Lepton Flavor violation within Simplest Little Higgs Model

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This work is based on *arXiv* : 2205.10420,

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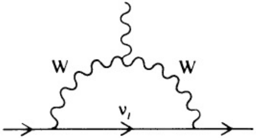
Motivation

After the discovery of neutrino oscillation, flavor violation only remains unmeasured in the charged lepton sector. In the minimal extension of the SM[a]:



$$\mathcal{BR}\,(\mu \to e\gamma) \lesssim 10^{-54} \qquad (1)$$

[a]Phys. Lett. B 67 (1977), Phys. Rev. D 16, (1977)

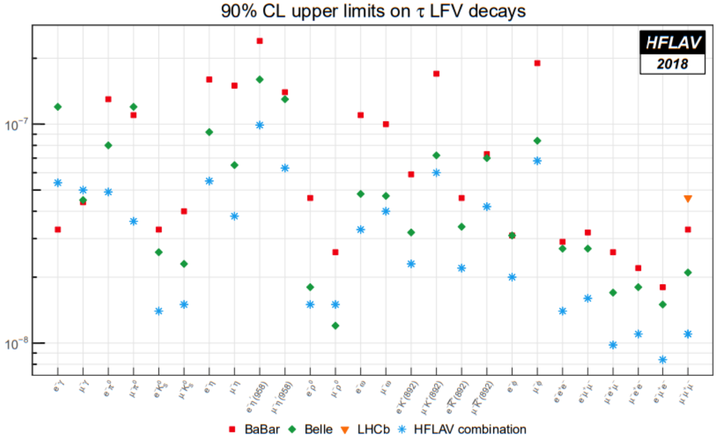
- MEG's present upper bound[a]: $\mathcal{BR}\,(\mu \to e\gamma) \leq 4.2 \times 10^{-13}$.

Many collaborations are interested in this type of process:

- MEG,
- MEGA,
- SINDRUM (II),
- Mu2e,
- Mu3e,
- Bell II.

[a]Phys. Rev. Lett. 110, (2013)

- τ leptons are expected to be coupled strongly with new physics and have many possible LFV decay modes due to their large mass[1].



[1] Eur.Phys.J.C 81 (2021)

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LH models[2] are an attempt to solve the hierarchy problem. This is done making the Higgs a pseudo-Goldstone boson of a new approximate global symmetry broken at a scale f $\mathcal{O}(TeV)$.

Two types of LH models:

- Product group models $[SU(2) \times U(1)]^N$
- Simple Groups Models $SU(N) \times U(1)$

Features of LH models:

- Loop-level generated Higgs mass.
- "Little" particles with masses of $\mathcal{O}(f)$, that cancel the main one loop corrections to the Higgs mass in the SM
- UV completion of the mode is expected at $\Lambda \sim 4\pi f$ TeV.

[2] Phys. Rev. Lett. 86 (2001),Phys. Lett. B513 (2001),Ann. Rev. Nucl. Part. Sci. 55(2005)

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Simplest Little Higgs Model

The first model of the "simple group" was constructed by Kaplan and Schmaltz[3]:

Global symmetry

- $[SU(3) \times U(1)]^2$.

Gauge symmetry

- $[SU(3)_L \times U(1)_X]$.

Spontaneous symmetry breaking

- $[SU(3) \times U(1)]^2 \to [SU(2) \times U(1)]^2$.

Explicit symmetry breaking

- $[SU(3)_L \times U(1)_X] \to [SU(2)_L \times U(1)_x]$.

This is achieved by two vacuum condensates:

$$\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ 0 \\ fc_\beta \end{pmatrix}, \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ 0 \\ fs_\beta \end{pmatrix} \qquad (2)$$

[3] JHEP 0310, 039 (2003)

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The dynamics of the fields at low energies can be parameterized as:

$$\phi_1 = \exp\left(\frac{i\Theta'}{f}\right)\exp\left(\frac{it_\beta\Theta}{f}\right)\begin{pmatrix}0\\0\\fc_\beta\end{pmatrix}_{(\mathbf{3},\mathbf{1})},$$

$$\phi_2 = \exp\left(\frac{i\Theta'}{f}\right)\exp\left(-\frac{i\Theta}{ft_\beta}\right)\begin{pmatrix}0\\0\\fs_\beta\end{pmatrix}_{(1,\mathbf{3})} \quad (3)$$

whit the misalignment angle:

$$c_\beta = \cos\beta,\quad s_\beta = \sin\beta,\quad t_\beta = \tan\beta. \quad (4)$$

This parametrization has the form of an $SU(3)$ (broken) transformation

$$\Theta = \frac{\eta}{\sqrt{2}}\mathbf{1}_{\mathbf{3}\times\mathbf{3}} + \begin{pmatrix}\mathbf{0}_{\mathbf{2}\times\mathbf{2}} & h\\ h^\dagger & 0\end{pmatrix},$$
$$\Theta' = \frac{\xi}{\sqrt{2}}\mathbf{1}_{\mathbf{3}\times\mathbf{3}} + \begin{pmatrix}\mathbf{0}_{\mathbf{2}\times\mathbf{2}} & k\\ k^\dagger & 0\end{pmatrix}, \quad (5)$$

where

$$h = \begin{pmatrix}h^0\\h^-\end{pmatrix},\quad h^0 = \frac{1}{\sqrt{2}}(v + H - i\chi)$$
$$k = \begin{pmatrix}y^0\\x^-\end{pmatrix},\quad h^\pm = -\phi^\pm. \quad (6)$$

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The $SU(3)_L \times U(1)_X$ is made a local symmetry by the introduction of the gauge-covariant derivative:

$$D_\mu = \partial_\mu - igA^a_\mu T_a + ig_x Q_x B^x_\mu, \qquad (7)$$

$$g_x = \frac{gt_w}{\sqrt{1 - t_w^2/3}}.$$

The kinetic terms for the Φ_i field can be written as:

$$\mathcal{L}_\Phi = (D^\mu \Phi_i)^\dagger (D_\mu \Phi_i) \quad i = 1, 2. \qquad (8)$$

The mass of the charged gauge bosons are

$$\begin{aligned} M_W &= \frac{gv}{2}\left[1 - \frac{v^2}{12f^2}\left(\frac{c_\beta^4}{s_\beta^2} + \frac{s_\beta^4}{c_\beta^2}\right)\right], \\ M_X &= \frac{gf}{\sqrt{2}}\left[1 - \frac{v^2}{4f^2}\right]. \end{aligned} \qquad (9)$$

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The neutral gauge bosons masses are:[4] :

$$
\begin{aligned}
M_A &= 0, \\
M_Y &= \frac{gf}{\sqrt{2}}, \\
M_{Z'} &= \frac{\sqrt{2}fg}{\sqrt{3-t_w^2}}\left(1-\frac{\left(3-t_w^2\right)v^2}{16c_w^2 f^2}\right), \\
M_Z &= \frac{gv}{2c_w}\left(1-\frac{v^2}{16f^2}\left(1-t_w^2\right)^2-\frac{v^2}{12f^2}\left(\frac{s_\beta^4}{c_\beta^2}+\frac{c_\beta^4}{s_\beta^2}\right)\right),
\end{aligned}
\tag{10}
$$

[4] JHEP 1103 (2011), 080

The SM fermions that are doublets under $SU(2)$ must be enlarged to triplets under $SU(3)$.

- **Leptons**

Each lepton family consists of an $SU(3)$ left-handed triplet **3** and two right-handed singlets **1**

$$L_m^T = (\nu_L \quad \ell_L \quad iN_L)_m \,, \quad \ell_{Rm}, \quad N_{Rm},$$

- **Quarks**

Universal embedding

$$\mathcal{Q}_m^T = (u_L \quad d_L \quad iU_L)_m \,, \quad u_{Rm}, \quad d_{Rm}, \quad U_{Rm}.$$

Anomaly Free embedding[a]

$$\begin{aligned} \mathcal{Q}_1^T &= (d_L \quad -u_L \quad iD_L)\,, \quad d_R, \quad u_R, \quad D_R, \\ \mathcal{Q}_2^T &= (s_L \quad -c_L \quad iS_L)\,, \quad s_R, \quad c_R, \quad S_R, \\ \mathcal{Q}_3^T &= (t_L \quad b_L \quad iT_L)\,, \quad t_R, \quad b_R, \quad T_R, \end{aligned}$$

[a] J. Korean Phys. Soc. 45 (2004)

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After EWSSB this lagrangian yields the lepton masses:

$$\mathcal{L}_Y \supset -fs_\beta \lambda_N^m \left[\left(1 - \frac{\delta_\nu^2}{2}\right) \bar{N}_{Rm} N_{Lm} - \delta_\nu \bar{N}_{Rm} \nu_{Lm} \right] + \xi_\beta \frac{fv}{\sqrt{2}\Lambda} \lambda_\ell^{mn} \bar{\ell}_{Rm} \ell_{Ln} + \mathrm{h.c.}, \quad (11)$$

where

$$\delta_\nu = -\frac{v}{\sqrt{2} f t_\beta},$$

$$\xi_\beta = \left[1 - \frac{v^2}{4f^2} - \frac{v^2}{12f^2} \left(\frac{s_\beta^4}{c_\beta^4} + \frac{c_\beta^4}{s_\beta^4} \right) \right]$$

the charged lepton mass eigenstates are related to the flavour eigenstates by the rotation:

$$\ell_{Lm} \longrightarrow (V_\ell \ell_L)_m = V_\ell^{mi} \ell_{Li}, \quad (12)$$

according to (11) each heavy neutrino is mixed just with the light neutrino of the same family.

$$\begin{pmatrix} \nu_L \\ N_L \end{pmatrix}_m = \left[\begin{pmatrix} 1 - \frac{\delta_\nu^2}{2} & -\delta_\nu \\ \delta_\nu & 1 - \frac{\delta_\nu^2}{2} \end{pmatrix} \begin{pmatrix} V_\ell \nu_L \\ N_\ell \end{pmatrix} \right]_m . \quad (13)$$

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General Structure of the LFV Processes

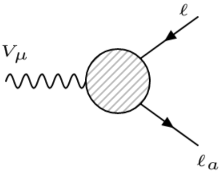

Figure 1: Effective LFV vertex, where $V_\mu = \gamma, Z, Z'$.

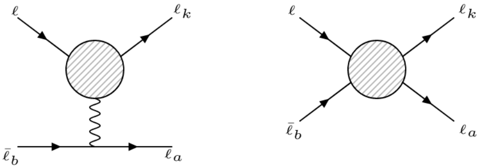

Figure 2: Generic penguin and box diagrams for $\ell \to \ell_k \ell_a \bar{\ell}_b$.

- We calculate our amplitudes as an expansion, until the second order of the parameter: v/f,
- $\ell \to \ell_a \gamma$
- $\mu - e$ nuclei conversion,
- *same-flavors* decays $(\ell \to 3\ell_a)$,
- *same-sign* decays, $(\ell \to \ell_k \ell_a \bar{\ell}_a)$,
- *wrong-sign* decays $(\ell \to \ell_k \ell_k \bar{\ell}_a)$,
- $\ell - \tau$ nuclei conversion.

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We can classify the contributions to $\ell \to \ell_a \gamma$ into two types of topologies:

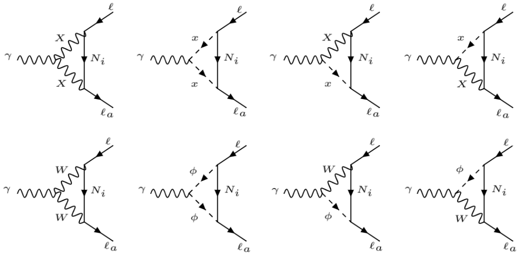

Figure 3: Feynman diagrams for $\ell \to \gamma \ell_a$

Defining the mass ratios:

$$x_i = \frac{M_{N_i}^2}{M_X^2} \simeq \mathcal{O}(1), \quad \omega = \frac{M_W^2}{M_X^2} \simeq \mathcal{O}\left(v^2/f^2\right), \tag{14}$$

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Feynman Diagrams: $\ell \to \ell_k \ell_a \bar{\ell}_b$

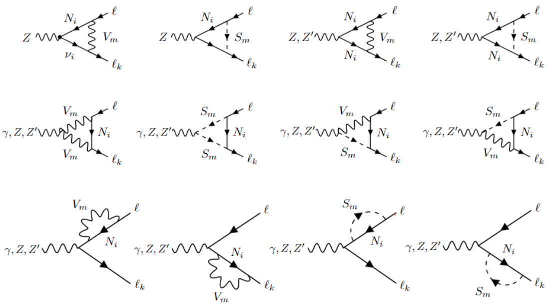

Figure 4: Diagrams for $\ell \to \ell_k \ell_a \bar{\ell}_b$ decays, where $V_m = X, W$ and $S_m = x, \phi$.

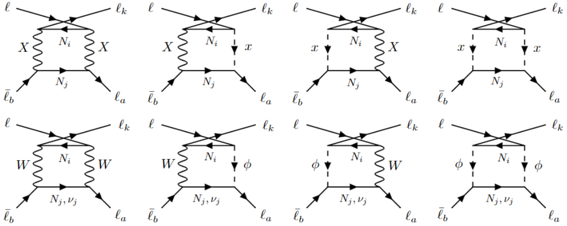

Figure 5: Box diagrams for $\ell \to \ell_k \ell_a \bar{\ell}_b$ decays.

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Feynman Diagrams: $\ell \to \ell_a$ nuclei conversion

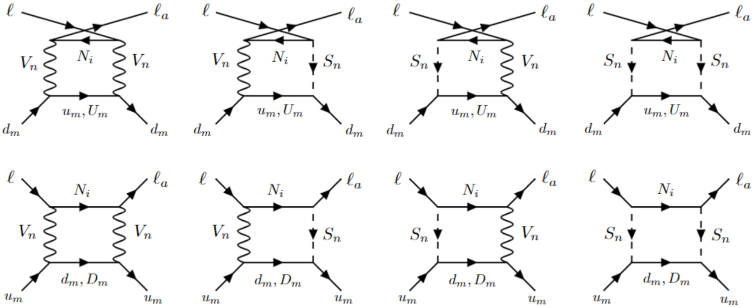

Figure 6: Relevant box diagrams for $\ell N \to \ell_a N$ conversion, where $V_n(S_n) = X, W(x, \phi)$, $u_m(d_m) = u, c(d, s)$ and $U_m(D_m) = U, C(D, S)$.

The first step is setting the range for the free parameters of SLH model: f, t_β, M_{N_i}, δ_ν, $V^{\ell i}$ and δ_q,

- **scale of compositeness, f**
  Direct search of Z' bosons at LHC[5], set the lower limit as[6] $f \gtrsim 7.5$ TeV at 95% C.L. We fix the upper limit[7] $f \lesssim 85$ TeV.

- **The ratio of the two vevs $t_\beta = f_1/f_2$**
  Perturbative unitarity analysis[8] binds $1 \leq t_\beta \leq 9$. For small f $(10 \leq f(\mathrm{TeV}) \leq 20)$, t_β can vary freely in this interval, while for $20 \leq f(\mathrm{TeV}) \leq 80$, the approximate relation $t_\beta = \frac{2}{15} f(\mathrm{TeV}) - \frac{25}{15}$ holds.

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[5] JHEP 10 (2017)
[6] Phys. Rev. D 97.7 (2018),
[7] Phys. Rev. D 97.11 (2018)
[8] Phys. Rev. D 97.11 (2018)

- **Heavy neutrinos**
  This "little" neutrino masses are unknown. We will take the ratios involving them as[9]: $0.1 \leq x_1 \leq 0.25$, $1.1x_1 \leq x_2 \leq 10x_1$, $1.1x_2 \leq x_3 \leq 10x_2$.
- **Neutrino mixing** δ_ν
  According to data[10], $\delta_\nu \lesssim 0.05$, that we will take.
- **Mixing matrix** $V^{\ell i}$
  We do not have any information of the mixing matrix $V^{\ell i}$. According to previous work[11], we have scanned over $-1 \leq s_{ij} \leq 1$ ensuring the low-energy restrictions.

[9] Phys. Rev. D 94.5 (2016)
[10] JHEP 03 (2011), New J. Phys. 17.7 (2015), EPJ Web Conf. 60 (2013)
[11] Phys. Rev. D 94.5 (2016)

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- **Quark mixing δ_q**
  We assume that the mixing effects are suppressed in the $t_\beta > 1$ regime[12], so it implies: $\delta_q = \mp\delta_\nu$.

The expected sensitivity of NA64 experiment we can express the conversion probability of $\ell - \tau$ conversion as the ratio[13]

$$\mathcal{R} = \frac{\sigma\left(\ell + N \to \tau + X\right)}{\sigma\left(\ell + N \to \ell + X\right)} \sim 10^{-13} - 10^{-12}, \tag{15}$$

where

$$\begin{aligned} &\sigma\left(e + Fe \to e + X\right) = 0.129 \times 10^5 \ \mathrm{GeV}^{-2}, \quad \sigma\left(\mu + Fe \to \mu + X\right) = 0.692 \ \mathrm{GeV}^{-2}, \\ &\sigma\left(e + Pb \to e + X\right) = 1.165 \times 10^5 \ \mathrm{GeV}^{-2}, \quad \sigma\left(\mu + Pb \to \mu + X\right) = 6.607 \ \mathrm{GeV}^{-2}. \end{aligned} \tag{16}$$

[12] JHEP 01 (2006)
[13] Phys. Rev. D 98.1 (2018)

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Mean values for muon physics

LFV decays	Experimental Limits	Mean values	Future sensitivity
$\mu \to e\gamma$	4.2×10^{-13}	2.1×10^{-14}	6×10^{-14}
$\mu \to ee\bar{e}$	1.0×10^{-12}	5.7×10^{-15}	10^{-16}
$\mu Ti \to eTi$	4.3×10^{-12}	6.8×10^{-14} (AF), 8.6×10^{-14} (U)	10^{-18}
$\mu Au \to eAu$	7.0×10^{-13}	8.2×10^{-14} (AF), 1.1×10^{-13} (U)	-

Table 2: Mean values of branching ratios and conversion rates for muon LFV processes against current upper limits[20] at 90 % C. L. and future sensitivities[21]

[20] PTEP 2020.8 (2020)
[21] Phys. Rev. D 101.7 (2020).

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Mean values for tau physics I

LFV decays	Experimental Limits	Our mean values	Future sensitivity
$\tau \to e\gamma$	3.3×10^{-8}	5.6×10^{-12}	3×10^{-9}
$\tau \to \mu\gamma$	4.4×10^{-8}	2.3×10^{-12}	10^{-9}
$\tau \to ee\bar{e}$	2.7×10^{-8}	3.2×10^{-12}	$(2-5) \times 10^{-10}$
$\tau \to \mu\mu\bar{\mu}$	2.1×10^{-8}	1.6×10^{-12}	$(2-5) \times 10^{-10}$
$\tau \to e\mu\bar{\mu}$	2.7×10^{-8}	2.1×10^{-12}	$(2-5) \times 10^{-10}$
$\tau \to \mu e\bar{e}$	1.8×10^{-8}	1.0×10^{-12}	$(2-5) \times 10^{-10}$
$\tau \to \mu\mu\bar{e}$	1.7×10^{-8}	3.8×10^{-18}	$(2-5) \times 10^{-10}$
$\tau \to ee\bar{\mu}$	1.5×10^{-8}	5.6×10^{-18}	$(2-5) \times 10^{-10}$

Table 3: Branching ratios against current upper limits at 90 % C. L. and future sensitivities.

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Mean values for tau physics II

LFV decays	Experimental Limits	Our mean values	Future sensitivity
$eFe \to \tau Fe$	-	9.2×10^{-20} (AF), 9.3×10^{-20} (U)	-
$ePb \to \tau Pb$	-	1.6×10^{-19} (AF), 1.6×10^{-19} (U)	-
$\mu Fe \to \tau Fe$	-	6.2×10^{-16} (AF), 6.2×10^{-16} (U)	-
$\mu Pb \to \tau Pb$	-	9.6×10^{-16} (AF), 9.8×10^{-16} (U)	-

Table 4: Conversion rates against current upper limits at 90 % C. L. and future sensitivities.

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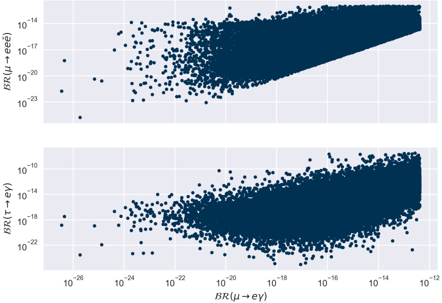
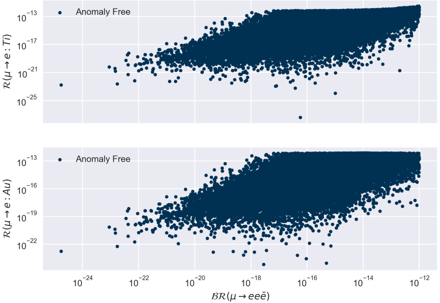
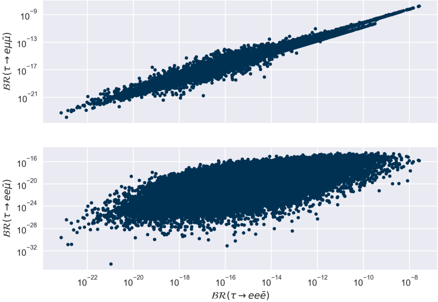


(a) $\mathcal{BR}(\mu \to e\gamma)$ vs. $\mathcal{BR}(\mu \to ee\bar{e})$, $\mathcal{BR}(\tau \to e\gamma)$

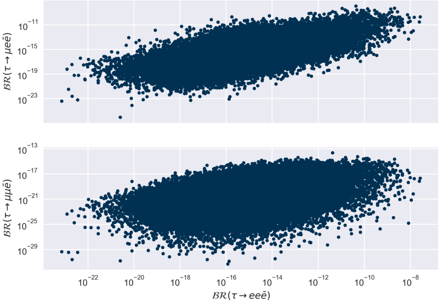


(b) $\mathcal{BR}(\mu \to ee\bar{e})$ vs. $\mathcal{R}(\mu \to e : Ti)$, $\mathcal{R}(\mu \to e : Au)$

Figure 7: Scatter plots for muon LFV processes.



(a) $\mathcal{BR}(\tau \to ee\bar{e})$ vs. $\mathcal{BR}(\tau \to ee\bar{\mu})$, $\mathcal{BR}(\tau \to e\mu\bar{\mu})$



(b) $\mathcal{BR}(\tau \to ee\bar{e})$ vs. $\mathcal{BR}(\tau \to \mu\mu\bar{e})$, $\mathcal{BR}(\tau \to \mu e\bar{e})$

Figure 8: Scatter plots for $\ell \to 3\ell'$ tau decays.

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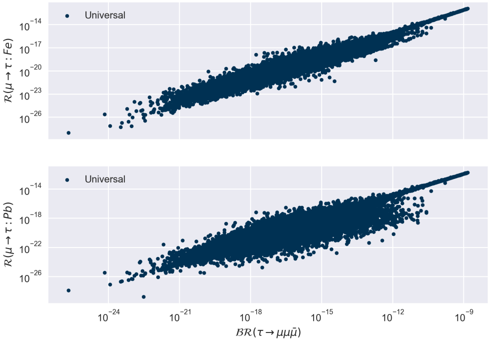

Figure 9: $\mathcal{BR}(\tau \to \mu\mu\bar{\mu})$ vs. $\mathcal{R}(\mu \to \tau : Fe)$, $\mathcal{R}(\mu \to \tau : Pb)$

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SLH does not have $SU(2)$ custodial symmetry, and the tree-level SM relation $\rho = 1$ is no longer valid:

$$\rho = 1 + \frac{v^2}{8f^2}\left(1 - t_W^2\right)^2, \quad \text{where} \quad \delta\rho = \frac{v^2}{8f^2}\left(1 - t_W^2\right)^2 \equiv \alpha T \tag{17}$$

A theory with Z' bosons can modify the oblique parameters[14] $(T,\, S,\, U)$

$$S = 4 s_W^2 \hat{S}/\alpha, \quad T = \hat{T}/\alpha \tag{18}$$

In SLH model

$$\hat{T} \approx 0, \quad \hat{S} \approx \frac{4M_W^2}{M_{Z'}^2\left(3 - t_W^2\right)} = 4W = \frac{4Y}{t_W^2}. \tag{19}$$

[14] Phys. Rev. D 72 (2005), arXiv: 2204.04191 [hep-ph]

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	SM	EWPD	$M_W = 80.357$ GeV	$M_W = 80.4242$ GeV
ρ	1	1.00038 ± 0.00020	1.0004758	1.0016013
$\hat{T}$	0	-	5×10^{-4}	1.6×10^{-3}
$\hat{S}$	0	-	7×10^{-5}	7×10^{-5}
T	0	0.03 ± 0.12 0.05 ± 0.06	0.07	0.22
S	0	-0.01 ± 0.10 0.00 ± 0.07	0.008	0.008

Table 2: Olique parameters according to EWPD and using instead M_W as in the PDG, or from the CDF measurement[21]. Two values are given for T and S. The upper one is obtained fitting also U (for which 0.02 ± 0.11 is obtained) and the second one setting $U = 0$[22]

[21] Science 376.6589 (2022)
[22] PTEP 2020.8 (2020

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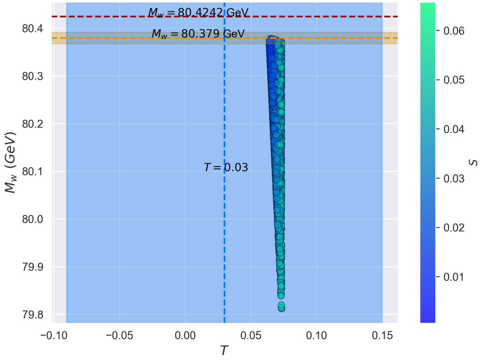
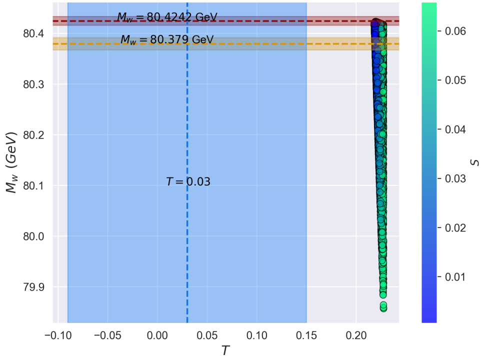


(a) Scatter plot using $M_W = 80.379$ GeV.



(b) Scatter plot using $M_W = 80.4242$ GeV.

Figure 10: Correction to the oblique parameters S and T in the SLH.

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Conclusion

- As is well-known, processes with muons would most likely be the discovery channels for LFV.
- Tau physics will be needed for characterizing the underlying new physics.
- We verified that although the SLH modifies the ρ parameter is in agreement with EWPD using the PDG W mass but if we take the recent CDF M_W measurement, this is in tension with electroweak precision data.

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Thank You

Questions?

Feynman Rules

$V_\mu \bar{f}_i f_m$ Vertex	$g_L^{V\bar{f}_i f_m}$	$g_R^{V\bar{f}_i f_m}$
$W^+\bar{\nu}_i\ell_i$	$\frac{1}{\sqrt{2}s_W}\left(1-\frac{\delta_\nu^2}{2}\right)$	0
$W^+\bar{N}_m\ell_i$	$-\delta_\nu\frac{1}{\sqrt{2}s_W}V_\ell^{mi}$	0
$Z\bar{\ell}_i\ell_i$	$\frac{2s_W^2-1}{2c_W s_W}+\frac{\delta_Z\left(2s_W^2-1\right)}{2s_W c_W^2\sqrt{3-t_W^2}}$	$t_W+\frac{\delta_Z s_W}{c_W^2\sqrt{3-t_W^2}}$
$Z\bar{\nu}_i\nu_i$	$\frac{1-\delta_\nu^2}{2c_W s_W}-\frac{\delta_Z\left(1-2s_W^2\right)}{2s_W c_W^2\sqrt{3-t_W^2}}$	0
$Z\bar{N}_i N_i$	$\frac{\delta_Z}{s_W\sqrt{3-t_W^2}}+\frac{\delta_\nu^2}{2c_W s_W}$	0
$Z\bar{N}_m\nu_i$	$-\delta_\nu\frac{1}{2c_W s_W}V_\ell^{mi}$	0

Table 1: Vertices $[V^\mu ff]=ie\gamma^\mu\left(g_L P_L+g_R P_R\right)$ for the lepton sector.

$Z'\,\bar{\ell}_i\ell_i$	$\frac{2s_W^2-1}{2s_W c_W^2\sqrt{3-t_W^2}}+\frac{\delta_Z\left(1-2s_W^2\right)}{2s_W c_W}$	$\frac{s_W}{c_W^2\sqrt{3-t_W^2}}-\delta_Z t_W$
$Z'\,\bar{\nu}_i\nu_i$	$\frac{2s_W^2-1}{2s_W c_W^2\sqrt{3-t_3^2}}\left(1-\frac{\left(3-t_W^2\right)\delta_\nu^2 c_W^2}{1-2s_W^2}\right)-\frac{\delta_Z}{2c_W s_W}$	0
$Z'\,\bar{N}_i N_i$	$\frac{1}{2s_W\sqrt{3-t_W^2}}\left[2-\delta_\nu^2\left(3-t_W^2\right)\right]$	0
$Z'\,\bar{N}_m\nu_i$	$\frac{\delta_\nu\sqrt{3-t_W^2}}{2s_W}V_\ell^{mi}$	0

Table 2: Vertices $[V^\mu ff]=ie\gamma^\mu\left(g_L P_L+g_R P_R\right)$ for the lepton sector.

$\ell \to \ell_a \gamma$

The amplitude $\ell \to \ell_a \gamma$ is proportional to the vertex in figure 1, only dipole form factors contribute to this decay[15]. Neglecting $m_{\ell_a} \ll m_\ell$, the total width for $\ell_j \to \ell_i \gamma$ is given by[16]:

$$\Gamma\left(\ell_j \to \ell_i \gamma\right) = \frac{\alpha m_{\ell_j}^3}{2} \left(\left| F_M^\gamma \right|^2 + \left| F_E^\gamma \right|^2 \right). \tag{20}$$

[15] JHEP 01 (2009), Nucl. Phys. B 551 (1999

[16] Phys. Lett. B**119**, Phys. Rev. D **53**, Phys. Rev. D **63**, Phys. Rev. D **67**.

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Amplitude for $\ell \to \ell_k \ell_a \bar{\ell}_b$

We define the amplitudes and form factors as:

$$\begin{aligned}
\mathcal{M}_{\gamma penguin} =& \frac{e^2}{Q^2}\bar{u}(p_1)\left[Q^2\gamma^\mu\left(A_1^L P_L + A_1^R P_R\right) + m_\mu i\sigma^{\mu\nu}Q_\nu\left(A_2^L P_L + A_2^R P_R\right)\right]u(p) \\
& \times \bar{u}(p_2)\gamma_\mu v(p_3) - (p_1 \leftrightarrow p_2)\,, \\
\mathcal{M}_{Zpenguin} =& \frac{e^2}{M_Z^2}\bar{u}(p_1)\left[\gamma^\mu\left(F_L P_L + F_R P_R\right)\right]u(p)\bar{u}(p_2)\left[\gamma_\mu\left(Z_L^e P_L + Z_R^e P_R\right)\right]v(p_3) \\
& - (p_1 \leftrightarrow p_2)\,, \\
\mathcal{M}_{Z' penguin} =& \frac{e^2}{M_Z^2}\bar{u}(p_1)\left[\gamma^\mu\left(F_L' P_L + F_R' P_R\right)\right]u(p)\bar{u}(p_2)\left[\gamma_\mu\left(Z_L'^e P_L + Z_R'^e P_R\right)\right]v(p_3) \\
& - (p_1 \leftrightarrow p_2)\,,
\end{aligned} \tag{21}$$

$$\mathcal{M}_{boxes} = e^2 B_1^L \left[\bar{u}(p_1)\gamma^{\mu} P_L u(p)\right]\left[\bar{u}(p_2)\gamma_{\mu} P_L v(p_3)\right] \tag{22}$$

where

$$A_1^L = F_L^{\gamma}/Q^2, \quad A_1^R = F_R^{\gamma}/Q^2, \quad A_2^L = -\left(F_M^{\gamma} + iF_E^{\gamma}\right)/m_{\ell}, \quad A_2^R = -\left(F_M^{\gamma} - iF_E^{\gamma}\right)/m_{\ell}$$
$$F_L = -F_L^Z, \quad F_R = -F_R^Z, \quad F_L' = -F_L^{Z'}, \quad F_R' = -F_R^{Z'}, \quad F_{LL} = \frac{F_L Z_L^e}{M_Z^2}, \quad F_{RR} = \frac{F_R Z_R^e}{M_Z^2},$$
$$F_{LR} = \frac{F_L Z_R^e}{M_Z^2}, \quad F_{RL} = \frac{F_R Z_L^e}{M_Z^2} \tag{23}$$

We define the box form factors for same-flavor decays:

$$\hat{B}_1^L = B_1^L + 2F_{LL}', \qquad \hat{B}_2^L = F_{LR}' \tag{24}$$

We define the box form factors for same-sign decays:

$$\hat{B}_1^L = B_1^L + F_{LL}', \qquad \hat{B}_2^L = F_{LR}' \tag{25}$$

Amplitude for $\mu - e$ nuclei conversion

$\mu - e$ nuclei conversion is similar to $\mu \to ee\bar{e}$.

$$\Gamma\left(\mu N \to eN\right) = \alpha^5 \frac{Z_{eff}^4}{Z} |F(q)|^2 m_\mu^5 \left| 2Z\left(A_1^L - A_2^R\right) - (2Z+N)\,\bar{B}_{1u}^L - (Z+2N)\,\bar{B}_{1d}^L \right|^2, \tag{26}$$

We have also defined:

$$\bar{B}_{1q}^L = B_{1q}^L + F_{LL}^q + F_{RL}^q + F_{LL}^{\prime q} + F_{RL}^{\prime q},$$

the conversion rate is obtained by dividing by the muon capture rate:

$$\mathcal{R} = \frac{\Gamma\left(\mu \to e\right)}{\Gamma_{capt}}. \tag{27}$$

parameters of the nuclei[a].

Nucleus	Z	N	Z_{eff}	$F(q)$	Γ_{capt} [GeV]
$^{22}_{48}\mathrm{Ti}$	22	26	17.6	0.54	1.7×10^{-18}
$^{79}_{197}\mathrm{Au}$	79	118	33.5	0.16	8.6×10^{-18}

Table 3: Relevant input parameters for the nuclei under study.

[a]Phys. Rev. D 66 (2002), Phys. Rev. C 35 (1987)

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Amplitude for $\ell - \tau$ nuclei conversion

The study of $\ell - \tau$ conversion is a deep inelastic scattering (DIS) of the initial lepton beam. We are only interested in the $\ell + \mathcal{N}(A, Z) \to \tau + X$ case. PDFs encoding the low-energy non-perturbative QCD effects[17].

$$\sigma_{\ell-\tau} = \hat{\sigma}\left(\xi, Q^2\right) \otimes H\left(\xi, Q^2\right) \ . \tag{28}$$

The perturbative cross sections are[18]:

$$\begin{aligned} \frac{d^2\hat{\sigma}\left(\ell q_i(\xi P) \to \tau q_i\right)}{d\xi dQ^2} &= \frac{1}{16\pi\lambda\left(s(\xi), m_\ell^2, m_i^2\right)}\overline{|\mathcal{M}_{qq}(\xi, Q^2)|^2}, \\ \frac{d^2\hat{\sigma}\left(\ell \bar{q}_i \to \tau \bar{q}_i(\xi P)\right)}{d\xi dQ^2} &= \frac{1}{16\pi\lambda\left(s(\xi), m_\ell^2, m_i^2\right)}\overline{|\mathcal{M}_{\bar{q}\bar{q}}(\xi, Q^2)|^2}, \end{aligned} \tag{29}$$

[17] PoS DIS2015 (2015), Comput. Phys. Commun. 216 (2017), Comput. Phys. Commun. 133 (2000).

[18]: JHEP 01 (2021

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The nucleon cross section is[19]

$$\sigma\left(\ell + N \to \tau + X\right) = \sum_i \int_{\xi_{min}}^{1} \int_{Q_-^2(\xi)}^{Q_+^2(\xi)} d\xi dQ^2 \Bigg[\frac{d^2\hat{\sigma}\left(\ell q_i(\xi P) \to \tau q_i\right)}{d\xi dQ^2} H_{q_i}\left(\xi, Q^2\right) + \frac{d^2\hat{\sigma}\left(\ell \bar{q}_i \to \tau \bar{q}_i(\xi P)\right)}{d\xi dQ^2} H_{\bar{q}_i}\left(\xi, Q^2\right) \Bigg], \tag{30}$$

The total cross section can be expressed as[20]:

$$\sigma\left(\ell + (A, Z) \to \tau + X\right) = Z\sigma\left(\ell + p \to \tau + X\right) + (A - Z)\sigma\left(\ell + n \to \tau + X\right), \tag{31}$$

[19] JHEP 01 (2021), Phys. Rev. D 98.1 (2018)
[20] Rev. D 98.1 (2018)

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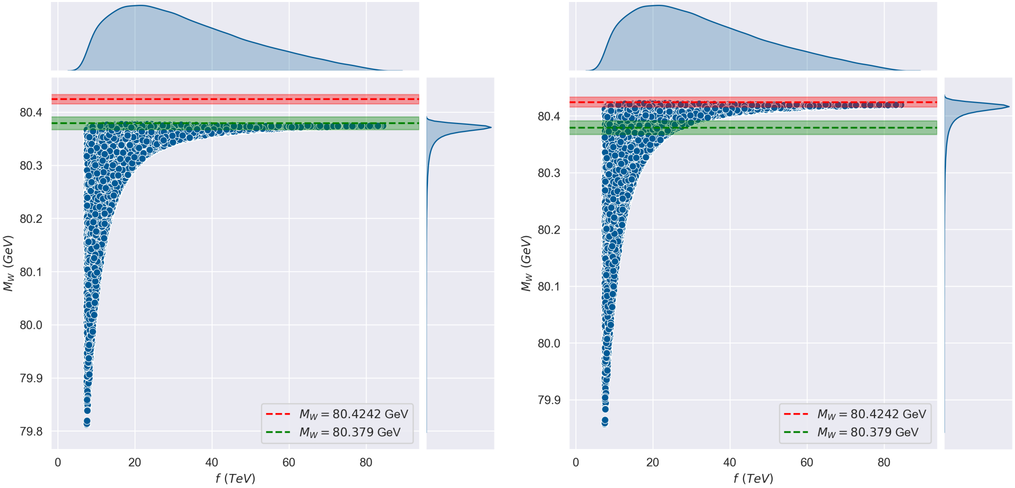

(a) Scatter plot using $M_W = 80.379$ GeV.

(b) Scatter plot using $M_W = 80.4242$ GeV.

Figure 11: Corrections to the W boson mass provided by the SLH compared to its measurement.

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