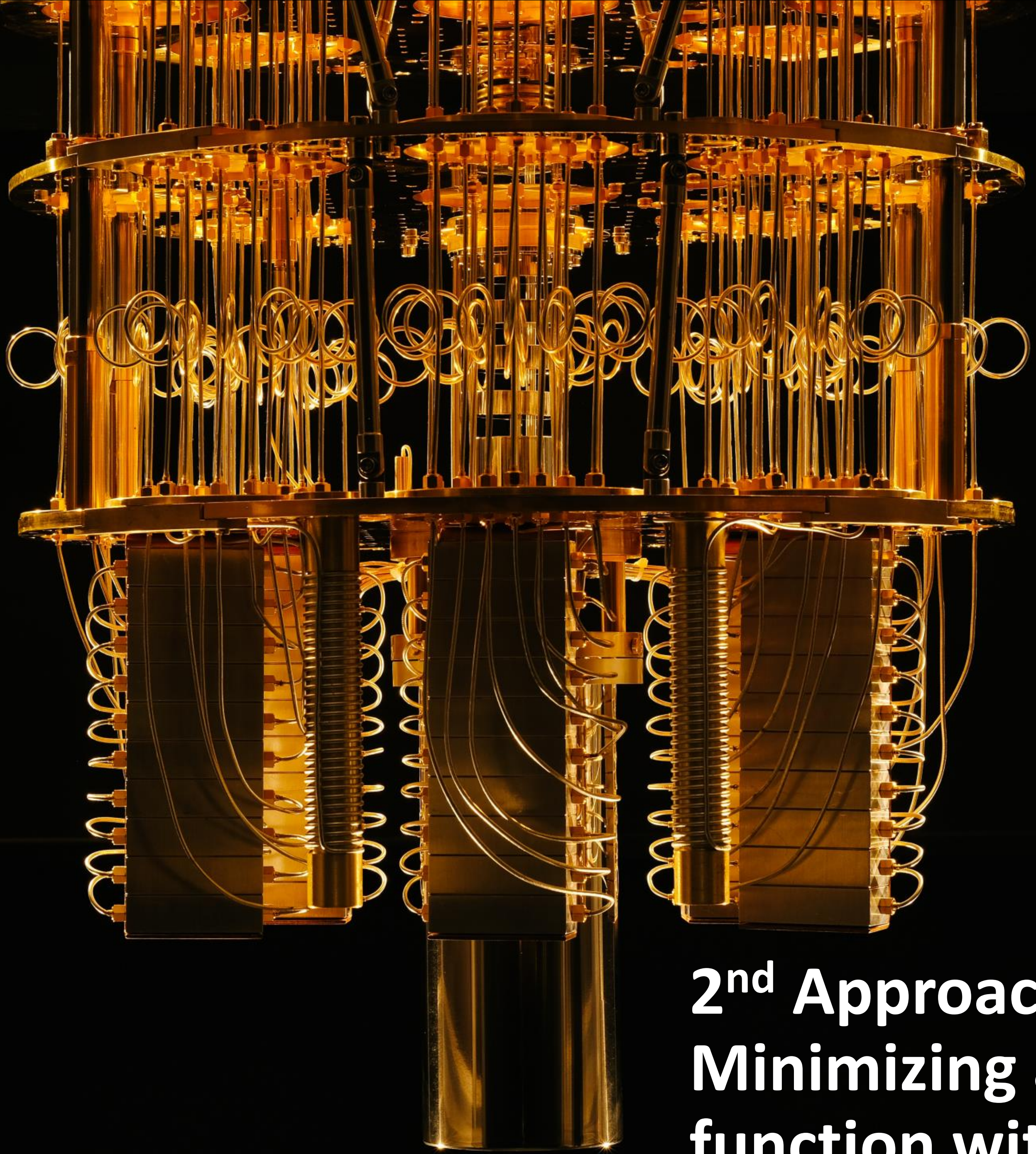
Good… but…

PROBLEM
Binary clauses and selection rules require several qubits (and they are not re-usable!). Resource consumption scales very fast!

QUESTION
Within QA, can we use other approach?

ANSWER
We can combine classical and quantum codes …

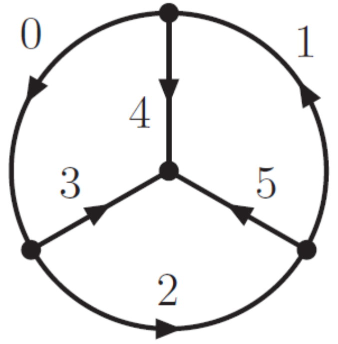
Quantum Minimization Algorithms!



2nd Approach: Minimizing a cost function with VQE!

Hamiltonian construction for minimization

- Geometrical information is codified in the **adjacency matrix**

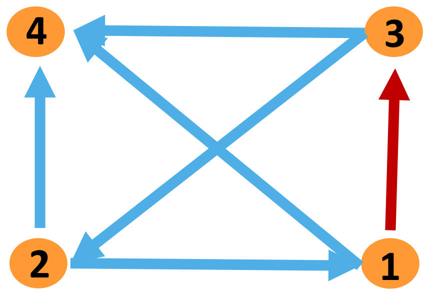


N2MLT topology:
Mercedes-Benz diagram

```
(*Configuracion momentos*)
NumeroVertices = 4; Orden = NumeroVertices - 1;
Eq[1] = {q[6] + q[4] - q[1] + p[1]};
Eq[2] = {q[1] + q[5] - q[2] + p[2]};
Eq[3]  = {q[3] - q[6] + q[2] + p[3]};
Eq[4] = {-q[3] - q[4] - q[5] - (p[1] + p[2] + p[3])};
```

$$\left(\begin{array}{ccccc|cccc} -1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1 & -1 & 0 & -1 & -1 & -1 \end{array}\right)$$

Vertex matrix
(from the conservation equations)



$$A = \begin{pmatrix} 0 & a_{12} & a_{13} & \cdots \\ \neg a_{12} & 0 & a_{23} & \cdots \\ \neg a_{13} & \neg a_{23} & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Adjacency matrix
(how are vertices connected)

- **Useful properties:**

$$f_3(A) \equiv \Sigma_{i_1,i_2,i_3}^{i_1 \neq i_2 \neq i_3} a_{i_1 i_2} \times a_{i_2 i_3} \times a_{i_3 i_1} \neq 0$$ ⟺ There are triangles

$$f_4(A) \equiv \Sigma_{i_1,i_2,i_3,i_4}^{i_1 \neq i_2 \neq i_3 \neq i_4} a_{i_1 i_2} \times a_{i_2 i_3} \times a_{i_3 i_4} \times a_{i_4 i_1} \neq 0$$ ⟺ There are boxes

In general, there are N-cycles **iif** $f_N(A)$ is non zero

A graph with associated adjacency matrix A is acyclic if:

$$\mathrm{tr}(A^N) = 0$$

Clemente, Crippa et al (2022) arXiv:2210.13240 [hep-ph]

Hamiltonian construction for minimization

- Exploit the **adjacency matrix** to build a **Hamiltonian** → **Ground state = Acyclic graph**
- **1st approach:** Penalize oriented cycles using projectors to build the **loop Hamiltonian**

$$H_G = \sum_{\gamma \in \Gamma_{G_0}} \prod_{e \in \gamma} \pi_e^{s(e;G_0)}$$

Orientation (0 or 1)

Cycles (within graph)

Edges (within cycle)

Projector

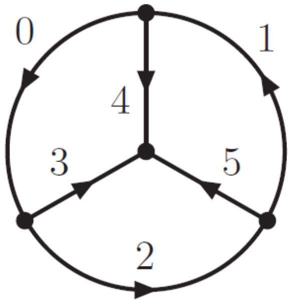
with

$$\pi_e^0 \equiv |0\rangle\langle 0|_e = \frac{1}{2}(I+Z)_e$$

$$\pi_e^1 \equiv |1\rangle\langle 1|_e = \frac{1}{2}(I-Z)_e$$

0 original direction
1 reversed direction

Need to fix a initial orientation, but results are independent of this choice!!



- **2nd approach (BETTER):** Promote adjacency matrix to operator, and *trace over all possible cycles*

$$A \equiv \sum_{e \equiv (v_0,v_1) \in \bar{E}} \left[\sigma_{v_0}^- \pi_e^0 \sigma_{v_1}^+ + \sigma_{v_1}^- \pi_e^1 \sigma_{v_0}^+\right]$$

Edge = (origin, end)

Pauli +/- operators

$$H_G = \sum_{n=1}^{M_G} \mathrm{Tr}_V[A^n]$$

Trace over vertex space

n = cycle length

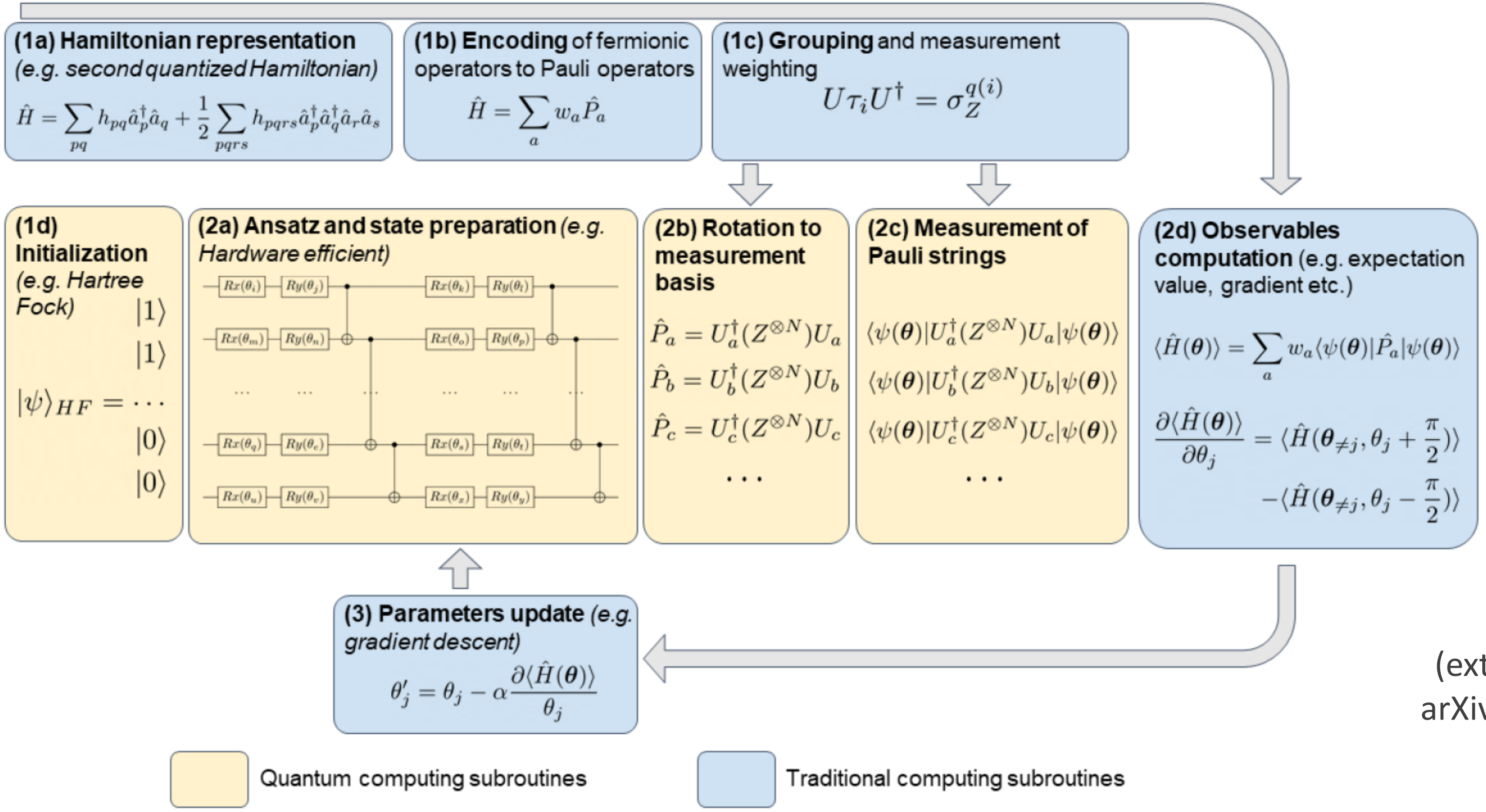
More details arXiv:2210.13240 [hep-ph]

Minimizing H, we find the acyclic graphs (0 energy)

Clemente, Crippa et al (2022) arXiv:2210.13240 [hep-ph]

Using a Variational Quantum Eigensolver

- *VQE is a hybrid quantum-classical algorithm, optimized for minimization problems*
- **<u>QUANTUM PART</u>: Evaluation of the Hamiltonian applied to an ansatz (parametrized quantum circuit)**
- **<u>CLASSICAL PART</u>: Modification of the parameters, through minimization algorithms**



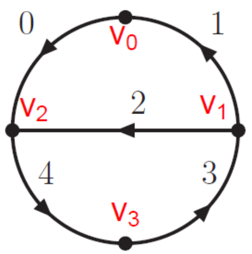
VQE pipeline

(extracted from J. Tilly et al, arXiv:2111.05176 [quant-ph])

J. Tilly et al (2022), Phys. Rep. 986, 1-128

Using a Variational Quantum Eigensolver

1. Our implementation with Qiskit: **Real Amplitudes** (ansatz) **+ COBYLA** (optimizer)
2. Improved results with **multi-run VQE**: set a selection **threshold**, collect **solutions** and modify the Hamiltonian with **penalization terms**



Two eloop
5-point

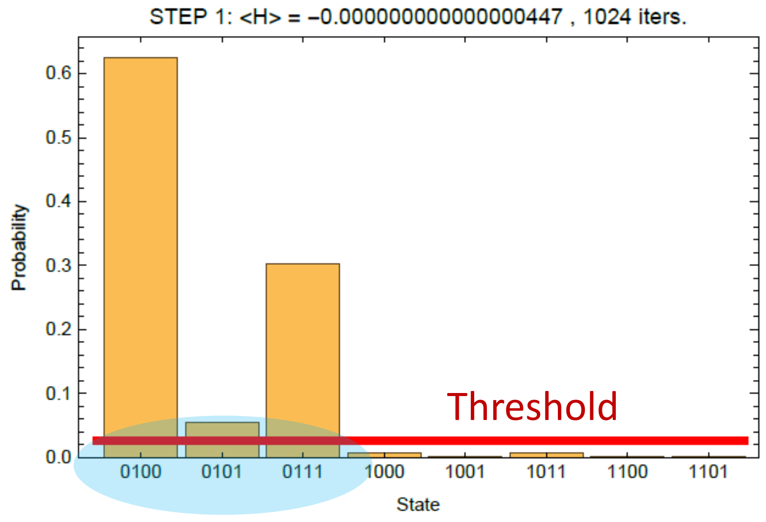
$$\begin{aligned} H_{E/\{e_0\}} = & \; 4\, I_4 \otimes I_3 \otimes I_2 \otimes I_1 + 2\, I_4 \otimes I_3 \otimes I_2 \otimes Z_1 - I_4 \otimes I_3 \otimes Z_2 \otimes I_1 \\ & - I_4 \otimes I_3 \otimes Z_2 \otimes Z_1 + I_4 \otimes Z_3 \otimes I_2 \otimes I_1 + I_4 \otimes Z_3 \otimes I_2 \otimes Z_1 \\ & + 2\, I_4 \otimes Z_3 \otimes Z_2 \otimes I_1 + Z_4 \otimes I_3 \otimes I_2 \otimes I_1 + Z_4 \otimes I_3 \otimes I_2 \otimes Z_1 \\ & + 2\, Z_4 \otimes I_3 \otimes Z_2 \otimes I_1 + 3\, Z_4 \otimes Z_3 \otimes I_2 \otimes I_1 + Z_4 \otimes Z_3 \otimes I_2 \otimes Z_1 \end{aligned}$$

Reduced Hamiltonian

Classical methods

Complete set of solutions

$$\{|0011\rangle, |0100\rangle, |0101\rangle, |0111\rangle, |1000\rangle, |1001\rangle, |1011\rangle, |1100\rangle, |1101\rangle\}$$



$$\mathcal{S}_1 = \{|0100\rangle, |0101\rangle, |0111\rangle\}$$

Selected solutions (1st run)

$$\Pi^{(1)} = \sum_{|\phi_l\rangle \in \mathcal{S}_1} b_l^{(1)} |\phi_l\rangle\langle\phi_l|$$

Penalization term ...

$$H^{(1)} = H^{(0)} + \Pi^{(1)}$$

... to be added to the Hamiltonian for next run!!

$$|\psi^{(1)}\rangle = \sum_j c_j^{(1)} |\phi_j\rangle$$

Approximated ground state found by VQE

$$\mathcal{S}_1 = \{|\phi_j\rangle \,|\, |c_j^{(1)}|^2 > \lambda\}$$

and subset of terms above the selection threshold

Clemente, Crippa et al (2022) arXiv:2210.13240 [hep-ph]

Using a Variational Quantum Eigensolver

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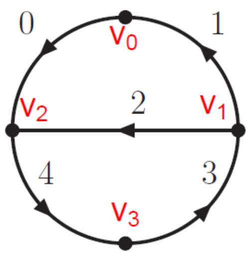
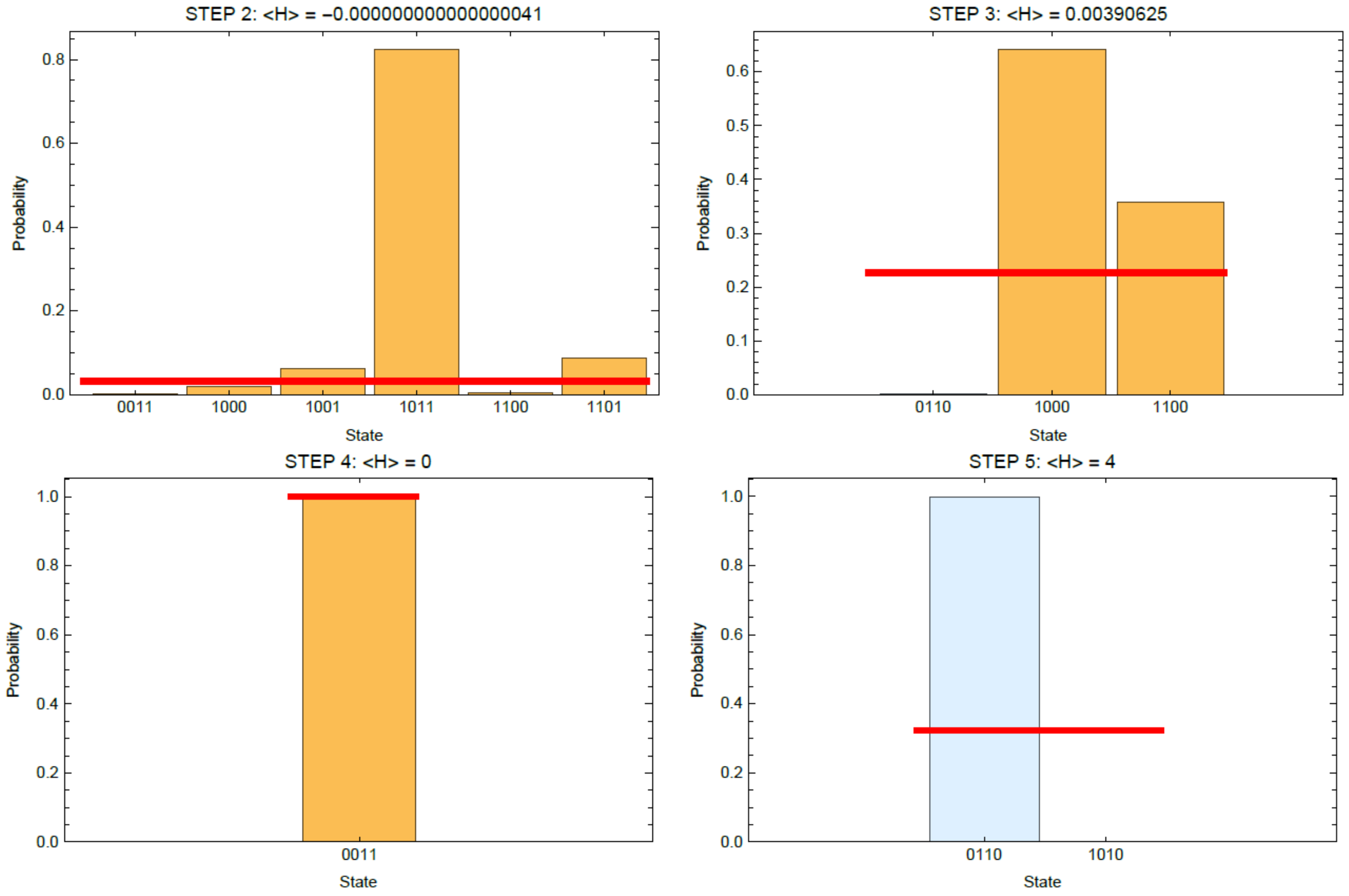
More details arXiv:2210.13240 [hep-ph]



Two eloop
5-point



- We collect solutions step by step, till the algorithm converges (if <H> >1)
- *Problem:* it is not guaranteed that all the solutions are collected (**work in progress!!**)

Clemente, Crippa et al (2022) arXiv:2210.13240 [hep-ph]

Conclusions

- Use LTD to cleverly rewrite Feynman integrals: **Minkowski to Euclidean**
- **Nested residues leads to manifestly causal representations** of scattering amplitudes!
- Very compact formulae **with strong physical/conceptual** motivation

- **Geometrical rules** select **entangled thresholds. Complete reconstruction** of multiloop amplitudes!
- **Quantum algorithms** to speed-up **causal flux selection**. *Exploring new disruptive tools for breaking the precision frontier!!*
- **Both Grover's search algorithm and VQE seem promising candidates to unveil the causal representations in (real) quantum devices**

qGEODAS

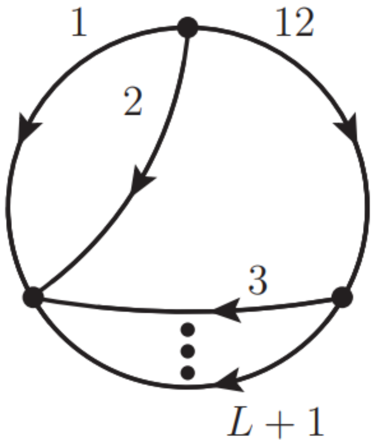

THANKS!
ieronimus

BACKUP.

Causality and nested residues

- Similar causal formulae can be found for NMLT and NNMLT to all loop orders!

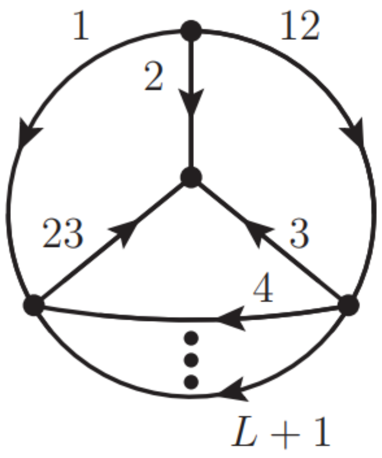
Next-to Maximal Loop Topology



$$\mathcal{A}^{(L)}_{\rm NMLT}(1,2,\ldots,L+2) = \int_{\vec{\ell}_1,\ldots,\vec{\ell}_L} \frac{2}{x_{L+2}} \left(\frac{1}{\lambda_1\lambda_2} + \frac{1}{\lambda_2\lambda_3} + \frac{1}{\lambda_3\lambda_1} \right)$$

with $\lambda_1 = \sum_{i=1}^{L+1} q_{i,0}^{(+)}$ $\quad \lambda_2 = q_{1,0}^{(+)} + q_{2,0}^{(+)} + q_{L+2,0}^{(+)}$ $\quad \lambda_3 = \sum_{i=3}^{L+2} q_{i,0}^{(+)}$

Next-to-Next-to Maximal Loop Topology



$$\mathcal{A}^{(L)}_{\rm N^2MLT}(1,2,\ldots,L+3) = -\int_{\vec{\ell}_1,\cdots,\vec{\ell}_L} \frac{2}{x_{L+3}} \left[\frac{1}{\lambda_1}\left(\frac{1}{\lambda_2}+\frac{1}{\lambda_3}\right)\left(\frac{1}{\lambda_4}+\frac{1}{\lambda_5}\right) + \frac{1}{\lambda_6}\left(\frac{1}{\lambda_2}+\frac{1}{\lambda_4}\right)\left(\frac{1}{\lambda_3}+\frac{1}{\lambda_5}\right) + \frac{1}{\lambda_7}\left(\frac{1}{\lambda_2}+\frac{1}{\lambda_5}\right)\left(\frac{1}{\lambda_3}+\frac{1}{\lambda_4}\right) \right]$$

with $\lambda_4 = q_{2,0}^{(+)} + q_{3,0}^{(+)} + q_{L+3,0}^{(+)}$ $\quad \lambda_6 = q_{1,0}^{(+)} + q_{3,0}^{(+)} + q_{L+2,0}^{(+)} + q_{L+3,0}^{(+)}$

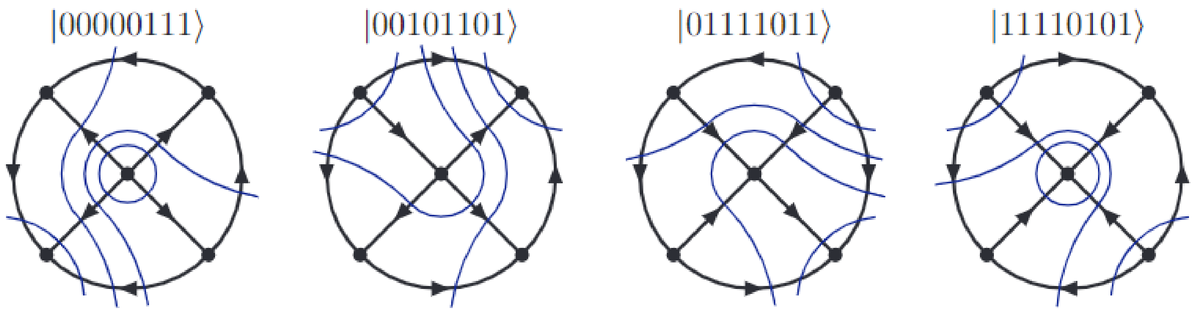
$\lambda_5 = q_{1,0}^{(+)} + q_{L+3,0}^{(+)} + \sum_{i=4}^{L+1} q_{i,0}^{(+)}$ $\quad \lambda_7 = q_{2,0}^{(+)} + \sum_{i=4}^{L+3} q_{i,0}^{(+)}$

Aguilera-Verdugo et al, JHEP 01 (2021) 069; JHEP 02 (2021) 112

Grover's Algorithm for Causal Reconstruction

- **Further examples:** four eloops (N3MLT and s & t-channels N4MLT)

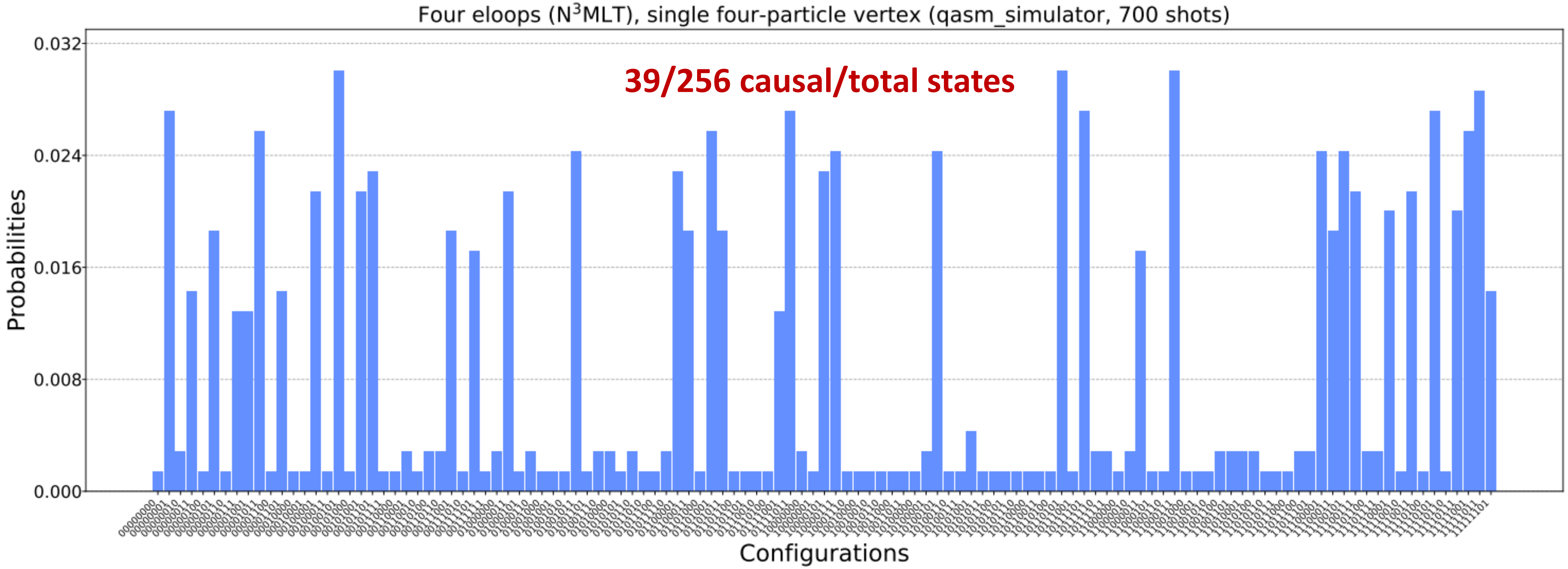
Pizza (N3MLT)



Boolean conditions (oracle definition)

$$a_0^{(4)} = \neg(c_{01} \wedge c_{12} \wedge c_{23}) \ ,$$
$$a_1^{(4)} = \neg(\bar{c}_{05} \wedge \bar{c}_{45}) \ ,$$
$$a_2^{(4)} = \neg(\bar{c}_{16} \wedge \bar{c}_{56}) \ ,$$
$$a_3^{(4)} = \neg(\bar{c}_{27} \wedge \bar{c}_{67}) \ ,$$
$$a_4^{(4)} = \neg(\bar{c}_{34} \wedge \bar{c}_{47}) \ ,$$

$$f^{(4)}(a, q) = (a_0^{(4)} \wedge \ldots \wedge a_4^{(4)}) \wedge q_0$$

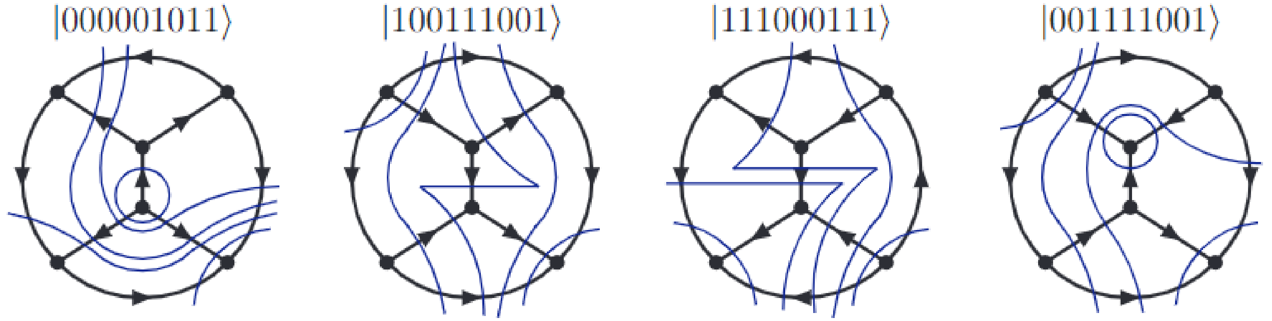


Ramírez-Uribe et al, JHEP 05 (2022) 100

- **Further examples:** four eloops (N3MLT and s & t-channels N4MLT)

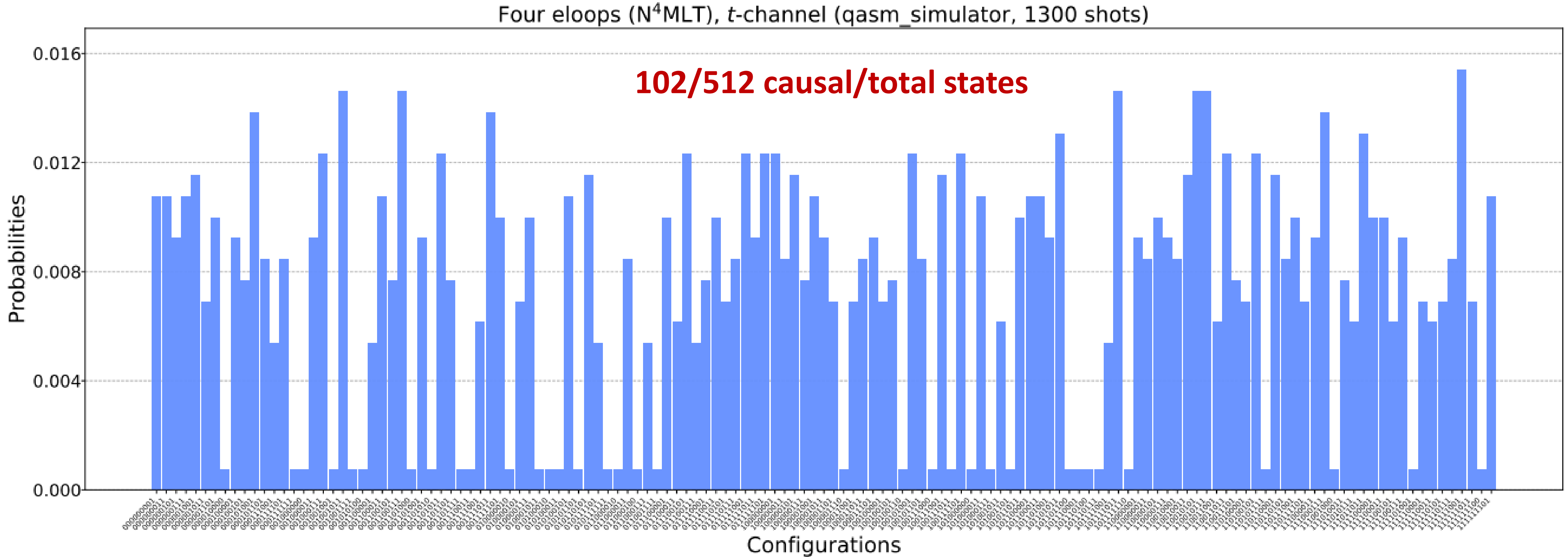
s-channel similar to t-channel BUT…
u-channel exceeds IBMQ capabilities

t-channel (N4MLT)



$$f^{(4,t)}(a,q) = \left(a_0^{(4)} \wedge a_1^{(t)} \wedge a_2^{(4)} \wedge a_3^{(t)} \wedge a_4^{(4)}\right) \wedge q_0 \, ,$$
$$f^{(4,s)}(a,q) = \left(a_0^{(4)} \wedge a_1^{(4)} \wedge a_2^{(s)} \wedge a_3^{(4)} \wedge a_4^{(s)}\right) \wedge q_0 \, ,$$
$$f^{(4,u)}(a,q) = \left(a_0^{(4)} \wedge a_1^{(t)} \wedge a_2^{(s)} \wedge a_3^{(u)} \wedge \ldots \wedge a_8^{(u)}\right) \wedge q_0$$

Boolean conditions (oracle definition)



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