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Entanglement entropy in high energy collisions of electrons and protons

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Based on

MH, K. Kutak, Eur.Phys.J.C 82 (2022) 2, 111 arXiv:2110.06156
MH, K. Kutak, R. Straka; arXiv:2207.0943

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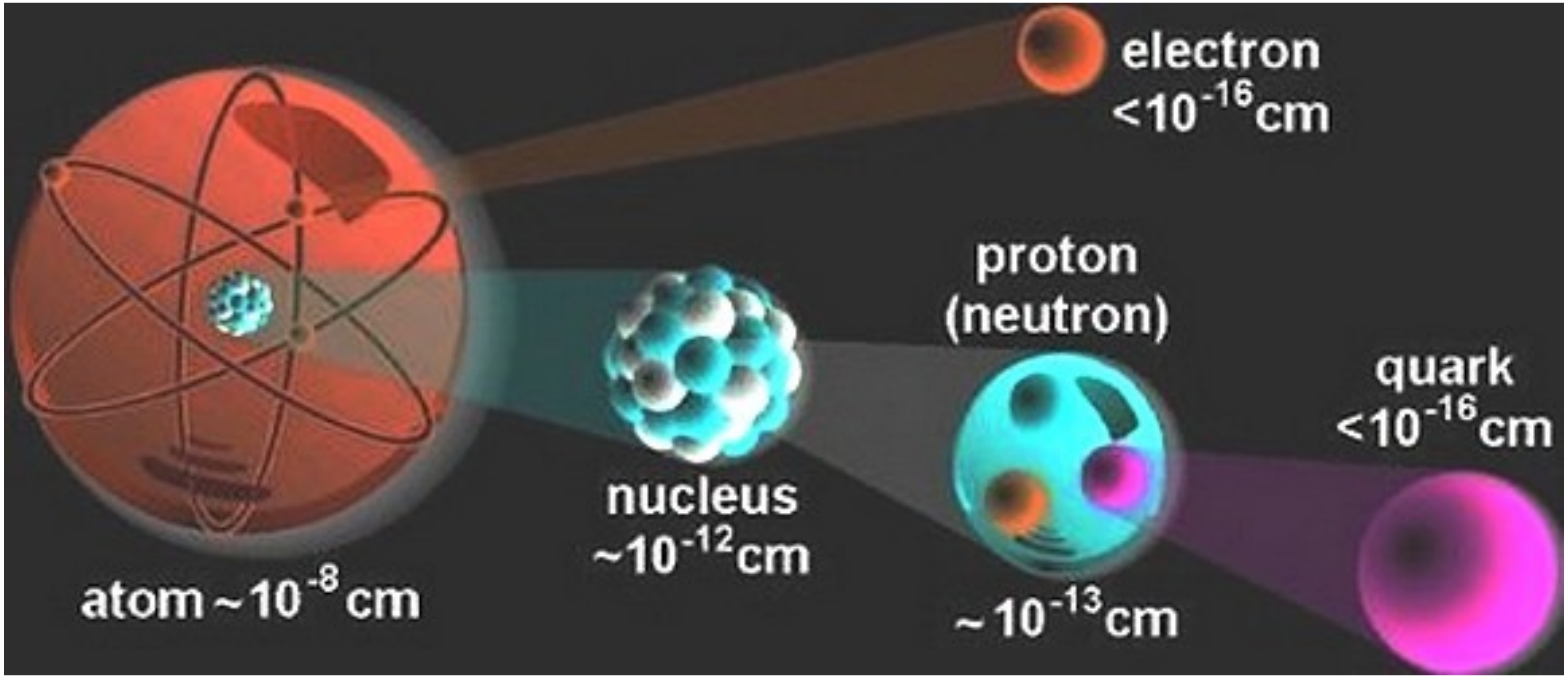
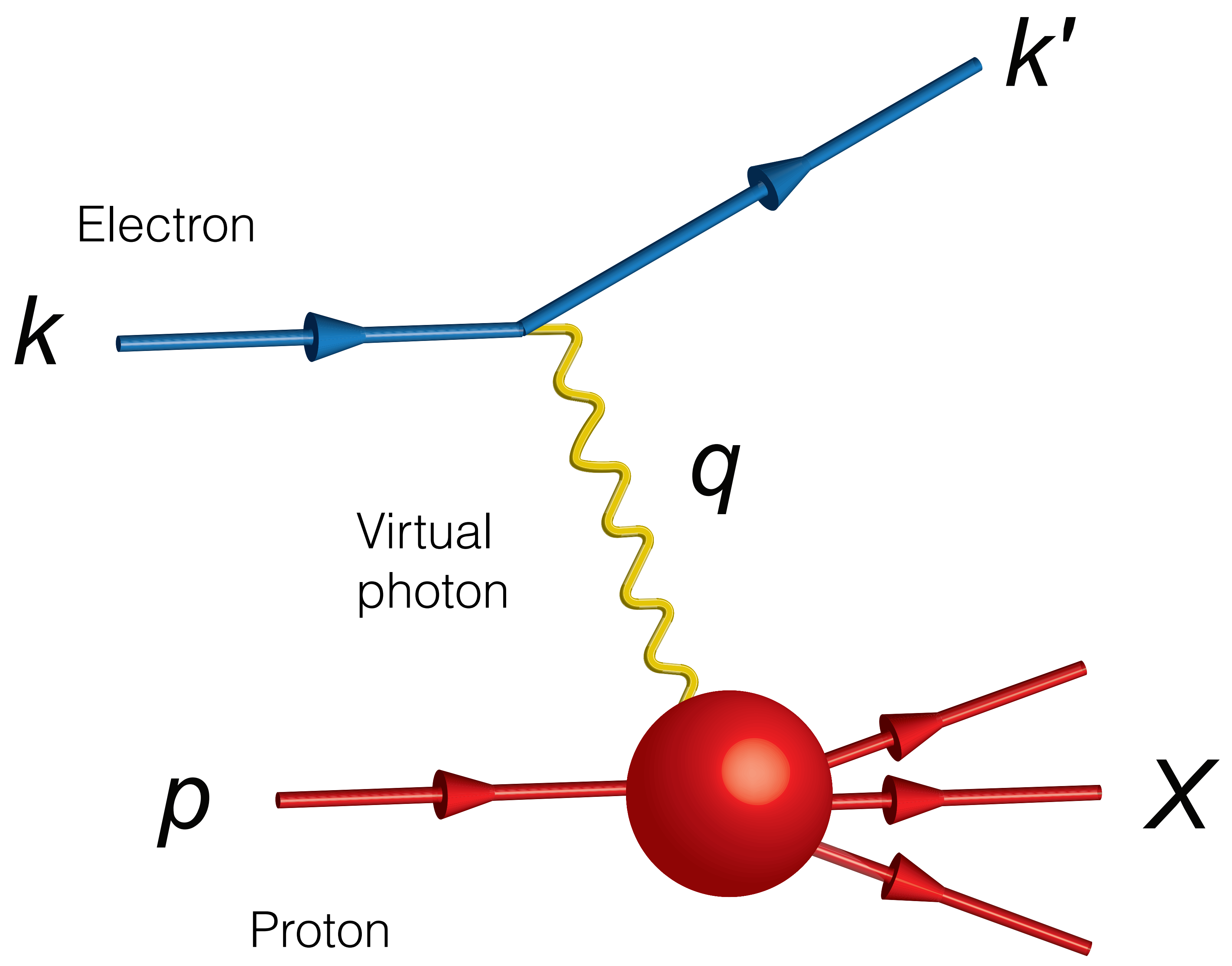
Exploring nuclear structure in electron nucleus collisions



Proton breaks up = Deep Inelastic Scattering (DIS)



Elastic scattering: either $Q = 0$ or $x = 1$

Photon virtuality (=resolution)

$$Q^2 = -q^2, \quad \lambda \sim \frac{1}{Q}$$

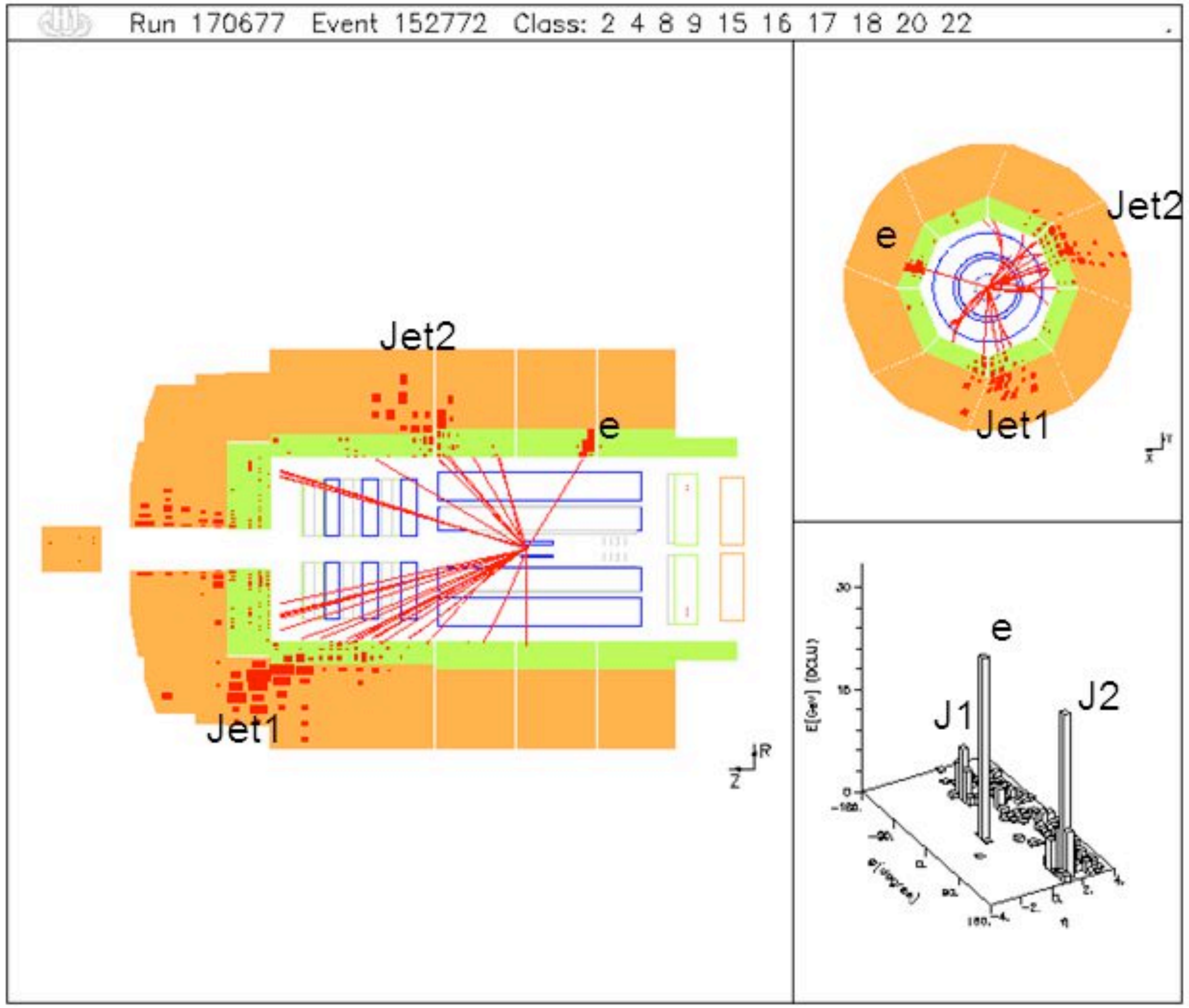
Bjorken x

$$x_{Bj.} = \frac{Q^2}{2p \cdot q}$$

“Mass" of the system X

$$W^2 = (p+q)^2 = M_p^2 + \frac{1-x}{x} Q^2$$

A NC-DIS event with two jets $ep \to e' Jet_1 Jet_2$



H1 Events Joachim Meyer DESY 2005

Puzzle:
proton = pure quantum state → zero von Neumann entropy
But produce a plethora of particles in DIS reaction

- [Tu, Kharzeev, Ullrich; 1904.11974]

Possible relation to the

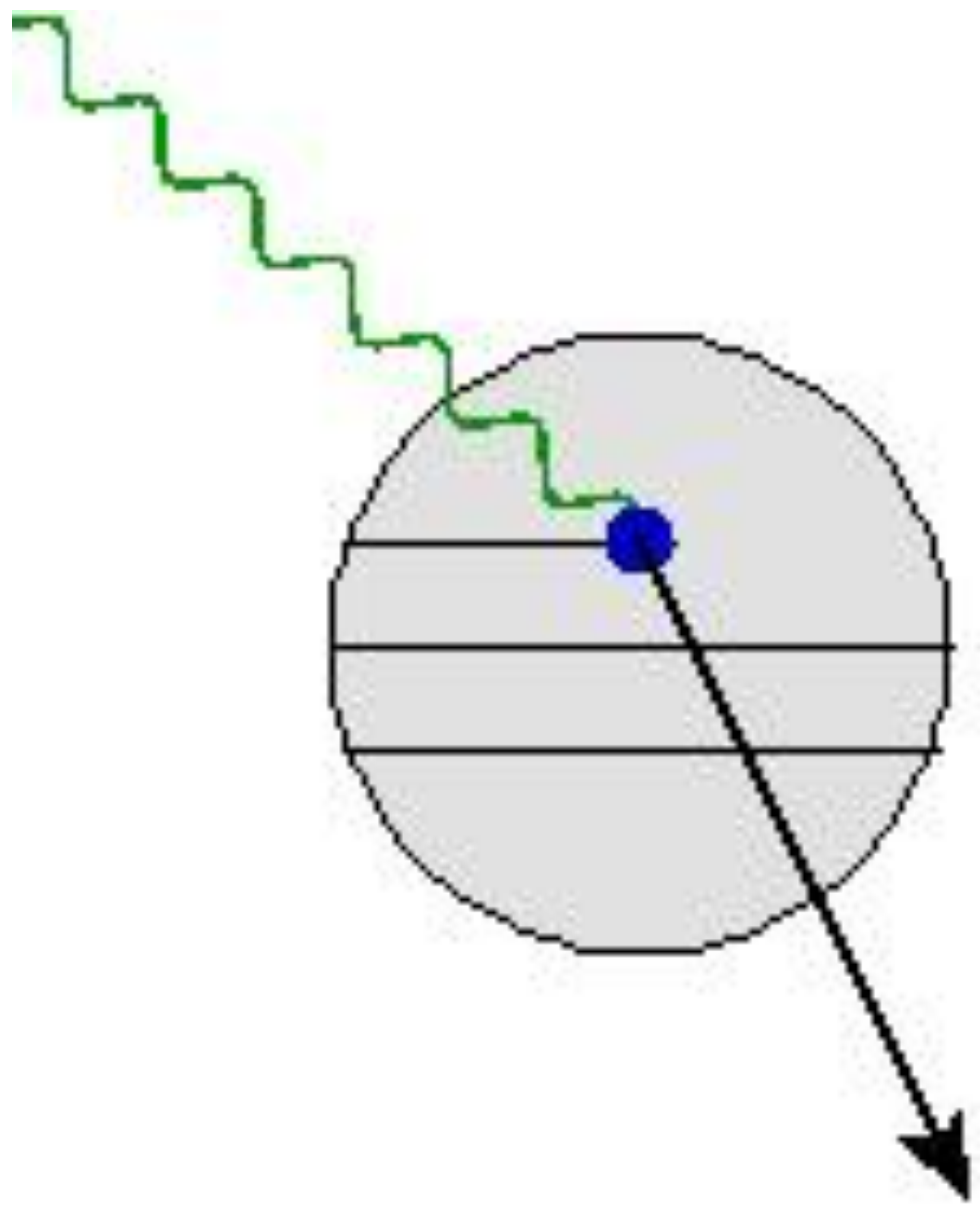
Einstein-Podolsky-Rosen (EPR) paradox

- 2 quantum systems are allowed to interact initially
- Later separated
- Measure physical observable of one system → immediate effect on conjugate observable in 2nd system

- Textbook example: 2 e^- in spin singlet *etc.*

$$|00\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right)$$

Einstein-Podolsky-Rosen (EPR) paradox in DIS



Interaction of virtual photon with 1 quark in Deep Inelastic electron proton Scattering (DIS)

Standard argument

- proton boosted to infinite momentum frame + probe 1 quark with virtual photon
- This quark is casually disconnected from the rest of the proton, *during the interaction*
- Reason why $\sigma_{hadron} = \hat{\sigma}_{parton} \otimes PDF$ works

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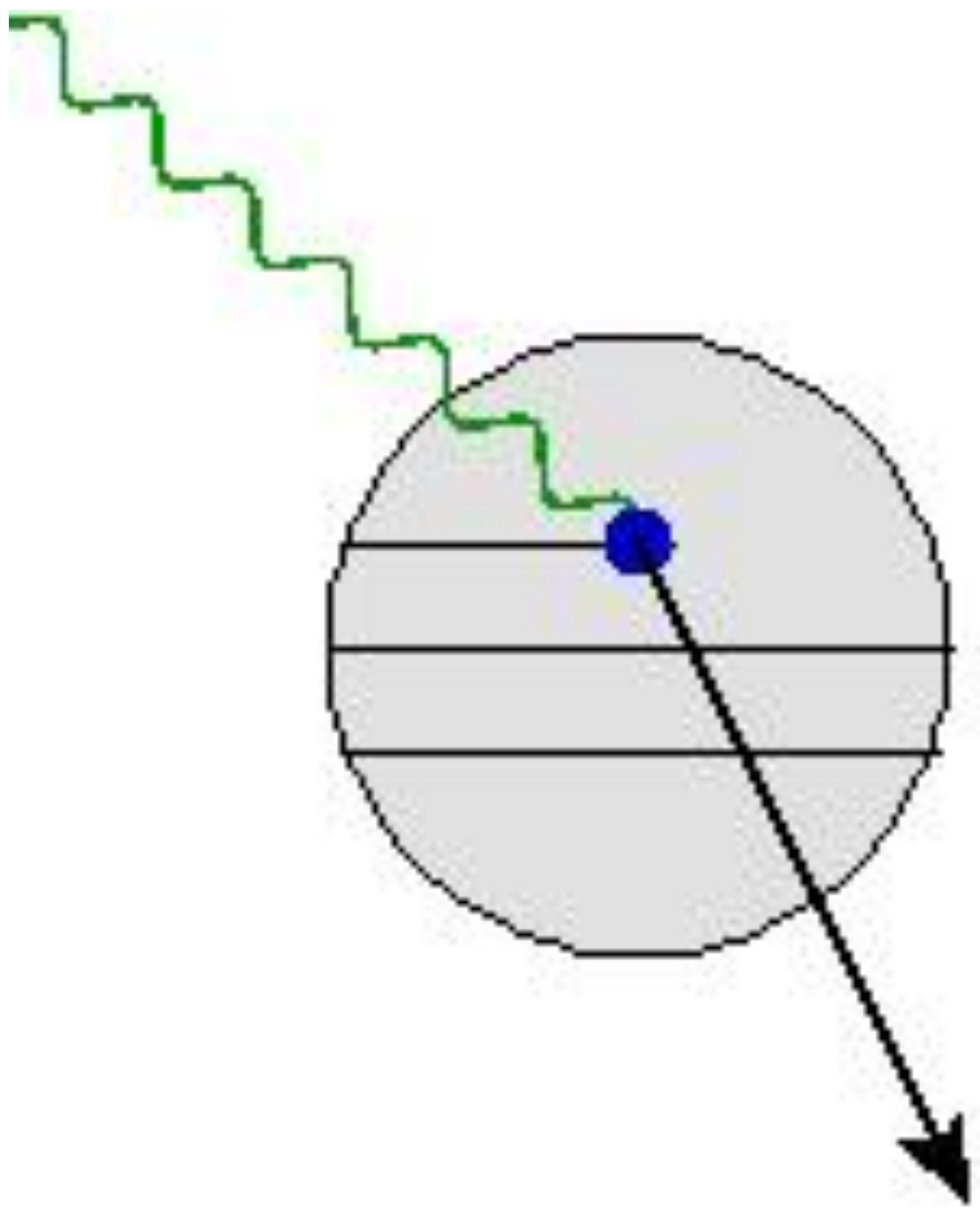


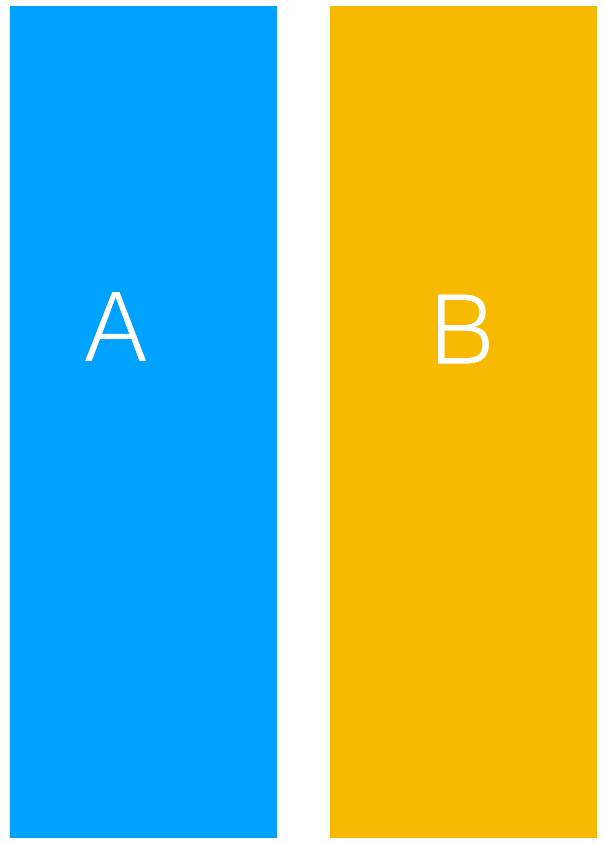
Interaction of virtual photon with 1 quark in Deep Inelastic electron proton Scattering (DIS)

But:

- struck quark + remainder form color singlet (confinement)→ strongly correlated quantum system
- EPR at subatomic scale: strongly correlated, but casually disconnected [Tu, Kharzeev, Ullrich; 1904.11974]
- Entangled system
- Observed entropy = entanglement entropy?

Entanglement entropy

Entanglement:
2 subsystems A and B



$|\Psi_{AB}\rangle = \sum_{j,k} \alpha_{jk} |\Psi_{A,j}\rangle \otimes |\Psi_{B,k}\rangle$ is entangled, but a pure state

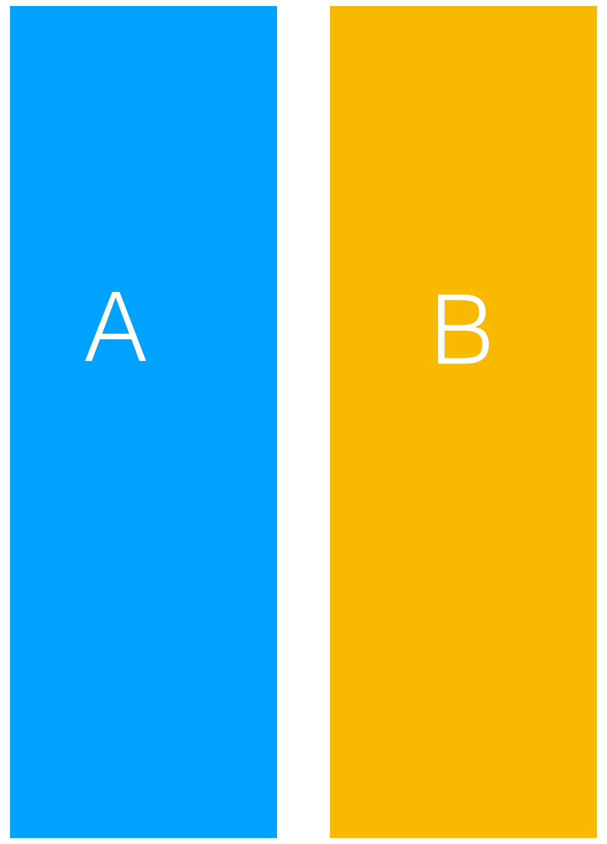
$$\rightarrow S_{AB} = -\text{tr}\hat{\rho}_{AB} \ln \hat{\rho}_{AB} = 0$$

$$\hat{\rho} = |\Psi\rangle\langle\Psi|$$

Density matrix of a pure state

Entanglement entropy

Entanglement:
2 subsystems A and B



Combined state can

- factorize $|\Psi_{AB}\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$
- Or not (it is “entangled”) $|\Psi_{AB}\rangle = \sum_{j,k} \alpha_{jk} |\Psi_{A,j}\rangle \otimes |\Psi_{B,k}\rangle$

Hilbert space: $\mathscr{H}_{AB} = \mathscr{H}_A \otimes \mathscr{H}_B$

$|\Psi_{AB}\rangle = \sum_{j,k} \alpha_{jk} |\Psi_{A,j}\rangle \otimes |\Psi_{B,k}\rangle$ is entangled, but a pure state

$\hat{\rho} = |\Psi\rangle\langle\Psi|$

Density matrix of a pure state

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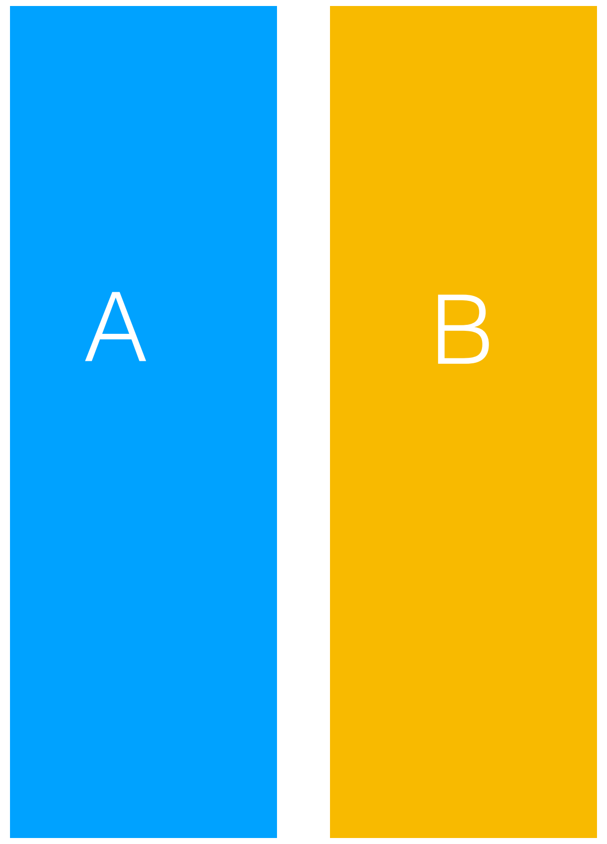
Entanglement & density matrix

Now: do not observe system B
QM: anything can happen in B → sum over all possibilities that can occur in the system B



For the density matrix of system A (observed): sum over all B states

Use Mathematical trick (Schmidt decomposition):

Density matrix of a mixed system, if state $|\Psi_{AB}\rangle$ was entangled

Entanglement & density matrix

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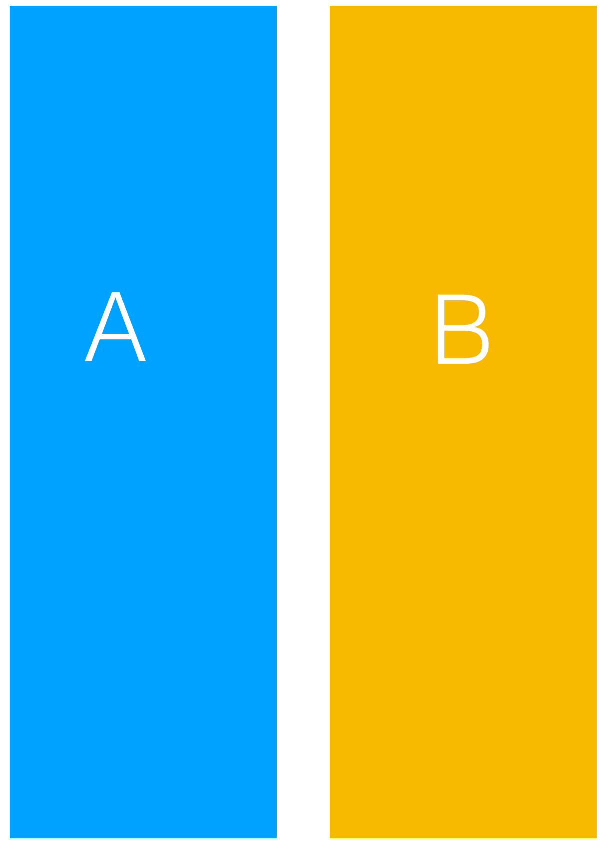


For the density matrix of system A (observed): sum over all B states

Use Mathematical trick (Schmidt decomposition):

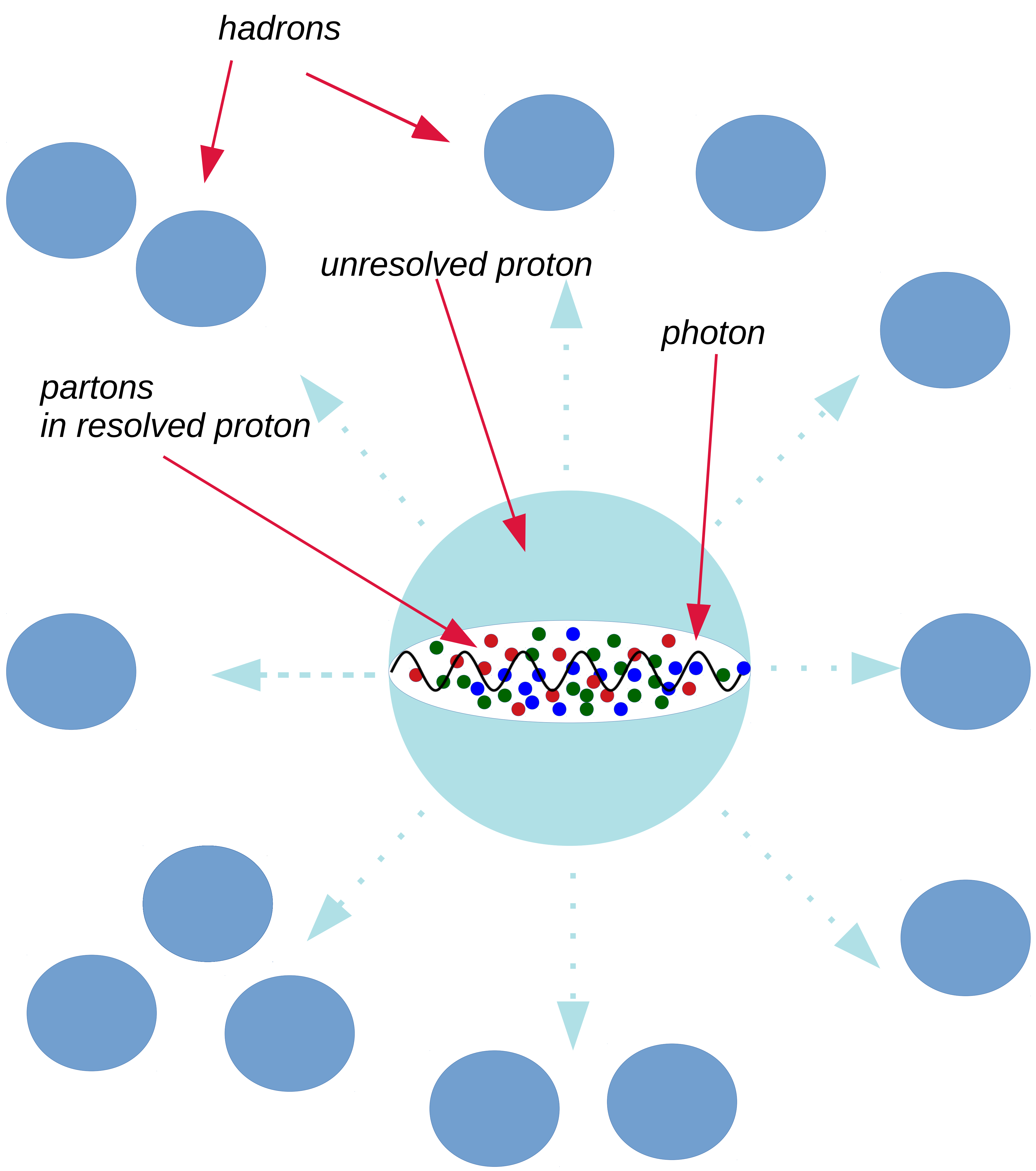
Density matrix of the subsystem A: $\hat{\rho}_A = \mathrm{tr}_B \hat{\rho}_{AB} = \sum_j p_j |\Psi_{A,j}\rangle\langle\Psi_{A,j}|, \qquad p_j = |\beta|_j^2$

Density matrix of a mixed system, if state $|\Psi_{AB}\rangle$ was entangled

Deep Inelastic Scattering

DIS: do not observe the entire proton, but only parts of it

[Gribov, Ioffe, Pomeranchuk, SJNP, 2, 549 (1966)];
[Ioffe, PLB 30B, 123, (1969)]



[Kharzeev, Levin; 1702.03489]

- Observed entropy = entanglement entropy

Demonstrating this, is a challenge …

- Pure state at $Q^2 \to 0$ = observe entire proton
- But this is the region, where $\alpha_s(Q)$ is not small $\neq$ perturbation theory; concept of quarks and gluons as degrees of freedom at least difficult
- Unobserved region subject to non-perturbative dynamics

Result by Kharzeev & Levin

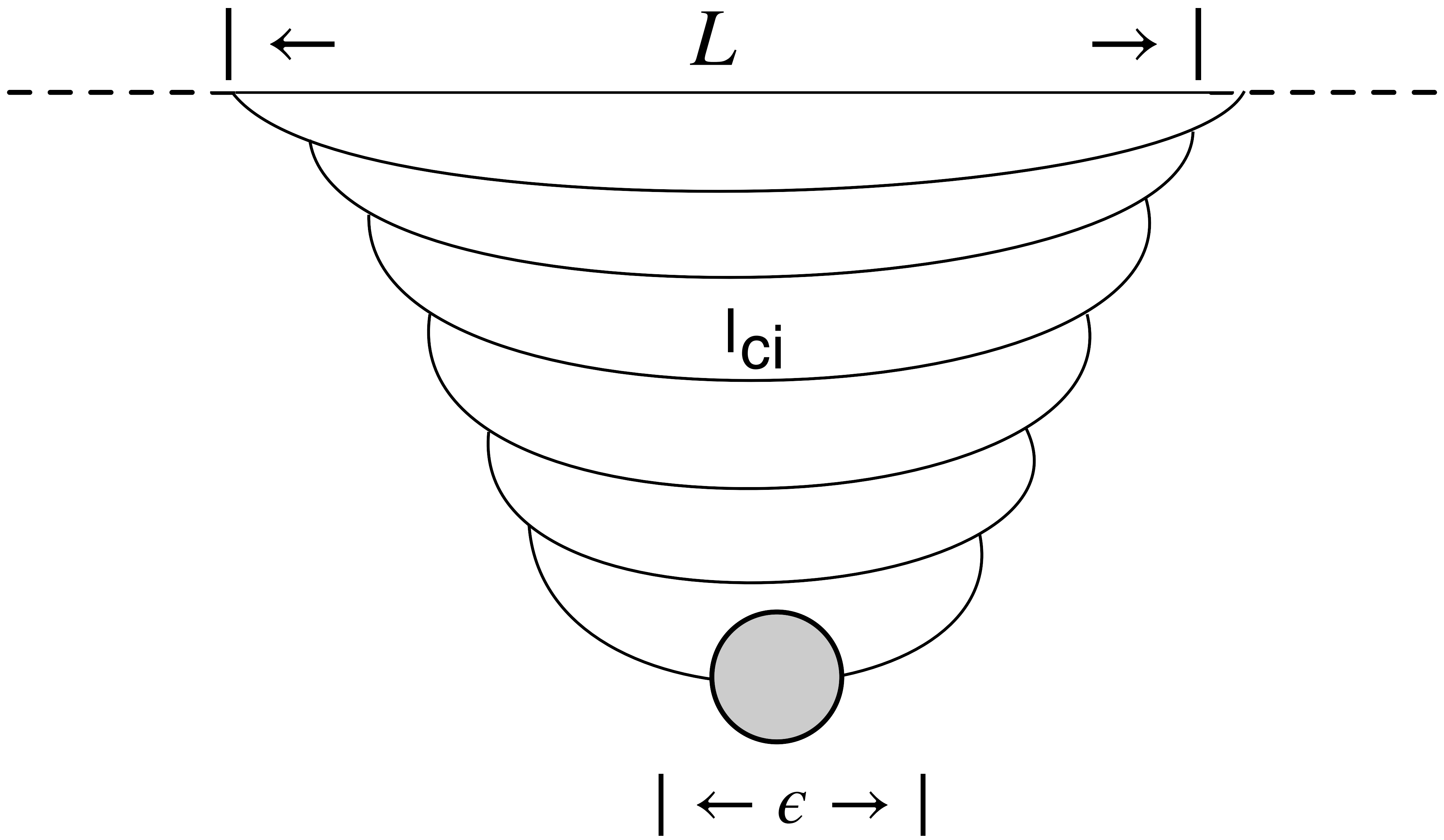
[Kharzeev, Levin; 1702.03489]

- Entanglement entropy was calculated for 2D conformal field theories [Holzhey, Larsen, Wilczek; 1994], [Calabrese, Cardi; 2006]
  L : studied region
  ϵ: regularization scale = resolution

$$S = \frac{c}{3} \ln \frac{L}{\epsilon}$$

- Identify $\epsilon = \frac{1}{m} \ll L = \frac{1}{x}\epsilon$, find $S = \frac{c}{3} \ln 1/x$

- Entropy in 1+1 toy model of non-linear QCD evolution (not entanglement): $S = \Delta \ln(1/x)$

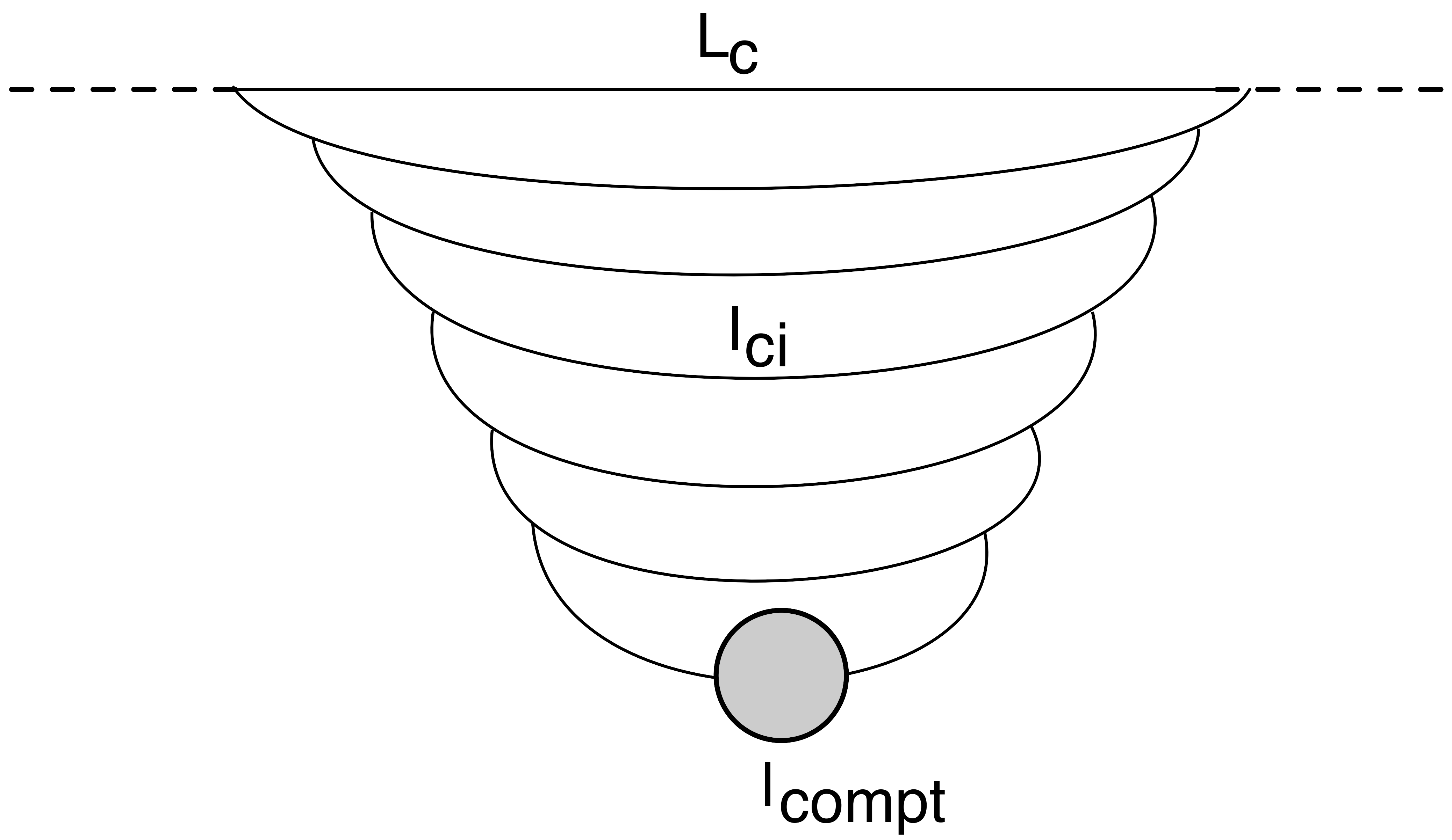


In the proton **rest frame**:

- parton (of the the photon) fluctuation over long. distance $L = \dfrac{1}{m_p x}$
- Proton probes partonic fluctuation with resolution $\epsilon = \dfrac{1}{m} \ll L = \dfrac{1}{x}\epsilon$
- Proton probes only region $\epsilon \ll L$ of the entire interaction

$$S = \frac{c}{3} \ln \frac{L}{\epsilon} = \frac{c}{3} \ln \frac{1}{x}$$

Figure taken from [Kharzeev, Levin; 1702.03489]



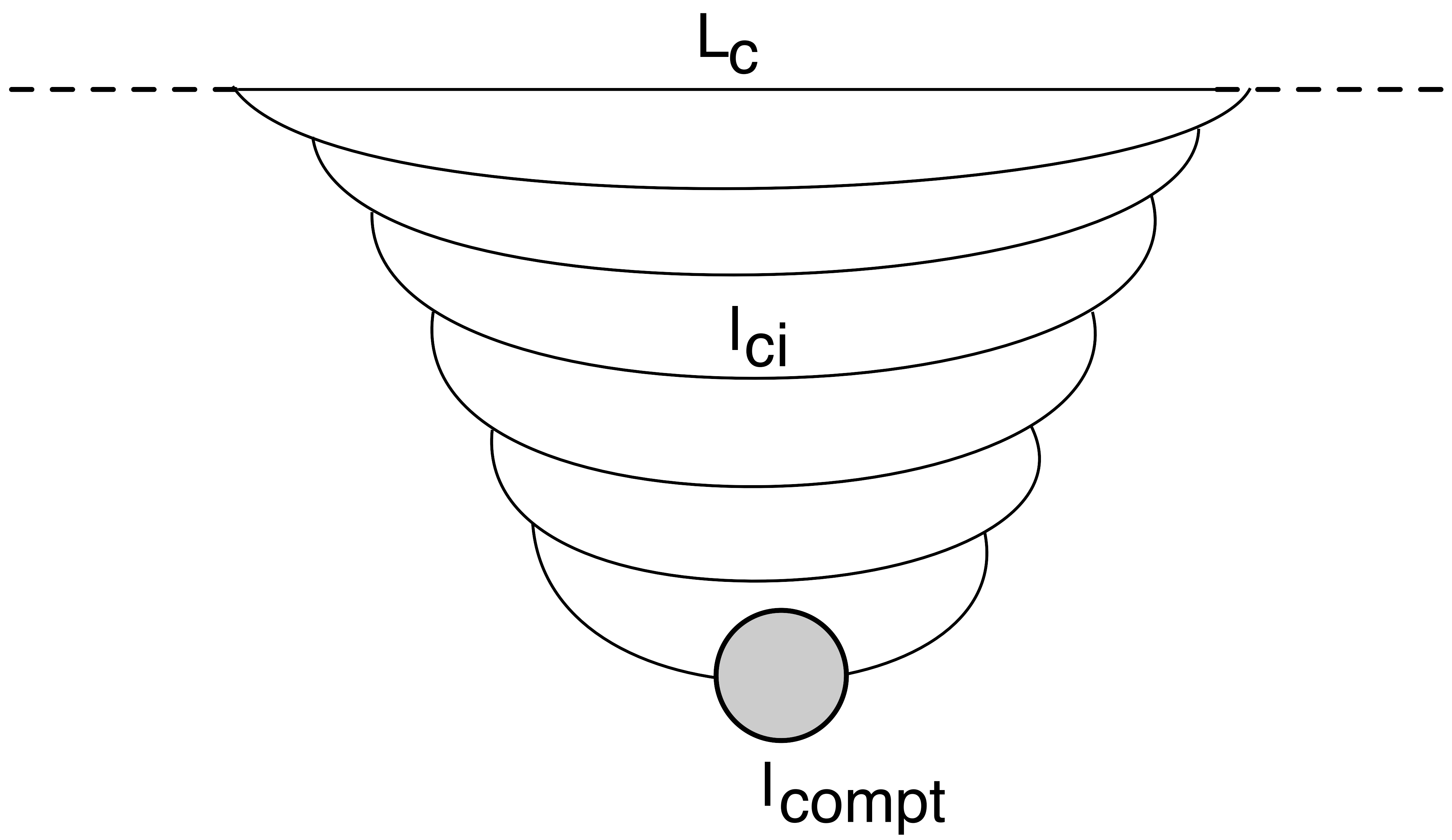
1+1 non-linear model of non-linear QCD evolution in $Y = \ln(1/x)$
[Levin, Lubinsky; arXiv:hep-ph/0308279]

$p_n(Y)$ probability to encounter n color dipoles (~gluons) in the proton

Subject to Cascade equation:

$$\frac{d}{dY}p_n(Y) = -\Delta n p_n(Y) + \Delta(n-1)p_{n-1}(Y)$$

Yields entropy $S = -\sum_n p_n \ln p_n \rightarrow \Delta Y = \Delta \ln 1/x$



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Conformal field theory $S = \frac{c}{3} \ln 1/x$

Result by Kharzeev & Levin

[Kharzeev, Levin; 1702.03489]

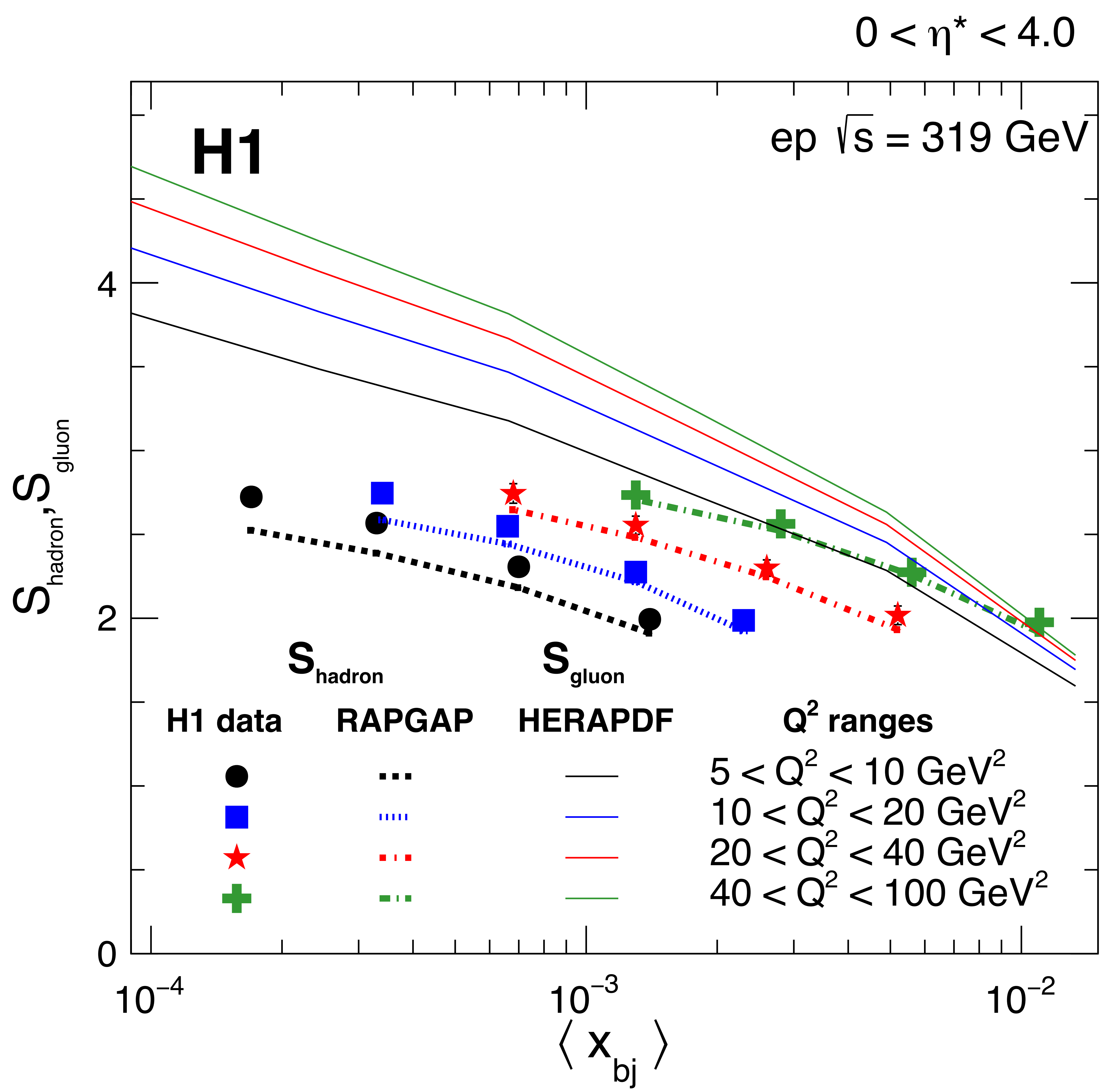
- 1+1 toy model of non-linear QCD evolution:
  - gluon distribution function $xg(x) = e^{\Delta \ln 1/x}$
  - $\langle n_{\text{gluons}} \rangle = xg(x)$
- Identification: $S = \ln \left[xg(x) \right] = \ln n_{\text{gluons}} \cdots\cdots$
- Additional proposal: (partonic) entropy = entropy of final state hadrons $S_h \sim S$
  → test this through event-by-event measurements of the hadronic final state in DIS

Where measure this?
- future: EIC
- Right now: analyze existing data of HERA
  → H1 Collaboration

$$S_{hadron} = \sum P(N) \ln P(N)$$

$P(N)$: particle multiplicity distribution

H1 collaboration: results [arXiv:2011.01812]



- [Kharzeev, Levin; 1702.03489] Particle # at certain $\ln 1/x$:

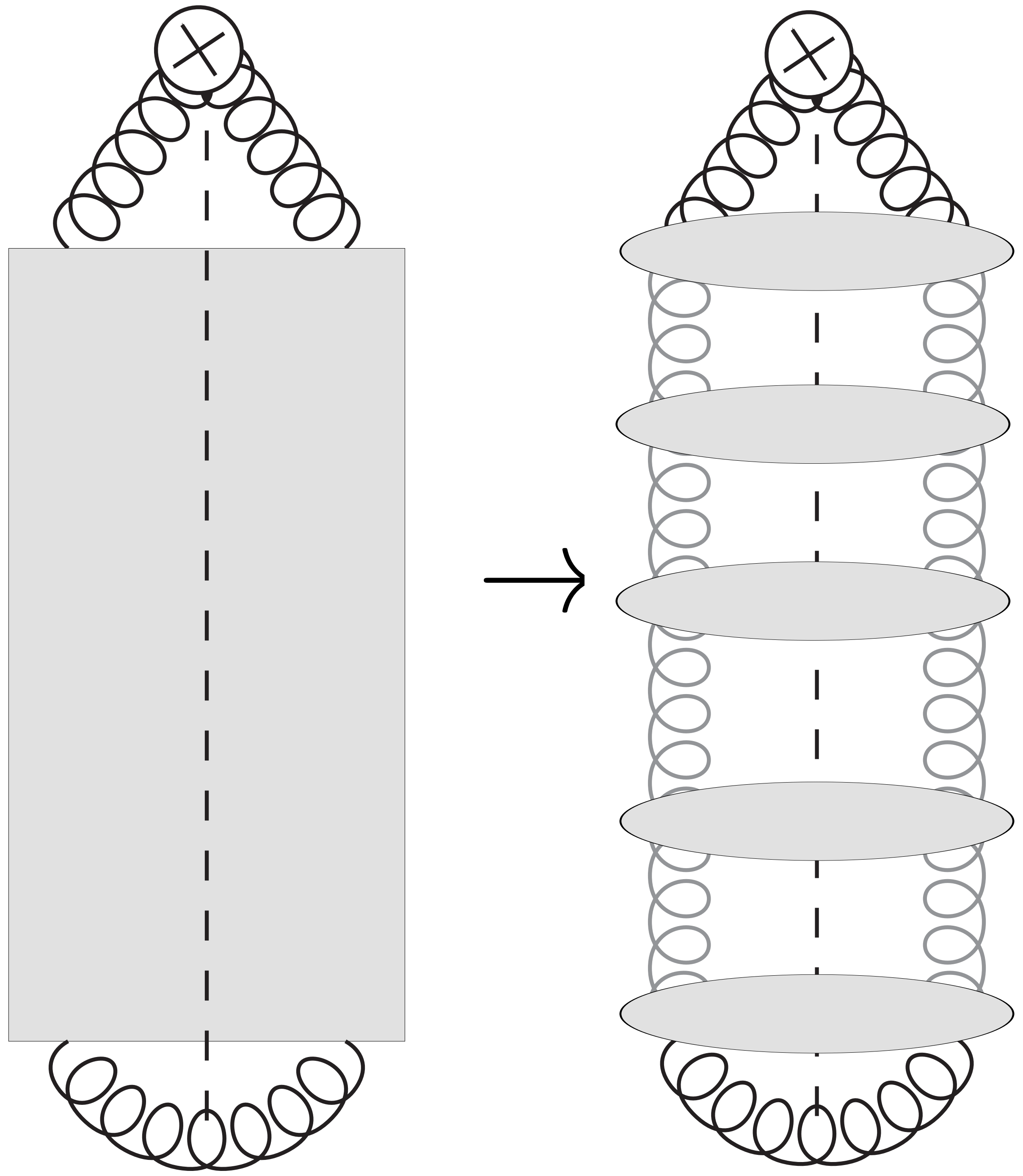
$$n_{partons} = xg(x, Q^2), \qquad S(x, Q) = \ln \left[xg(x, Q^2)\right]$$

- Reason: glue dominates at low x
- H1 collaboration: LO HERAPDF
- "The predictions from the entanglement approach based on the gluon density again fail to describe S_{hadron} in magnitude. However, at low Q the slope of S_{gluon} has some similarities with that observed for S_{hadron}, while it becomes steeper than observed with increasing Q"

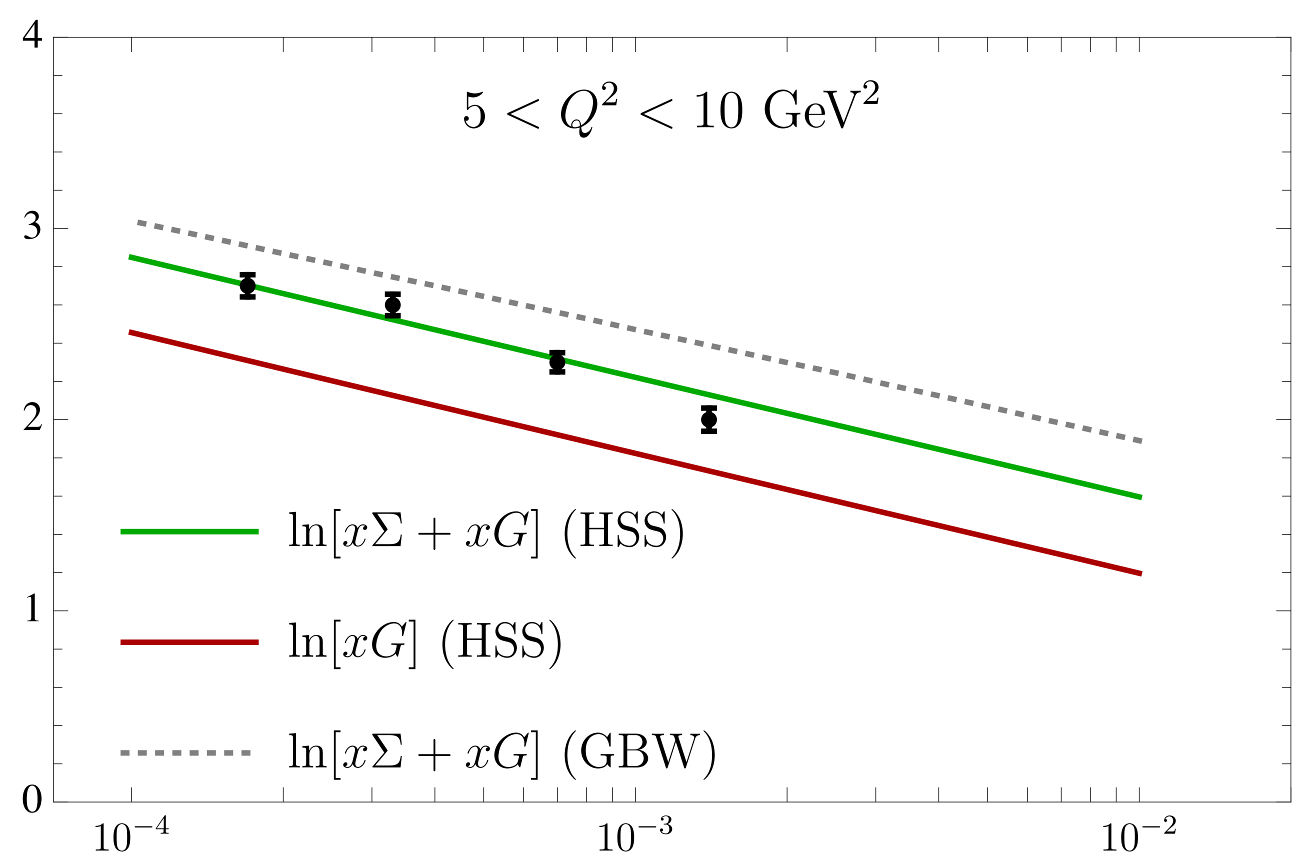
[Kharzeev, Levin; 2102.09773]: try something based on LO BFKL & seaquarks

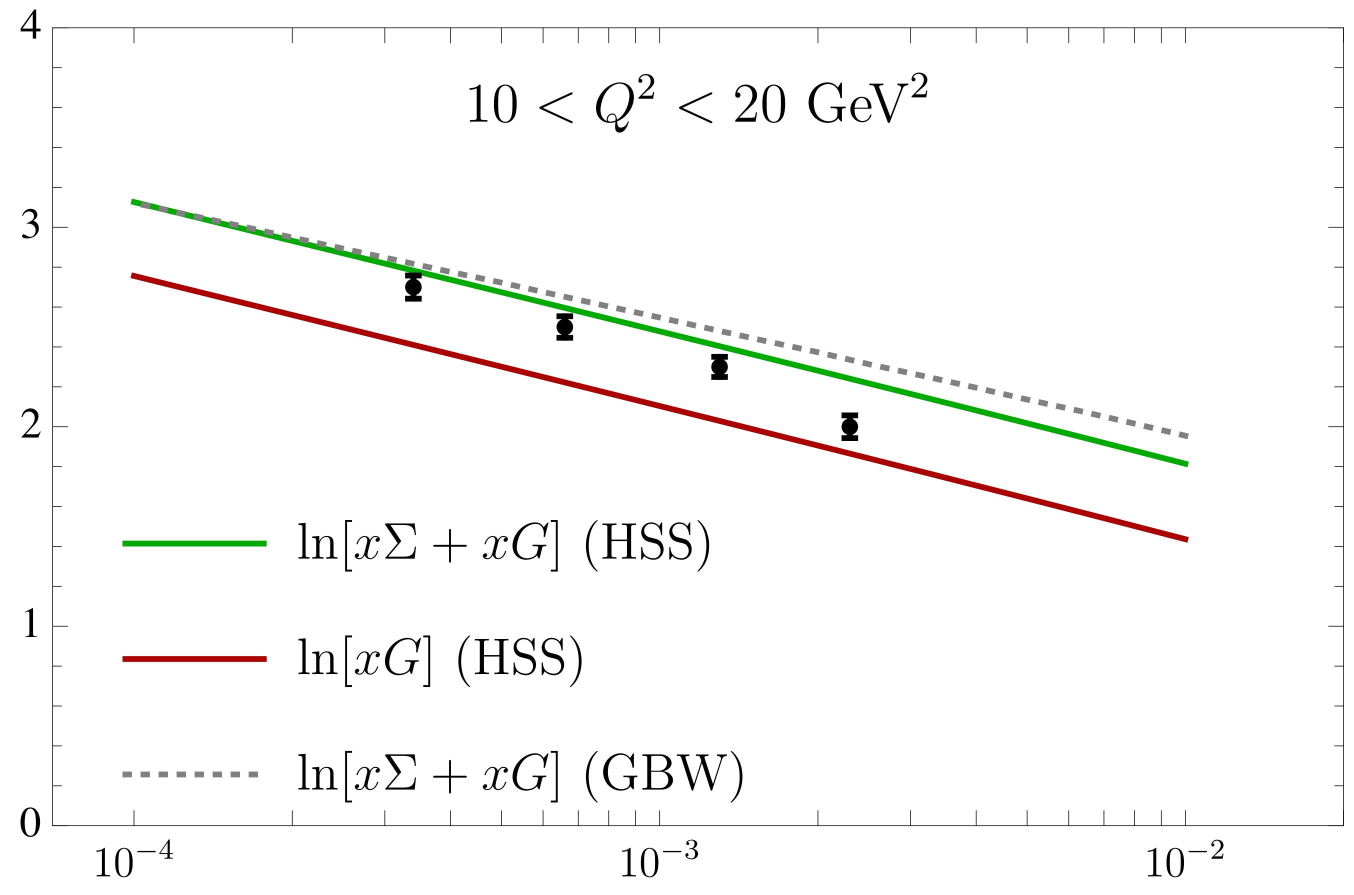
Our approach: PDFs from unintegrated gluon

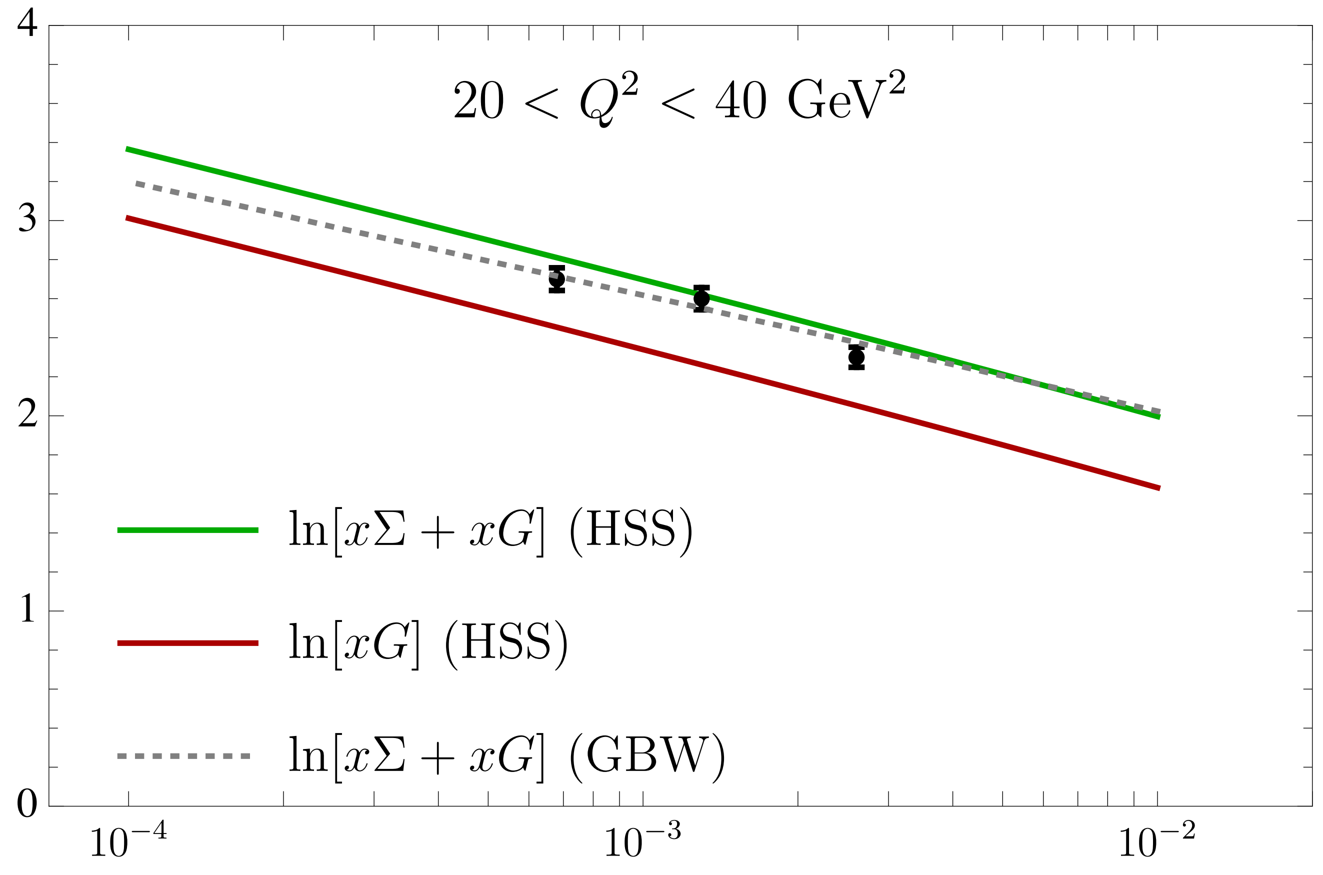
[Catani, Hautmann, NPB 427 (1994) 475]: idea: use collinear factorization in light-cone gauge
[Curci, Furmanski, Petronzio; NPB 175 (1980) 27]
→ calculate all order low x resumed DGLAP splitting functions

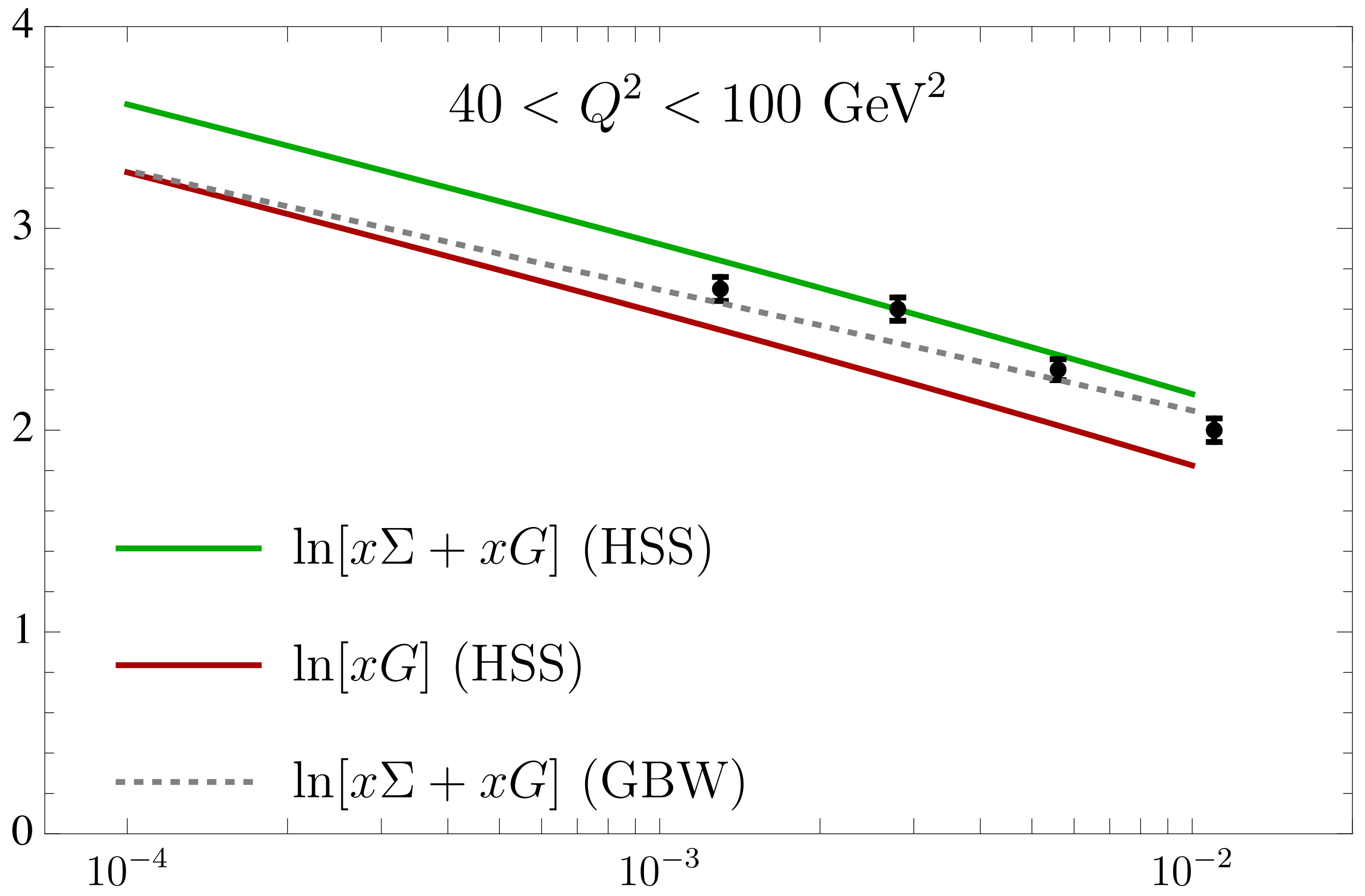


- Yields Transverse Momentum splitting function for gluon - quark splitting
- Splitting = collinear PDF with partonic initial state
- Can calculate gluon and seaquark PDFs from BFKL unintegrated gluon distribution, subject to $\ln(1/x)$ evolution
  see also [Hautmann, MH, Jung; 1205.1759]









Based on [Kharzeev, Levin; 2102.09773]: only seaquark → not even close to data
Gluon alone is better

Proposal: why # of gluons, better: # of partons = quarks + gluons

Great happiness, but there are some flaws …

- incorrect normalization constant for HSS gluon → correct constant overshoots data

- H1 collaboration measures charged hadron multiplicity, yet we calculate entropy for all hadrons
  roughly related by a factor 2/3

$$S_{part.}(x) = \ln\left[\frac{xg(x)}{B}\right] + 1 + \mathcal{O}\left[\frac{B}{xg(x)}\right]$$

- In the model: $xg(x) = C \cdot e^{\Delta Y}$ possible + possible (pre-asymptotic constant in expansion of entropy)

  for $S \sim 3.5$, this makes a difference

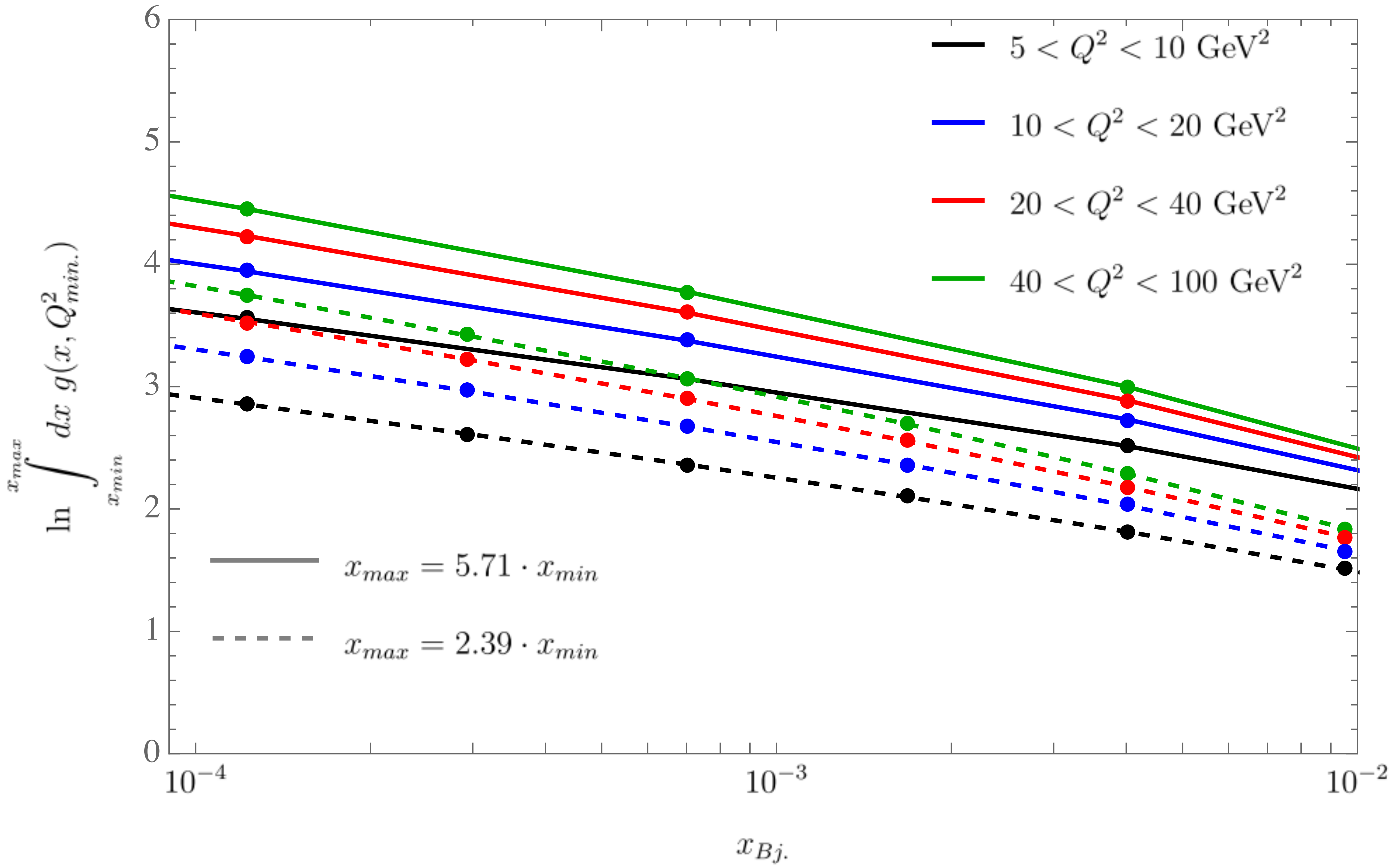
Integrate PDF (somehow) number of patrons

$$n_g(Q^2) = \int_0^1 dx\, g(x, Q^2),$$

H1: (seems) # of partons in a certain bin

$$n_g(\bar{x}) = \int_{x_{\rm min}}^{x_{\rm max}} dx g(x, Q^2),$$

Problem: depends on bin size

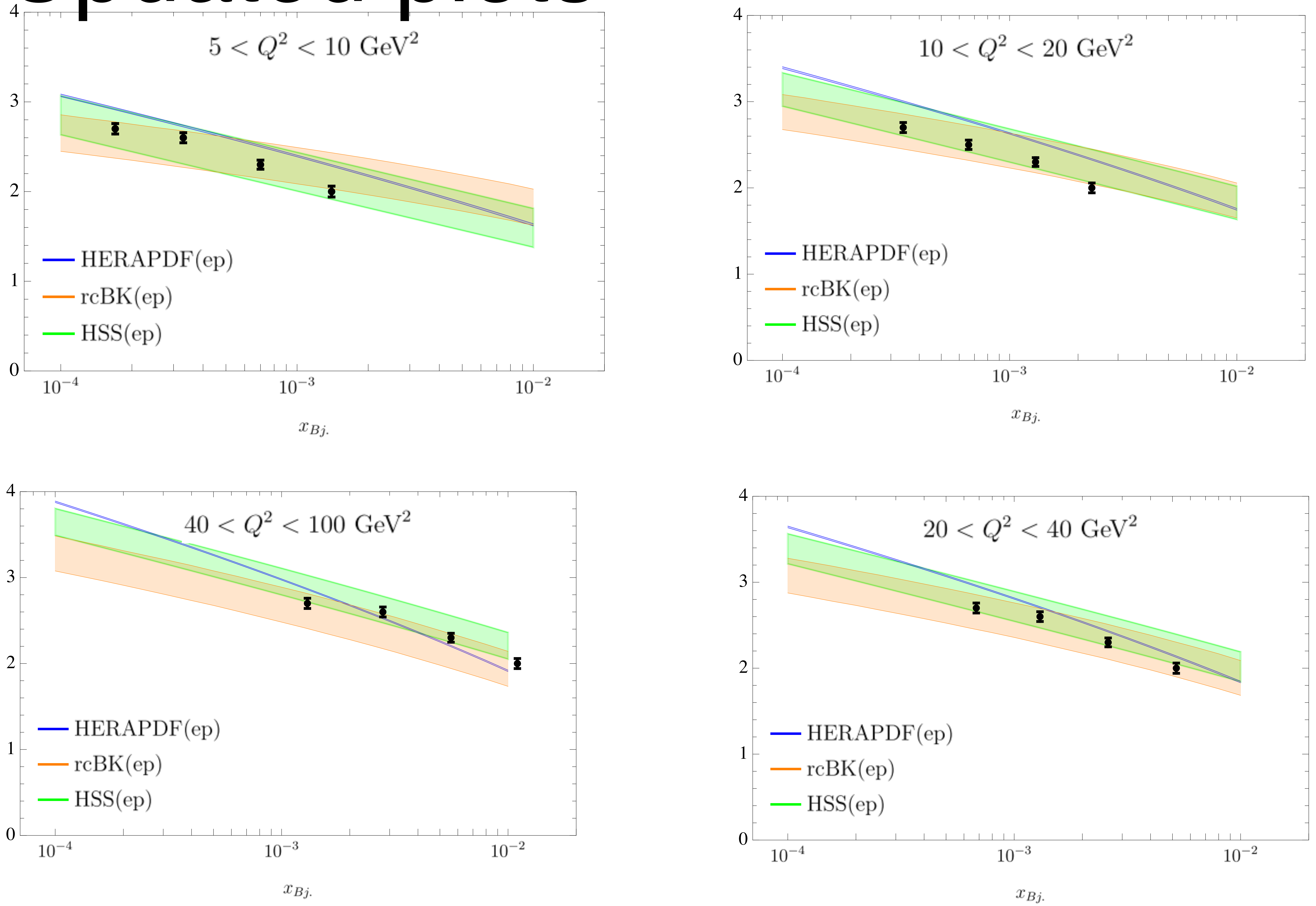


# of partons/bin size (and infinitesimal limit)

$$\bar{n}_g(x, Q^2) = \frac{dn_g}{d\ln(1/x)} = xg(x, Q^2)\,.$$

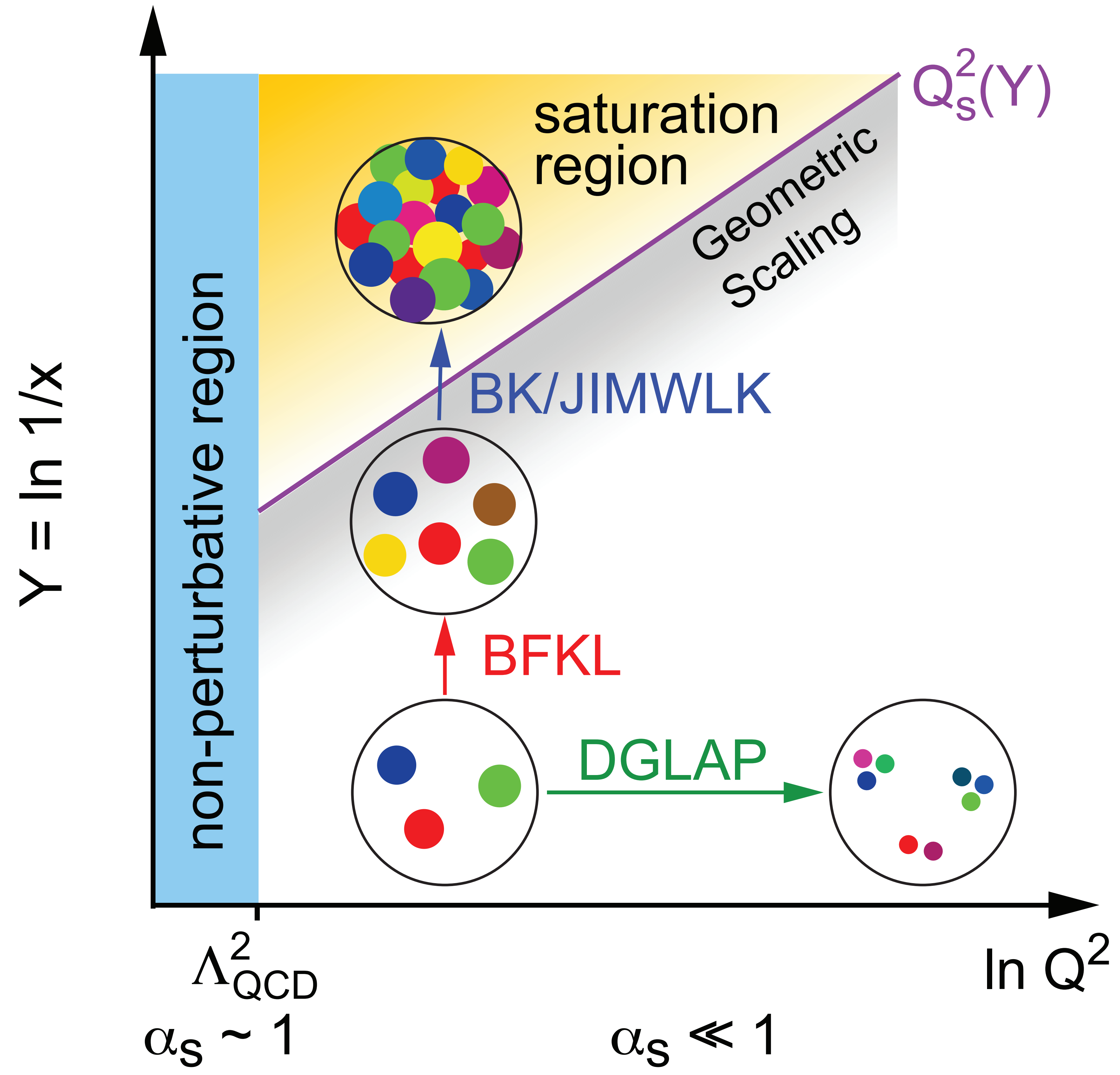
Updated plots

Still with normalization issue, x-dependence well described



Include now LO HERAPDF — works actually pretty well!

Why do we care?

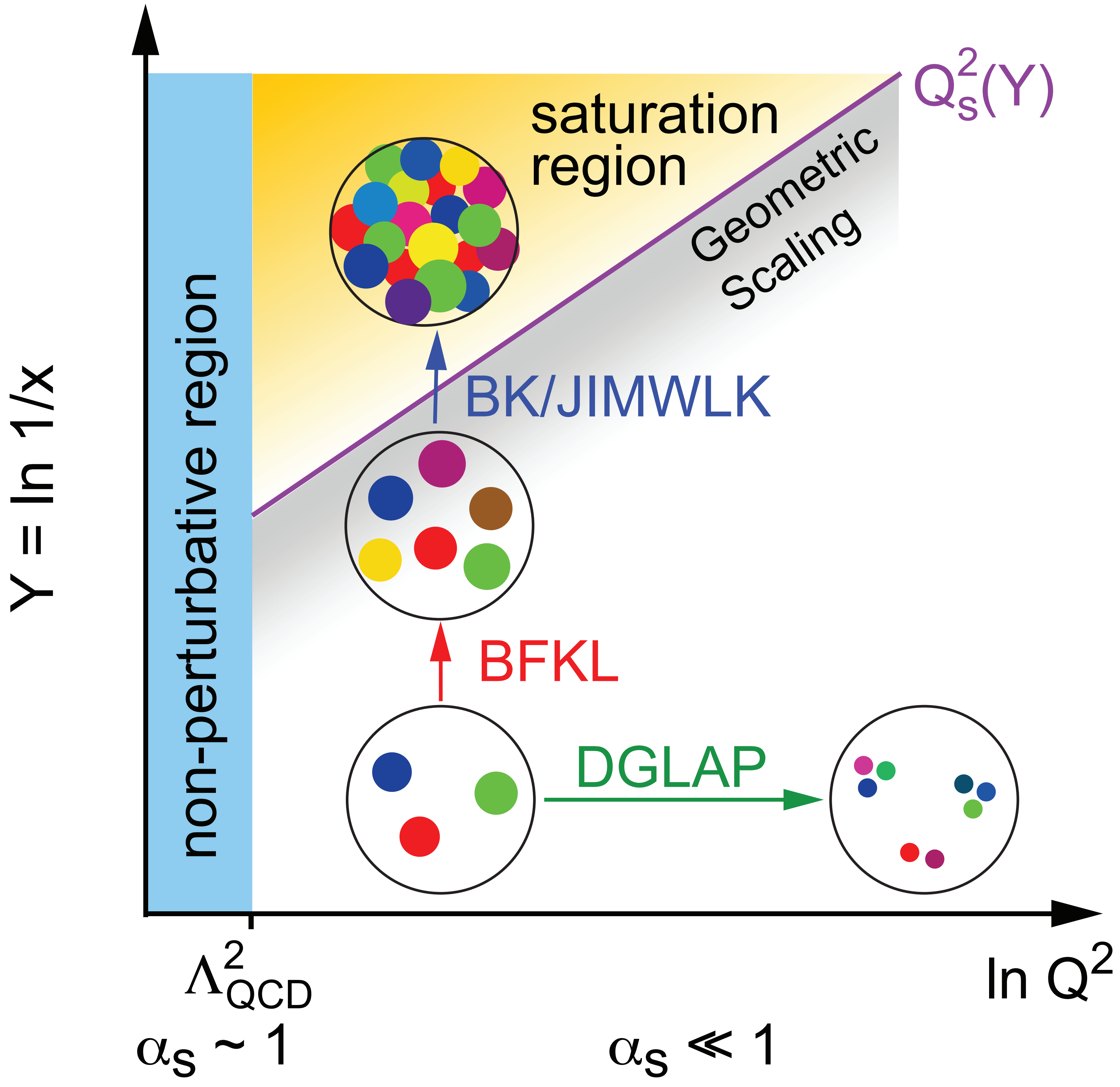


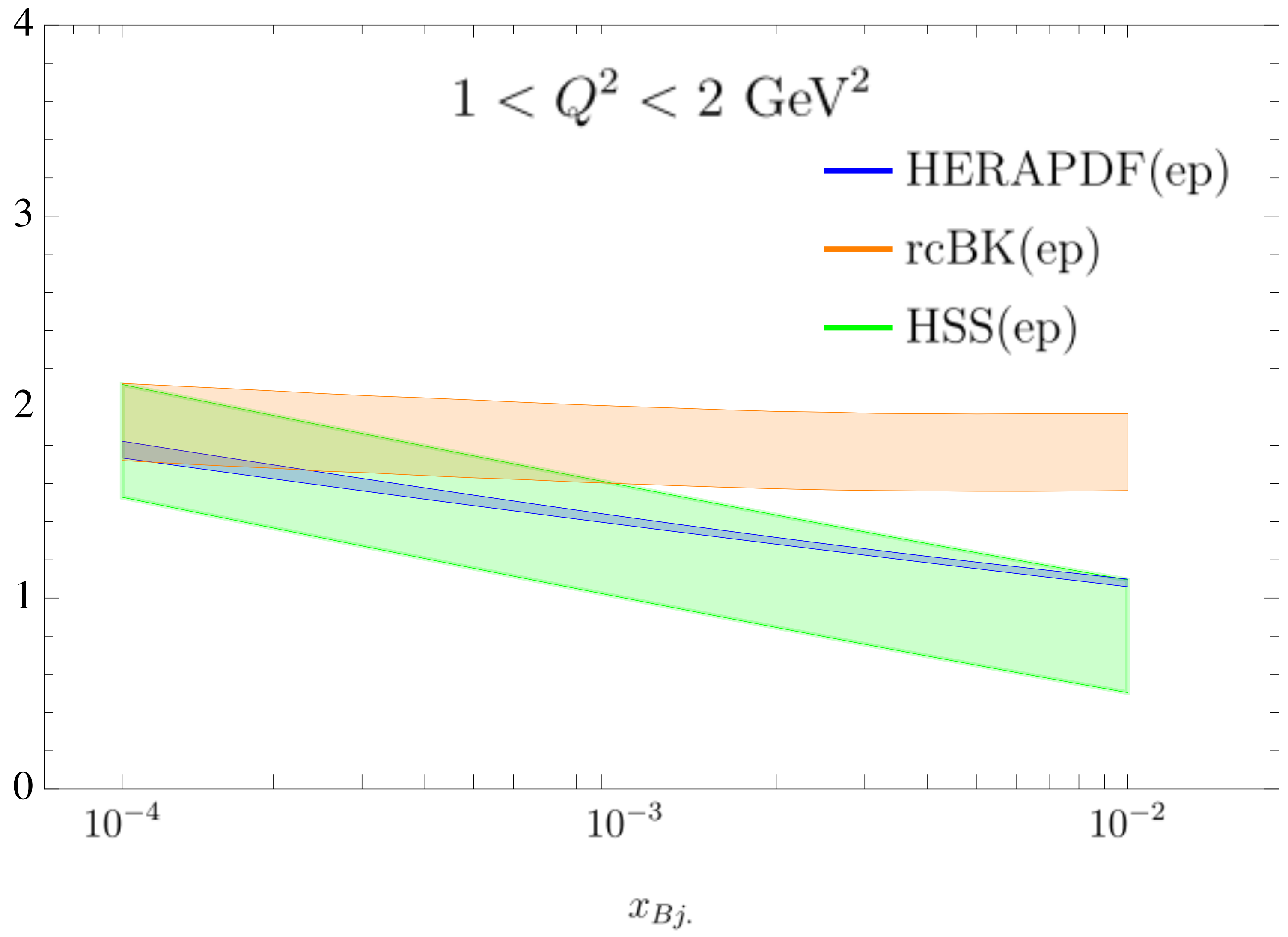
- low x drives us into a overoccupied and saturated system of gluons ↔ quantum bounds on entropy, Bekenstein bound etc.?
- Unobserved system is non-perturbative … can perturbative physics tells us something new about it? $'S_A = S_B'$
- If $S_h = \ln \sum_{a=q,g} x f_a(x, Q)$, does this constraint further parton distribution functions?
- Heavy ion collisions & entropy?
- Calculate p_n? Diffraction?
- ….

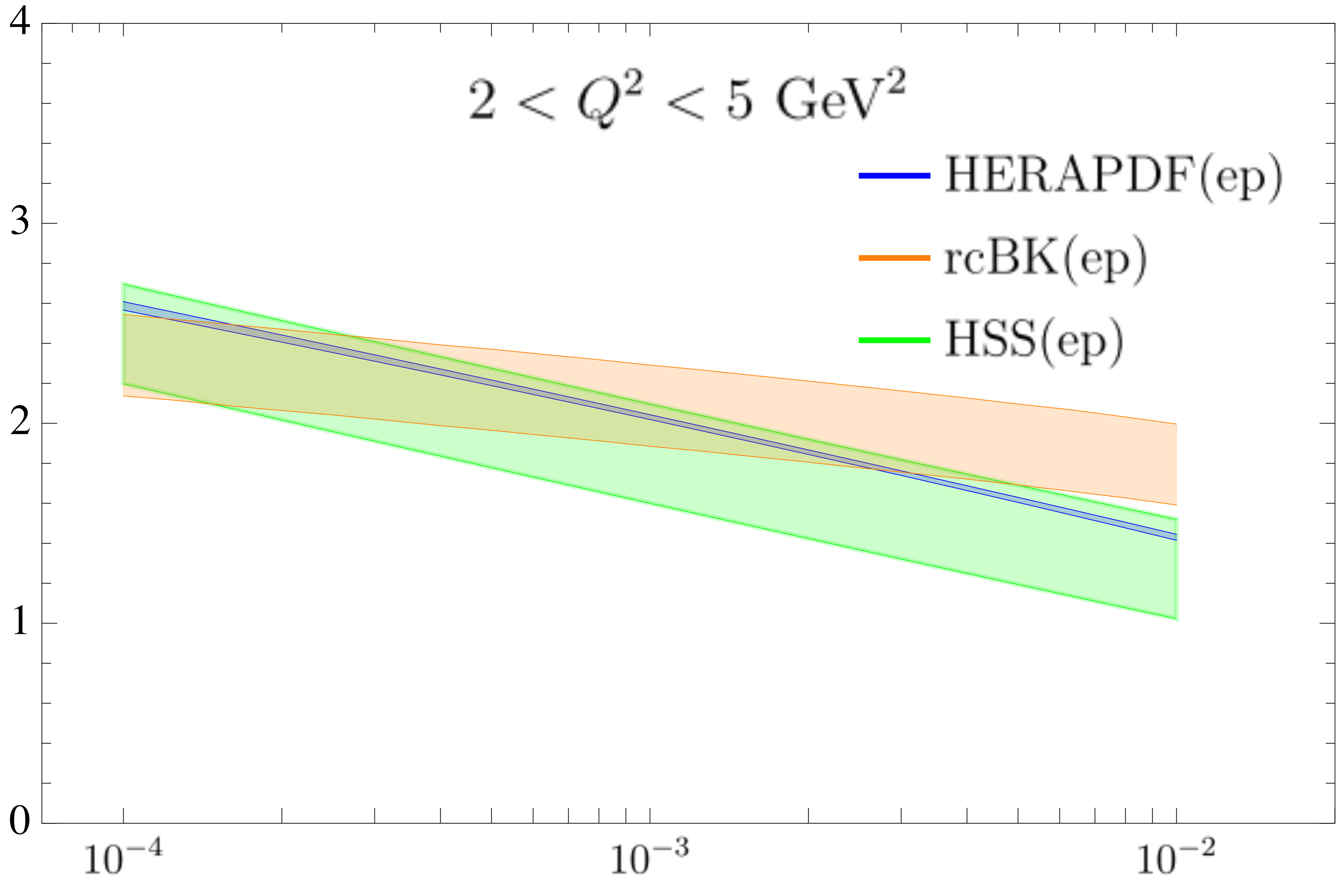
Thanks a lot!

Work in progress

First steps: towards low Q^2







Appendix