Beyond Rainbow-Ladder calculations from Dyson-Schwinger and Bethe-Salpeter equations

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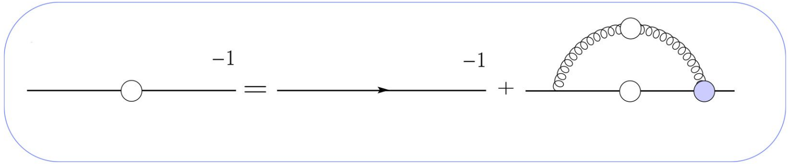
Outline

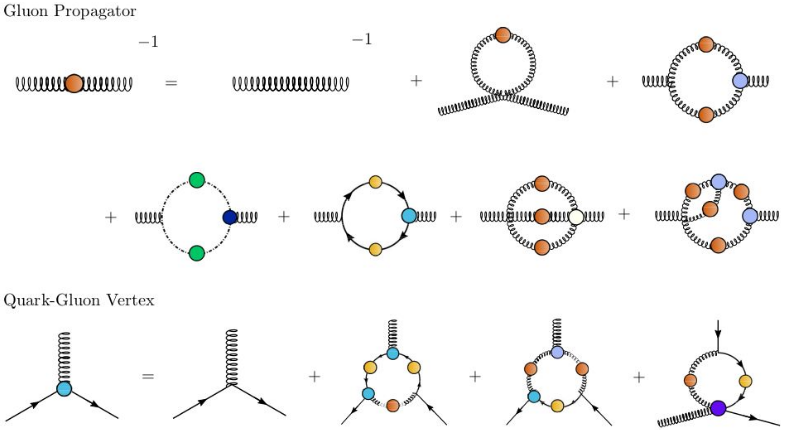
- ❑ Formalism
- ❑ Beyond-Rainbow-Ladder calculation
- ❑ Form Factors
- ❑ Contribution to muon g-2
- ❑ Isospin Breaking effects

Formalism: DSE/BSE

Dyson-Schwinger equations

- They are the equations of motion of a quantum field theory
  - Propagators
  - Vertices
- Exact equations derived from the path integral.
- Infinite set of integral equations.



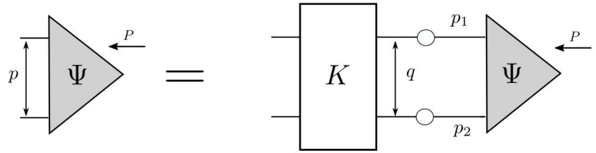


$$S^{-1} = S_0^{-1} - Z_{1f} \int_q \gamma_\mu S(q) \Gamma_\nu^{qgl}(q,k) D_{\mu\nu}(k) \ ,$$

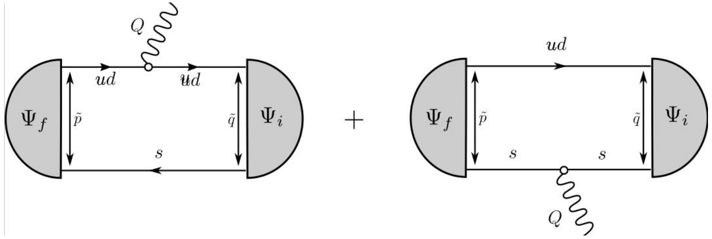
Formalism: DSE/BSE

Bethe-Salpeter equations

- ❑ Encode the information about the hadron
  - ❑ Mass
  - ❑ Decay constants
  - ❑ Decay width
- ❑ When coupled to an external field, we can have access to form factors
- ❑ Also an infinite set of diagrams.



$$(\Gamma)_{a\alpha,b\beta}(p,P) = \int_q K^{r\rho,s\sigma}_{a\alpha,b\beta}(P,p,q) \times S_{r\rho,e\epsilon}(k_1)\,(\Gamma)_{e\epsilon,n\nu}(q,P)\,S_{n\nu,s\sigma}(k_2)$$

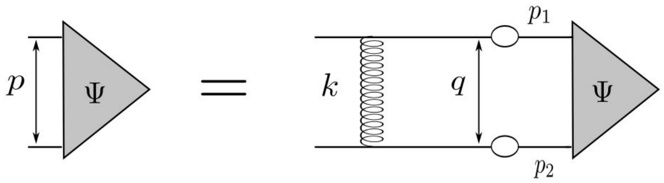


Ingredients:

- ❑ Quark propagator
- ❑ Meson Bethe-Salpeter amplitudes
- ❑ Interaction kernel
- ❑ Quark-photon vertex

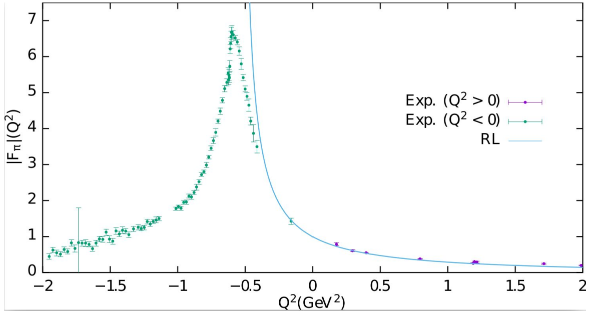
Rainbow-ladder truncation

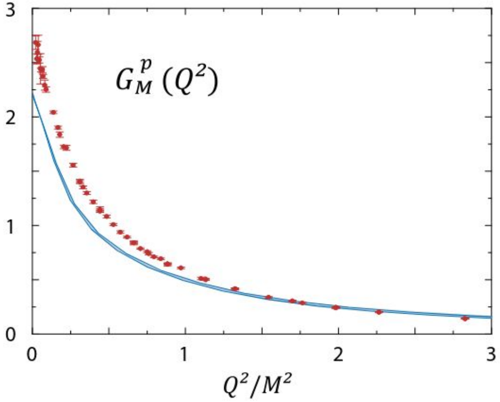
- Simples truncation to preserve chiral symmetry.
- One collects all the correction of the gluon propagator and quark gluon vertex into an effective coupling.
- A model to describe this effective coupling will be necessary.
- We employ the Maris-Tandy model, it includes two free parameters to be fitted to reproduce the pion mass and decay constant.
- Works very good on the description of hadron spectroscopy.



$$\alpha(q^2) = \pi\eta^7\left(\frac{q^2}{\Lambda^2}\right)^2 e^{-\eta^2\frac{q^2}{\Lambda^2}} + \frac{2\pi\gamma_m(1-e^{-q^2/\Lambda_t^2})}{\ln[e^2-1+(1+q^2/\Lambda_{QCD}^2)^2]}$$

Problems with RL truncation





G. Eichmann, Helios Sanchis-Alepuz, R. Williams, R. Alkofer and C. Fischer

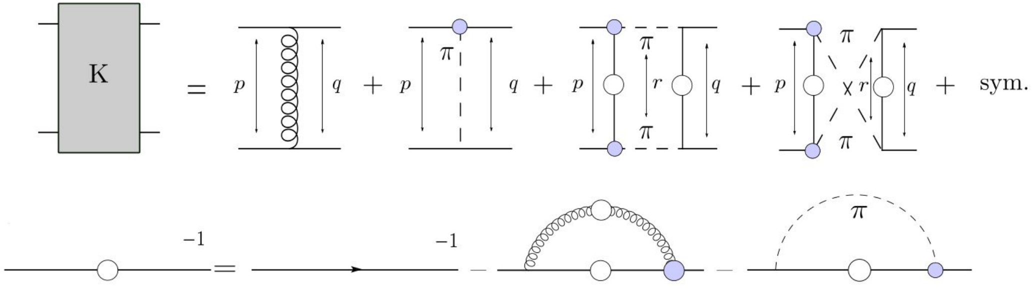
- The solutions are stable bound states, do not describe the decay width.
- Not very good description of nucleon space-like form factors
- Poor description of excited state masses

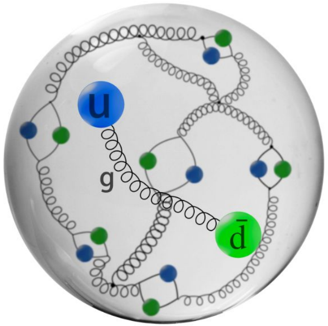
E. Rojas, B. El-Bennich and J.P de Melo, Phys. Rev. D 90, 074025

	Model 1	Model 2	Reference
m_π	0.138	0.153	0.139 [69]
f_π	0.139	0.189	0.1304 [69]
$m_{\pi(1300)}$	0.990	1.414	1.30±0.10 [69]

Beyond the Rainbow: non-valence contributions

- ❑ Mesonic effects are considered by including explicit mesons as degrees of freedom.
- ❑ Besides the gluon contribution, a meson loop and meson exchange will appear on the DSE/BSE system.





C. S. Fischer, D. Nickel and R. Williams, Eur. Phys. J. C 60 (2009) 47 doi:10.1140/epjc/s10052-008-0821-1

A. S. Miramontes and H. Sanchis-Alepuz, Eur. Phys. J. A 55 (2019) no.10, 170 doi:10.1140/epja/i2019-12847-6

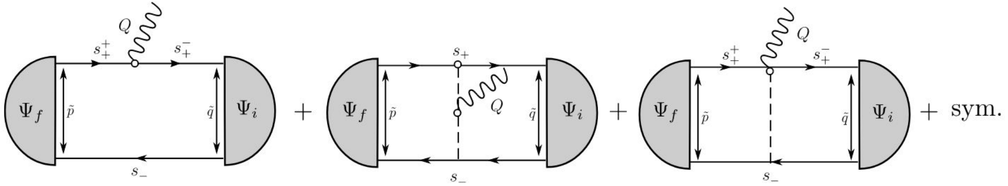
Figure from Z.-F. Cui et. al

Numerical Results

Electromagnetic FF from DSE/BSE

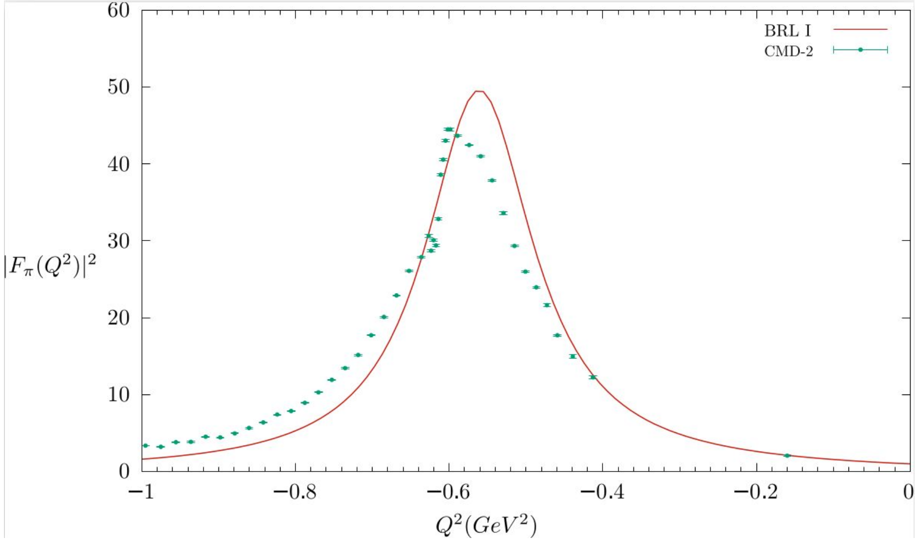
- We extract the electromagnetic form factor from the electromagnetic current
- Ingredients:
  - Meson Bethe Salpeter amplitude,
  - Quark propagator
  - Quark-photon vertex
- We restrict the calculation to the impulse approximation.

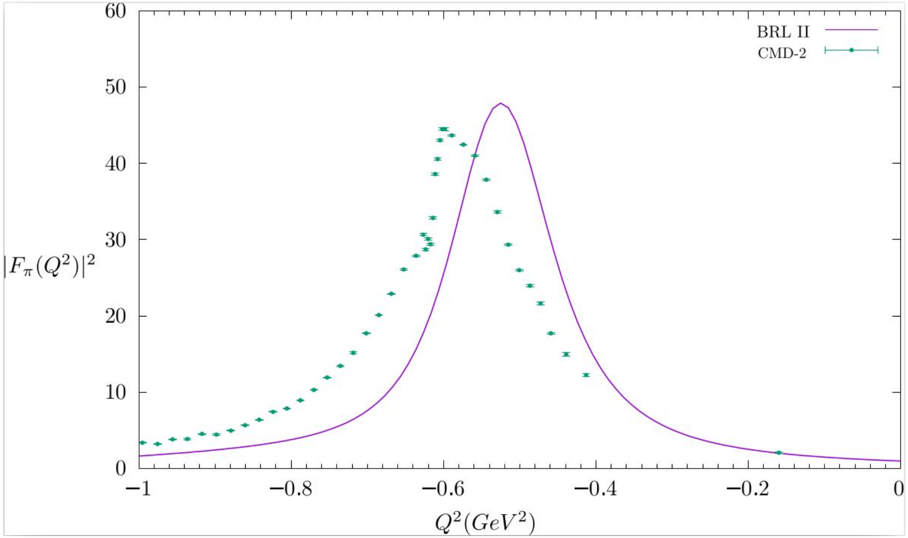
$$J^\mu = \bar{\Psi}_f G_0 (\mathbf{\Gamma}^\mu - K^\mu) G_0 \Psi_i \ ,$$



Time-like pion Form factor.

- We observe that employing BRL truncation we describe very well, at qualitative level, the behaviour of the FF.
- Calculation have been done in isospin symmetric limit.
- The pole position depends on the free parameters of the effective Maris Tandy interaction.

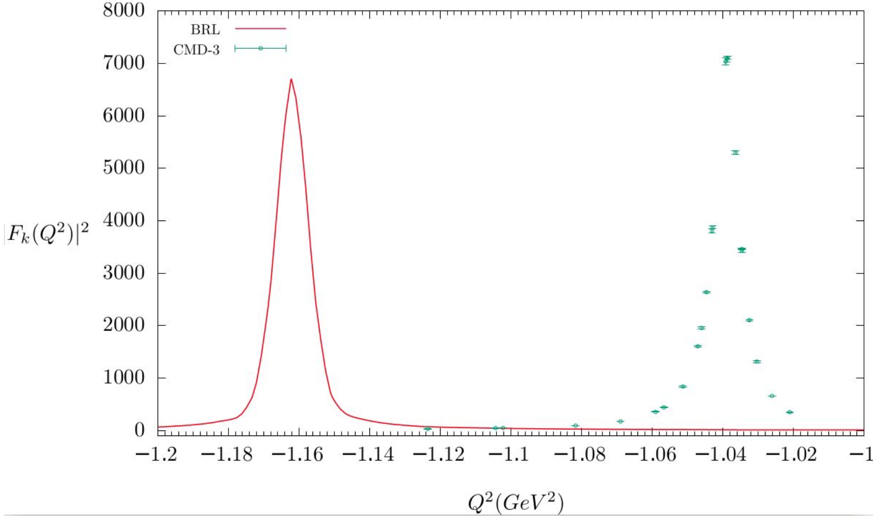


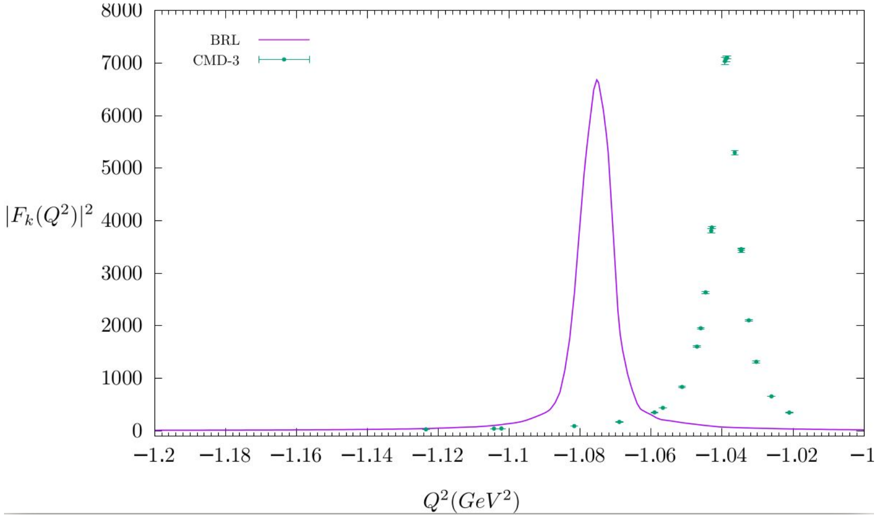


A.S Miramontes, Helios Sanchis-Alepuz and Reinhard Alkofer.**Phys.Rev.D** 103 (2021) 11, 116006

Kaon Form Factor: Time-like (To be published)

- The position of the phi resonance pole is sensitive to the parameters in the Maris-Tandy model.





A.S Miramontes, Adnan Bashir, to be published

Pion and Kaon box contributions to g-2

- The main non-perturbative object to compute these kind of contributions are the pion and kaon electromagnetic form factors.
- We calculated the space-like kaon form factor employing rainbow-ladder and beyond rainbow-ladder truncation .
- Once the form factor are calculated one can compute their box contribution to the muon g-2.

Pion and Kaon Box contributions to g-2

$\pi^{\pm}-$box contributions are:

$$a_{\mu}^{\pi^{\pm}-\text{box}} = -(15.4 \pm 0.3) \times 10^{-11} \quad [\text{RL-direct}]\,,$$
$$a_{\mu}^{\pi^{\pm}-\text{box}} = -(15.6 \pm 0.3) \times 10^{-11} \quad [\text{RL-PTIR}]\,,$$
$$a_{\mu}^{\pi^{\pm}-\text{box}} = -(15.7 \pm 0.2) \times 10^{-11} \quad [\text{BRL}]\,.$$

Analogous results for the $K^{\pm}$ case yield:

$$a_{\mu}^{K^{\pm}-\text{box}} = -(0.47 \pm 0.03) \times 10^{-11} \quad [\text{RL-direct}]\,,$$
$$a_{\mu}^{K^{\pm}-\text{box}} = -(0.48 \pm 0.03) \times 10^{-11} \quad [\text{RL-PTIR}]\,,$$
$$a_{\mu}^{K^{\pm}-\text{box}} = -(0.48 \pm 0.02) \times 10^{-11} \quad [\text{BRL}]\,.$$

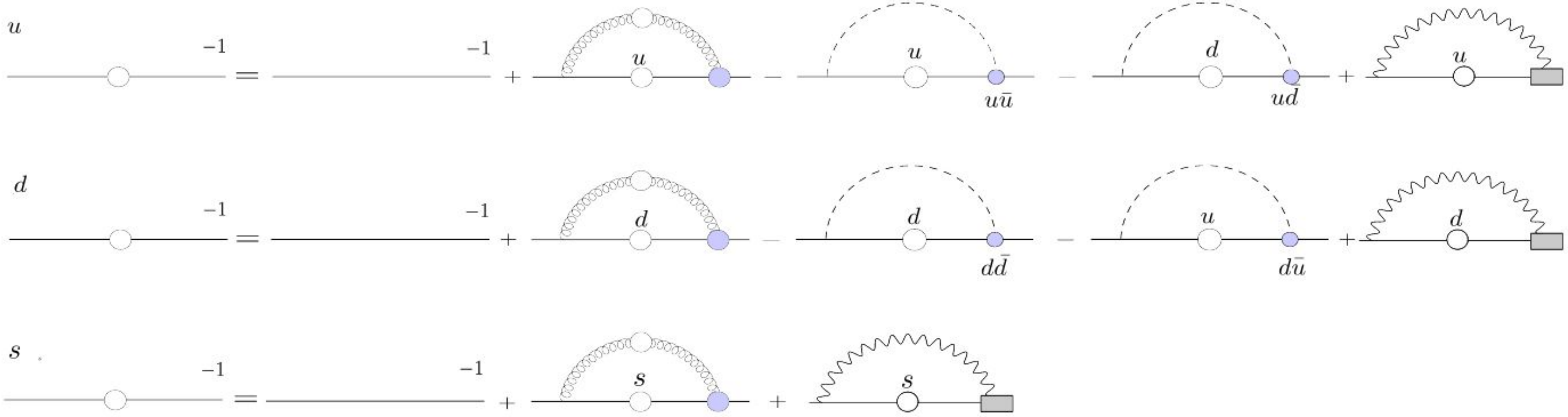
- Our results are compatible with contemporary investigations for the pion and kaon box contribution to the Hadronic Light by Light (HLbL).

A.S. Miramontes, Khépani Raya, Adnan Bashir and Pablo Roig
Phys.Rev.D 105 (2022) 7, 074013

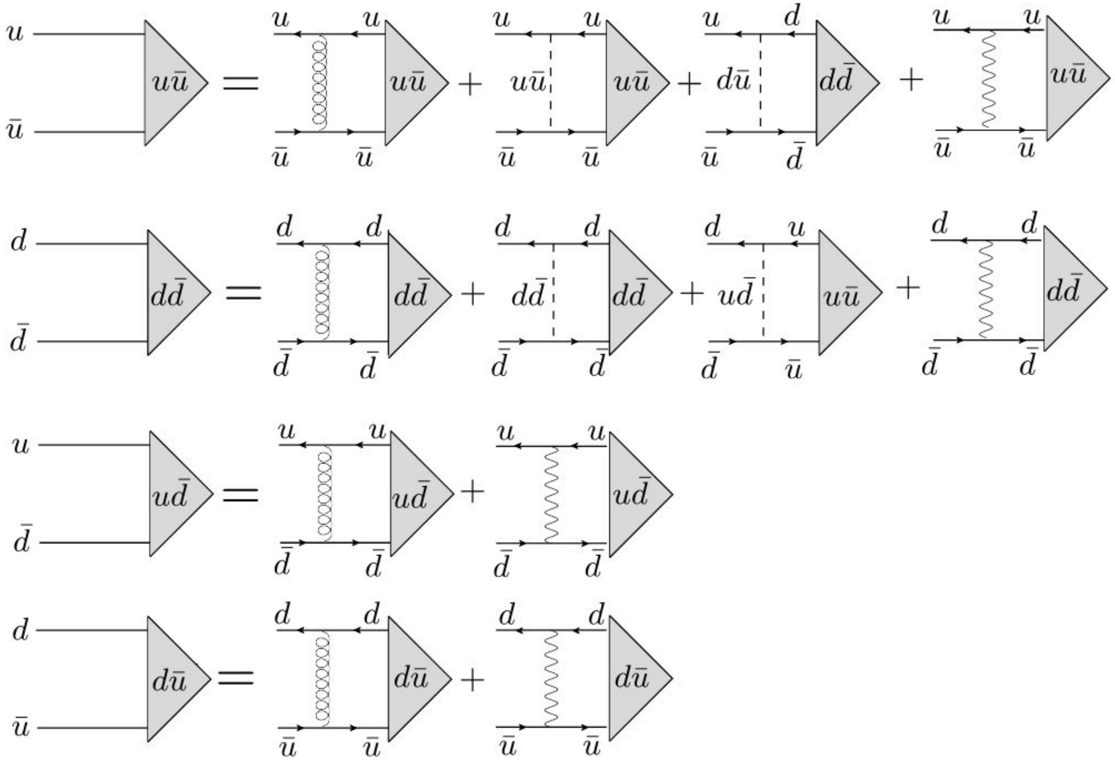
Isospin Breaking effects in light meson masses

- Isospin symmetry is broken at the level of QED due to the difference in the charges between the up and down quarks
- Also broken in QCD through the small mass difference between up and down quarks in the QCD lagrangian.
- Almost an exact symmetry, mass difference much smaller than QCD scale.
- For this reason, theoretical predictions of different physical quantities assume isospin symmetry taking the mass of the quarks up and down to be the same and neglecting electromagnetic effects.

DSE/BSE Setup



DSE/BSE setup



Results of mass splitting

M [MeV]	$m_u = m_d$ $\alpha_0 = 0$	$m_u \neq m_d$ $\alpha_0 = 0$	$m_u = m_d$ $\alpha_0 = 1/137$	$m_u \neq m_d$ $\alpha_0 = 1/137$
π^0	134.5	132.5	136.0	133.4
$\pi^\pm$	134.5	134.2	139.6	$139.7^\dagger$
$\pi^\pm - \pi^0$	0	1.7	3.6	6.3
K^0	494.7	497.5	495.2	$497.7^\dagger$
$K^\pm$	494.7	492.1	497.2	$493.7^\dagger$
$K^0 - K^\pm$	0	5.4	−2.0	4.0
ρ^0	720.3	721.5	721.1	721.4
$\rho^\pm$	720.3	719.9	722.0	720.9
$\rho^\pm - \rho^0$	0	−1.6	0.9	−0.5

A.S Miramontes, Reinhard Alkofer, Christian Fischer and Helios Sanchis-Alepuz, **Phys.Lett.B** 833 (2022) 13729

Conclusions

The DSE/BSE have reached a degree of sophistication that allows to investigate and describe problems that we were not able in the past using functional methods, like: Time-like form factors and isospin symmetry breaking.

Further studies on:

- Hadron Vacuum polarization contributions to g-2
- Excited states
- Anomalous form factor gamma-3pi
- Eta-pion mass splitting