

SYMMETRIES AND THEIR MASSIVE CONSEQUENCES

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SYMMETRIES

- Modern physics is built on the observation that there are symmetries in Nature (exact or broken)
- Symmetry is a transformation that leaves the system invariant

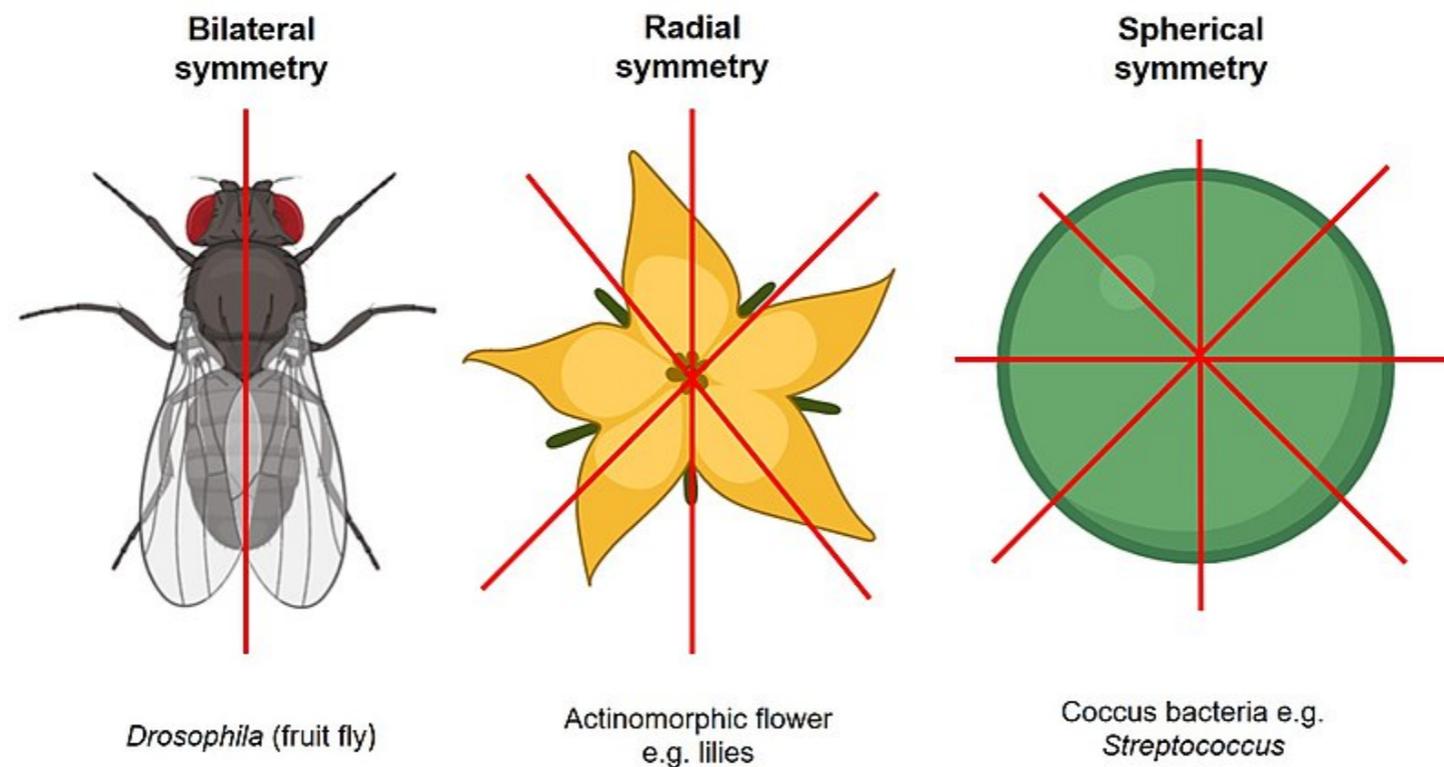
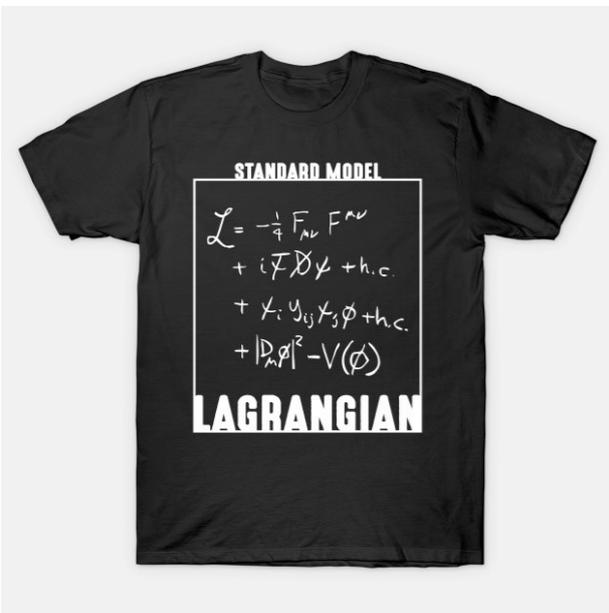


Fig: Wikipedia

SYMMETRIES

- Quantum field theory - combines quantum mechanics and special relativity
- Space-time symmetries:
rotations, translations, Lorenz and Poincaré transformations
- Internal symmetries:
transformation of the fields in the theory → gauge symmetries
- Global → spacetime momentum, angular momentum, spin
- Local → gauge symmetries
- Continuous symmetries → conserved quantities
 - rotational symmetry
angular momentum conservation
 - translational symmetry
momentum and energy conservation
- Discrete → charge and parity conjugation CP
- Label and classify particles
- Determine interactions among particles → they must respect the symmetries
- Exact, broken, a little bit broken (softly), hidden



WHAT PART OF

$$\begin{aligned} & -\frac{1}{2}\partial_\nu g_\mu^a \partial_\nu g_\mu^a - g_s f^{abc} \partial_\mu g_\nu^b g_\mu^c g_\nu^a - \frac{1}{4}g_s^2 f^{abc} f^{ade} g_\mu^b g_\nu^c g_\mu^d g_\nu^e + \frac{1}{2}ig_s^2 (\bar{q}_i \gamma^\mu q^i) g_\mu \\ & \bar{G}^a \partial^2 G^a + g_s f^{abc} \partial_\mu G^a G^b g_\mu^c - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- - M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2c_w^2} M^2 Z_\mu^0 Z_\mu^0 \\ & - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H - \frac{1}{2}m_H^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \\ & \frac{1}{2c_w^2} M \phi^0 \phi^0 - \beta_h \left[\frac{2M^2}{g^2} + \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) \right] + \frac{2M}{g^2} \alpha_h - ig_{c_w} [\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- \\ & W_\nu^+ W_\mu^-) - Z_\nu^0 (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + Z_\nu^0 (W_\nu^+ \partial_\mu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+)] - ig_{s_w} \partial_\nu A_\mu (W_\mu^+ W_\nu^- \\ & W_\nu^+ W_\mu^-) - A_\nu (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + A_\mu (W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+) - \frac{1}{2}g^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- + \\ & \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\mu^+ W_\nu^- + g^2 c_w^2 (Z_\mu^0 W_\mu^+ Z_\mu^0 W_\nu^- - Z_\mu^0 Z_\mu^0 W_\nu^+ W_\nu^-) + g^2 s_w^2 (A_\mu W_\mu^+ A_\nu W_\nu^- - \\ & A_\mu A_\nu W_\mu^+ W_\nu^-) + g^2 s_w c_w A_\mu Z_\nu^0 (W_\mu^+ W_\nu^- - W_\nu^- W_\mu^-) - 2A_\mu Z_\mu^0 W_\nu^+ W_\nu^- - g\alpha [H^3 + \\ & H\phi^0 \phi^0 + 2H\phi^+ \phi^-] - \frac{1}{2}g^2 \alpha_h H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + \\ & 2(\phi^0)^2 H^2] - g M W_\mu^+ W_\mu^- H - \frac{1}{2}g \frac{M}{c_w} Z_\mu^0 Z_\mu^0 H - \frac{1}{2}ig [W_\mu^+ (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W_\mu^- (\phi^0 \partial_\mu \phi^+ \\ & \phi^+ \partial_\mu \phi^0)] + \frac{1}{2}g [W_\mu^+ (H \partial_\mu \phi^- - \phi^- \partial_\mu H) - W_\mu^- (H \partial_\mu \phi^+ - \phi^+ \partial_\mu H)] + \frac{1}{2}g \frac{1}{c_w} (Z_\mu^0 (H \partial_\mu \phi^0 - \\ & \phi^0 \partial_\mu H) + ig \frac{s_w}{c_w} M Z_\mu^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+) + ig_{s_w} M A_\mu (W_\mu^+ \phi^- - W_\mu^- \phi^+) - ig \frac{1-2c_w^2}{2c_w} Z_\mu^0 (\phi^+ \partial_\mu \phi^- - \\ & \phi^- \partial_\mu \phi^+) + ig_{s_w} A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \frac{1}{2}g^2 W_\mu^+ W_\mu^- H^2 + (\phi^0)^2 + 2\phi^+ \phi^-] - \\ & \frac{1}{2}g^2 \frac{1}{c_w} Z_\mu^0 Z_\mu^0 [H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)\phi^+ \phi^-] - \frac{1}{2}g^2 \frac{s_w^2}{c_w} Z_\mu^0 \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+) - \\ & \frac{1}{2}ig^2 \frac{s_w}{c_w} Z_\mu^0 H (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+) + \frac{1}{2}ig^2 s_w A_\mu H (W_\mu^+ \phi^- \\ & W_\mu^- \phi^+) - g^2 \frac{s_w^2}{c_w} (2c_w^2 - 1) Z_\mu^0 A_\mu \phi^+ \phi^- - g^2 s_w^2 A_\mu A_\mu \phi^+ \phi^- - \bar{e}^\lambda (\gamma^\theta + m_e^\lambda) e^\lambda - \\ & \bar{\nu}^\lambda \gamma^\theta \nu^\lambda - \bar{u}_j^\lambda (\gamma^\theta + m_u^\lambda) u_j^\lambda - d_j^\lambda (\gamma^\theta + m_d^\lambda) d_j^\lambda + ig_{s_w} A_\mu [-(e^\lambda \gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}_j^\lambda \gamma^\mu u_j^\lambda) - \\ & \frac{1}{3}(\bar{d}_j^\lambda \gamma^\mu d_j^\lambda)] + \frac{ig}{4c_w} Z_\mu^0 [(\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (e^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) - (\bar{u}_j^\lambda \gamma^\mu (\frac{4}{3}s_w^2 - \\ & 1 - \gamma^5) u_j^\lambda) + (\bar{d}_j^\lambda \gamma^\mu (1 - \frac{8}{3}s_w^2 - \gamma^5) d_j^\lambda)] + \frac{ig}{2\sqrt{2}} W_\mu^+ [(\nu^\lambda \gamma^\mu (1 + \gamma^5) e^\lambda) - (u_j^\lambda \gamma^\mu (1 + \\ & \gamma^5) C_{\lambda k} d_k^\lambda)] + \frac{ig}{2\sqrt{2}} W_\mu^- [(e^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{d}_j^\lambda C_{\lambda k} \gamma^\mu (1 + \gamma^5) u_j^\lambda)] + \frac{ig}{2\sqrt{2}} \frac{m_\tau}{M} [-\phi^+ (\bar{\nu}^\lambda (1 - \\ & \gamma^5) e^\lambda) + \phi^- (e^\lambda (1 + \gamma^5) \nu^\lambda)] - \frac{g}{2} \frac{m_\tau^2}{M} [H (\bar{e}^\lambda e^\lambda) + i\phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}} \phi^+ [-m_\tau^2 (\bar{u}_j^\lambda C_{\lambda k} (1 - \\ & \gamma^5) d_k^\lambda) + m_\tau^2 (\bar{u}_j^\lambda C_{\lambda k} (1 + \gamma^5) d_k^\lambda)] + \frac{ig}{2M\sqrt{2}} \phi^- [m_\tau^2 (\bar{d}_j^\lambda C_{\lambda k} (1 + \gamma^5) u_j^\lambda) - m_\tau^2 (\bar{d}_j^\lambda C_{\lambda k} (1 - \\ & \gamma^5) u_j^\lambda)] - \frac{g}{2} \frac{m_\tau^2}{M} H (u_j^\lambda u_j^\lambda) - \frac{g}{2} \frac{m_\tau^2}{M} H (d_j^\lambda d_j^\lambda) + \frac{ig}{2} \frac{m_\tau^2}{M} \phi^0 (\bar{u}_j^\lambda \gamma^5 u_j^\lambda) - \frac{ig}{2} \frac{m_\tau^2}{M} \phi^0 (\bar{d}_j^\lambda \gamma^5 d_j^\lambda) + \\ & X^+ (\partial^2 - M^2) X^+ + X^- (\partial^2 - M^2) X^- + X^0 (\partial^2 - \frac{M^2}{c_w^2}) X^0 + Y \partial^2 Y + ig_{c_w} W_\mu^+ (\partial_\mu X^0 X^- - \\ & \partial_\nu X^+ X^0) + ig_{s_w} W_\mu^+ (\partial_\mu \bar{Y} X^- - \partial_\mu X^+ \bar{Y}) + ig_{c_w} W_\mu^- (\partial_\mu X^- X^0 - \partial_\mu \bar{X}^0 X^+) + \\ & ig_{s_w} W_\mu^- (\partial_\mu X^- Y - \partial_\mu Y X^+) + ig_{c_w} Z_\mu^0 (\partial_\mu X^+ X^+ - \partial_\mu X^- X^-) + ig_{s_w} A_\mu (\partial_\mu X^+ X^+ - \\ & \partial_\mu \bar{X}^- X^-) - \frac{1}{2}g M [\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w} \bar{X}^0 X^0 H] + \frac{1-2c_w^2}{2c_w} ig M [\bar{X}^+ X^0 \phi^+ - \\ & X^- X^0 \phi^-] + \frac{1}{2c_w} ig M [X^0 X^- \phi^+ - X^0 X^+ \phi^-] + ig M s_w [X^0 X^- \phi^+ - X^0 X^+ \phi^-] + \\ & \frac{1}{2}ig M \bar{X}^+ X^+ \phi^0 - X^- X^- \phi^0] \end{aligned}$$

DO YOU NOT UNDERSTAND?



WHY GO BEYOND?

- The hierarchy problem
- Neutrino masses
- All masses
- Origin of gauge interactions
- Dark matter
- Matter over anti-matter abundance
- Cosmological constant
- Inflation
- ...

Higgs sector not natural

Fermion masses vastly different

Origin of electroweak symmetry breaking unknown

Dirac or Majorana neutrinos

Strong CP problem

Not enough CP in SM for Baryogenesis

Value of cosmological constant

Inflation inconsistent with non-zero baryon number

Is DM a particle, then which, is it only one

HOW TO GO BEYOND THE STANDARD MODEL (BSM)?

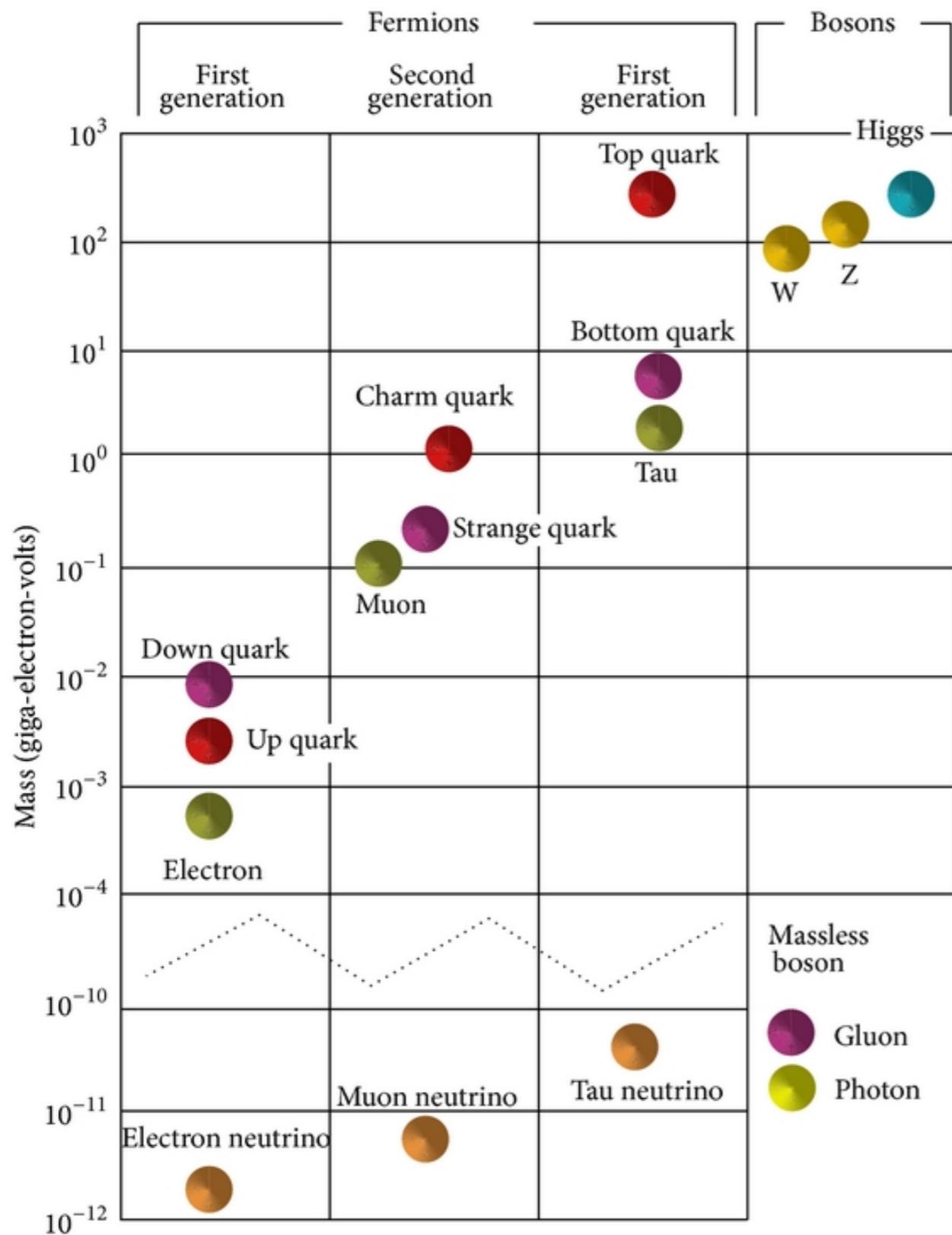
- Addition of symmetries
- Addition of particles
- Addition of interactions
- Addition of space-time dimensions
- All of the above...

Can get messy...

I will concentrate on masses and mixings.

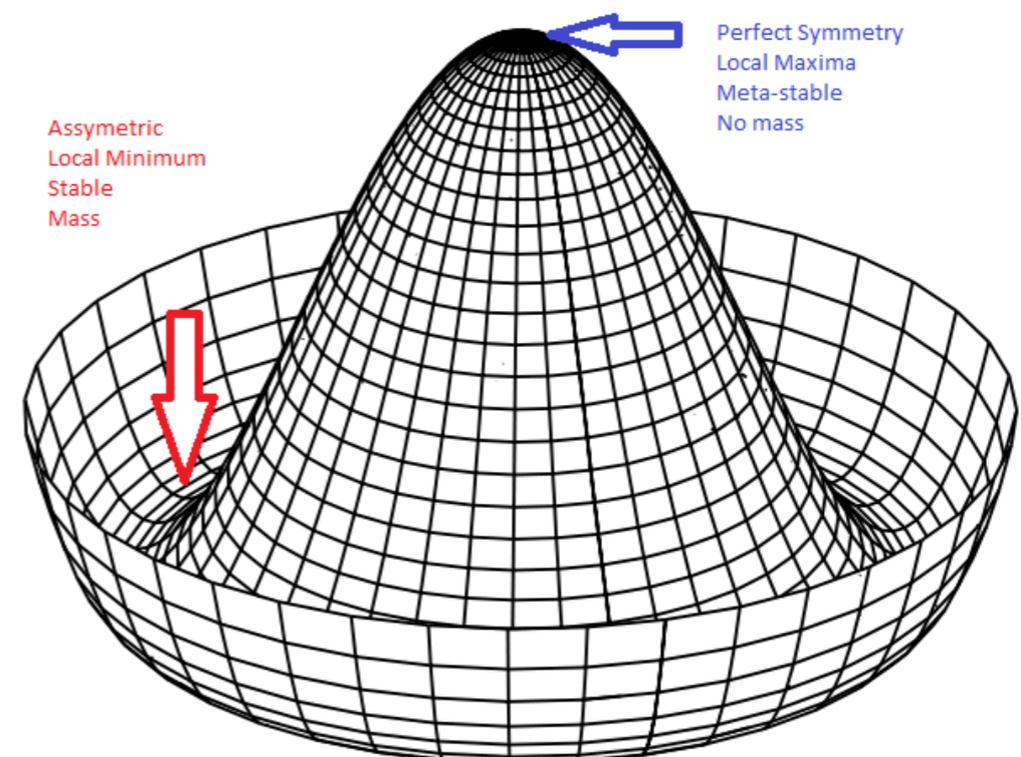
And perhaps dark matter and leptogenesis, and...



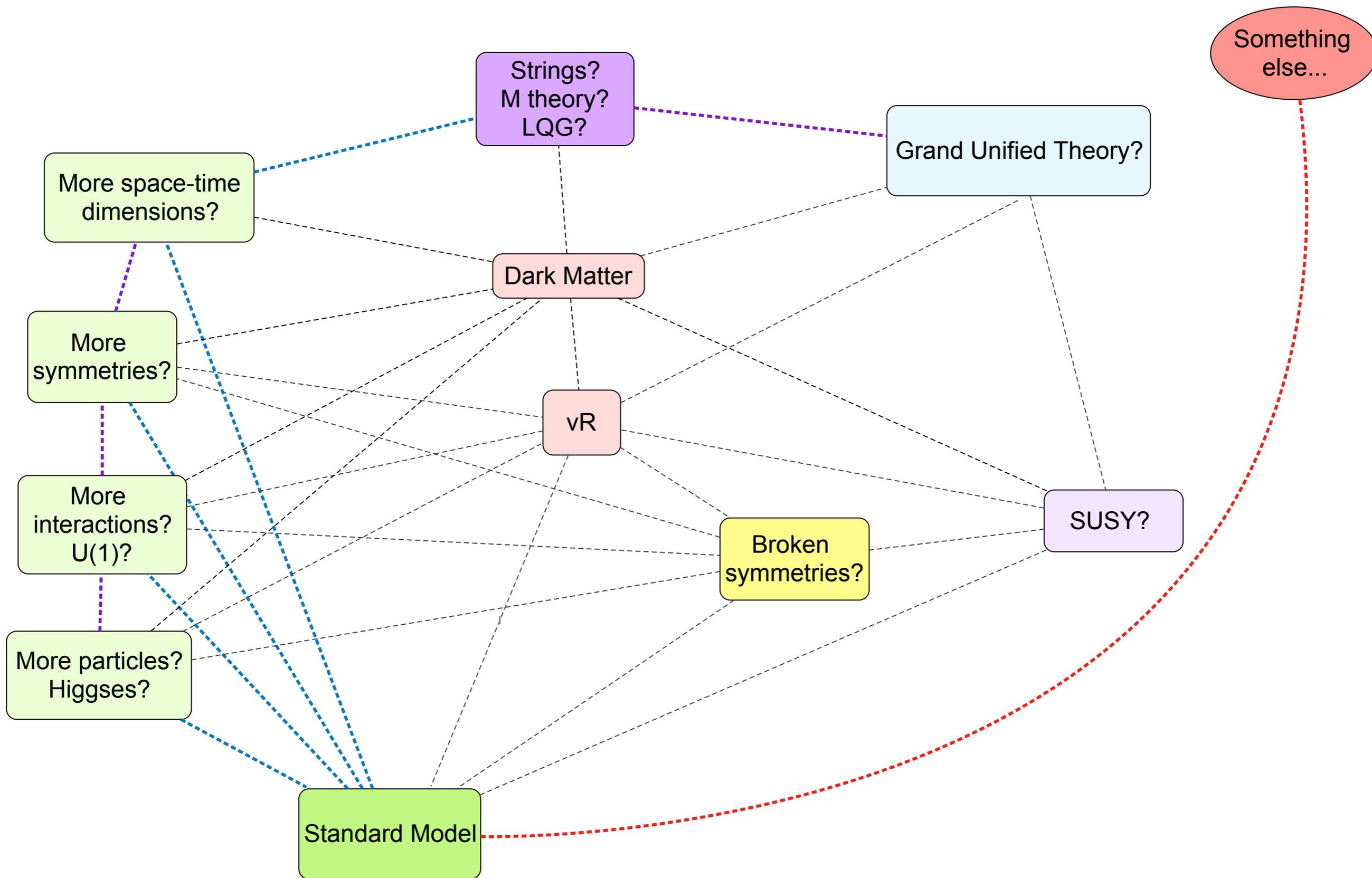


MASSES ARE VERY DIFFERENT...

- Quark and lepton masses hierarchical and very different
- Neutrino masses enhance the problem, tiny!
- Higgs boson crucial for mass generation



Picture from:
 Neutrino Oscillations in the Atmospheric Parameter Region: From the Early Experiments to the Present
 BGiacomelli, M. Giorgini, L. Patrizii, M. Sioli.
 Adv.High Energy Phys. 2013 (2013) 464926.



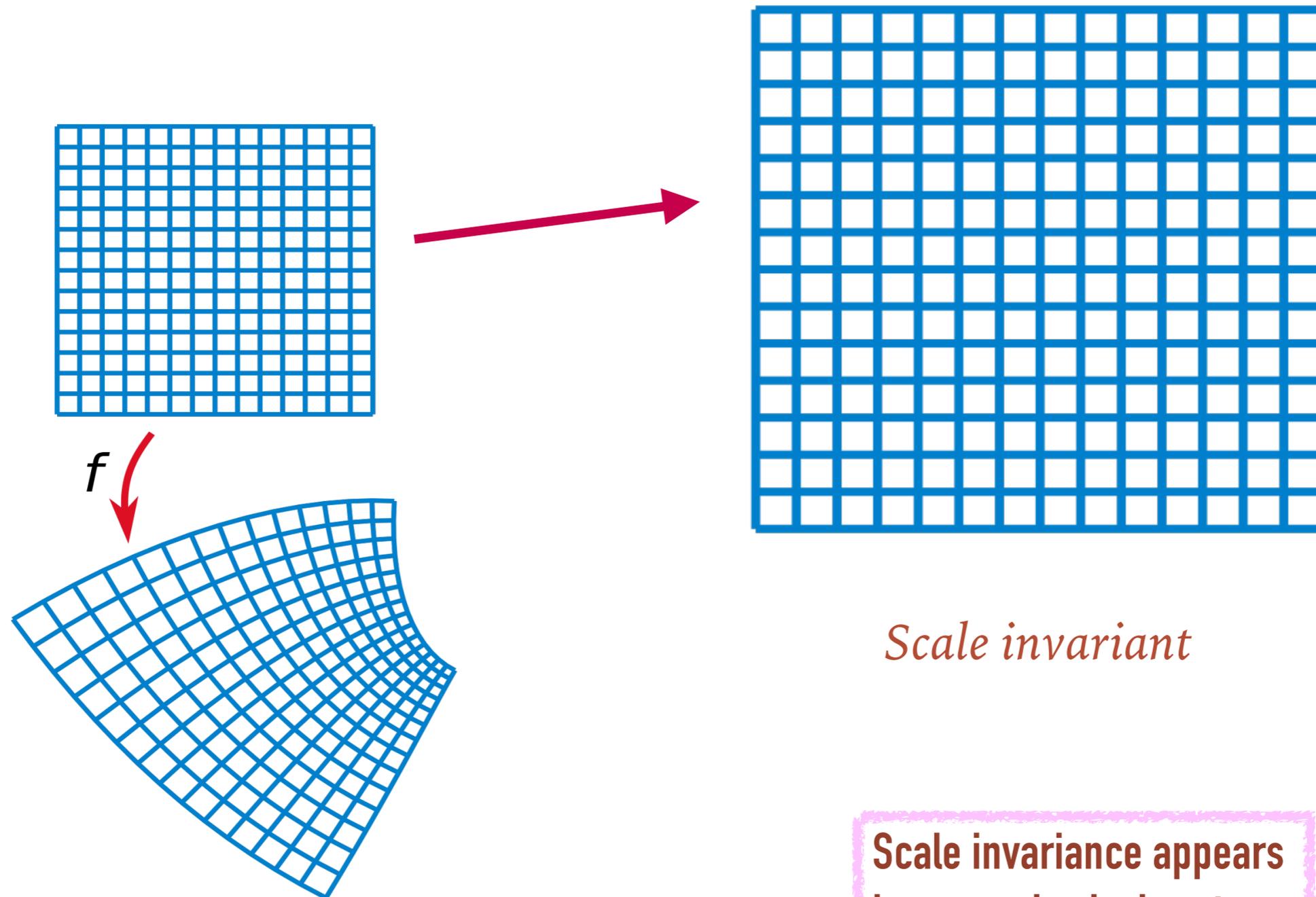
HOW DO WE MOVE UP (OR DOWN) IN ENERGY?

- We know how a QFT behaves at different scales through the renormalization group RG
- The theory has the same structure at different energy scales, but the parameters — couplings and masses — change with energy
- Related to scale invariance and conformal invariance

$$\beta(g) = \mu \frac{\partial g}{\partial \mu} \qquad \gamma(\phi) = \mu \frac{\partial \ln Z}{\partial \mu}$$

**Set of differential equations that describe the behaviour of the theory at different scales —
Renormalization Group Equations RGE**

SCALE INVARIANCE AND CONFORMAL INVARIANCE



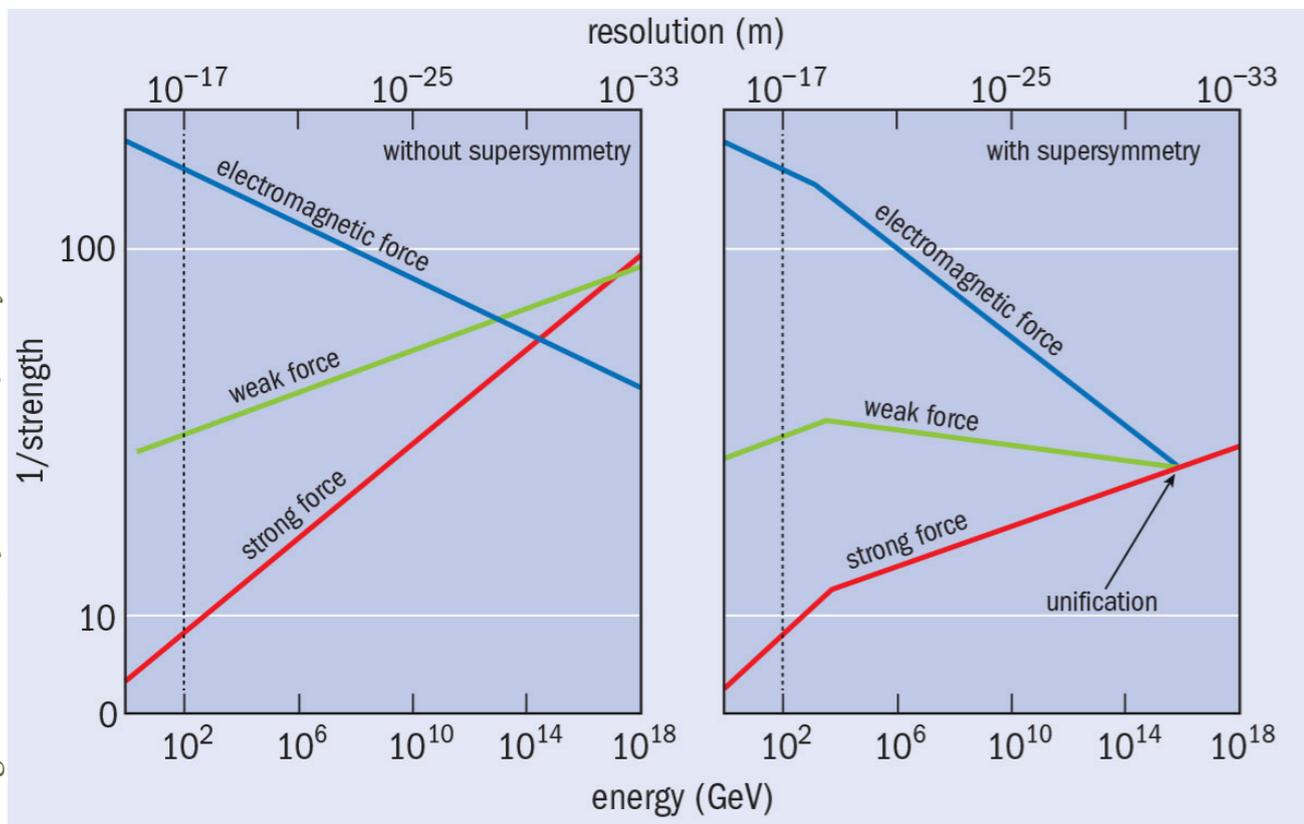
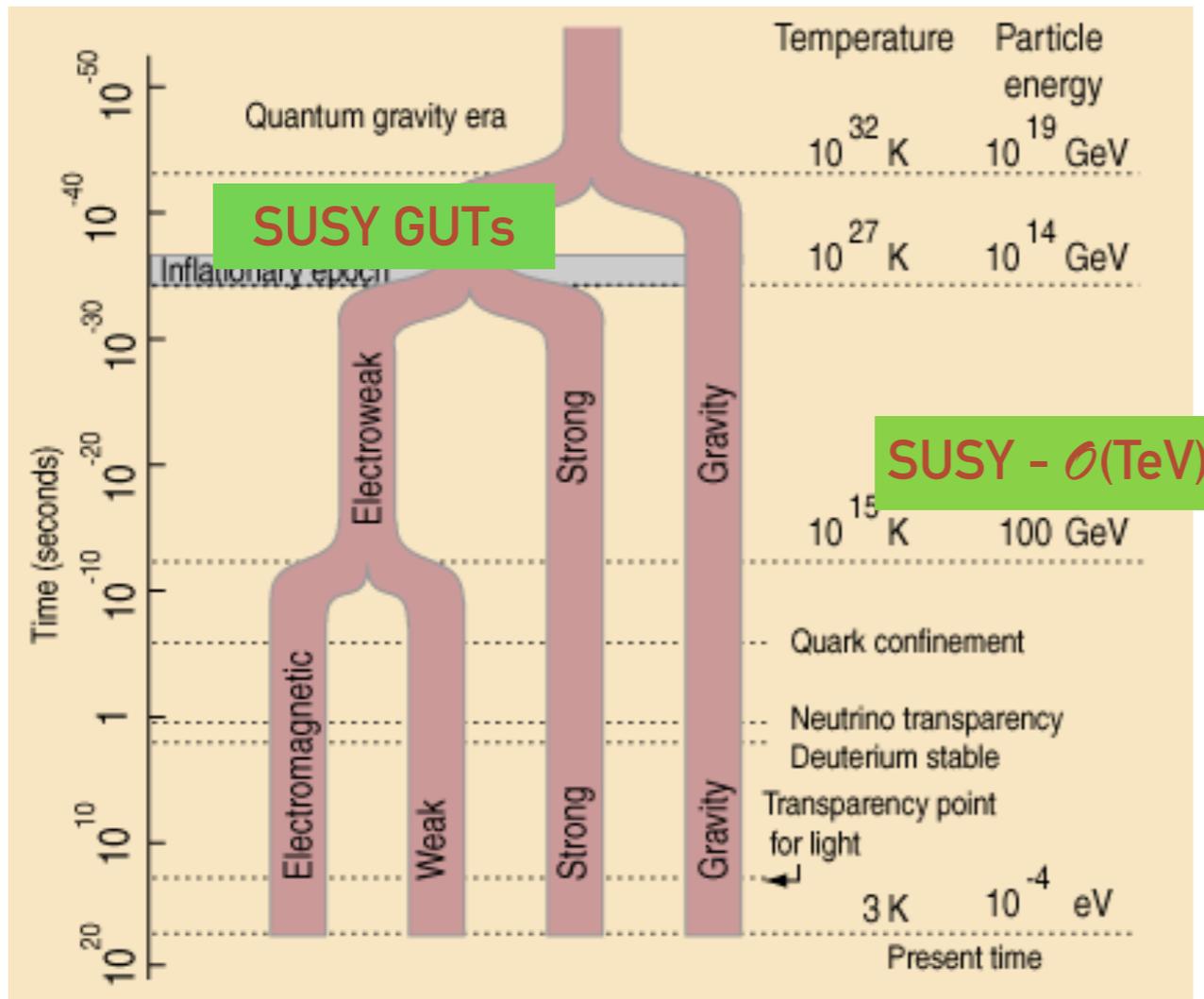
Scale invariant

Scale invariance appears
in many physical systems

Conformally invariant

SUSY GUTS: MORE SYMMETRY

- GUTs: at high energies only one fundamental interaction (excluding gravity)
- SUSY: relates bosons (interactions) and fermions (matter)
- At low energies, most studied: MSSM Extended Higgs sector: → H_u & H_d , 5 physical Higgs bosons, $\tan\beta = v_u/v_d$
- **Broken SUSY...**



MORE SYMMETRY NOT ALWAYS HELPFUL...

- Larger symmetry groups → more (unobserved) particles
- Different ways to break the symmetry → more parameters
- Examples:

GUTs — leptoquarks, heavy bosons

SUSY — spartners

Superstrings — all of the above plus extra dimensions that have to be compactified

- Super interesting, but need extra guidance



RENORMALIZATION GROUP INVARIANTS RGI

- Search for more fundamental theory → less parameters

Renormalization Group Invariants (RGI)

$$\Phi(g_1, \dots, g_N) = 0$$

$$\mu d\Phi/d\mu = \sum_{i=1}^N \beta_i \partial\Phi/\partial g_i = 0$$

- Equivalent to solve reduction equations

$$\beta_g (dg_i/dg) = \beta_i$$

$$i = 1, \dots, N$$

- **Reduced theory has only one coupling and its beta function**
- **Reduction → power series solution**
- **Uniqueness of solution can be studied at one-loop**
Zimmermann (1985); Zimmermann, Oehme, Sibold (1984–1985)

REDUCTION OF COUPLINGS

- May indicate an underlying symmetry
- There are solutions to the RE without apparent symmetry
- SUSY solutions appear often as solutions to the RE
- In SM, reducing the Higgs and top couplings in terms of the strong coupling constant leads to

$$\alpha_t/\alpha_s = \frac{2}{9} ; \quad \alpha_\lambda/\alpha_s = \frac{\sqrt{689} - 25}{18} \simeq 0.0694$$

$$M_t = 98.6 \pm 9.2 \text{ GeV}, \quad M_h = 64.5 \pm 1.5 \text{ GeV}$$

Outside experimental range, but still remarkable...

Kubo, Sibold, Zimmermann (1984–1987)

REDUCTION OF COUPLINGS WITHOUT SUSY

- It's also possible to do reduction of couplings without SUSY
- Analysis done in the past for the 2HDM Denner
- New systematic analysis for 2HDM done now, facilitates extension to different models
M.A. May Pech M.Sc. Thesis (2022)
- Miguel Angel May talk on Friday for details...

FINITENESS = SCALE INVARIANCE

- ▶ Finiteness $\Rightarrow \beta = 0$
to all orders in perturbation theory
- ▶ **Scale or conformal invariance**
- ▶ Couplings do not depend on energy scale
- ▶ Based on RGI and reduction of couplings
- ▶ Reduces greatly the number of free parameters
→ **new symmetries**
- ▶ Partial reduction → predictions for 3rd generation masses



FINITNESS \Rightarrow GAUGE YUKAWA UNIFICATION

Grand Unified SUSY N=1, no gauge anomalies:

$$W = \frac{1}{2} m^{ij} \Phi_i \Phi_j + \frac{1}{6} C^{ijk} \Phi_i \Phi_j \Phi_k$$

$$\beta_g^{(1)} = 0 = \gamma_i^{j(1)}$$

$$\sum_i T(R_i) = 3C_2(G), \quad \frac{1}{2} C_{ipq} C^{jpr} = 2\delta_i^j g^2 C_2(R_i)$$

T Dynkin index of irrep, C_2 Casimir invariant of group

C_{ijk} Yukawa couplings, g gauge coupling

- Restricts the gauge group
- Relates gauge and Yukawa couplings
- Can be made finite to all orders

Conformal invariance

- Just analyze one-loop solution
- Isolated and non-degenerate solution
Lucchesi, Piguet, Sibold
- Implies extra symmetries, in this case discrete

$\beta = 0$ non-renormalization of coupling constants, not complete UV finiteness where field renormalization is absent

MANY ASPECTS OF FINITENESS STUDIED

- SU(5) models extensively studied Rabi et al; Kazakov et al; Quirós et al;
MM, Zoupanos et al
- One coincides with a non-standard Calabi-Yau MM, Zoupanos
- Finite string theories and criteria for branes Ibáñez
- Models with three generations Babu, Enkhbat, Gogoladze; MM & Jiménez
- $SU(N)^k$ models finite \iff 3 generations
SU(3)³ finite MM, Ma, Zoupanos
- Relations non-commutative theories and finiteness Jack, Jones
- Proof of conformal invariance (dimensionless part) Kazakov, Bork; MM & Reyes
- Relation between finiteness and QFT in curved space-time & inflation
Elizalde et al
- Recent reviews Heinemeyer, M.M, Tracas, Zoupanos, Phys.Rept. 814 (2019); Fortsch.Phys. 68 (2020)

BOUNDARY CONDITIONS AT GUT SCALE

Model A

- ▶ $g_t^2 = \frac{8}{5} g^2$
- ▶ $g_{b,\tau}^2 = \frac{6}{5} g^2$
- ▶ $m_{H_u}^2 + 2m_{10}^2 = M^2$
- ▶ $m_{H_d}^2 + m_{\frac{5}{5}}^2 + m_{10}^2 = M^2$

- ▶ **3 free parameters:**
 $M, m_{\frac{5}{5}}^2$ and m_{10}^2

Model B

- ▶ $g_t^2 = \frac{4}{5} g^2$
- ▶ $g_{b,\tau}^2 = \frac{3}{5} g^2$
- ▶ $m_{H_u}^2 + 2m_{10}^2 = M^2$
- ▶ $m_{H_d}^2 - 2m_{10}^2 = -\frac{M^2}{3}$
- ▶ $m_{\frac{5}{5}}^2 + 3m_{10}^2 = \frac{4M^2}{3}$

- ▶ **2 free parameters:**
 $M, m_{\frac{5}{5}}^2$

*Each model implies different discrete symmetries
→ isolated and non-degenerate solution of the RE*

STANDARD MODEL

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + i\bar{\psi} \not{D} \psi + \text{h.c.} \\ & + \psi^\dagger y_{ij} \not{L}_j \phi + \text{h.c.} \\ & + |D_\mu \phi|^2 - V(\phi) \end{aligned}$$

LAGRANGIAN

Yukawa couplings
free parameters

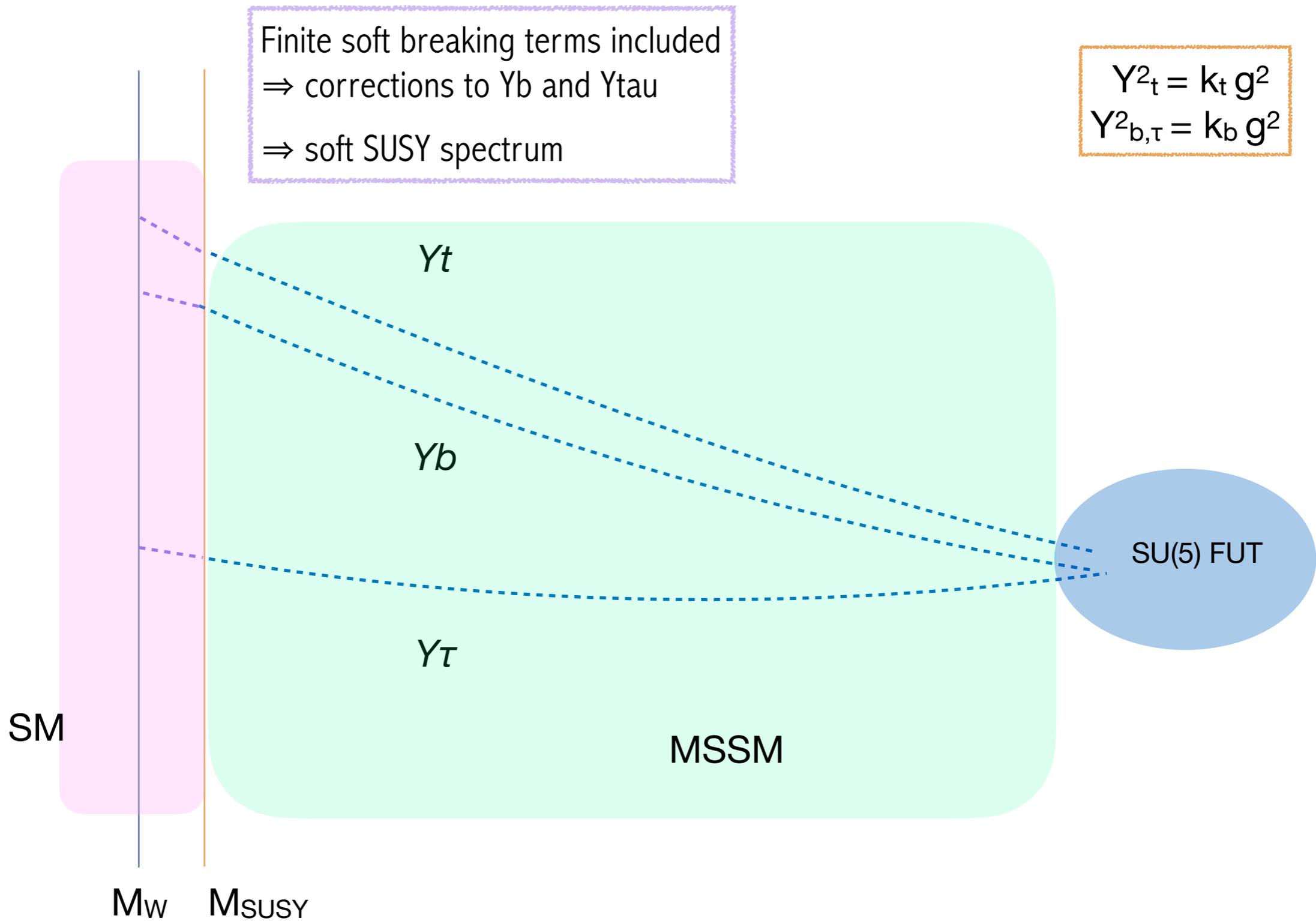
$$M_H^2 = \lambda v^2$$

$$m_f = g_f v / \sqrt{2}$$

Finite soft breaking terms included
 \Rightarrow corrections to Y_b and Y_τ
 \Rightarrow soft SUSY spectrum

$$Y_t^2 = k_t g^2$$

$$Y_{b,\tau}^2 = k_b g^2$$



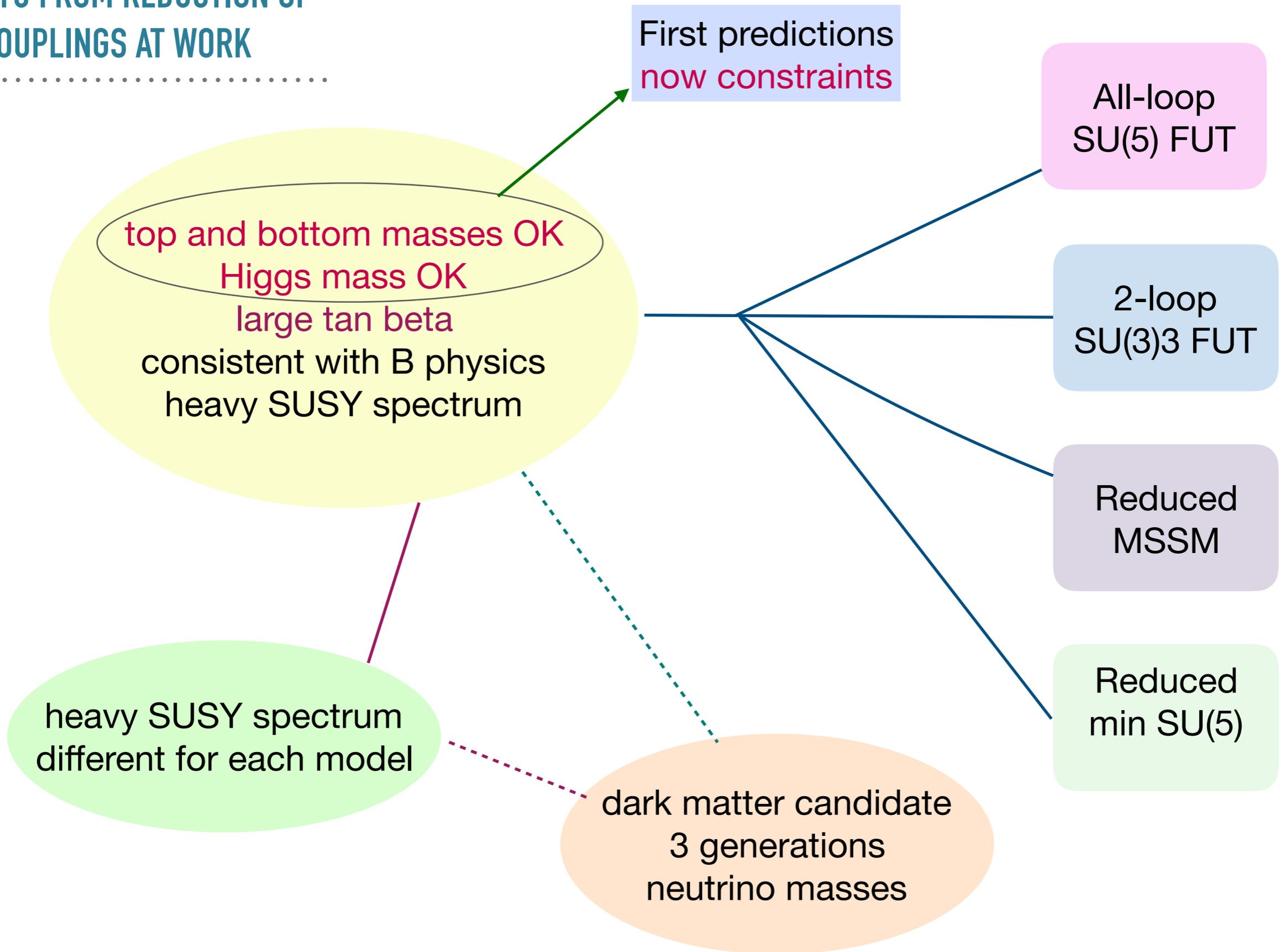
Results confronted to experimental constraints \Rightarrow
 gives available parameter space

$$m_t = Y_t v_u \quad v_u / v_d = \tan \beta$$

$$m_{b,\tau} = Y_{b,\tau} v_d \quad v_d = m_\tau^{\text{exp}} / Y_\tau$$

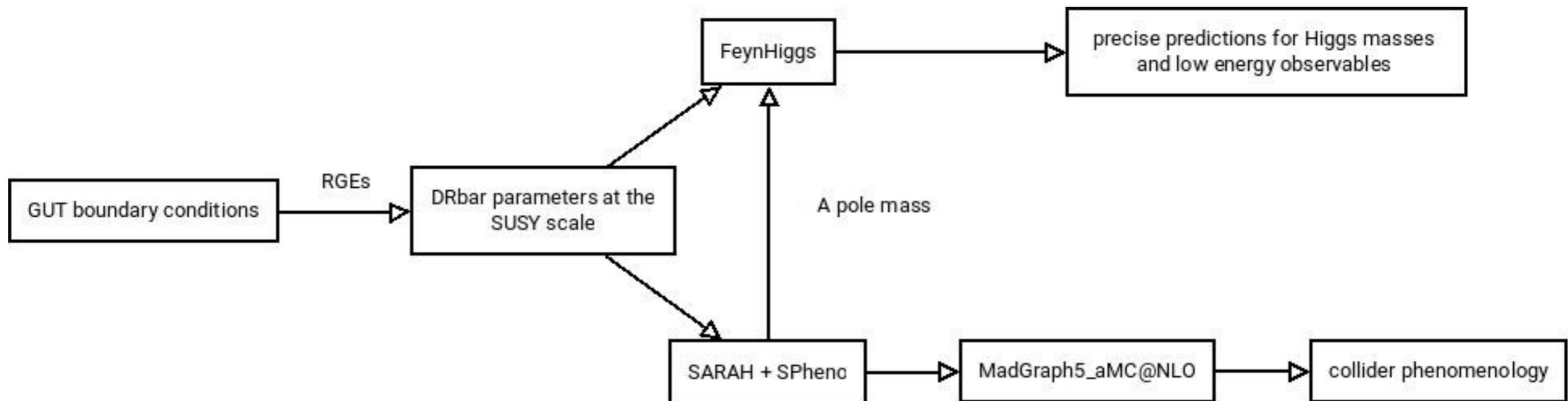
GYU FROM REDUCTION OF COUPLINGS AT WORK

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PREDICTIONS – NEW 2018 AND 2022 ANALYSES

- ▶ Results consistent with B physics constraints (not trivial)
- ▶ Predictions for top and bottom quark masses in experimental range
 - M_{top} - 171-174 GeV exp 173.3 ± 0.76 GeV
 - M_{bot} - 2.6-2.9 (M_Z) GeV exp 2.8 ± 0.1 GeV
- ▶ Large $\tan\beta$ 48-52
- ▶ Higgs mass 122-129 GeV exp 125.1 ± 0.3
- ▶ Heavy SUSY spectrum in TeV region — Collider phenomenology, challenging even for FCC
- ▶ Not all models survive, non minimal FUT SU(5) and SU(3) 3 FUT do



RECENT DEVELOPMENTS

- Updated phenomenological analysis still consistent with 3rd generation masses, large $\tan\beta$ and **very heavy SUSY spectrum**
S. Heinemeyer, J. Malinowski, W. Kotlarski, M. Mondragón, N. Tracas, G. Zoupanos 2018 and 2022
- Finiteness implies conformal invariance and phase transition
L.E. Reyes Rodríguez, Lic. Thesis (2018)
- Three generation analysis SU(5):
Diagonal quark mass matrix compatible with data and proton decay
Luis Odín Estrada, M.Sc. Thesis (2018)
- Three generation solution for SU(5) with Z symmetries compatible with good textures at high energies
Luis Odín Estrada, Ph.D. Thesis
- **Finiteness in** Soft breaking terms lead to anomaly mediated type breaking
L.E. Reyes Rodríguez, M.Sc. Thesis (2021)
- SU(3) [^] 3 finite split susy model in progress
L.E. Reyes Rodríguez, Ph. D. Thesis

OUTLOOK AND TO PACK...

- Inclusion of neutrino masses through \mathcal{R}
- Will not change much the collider phenomenology presented
- Will impact the DM candidate \Rightarrow **gravitino + ?**
- Detailed analysis of three gen solutions with discrete symmetries
- Detailed analysis of phase transition and SUSY breaking
- Lately: FUTs in curved space time \Rightarrow **successful inflation**
E. Elizalde, S. Odintsov, E. Pozdeeva, S. Vernov (2015)
- New type of finite theories imply duality between UV and IR fixed points. Connection to FUTs?
Y. Kawamura (2015)

➤

**AND AT LOWER
ENERGIES . . . ?**

HOW DO WE CHOOSE A FLAVOUR SYMMETRY?

- Several ways:
- Look for inspiration in a high energy extension of SM, i.e. strings or GUTs
- Look at low energy phenomenology
- At some point they should intersect...

- In here:
 - Find the smallest flavour symmetry suggested by data
 - Explore how generally it can be applied (universally)
 - Follow it to the end
 - Compare it with the data

SOME ASPECTS OF THE FLAVOUR PROBLEM

- ▶ Quark and charged lepton masses very different, very hierarchical

$$m_u : m_c : m_t \sim 10^{-6} : 10^{-3} : 1$$

$$m_d : m_s : m_b \sim 10^{-4} : 10^{-2} : 1$$

$$m_e : m_\mu : m_\tau \sim 10^{-5} : 10^{-2} : 1$$

- ▶ Neutrino masses unknown, only difference of squared masses.
- ▶ Type of hierarchy (normal or inverted) also unknown
- ▶ Higgs sector under study

- ▶ Quark mixing angles

$$\theta_{12} \approx 13.0^\circ$$

$$\theta_{23} \approx 2.4^\circ$$

$$\theta_{13} \approx 0.2^\circ$$

- ▶ Neutrino mixing angles

$$\Theta_{12} \approx 33.8^\circ$$

$$\Theta_{23} \approx 48.6^\circ$$

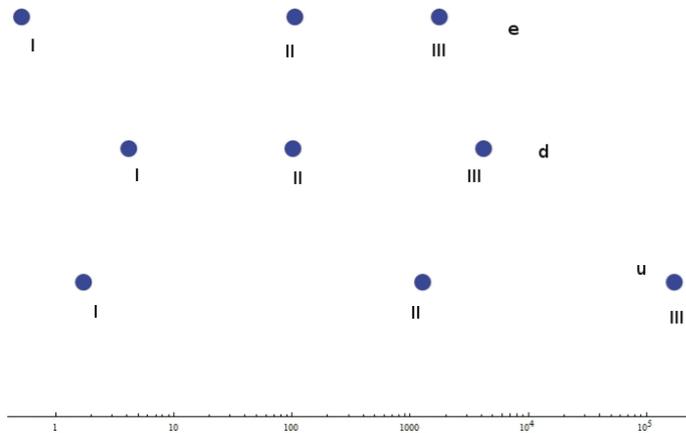
$$\Theta_{13} \approx 8.6^\circ$$

- ▶ Small mixing in quarks, large mixing in neutrinos.

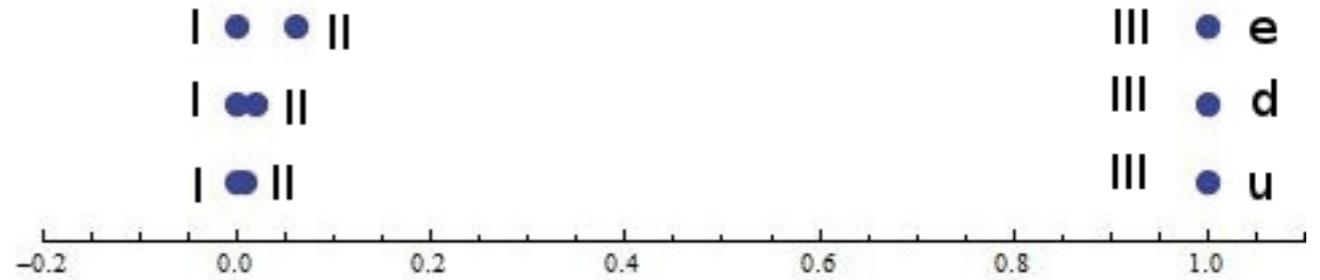
Very different

- ▶ Is there an underlying symmetry?





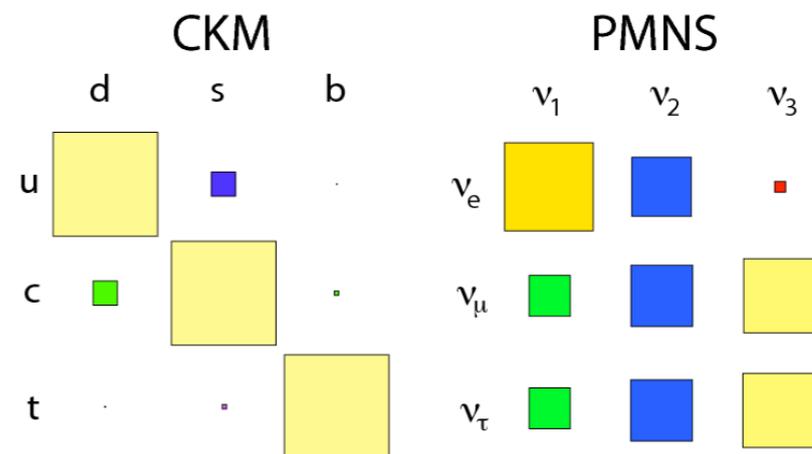
Plot of mass ratios



Logarithmic plot of quark masses

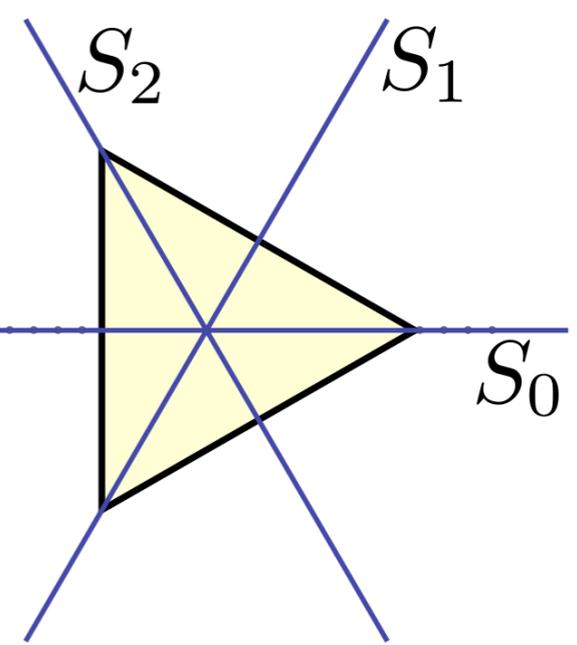
$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} \approx \begin{bmatrix} 0.974 & 0.225 & 0.003 \\ 0.225 & 0.973 & 0.041 \\ 0.009 & 0.040 & 0.999 \end{bmatrix},$$

Suggests a $2 \oplus 1$ structure



S3-3HDM

- Smallest non-Abelian discrete group
- Has irreducible representations, 2, 1_S and 1_A
- We add three right-handed neutrinos to implement the see-saw mechanism
- We apply the symmetry “universally” to quarks, leptons and Higgs-es
 - First two families in the doublet
 - Third family in symmetric singlet
- Three sectors related, we treat them simultaneously



PREDICTIONS, ADVANTAGES?

- Possible to reparametrize mixing matrices in terms of mass ratios, successfully
- CKM has NNI and Fritzsch textures
- PMNS → fix one mixing angle, predictions for the other two within experimental range
- Reactor mixing angle
 $\theta_{13} \neq 0$
- Some FCNCs suppressed by symmetry
- Higgs potential has 8 couplings
- Underlying symmetry in quark, leptons and Higgs
→ residual symmetry of a more fundamental one?
- Lots of Higgses:
3 neutral, 4 charged,
2 pseudoscalars
- Further predictions will come from Higgs sector:
decays, branching ratios

A. Mondragón, M. M., F. González, E. Peinado, U. Saldaña, O. Félix, E. Rodríguez, A. Pérez, H. Reyes, C. Espinoza, E. Garcés,...; Das, Dey et al; Teshima et al; E. Barradas, O. Félix, E. Rodríguez; M. Rebelo, P. Osland et al; many many more

Quarks

3HDM: $G_{SM} \otimes S_3$

	ψ_L^f	ψ_R^f	Mass matrix	Possible mass textures	
A	$\mathbf{2}, 1_S$	$\mathbf{2}, 1_S$	$\begin{pmatrix} \mu_1^f + \mu_2^f & \mu_4^f & \mu_6^f \\ \mu_4^f & \mu_1^f - \mu_2^f & \mu_7^f \\ \mu_8^f & \mu_9^f & \mu_3^f \end{pmatrix}$	$\begin{pmatrix} 0 & \mu_2^f sc(3-t^2) & 0 \\ \mu_2^f sc(3-t^2) & -2\mu_2^f c^2(1-3t^2) & \mu_7^f/c \\ 0 & \mu_7^{f*}/c & \mu_3^f - \mu_1^f - \mu_2^f c^2(1-3t^2) \end{pmatrix}$	
A'				$\begin{pmatrix} 0 & \frac{2}{\sqrt{3}}\mu_2^f & 0 \\ \frac{2}{\sqrt{3}}\mu_2^f & 0 & \frac{2}{\sqrt{3}}\mu_7^f \\ 0 & \frac{2}{\sqrt{3}}\mu_9^f & \mu_3^f - \mu_1^f \end{pmatrix}$	NNI
B	$\mathbf{2}, 1_A$	$\mathbf{2}, 1_A$	$\begin{pmatrix} \mu_1^f + \mu_2^f & \mu_4^f & \mu_7^f \\ \mu_4^f & \mu_1^f - \mu_2^f & -\mu_6^f \\ -\mu_9^f & \mu_8^f & \mu_3^f \end{pmatrix}$	$\begin{pmatrix} 0 & -\mu_4^f c^2(1-3t^2) & 0 \\ -\mu_4^f c^2(1-3t^2) & 2\mu_4^f sc(3-t^2) & -\mu_6^f/c \\ 0 & -\mu_6^{f*}/c & \mu_3^f - \mu_1^f + \mu_4^f sc(3-t^2) \end{pmatrix}$	
B'				$\begin{pmatrix} 0 & -2\mu_4^f & 0 \\ -2\mu_4^f & 0 & -2\mu_6^f \\ 0 & 2\mu_8^f & \mu_3^f - \mu_1^f \end{pmatrix}$	NNI

Table 2: Mass matrices in S_3 family models with three Higgs $SU(2)_L$ doublets: H_1 and H_2 , which occupy the S_3 irreducible representation $\mathbf{2}$, and H_S , which transforms as 1_S for the cases when both the left- and right-handed fermion fields are in the same assignment. The mass matrices shown here follow a normal ordering of their mass eigenvalues (m_1^f, m_2^f, m_3^f) . We have denoted $s = \sin \theta$, $c = \cos \theta$ and $t = \tan \theta$. The third column of this table corresponds to the general case, while the fourth column to a case where we have rotated the matrix to a basis where the elements $(1, 1)$, $(1, 3)$ and $(3, 1)$ vanish. The primed cases, A' or B' , are particular cases of the unprimed ones, A or B , with $\theta = \pi/6$ or $\theta = \pi/3$, respectively.

Mass matrices reproduce the NNI or the Fritzsch forms

NEW RESULTS S3-3H

- Adriana's talk: full scalar potential analysis
- 2 scenarios:
 - A: SM Higgs lightest one
 - B: Neutral scalar lighter than SM ~ 100 GeV possible neutral scalar signal?
- Both compatible with SM limit for trilinear and quartic couplings
- Small deviations from SM in trilinear and quartic couplings compatible with recent phenomenological analyses in the modifier or κ framework

$$g_{H_2H_2H_2} \equiv \lambda_{SM} \kappa_\lambda = \frac{m_{H_2}^2}{2v} \left[(1 + 2\delta^2) \sqrt{1 - \delta^2} + \delta^3 (\tan \theta - \cot \theta) - \frac{m_{h_0}^2}{m_{H_2}^2} \frac{\delta^3}{9s_\theta c_\theta^3} \right]$$

S3-3H FUTURE ANALYSIS

- Mass spectrum in reach of future runs of the LHC
 - Inclusion of one-loop corrections necessary
 - Calculation of decays and branching ratios of scalars
- New neutral scalar not coupled to gauge bosons, DM?
- Residual symmetry \Rightarrow problematic in fermionic sector
Solutions:

- break S3
- modular S3
- high energy sector terms
- 4 Higgs doublets
- Higgs singlets

*Possible to return
to good previous results
for mixing matrices*

S3-4H

- “Saturate” the irreps: add an extra inert Higgs doublet in the 1_A
- Natural DM candidates from inert part
the non-inert part same as S3-3H in SM alignment limit
- Full analysis for DM, with Higgs bounds, relic density, indirect detection done

E. Garcés, C. Espinoza, M.M, H. Reyes, PLB (2018), M.Sc. Thesis H. Reyes

- Multi-component DM in progress

With scalar and neutrino M. Valenzuela, Ph.D. Thesis in progress, with E. Espinoza, A. Ramírez

With 2 scalars J. Pacheco, Ph.D. Thesis in progress, with E. Espinoza, E. Barradas, T. Valencia

- Neutrino portal in progress with E. Espinoza, E. Barradas, T. Valencia

- LFV analysis in progress J. Pacheco, Ph.D. Thesis in progress

- S3 with 3H and $U(1)_{B-L}$ multi component DM in progress

L. E. Gutiérrez-Luna, Ph.D. Thesis in progress with J.C. Gómez-Izquierdo, C. Espinoza

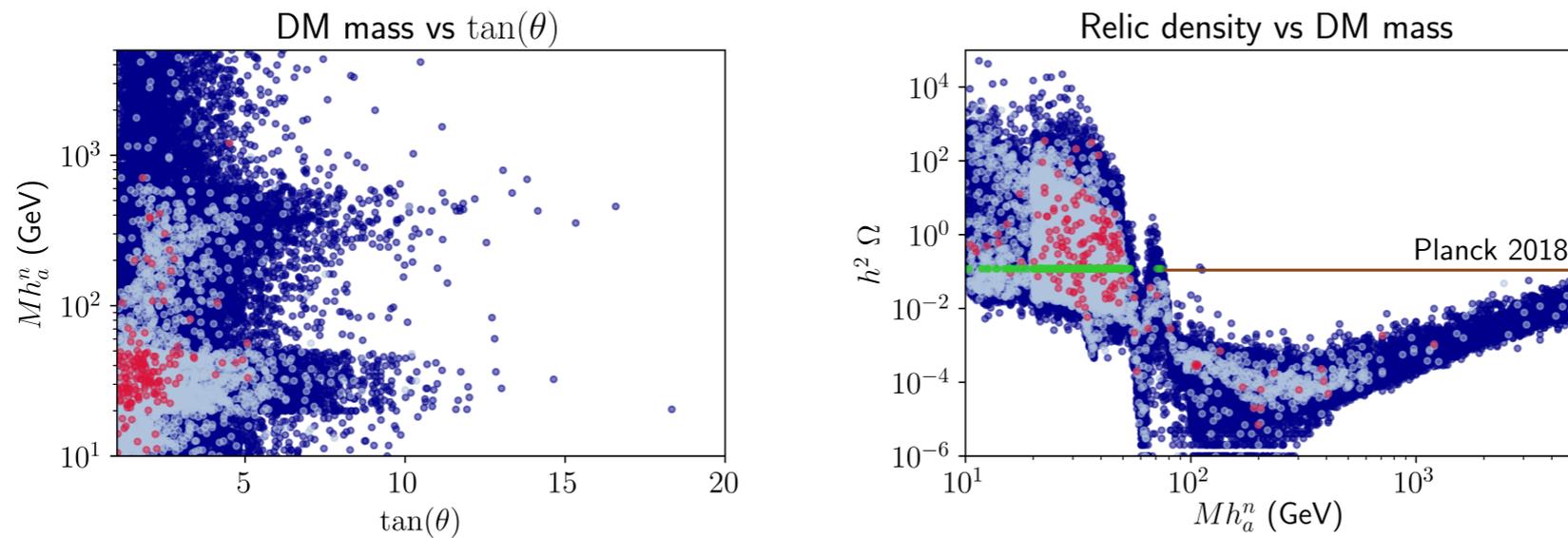


Figure 1: Mass of the DM candidate as a function of $\tan\theta$ (left panel), and value of the DM relic density as a function of the DM mass. The dark blue points (set A) are the ones that comply with stability and unitarity constraints, the light blue points (set B) are also compatible with the experimental bounds for extra scalar searches (see text), the red points also satisfy the decoupling limit and the green points in the right panel lie within the experimental Planck bound.

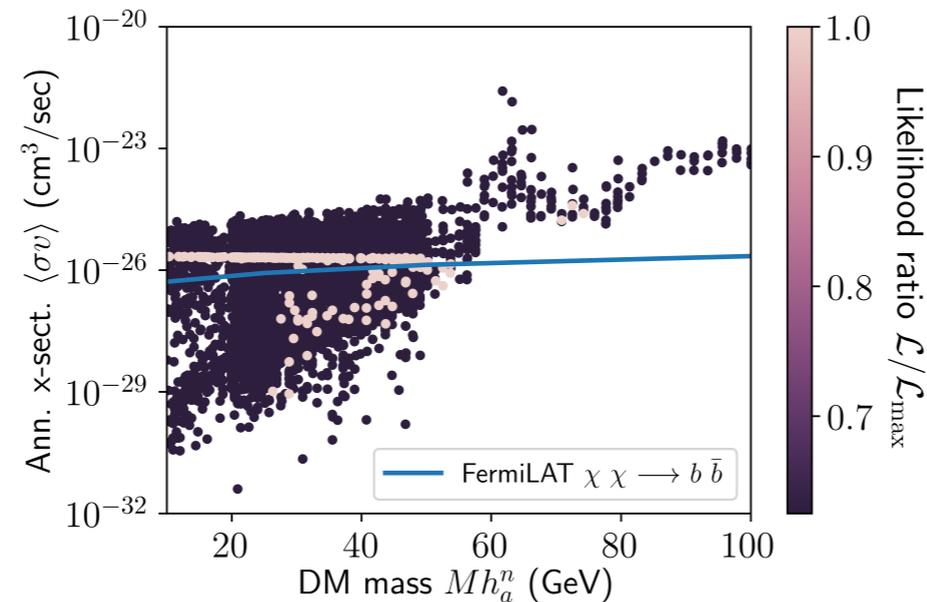


Figure 3: Annihilation cross section as a function of the DM mass for small DM masses, the points are colored according to their (normalized) likelihood (with respect to the relic density) value. Also shown is the FermiLAT dwarf spheroidal combined DM exclusion curve.

C. Espinoza, E. Garcés, M.M.,
H Reyes-González
Phys.Lett.B 788 (2019) 185-19
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WHAT CAN GUIDE US IN BETWEEN?

- $SU(5) \times Q6$ (non-FUT) \implies good CKM, similar predictions to $S3-3H$ for neutrino sector

J.C. Gómez-Izquierdo, F. González-Canales, MM (2015)

- Breaking of mu-tau symmetry through $Q6$ SUSY and $S3$ non-SUSY

J.C. Gómez-Izquierdo, F. González-Canales, M.M. (2017);(2018)

- $g-2$ solution through LFV in extended MSSM, with discrete symmetry inspired terms

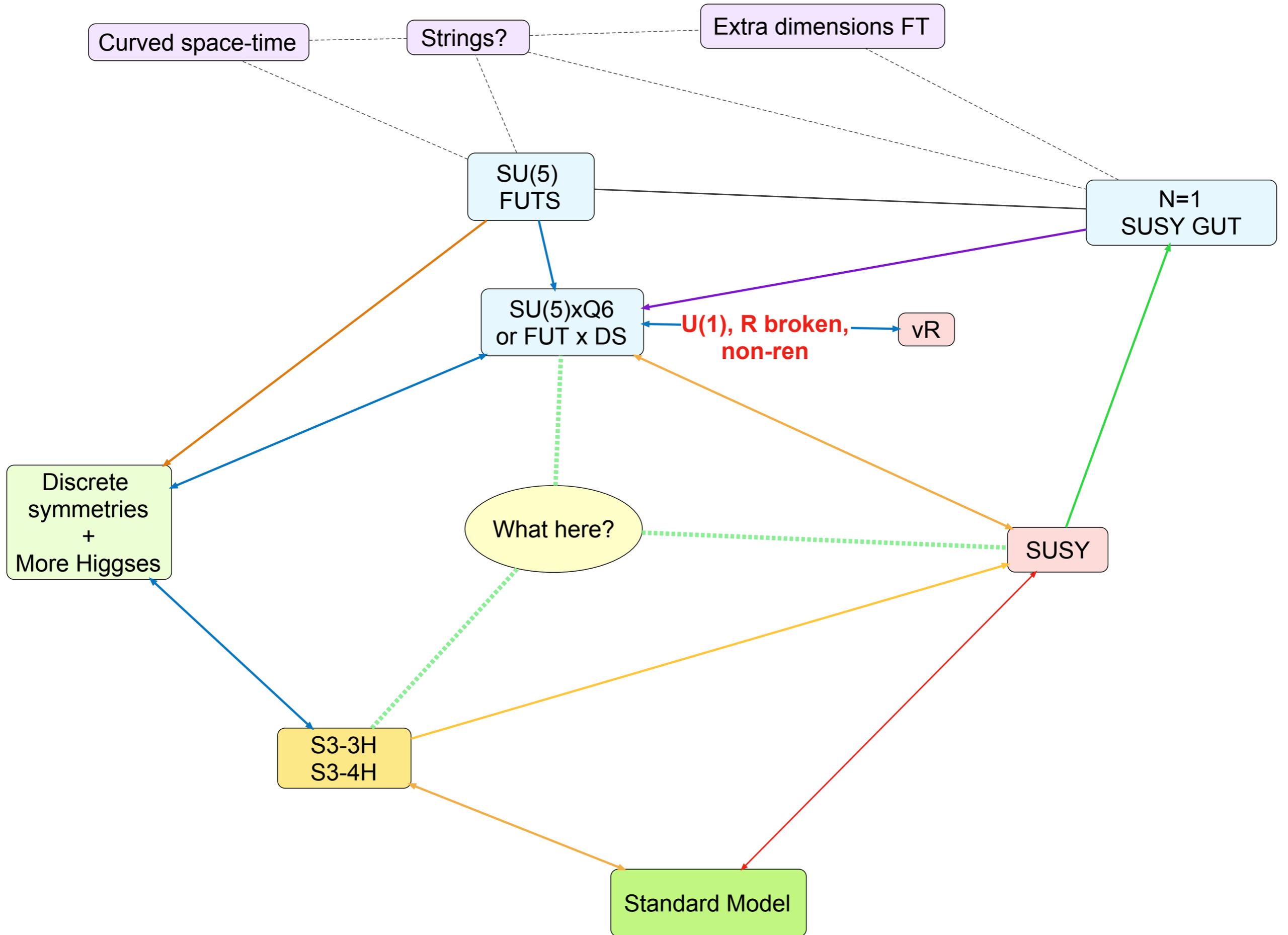
M. Gómez-Bock, F. Flores-Báez, MM (2016)

- And of course experimental data and observations...

MORE MODELS?

- 2+1 successful in quark sector
- Neutrino sector also, but more flexibility
- Q4 2HDM and singlets
see Catalina Espinoza's talk on Thursday:
quarks, leptons, DM, leptogenesis, g-2
Gatto-Sartori-Tonin relation between quark masses and mixing angles
[A. Cárcamo, E. Espinoza, J.C. Gómez-Izquierdo, M.M. *Eur.Phys.J.Plus* 137 \(2022\) 11, 1224](#)
- S3-3H with S3 as modular symmetry, nice results without residual Z2
in quark sector [M.C. Cerón, M.Sc. Thesis \(2021\)](#)
- SUSY SU(5) non-minimal with modular S3 [A.C. Samaniego, M.Sc. Thesis](#)
- S4 3HDM and 4HDM, plus singlets, with very predictive neutrino
sector [A. Cárcamo, C. Espinoza, J.C. Gomez, J. Marchant, M.M.](#)

Explore other models



OUTLOOK AND CONCLUSIONS

- Among the different ways to go BSM **finiteness** proves to be a good guiding principle.
Reduces greatly the number of free parameters, RG flow of the third family in the right direction
- Needs extended Higgs sector and discrete flavour symmetries
- At low energies S_3 , S_4 , Q_4 , Q_6 theories with extended Higgs sector explain well CKM and have predictions for neutrino sector.
Provide baryogenesis through leptogenesis and good DM candidates
- **Maybe is possible to connect both approaches**



For our journeys
Beyond the Standard Model
there is a lot to pack...

RGIs: Reduction of couplings, finiteness

Symmetries: SUSY, GUTs, discrete symmetries

Allow us to put some order...

They all require extended Higgs sectors for success

They all have consequences for the particle masses



¡GRACIAS!