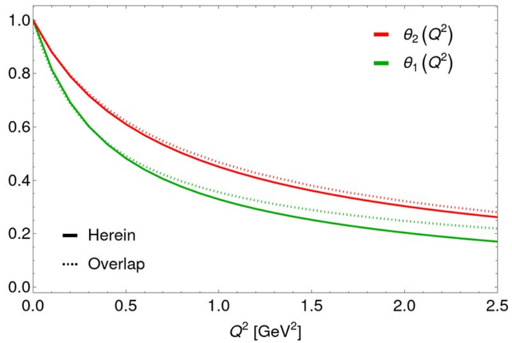
Mass distribution and forces inside the pion

Khépani Raya Montaño

Yin-Zhen Xu, José Rodríguez-Quintero...



Universidad
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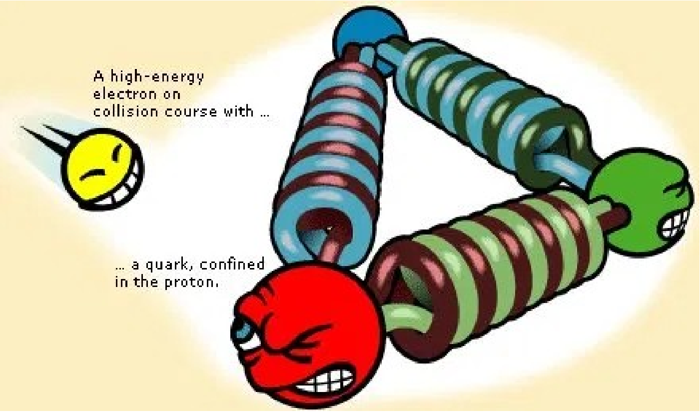
QCD: Basic Facts

- **QCD** is characterized by two **emergent** phenomena: confinement and dynamical generation of mass (DGM).

$$\mathcal{L}_{\rm QCD} = \sum_{j=u,d,s,\ldots} \bar{q}_j[\gamma_\mu D_\mu + m_j]q_j + \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu},$$
$$D_\mu = \partial_\mu + ig\frac{1}{2}\lambda^a A^a_\mu\,,$$
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu + \partial_\nu A^a_\mu - \underline{g f^{abc}A^b_\mu A^c_\nu},$$

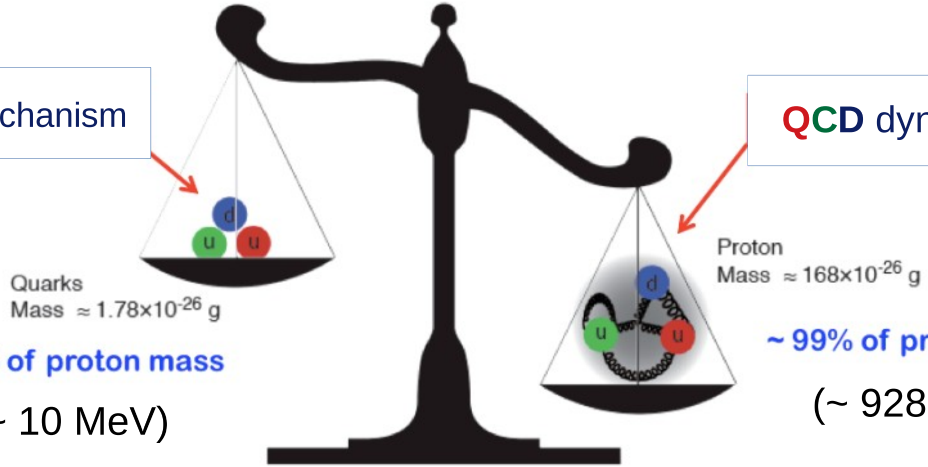
- Quarks and gluons not *isolated* in nature.
  - Formation of colorless bound states: “**Hadrons**”
  - **1-fm scale** size of hadrons?

- Emergence of hadron masses (EHM) from QCD **dynamics**



Higgs mechanism

QCD dynamics



~ 1% of proton mass

(~ 10 MeV)

~ 99% of proton mass

(~ 928 MeV)

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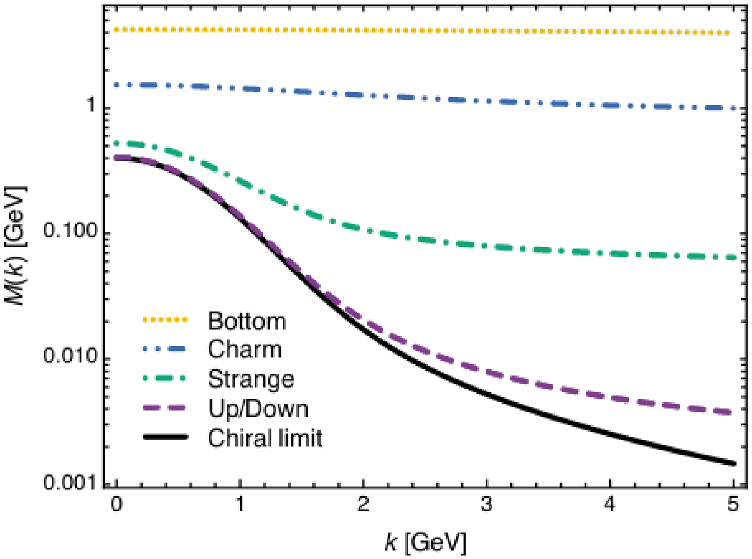
Can we trace them down to fundamental d.o.f?

Dynamical masses

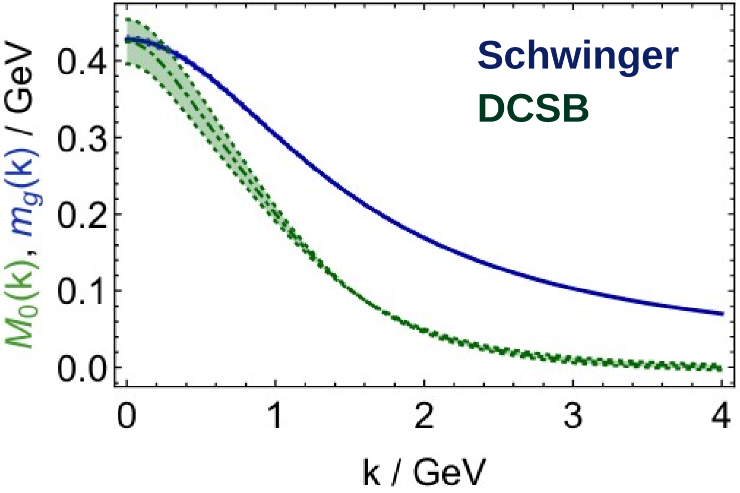
(Dynamical Chiral Symmetry Breaking)



“Higgs” masses

$$S_f^{-1}(p) = Z_f^{-1}(p^2)(i\gamma\cdot p + \mathbf{M_f(p^2)})$$

- Emergence of hadron masses (EHM) from QCD **dynamics**



Gluon and quark *running masses*

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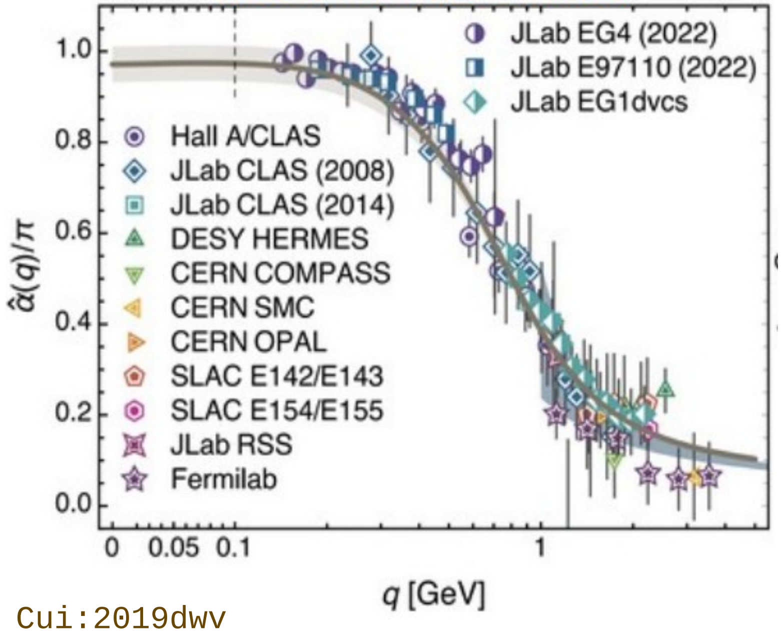
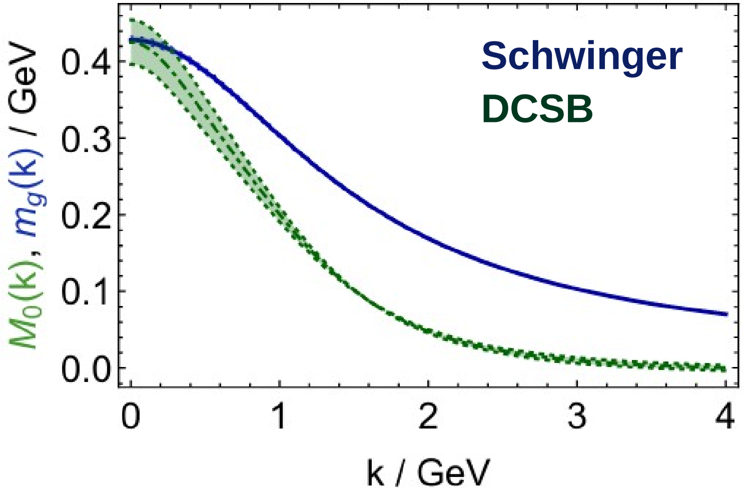
Can we trace them down to fundamental d.o.f?

◆ Emergence of hadron masses (EHM) from QCD **dynamics**

(figure: D. Binosi's courtesy!)

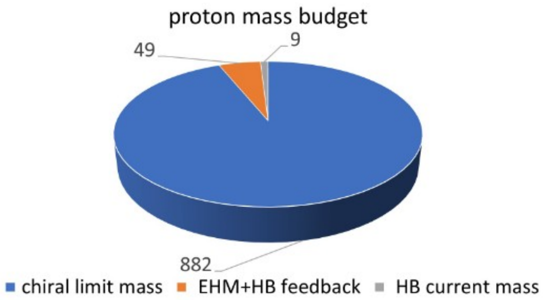


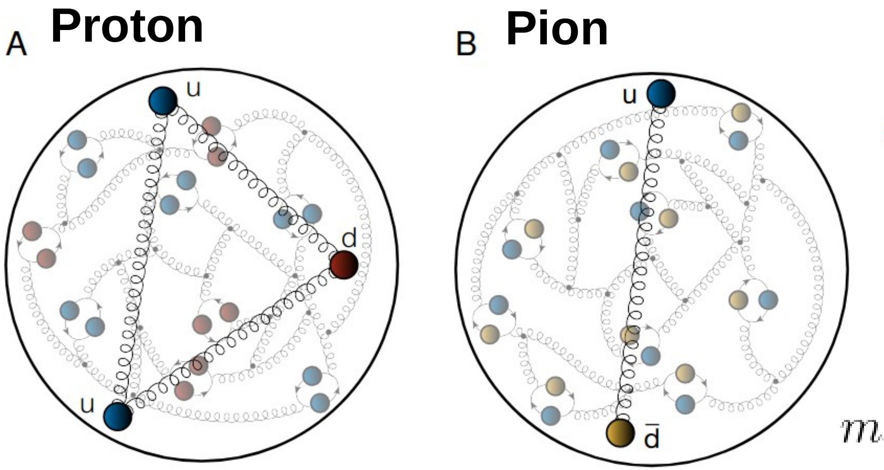
Cui:2019dwv

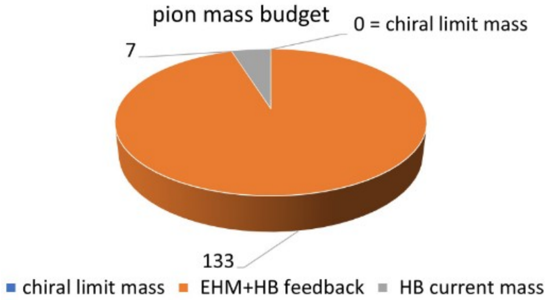


Gluon and quark *running masses*

QCD: Understanding EHM







$m_p \approx 0.940\ \mathrm{GeV}$

$m_\pi \approx 0.140\ \mathrm{GeV}$

Massive, regardless of **Higgs** mass generation

Massless in the absense of **Higgs** mass generation

- **QCD** should explain both the *massiveness* of the proton and the *masslessness* of the pion

Gravitational form factors

Gravitational form factors

- For a given parton class, the **spin-0** energy-momentum tensor (**EMT**) can take the following form:

$$\underbrace{\Lambda^{a}_{\mu\nu}(P,Q)}_{\langle P_f|T_{\mu\nu}(0)|P_i\rangle} = 2P_\mu P_\nu \theta^{a}_2(Q^2) + \frac{1}{2}\left(Q^2 g_{\mu\nu} - Q_\mu Q_\nu\right)\theta^{a}_1(Q^2) + 2m_\pi^2 g_{\mu\nu}\bar{c}^{a}(Q^2)$$

With: $P = [P_f + P_i]/2 \text{ and } Q = P_f - P_i$

- Such that $\theta_{1,2}(Q^2),\ \bar{c}(Q^2)$ define the so called **gravitational form factors** (**GFFs**).

(these are extracted by sensible projection operators)

$$\int d^3r\, T_q^{00}(\vec{r}) = m_\pi \Theta_{2,q}(0)$$ → $\theta_2(Q^2)$ Is connected with the **mass** distribution inside the hadron

$$T_q^{ij}(\vec{r}) = p_q(r)\,\delta_{ij} + s_q(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right)$$
$$(i,\, j = 1,\, 2,\, 3)$$
→ $\theta_1(Q^2)$ Is connected with the **mechanical** properties of the hadron

p(r) : pressure
s(r) : shear forces

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- Energy-momentum **conservation** entail the following **sum rules**:

$$\sum_{q,g} \theta_2(0) = 1 \qquad \sum_{q,g} \bar{c}(t) = 0$$

- While, in the **chiral limit**, the **soft-pion theorem** constraints:

$$\sum_{q,g} \theta_1(0) = 1$$

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$$\sum_{q,\bcancel{g}} \theta_1(0) = 1$$

- At the **hadronic scale**, ζ_H, all properties of the hadron are contained within the valence quarks.

Here we shall work...

Continuum Schwinger-Methods

(Dyson-Schwinger equations)

The DSE approach

- Equations of motion of a **quantum field theory**
- Relate Green functions with higher-order Green functions

➡
  - **Infinite** tower of coupled equations.
  - ✗ Systematic **truncation** required

✓ **No assumptions** on the **coupling** for their derivation.

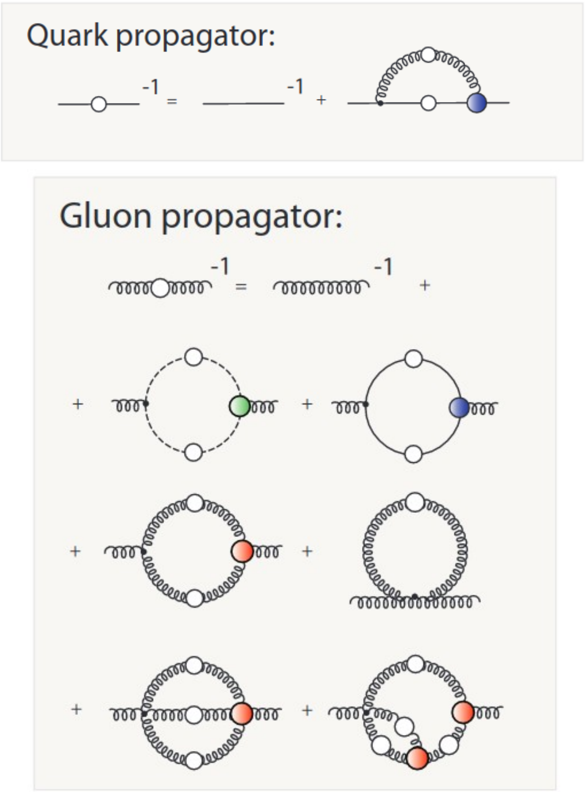
➡
  - ✓ Capture both **perturbative** and **non-perturbative** facets of **QCD**

✓ **Not limited** to a certain domain of current **quark masses**

✓ Maintain a **traceable connection** to QCD.

C.D. Robert and a A.G. Williams,
Prog.Part.Nucl.Phys. 33 (1994) 477-575

Example DSEs



Eichmann:2009zx

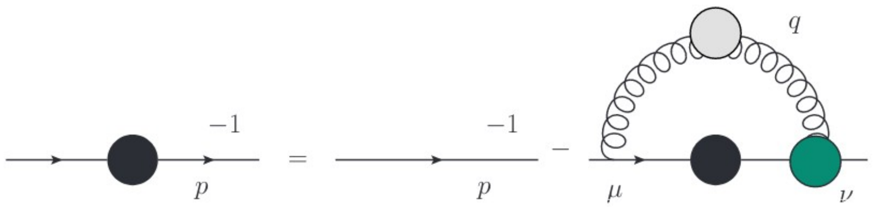
The DSE approach

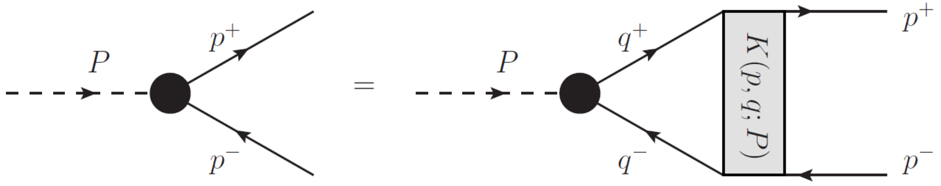
- **BSWF**: sandwich of the Bethe-Salpeter amplitude and quark propagators:

$$\chi_H(k_-^H; P_H) = S_q(k)\Gamma_H(k_-^H; P_H)S_{\bar{q}}(k - P_H)\ ,\ k_-^H = k - P_H/2\ .$$

$P^2 = -m_H^2$: **meson's mass**; Γ_H **BS amplitude**; $S_{q(\bar{q})}$ **quark (antiquark) propagator**

- Quark **propagator** and **BSA** should come from solutions of:





Quark DSE

- Relates the quark propagator with **QGV** and **gluon propagator.**

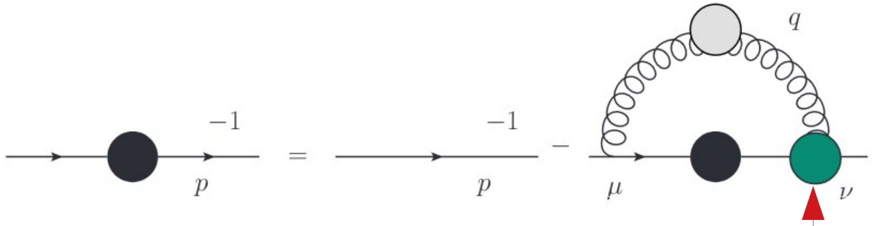
Meson BSE

- Contains **all interactions** between the quark and antiquark

The DSE approach

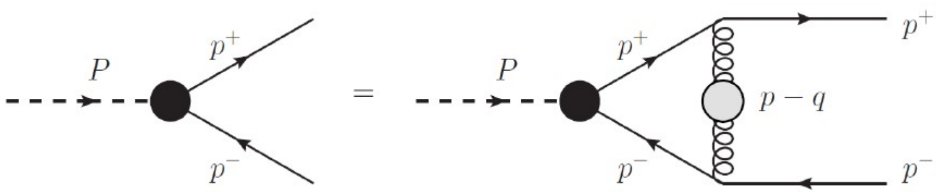
- For the ground-state pseudoscalar and vector mesons, it is typical to employ the so called Rainbow-Ladder (**RL**) truncation:

Y-Z Xu et al., PRD 100 (2019) 11, 114038.
K. Raya et al., PRD 101 (2020) 7, 074021.



Quark DSE

bare



Meson BSE

- It preserves the **Goldstone’s Theorem**, whose most fundamental expression is captured in:

$$f_\pi E_\pi(k;P=0) = B(k^2)$$

Leading BSA

“***Mass*** Function”

> “**Pions** exists, if and only if, **DCSB** occurs.”

The DSE approach

- Recall the relation between **GFFs** and **EMT**:

$$\Lambda_{\mu\nu}(P,Q) = 2P_\mu P_\nu \theta_2(Q^2) + \frac{1}{2}\left(Q^2 g_{\mu\nu} - Q_\mu Q_\nu\right)\theta_1(Q^2) + 2m_\pi^2 g_{\mu\nu}\bar{c}(Q^2)$$

- The matrix element can be expressed in terms of **propagators** and **vertices**:

$$\Lambda_{\mu\nu}(P,Q) = N_c \int_{dk} \mathrm{Tr}\left[\Gamma_\pi\left(k-\frac{Q}{4}, P-\frac{Q}{2}\right) S\left(k-\frac{P}{2}\right) \Gamma_\pi\left(k+\frac{Q}{4}, P+\frac{Q}{2}\right)\right. \quad + \text{ beyond } \textbf{I.A.}$$

$$\left. S\left(k+\frac{P}{2}+\frac{Q}{2}\right) \Gamma_{\mu\nu}\left(k+\frac{P}{2}, Q\right) S\left(k+\frac{P}{2}-\frac{Q}{2}\right)\right]$$

- **EM** conservation implies:

$$Q_\mu \Lambda_{\mu\nu}(P,Q) = 0$$

- Also **note** that:

$$Q_\mu \Lambda_{\mu\nu}(P,Q) \sim \bar{c}(Q^2)$$

This **restricts** the structure of the beyond **I.A. contribution**.

The DSE approach

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+ beyond **I.A.**

$$\left. S\left(k+\frac{P}{2}+\frac{Q}{2}\right) \underbrace{\Gamma_{\mu\nu}\left(k+\frac{P}{2}, Q\right)} S\left(k+\frac{P}{2}-\frac{Q}{2}\right)\right]$$

Quark-tensor vertex (QTV): The brand-new ingredient

To be determined by its **DSE**, but also its Ward-Green-Takahashi identity (**WGTI**) and other **symmetry properties**.

Quark-tensor vertex

- The interaction of a **quark** with a **spin-2 probe** is encoced in the **QTV**, $\Gamma^{\mu\nu}$
- As the **quark-photon** vertex (**QPV**), the **QTV** obeys a **DSE**:

$$i\Gamma^{\mu\nu}(P,Q) = \underbrace{i\Gamma_0^{\mu\nu}(P,Q)}_{\text{Tree level QTV}} + \int \underbrace{K^{(2)}(P,Q|P',Q')}_{\text{IA kernel}}\, i\Gamma^{\mu\nu}(P',Q') + \underbrace{\Delta^{\mu\nu}(P,Q)}_{\text{Symmetry restoring term}}$$

$$i\Gamma_0^{\mu\nu}(P,Q) = i\gamma^{\mu}P_i^{\nu} - g^{\mu\nu}S_0^{-1}(P_i)$$

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- Again, as the **QPV**, symmetry principles (**WGTIs**) partially constraint its structure:

$$iQ_\mu\Gamma^{\mu\nu}(P,Q) = P_i^\nu S^{-1}(P_f) - P_f^\nu S^{-1}(P_i) \qquad \text{(QTV WGTI)}$$

Quark-tensor vertex

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(QTV WGTI)

- Thus, the **QTV** can be expressed as:

$$i\Gamma^{\mu\nu}(P,Q) = \underbrace{\overset{\textbf{QPV}}{i\Gamma^{\mu}_{L}(P,Q)}P_i^{\nu} - g^{\mu\nu}S^{-1}(P_i) + \overset{\textbf{QPV}}{i\Gamma^{\mu}_{T}(P,Q)}P_i^{\nu}}_{i\Gamma^{\mu\nu}_{L}(P,Q)} + i\Gamma^{\mu\nu}_{T}(P,Q)$$

$$Q_\mu\Gamma^{\mu\nu}_{T} = 0$$

Quark-tensor **vertex**

$$Q_\mu \Gamma_T^{\mu\nu} = 0$$

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This part being fully determined by the **quark-propagator** and **QPV**, Γ^μ

The **QPV** also obeys its own **WGTI**:

$$iQ_\mu \Gamma_L^{\mu}(P,Q) = S^{-1}(P_f) - S^{-1}(P_i)$$

$$\text{with } \Gamma^\mu = \Gamma_L^\mu + \Gamma_T^\mu \text{ and } Q_\mu \Gamma_T^\mu = 0$$

This is *a priori* **unknown**

Can be obtained once a basis is specified and the **QTV DSE** is solved.

So, in general, the **non-transverse** piece can be written as:

$$i\Gamma_L^{\mu\nu}(P,Q) = \sum_{i=1}^{14} F_i(P^2, Q^2, P\cdot Q)\ \tau_i^{\mu\nu}(P,Q)$$

Quark-tensor **vertex**

$$Q_\mu \Gamma_T^{\mu\nu} = 0$$

- Thus, the **QTV** can be expressed as:

$$i\Gamma^{\mu\nu}(P,Q) = \underbrace{i\Gamma^{\mu}_{L}(P,Q)P_i^{\nu} - g^{\mu\nu}S^{-1}(P_i) + i\Gamma^{\mu}_{T}(P,Q)P_i^{\nu}}_{i\Gamma_L^{\mu\nu}(P,Q)} + i\Gamma_T^{\mu\nu}(P,Q)$$

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This is *a priori* **unknown**

Can be obtained once a basis is specified and the **QTV DSE** is solved.

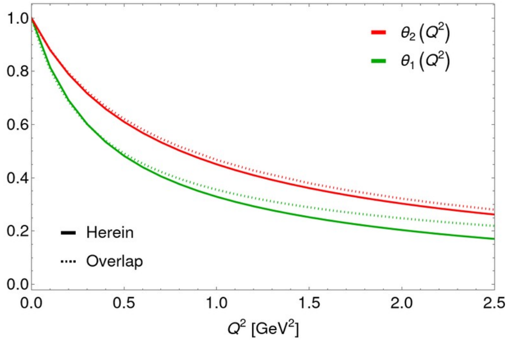
Setting $\Gamma^{\mu\nu} \equiv \Gamma_L^{\mu\nu}$ is sufficient to produce a sensible result for $\theta_2(Q^2)$

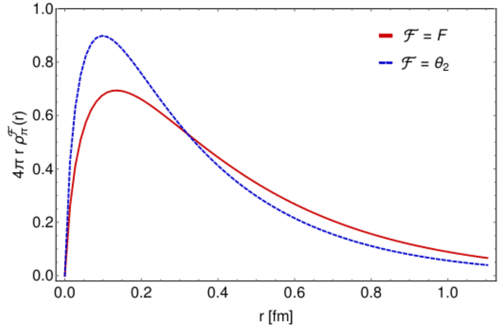
- So, it is convenient that the pieces entering $\Gamma_T^{\mu\nu}$ don’t affect this outcome.
- Capitalizing on the latter, we propose the following **minimal representation**:

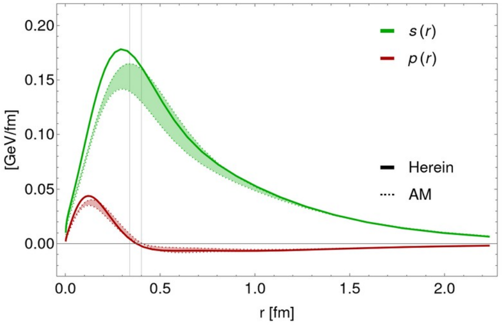
$$i\Gamma_T^{\mu\nu}(P,Q) \;=\; F_{15}(P^2,Q^2,P\cdot Q)\,\tau_{15}^{\mu\nu}(P,Q) = i\mathbb{1}\left(Q^2 g^{\mu\nu} - Q^\mu Q^\nu\right)F_{15}(P^2,Q^2,P\cdot Q)$$

- Then we proceed to solve the **QTV DSE**.

NUMERICAL RESULTS





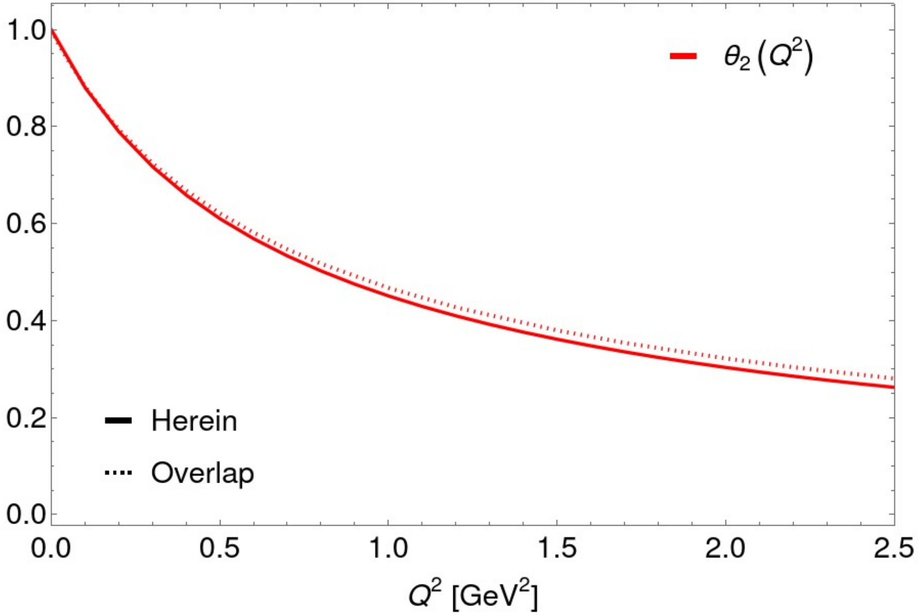


Results: GFFs

- Recall the **GFFs** are extracted from: $\Lambda_{\mu\nu}(P,Q) = 2P_\mu P_\nu \theta_2(Q^2) + \frac{1}{2}\left(Q^2 g_{\mu\nu} - Q_\mu Q_\nu\right)\theta_1(Q^2) + 2m_\pi^2 g_{\mu\nu}\cancelto{0}{\bar{c}(Q^2)}$
- $\theta_2(Q^2)$ Is well described by the part of the **QTV** that satisfies its **WGTI** alone:

$$iQ_\mu\Gamma^{\mu\nu}(P,Q) = P_i^\nu S^{-1}(P_f) - P_f^\nu S^{-1}(P_i)$$

Which is fully determined by the **QPV** and the **quark propagator**



$$i\Gamma^{\mu\nu}(P,Q) = \underbrace{i\Gamma_L^{\mu}(P,Q)P_i^\nu - g^{\mu\nu}S^{-1}(P_i) + i\Gamma_T^{\mu}(P,Q)P_i^\nu}_{i\Gamma_L^{\mu\nu}(P,Q)}$$

Overlap: Result obtained via the computation of the pion **LFWF** and **GPD**

$$\int_{-1}^{1} dx\, x\, H_{\mathsf{P}}^{q}(x,\xi,-\Delta^2;\zeta_{\mathcal{H}}) = \theta_2^{\mathsf{P}}(\Delta^2) - \xi^2\theta_1^{\mathsf{P}}(\Delta^2)$$

`Raya:2021zrz`

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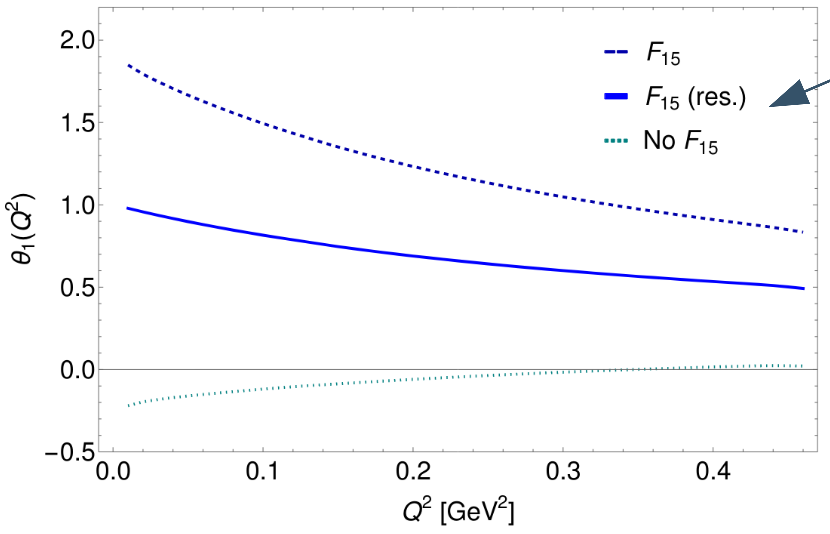
  0

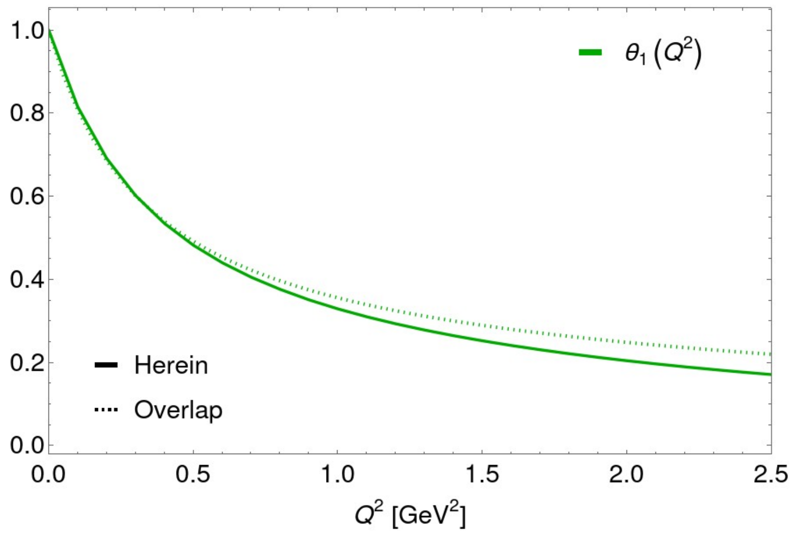
- $\theta_1(Q^2)$ Requires the inclusion of fully transverse pieces in the **QTV;** our *minimal* extension*:*

$$i\Gamma_T^{\mu\nu}(P,Q) \;=\; F_{15}(P^2,Q^2,P\cdot Q)\,\tau_{15}^{\mu\nu}(P,Q) = i\mathbb{1}\left(Q^2 g^{\mu\nu} - Q^\mu Q^\nu\right)F_{15}(P^2,Q^2,P\cdot Q)$$



Rescaled to account for soft-pion theorem: $\sum_{q,g}\theta_1(0) = 1$

- The **complete** result:



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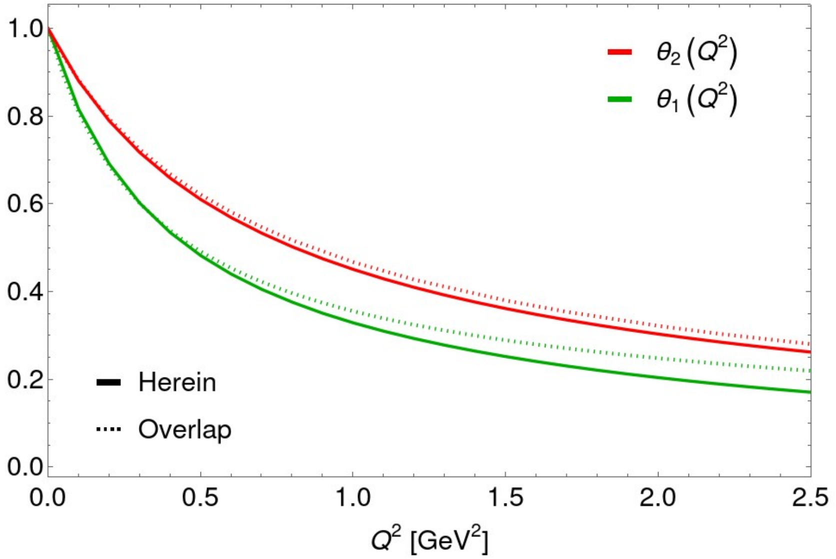
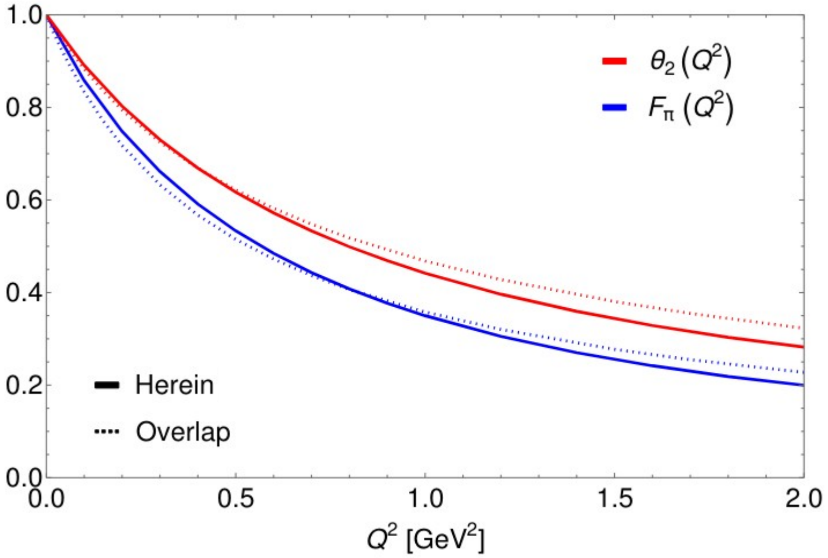
0

- $\theta_2(Q^2)$ Is harder than $\theta_1(Q^2)$ (and than the pion electromagnetic form factor):
  - In fact, one finds: $r_{\theta_2} \approx 0.8 r_\pi, \quad r_{\theta_2} \langle r_\pi \langle r_{\theta_1}$



Not an accident! Can be proven via GPD

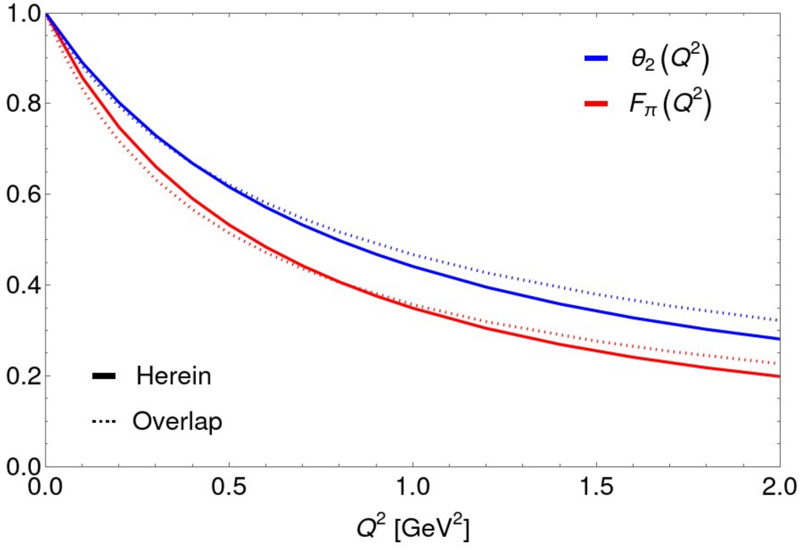
Raya:2021zrz



Overlap: Result obtained via the computation of the pion **LFWF** and **GPD**

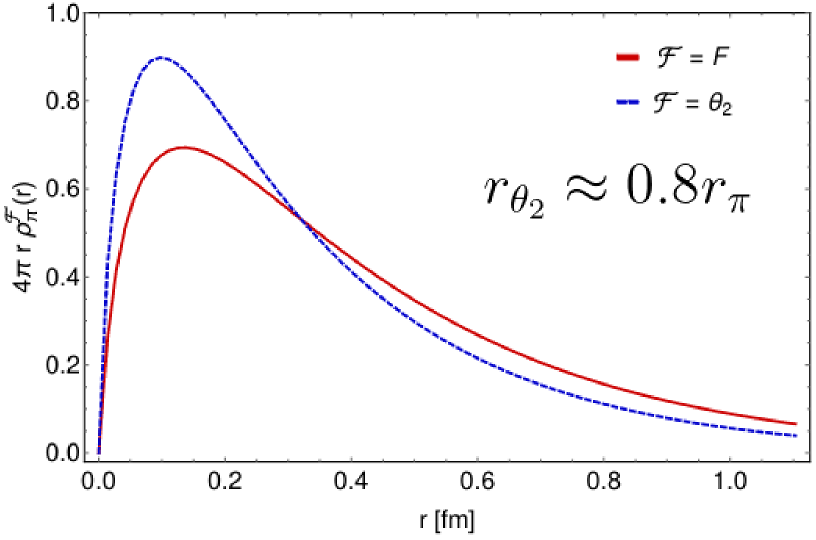
Results: Mass distribution

- Recall the **GFFs** are extracted from: $\Lambda_{\mu\nu}(P,Q) = 2P_\mu P_\nu \theta_2(Q^2) + \frac{1}{2}\left(Q^2 g_{\mu\nu} - Q_\mu Q_\nu\right)\theta_1(Q^2) + 2m_\pi^2 g_{\mu\nu}\underset{\swarrow\, 0}{\bar{c}(Q^2)}$
- $\theta_2(Q^2)$ Is harder than $\theta_1(Q^2)$ (and than the pion electromagnetic form factor):



- The **charge** and **mass** distributions:

$$\rho_{\mathbf{P}}^{\mathfrak{F}}(r) = \frac{1}{2\pi}\int_0^\infty d\Delta\Delta J_0(\Delta r)\mathfrak{F}_{\mathbf{P}}(\Delta^2)$$



$r_{\theta_2} \approx 0.8 r_\pi$

Overlap: Result obtained via the computation of the pion **LFWF** and **GPD**

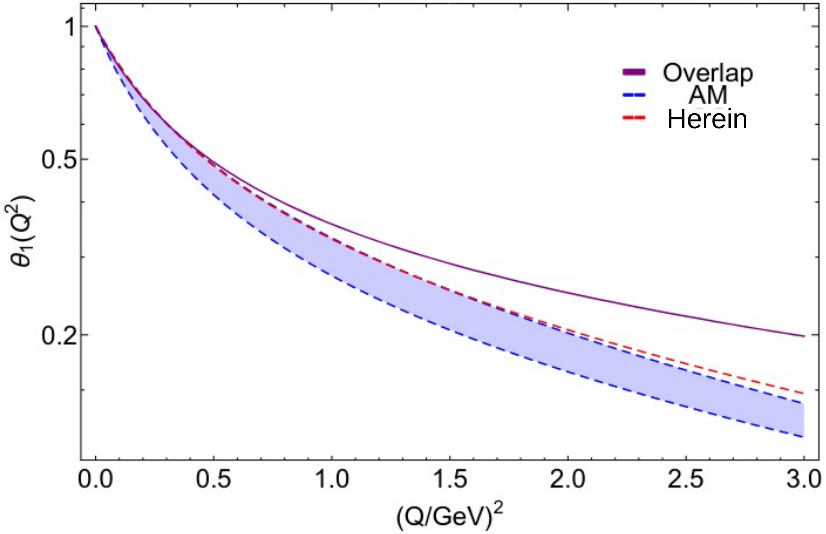
Raya:2021zrz

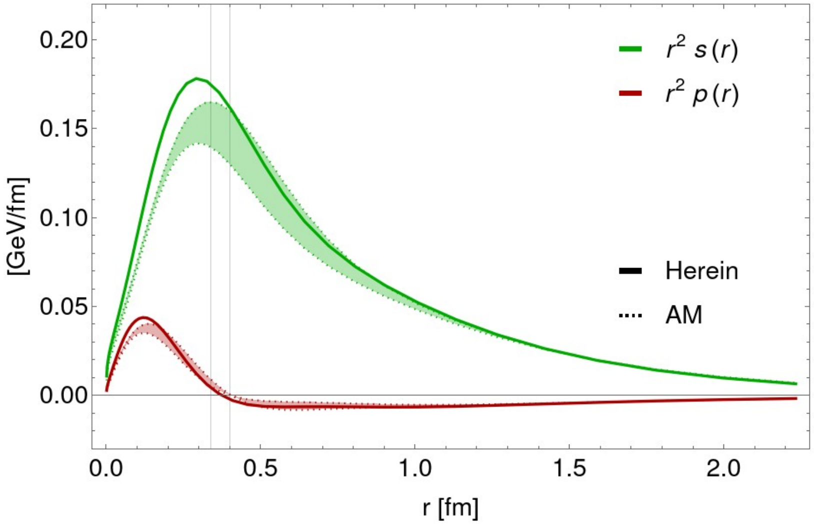
Results: Pressure profiles

- **Pressure** and **shear** forces are obtained from $\theta_1(Q^2)$:

$$p_{\mathbf{P}}(r) = \frac{1}{6\pi^2 r}\int_0^\infty d\Delta\,\frac{\Delta}{2E(\Delta)}\,\sin(\Delta r)[\Delta^2\theta_1^{\mathbf{P}}(\Delta^2)]$$

$$s_{\mathbf{P}}(r) = \frac{3}{8\pi^2}\int_0^\infty d\Delta\,\frac{\Delta^2}{2E(\Delta)}\,j_2(\Delta r)\,[\Delta^2\theta_1^{\mathbf{P}}(\Delta^2)]$$



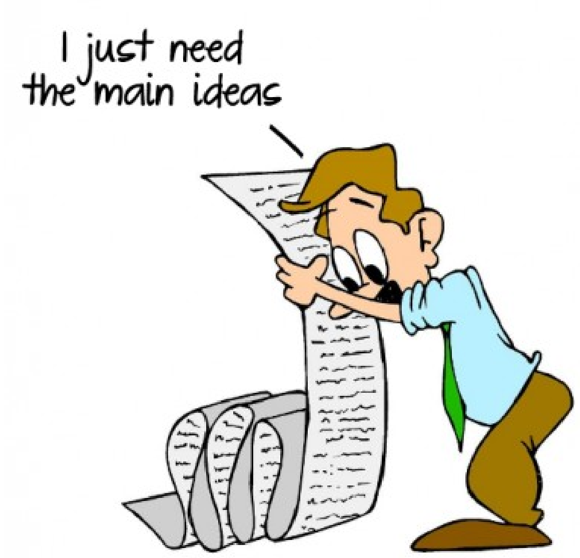


AM: Ingredients from **Overlap** but using the diagrammatic approach discussed herein.

`Raya:2021zrz`

Shear forces are maximal where the **pressure** shifts sign, i.e. where **confinement** forces become dominant.

Summary and Scope



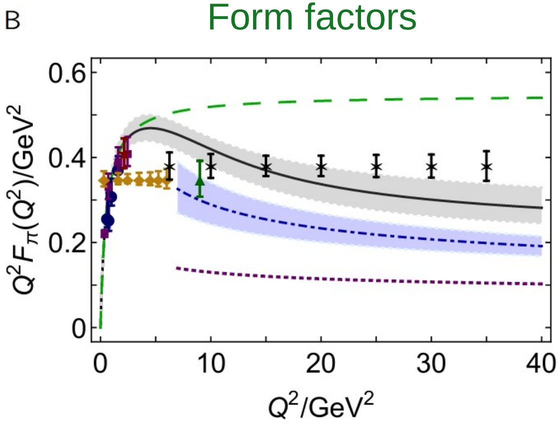
Summary and Scope

- We have described a **CSM based** computation of the pion **GFFs**.
    - The brand-new ingredient is the **QTV** entering the **game**.

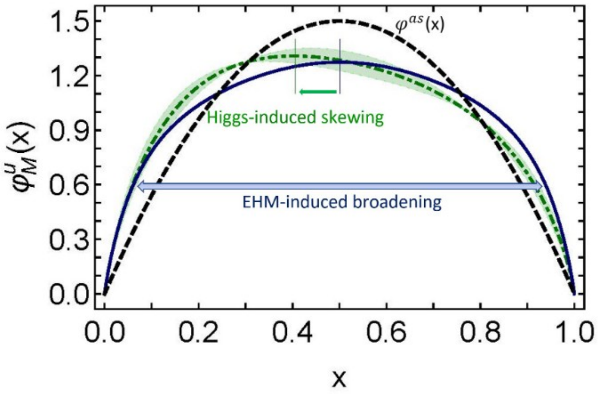
- The obtained results expose the **robustness** of the framework:
    - Many **pion-related** predictions exist within this formalism, *e. g.*:

Arrington:2021biu, Raya:2021zrz

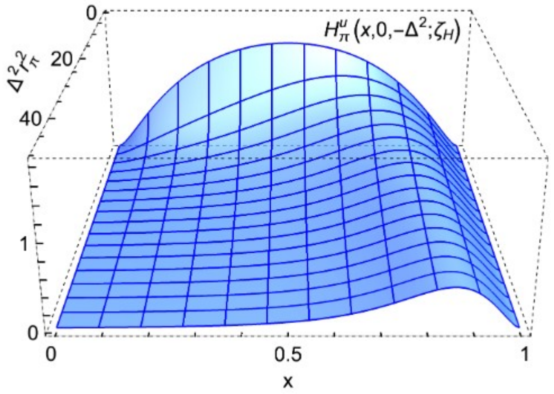
Form factors



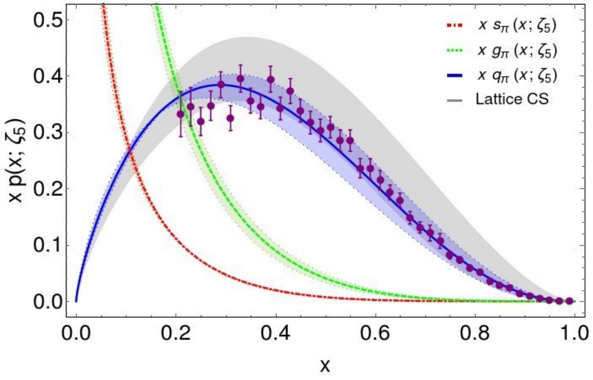
Distribution amplitudes



Generalized parton distributions



Distribution functions



Summary **and Scope**

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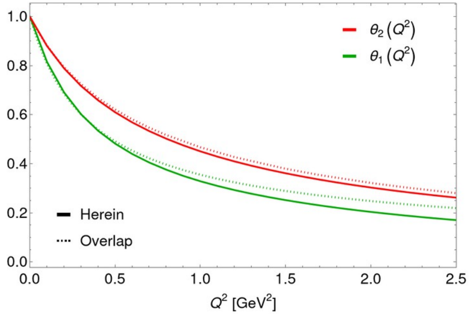
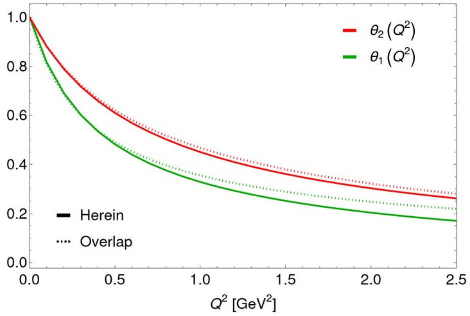
    - The brand-new ingredient is the **QTV** entering the **game**.



- The obtained results expose the **robustness** of the framework and the importance of **symmetries**:
    - Both **QPV** and **QTV** obey their own **WGTI** (current conservation)
    - This is sufficient to produce a sensible $\theta_2(Q^2)$ (mass distribution)
    - The **QTV** is completed by accounting for the **soft-pion** theorem, fixing the normalization of $\theta_1(Q^2)$ (internal forces)
    - **Beyond I.A.** diagrams are crucial to ensure $\sum_{q,g} \bar{c}(t) = 0$ (EMT conservation)

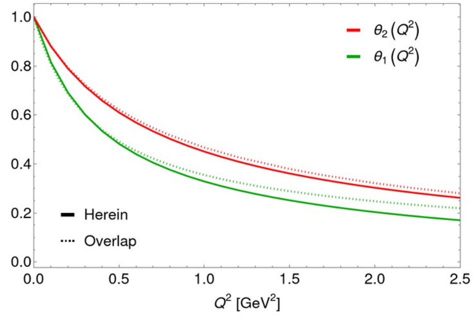
(irrelevant for the other 2 GFFs)

Summary **and Scope**



- We have described a **CSM based** computation of the pion **GFFs**.
- The obtained results expose the **robustness** of the framework and the importance of **symmetries.**
- **Physically** meaningful pictures are **drawn**:
  - **Charge** effects span over a larger domain than **mass** effects
  - **Shear** forces are maximal where **confinement** forces become dominant

Summary **and Scope**



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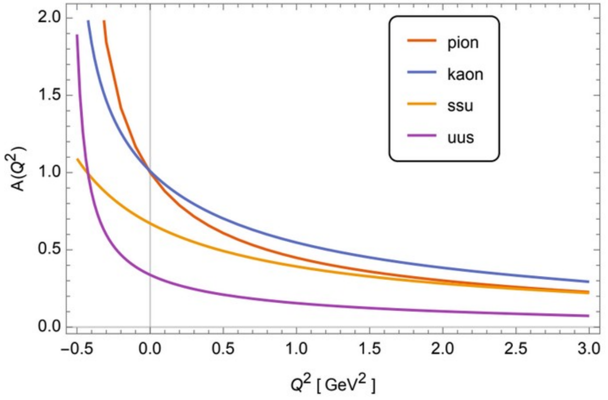
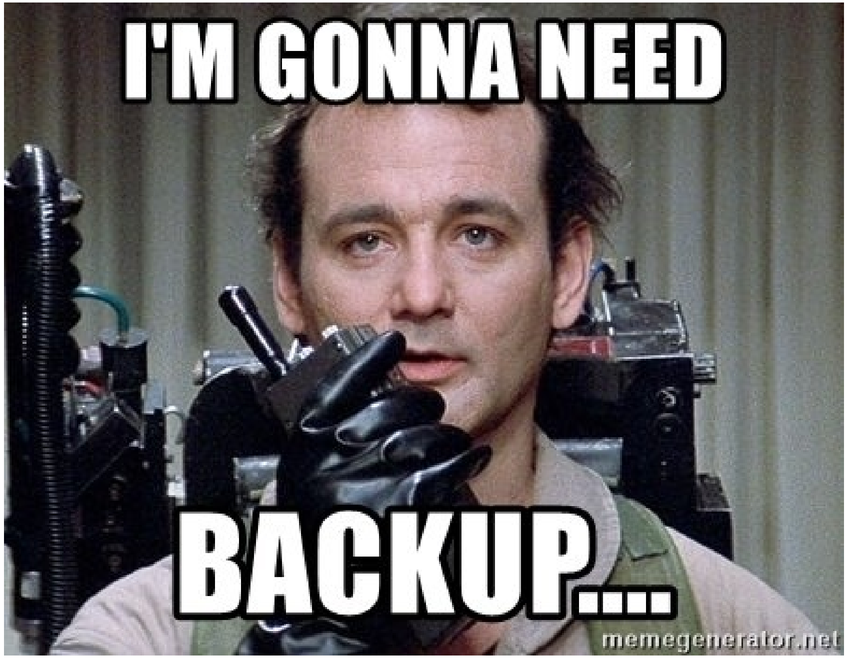


- Other hadrons are **within reach:**
  - Analogous investigations on the **kaon** are being performed
  - Subsequently, we will proceed with **heavy quarkonia**
- The **QTV** derived herein can also be used to produce a realistic picture of the **tensor form factor**.

Wang:2022mrh

Alexandrou:2021ztx

Backup



Scales and **Invariance**

- The **sum** of the **GFFs** of all **parton classes** within the hadron is **scale invariant**

$$\Lambda^{a}_{\mu\nu}(P,Q) = 2P_\mu P_\nu \theta^{a}_2(Q^2) + \frac{1}{2}\left(Q^2 g_{\mu\nu} - Q_\mu Q_\nu\right)\theta^{a}_1(Q^2) + 2m_\pi^2 g_{\mu\nu}\bar{c}^{a}(Q^2)$$

… but not the indivitual GFFs

- We perform our computations at the **hadronic scale**: **valence quarks** contain **all** of the properties of the hadron.
- Thus, our **symmetry constraints** are: $\theta_2^{u+\bar{d}}(0) = 1 = \theta_1^{u+\bar{d}}(0)\,,\ \bar{c}^{u,d}(Q^2) = 0$
- Evolution will unveil gluon and sea content!

ζ_H

$\zeta > \zeta_H$

Evolution equations

Experimental Access

- The natural way to address **GFFs** is via a **spin-2 probe**; this is rather **unviable** from the experimental point of view...

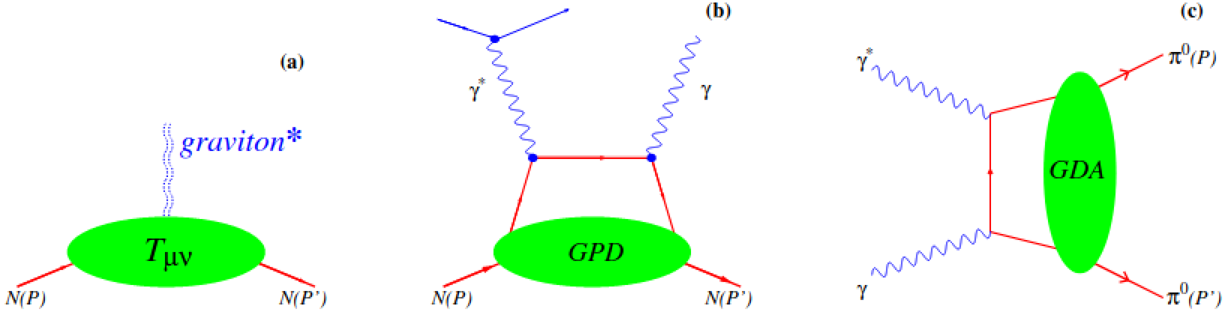
- However, those can also be accessed via **generalized parton distributions** and **generalized distribution amplitudes**.

$$\int_{-1}^{1} dx\, x\, H_{\mathsf{P}}^{q}(x,\xi,-\Delta^2;\zeta_{\mathcal{H}}) = \theta_2^{\mathsf{P}}(\Delta^2) - \xi^2\theta_1^{\mathsf{P}}(\Delta^2)$$

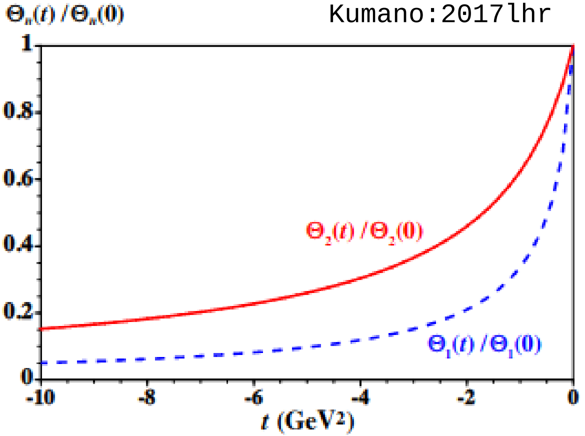
(e.g. first moment of the GPD)

(e.g. from GDA)

$$\int_0^1 dz(2z-1)\,\Phi_q^{\pi^0\pi^0}(z,\,\zeta,\,W^2) = \frac{2}{(P+)^2}\langle\,\pi^0(p)\,\pi^0(p')\,|\,T_q^{++}(0)\,|\,0\,\rangle$$



Polyakov:2018zvc



Kumano:2017lhr