

The dark matter halo as a particle detector



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University of Guanajuato

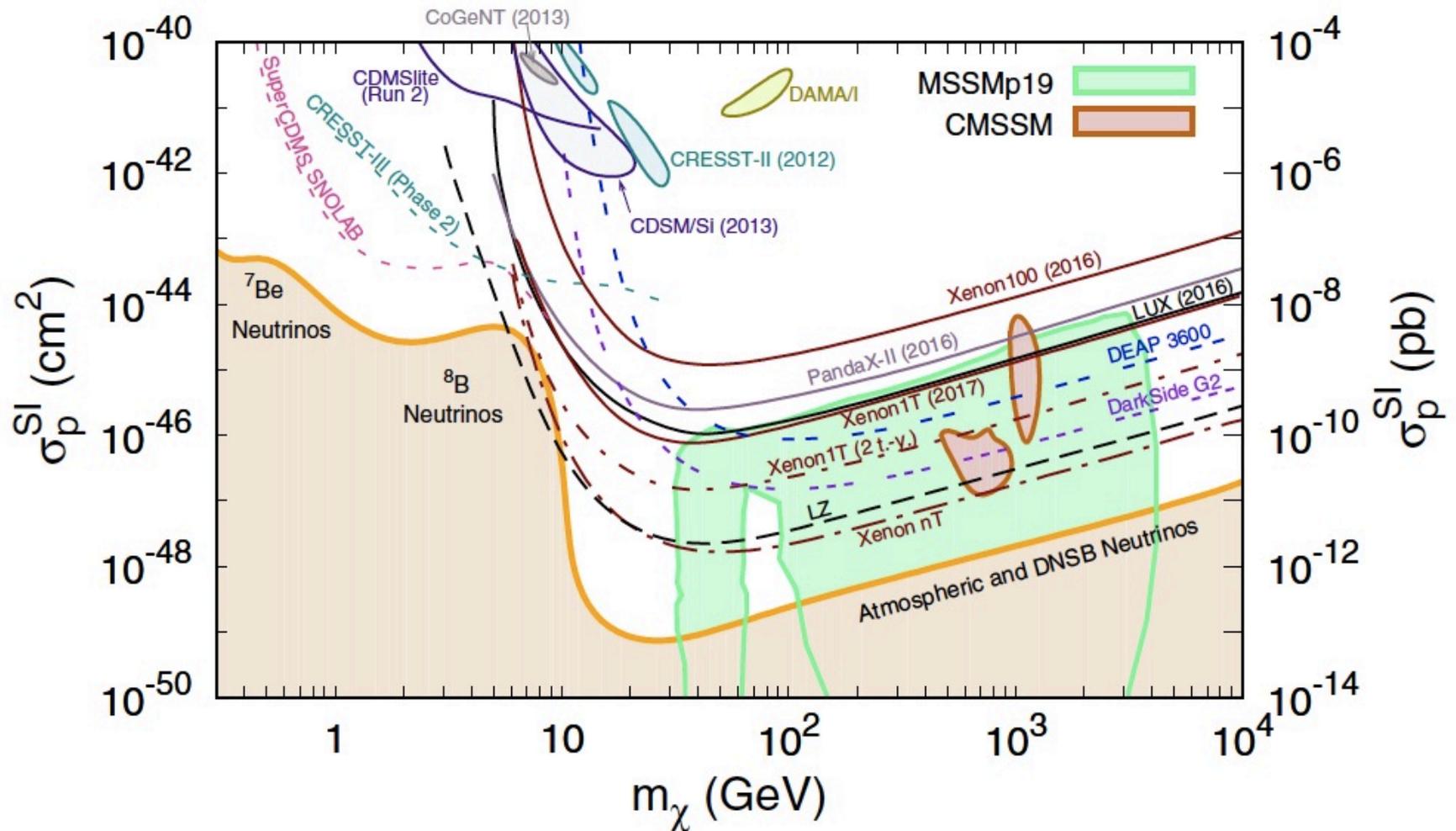
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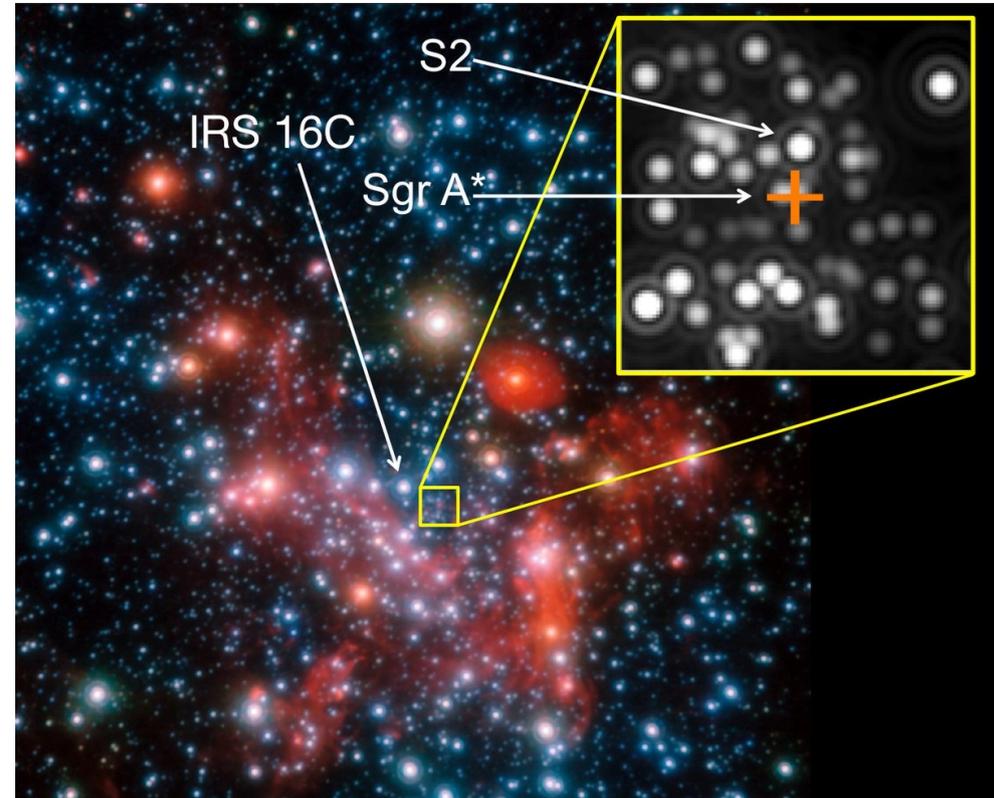
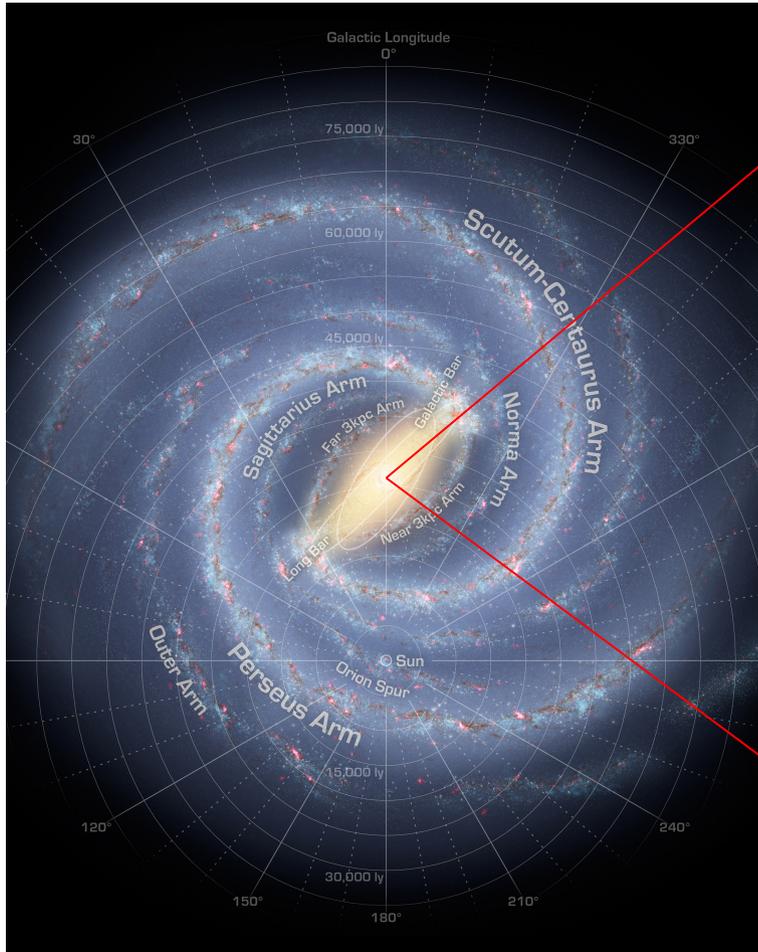
Outline

1. The lightness is the new darkness
2. What if DM is completely dark
3. The simplest models of dark matter
 1. Ultra-light scalar dark matter
 2. Ultra-light fermionic dark matter
4. Constrains from the Milky way
5. Some surprises in self-gravitating bosonic configurations
6. Conclusions

Direct dark matter detection status



The universe is transparent to light...

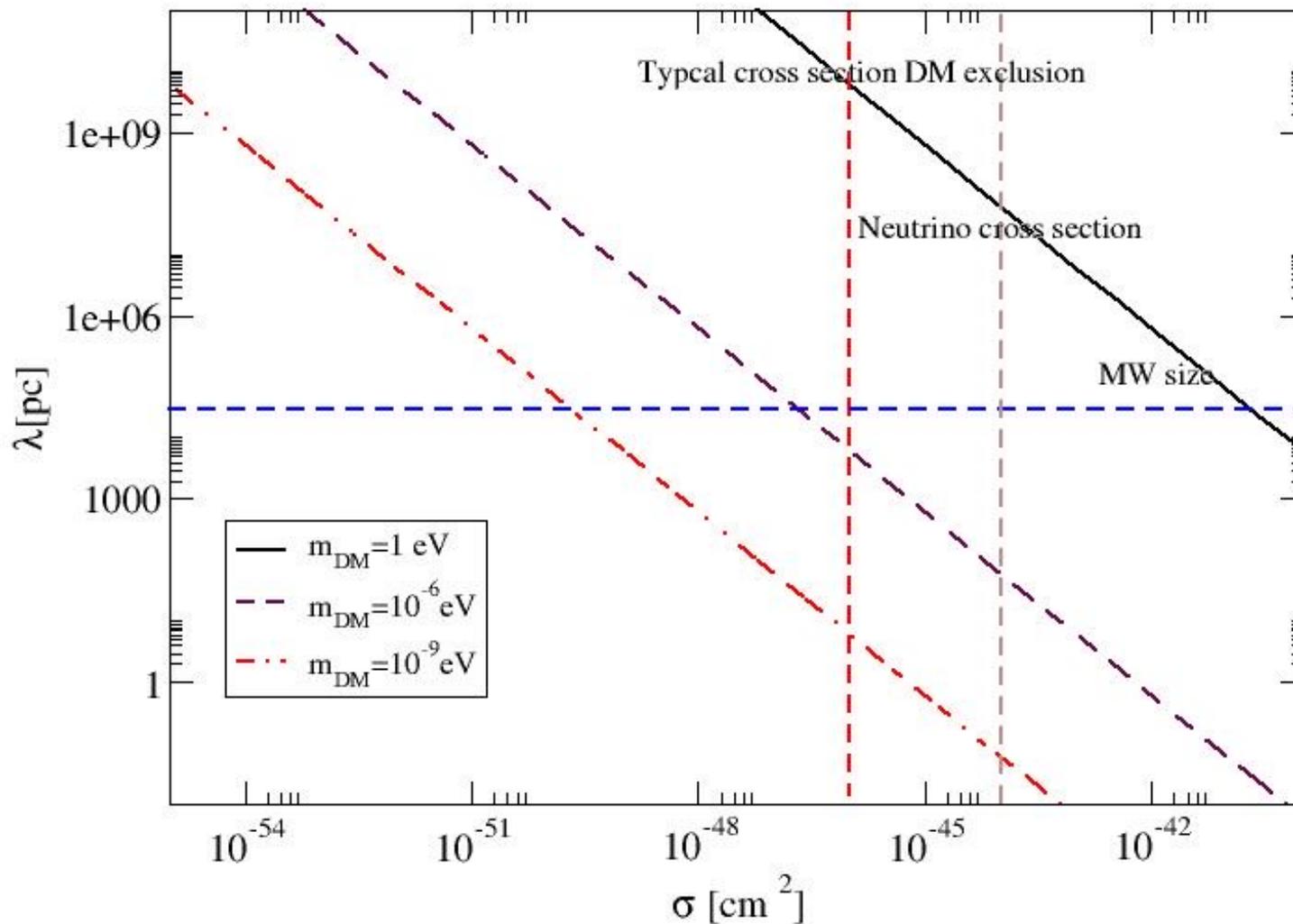


$$\lambda = (n\sigma)^{-1}$$

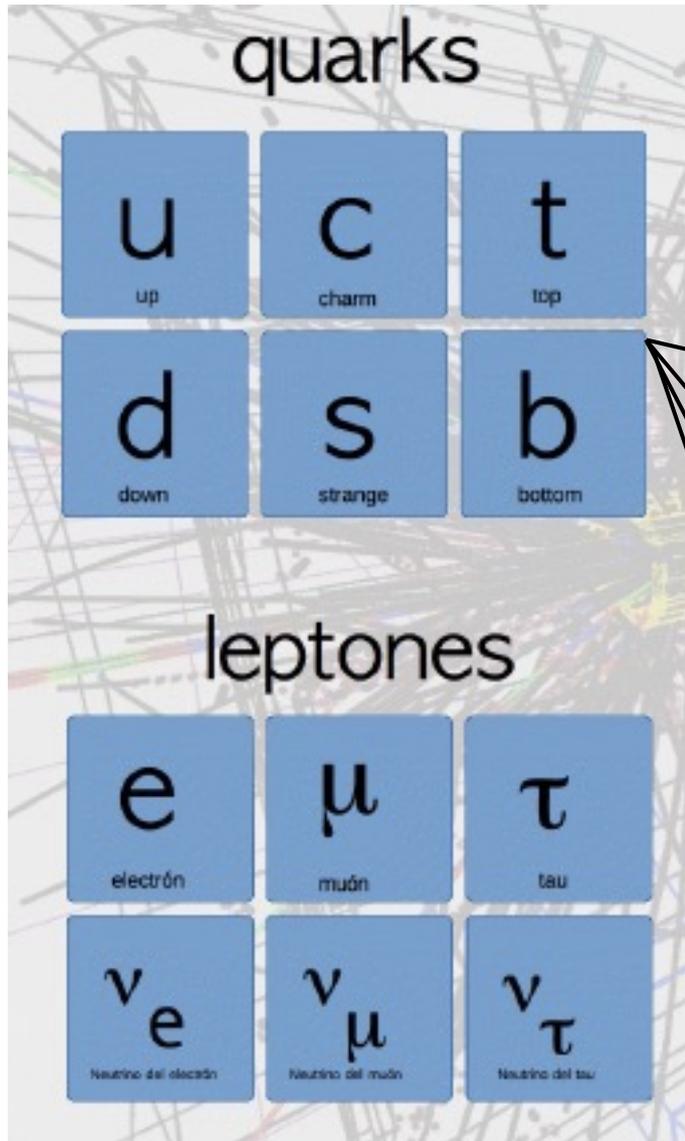
$$F(L) = F_0 e^{-L/\lambda}$$

For heavy dark matter candidates,
the universe is transparent

Lightness is the new darkness



Could it be that simple?

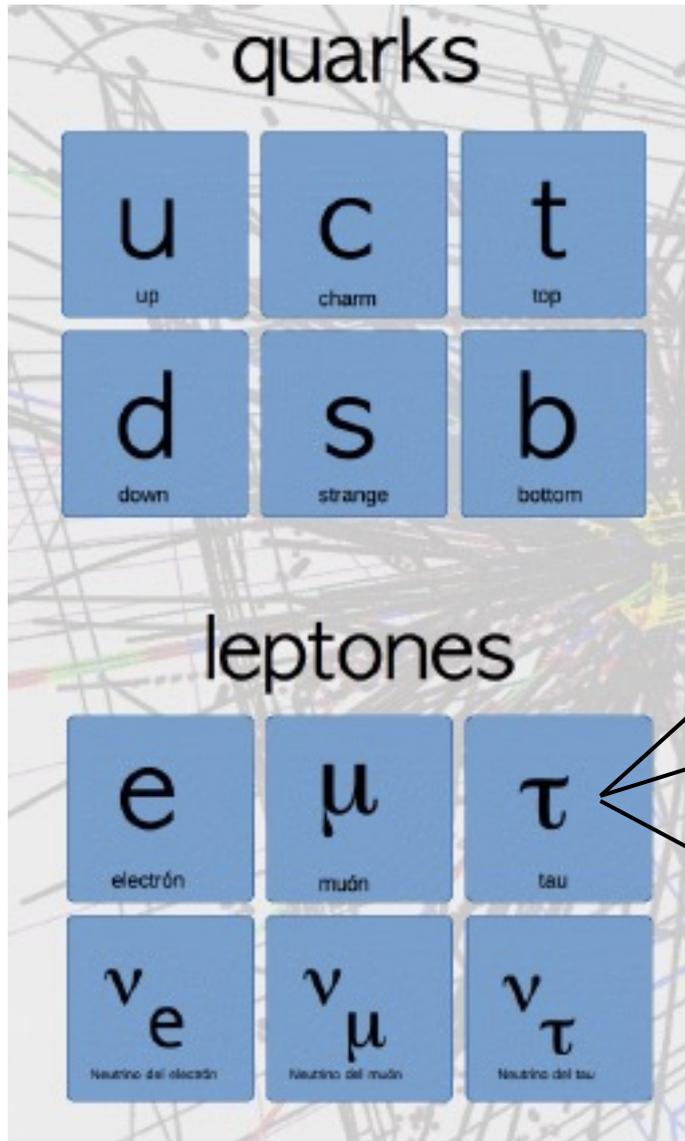


Fundamental interactions

1. Strong force
2. Electromagnetic force
3. Weak force
4. Gravitational force

Quarks: Too much interaction

Could it be that simple?



Fundamental interactions

1. Strong force

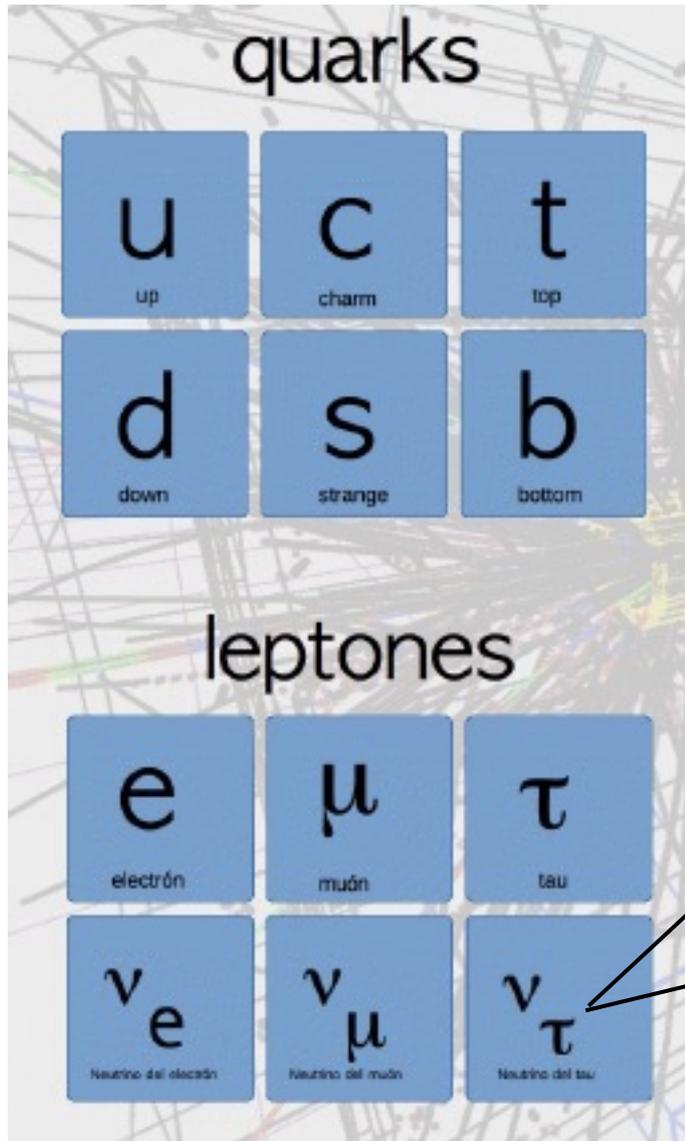
2. Electromagnetic force

3. Weak force

4. Gravitational force

Charged leptons: Good enough

Could it be that simple?



Fundamental interactions

1. Strong force

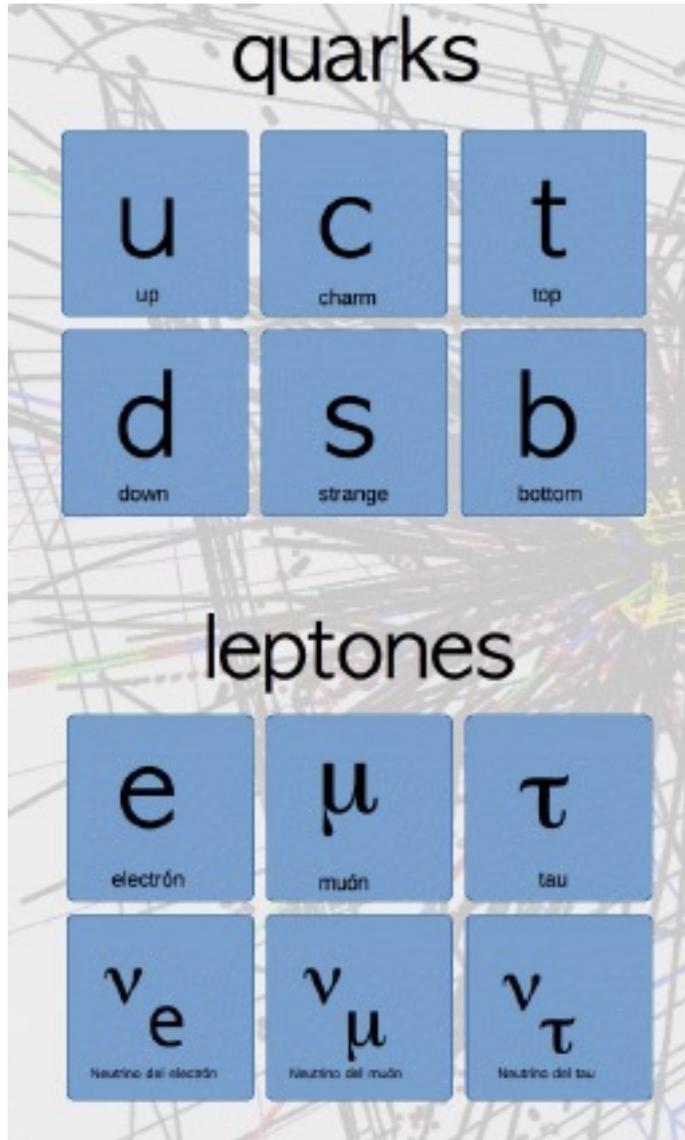
2. Electromagnetic force

3. Weak force

4. Gravitational force

Neutrinos: Difficult to see, but observable

Could it be that simple?



Fundamental interactions

1. Strong force
2. Electromagnetic force
3. Weak force
4. Gravitational force



Is dark matter the particle that interacts only through gravitational interactions?

The darkest scenario:

Dark matter interacts only through gravitational interactions:

1) Forget how to detect it on Earth

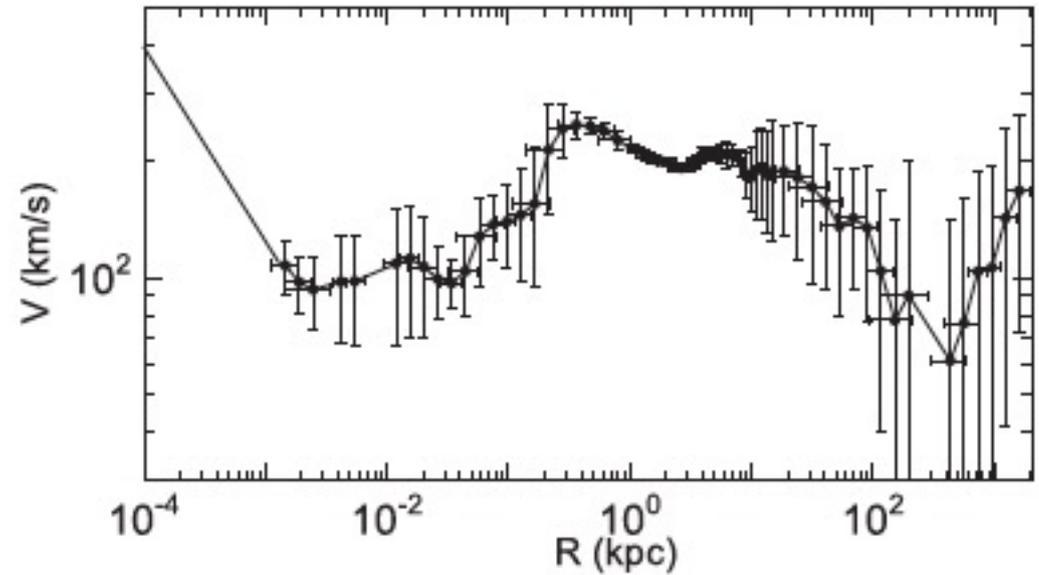
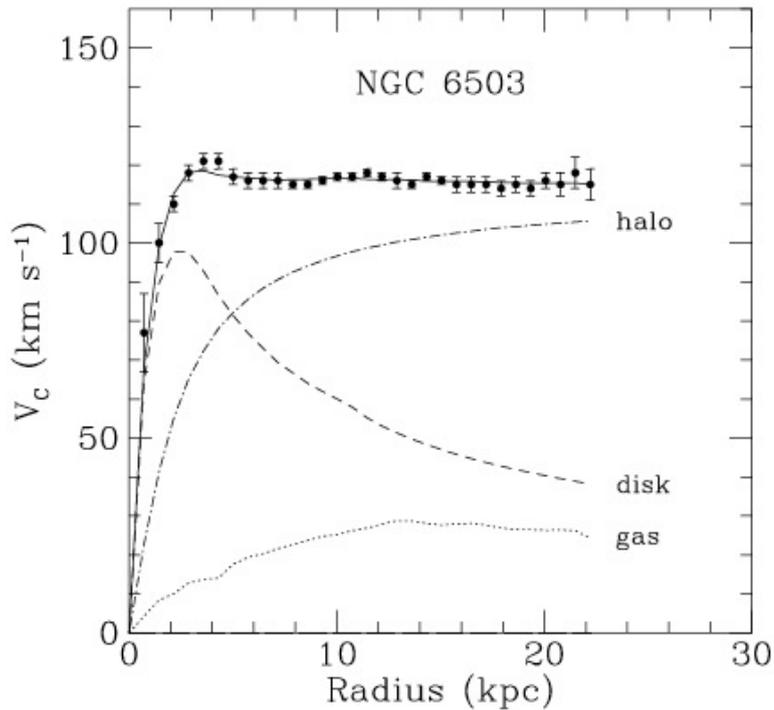
2) Main properties: The mass and the spin:

Ultralight fermionic particles

Ultralight spin-zero particles

The Milky way as a detector

Rotational velocity curve



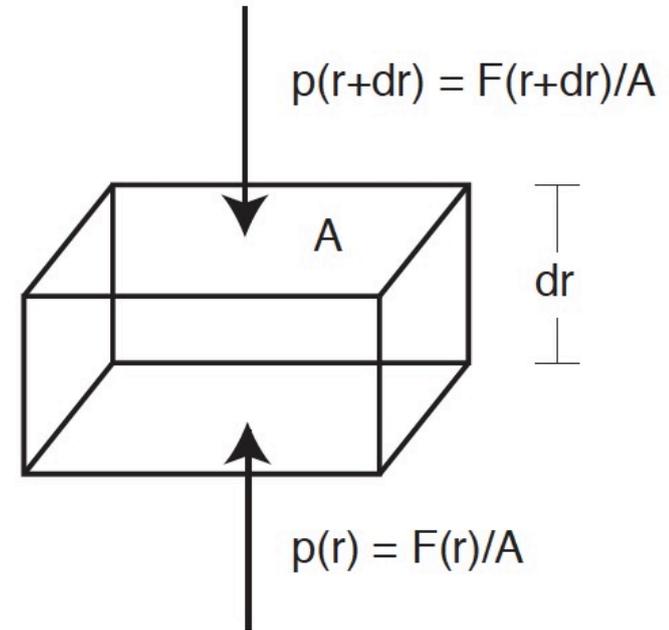
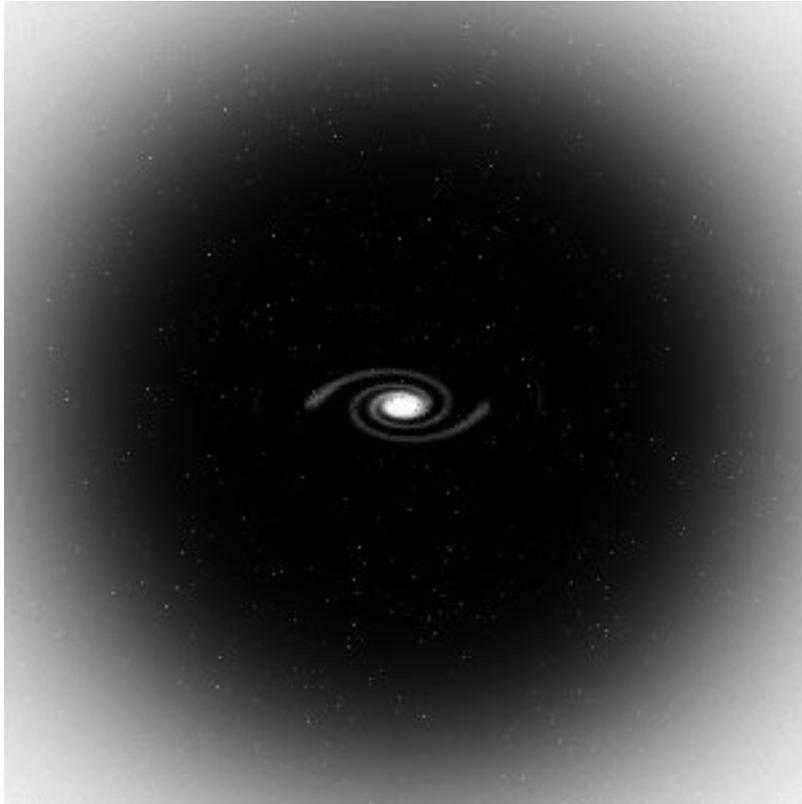
PASJ: Publ. Astron. Soc. Japan 65, 118, 2013 December 25
© 2013. Astronomical Society of Japan.

**Rotation Curve and Mass Distribution in the Galactic Center
—From Black Hole to Entire Galaxy—**

Yoshiaki SOFUE

Ultralight Fermionic dark matter

The halo is surrounding the galaxy, and perhaps in hydrostatic equilibrium with gravity



$$\frac{dM}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{dp}{dr} = -\frac{G\rho(r)M(r)}{c^2 r^2} \left[1 + \frac{p(r)}{\rho(r)} \right] \left[1 + \frac{4\pi r^3 p(r)}{M(r)c^2} \right] \left[1 - \frac{2GM(r)}{c^2 r} \right]^{-1}$$

An equation of state is needed: A free gas of fermions at zero temperature:

$$\begin{aligned}\rho &= \frac{1}{\pi^2} \int_0^{k_F} k^2 \sqrt{m_F^2 + k^2} dk \\ &= \frac{m_f^4}{8\pi^2} \left((2z^3 + z)(1 + z^2)^{1/2} - \sinh^{-1}(z) \right) \\ p &= \frac{1}{3\pi^2} \int_0^{k_F} \frac{k^4}{\sqrt{m_F^2 + k^2}} \\ &= \frac{m_f^4}{24\pi^2} \left((2z^3 - 3z)(1 + z^2)^{1/2} + 3 \sinh^{-1}(z) \right),\end{aligned}$$

In the non relativistic limit: $z \ll 1$

$$p = \frac{34}{65} \left(\frac{6\pi^2}{13} \right)^{2/3} \frac{\rho^{5/3}}{m_f^{8/3}}$$

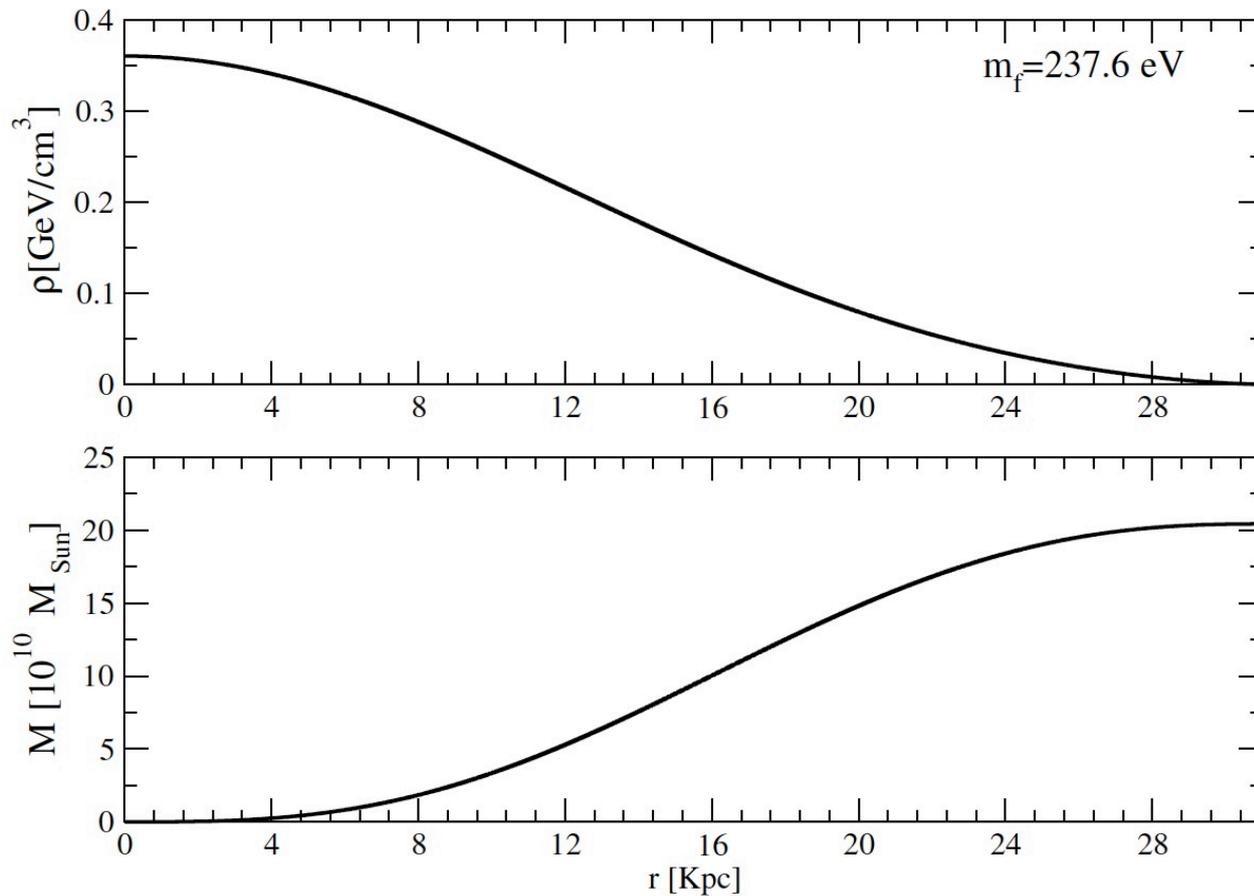
$$\frac{d\rho}{dr} = -\frac{39}{34} \left(\frac{13}{6\pi^2} \right)^{2/3} \frac{GM}{r^2} m_f^{8/3} \rho^{1/3}$$

$$\frac{dM}{dr} = 4\pi r^2 \rho.$$

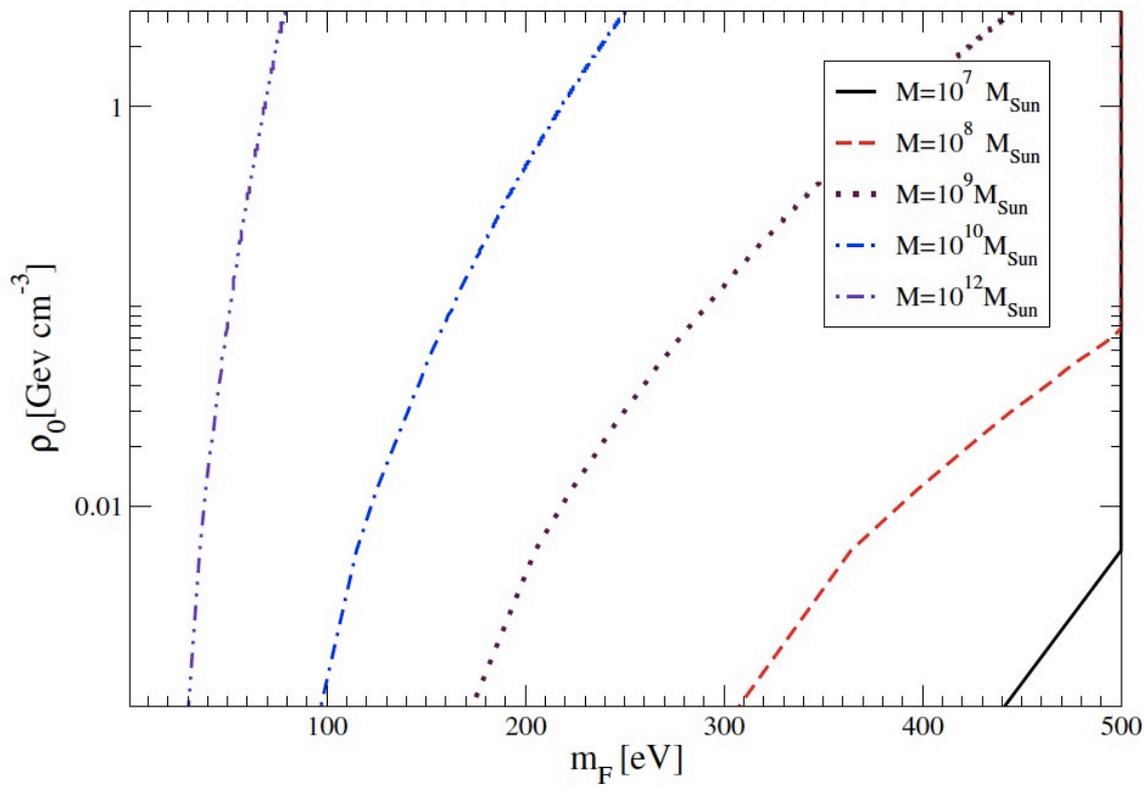
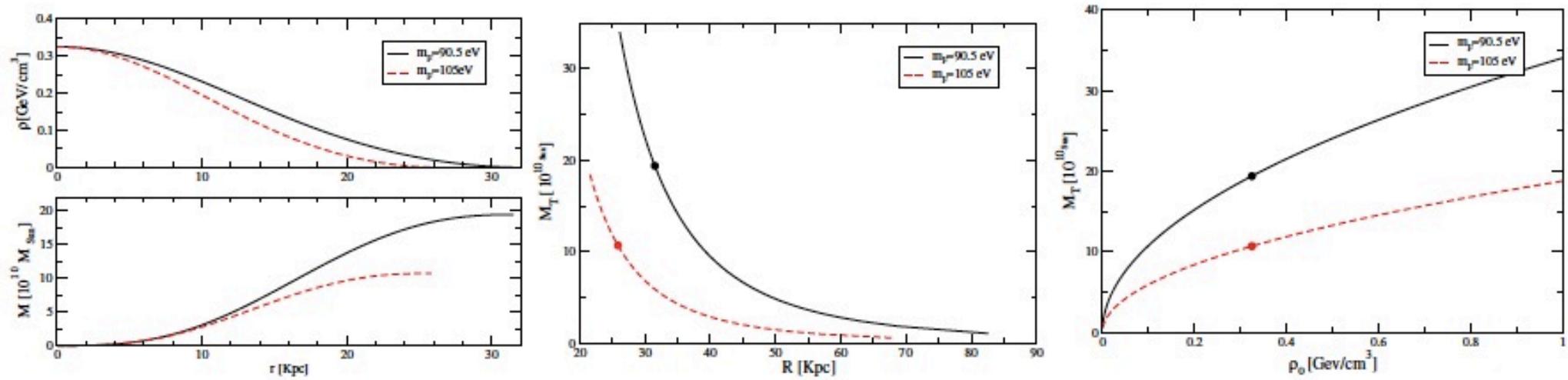
Boundary conditions

$$M(r = 0) = 0$$

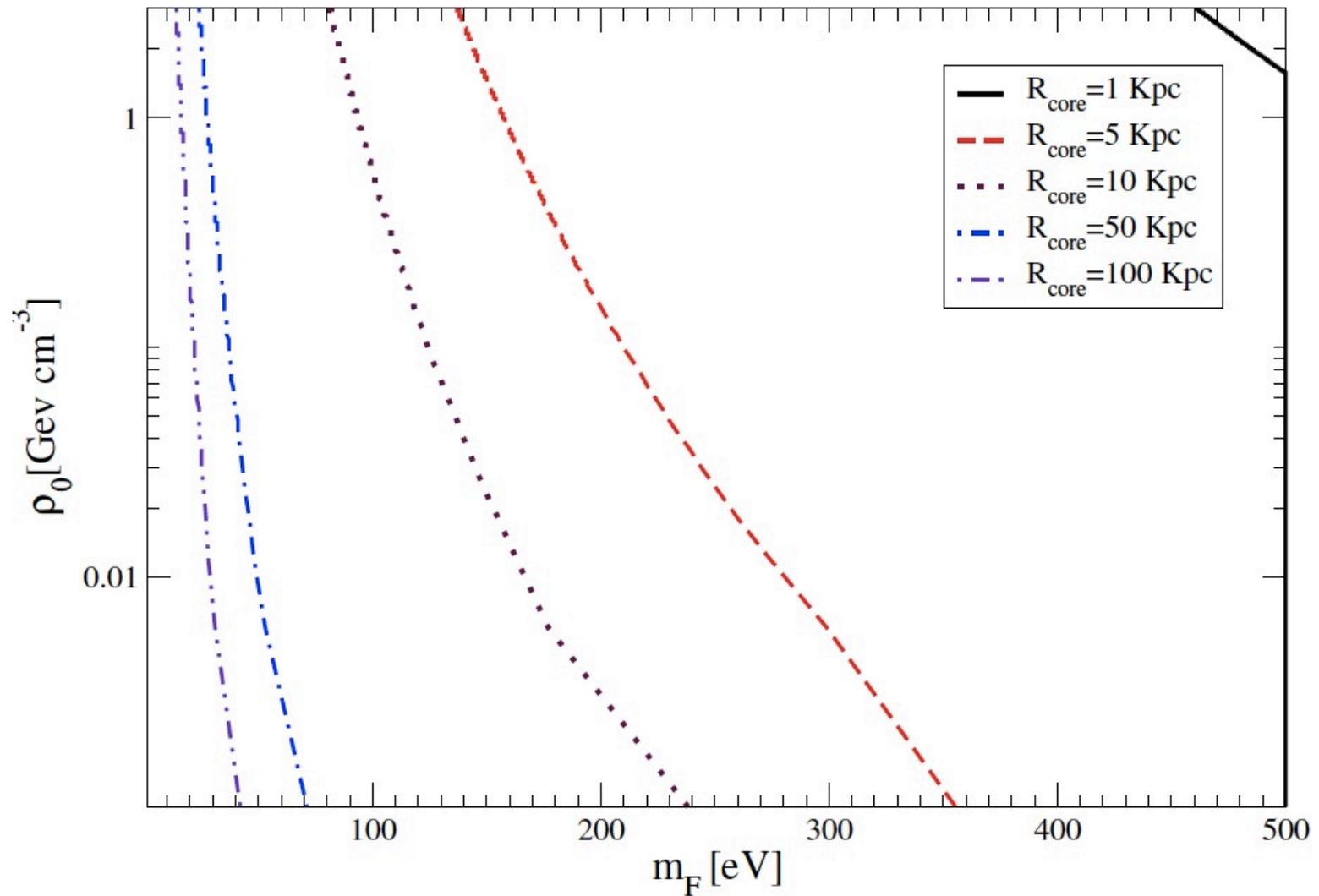
$$\rho(r = 0) = \rho_0$$



Ultralight fermionic DM halos



Cores (due to Fermi repulsion)



The rotational curve for the Milky Way can be computed:

Region (R)	Central density ρ_R^c [$10^{10} M_\odot \text{Kpc}^{-3}$]	Scale radius a_R [Kpc]
Inner Bulge (IB)	$\rho_{IB}^c = 3.6 \times 10^3$	$a_{IB} = 3.8 \times 10^{-3}$
Main Bulge (MB)	$\rho_{MB}^c = 19.0$	$a_{MB} = 1.2 \times 10^{-1}$
Disk (D)	$\rho_D^c = 1.50$	$a_D = 1.2$

$$\rho_R(r) = \rho_R^c \exp(-r/a_R)$$

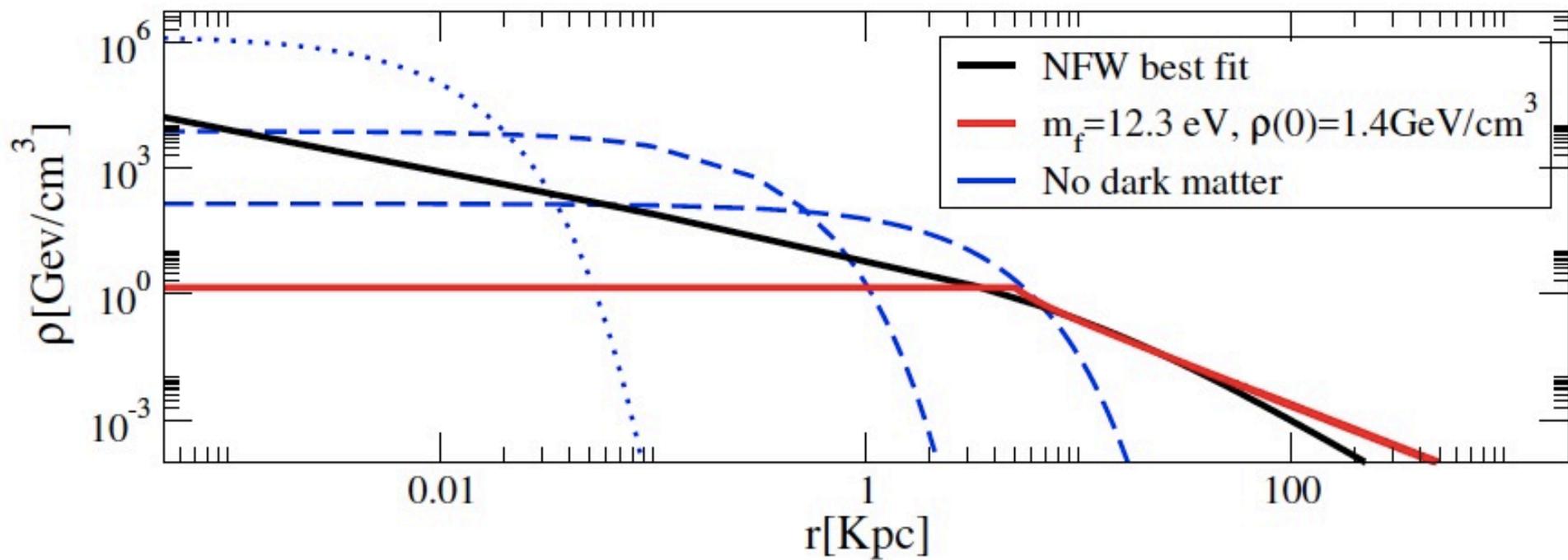
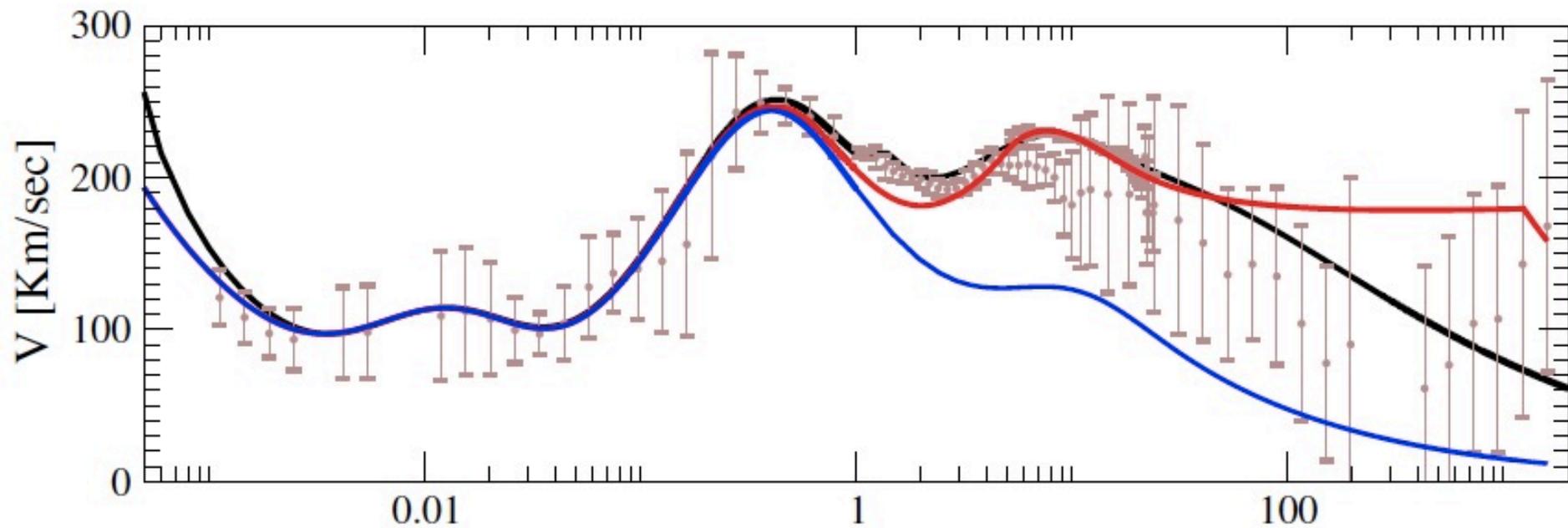
$$v_{BH}(r) = \sqrt{G \frac{M_{BH}}{r}}$$

$$v_R(r) = \sqrt{\frac{GM_R}{r}}, R = \text{IB, MB, D}$$

$$v_{DM}(r) = \sqrt{\frac{GM_{DM}}{r}}.$$

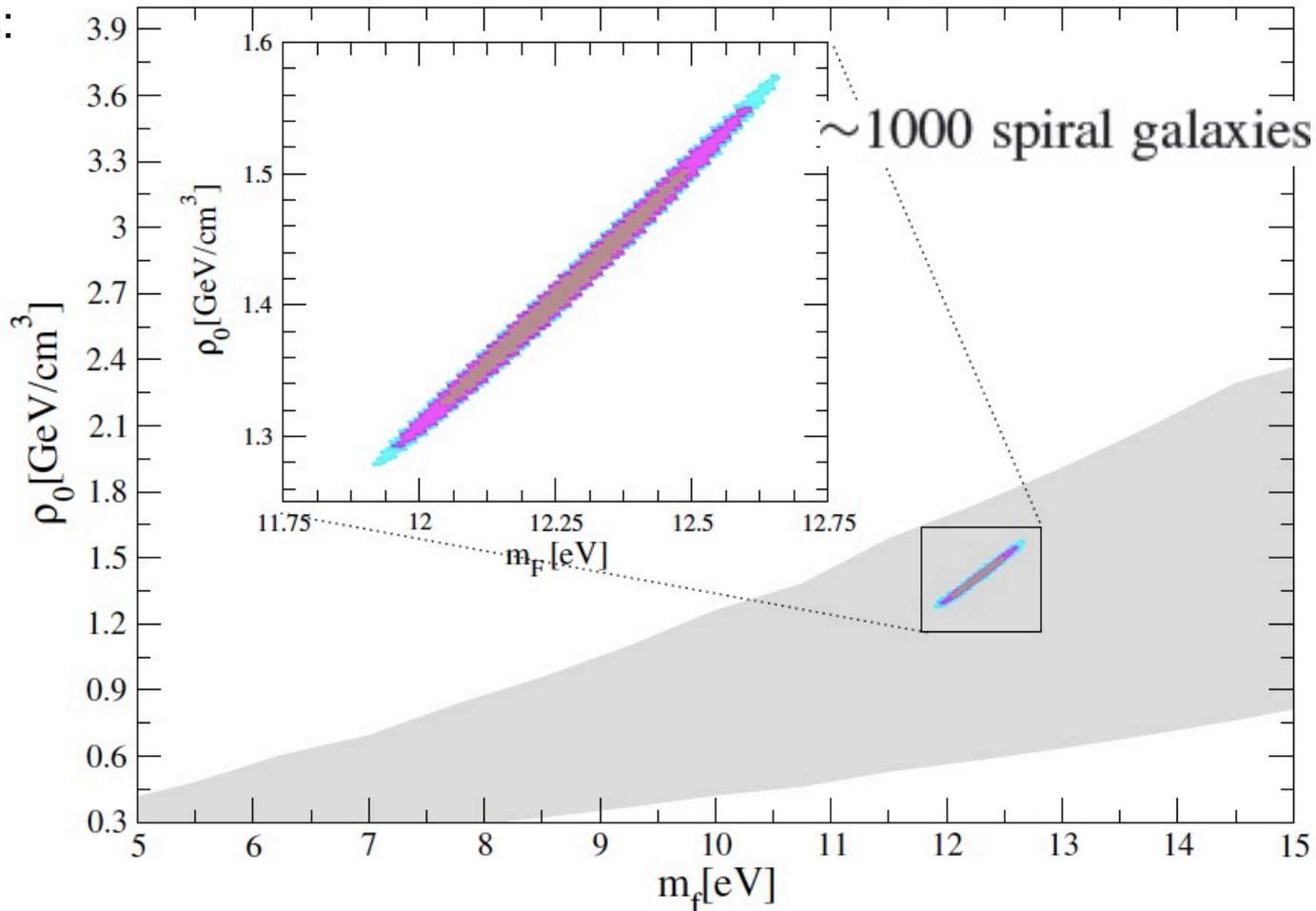
$$v^{th}(r) = \sqrt{\sum_{R=\text{IB, MB, D}} v_R^2(r) + v_{BH}^2(r) + v_{DM}^2(r)}$$

A fermion with very low mass, few eV can be a good candidate for DM!!!



Constraining ultralight fermionic DM

Strong constraints:



Core halos

$$\log \frac{\mu_{0D}}{M_{\odot} \text{pc}^{-2}} = 2.2 \pm 0.25$$

Scalar field as dark matter?

- A different approach: The Scalar Field Dark Matter model (SFDM)
The Dark Matter is modeled by a scalar field with a ultra-light associated particle. ($m \sim 10^{-23} \text{eV}$)
 - At cosmological scales it behaves as cold dark matter
T. Matos, L.A. Urena-Lopez, *Class. Quant. Grav.* 17 L75 (2000),
V. Sahni and L.M. Wang, *Phys. Rev D* 62, 103517 (2000).
 - At galactic scales, it does not have its problems: neither a cuspy profile, nor a over-density of satellite galaxies.

Ultralight scalars as cosmological dark matter

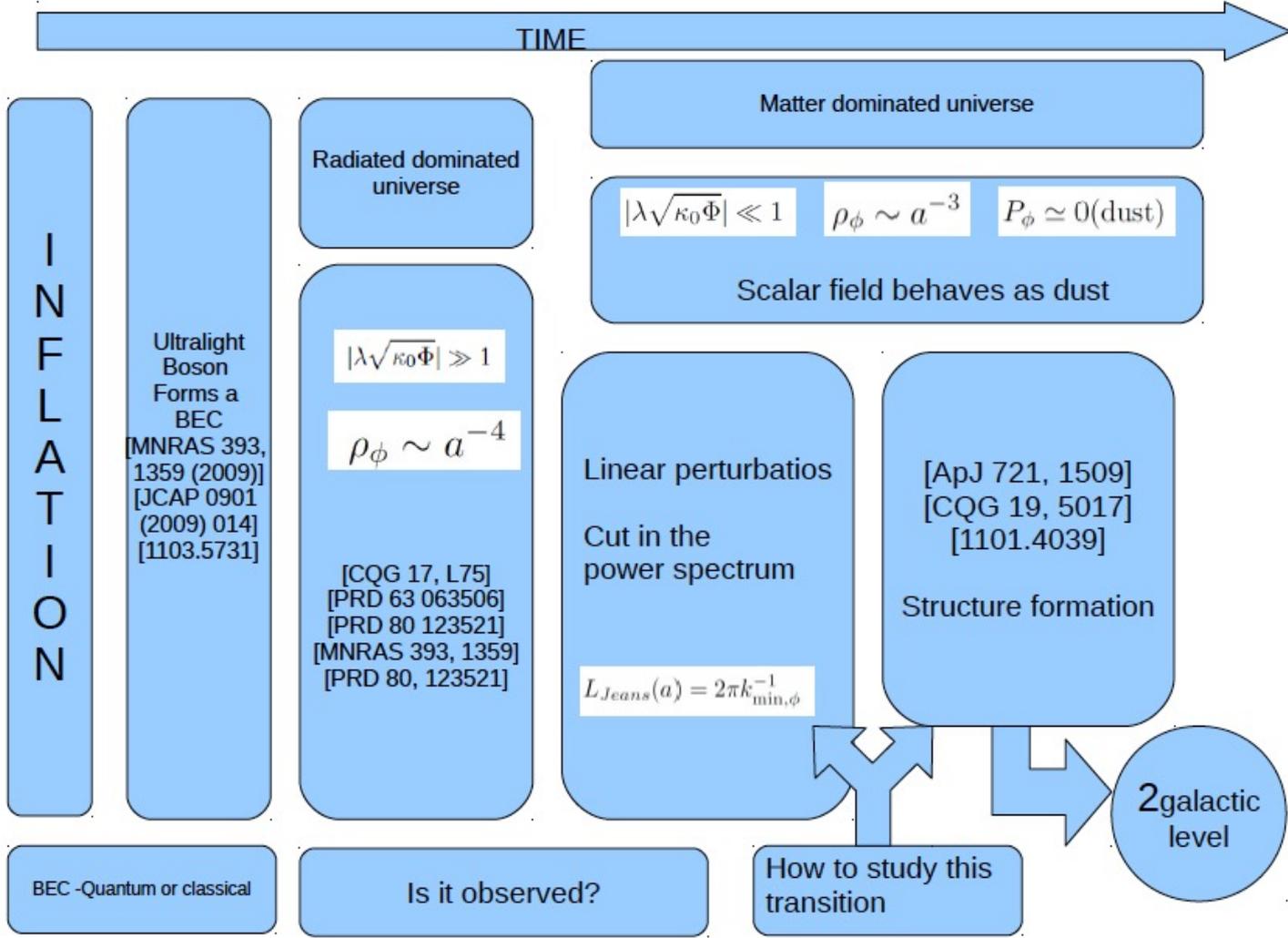
Lam Hui (Columbia U.), Jeremiah P. Ostriker (Columbia U. & Princeton U. Observ.), Scott Tremaine, Edward Witten (Princeton, Inst. Advanced Study). Oct 26, 2016. 32 pp.

Published in **Phys.Rev. D95 (2017) no.4, 043541**

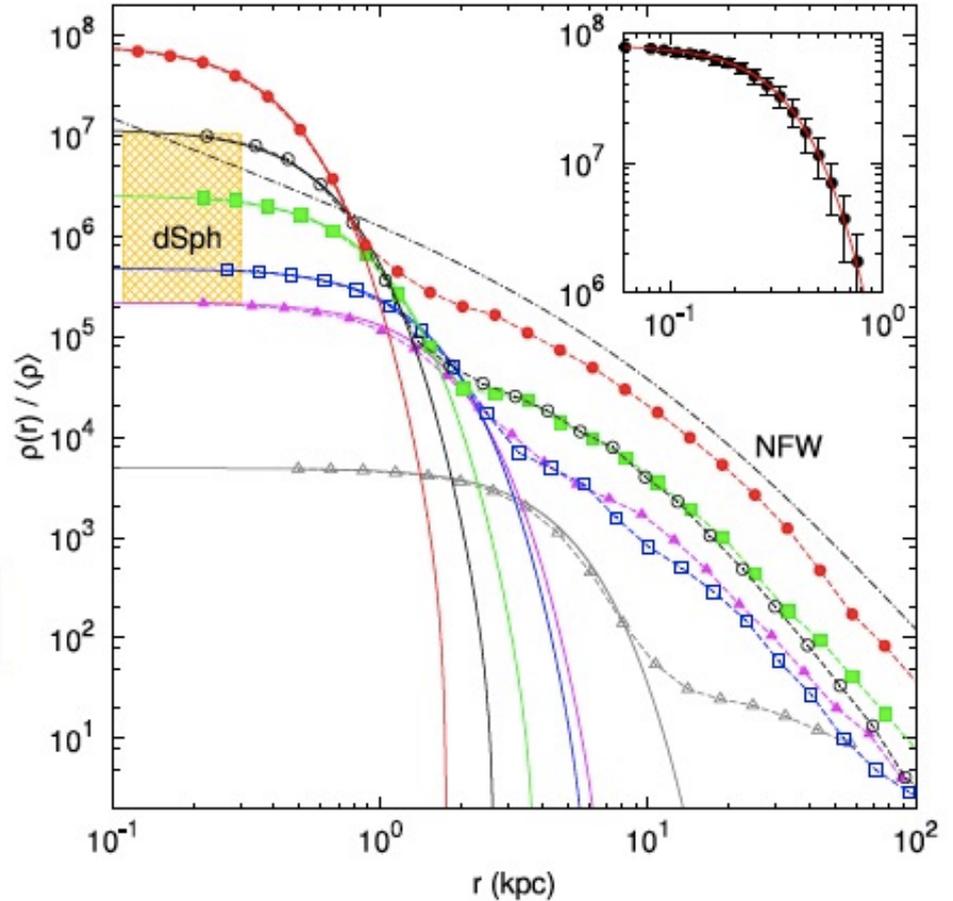
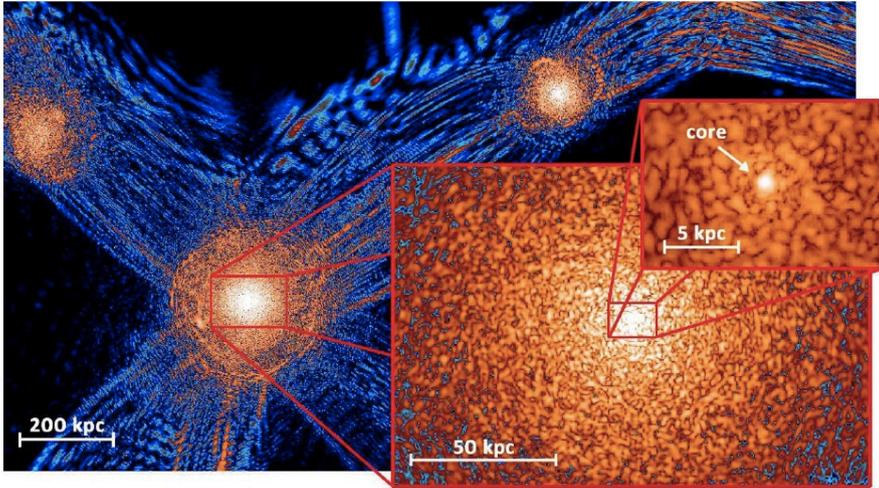
$$\mathcal{L}_{\text{L-SFDM}} = \mathcal{L}_{\text{GR}} + \mathcal{L}_B + \mathcal{L}_\Lambda - \sqrt{-g}[\Phi'^{\mu}\Phi_{,\mu} + 2V(\Phi)]$$

Ultralight bosonic dark matter?

$$\mathcal{L}_{L-SFDM} = \mathcal{L}_{GR} + \mathcal{L}_B + \mathcal{L}_\Lambda - \sqrt{-g}[\Phi^{,\mu}\Phi_{,\mu} + 2V(\Phi)]$$



The dark matter halo in SFDM



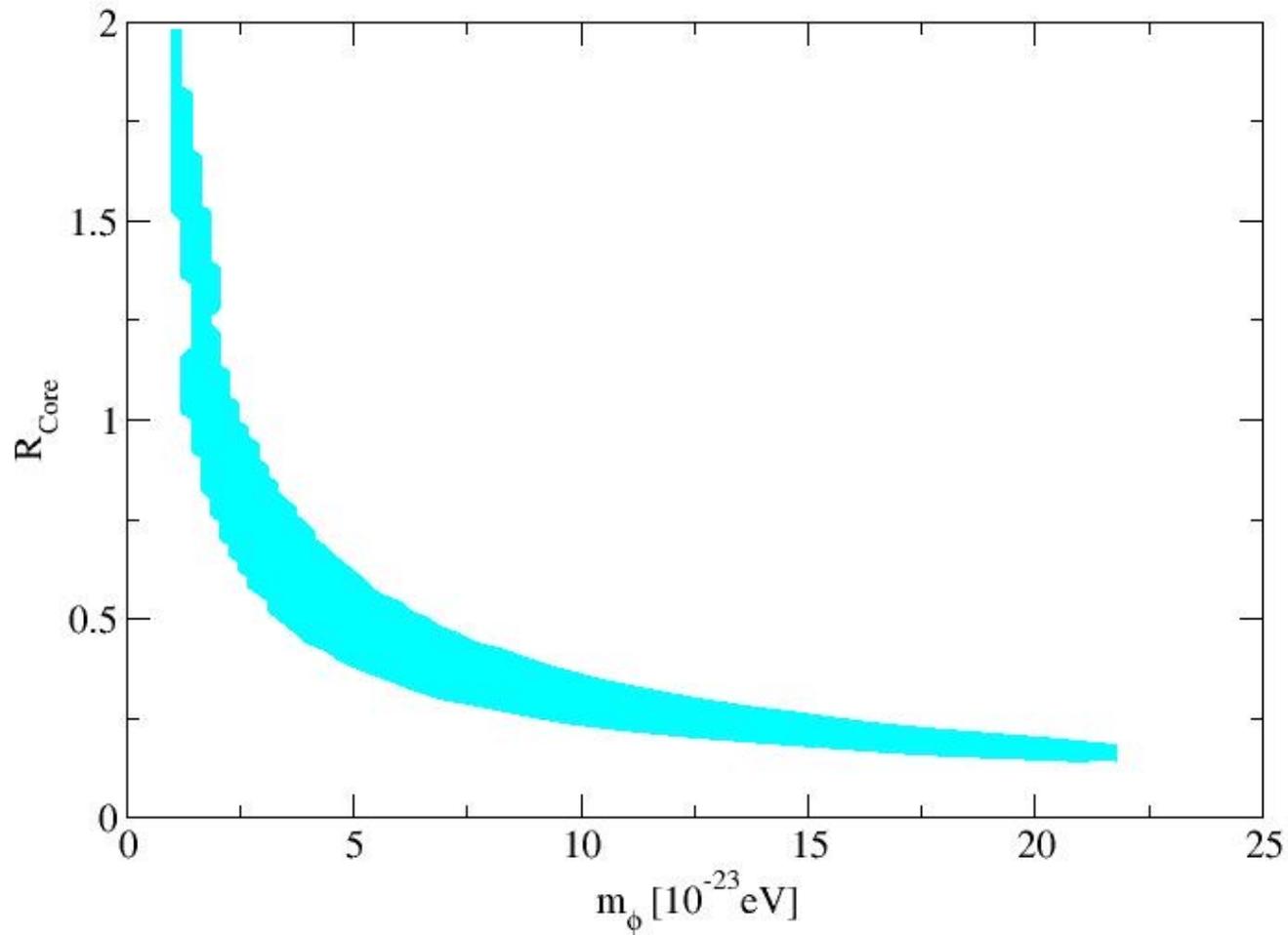
LETTERS
 PUBLISHED ONLINE: 22 JUNE 2014 | DOI: 10.1038/NPHYS2996
 nature physics

Cosmic structure as the quantum interference of a coherent dark wave

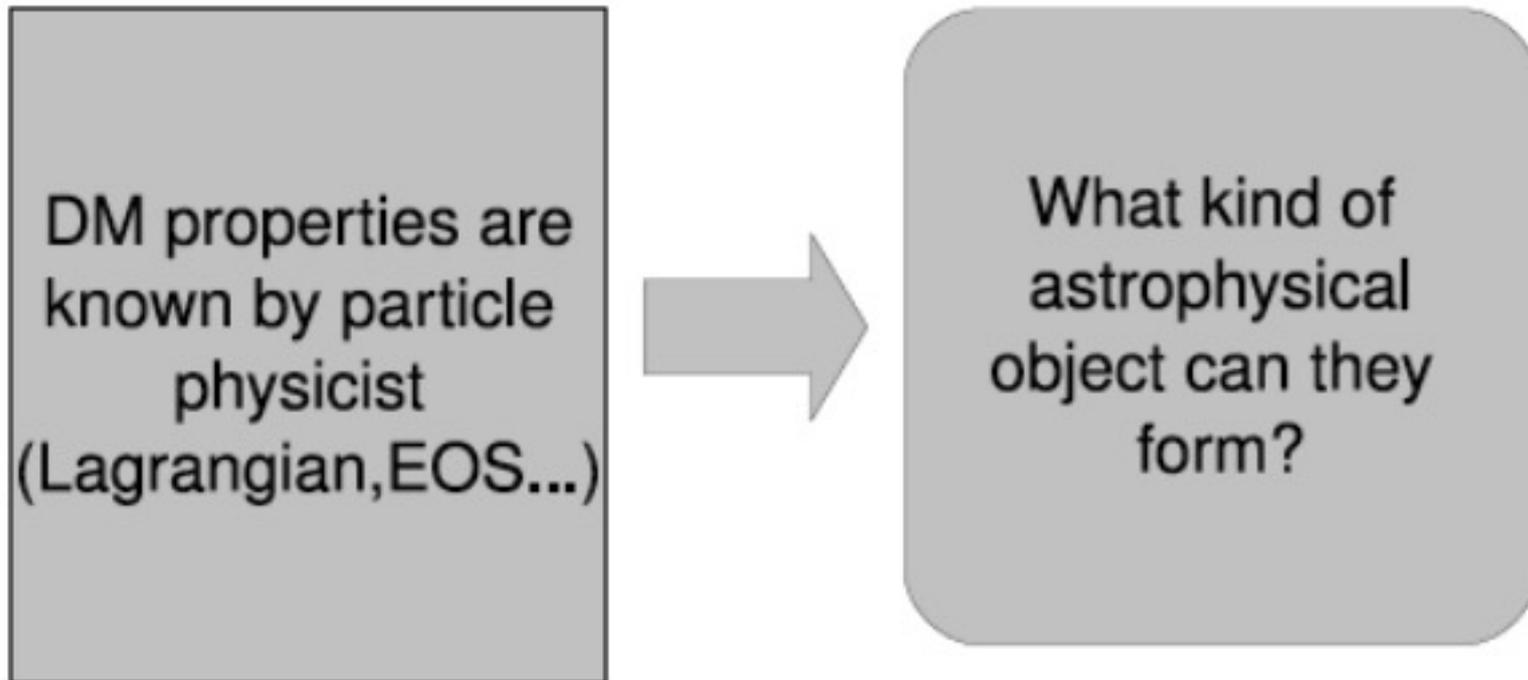
Hsi-Yu Schive¹, Tzihong Chiueh^{1,2*} and Tom Broadhurst^{3,4}

$$\rho(r) = \begin{cases} \frac{\rho_{sol}}{[1+b(r/r_{sol})^2]^8} & \text{for } r < r_\epsilon \\ \frac{\rho_{NFW}}{(r/r_s)[1+r/r_s]^2} & \text{for } r \geq r_\epsilon \end{cases} \quad \text{Newtonian Schrodinger-Poisson}$$

Constraints on bosonic dark matter from the Milky Way



Some surprises in the shape of DM halos



Systems of Self-Gravitating Particles in General Relativity and the Concept of an Equation of State*

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and

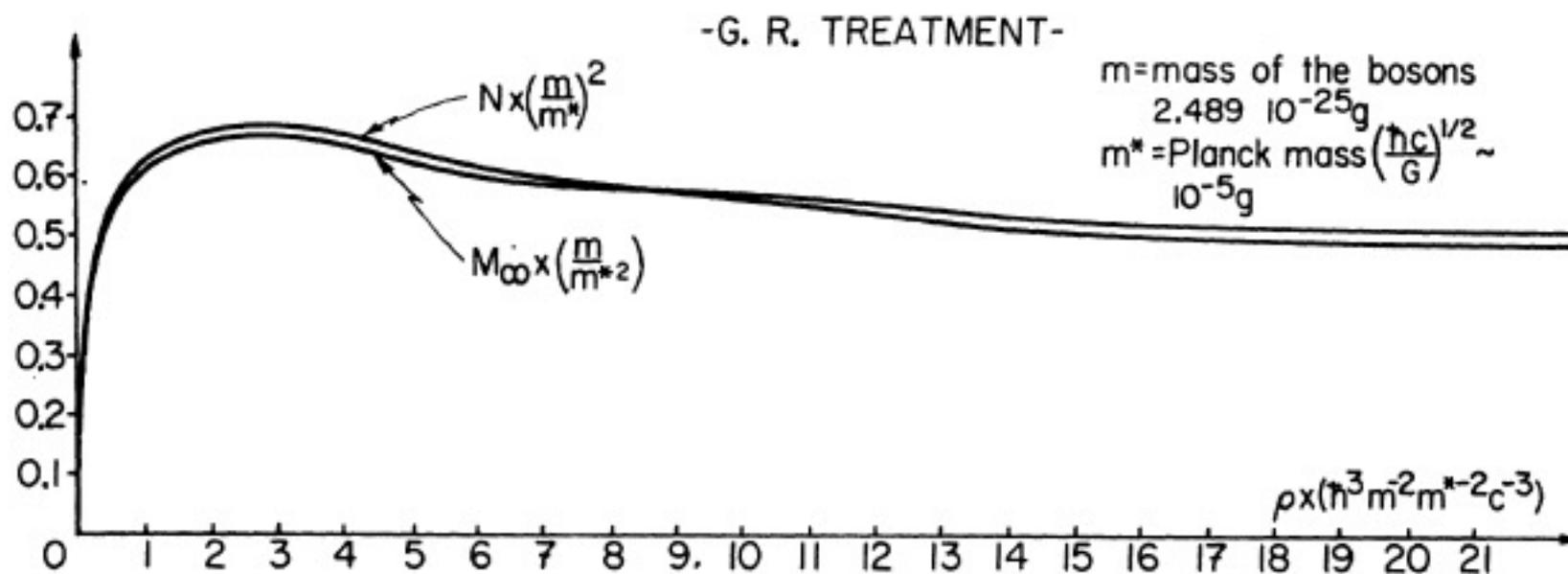
Institute for Advanced Study, Princeton, New Jersey 08540

AND

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Facoltà di Matematica, Università di Roma, Roma, Italy

(Received 4 February 1969)



Spherical symmetry

$$ds^2 = B(r)c^2 dt^2 - A(r)dr^2 - r^2(\sin^2\theta d\varphi^2 + d\theta^2)$$

Harmonic time dependence

$$\phi(r, \theta, \varphi, t) = R(r) Y_l^m(\theta, \varphi) e^{-i(E/\hbar)t}$$

The Klein-Gordon equations reads:

$$R_{ln}'' + (2/r + \frac{1}{2}B'/B - \frac{1}{2}A'/A)R_{ln}' + A[E_{nl}^2 B^{-1} \hbar^{-2} c^{-2} - \mu^2 - l(l+1)A^{-1}r^{-2}]R_{ln} = 0$$

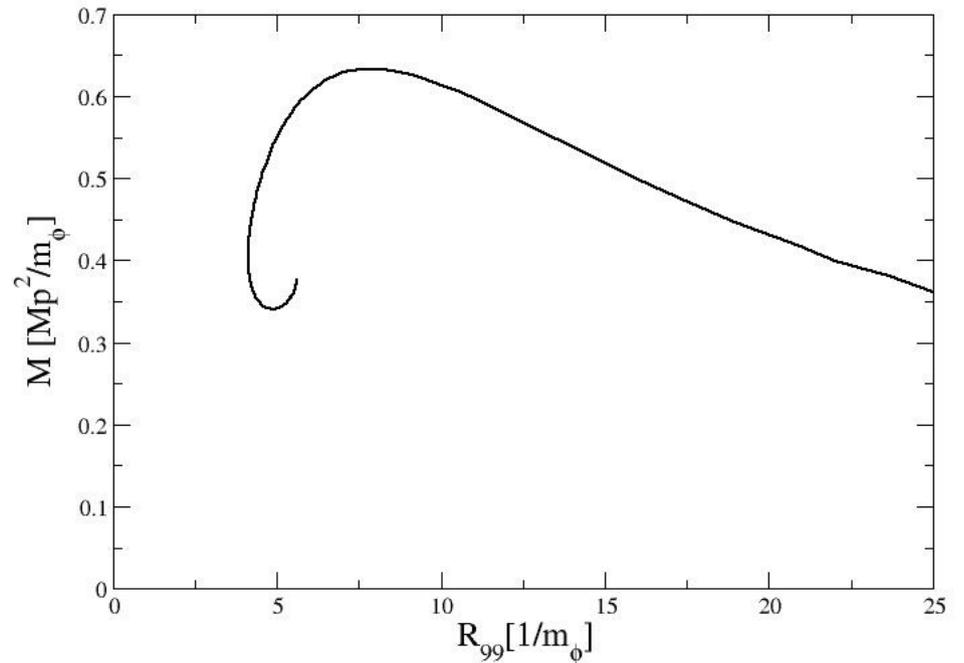
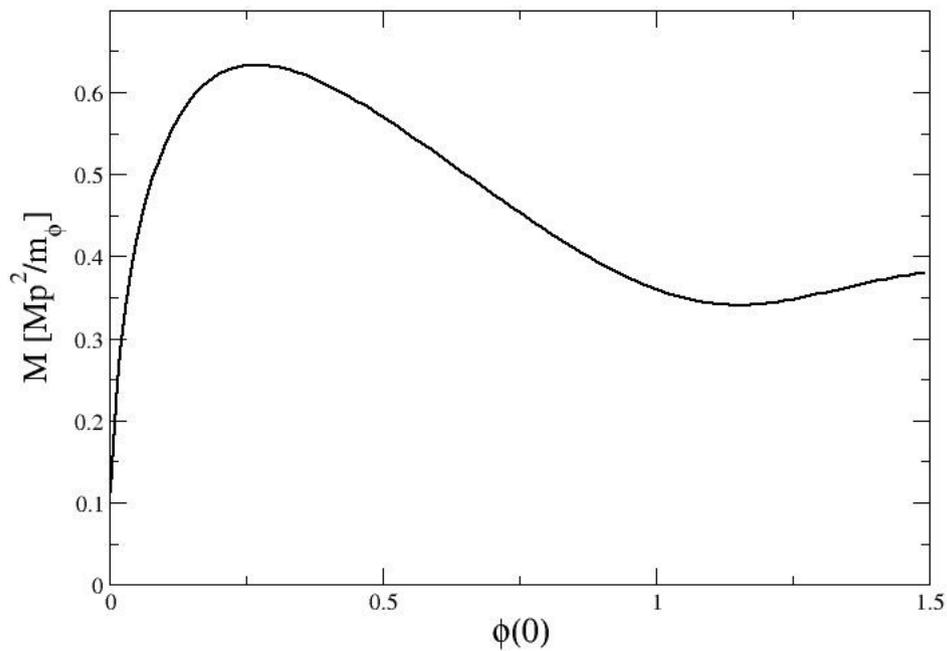
And the metric coefficients satisfies:

$$A'/(A^2 r) + (1/r^2)(1 - 1/A) = \epsilon \{ [B^{-1} E_{01}^2 / (\hbar^2 c^2) + \mu^2] R_{01}^2 + A^{-1} R_{01}'^2 \}$$

$$B'/(ABr) - (1/r^2)(1 - 1/A) = \epsilon \{ [B^{-1} E_{01}^2 / (\hbar^2 c^2) - \mu^2] R_{01}^2 + A^{-1} R_{01}'^2 \}$$

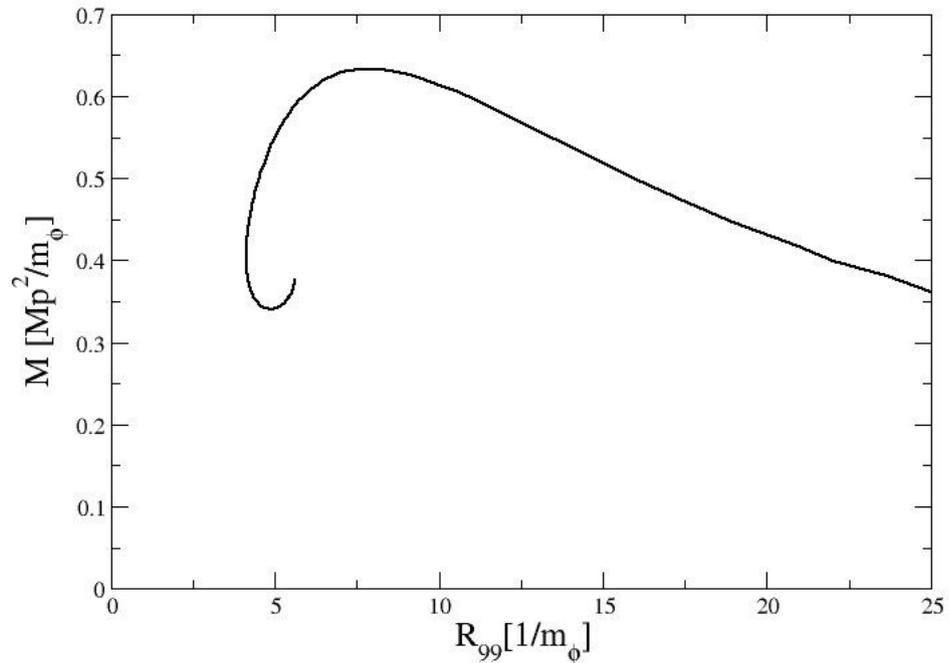
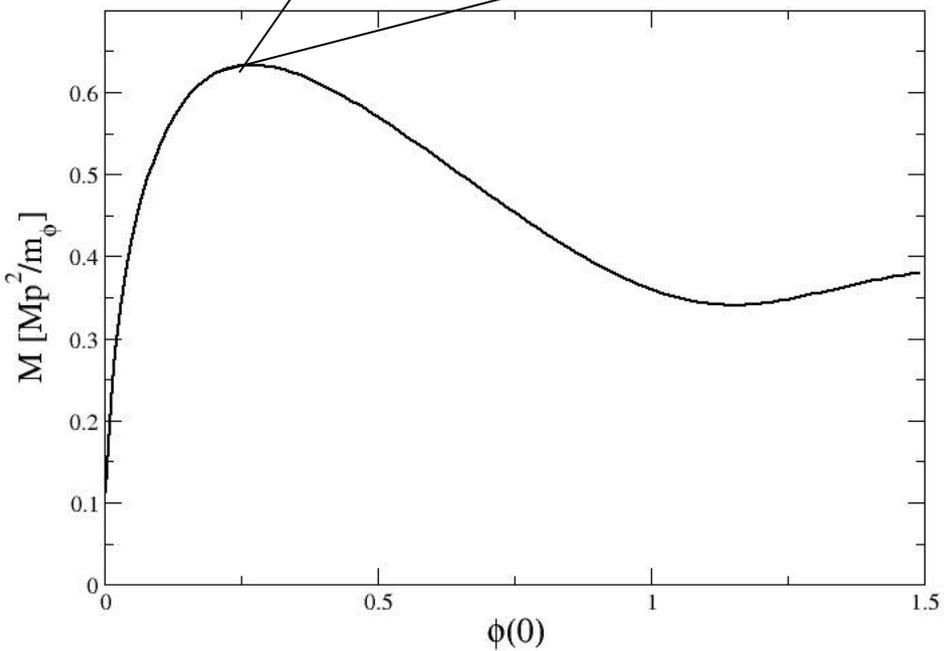
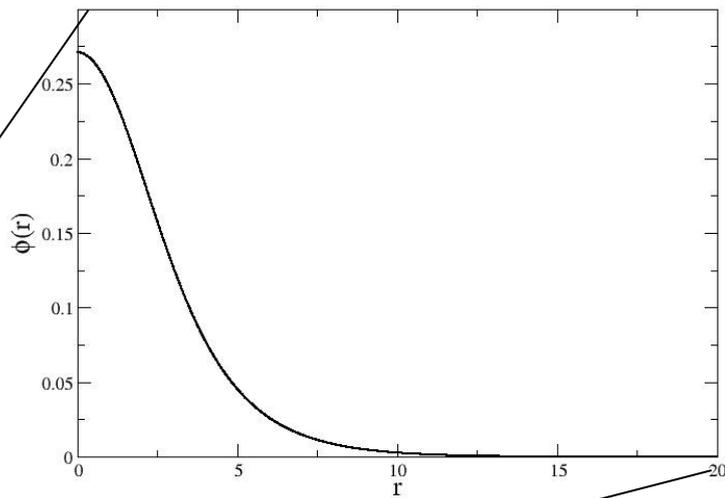
$$\epsilon = 4\pi G c^{-4} \hbar^2 m^{-1} N$$

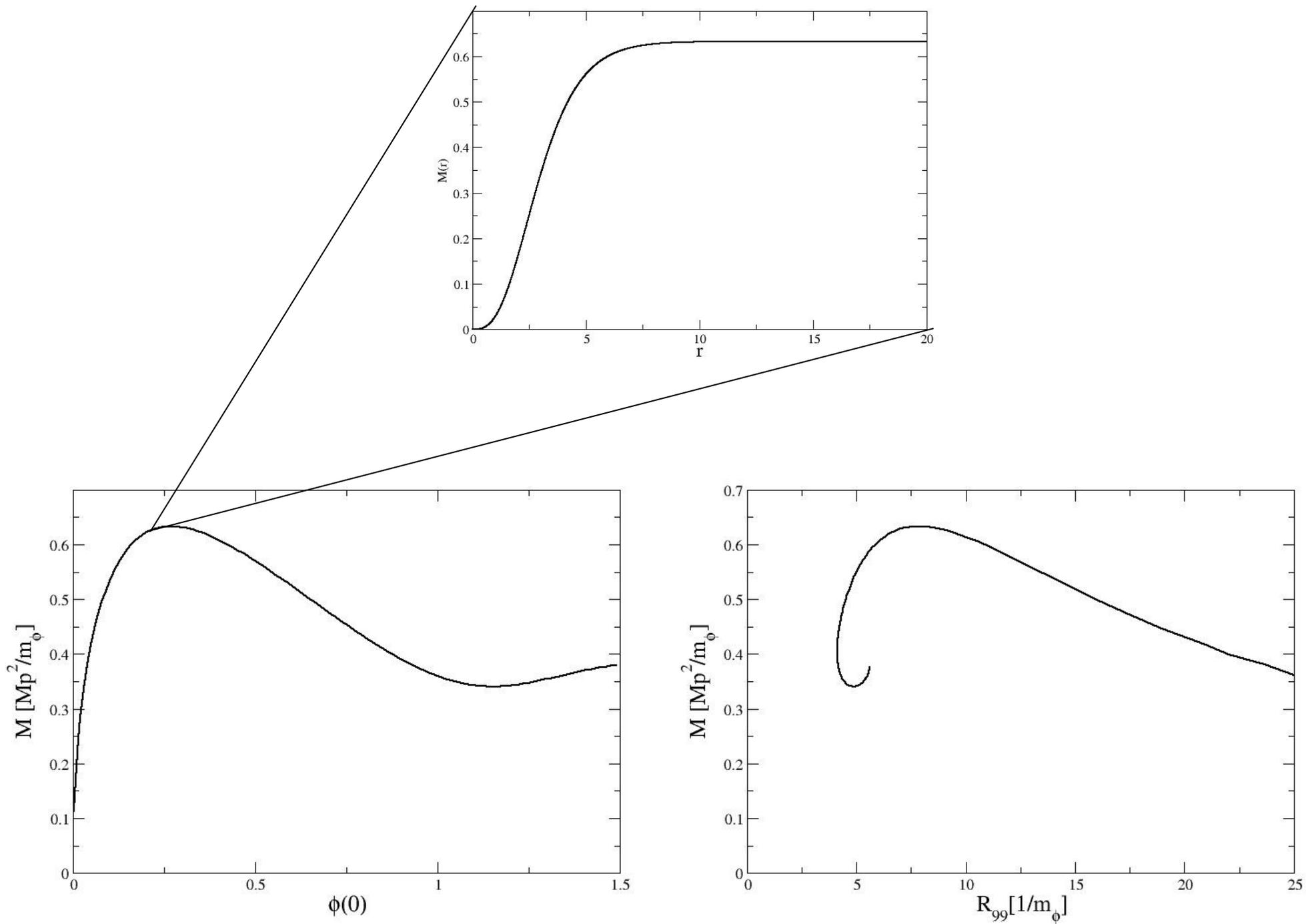
Possible boson star configurations



Possible bc

urations





Semiclassical description of the Einstein-Klein-Gordon system

$$G_{\mu\nu} = 8\pi G \langle \hat{T}_{\mu\nu} \rangle$$

Here $G_{\mu\nu}$ is the Einstein tensor, and $\langle \hat{T}_{\mu\nu} \rangle = \langle \psi | \hat{T}_{\mu\nu} | \psi \rangle$

$$\hat{T}_{\mu\nu} = \partial_\mu \hat{\phi} \partial_\nu \hat{\phi} - \frac{1}{2} g_{\mu\nu} (\partial_\alpha \hat{\phi} \partial^\alpha \hat{\phi} + m_0^2 \hat{\phi} \hat{\phi})$$

$$\hat{\phi}(x) = \sum_i (\hat{a}_i u_i(x) + \hat{a}_i^\dagger u_i^*(x)) \quad \pi(x) \equiv \sqrt{\det(\gamma_{mn})} (n^\mu \partial_\mu \phi)$$

$$[\hat{\phi}(t, \vec{x}), \hat{\pi}(t, \vec{y})] = i\delta(\vec{x} - \vec{y}),$$

$$[\hat{\phi}(t, \vec{x}), \hat{\phi}(t, \vec{y})] = [\hat{\pi}(t, \vec{x}), \hat{\pi}(t, \vec{y})] = 0.$$

$$\hat{T}_{\mu\nu} = \sum_{ij} \left[\hat{a}_i \hat{a}_j T_{\mu\nu}(u_i, u_j^*) + \hat{a}_i \hat{a}_j^\dagger T_{\mu\nu}(u_i, u_j) + h.c. \right]$$

$$T_{\mu\nu}(u_i, u_j) = \partial_\mu u_i \partial_\nu u_j^* - \frac{1}{2} g_{\mu\nu} (\partial_\alpha u_i \partial^\alpha u_j^* + m_0^2 u_i u_j^*)$$

$$\hat{N}_i = \hat{a}_i^\dagger \hat{a}_i$$

$$|N_1, N_2, \dots\rangle = \frac{(\hat{a}_1^\dagger)^{N_1}}{\sqrt{N_1!}} \frac{(\hat{a}_2^\dagger)^{N_2}}{\sqrt{N_2!}} \dots |0\rangle$$

A simple way to guarantee a static configuration is populating all the mode-functions with a definite number of particles,¹

$$|N_{000}, N_{100}, N_{11-1}, N_{110}, N_{111}, \dots\rangle.$$

$$ds^2 = -\alpha^2 dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega^2$$

This leads to:

$$\psi''_{nl} = - \left[\gamma^2 + 1 - (2l + 1)r^2\gamma^2 \left(\frac{l(l+1)}{r^2} + m_0^2 \right) \psi_{nl}^2 \right] \frac{\psi'_{nl}}{r} - \left(\frac{\omega_{nl}^2}{\alpha^2} - \frac{l(l+1)}{r^2} - m_0^2 \right) \gamma^2 \psi_{nl}$$

$$\frac{d\gamma}{dr} = \sum_{nl} \frac{2l+1}{2} r\gamma \left[\left(\frac{\omega_{nl}^2}{\alpha^2} + \frac{l(l+1)}{r^2} + m_0^2 \right) \gamma^2 \psi_{nl}^2 + \psi_{nl}'^2 \right] - \left(\frac{\gamma^2 - 1}{2r} \right) \gamma,$$

$$\frac{d\alpha}{dr} = \sum_{nl} \frac{2l+1}{2} r\alpha \left[\left(\frac{\omega_{nl}^2}{\alpha^2} - \frac{l(l+1)}{r^2} - m_0^2 \right) \gamma^2 \psi_{nl}^2 + \psi_{nl}'^2 \right] + \left(\frac{\gamma^2 - 1}{2r} \right) \alpha,$$

$$n = 1, l = 0$$

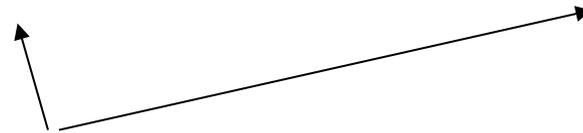
$$n = 1, 2, 3\dots, l = 0$$

$$n = 1, l = 1, 2, 3\dots$$

Boson stars

Multistate Boson stars

l -Boson Star



Boson stars relatives

Multi-State Boson stars (MSBS)

$$\hat{\Phi} = \sum_{nlm} \hat{b}_{nlm} \Phi_{nlm}(t, \mathbf{x}) + \hat{b}_{nlm}^\dagger \Phi_{nlm}^*(t, \mathbf{x})$$

$$\hat{T}_{ab} = \partial_a \hat{\Phi} \partial_b \hat{\Phi} - \frac{1}{2} g_{ab} (g^{cd} \partial_c \hat{\Phi} \partial_d \hat{\Phi} + \mu^2 |\hat{\Phi}|^2)$$

$$G_{ab} = 8\pi \langle Q | \hat{T}_{ab} | Q \rangle$$

$$ds^2 = -\alpha^2(r) dt^2 + a^2(r) dr^2 + r^2 d\Omega.$$

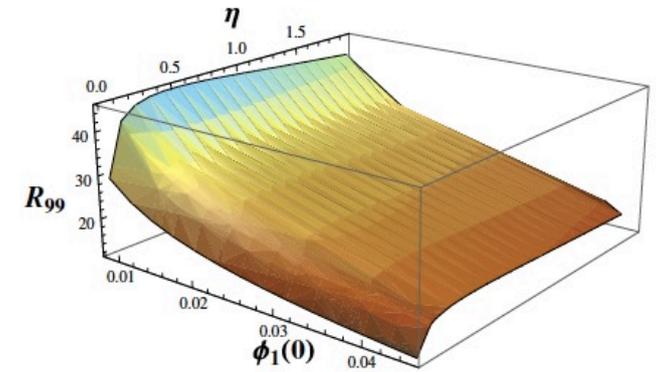
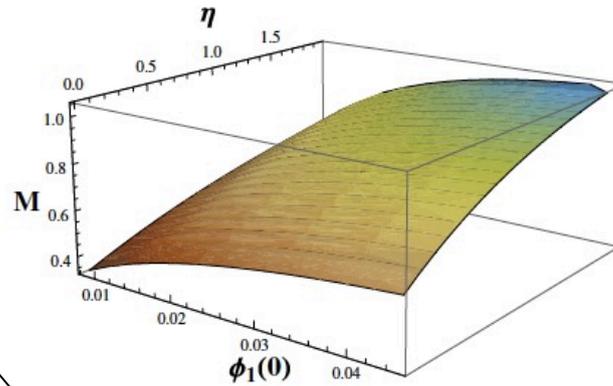
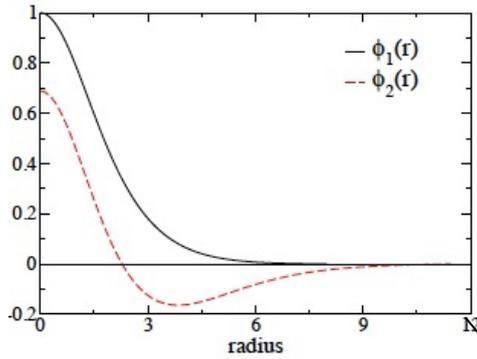
$$\partial_r a = \frac{a}{2} \left\{ -\frac{a^2 - 1}{r} + 4\pi r \sum_{n=1}^{\mathcal{J}} \left[\left(\frac{\omega_n^2}{\alpha^2} + m^2 \right) a^2 \phi_n^2 + \Phi_n^2 \right] \right\},$$

$$\partial_r \alpha = \frac{\alpha}{2} \left\{ \frac{a^2 - 1}{r} + 4\pi r \sum_{n=1}^{\mathcal{J}} \left[\left(\frac{\omega_n^2}{\alpha^2} - m^2 \right) a^2 \phi_n^2 + \Phi_n^2 \right] \right\},$$

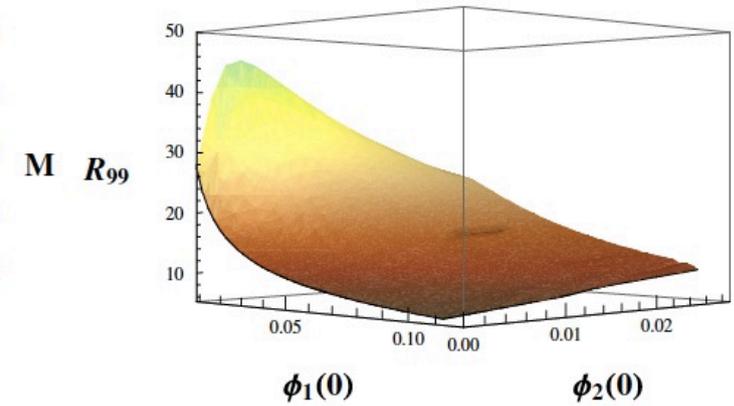
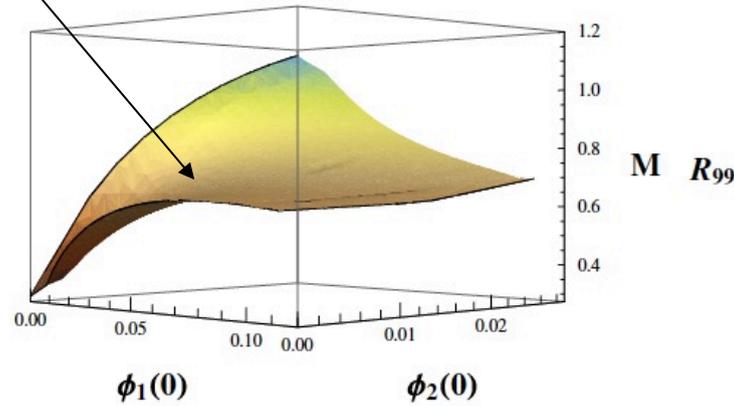
$$\partial_r \phi_n = \Phi_n,$$

$$\partial_r \Phi_n = - \left\{ 1 + a^2 - 4\pi r^2 a^2 m^2 \left(\sum_{s=1}^{\mathcal{J}} \phi_s^2 \right) \right\} \frac{\Phi_n}{r} - \left(\frac{\omega_n^2}{\alpha^2} - m^2 \right) \phi_n a^2.$$

n=2



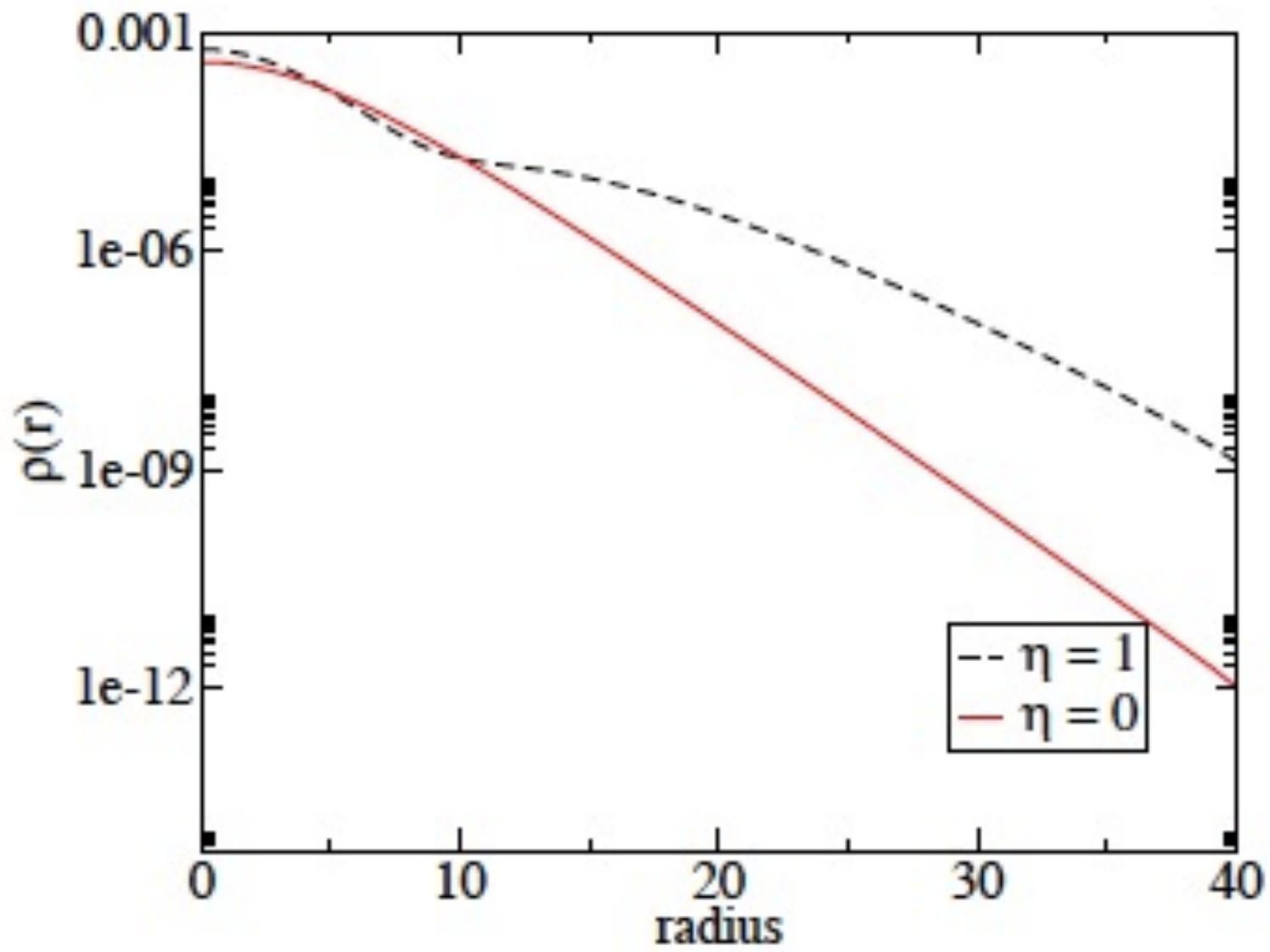
$$\eta = \frac{N^{(2)}}{N^{(1)}}$$



PHYSICAL REVIEW D 81, 044031 (2010)

Multistate boson stars

A. Bernal,¹ J. Barranco,¹ D. Alic,^{1,2} and C. Palenzuela^{1,3}



ℓ -Boson Star

$$\sum_{m=-\ell}^{\ell} |Y^{\ell m}(\vartheta, \varphi)|^2 = \frac{2\ell + 1}{4\pi}$$

$$T_{\mu\nu} = \frac{1}{2} \sum_i [\nabla_{\mu} \Phi_i^* \nabla_{\nu} \Phi_i + \nabla_{\mu} \Phi_i \nabla_{\nu} \Phi_i^* - g_{\mu\nu} (\nabla_{\alpha} \Phi_i^* \nabla^{\alpha} \Phi_i + \mu^2 \Phi_i^* \Phi_i)]$$

$$\Phi_{\ell m}(t, r, \vartheta, \varphi) = \phi_{\ell}(t, r) Y^{\ell m}(\vartheta, \varphi)$$

$$ds^2 = -\alpha^2 dt^2 + \gamma^2 dr^2 + r^2 d\Omega^2, \quad \gamma^2 := \frac{1}{1 - \frac{2M}{r}},$$

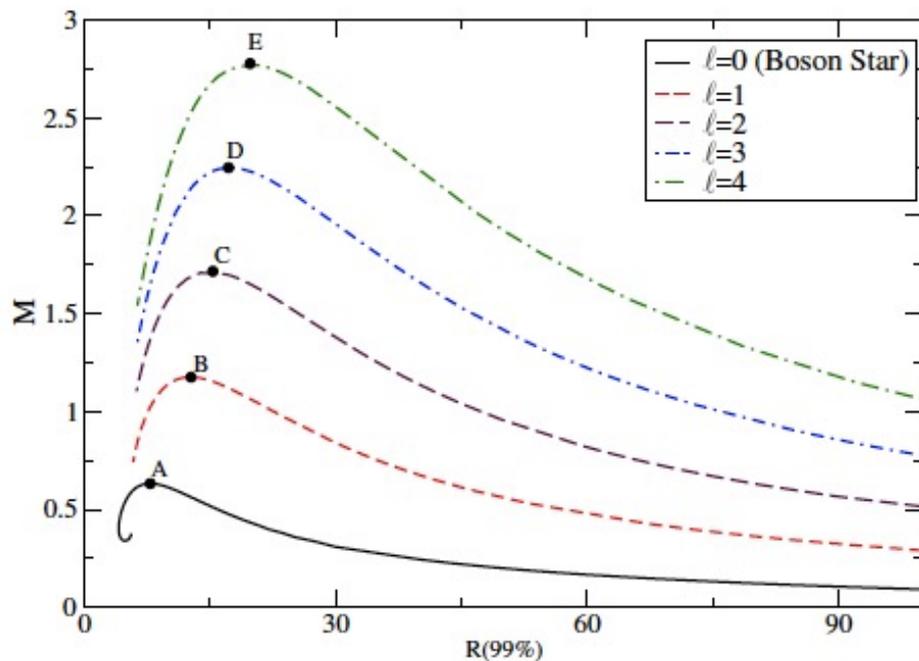
$$u_{\ell} := \psi_{\ell} / r^{\ell}$$

ℓ -Boson Star

$$\gamma' = \frac{2\ell + 1}{2} r \gamma \left[\left(\frac{\omega^2}{\alpha^2} + \frac{\ell(\ell + 1)}{r^2} + \mu^2 \right) \gamma^2 u_\ell^2 r^{2\ell} + (u_\ell' r^\ell + \ell u_\ell r^{\ell-1})^2 \right] - \left(\frac{\gamma^2 - 1}{2r} \right) \gamma,$$

$$\alpha' = \frac{2\ell + 1}{2} r \alpha \left[\left(\frac{\omega^2}{\alpha^2} - \frac{\ell(\ell + 1)}{r^2} - \mu^2 \right) \gamma^2 u_\ell^2 r^{2\ell} + (u_\ell' r^\ell + \ell u_\ell r^{\ell-1})^2 \right] + \left(\frac{\gamma^2 - 1}{2r} \right) \alpha,$$

$$u_\ell'' = \left(\mu^2 - \frac{\omega^2}{\alpha^2} \right) \gamma^2 u_\ell - (\gamma^2 + 2\ell + 1) \frac{u_\ell'}{r} + \ell^2 (\gamma^2 - 1) \frac{u_\ell}{r^2} + (2\ell + 1) \left(\mu^2 + \frac{\ell(\ell + 1)}{r^2} \right) \gamma^2 (r u_\ell' + \ell u_\ell) u_\ell^2 r^{2\ell},$$



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Classical and Quantum Gravity

Class. Quantum Grav. 35 (2018) 19LT01 (13pp)

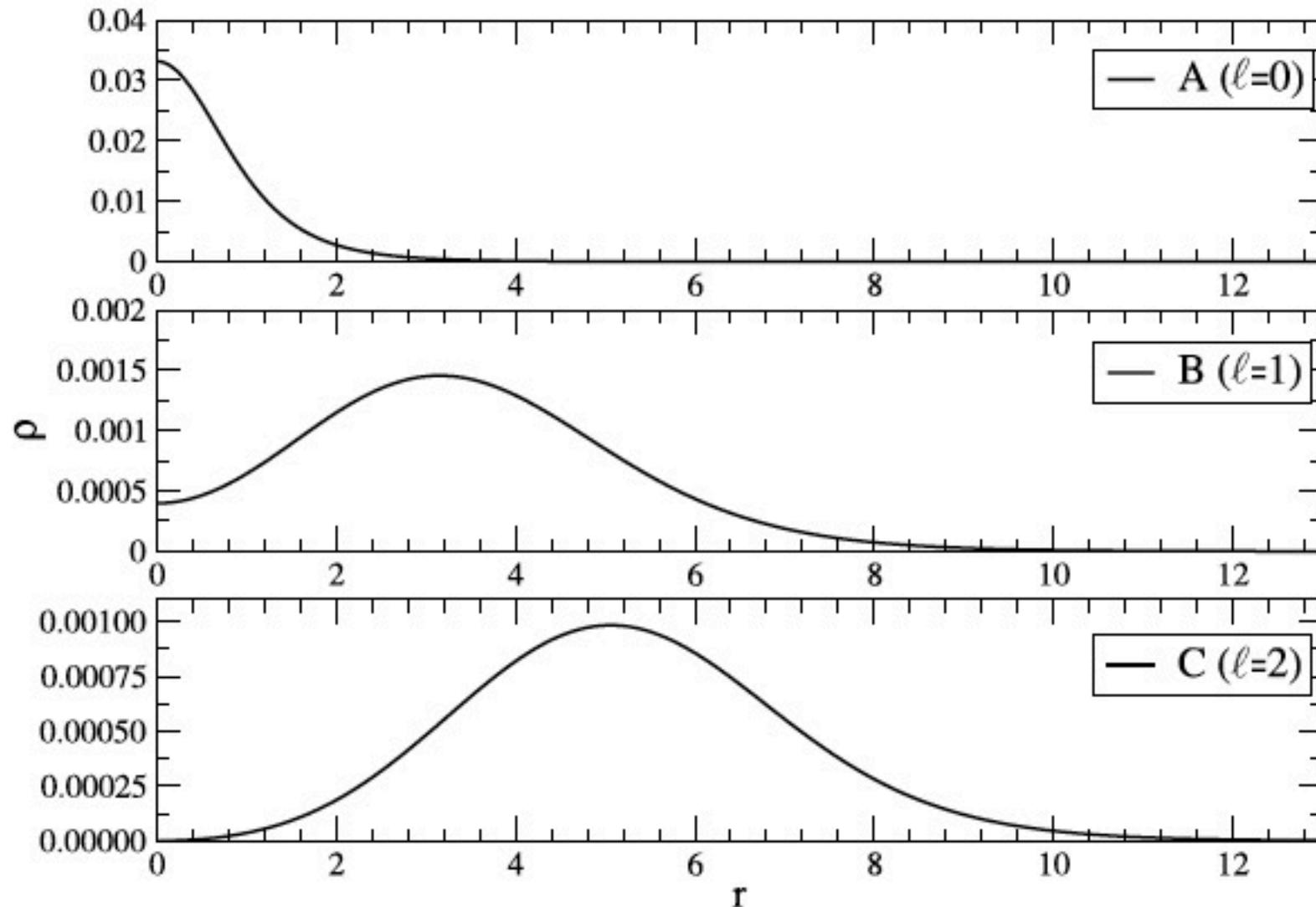
<https://doi.org/10.1088/1361-6382/aadc66>

Letter

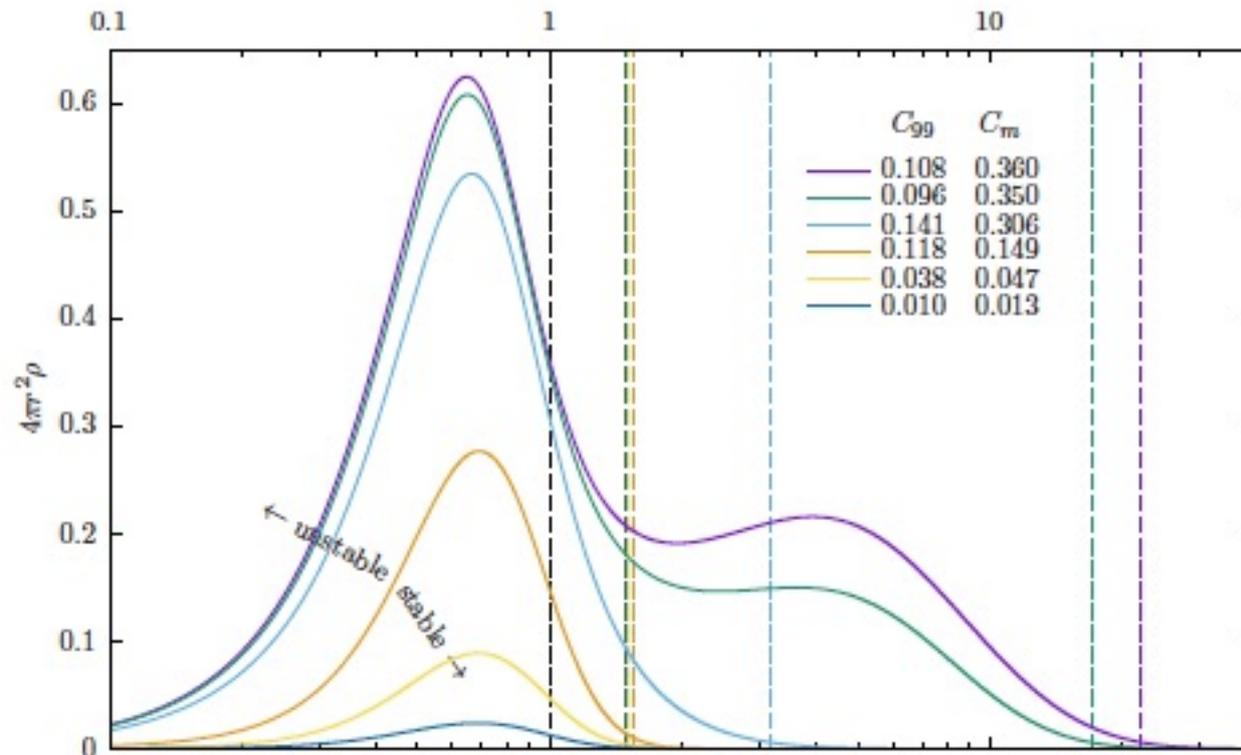
ℓ -boson stars

Miguel Alcubierre¹, Juan Barranco², Argelia Bernal²,
 Juan Carlos Degollado³, Alberto Diez-Tejedor²,
 Miguel Megevand⁴, Darío Núñez¹ and Olivier Sarbach^{5,6}

L-Boson stars can have interesting profiles



Different density profiles as a function of the “central density”



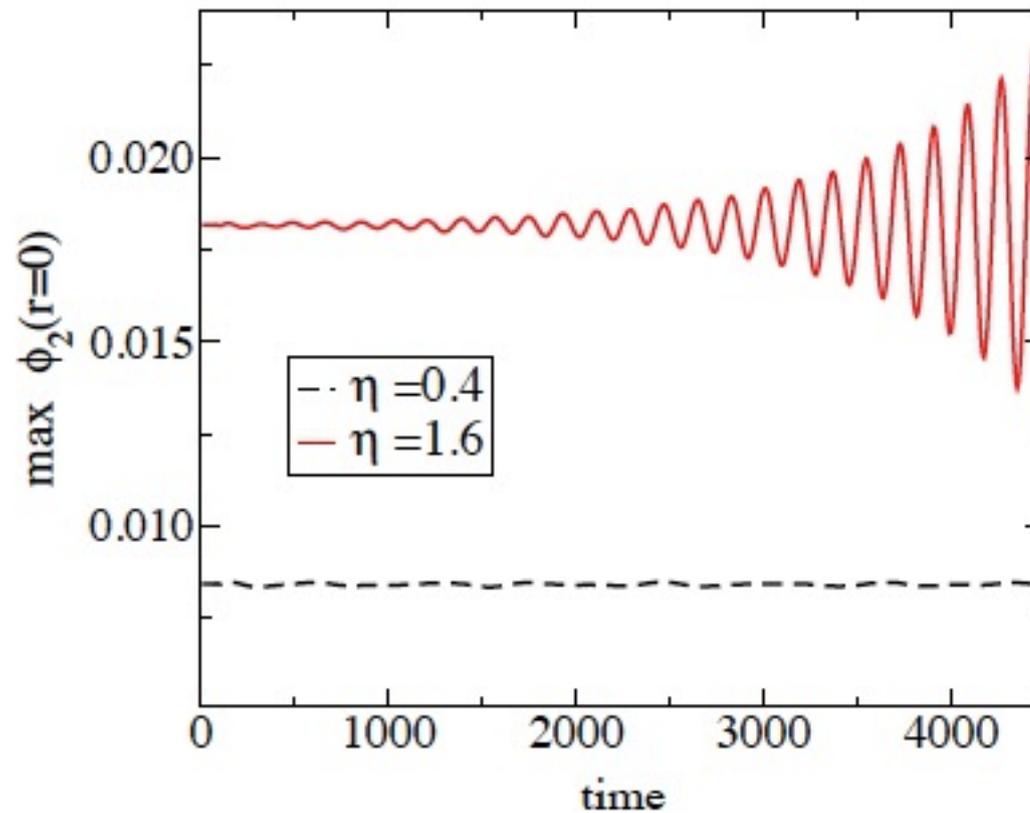
Astrophysical realization of boson
stars relative demands stability

Numerical perturbation analysis: Multistate boson stars

Boson stars in excited states are unstable under numerical perturbations.

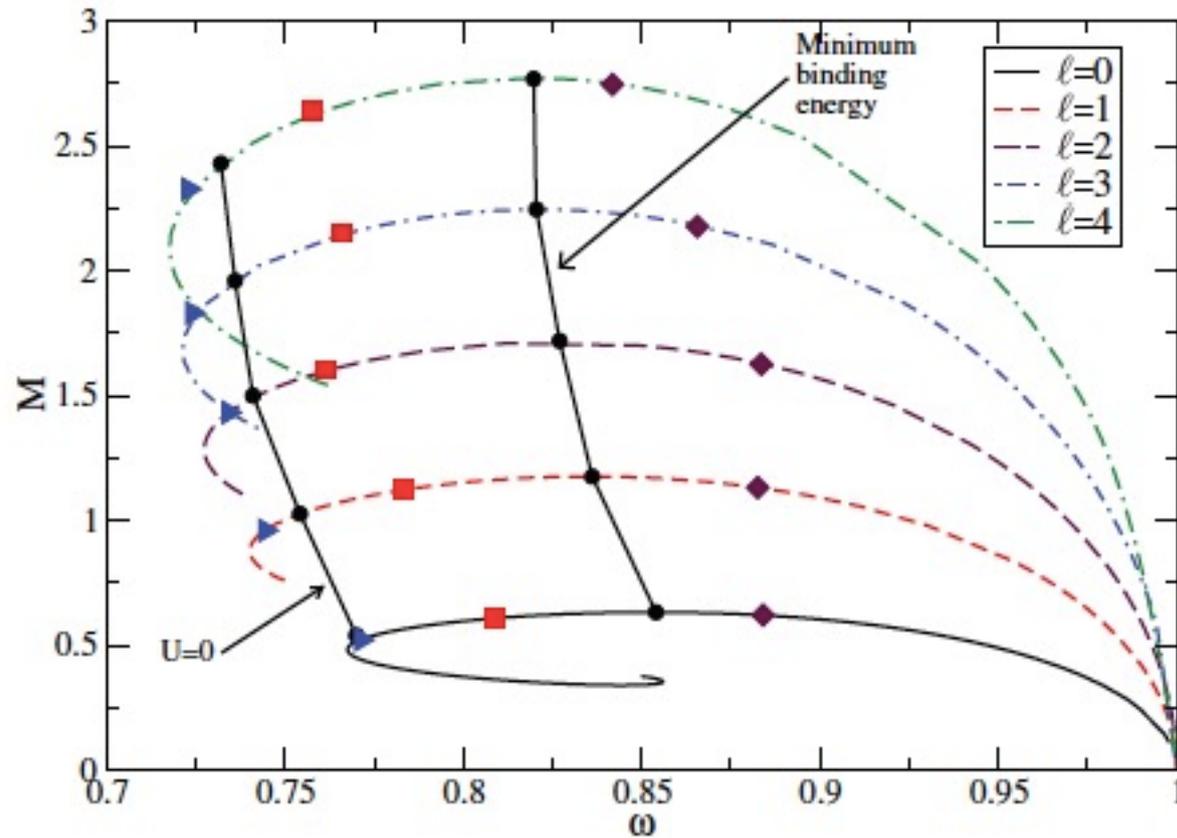
Multistate boson stars, even with particles in the excited states, can be stable

$$\eta = \frac{N^{(2)}}{N^{(1)}}$$



Stable if $\eta < 1$

Three fates of ℓ -Boson Star



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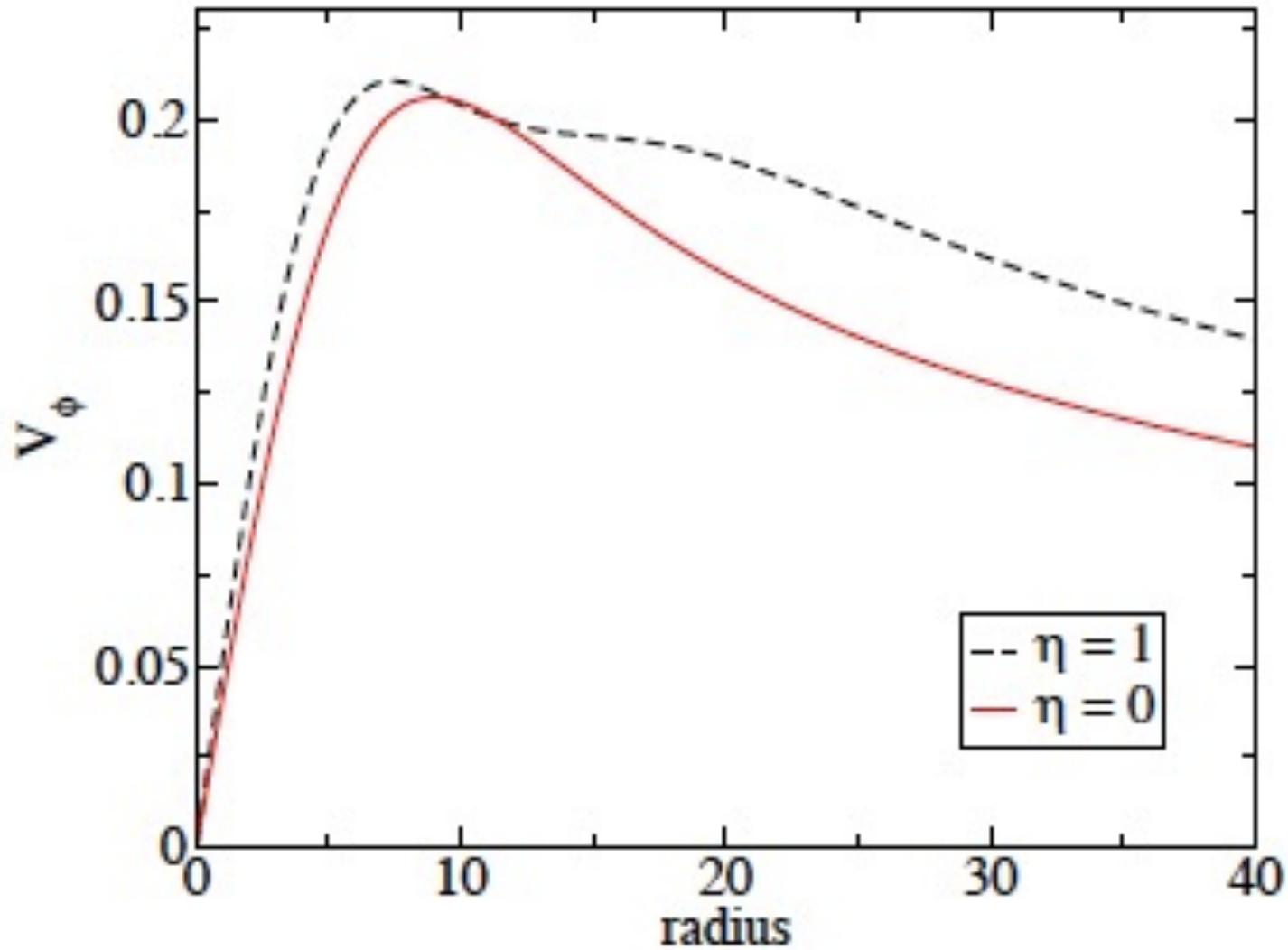
Class. Quantum Grav. **36** (2019) 215013 (26pp)

<https://doi.org/10.1088/1361-6382/ab4726>

Dynamical evolutions of ℓ -boson stars in spherical symmetry

Miguel Alcubierre¹, Juan Barranco², Argelia Bernal²,
Juan Carlos Degollado³, Alberto Diez-Tejedor²,
Miguel Megevand⁴, Darío Núñez¹ and Olivier Sarbach^{5,6}

Dark matter halos



How robust are the constraints on the scalar field mass? 45

Conclusions

1. It is possible that DM do not interact with the standard model
2. Stellar dynamics and astrophysical data can give information about the relevant parameters for ultralight candidates for dark matter
3. Care must be taken: there is a great diversity of the dark matter profile even in spherical symmetry