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Study of the growth of the total differential effective cross section as a function of the scale parameter, in the limit of low x for an energy dependent geometric scaling model.

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Outline

Optical Theorem

In scattering theory, the optical theorem relates the imaginary part of the scattering amplitude with the differential cross section with:

$$\sigma_{\text{tot}} = \frac{4\pi}{k}\Im[f(0)] \tag{1}$$

Froissart-Martin Limit

In 1961, Marcel Froissart proves that the total scattering cross-section do not increase faster than the square of the logarithm of the energy as this energy increases.
For a pair of interacting particles via a Yukawa potential $V_{Yuk}(r)$:

$$V_{Yuk}(r) = \frac{ge^{-\kappa r}}{r} \tag{2}$$

For the $b \to \infty$ limit:

$$\lim_{b \to \infty} e^{-\kappa b} = 0$$

For the other case $b \to 0$:

$$e^{-\kappa b} \to 1$$

Froissart-Martin Limit

Taking the minimum for b :

$$\begin{aligned} ge^{-\kappa b} &= 1 \\ b\kappa &= \ln g \end{aligned} \tag{3}$$

Putting this into the equation for σ_{tot}:

$$\sigma_{tot} \simeq \left(\pi/\kappa^2\right) \ln^2 |g|$$

doing g be the energy s :

$$\sigma_{tot} \simeq \left(\pi/\kappa^2\right) \ln^2 |s|$$

Grey Disc Model

Based on this two concepts we follow the proposed model which aims to explain cross section energy growing from MeV energy scale to black disk limit. In this model the σ_{total} are described with a parametrization of the apparent proton radius $R(s)$ and the density of gluonic matter $f(s)$ [4]:

$$\begin{aligned} \sigma_{\text{total}} &= 2\pi R^2(s) f(s) \\ \sigma_{\text{elastic}} &= \pi R^2(s) f^2(s) \end{aligned} \tag{4}$$

$R(s)$ is given by[5]:

$$R(s) = R_0 + \beta \ln\left(\frac{s}{s_0}\right) \tag{5}$$

Grey Disc Model

With s_0 an energy dependent parameter, R_0 a constant related with the valence quarks of the projectiles and the initial distribution.
We can relate σ_{total} with this $f(s)$:
First, the σ_{total} is:

$$\sigma_{total}(s) = 2\pi \int db^2 \, \mathrm{Im}\, G(s,b) \tag{6}$$

Where $G(s,b)$ is the elastic amplitude, in geometrical scaling we have:

$$\longrightarrow 2\pi R^2(s) \int^1 d\beta^2 \, \mathrm{Im}\, G(\beta) \tag{7}$$

Grey Disc Model

Where:

$$\beta^2 = \frac{b^2}{\sigma_{total}} \tag{8}$$

With b impact parameter.
We define the scalling variable τ[6]:

$$\tau \equiv -t\sigma_{total} \tag{9}$$

With t as a momentum conjugate variable of the impact parameter.
Combining equations (6), (9)and doing the variable change between (10) and (11) we have:

Grey Disc Model

$$-\int \frac{b^2}{\tau} dt \, \mathrm{Im}\, G(t) = f(s) \tag{10}$$

For the $f(s)$ we expect that in the high energy limit it tends to 1, in the eikonal representation:

$$f(s) = 1 - \exp[-2 \left(\gamma_1 + \gamma_2 \ln s + \gamma_3 \ln^2 s\right)] \tag{11}$$

With $\gamma_1 = 0.29, \gamma_2 = -0.0191, \gamma_3 = 0.0013352$.

Black Disc Limit

This limit occurs when in the high-energy limit, the elastic cross-section reaches the maximum fraction of the total cross-section and is related to the picture of scattering as one of total or partial absorption. As we are talking about an asymptotic limit, the ratio of the elastic to the total cross-section plays an important role.

Black Disc Limit

The quotient between them should be bounded by [10] :

$$\frac{\sigma_{\text{elastic}}}{\sigma_{\text{total}}} \leq 1/2$$

putting this into the previous expressions:

$$\frac{\sigma_{\text{elastic}}}{\sigma_{\text{total}}} \leq 3/4$$

Geometrical Scaling

The idea of geometric scaling is originally due to Dias de Deus [14] and describes the point in wich a minimum diffractive occurs. The black-disk limit is not reached even until $\sqrt{s} = 57TeV$ [15] and the dip structure being anchored upon it, is hence violated.

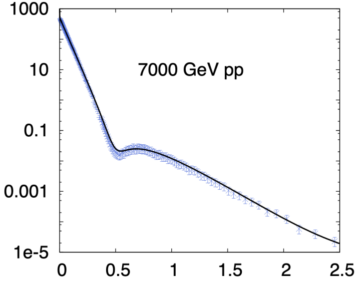

Figure: dip posotion[16] fitted to experimental data *pp* of elastic scattering: $d\sigma/dt$ in mbGeV^{-2} vs t in GeV^2.

Geometrical Scaling

Our job, is try to find the point in wich the dip are located with the equation(11):

$$\tau = -t\sigma_{total} \tag{12}$$

In G.S. the dip position is:

$$\tau_{DB} = -t_{dip}\sigma_{total} \tag{13}$$

With $\tau_{\text{BD}} = 35.92 GeV^2 mb$, we solved for $-t_{dip}$ and put it into σ_{total} (6) to obtain:

Geometrical Scaling

$$-t_{dip} = \frac{1}{2\pi R^2(s)} \frac{1}{f(s)} \tau_{BD} \tag{14}$$

substituting it into $R(s), f(s)$:

$$-t_{dip} = \frac{\tau_{BD}}{2\pi} \frac{1}{\left[R_0 + \beta \ln\left(\frac{s}{s_0}\right)\right]^2} \frac{1}{1 - \exp[-2\left(\gamma_1 + \gamma_2 \ln s + \gamma_3 \ln^2 s\right)]} \tag{15}$$

We fit this equation for differential cross section $d\sigma_{tot}/dt$

Results

A fit for σ_{tot} were made with data from Particle Data Group [17] in a rank for $\sqrt{s} = 10 - 4.8 \times 10\ GeV$

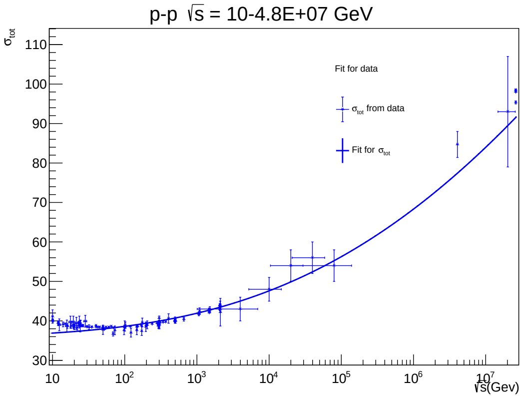

Figure: Fit for σ_{tot}.

Results

Then, we made the fit for $-t_{dip}$ with data from $d\sigma_{tot}/dt$ for [18,19,20,21,22,23]

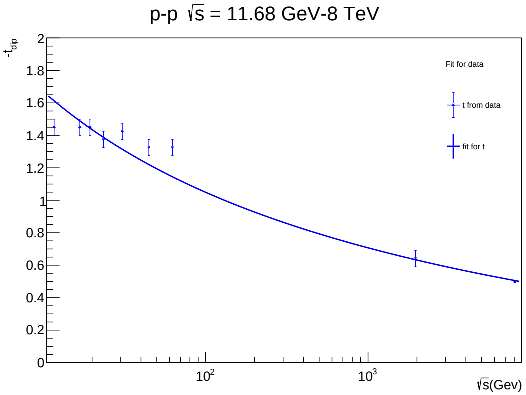

Figure: Fit for $-t_{dip}$.

Discussion

In the Fit for the σ_{tot} our model reproduces in good way the data at low energies, in the range $\sqrt{s}$ from 10 to 4.8×10^5 GeV. We need more data for energies beyond 10^5.

For the fit for $-t_{dip}$ from 11.68 to 8×10^3 GeV the fit is moderately good at low energies, unfortunately there is no enough data to compare with our model.

Thank you

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