Electromagnetic Form Factors in a Contact Interaction: Scalar and Pseudoscalar mesons

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Abstract

The determination of fundamental properties of hadrons is important to clearly understand the experimental measurements such as the anomalous magnetic moment of the muon. However, since a first principle approach is hard to find, several groups among the world are approaching phenomenologically to such description of hadrons. In this poster, we present a comprehensive survey of electromagnetic form factors of all light, heavy and heavy-light ground-state pseudoscalar and scalar mesons. Specifically, in this work we are interested on the determination of Elastic Meson Form Factors (EFF) for Scalar and Pseudoscalar mesons. However, the calculation of all EFF requires the computation of the quark propagator, the Bethe-Salpeter (BS) equation and the Bethe-Salpeter Amplitudes (BSA) of mesons, their masses as well as the knowledge of the quark-photon interaction at different probing momenta.

GAP equation

Hadrons are conformed by quarks in a $q\bar{q}$ pair, so-called mesons, or by qqq condensate, so-called baryons. It is known that the masses of hadrons are not simply the addition of the bare masses of constituent quarks but they acquire their mass from the dressed quark masses via the Bethe-Salpeter equation. Dressed masses of quarks are obtained by use of the equation, $S(p)^{-1} = i\,\gamma\cdot p + m_f + \Sigma(p)$, where the one-loop contribution to the quark propagator, $S(p)^{-1} = p^2 + M_f^2$, is given by [1, 2],

$$\Sigma(p) = \frac{4}{3}\int \frac{d^4q}{(2\pi)^4}\, g^2\, D_{\mu\nu}(p-q)\,\gamma_\mu\, S(q)\,\Gamma_\nu(p,q)\,, \tag{1}$$

with m_f and M_f the current and dressed masses of the f-quark, respectively, $D_{\mu\nu}$ the vector boson propagator, g the coupling of the interaction and Γ_ν the quark-vector boson vertex. By means of the Contact Interaction (CI),

$$g^2 D_{\mu\nu}(q) = \delta_{\mu\nu}\frac{4\pi\alpha_{\rm IR}}{m_G^2}\,, \qquad \Gamma_\nu = \gamma_\nu\,, \tag{2}$$

it is possible to find dressed masses of mesons M_f by,

$$M_f = m_f + M_f\,\frac{4\alpha_{\rm IR}}{3\pi m_G^2}\,\mathcal{C}(M_f^2)\,, \tag{3}$$

where $\mathcal{C}(z)/z = \Gamma(-1, z\,\tau_{\rm UV}^2) - \Gamma(-1, z\,\tau_{\rm IR}^2)$, with $\Gamma(\alpha, z)$ the incomplete gamma-function, $\tau_{\rm IR}$ and $\tau_{\rm UV}$ the Infrared and Ultraviolet regulators, respectively [3].

Bethe-Salpeter equation

The bound-state problem for Hadrons characterized by two valence-fermions may be studied using the homogeneous BS equation [4],

$$[\Gamma(k;P)]_{tu} = \int \frac{d^4q}{(2\pi)^4}[\chi(q;P)]_{sr}\mathcal{K}^{rs}_{tu}(q,k;P)\,, \tag{4}$$

where $[\Gamma(k;P)]_{tu}$ represents the bound-state's BSA and $\chi(q;P) = S(q+P)\Gamma S(q)$ is the BS wave-function; r,s,t,u represent colour, flavor and spinor indices; and $\mathcal{K}$ is the relevant quark-antiquark scattering kernel. This equation possesses solutions on that discrete set of P^2-values for which bound-states exist.

Decomposition of the BSA for the PS and the S mesons ($f_1\bar{f}_2$) in the CI has the following form [5],

$$\begin{aligned}\Gamma_{PS}(P) &= i\gamma_5\,E_{PS}(P) + \frac{1}{2M_R}\gamma_5\gamma\cdot P\,F_{PS}(P)\,,\\ \Gamma_S(P) &= I_D\,E_S(P)\,.\end{aligned} \tag{5}$$

Note that $E_i(P)$ and $F_i(P)$ with $i\in\{PS,S\}$ are known as the BSAs of the meson under consideration, P is its total momentum, I_D is the identity matrix and $M_R = M_{f_1}M_{\bar{f}_2}/[M_{f_1}+M_{\bar{f}_2}]$ is the reduced mass of the system. Eq. (4) has a solution when $P^2 = -M_M^2$ with M_M being the meson mass.

Electromagnetic Form Factors

EFF can be extracted by studying the process $M\gamma M$, where M is a general meson. The kinematics of the process is given by the Feynman diagram depicted in Fig. 1.

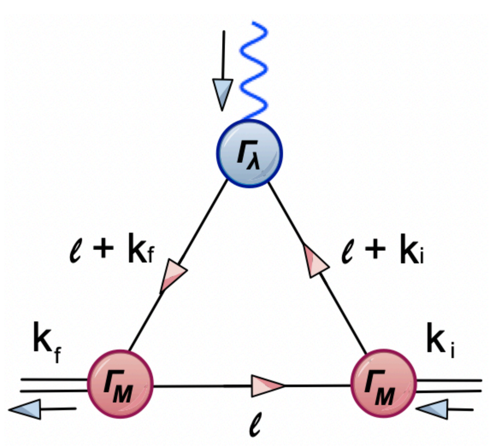


Figure 1: The triangle diagram for the impulse approximation to the $M\gamma M$ vertex.

Using Feynman rules, we find that for a general meson, this process can be written as [6],

$$\Lambda^{M,f_1} = N_c\int\frac{d^4\ell}{(2\pi)^4}{\rm Tr}\,\mathcal{G}^{M,f_1}\,, \tag{6}$$

where we have omitted the Dirac indices, and,

$$\mathcal{G}^{M,f_1} = i\Gamma_M(k_f)\,S(\ell+k_i,M_{f_1})\,i\Gamma_\lambda(Q,M_{f_1})\,S(\ell+k_f,M_{f_1})i\bar{\Gamma}_M(-k_i)\,S(\ell,M_{\bar{f}_2})\,.$$

The function Λ^{M,f_1} labels all M kind of mesons. Besides, the photon interacts with the quark with flavor f_1, and the fermion $\bar{f}_2$ is a spectator. Λ^{M,f_1} can be written as a function of the EFF as,

$$\Lambda^{S,f_1} = -2k_\lambda F^{S,f_1}\,,\ \Lambda^{PS,f_1} = -2k_\lambda F^{PS,f_1}\,. \tag{7}$$

In order to consider the total elastic form factor of mesons, we use,

$$F^M(Q^2) = e_{f_1}F^{M,f_1}(Q^2) + e_{\bar{f}_2}F^{M,\bar{f}_2}(Q^2)\,, \tag{8}$$

Meson masses

In the following tables, we present the solution of the Bethe-Salpeter equation for scalar and pseudoscalar mesons used for the calculations of EFF [7].

• **Scalar Mesons**

	Mass [GeV]	E_S
$u\bar{d}$	1.22	0.66
$u\bar{s}$	1.38	0.65
$s\bar{s}$	1.46	0.64
$c\bar{u}$	2.31	0.39
$c\bar{s}$	2.42	0.42
$u\bar{b}$	5.30	1.53
$s\bar{b}$	5.64	0.26
$c\bar{b}$	6.36	1.23
$c\bar{c}$	3.33	0.16
$b\bar{b}$	9.57	0.69

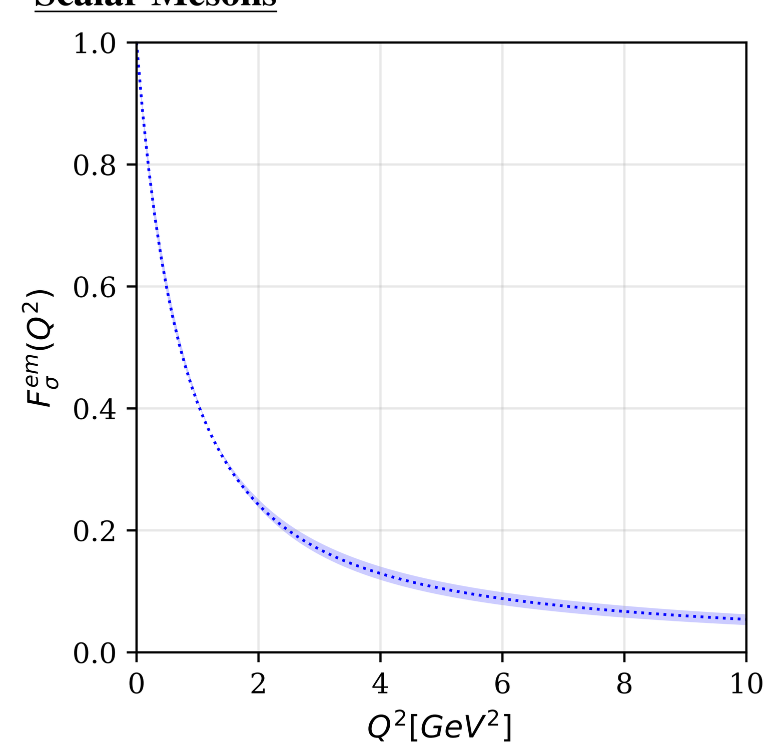
• **Pseudoscalar Mesons**

	Mass[GeV]	E_{PS}	F_{PS}
$u\bar{d}$	0.139	3.59	0.47
$u\bar{s}$	0.499	3.81	0.59
$s\bar{s}$	0.701	4.04	0.75
$c\bar{u}$	1.855	3.03	0.37
$c\bar{s}$	1.945	3.24	0.51
$u\bar{b}$	5.082	3.72	0.21
$s\bar{b}$	5.281	2.85	0.21
$c\bar{b}$	6.138	2.58	0.39
$c\bar{c}$	2.952	2.15	0.40
$b\bar{b}$	9.280	2.04	0.39

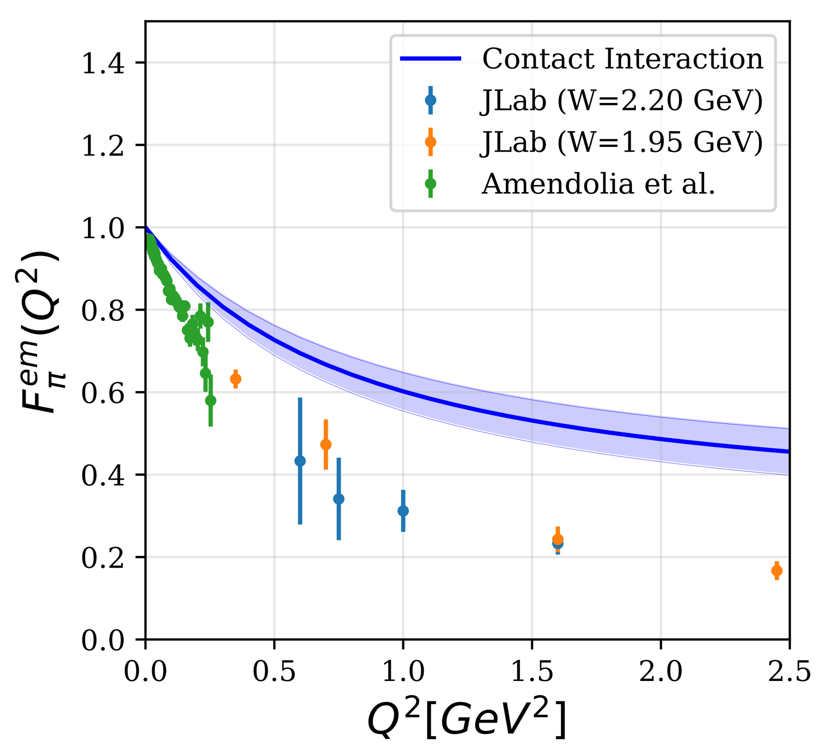
Electromagnetic Form Factors

For the sake of simplicity, we depicted in the following figures EFF for the lightest scalar and pseudoscalar mesons, sigma and pion meson.

• **Scalar Mesons**



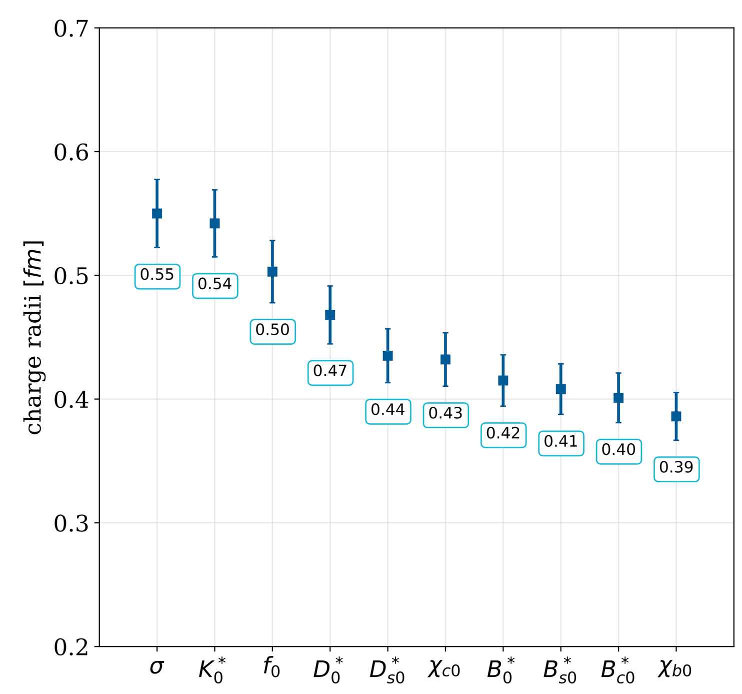
• **Pseudoscalar Mesons**



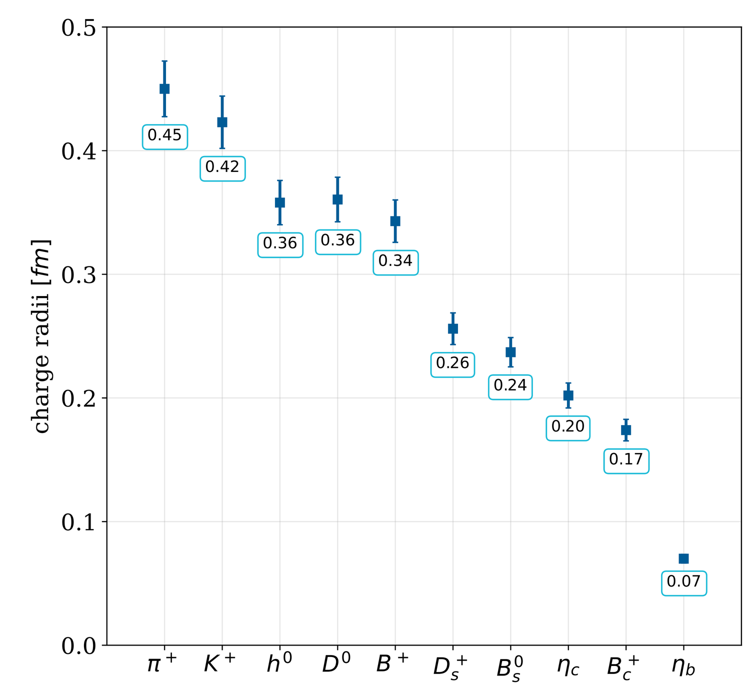
Charge radii of mesons

Following the definition of the charge radii of mesons, $r_M^2 = -6\left.\frac{dF_M(Q^2)}{dQ^2}\right|_{Q^2=0}$, we can compute the charge radii of scalar and pseudoscalar mesons.

• **Scalar Mesons**



• **Pseudoscalar Mesons**



Conclusions

In this work we present an exhaustive computation of EFFs employing the CI model for twenty ground state PS and S mesons. Note that the CI findings for light mesons and heavy quarkonia are already found in the literature as mentioned before [3, 8, 9, 10]. We expect these new EFFs to be harder than the exact QCD predictions, especially for the PS mesons due to the necessary inclusion of the F-amplitude. We also anticipate the charge radii to be in the ballpark of a (20-25)% error in light of the results where comparison with realistic studies and/or experiment has been possible.

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