

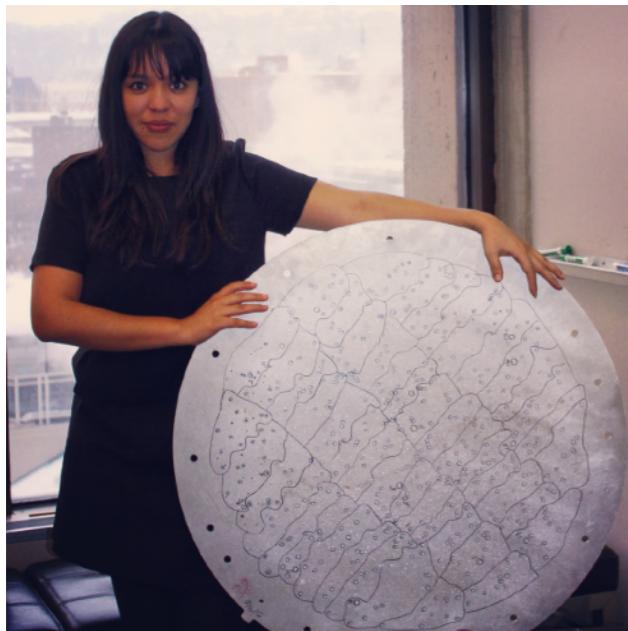
# Proposal of local bias estimation for forward modeling Reconstruction in the BAO Peak analysis.

**SEBASTIEN FROMENTEAU**  
**ICF-UNAM**

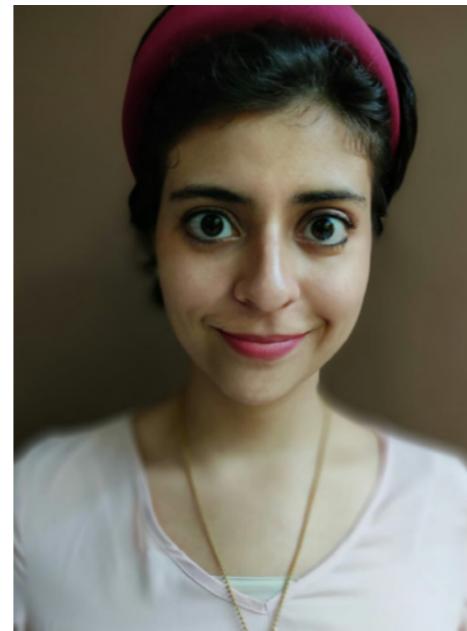


INSTITUTO DE  
CIENCIAS  
FÍSICAS

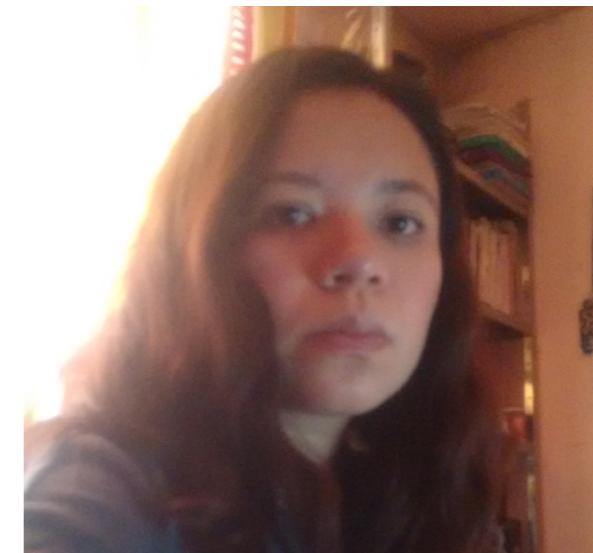
# GROUP PARTICIPATING ON THIS WORK



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**BRENDA TAPIA BENAVIDES**  
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**MATIAS**  
**RODRIGUEZ OTERO**  
**UNAM**



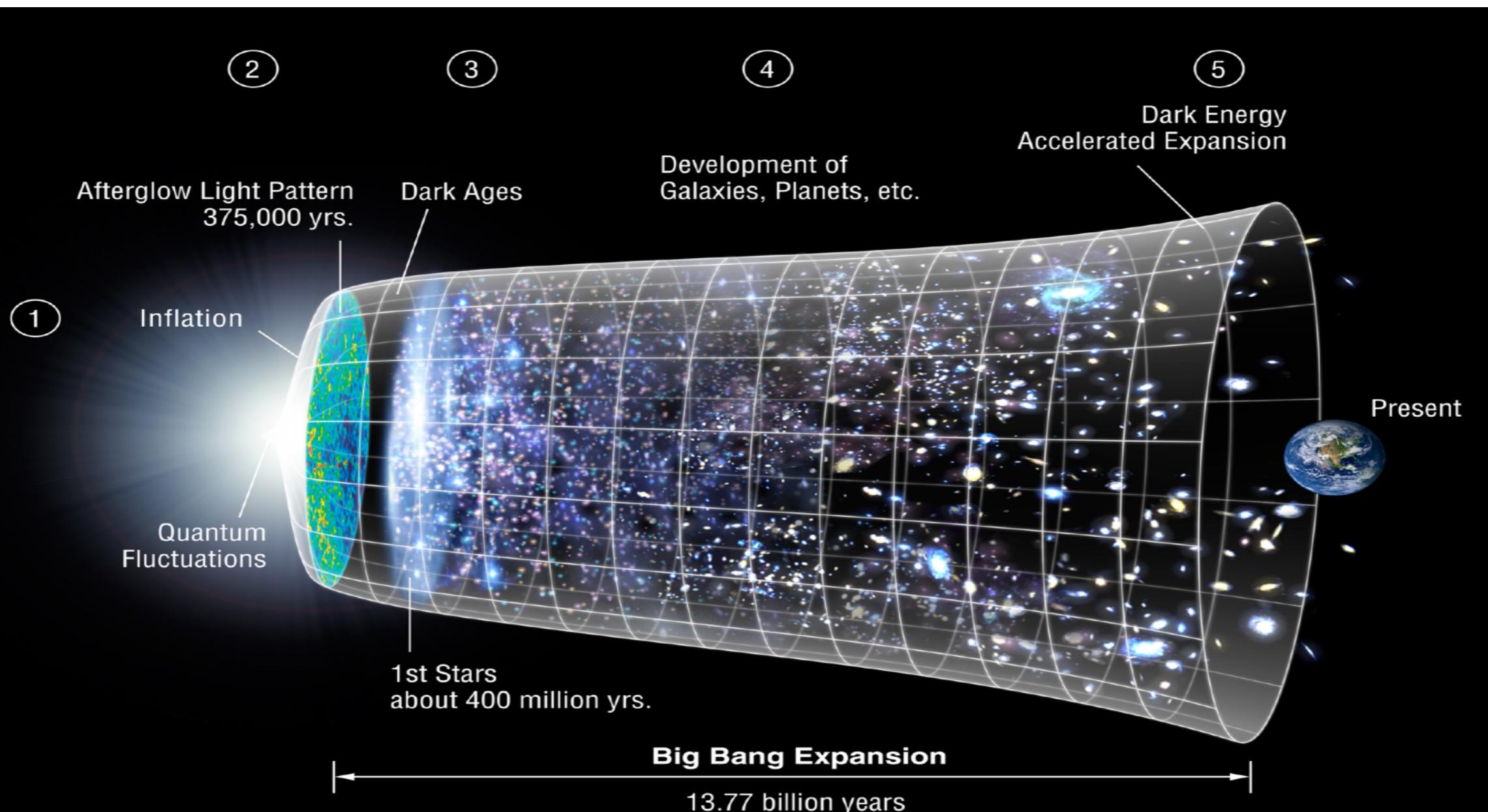
**MARCOS**  
**TOLEDO ORTIZ**  
**UAEM**

# Summary

- **Cosmology**
  - Perturbations description and evolution
  - Halo bias and Galaxy bias
- **Use of astrophysical local properties to enhance bias determination**
  - Galaxy luminosity as a degeneracy break information
  - A direct use for the CDM mapping - BAO Reconstruction
  - Emission line galaxies

# Cosmology Standard Model

$$\Lambda - CDM$$

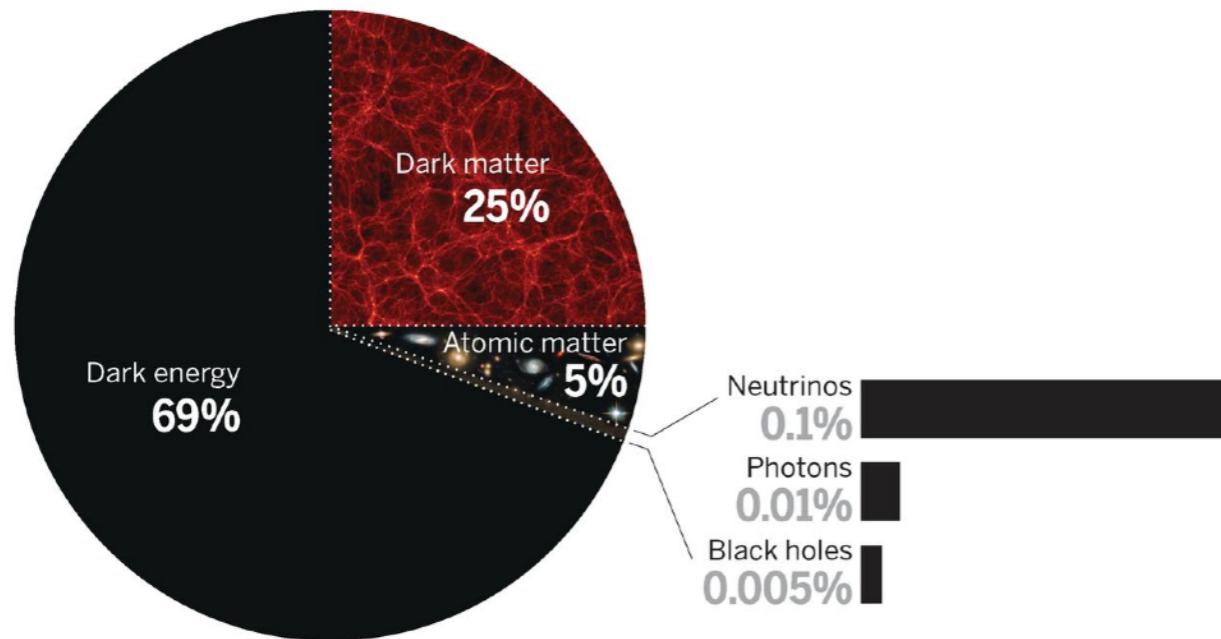


NASA/WMAP Science Team

# Cosmological Parameters

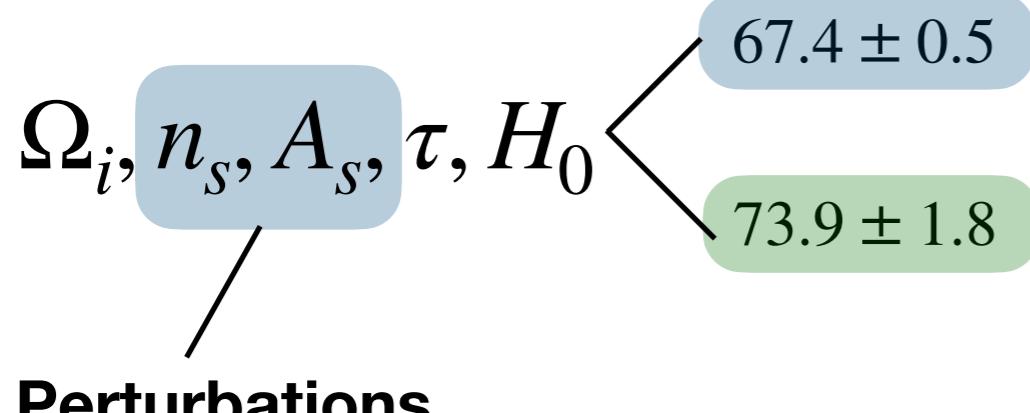
The multiple components that compose our universe

Current composition (as the fractions evolve with time)



**EARLY ROUTE**

**Cosmic Microwave Background**  
**Galaxy clustering:**  
**BAO**  
**RSD**  
**Galaxy weak lensing**



**LATE ROUTE**

**Supernovae Ia and all other standardizable observable based on Cosmic Ladder**

# Quantum Fluctuations during inflation

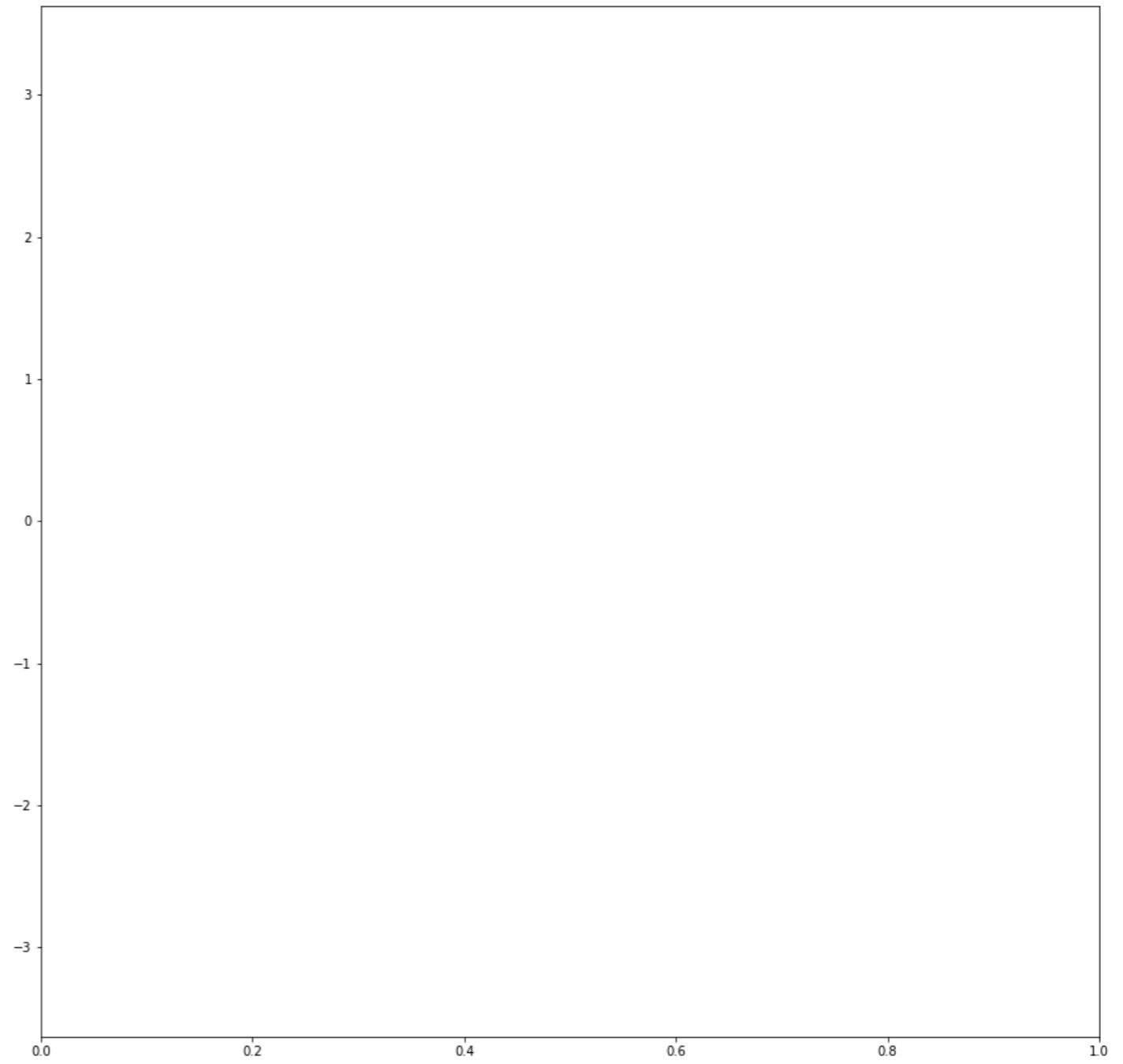
Heisenberg incertitude

$$\Delta x \times \Delta p \geq \hbar$$

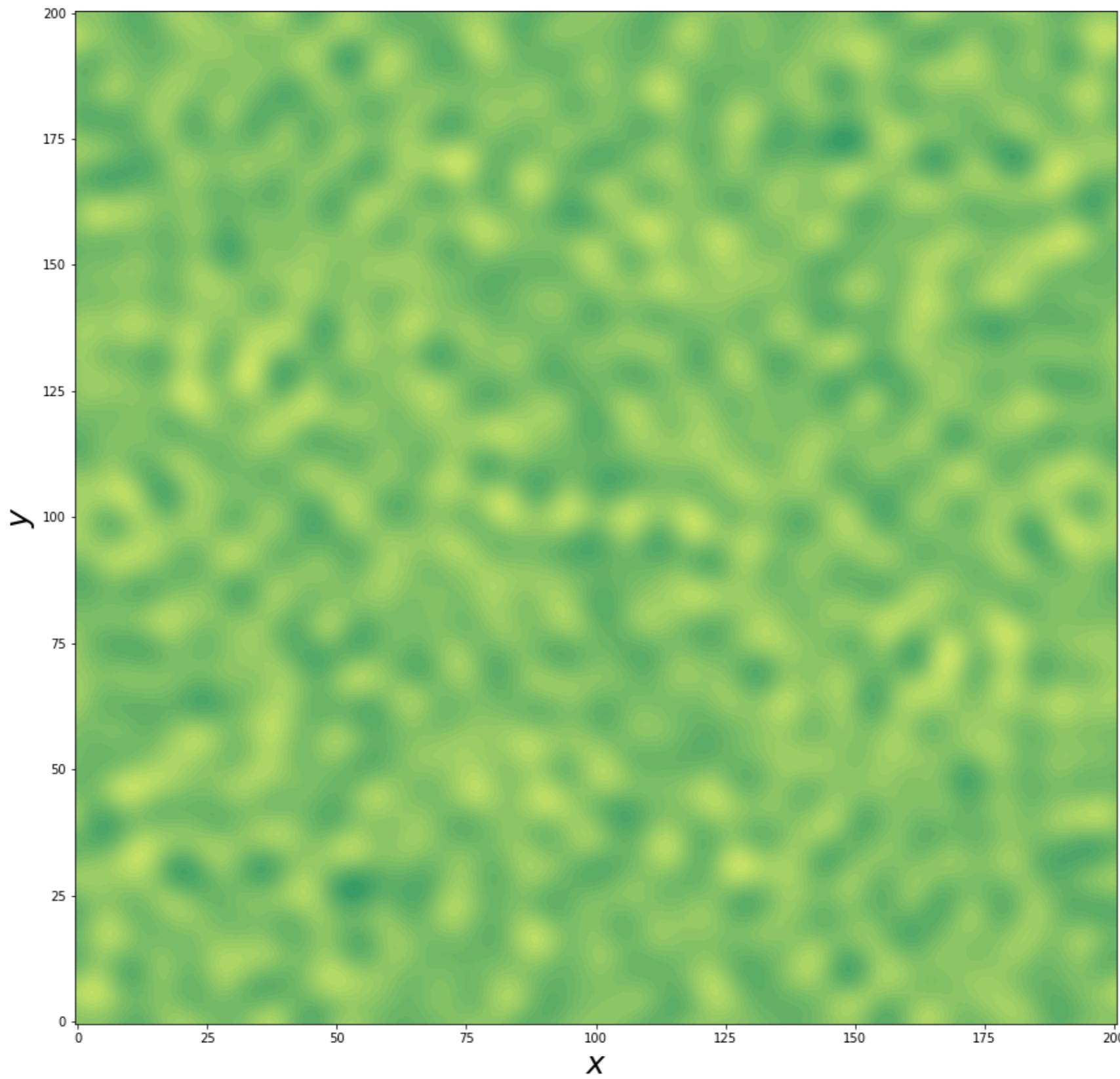


$$\Delta T \times \Delta E \geq \hbar$$

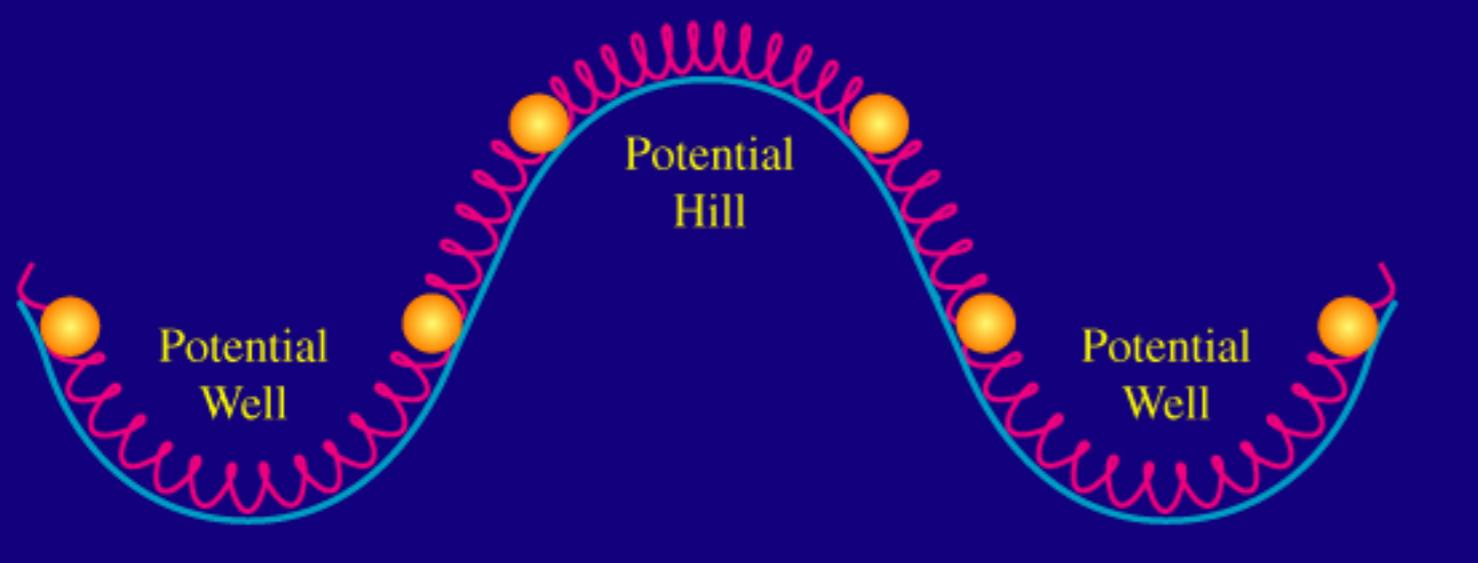
Freeze perturbations !!!!



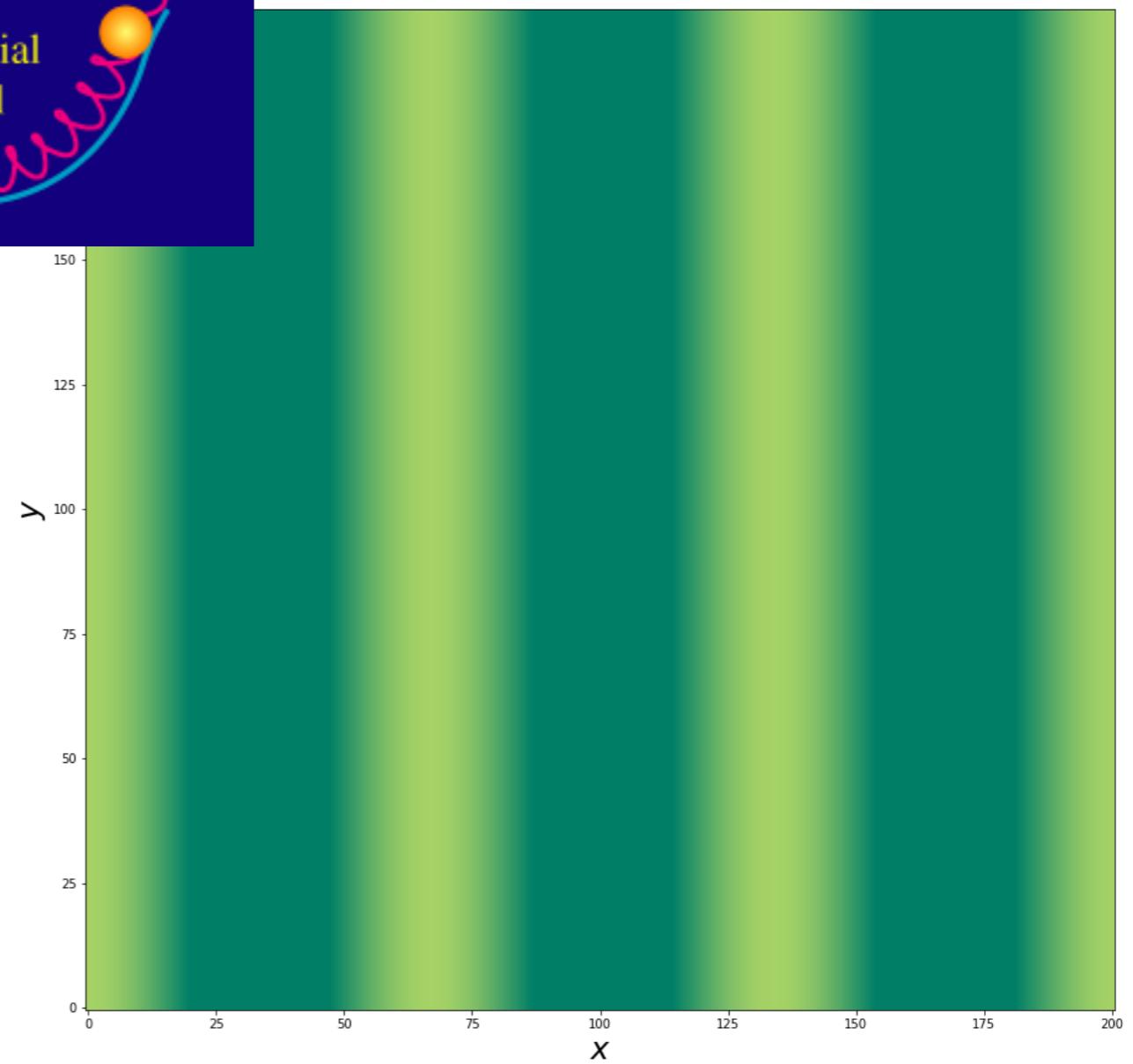
# From Reheating to CMB

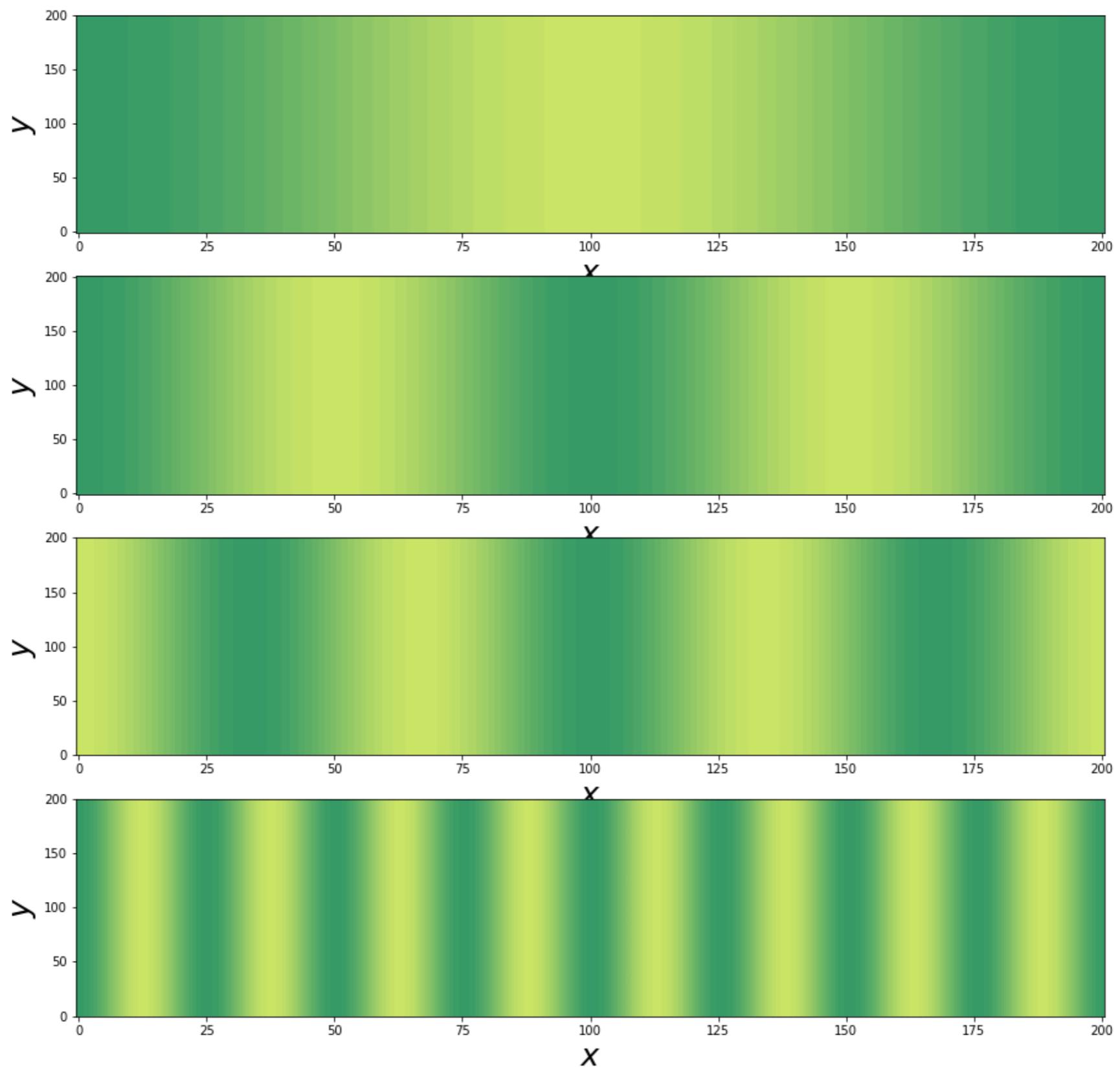


# Baryon-photon oscillates

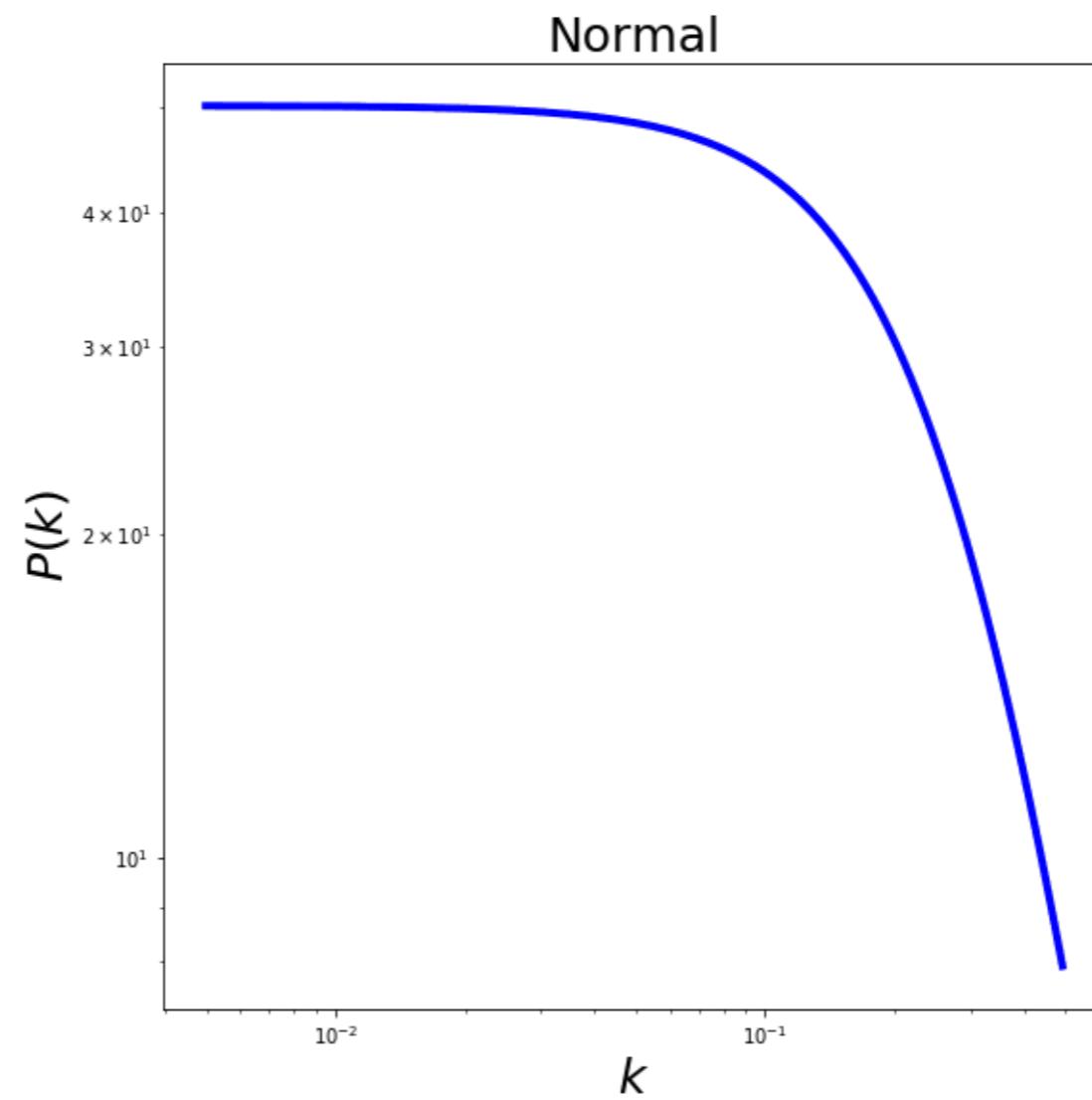
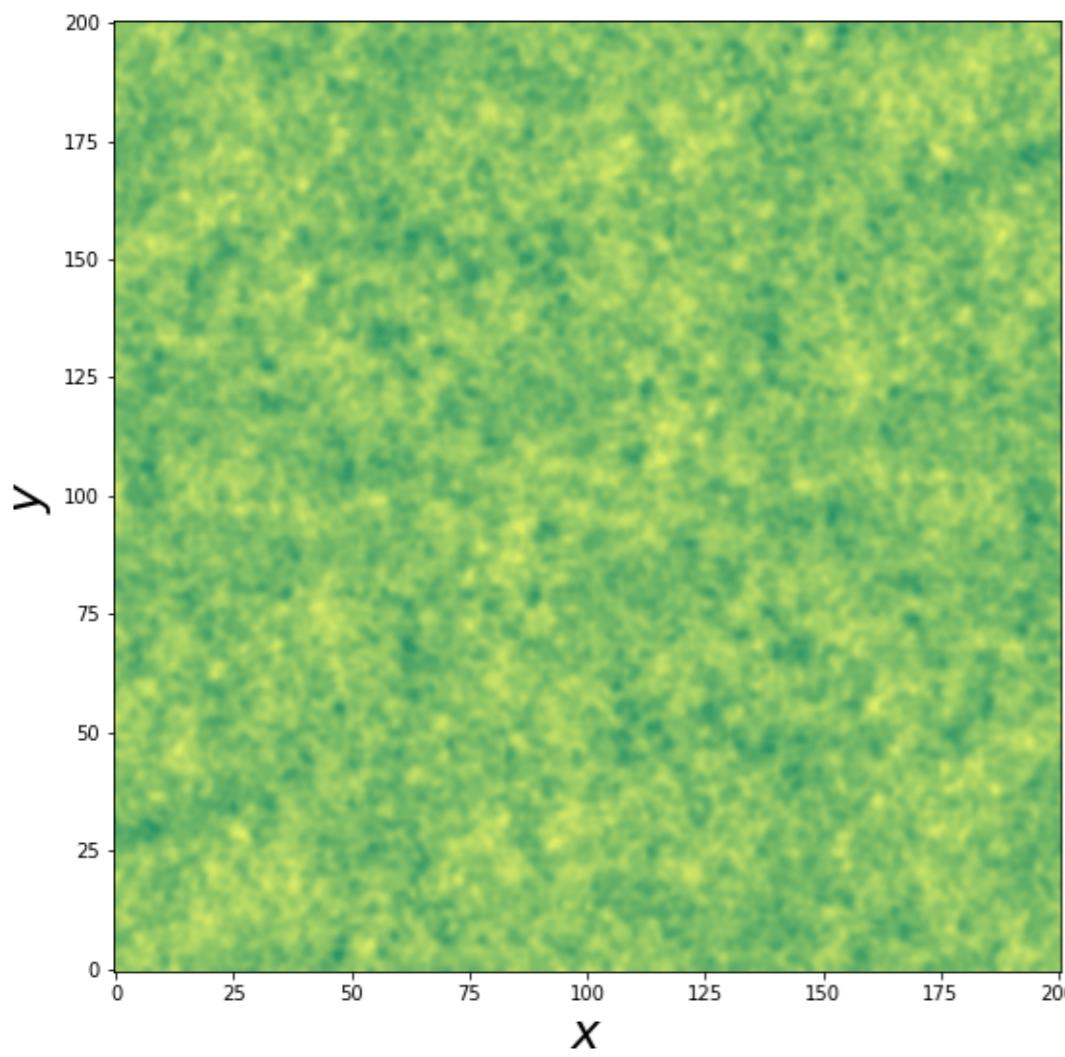


**Wayne Hu's tutorial**

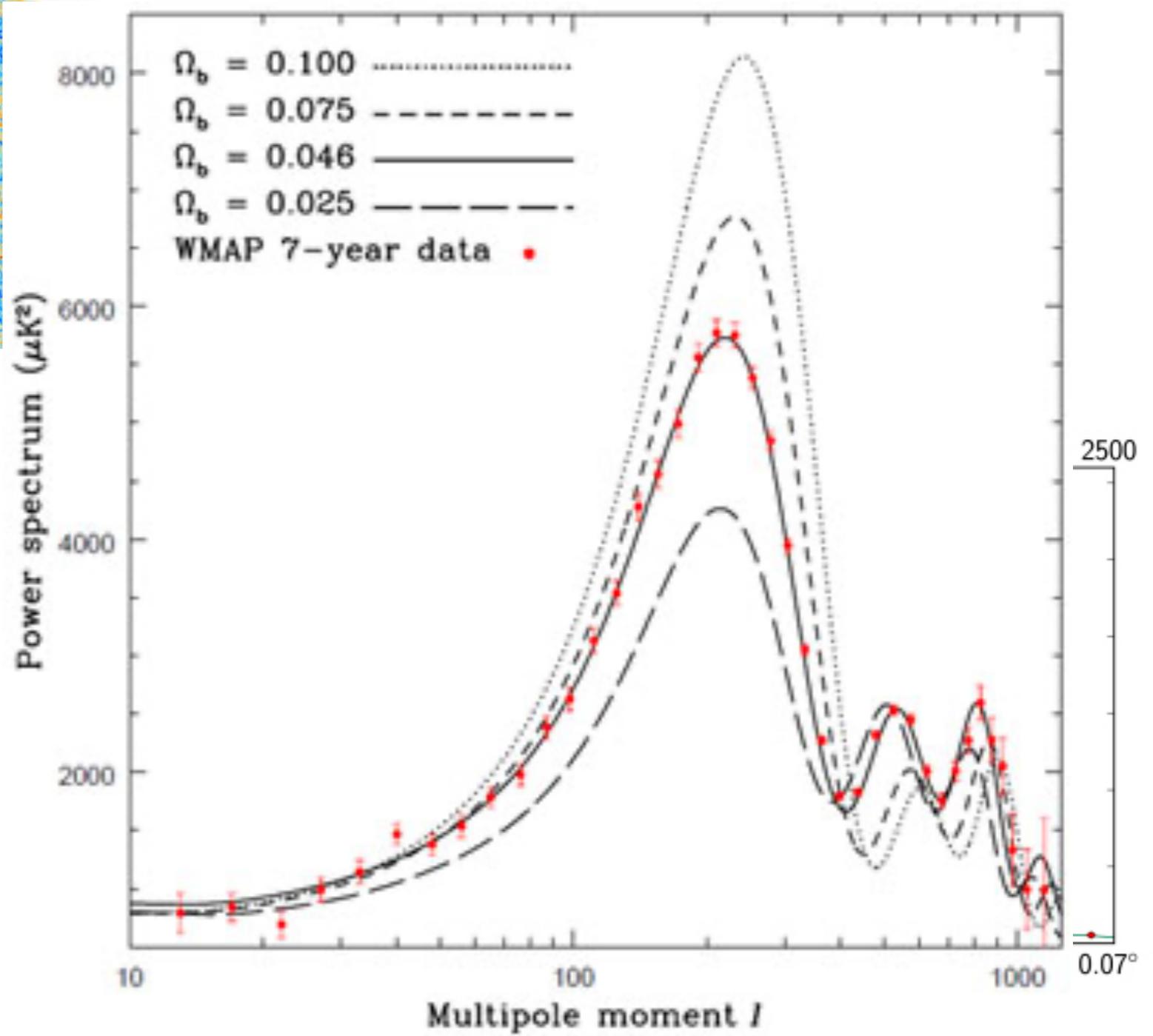
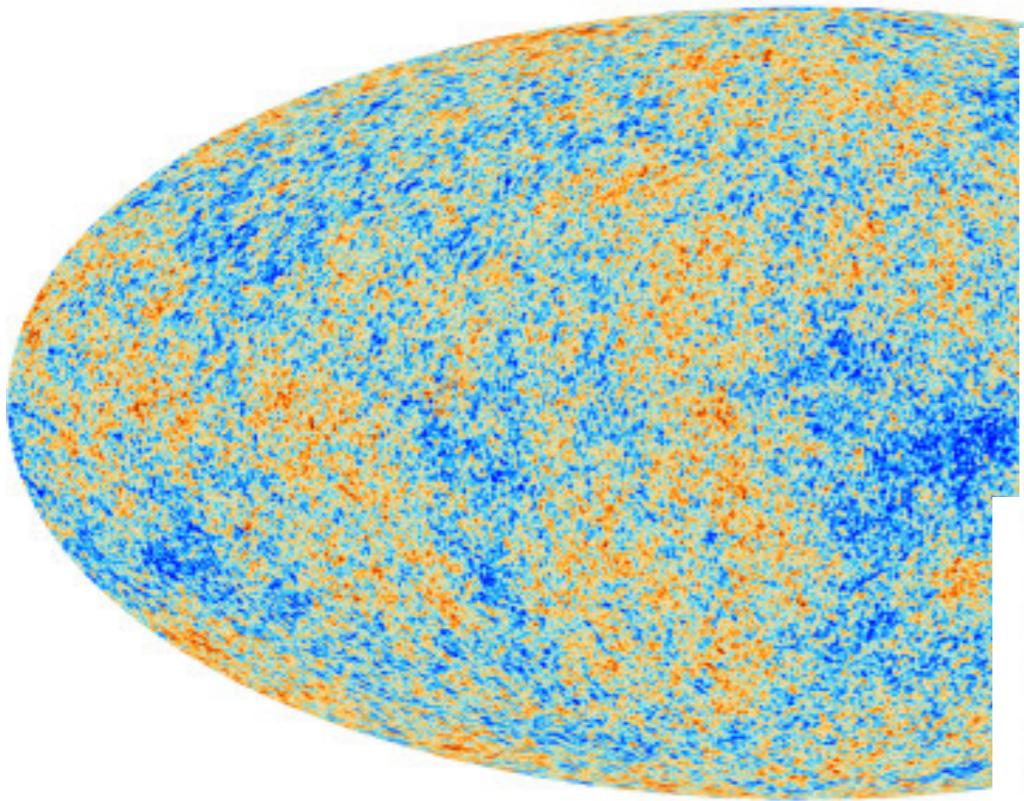




# Fourier / Spherical Harmonics (CMB)



# Cosmic Microwave Background (CMB)

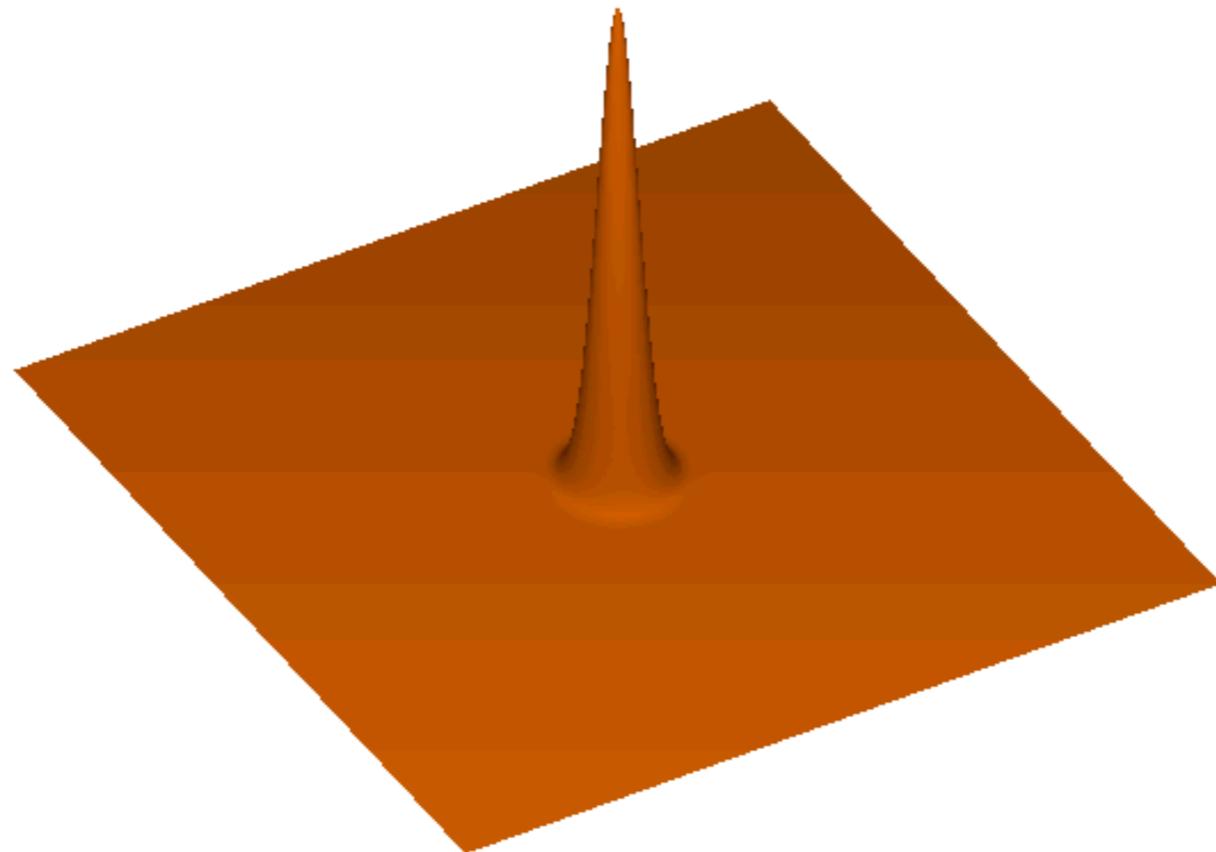


# Baryonic Acoustic Oscillations (BAO) Real Space

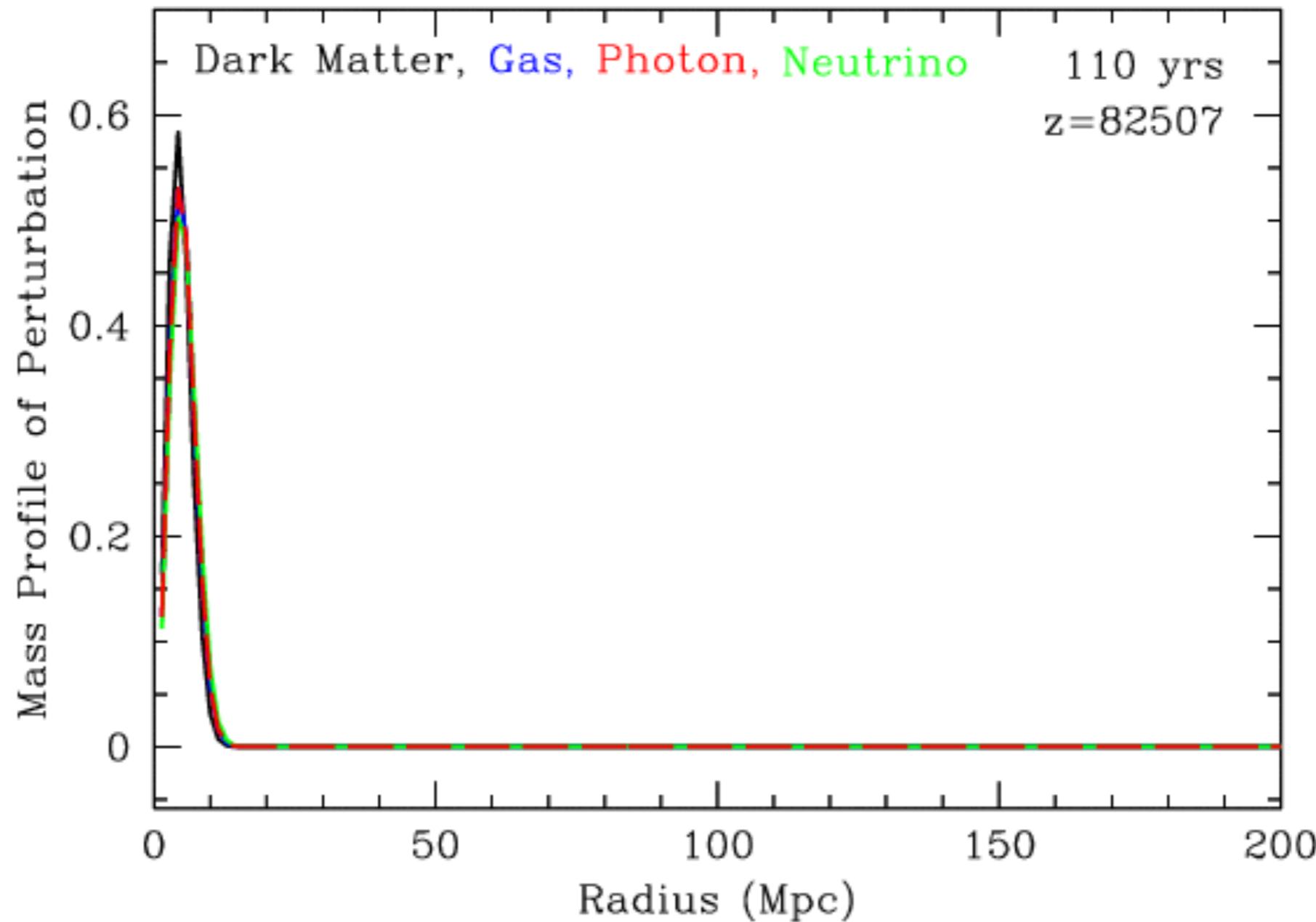
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Around all over-densities

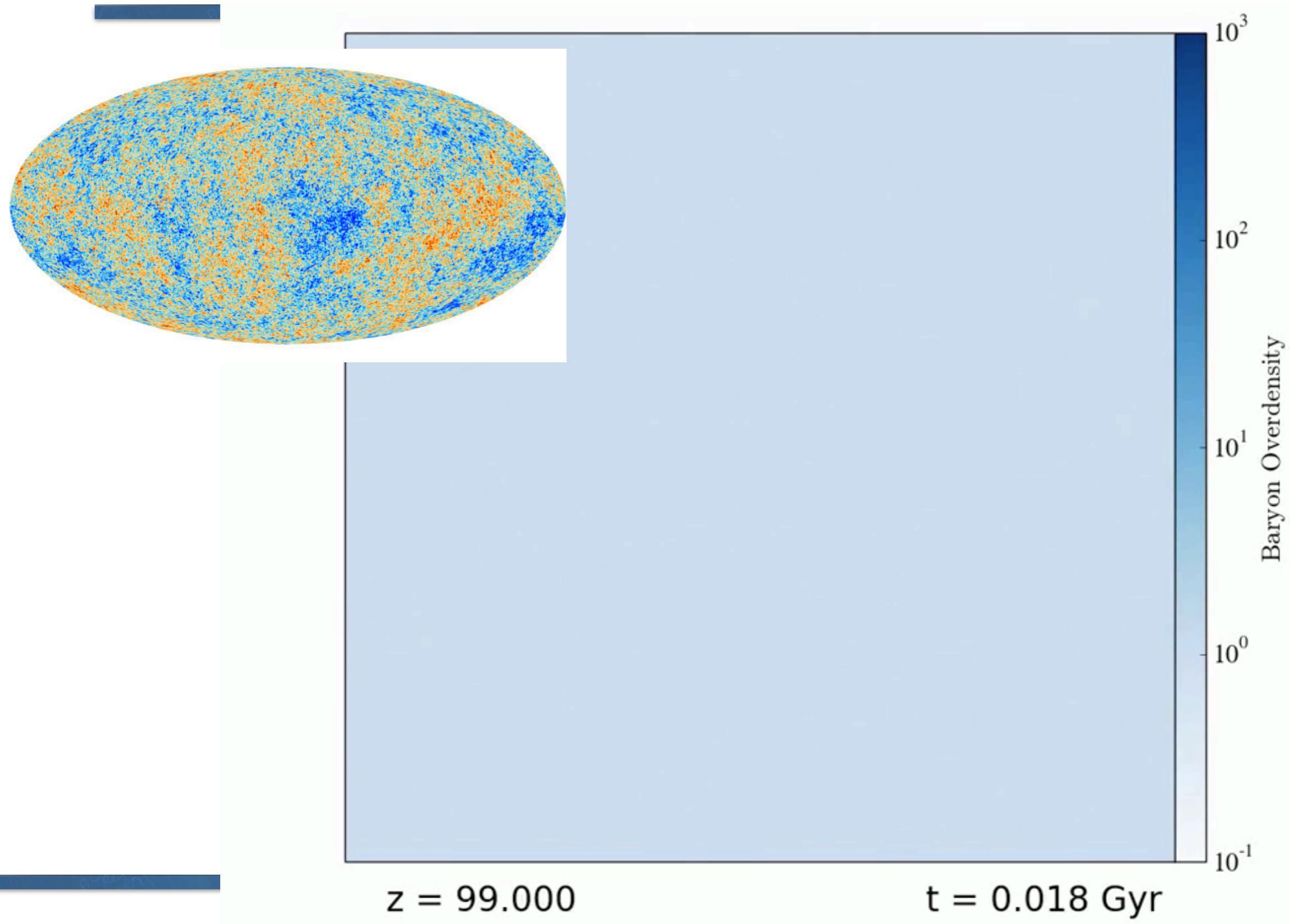
Stop to propagate when the plasma disappear



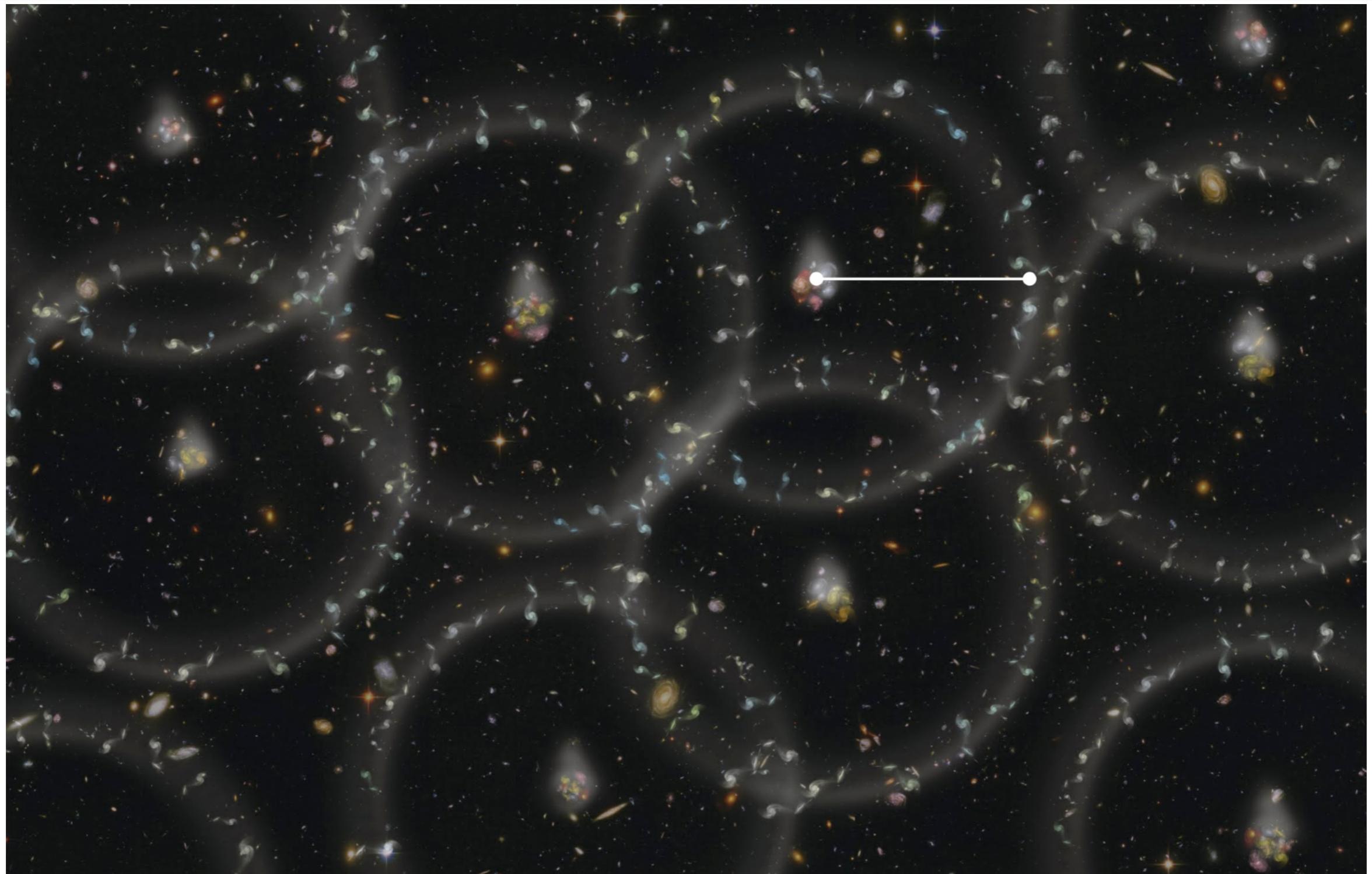
# BAO in real space



# Perturbations after CMB



# BAO in galaxies

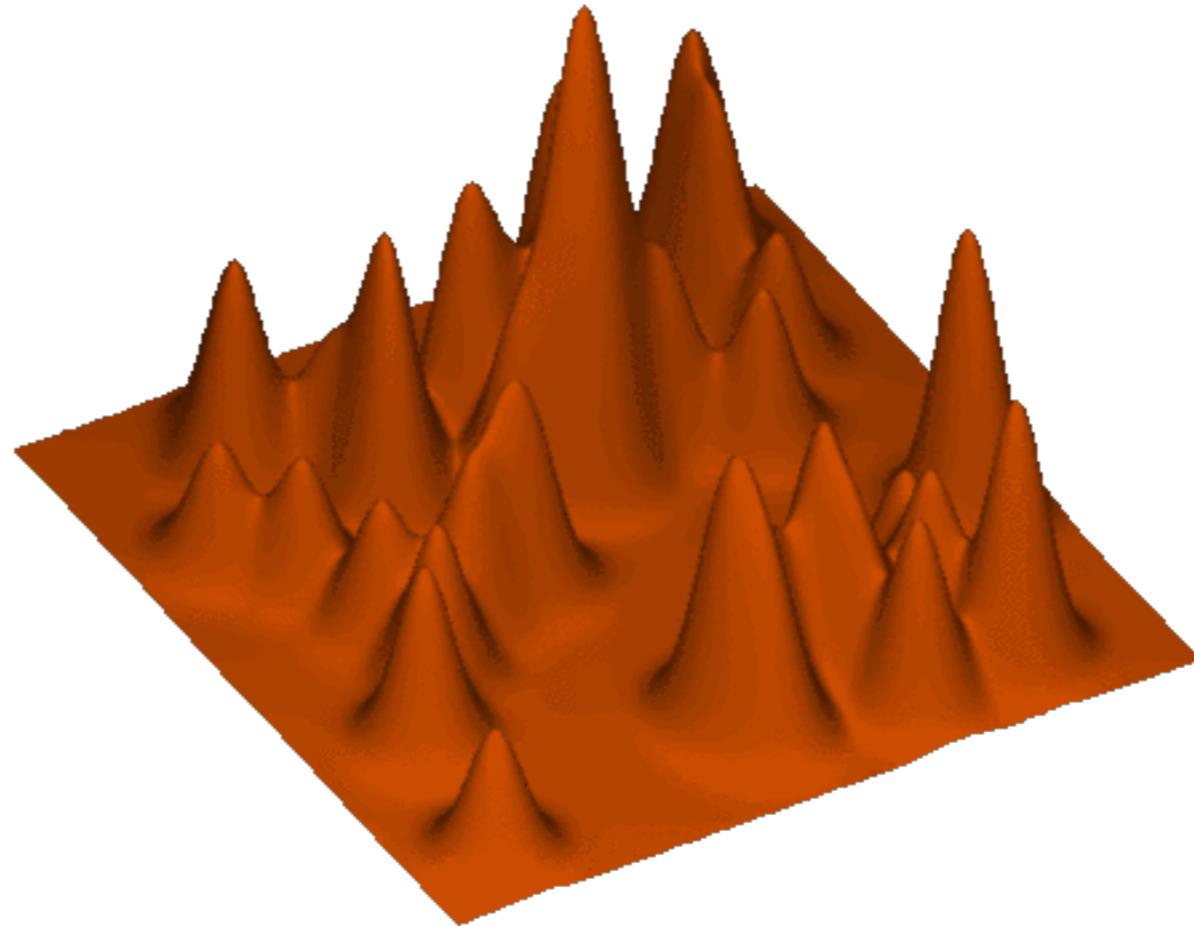


# Baryonic Acoustic Oscillations (BAO) Real Space

---

Around all over-densities

Stop to propagate when the plasma disappear



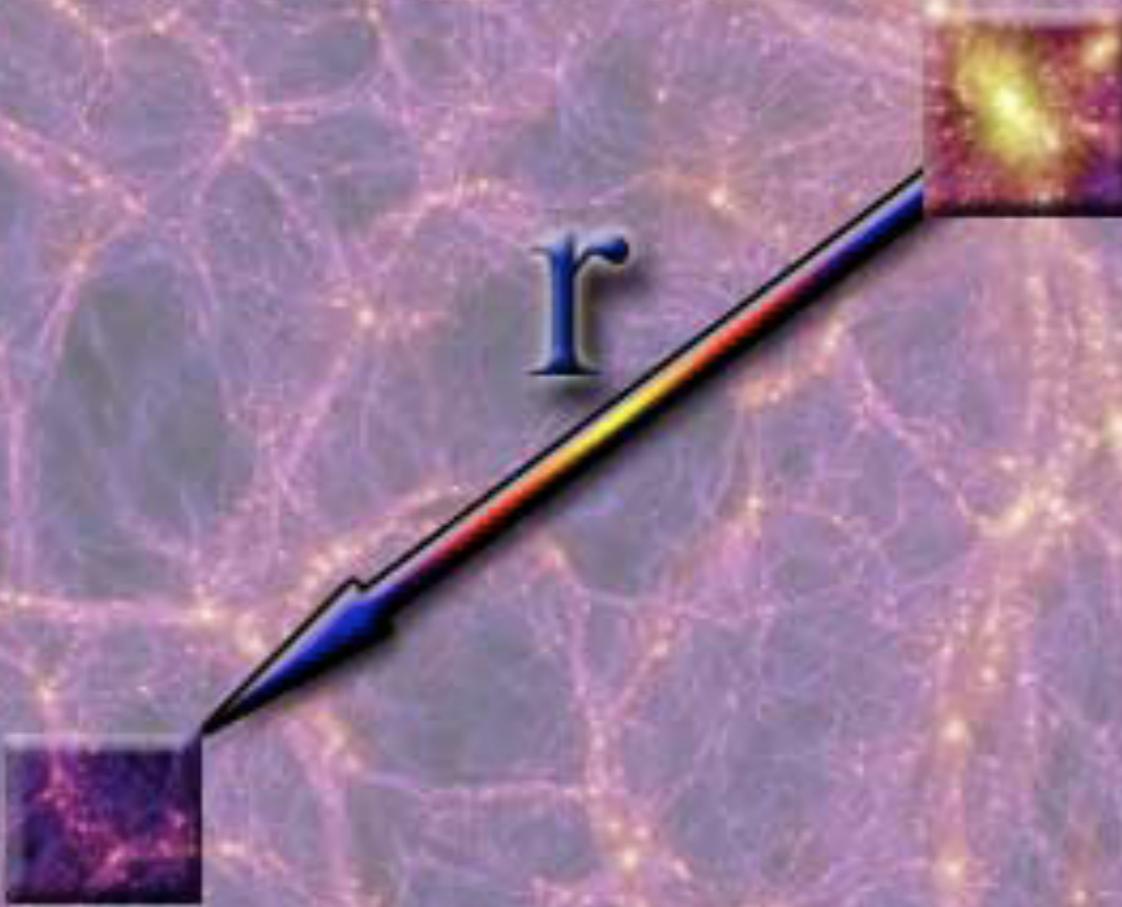
# Reality



# Measuring Clustering

$$dP_{12} = \bar{n}_g^2 [1 + \xi(\vec{r}_{12})] dV_1 dV_2$$

We estimate it using galaxy pair counts



# 2-pt statistics

$$\xi(r) = \langle \delta(x)\delta(x+r) \rangle_{volumen}$$

$\delta(x)$  is the contrast density

$$\delta(x) = \frac{\rho(x) - \langle \rho \rangle}{\langle \rho \rangle}$$

Power Spectrum  $P(k)$  is the FT of  $\xi(r)$ :

$$P(k) = \int d^3x e^{-ik \cdot r} \xi(r)$$

# Power Spectrum and 2pt-correlation function

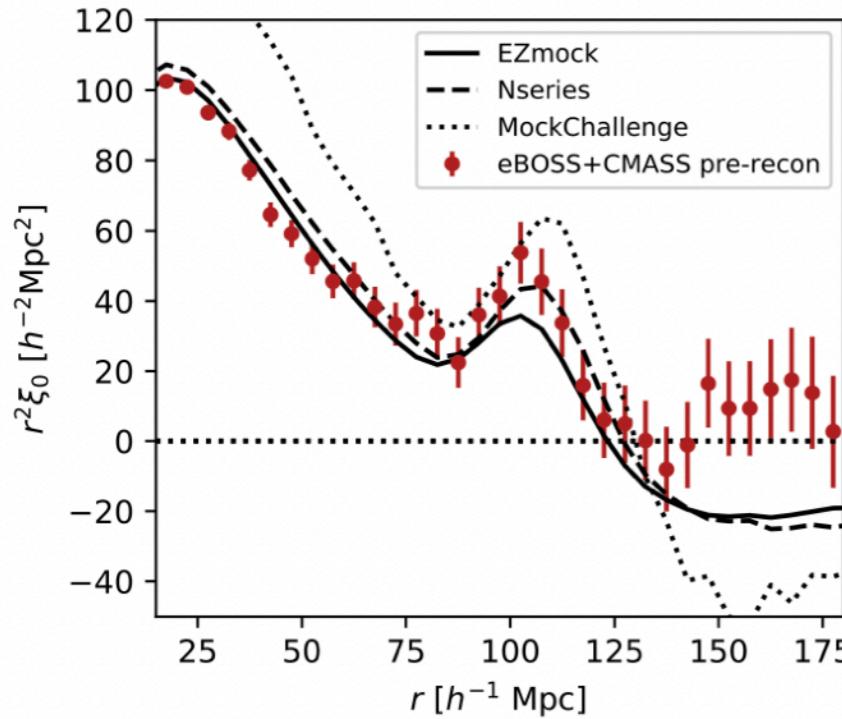
$$\xi(\vec{r}) = \langle \delta(\vec{x}) \cdot \delta(\vec{x} + \vec{r}) \rangle$$

$$P(\vec{k}) = \frac{1}{(2\pi)^3} \int d^3 \vec{r} \xi(\vec{r}) e^{-i \vec{k} \cdot \vec{r}}$$

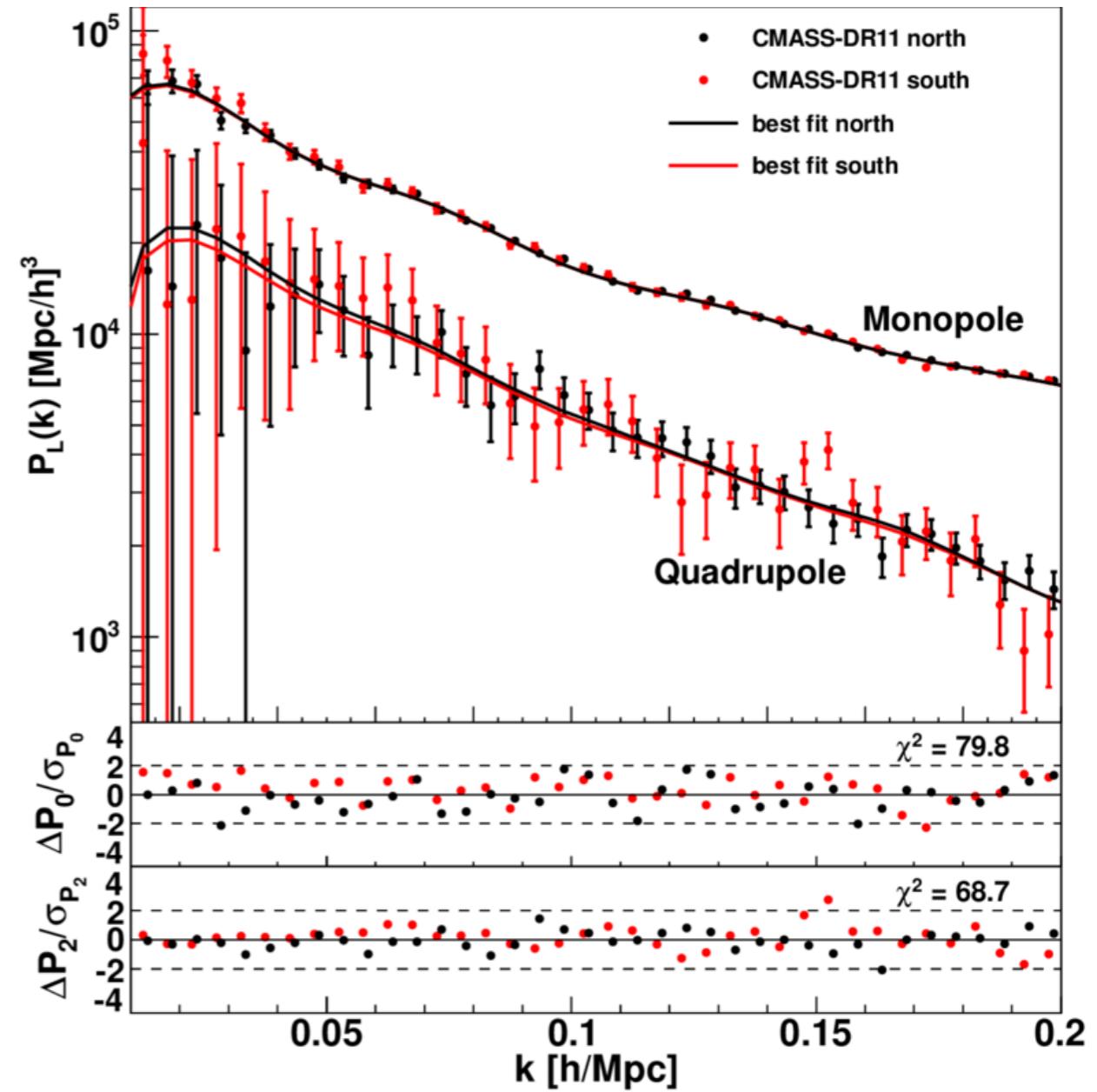
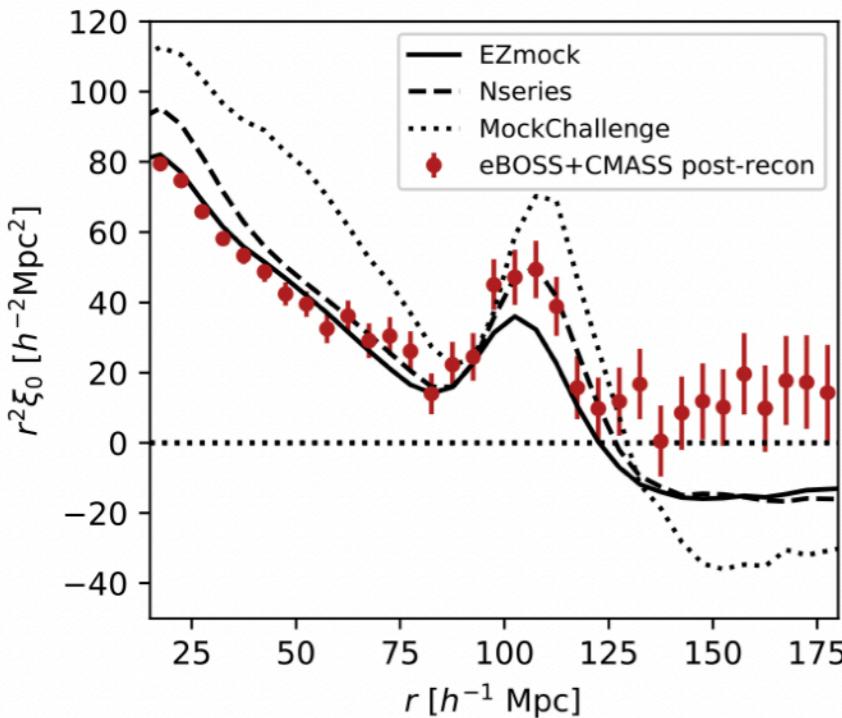
**Real field condition :**  $\delta(-\vec{k}) = \delta^*(\vec{k})$

$$(2\pi)^3 P(\vec{k}) = \langle \delta(\vec{k}) \cdot \delta^*(\vec{k}) \rangle = \langle |\delta(\vec{k})|^2 \rangle$$

# Galaxy 2pt-statistics measurement

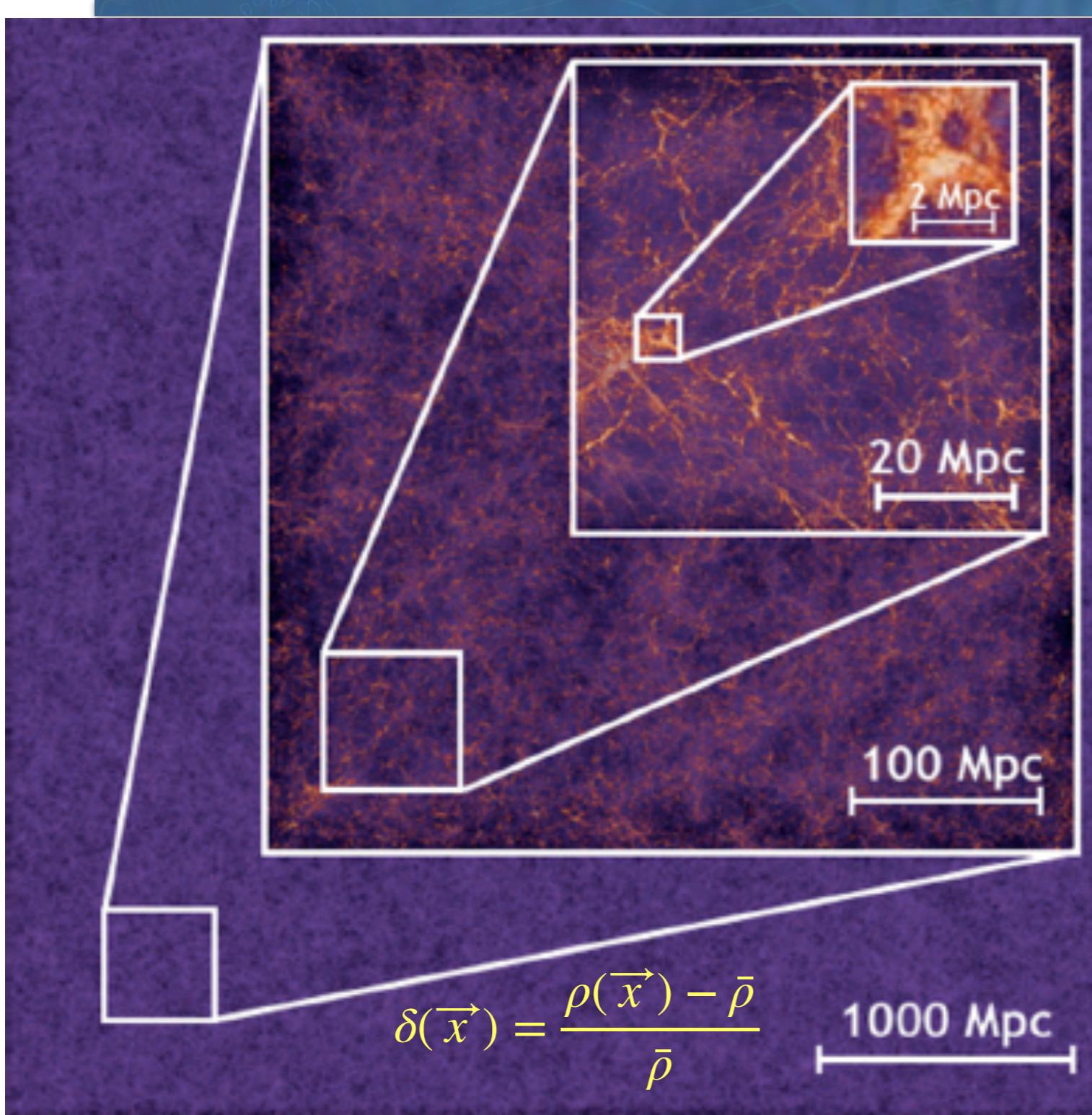


**Bautista et al. 2021**



**Beutler et al. 2013**

# Scales in Cosmology



Millenium simulation

$\delta > 1$  : Non-Linear

- **Simulations**

$\delta \sim 1$  : Quasi-Linear

- **Simulations**
- **2-LPT**

$\delta \ll 1$  : Linear

- **Simulations**
- **LPT** (Lagrangian Pert. Th)
- **Analytical approx.**

# Linear evolution of Perturbations

$$\dot{\rho} + \nabla_{\vec{r}} \cdot (\rho \vec{u}) = 0,$$

: Continuity equation

$$\rho \left[ \ddot{\vec{u}} + (\vec{u} \cdot \nabla_{\vec{r}}) \vec{u} \right] = -\nabla_{\vec{r}} p - \rho \nabla_{\vec{r}} \Phi,$$

: Euler's equation

$$\nabla_{\vec{r}}^2 \Phi = 4\pi G(\rho + 3p) - \Lambda.$$

: Poisson's equation

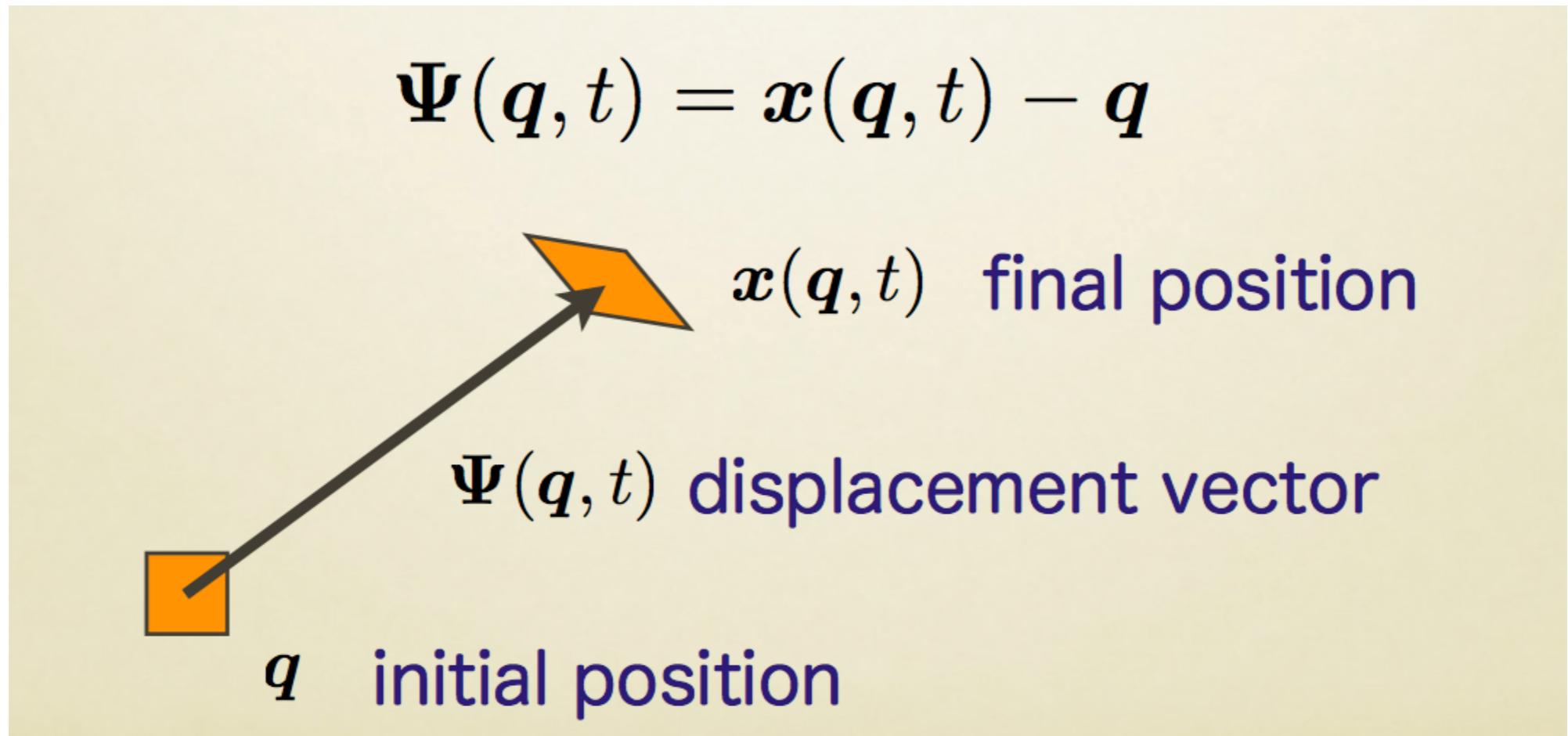
$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\rho_m \delta$$

**Solution for whole matter  
as Dark Matter**

$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}} \ll 1$$

linearity condition

# Lagrangian Perturbation Theory



Credit to Matsubara's presentation

# Lagrangian Perturbation Theory

## Equation of motion & Poisson's Equation

$$\ddot{\Psi} + \frac{\dot{a}}{a} \dot{\Psi} = -\frac{1}{a^2} \nabla_x \phi$$

$$\Delta_x \phi = 4\pi G \bar{\rho} a^2 \delta(x, t)$$

## Linearization & Zeldovich Approximation

$$\delta(x, t) = \left[ \det \left( I + \frac{\partial \Psi}{\partial q} \right) \right]^{-1} - 1 \approx -\nabla_q \cdot \Psi$$

$$\Psi \approx -D(t) \nabla_q \rho_0(q)$$

Credit to Lile Wang's presentation

# Lagrangian Perturbation Theory

- Taking into account the higher-order perturbations in the displacement

$$\Psi = \sum_{n=1}^{\infty} \Psi^{(n)} = \Psi^{(1)} + \Psi^{(2)} + \Psi^{(3)} + \dots$$



$$\Psi^{(1)} = -D(t) \nabla \varphi_0(\mathbf{q})$$

(First order: Zel'dovich approx.)

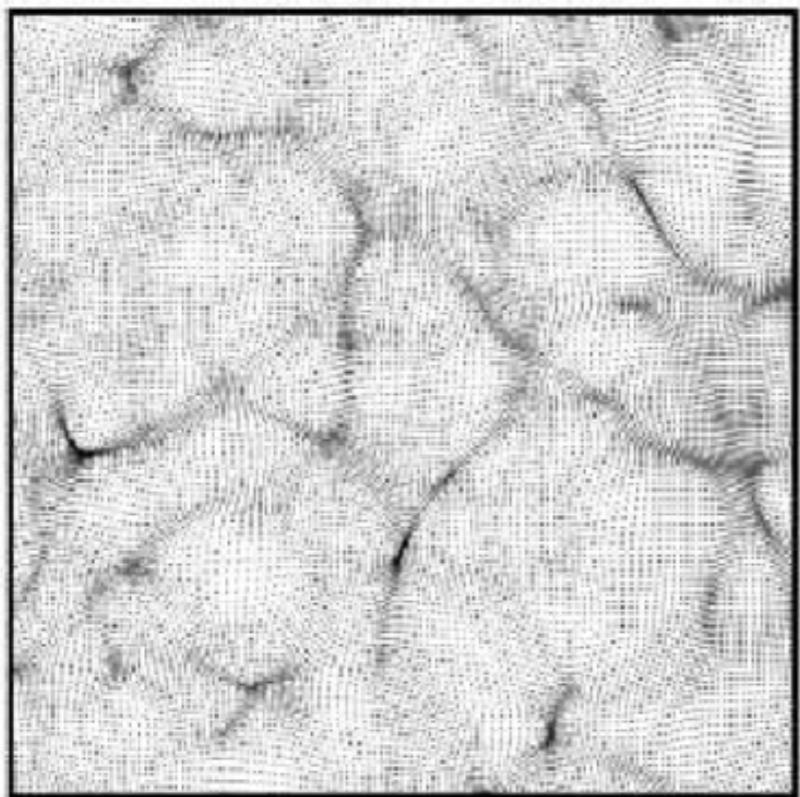
$$\Psi^{(2)} = -\frac{1}{2} D_2(t) \nabla \Delta^{-1} [\Psi_{i,i}^{(1)} \Psi_{j,j}^{(1)} - \Psi_{i,j}^{(1)} \Psi_{j,i}^{(1)}]$$

$$\begin{aligned} \Psi^{(3)} = & -\frac{1}{3!} \left[ D_{3a}(t) \nabla \Delta^{-1} \left( \Psi_{i,i}^{(1)} \Psi_{j,j}^{(2)} - \Psi_{i,j}^{(1)} \Psi_{j,i}^{(2)} \right) + D_{3b}(t) \nabla \Delta^{-1} \det \left( \Psi_{i,j}^{(1)} \right) \right. \\ & \left. + D_{3c}(t) \Delta^{-1} \left( \Psi_{,j}^{(1)} \Psi_{i,j}^{(2)} - \Psi_{i,j}^{(1)} \Psi_{,j}^{(2)} \right)_{,i} \right] \end{aligned}$$

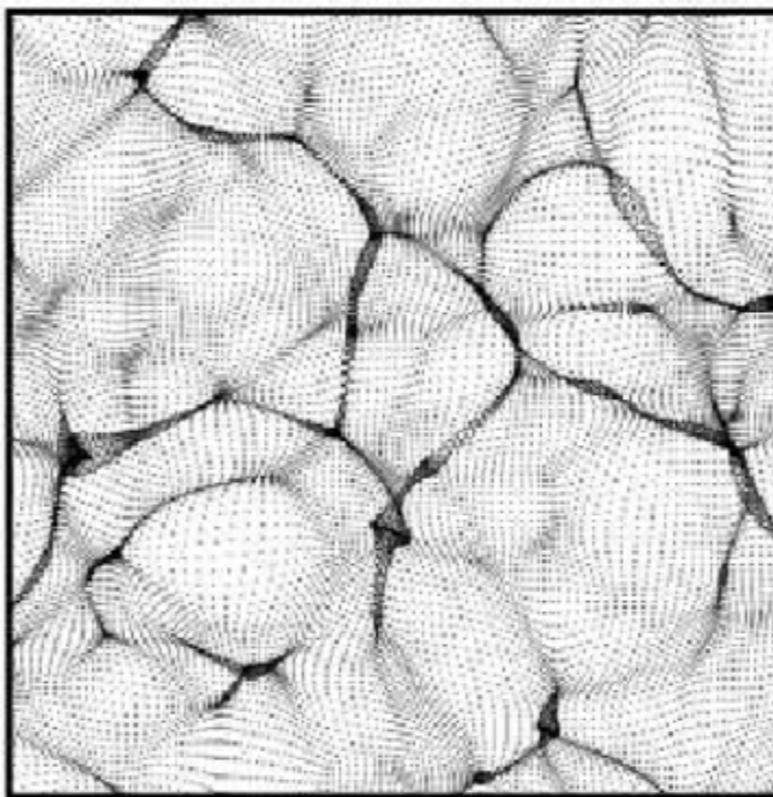
Recursive solutions

Credit to Matsubara's presentation

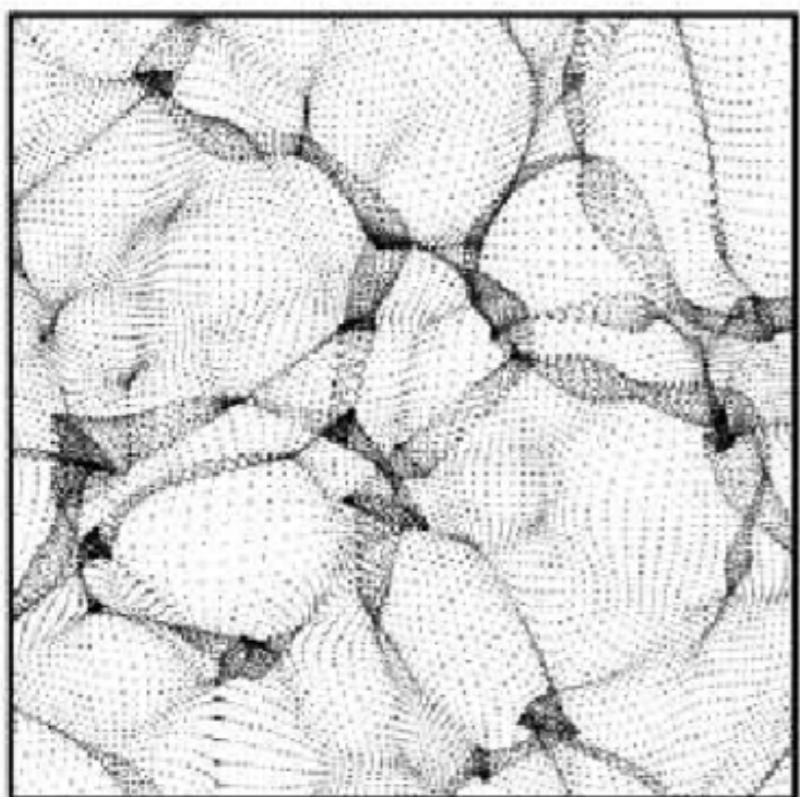
$\sigma=0.5$



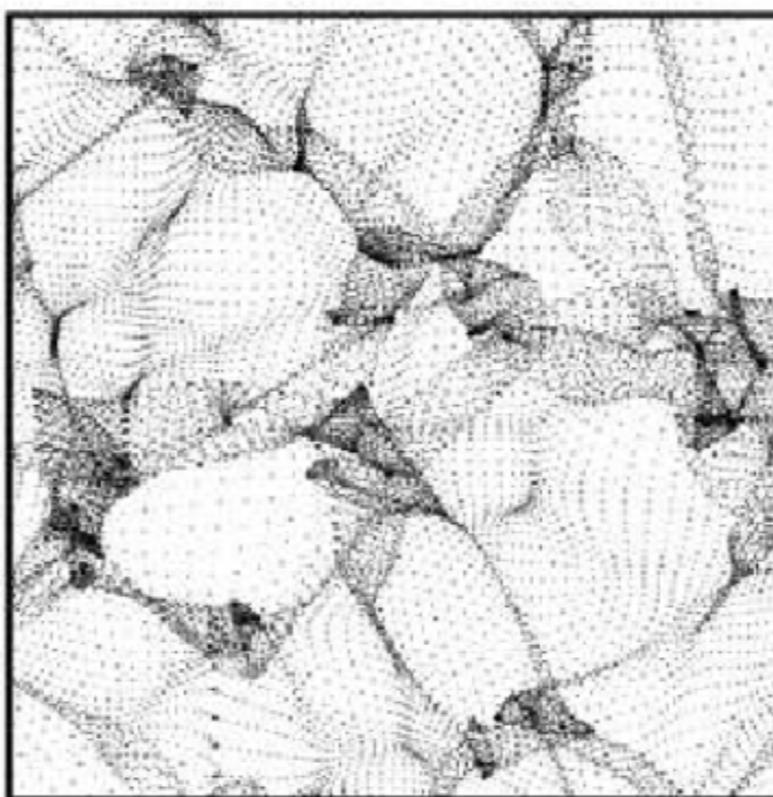
$\sigma=1.$



$\sigma=1.5$



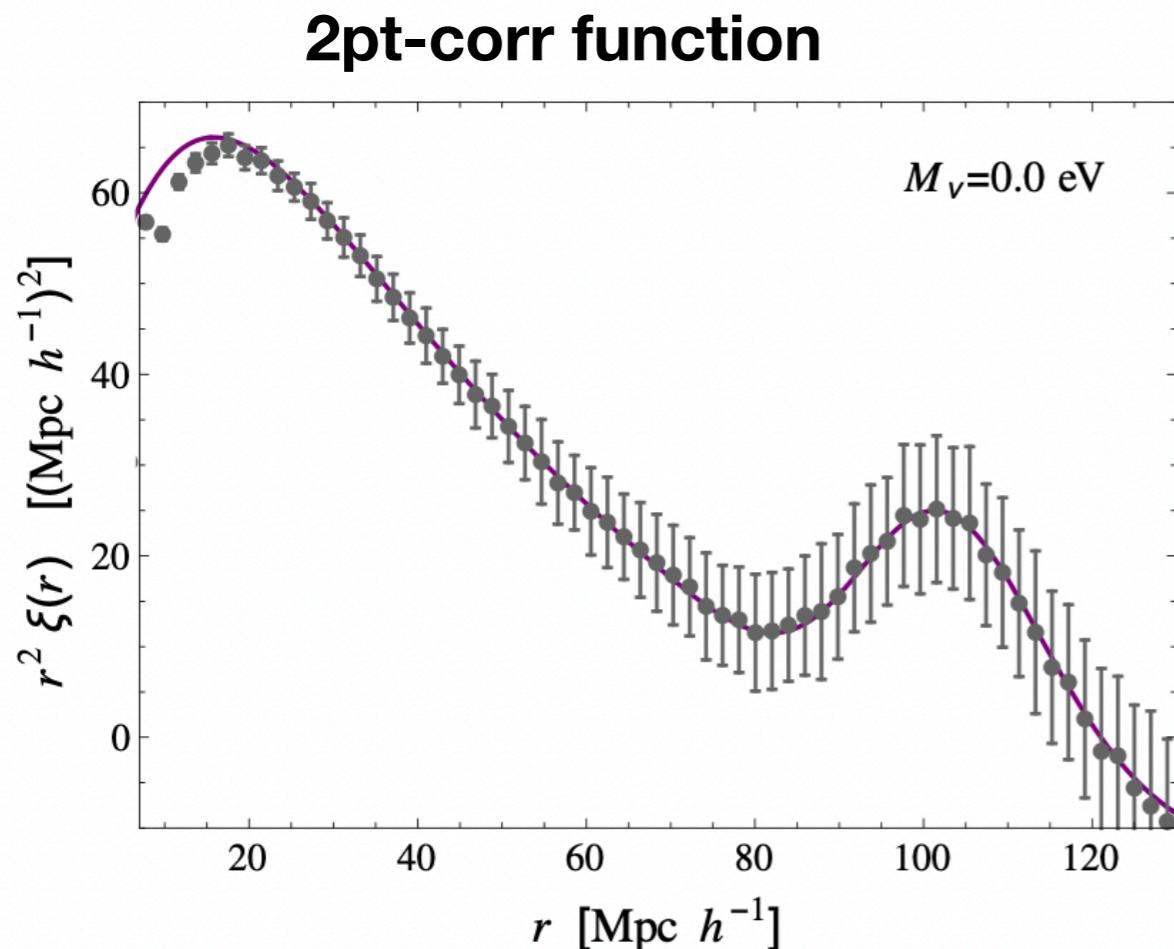
$\sigma=2.$



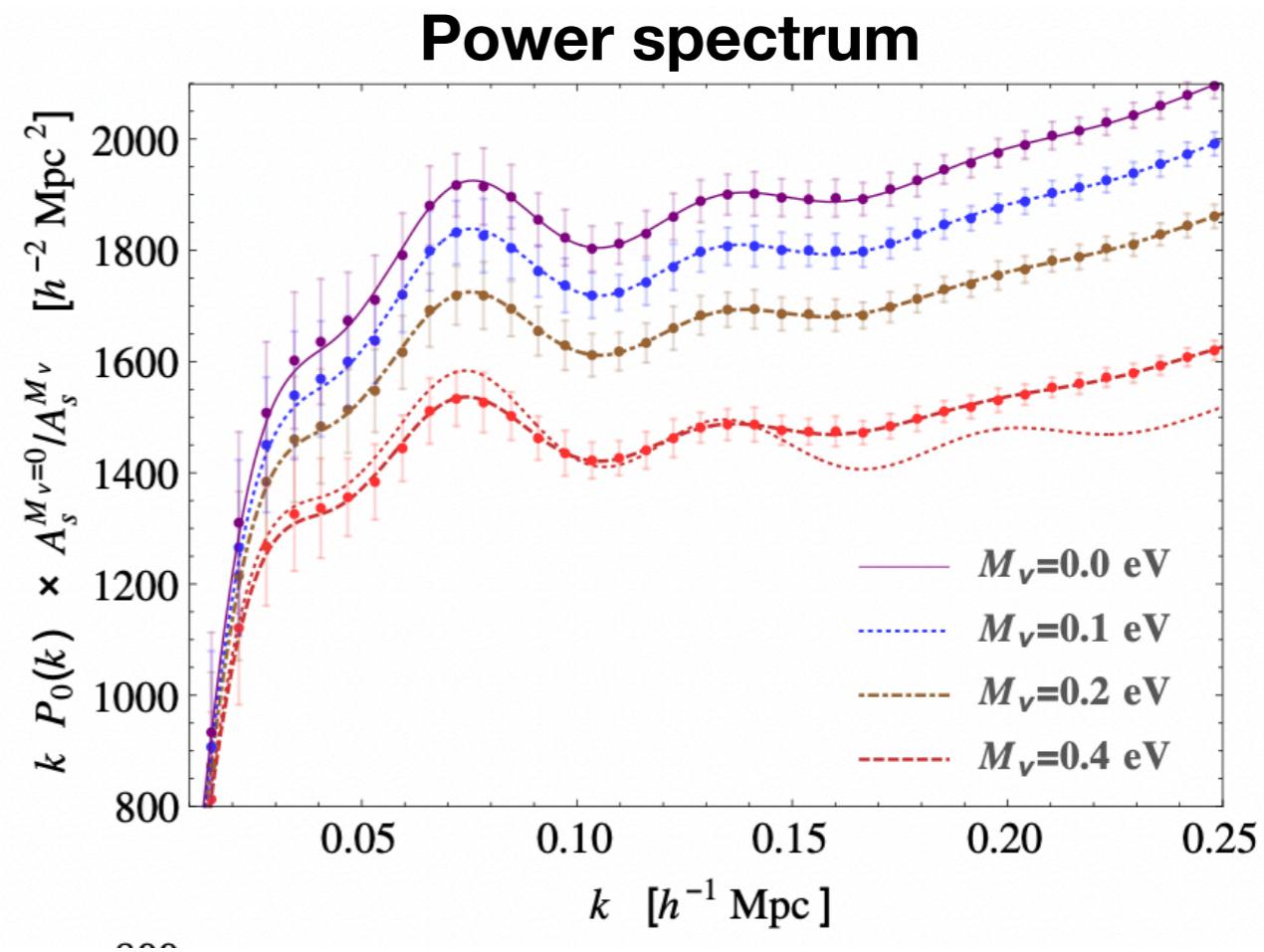
**Applying the first order displacement field on initial particle positions.**

**Perturbation theory can evaluate in average how these displacements modify the correlation function and Power Spectrum**

# Perturbation Theory on stats Vs CDM simulations



A, Aviles & A, Banerjee  
2020



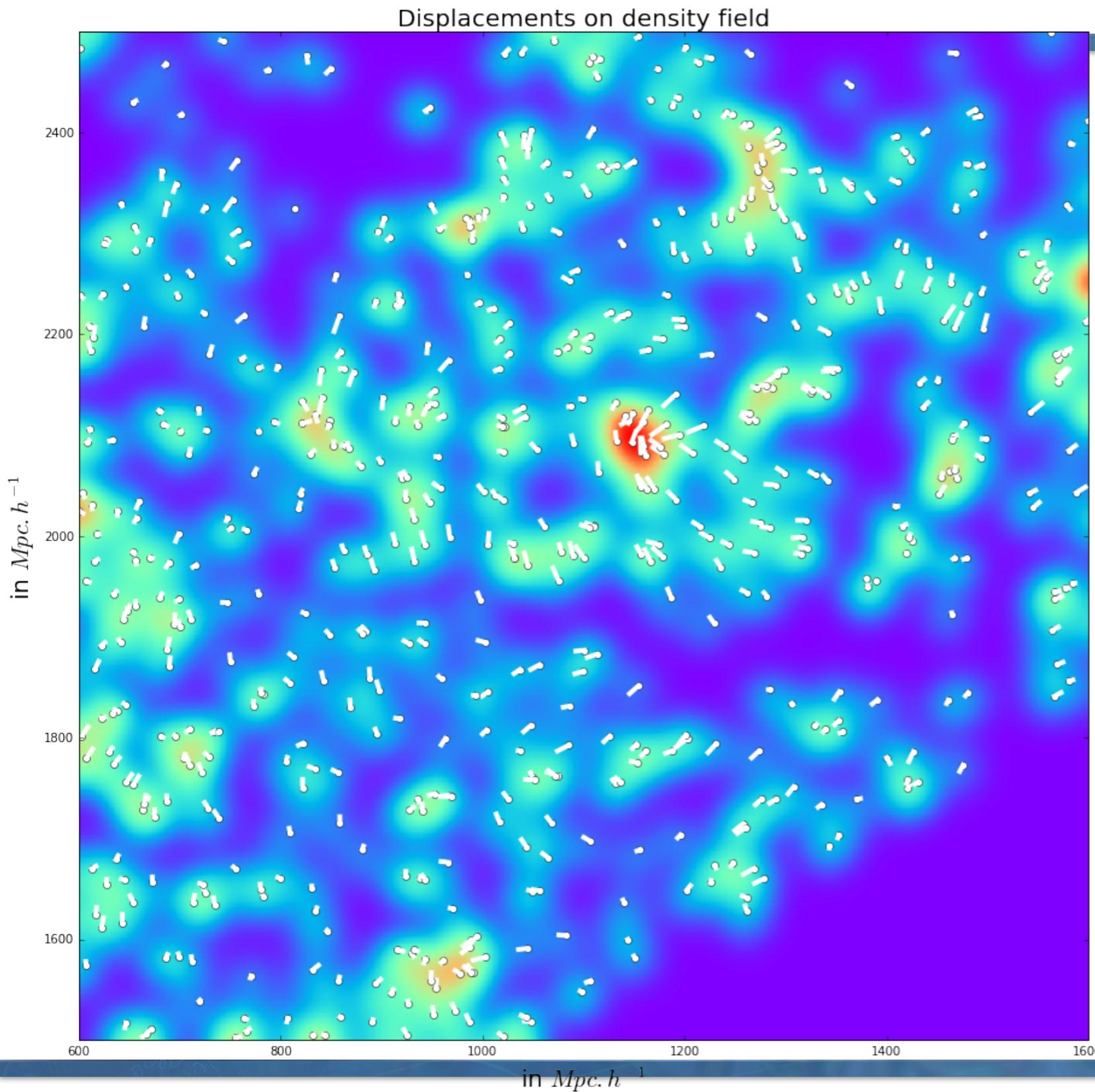
Alejandro Aviles et al 2021

**Works very well up to :**

$$r \sim 25 \text{ Mpc}/h$$

$$k \sim 0.3 \text{ h/Mpc}$$

# Reconstruction to enhance BAO peak



Vargas, Ho, Fromenteau, Cuesta (2017)

We estimate the smoothed galaxy density field



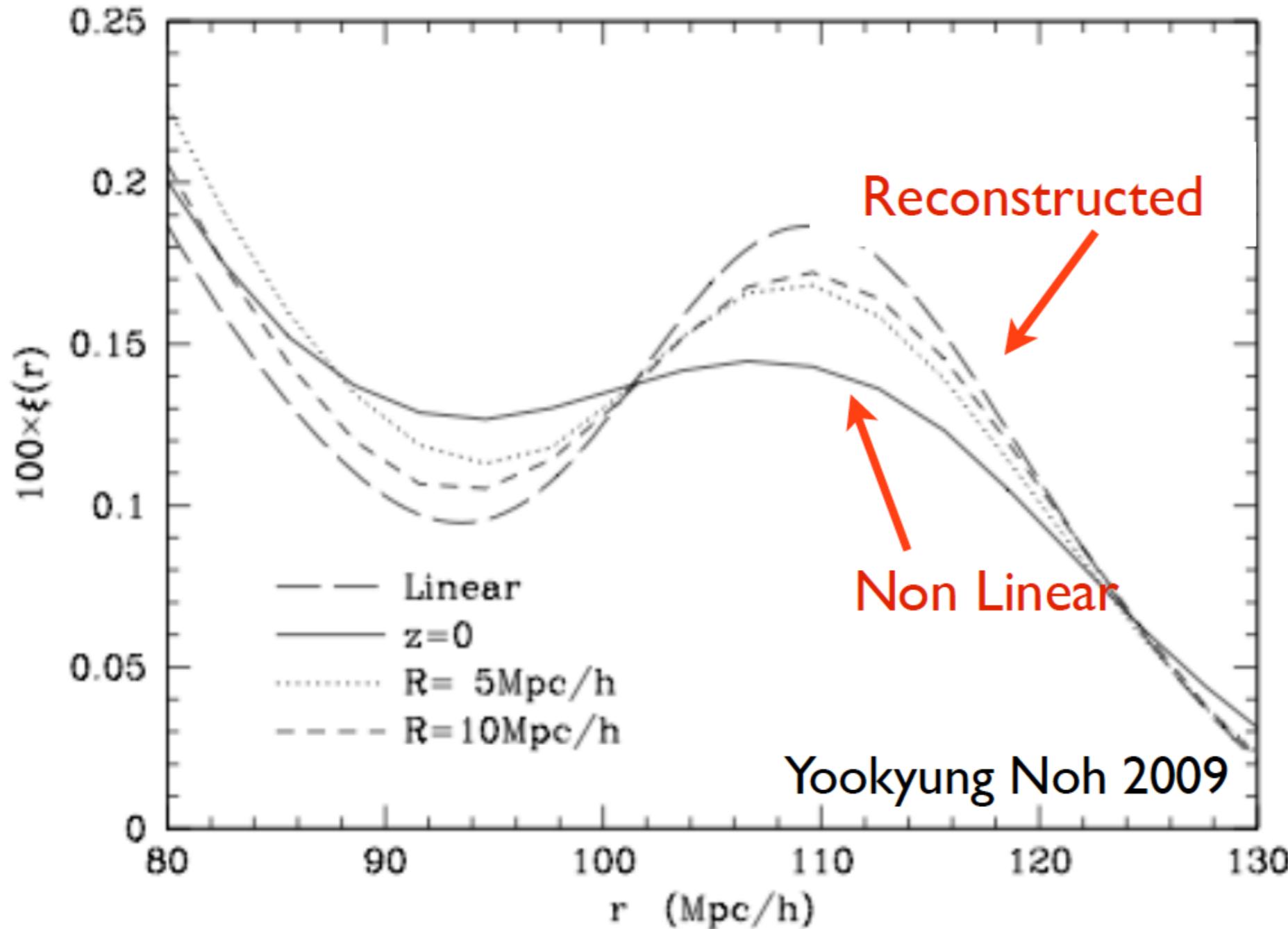
Using linear bias, we derive the matter density field



We move back the galaxies to their original position

$$\vec{\psi}(\vec{k}) = \frac{-i\vec{k}}{k^2} \frac{\delta_g(\vec{k})}{b_g}$$

# Reconstruction to enhance BAO peak

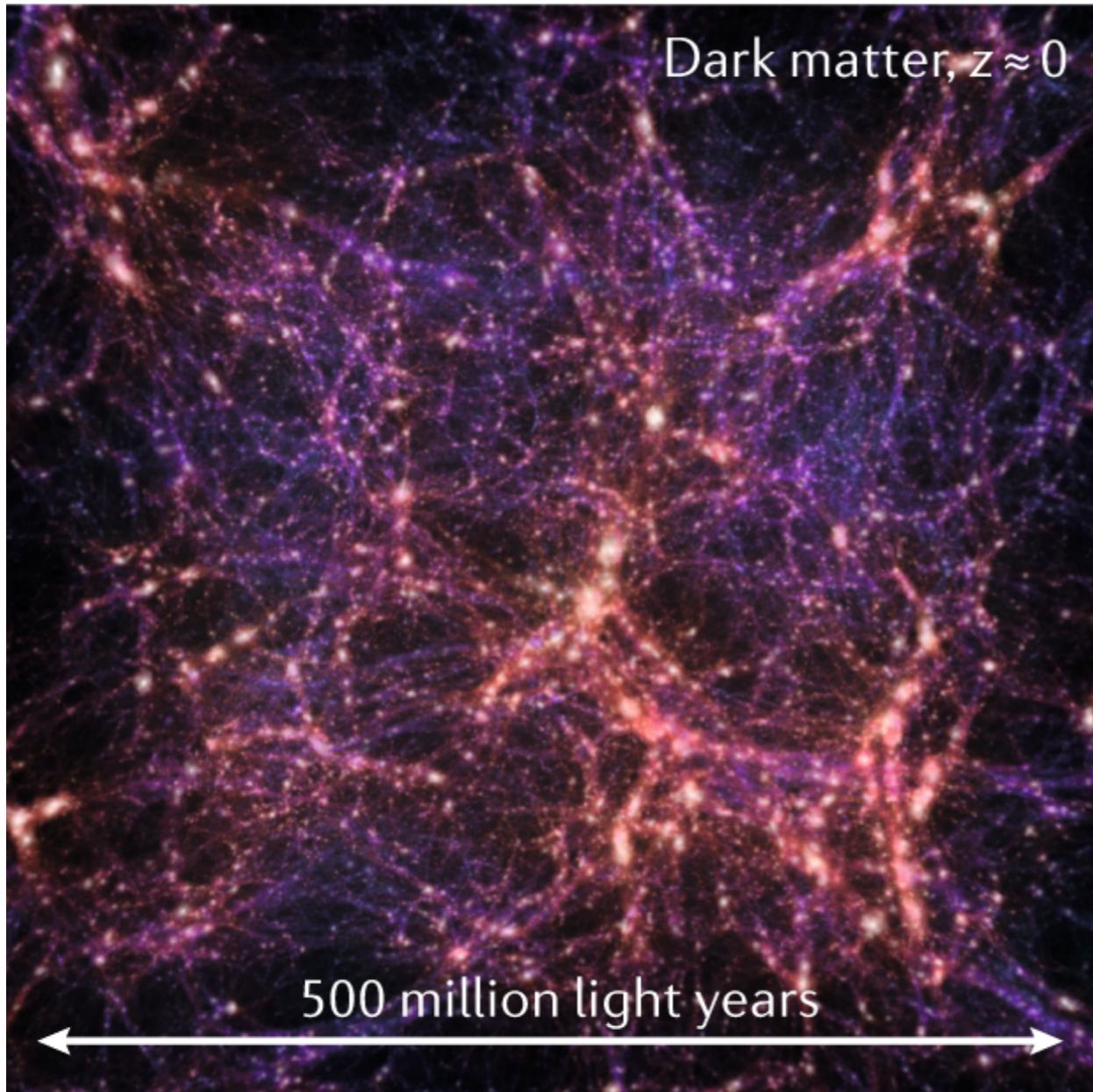


Is part of the standard  
BAO analysis

# What we do observe?

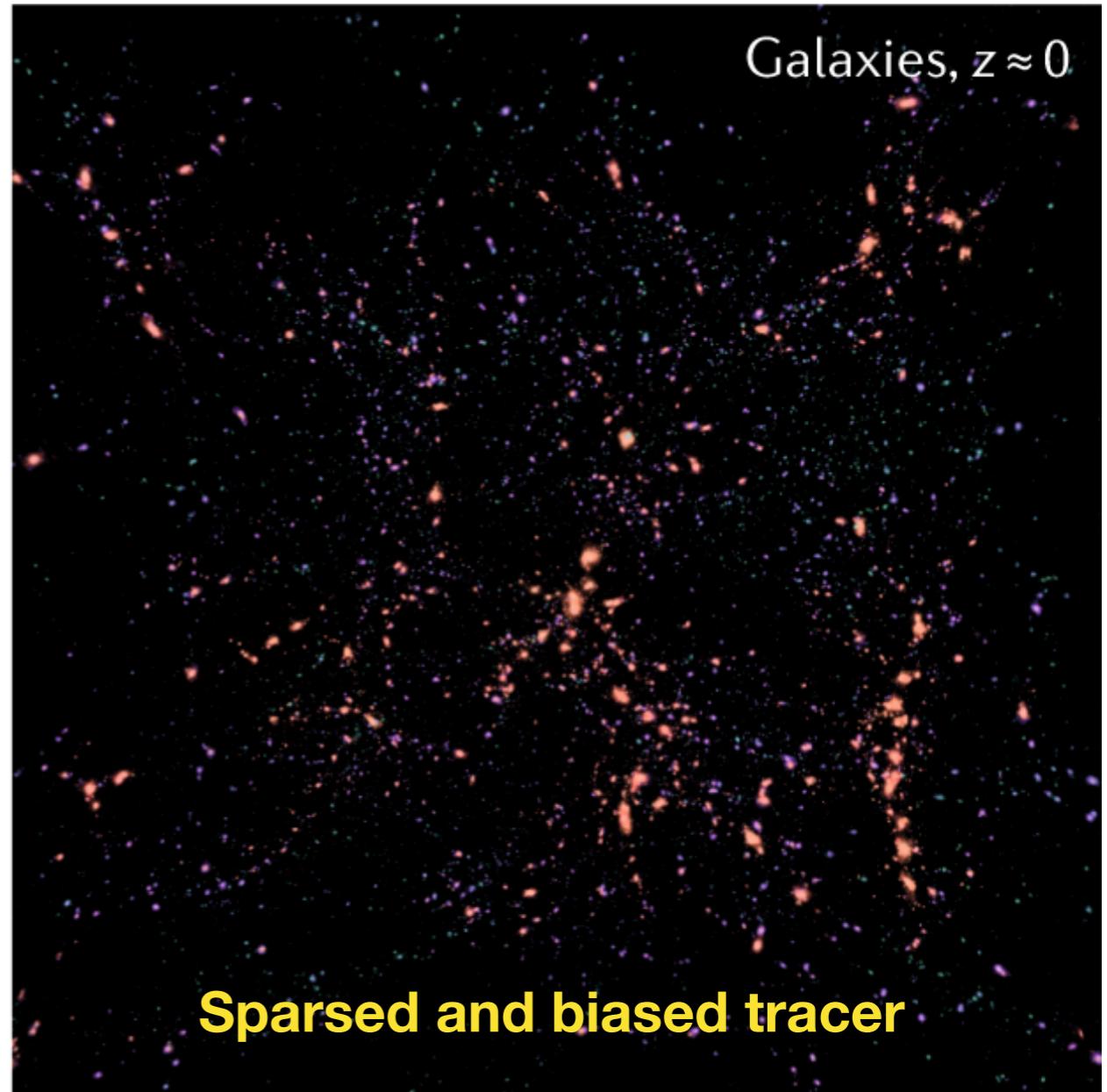
a

**That's what we need**



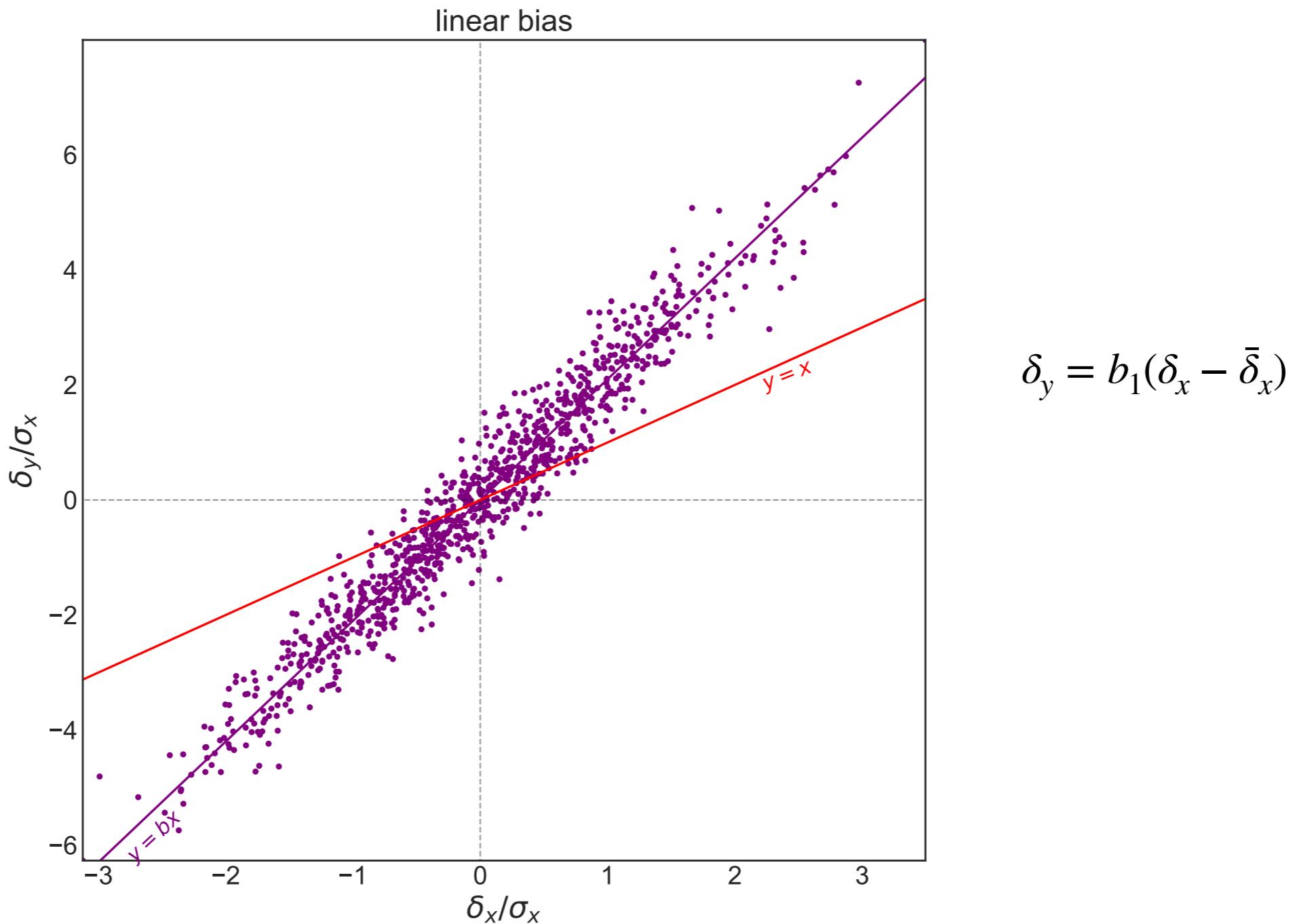
b

**That's what we have**



(Image from Robertson et al. 2019)

# Global bias (linear)



# 2pt-statistics and linear bias

---

$$\xi_{mm}(\vec{r}) = \left\langle \delta_m(\vec{x} + \vec{r}) \cdot \delta_m(\vec{x}) \right\rangle_{\vec{x}}$$

$$\delta_x = \frac{\rho_x - \bar{\rho}_x}{\bar{\rho}_x}$$

$$\xi_{gg}(\vec{r}) = \left\langle \delta_g(\vec{x} + \vec{r}) \cdot \delta_g(\vec{x}) \right\rangle_{\vec{x}}$$

$$\xi_{gg}(\vec{r}) = \left\langle b_g \delta_m(\vec{x} + \vec{r}) \cdot b_g \delta_m(\vec{x}) \right\rangle_{\vec{x}}$$

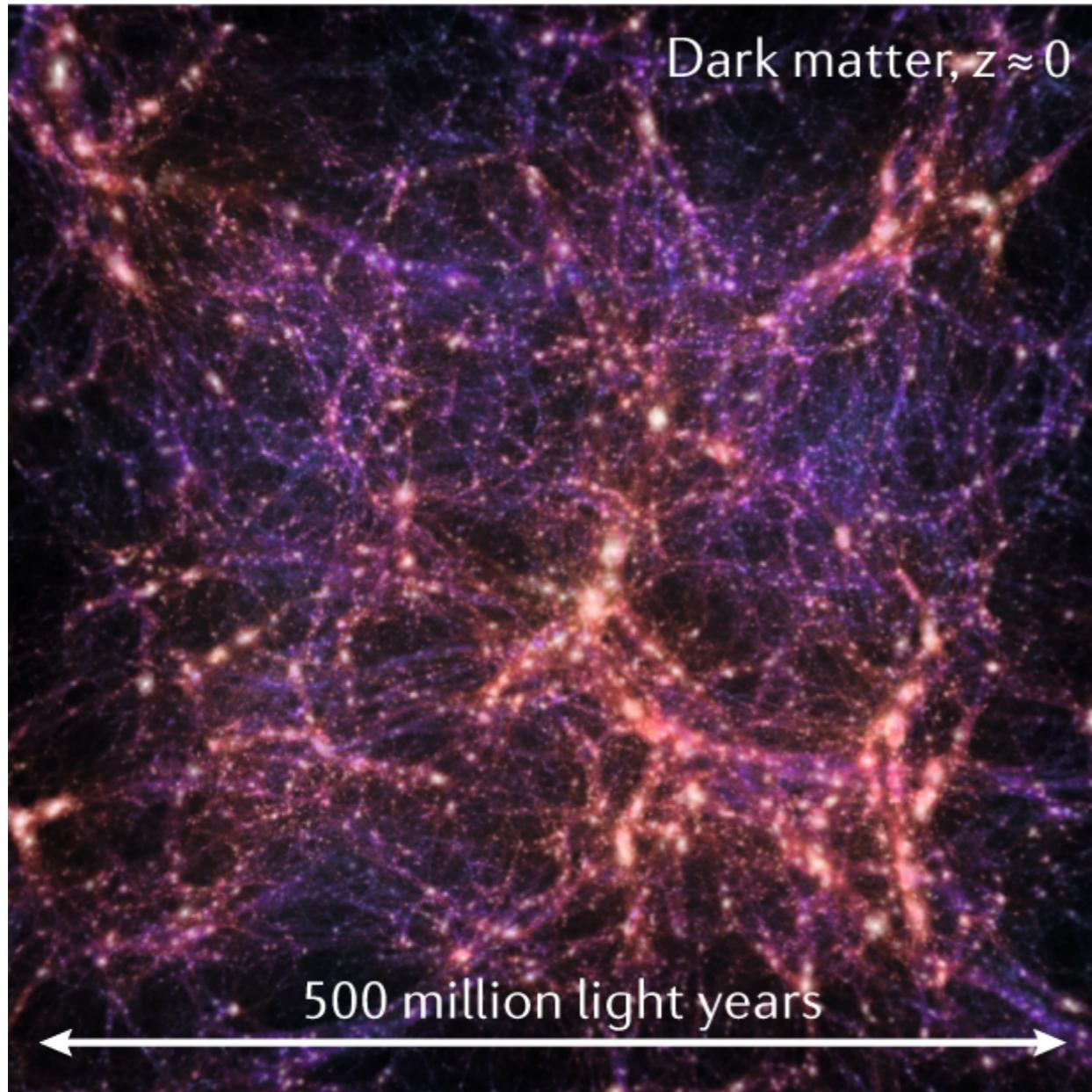
b = linear bias

$$\xi_{gg}(\vec{r}) = b_g^2 \xi_{mm}(\vec{r})$$

# Complicated because bias can be complex

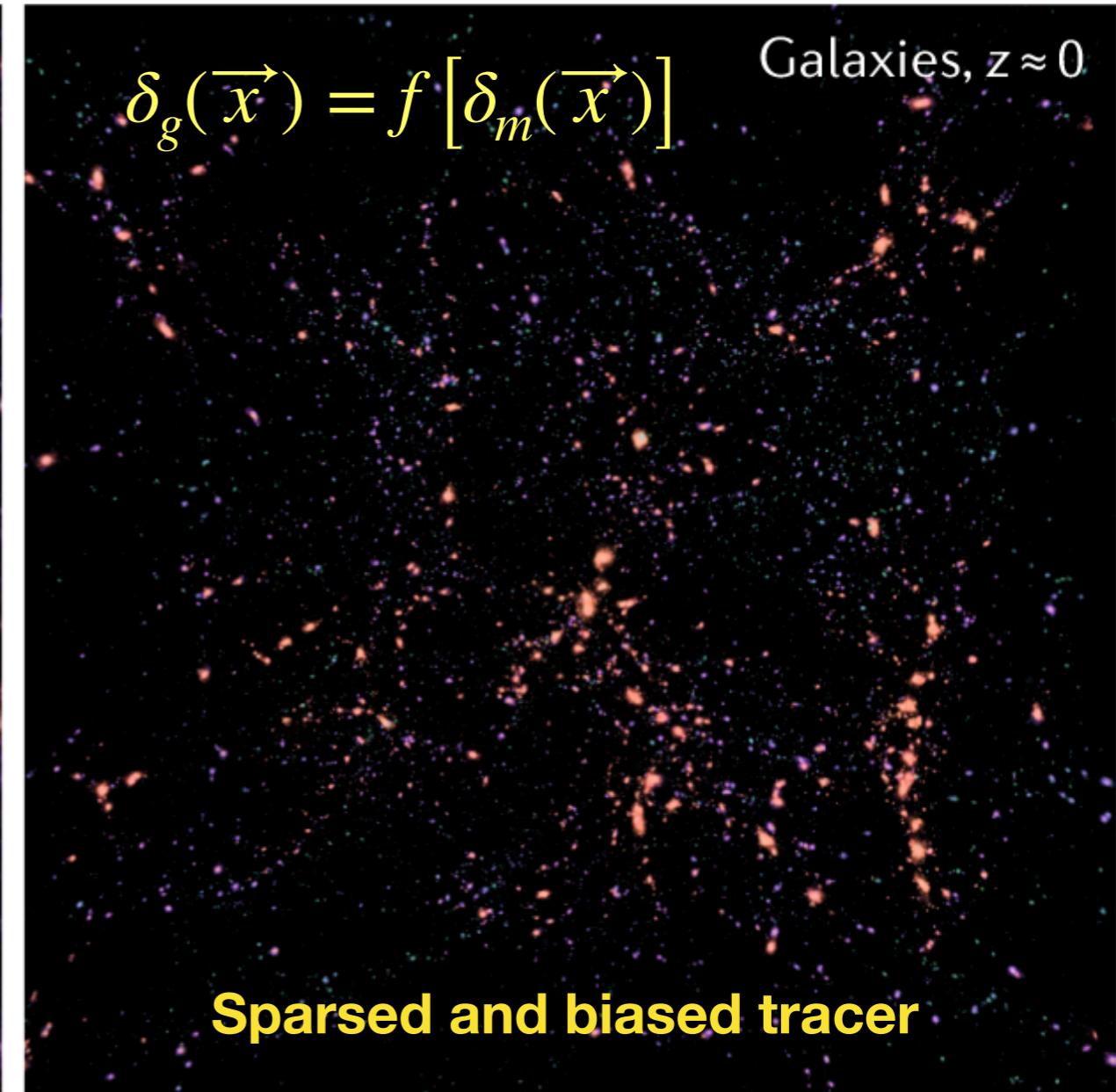
a

**That's what we need**



b

**That's what we have**



(Image from Robertson et al. 2019)

# But what to do?

---

$$\xi_{mm}(\vec{r}) = \left\langle \delta_m(\vec{x} + \vec{r}) \cdot \delta_m(\vec{x}) \right\rangle_{\vec{x}}$$

$$\delta_x = \frac{\rho_x - \bar{\rho}_x}{\bar{\rho}_x}$$

$$\xi_{gg}(\vec{r}) = \left\langle \delta_g(\vec{x} + \vec{r}) \cdot \delta_g(\vec{x}) \right\rangle_{\vec{x}}$$

$$\xi_{gg}(\vec{r}) = \left\langle f[\delta_m(\vec{x} + \vec{r})] \cdot f[\delta_m(\vec{x})] \right\rangle_{\vec{x}}$$

???

# Standard bias scheme

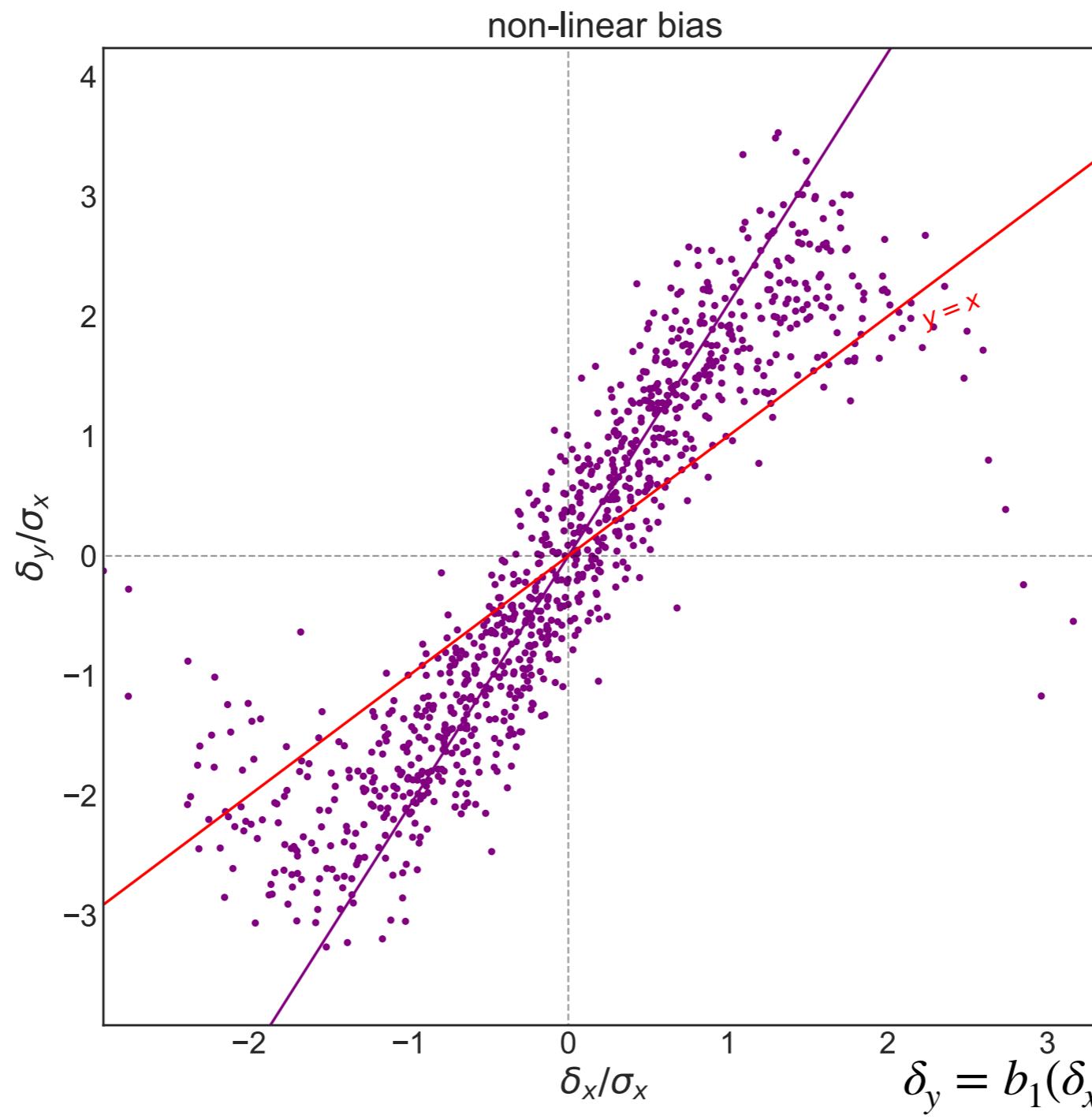
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$$\delta_g(\vec{x}) = f[\delta_m(\vec{x})] \longrightarrow \delta_g(\vec{x}) = \sum_{i=0}^{\infty} \frac{b_i}{i!} \delta_m^i(\vec{x})$$

**Local**  
**No specific form**

**non-Local**  
**Taylor expansion**

# Non-local bias orden 2



# Real application : Hard!!

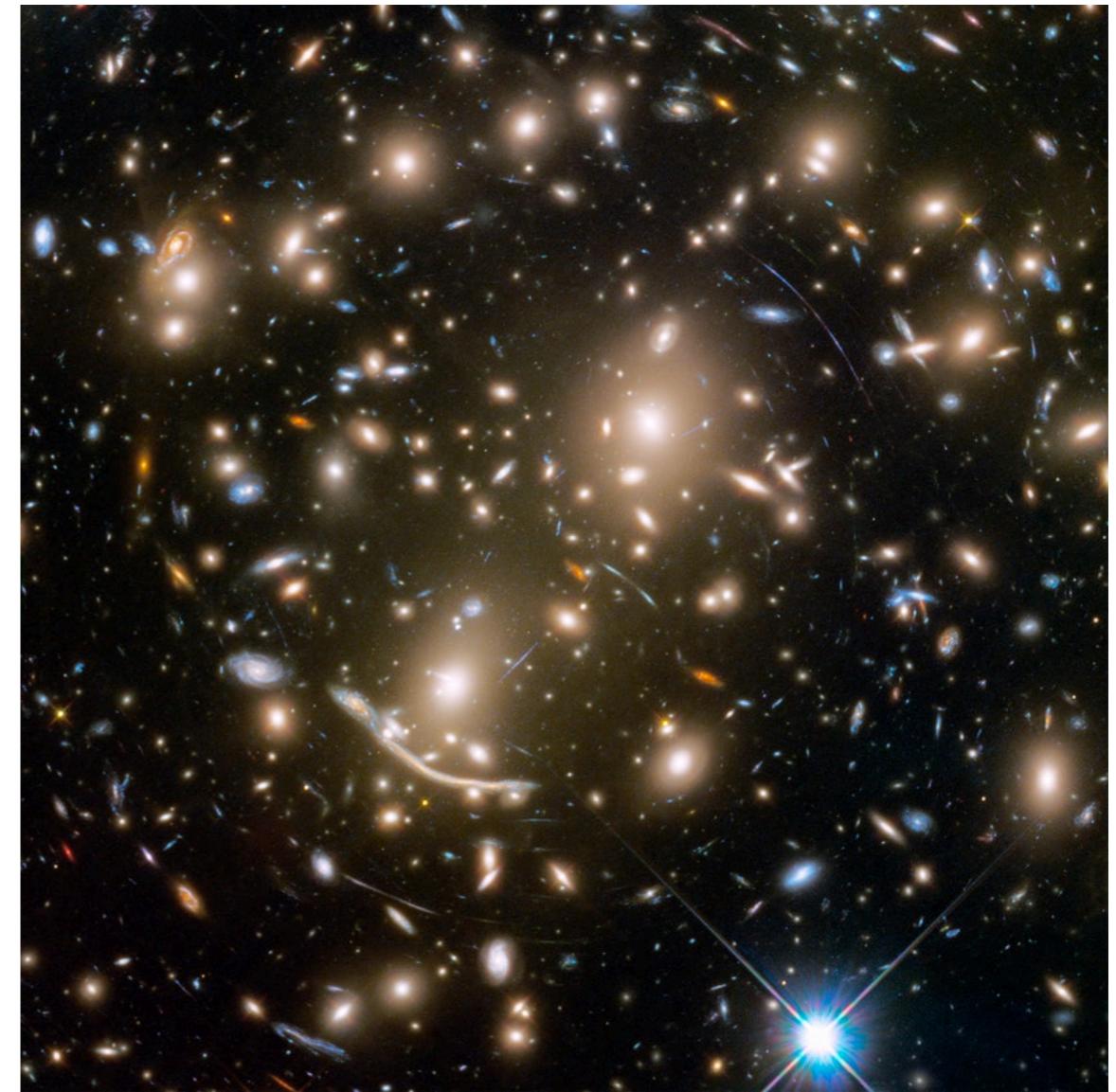
$$\begin{aligned}\delta_g = & c_\delta \delta + \frac{1}{2} c_{\delta^2} (\delta^2 - \sigma^2) + \frac{1}{2} c_{s^2} \left( s^2 - \frac{2}{3} \sigma^2 \right) + \frac{1}{3!} c_{\delta^3} \delta^3 + \frac{1}{2} c_{\delta s^2} \delta s^2 + c_\psi \psi + c_{st} st + \frac{1}{3!} c_{s^3} s^3 \\ & + c_\epsilon \epsilon + c_{\delta\epsilon} \delta\epsilon + \frac{1}{2} c_{\delta^2\epsilon} \delta^2\epsilon + \frac{1}{2} c_{s^2\epsilon} s^2\epsilon + \frac{1}{2} c_{\epsilon^2} (\epsilon^2 - \sigma_\epsilon^2) + \frac{1}{2} c_{\delta\epsilon^2} \delta\epsilon^2 + \frac{1}{3!} c_{\epsilon^3} \epsilon^3 + \dots\end{aligned}$$

$$\begin{aligned}P_{mg}(k) = & c_\delta P_{\text{NL}}(k) \\ & + c_{\delta^2} \int \frac{d^3\mathbf{q}}{(2\pi)^3} P(q) P(|\mathbf{k} - \mathbf{q}|) F_S^{(2)}(\mathbf{q}, \mathbf{k} - \mathbf{q}) + \frac{34}{21} c_{\delta^2} \sigma^2 P(k) \\ & + c_{s^2} \int \frac{d^3\mathbf{q}}{(2\pi)^3} P(q) P(|\mathbf{k} - \mathbf{q}|) F_S^{(2)}(\mathbf{q}, \mathbf{k} - \mathbf{q}) S(\mathbf{q}, \mathbf{k} - \mathbf{q}) \\ & + 2 c_{s^2} P(k) \int \frac{d^3\mathbf{q}}{(2\pi)^3} P(q) F_S^{(2)}(-\mathbf{q}, \mathbf{k}) S(\mathbf{q}, \mathbf{k} - \mathbf{q}) \\ & + \frac{1}{2} c_{\delta^3} \sigma^2 P(k) + \frac{1}{3} c_{\delta s^2} \sigma^2 P(k) \\ & + 2 c_\psi P(k) \int \frac{d^3\mathbf{q}}{(2\pi)^3} P(q) \left[ \frac{3}{2} D_S^{(3)}(\mathbf{q}, -\mathbf{q}, -\mathbf{k}) - 2 F_S^{(2)}(-\mathbf{q}, \mathbf{k}) D_S^{(2)}(\mathbf{q}, \mathbf{k} - \mathbf{q}) \right] \\ & + 2 c_{st} P(k) \int \frac{d^3\mathbf{q}}{(2\pi)^3} P(q) D_S^{(2)}(-\mathbf{q}, \mathbf{k}) S(\mathbf{q}, \mathbf{k} - \mathbf{q}) \\ & + \frac{1}{2} c_{\delta\epsilon^2} \sigma_\epsilon^2 P(k).\end{aligned}$$

**McDonald & Roy 2009**

# All galaxies are leaving in halos

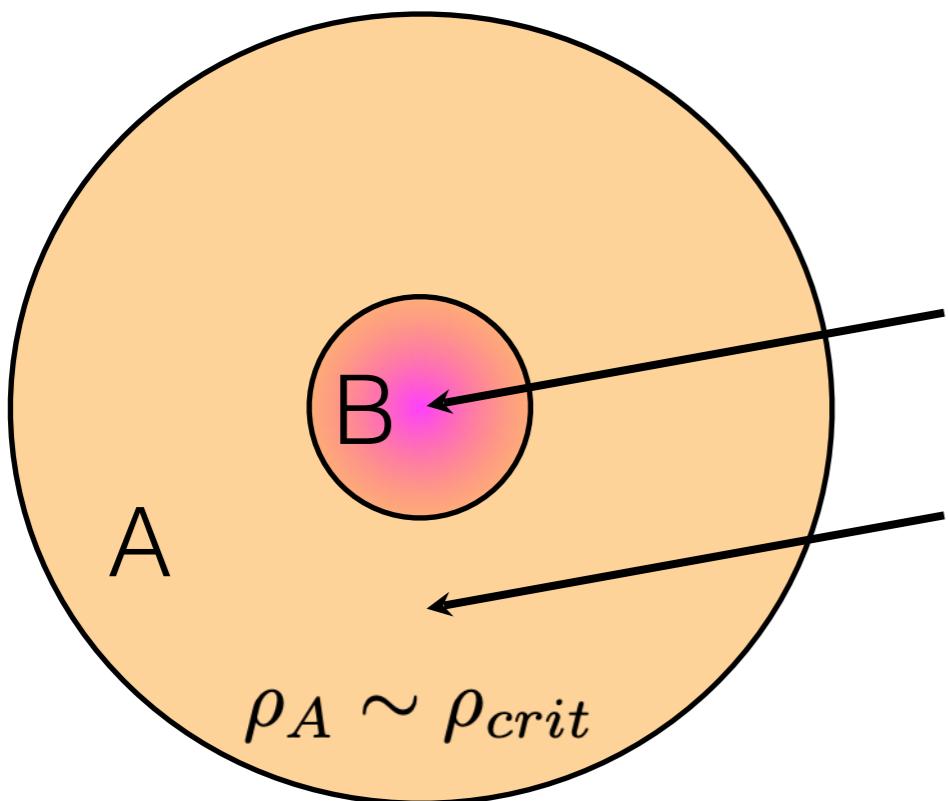
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# Spherical halo model

The 2 areas A & B are  
considered as local universe  
with proper scale factor  
evolution

(Gunn & Gott 1972)

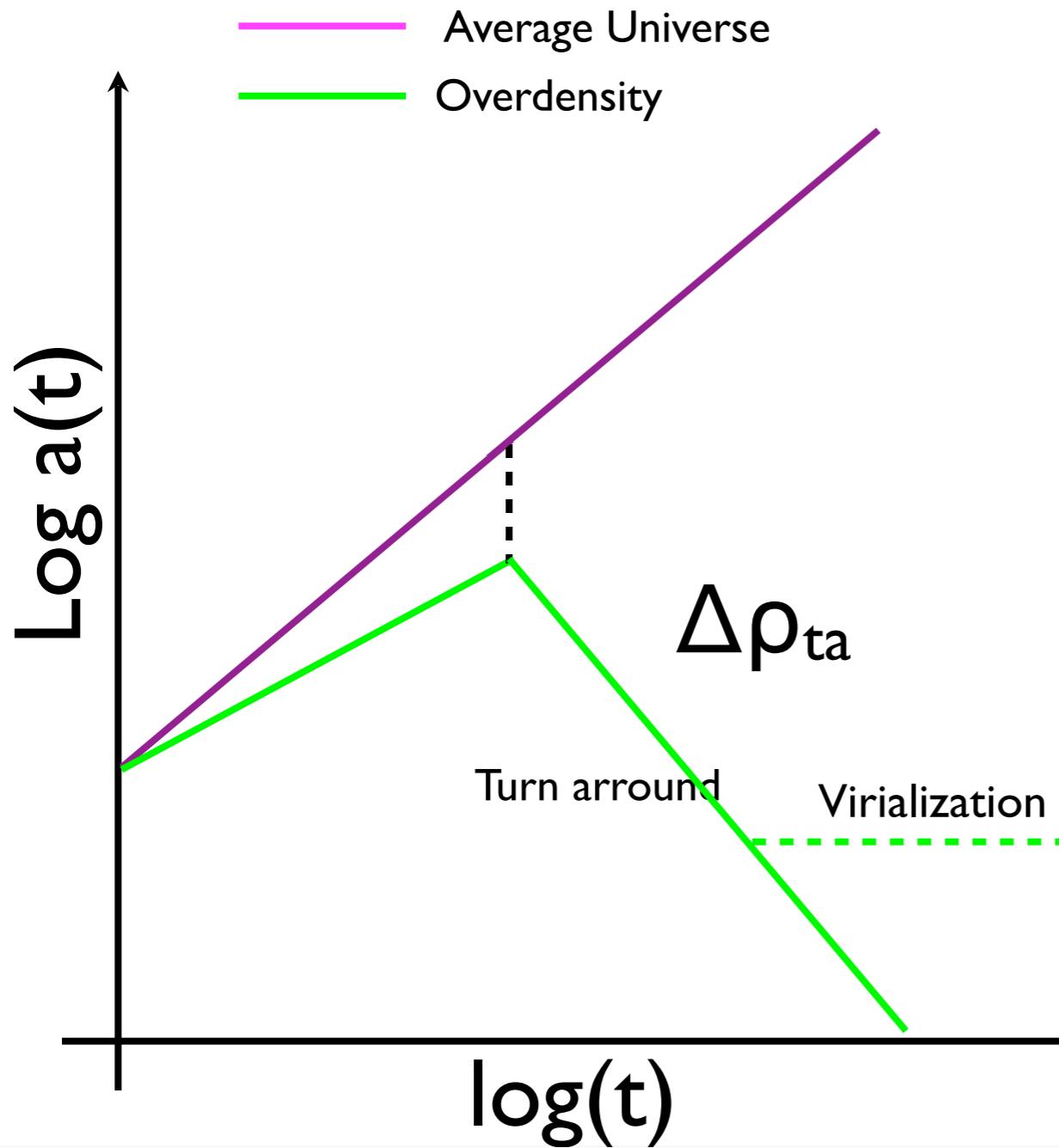


$$H_B^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_B + \frac{\Lambda}{3} - \frac{k}{a^2}$$

$$H_A^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_A + \frac{\Lambda}{3}$$

Curvature

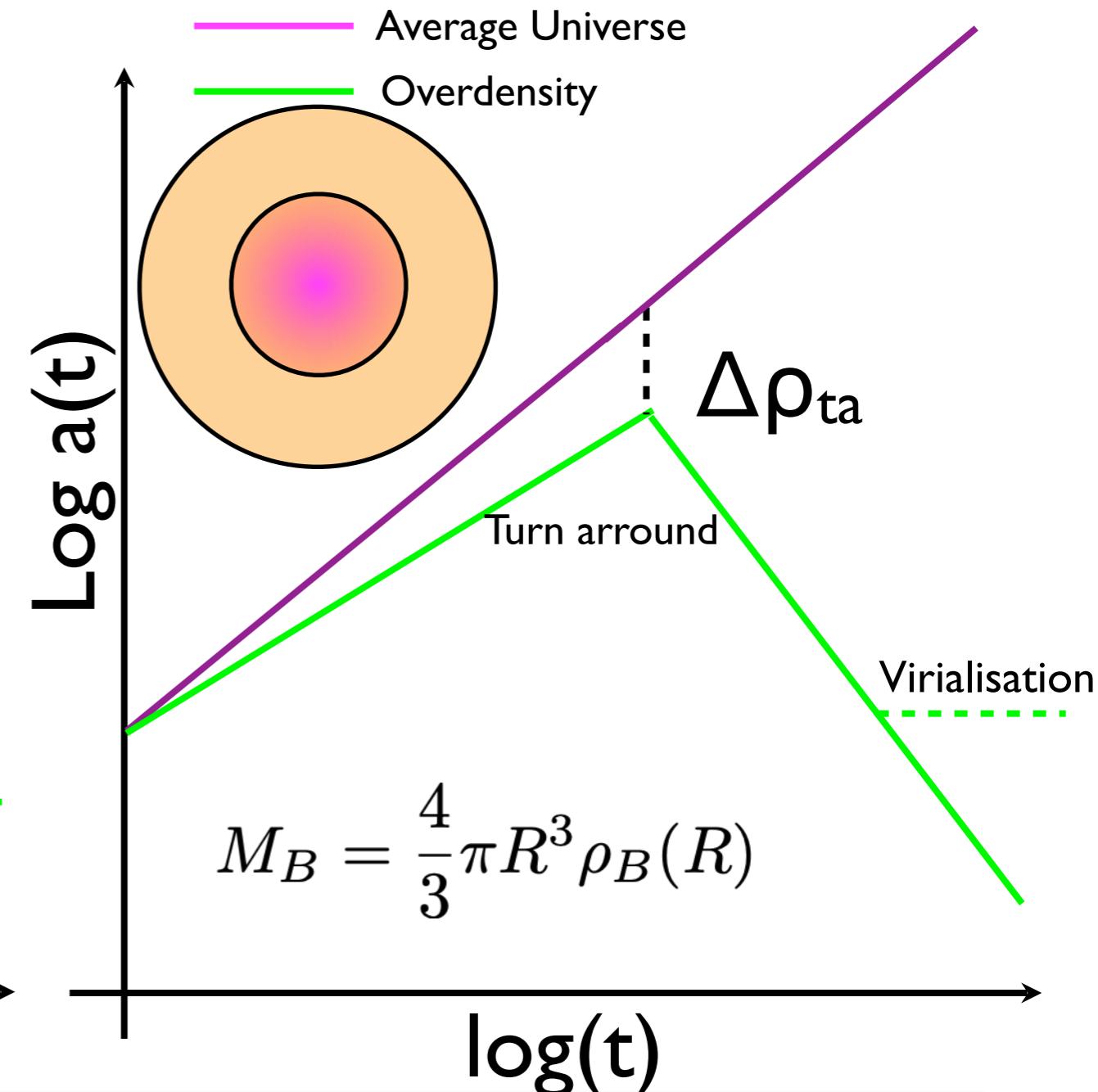
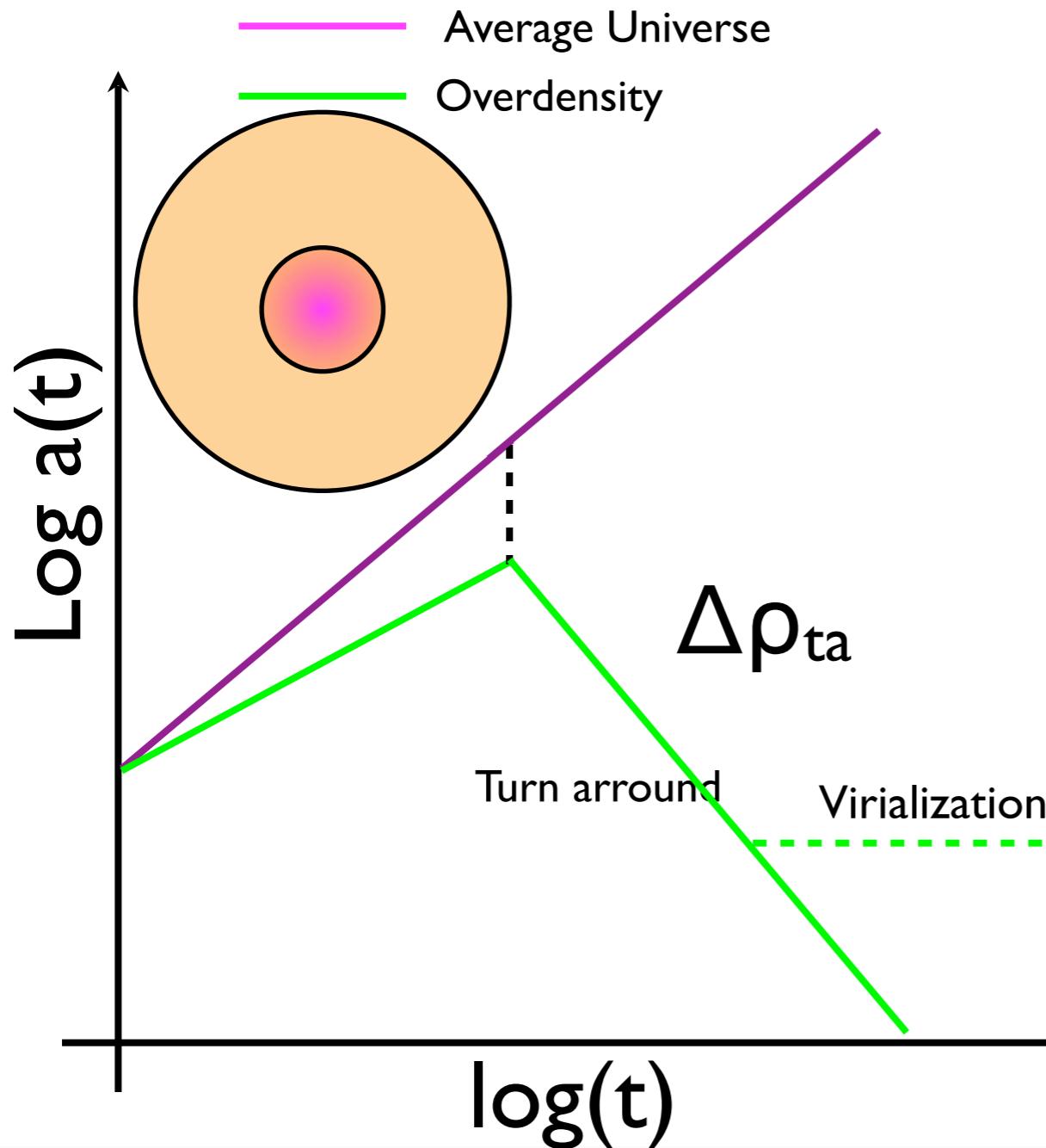
# Galaxy clusters



$$M_B = \frac{4}{3}\pi R^3 \rho_B(R)$$

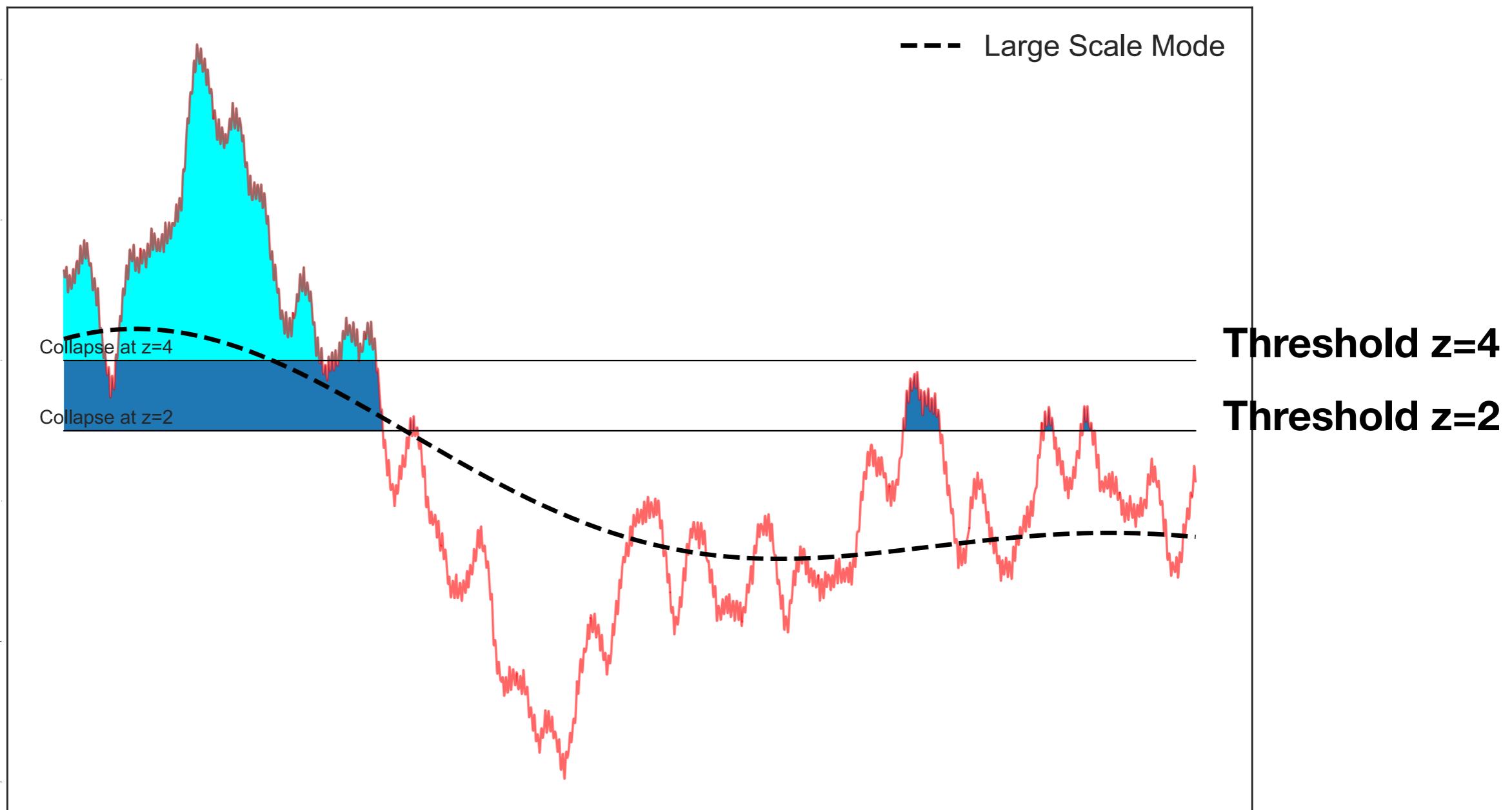
The number of virialized halos of a given mass at a given redshift

# Galaxy clusters



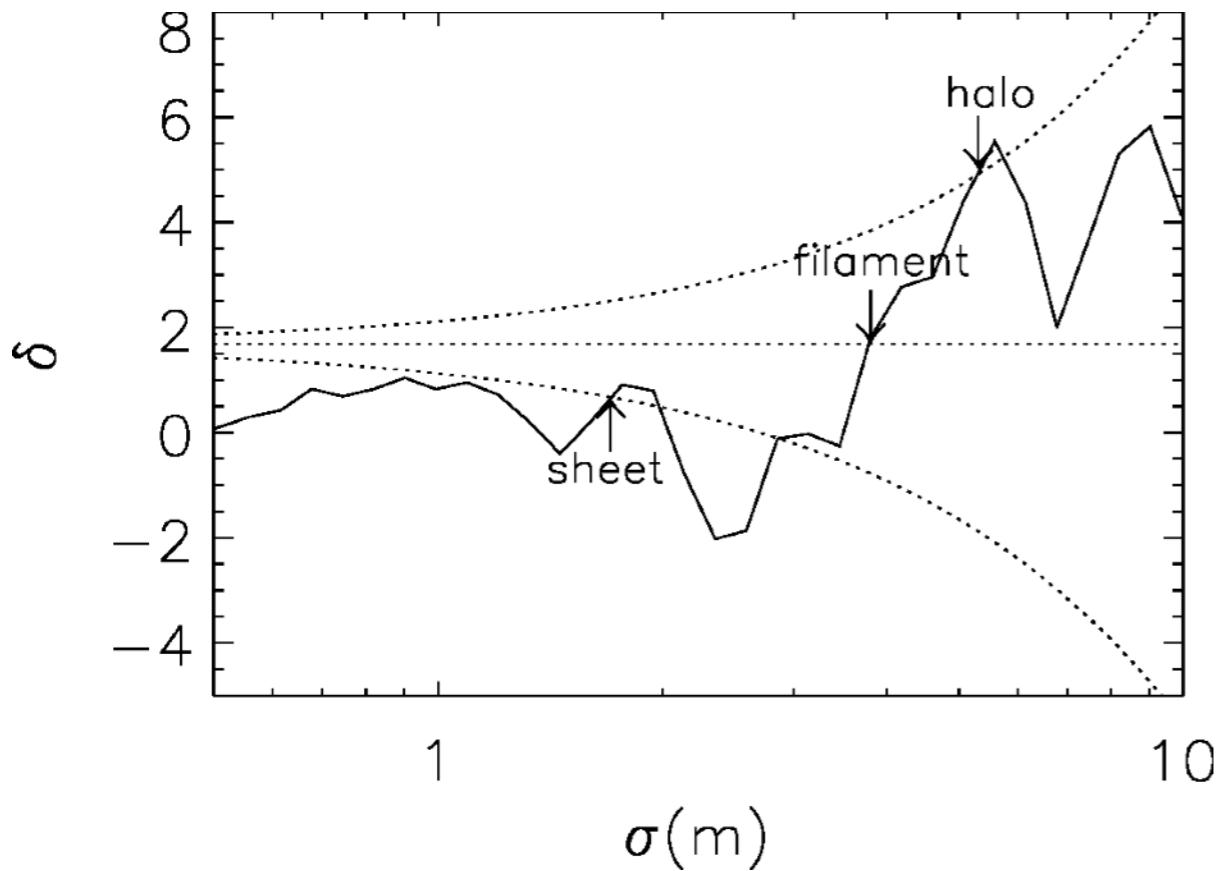
The number of virialized halos of a given mass at a given redshift

# More massive halos formed in large scale over densities



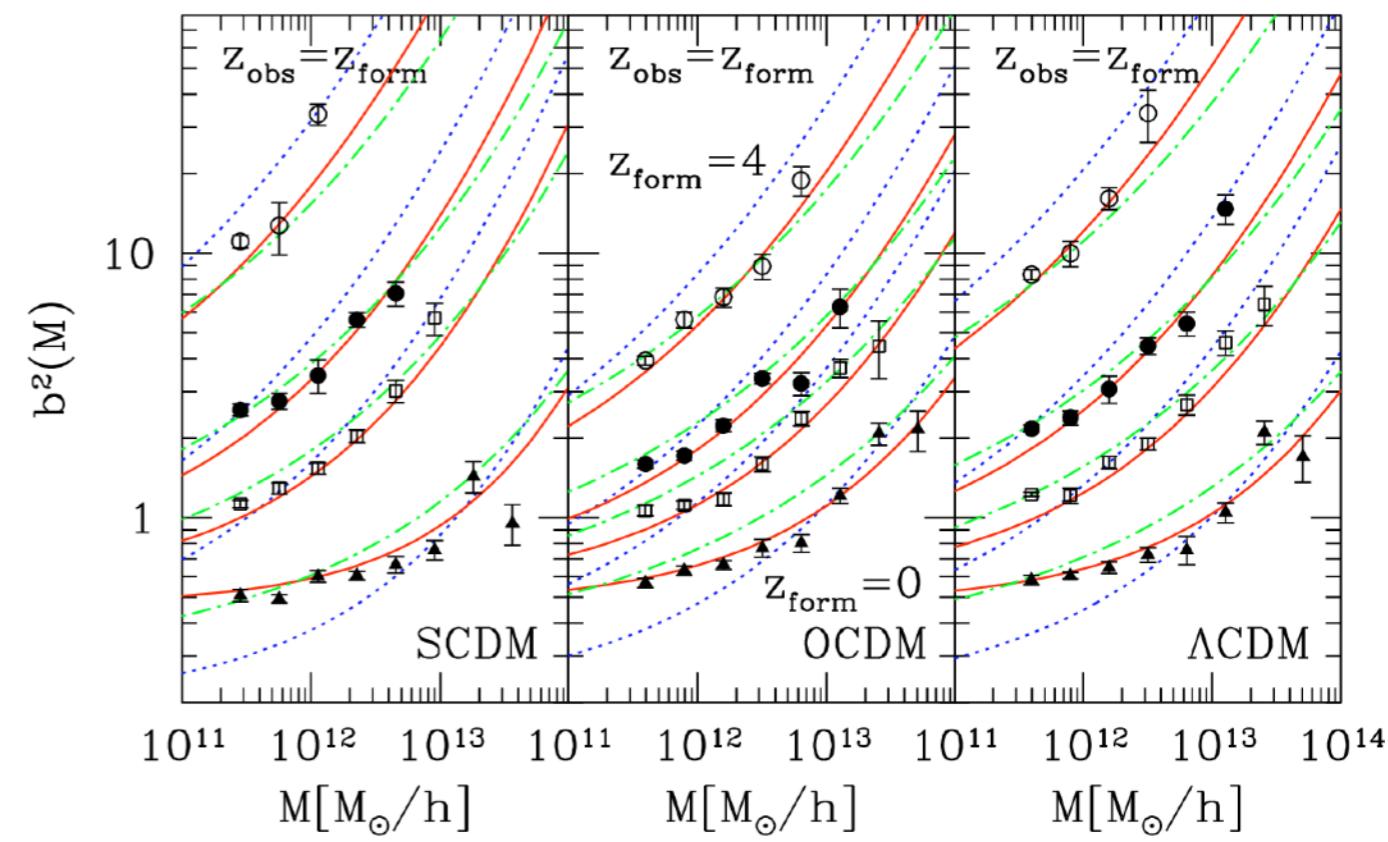
# Conexión halo - materia

**Excursion Set**



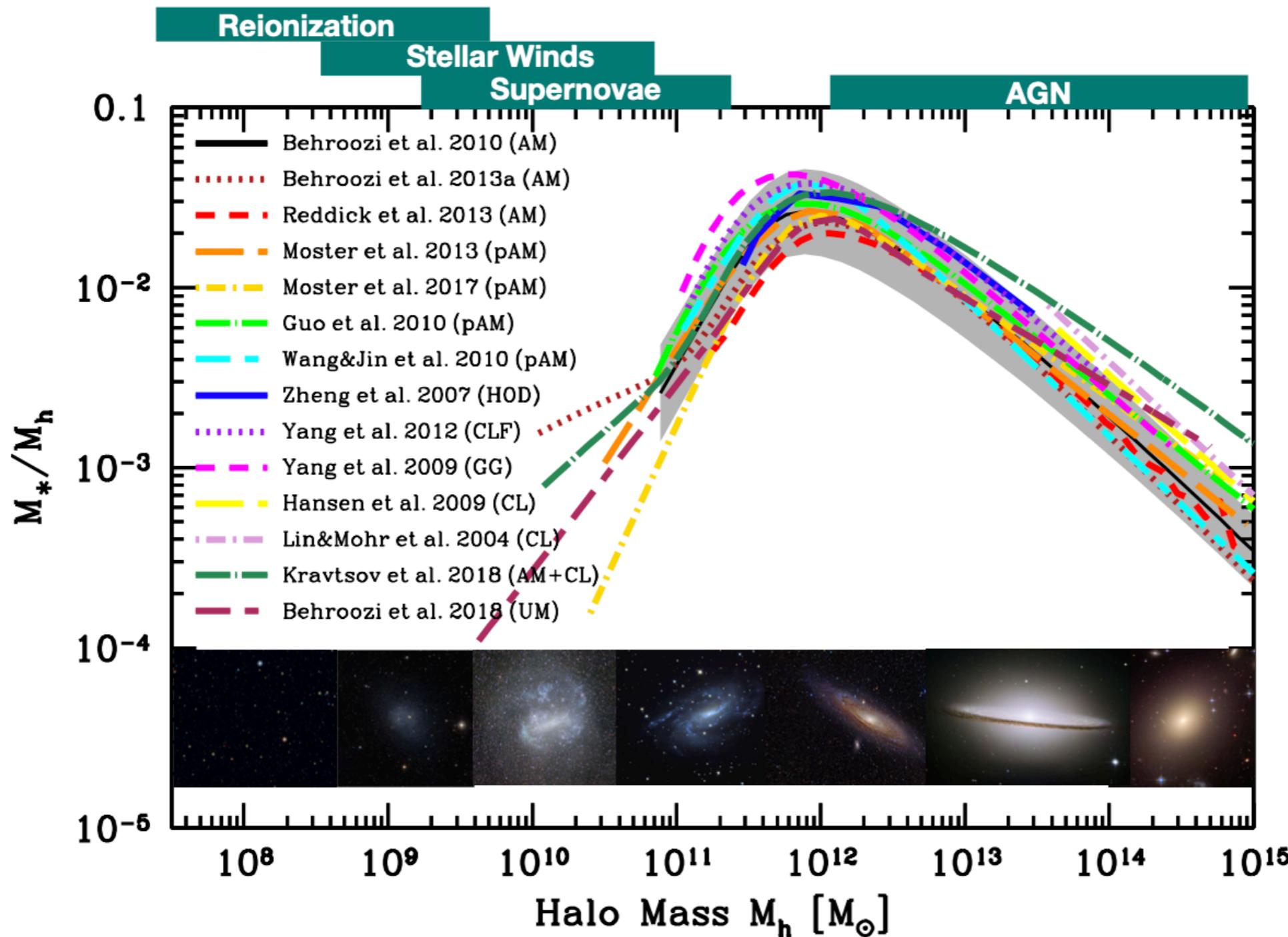
Bond el. 1991

**Peak Background Split**



Sheth & Tormen 1999

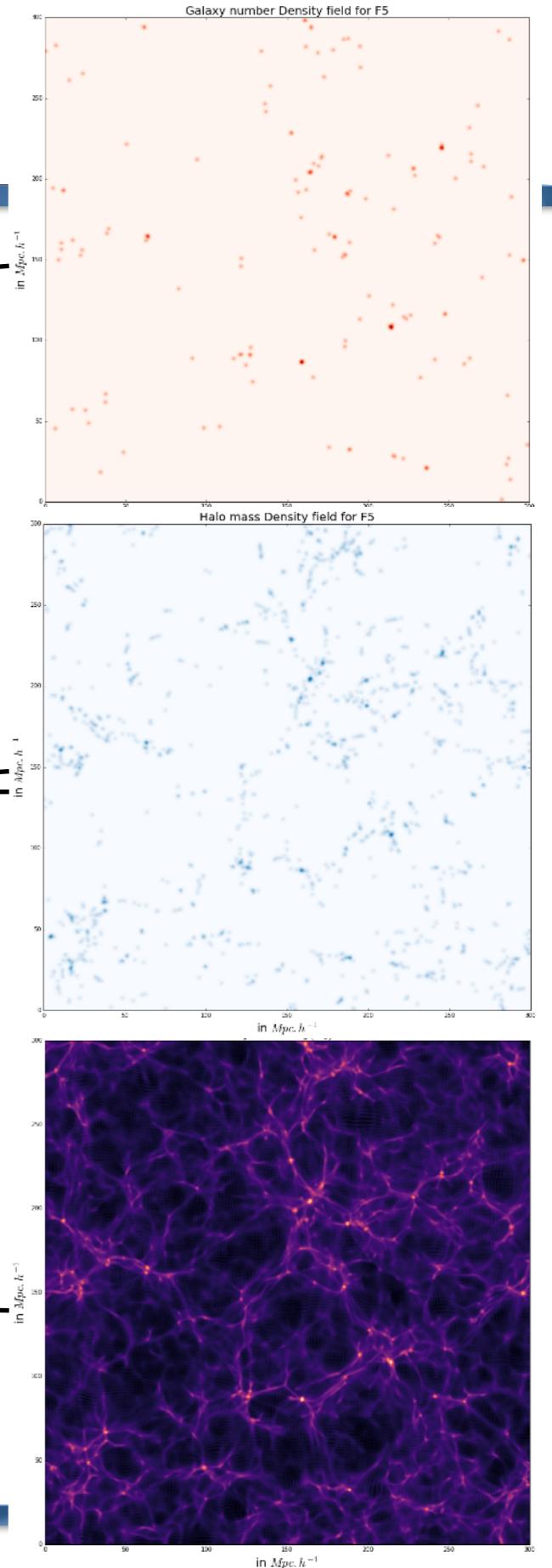
# Stellar formation and galaxy evolution is much more complex

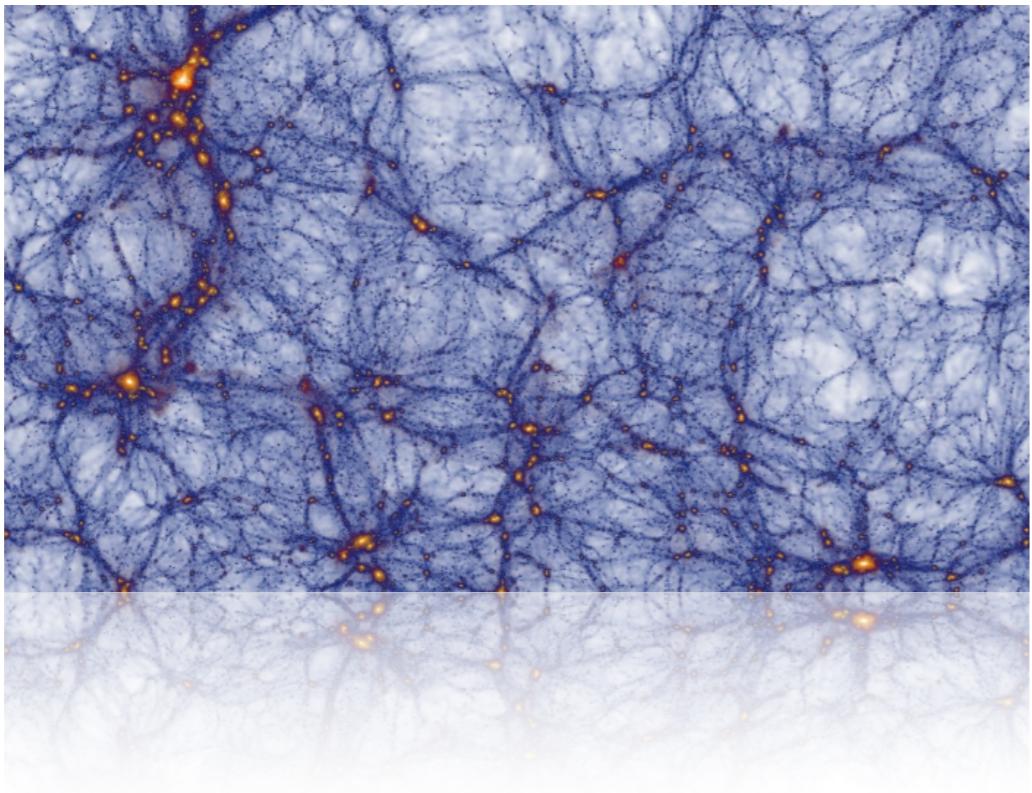


# Idea

Use local astrophysical information  
to infer the hosting halo mass

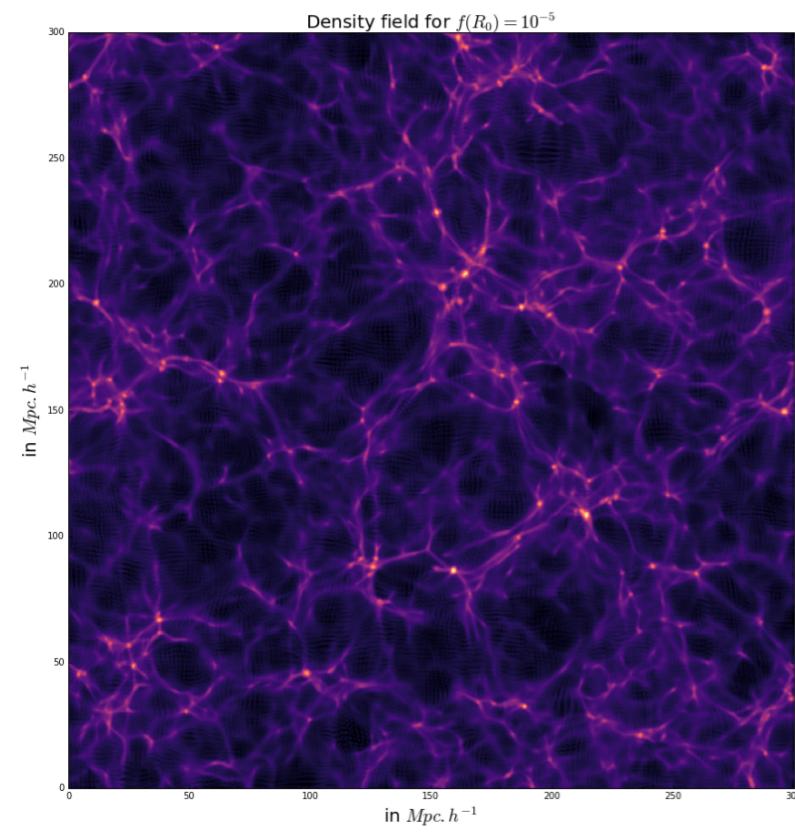
Use the halo mass bias to connect  
with the matter density field





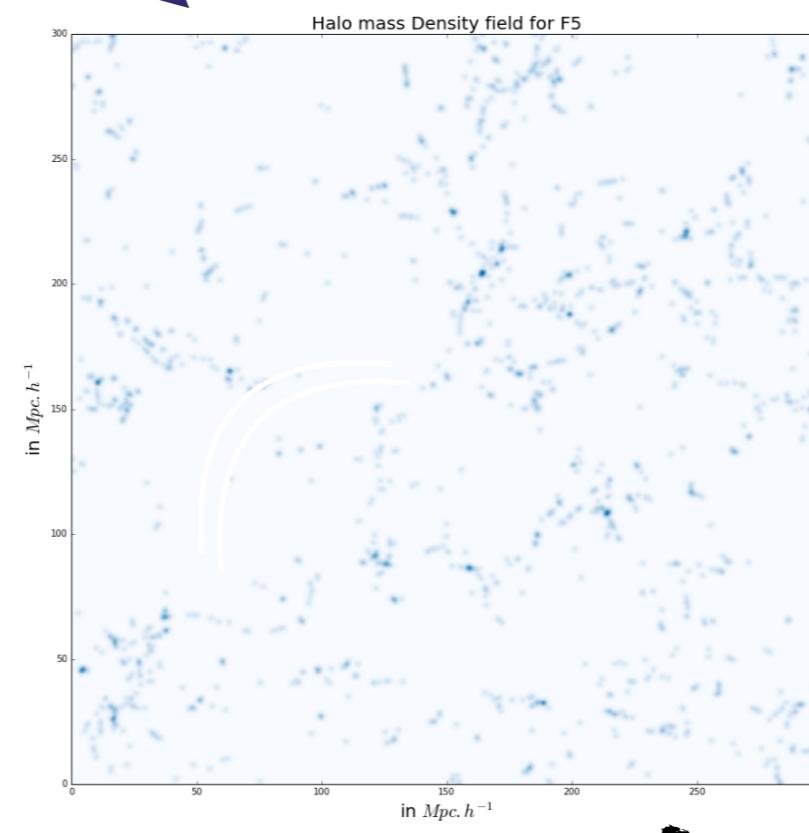
Invert HOD  
statistics

ROCKSTAR

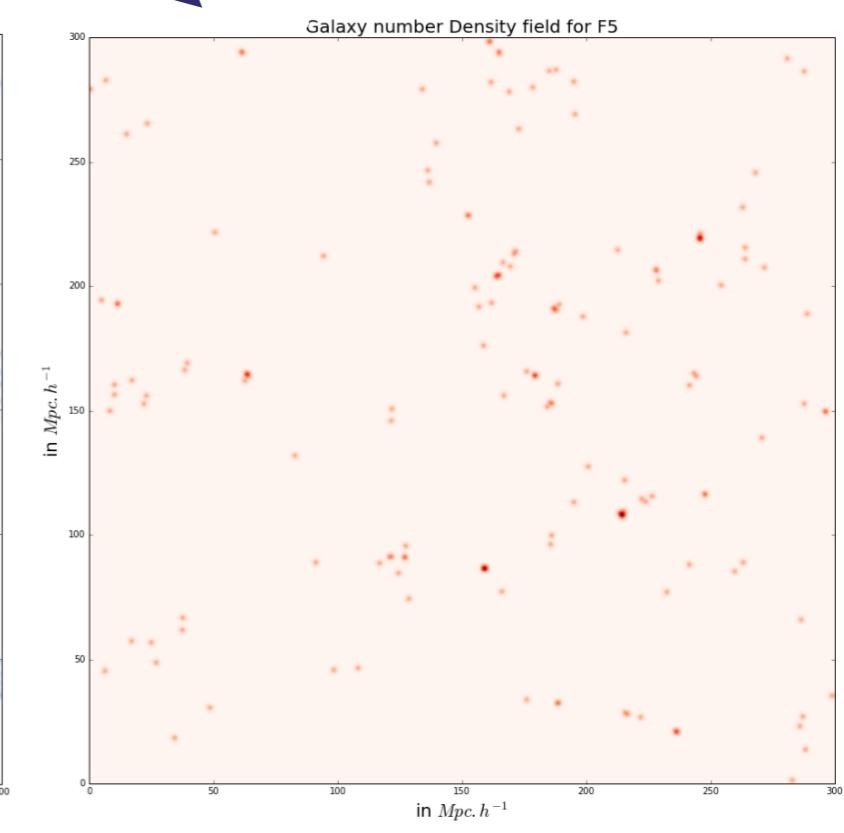


Dark Matter  
particles

HOD + CLF

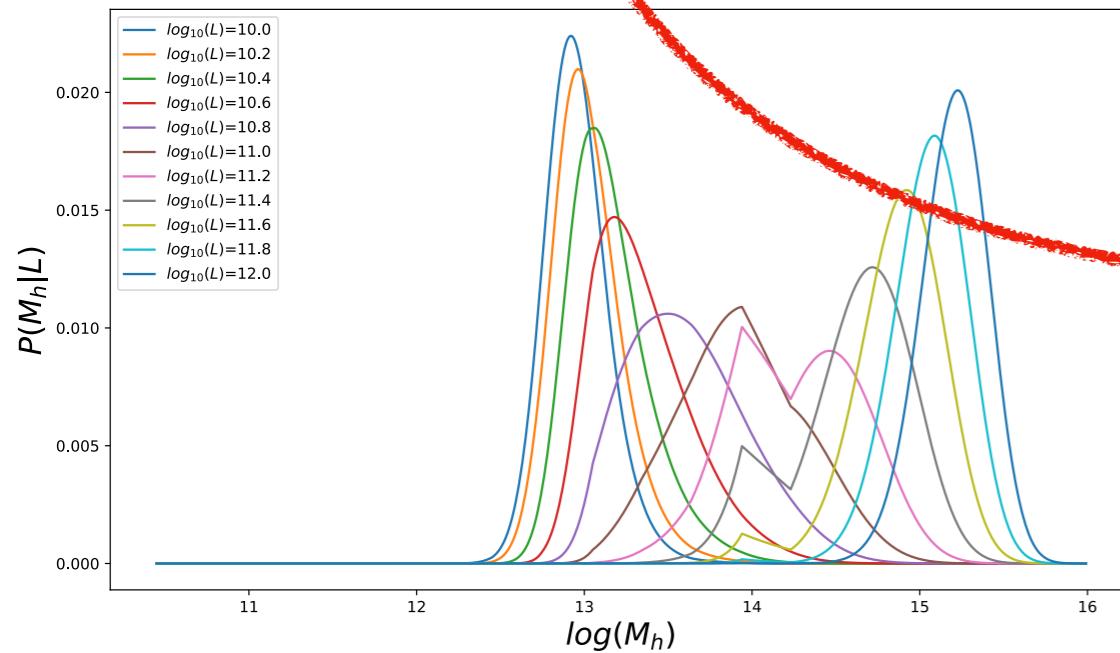


Dark Matter  
Halos



Galaxies with  
Luminosity

DATA



Infer the underlying CDM field

# Reverting the information

---

$$\mathcal{P}(M_h | L) = \frac{\mathcal{P}(L | M_h) \cdot \mathcal{P}(M_h)}{\mathcal{P}(L)}$$

**POSTERIOR WE WANT TO EVALUATE**

**CONDITIONAL LUMINOSITY FUNCTION**

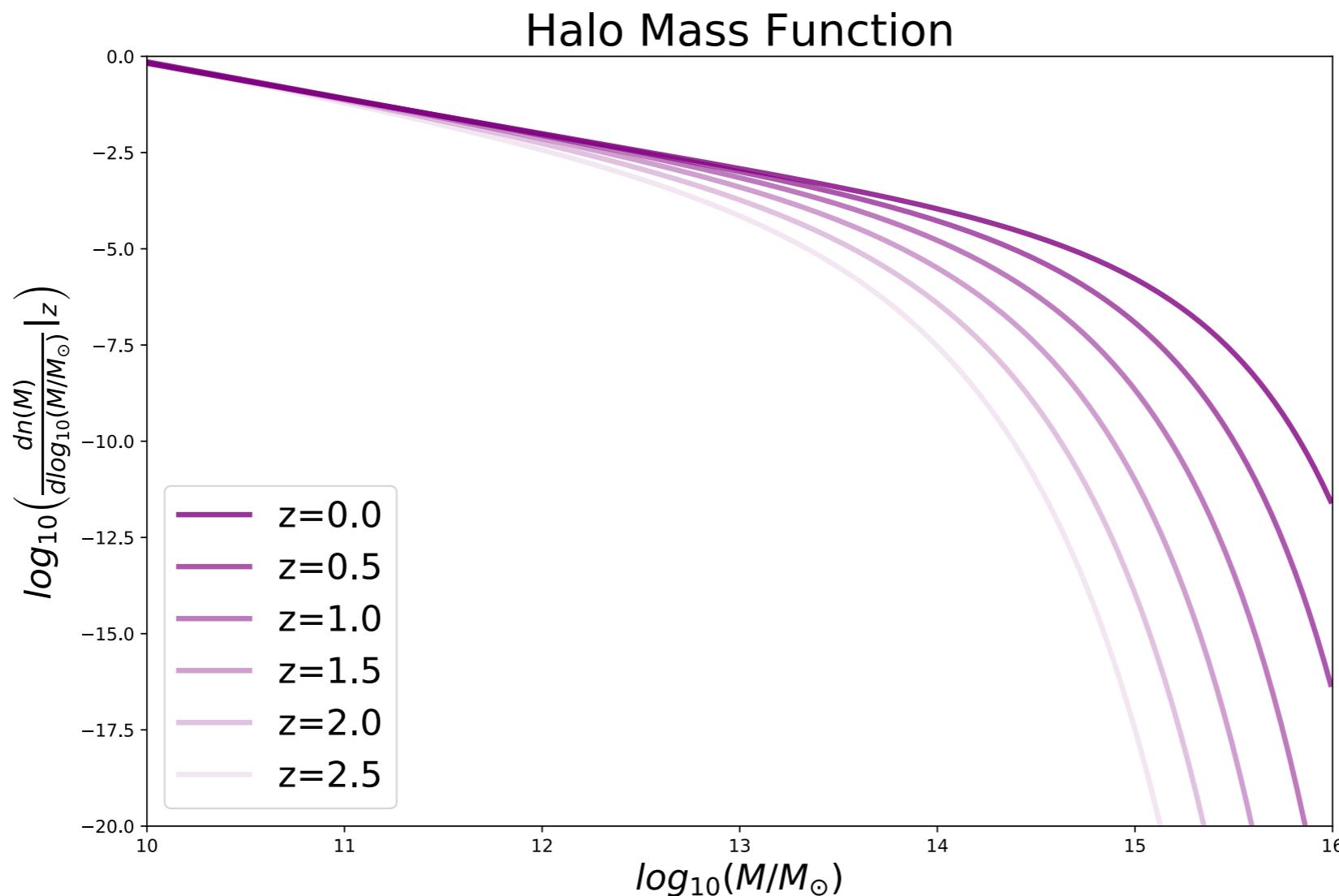
**CONVOLUTION OF HOD AND HMF**

**NORMALIZATION WHICH HAVE NOT IMPACT HERE**

---

# Halo Mass Function

1pt-statistics based on Press-Schechter and spherical collapse (Gunn & Gott) formalisms  
+ Semi-analytical corrections



# Halo Occupation Distribution

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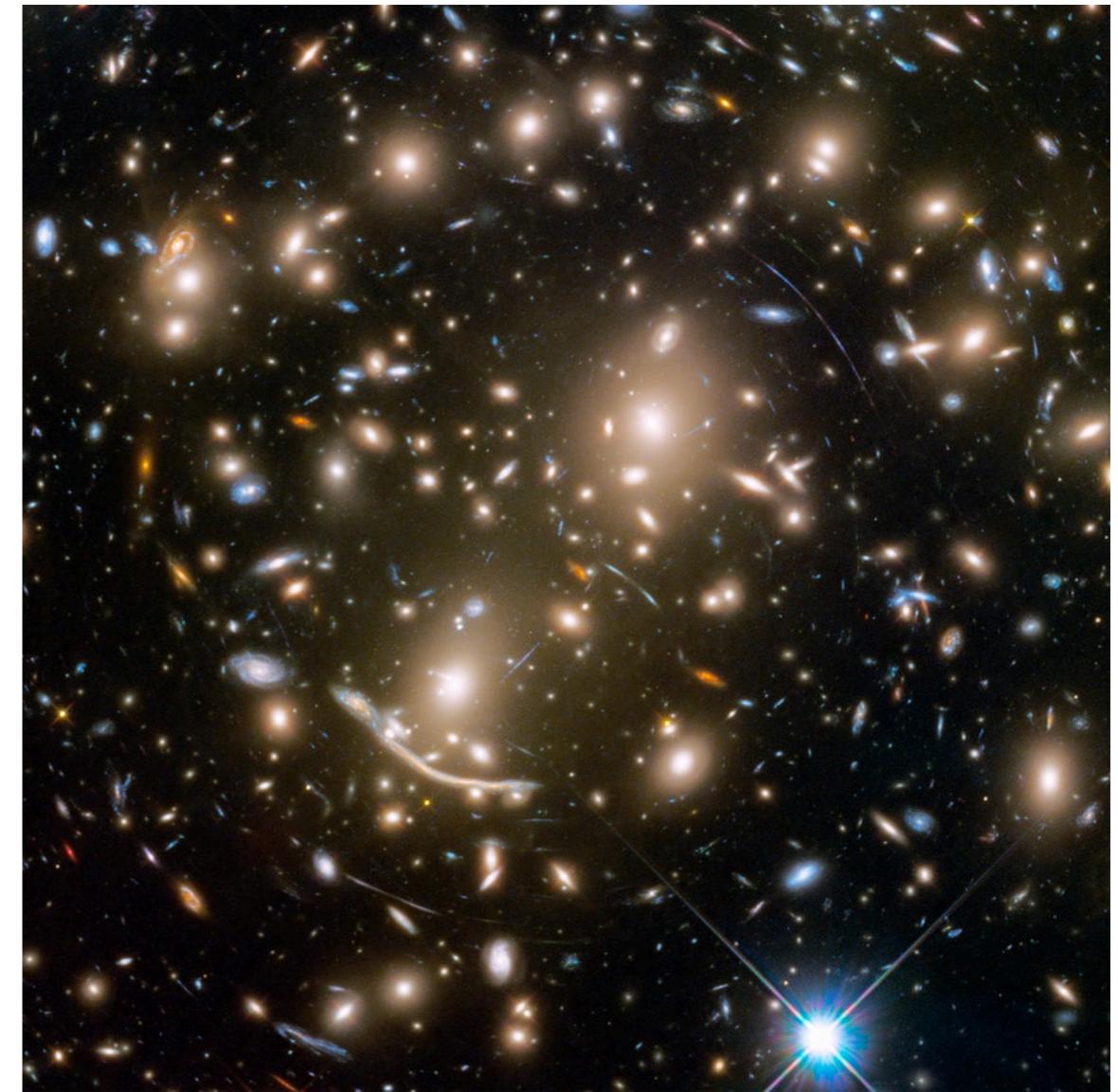
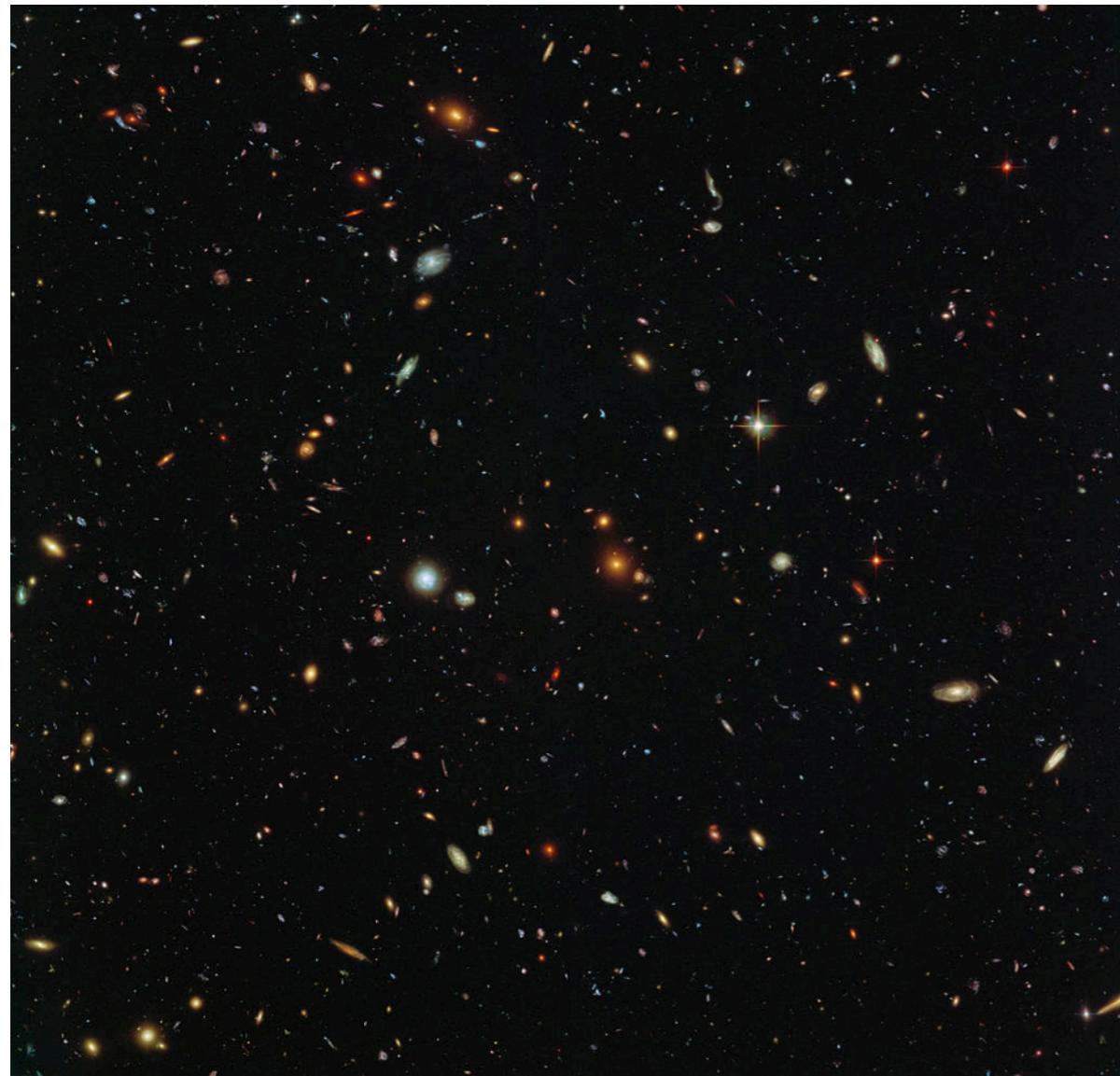
Link between the clustering of observed galaxies and the theoretical correlation function and the Halo profile

Associate a mean number of galaxies at a given halo mass

Each sample of galaxies has its proper HOD

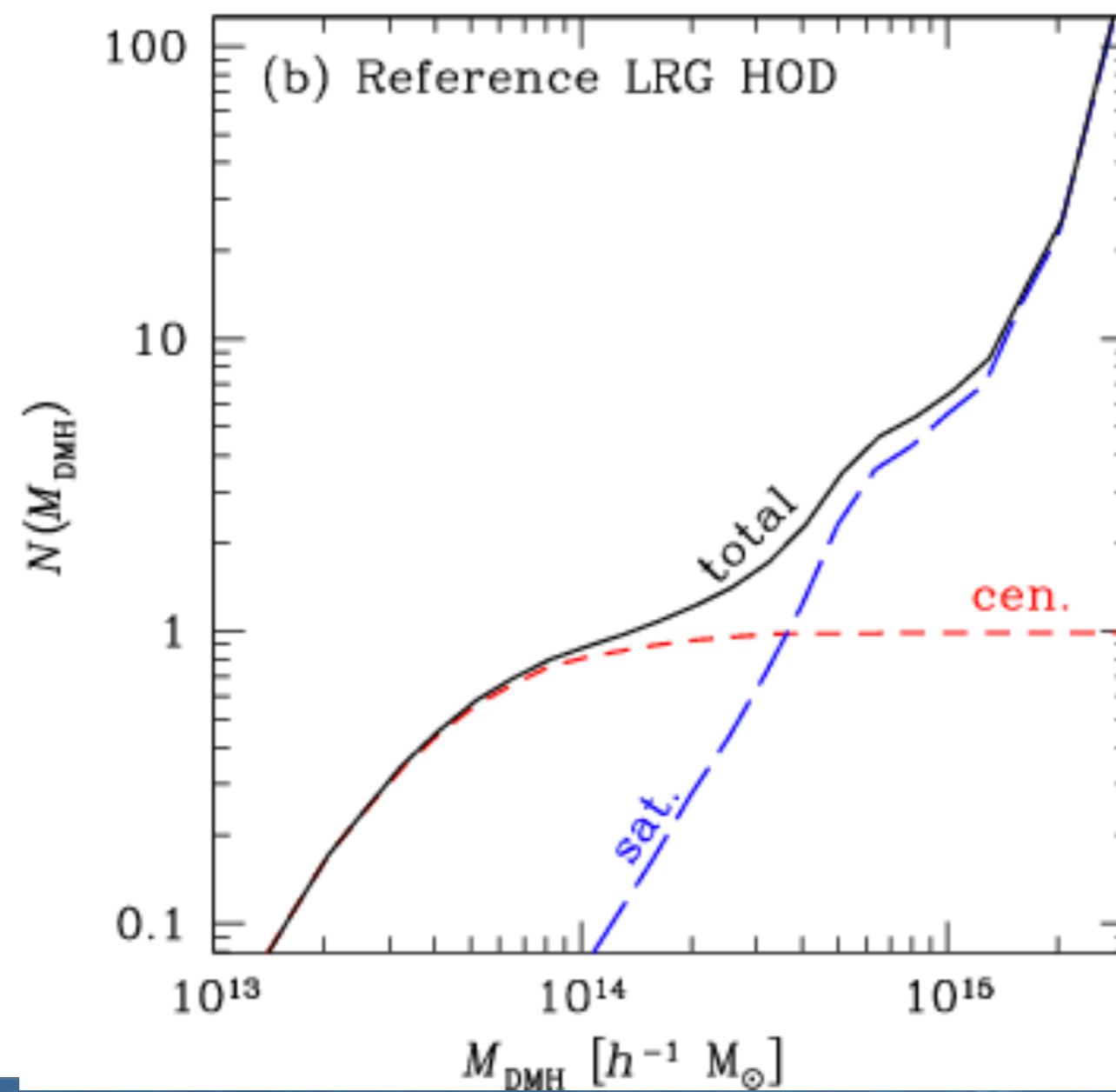
$$\langle N(M) \rangle = \langle N_{cen}(M) \rangle + \langle N_{sat}(M) \rangle$$

# Halo Occupation Distribution



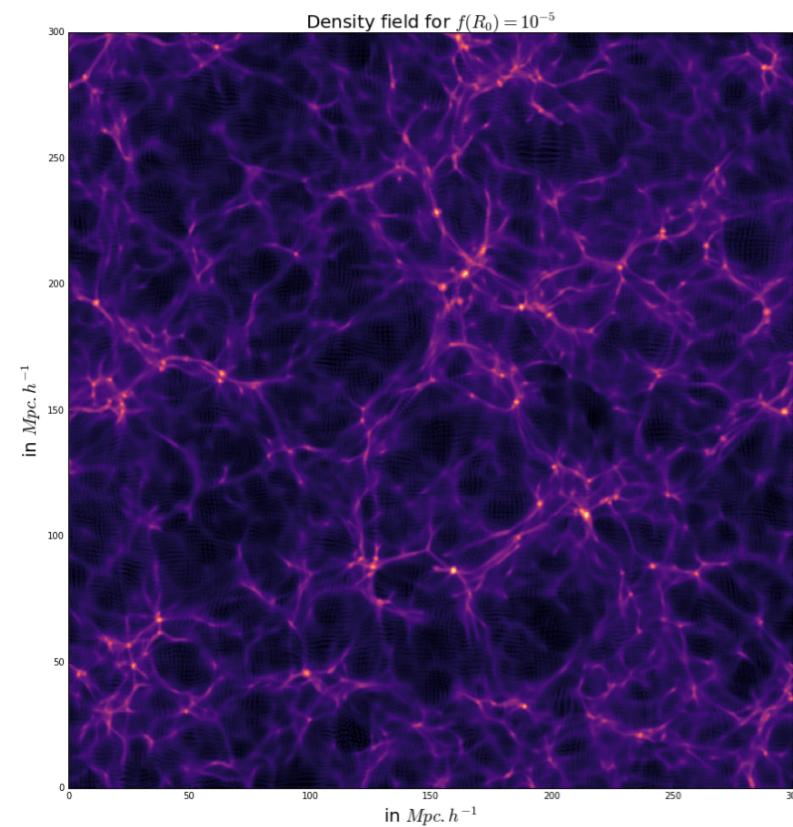
# Example of HOD parametrization

$$\langle N_{cen}(M) \rangle = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{\log M - \log M_{min}}{\sigma_{\log M}} \right) \right] \quad \langle N_{sat}(M) \rangle = \left( \frac{M - M_0}{M_1} \right)^\alpha$$

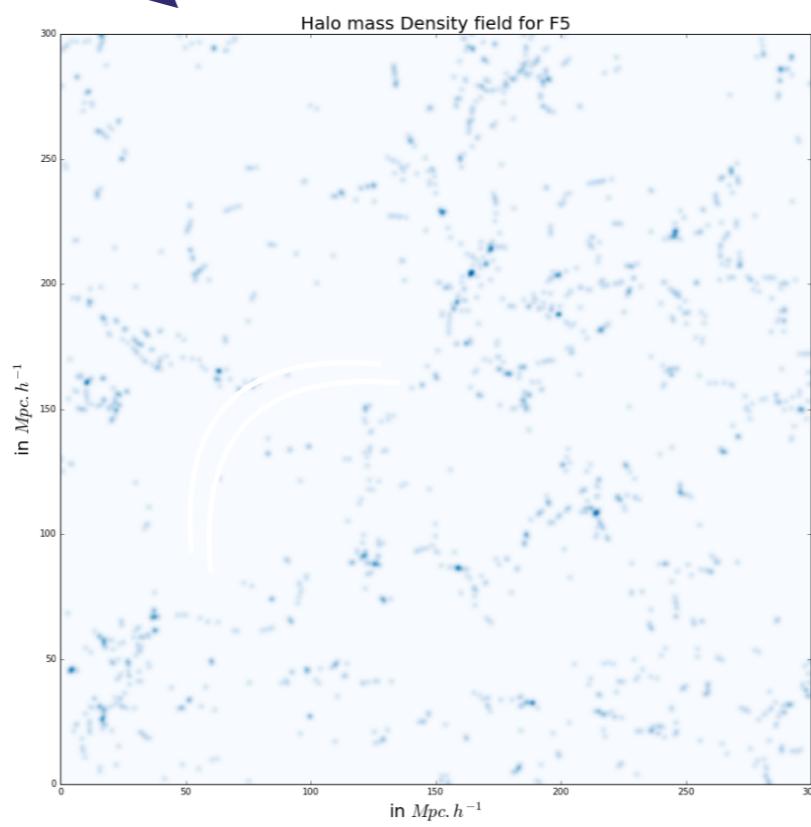


**ROCKSTAR**

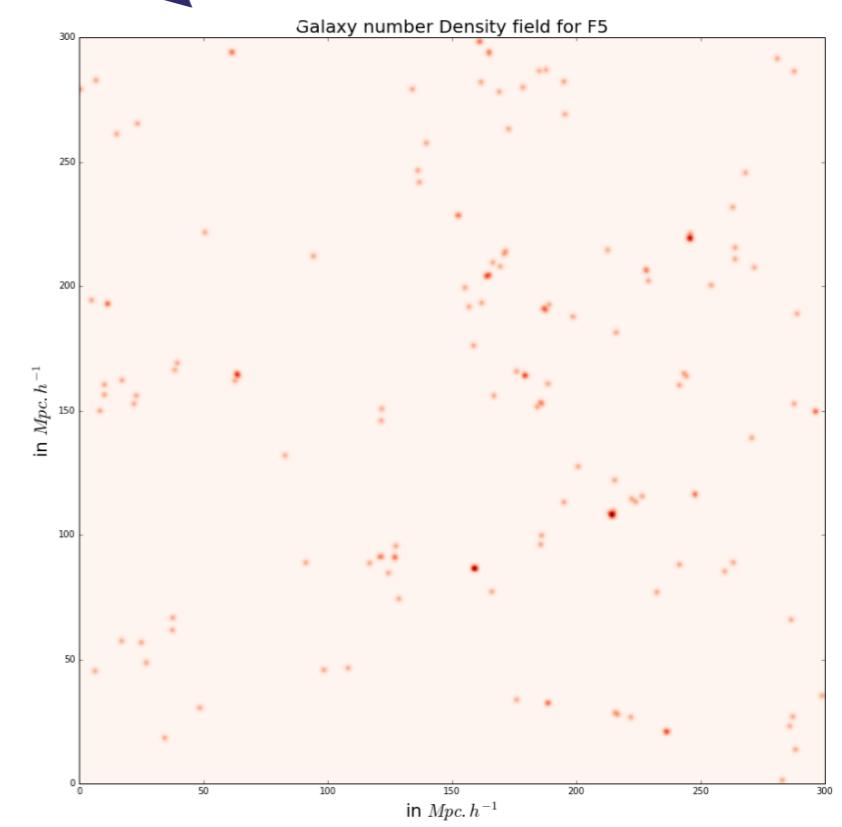
**HOD + CLF**



Dark Matter  
particles

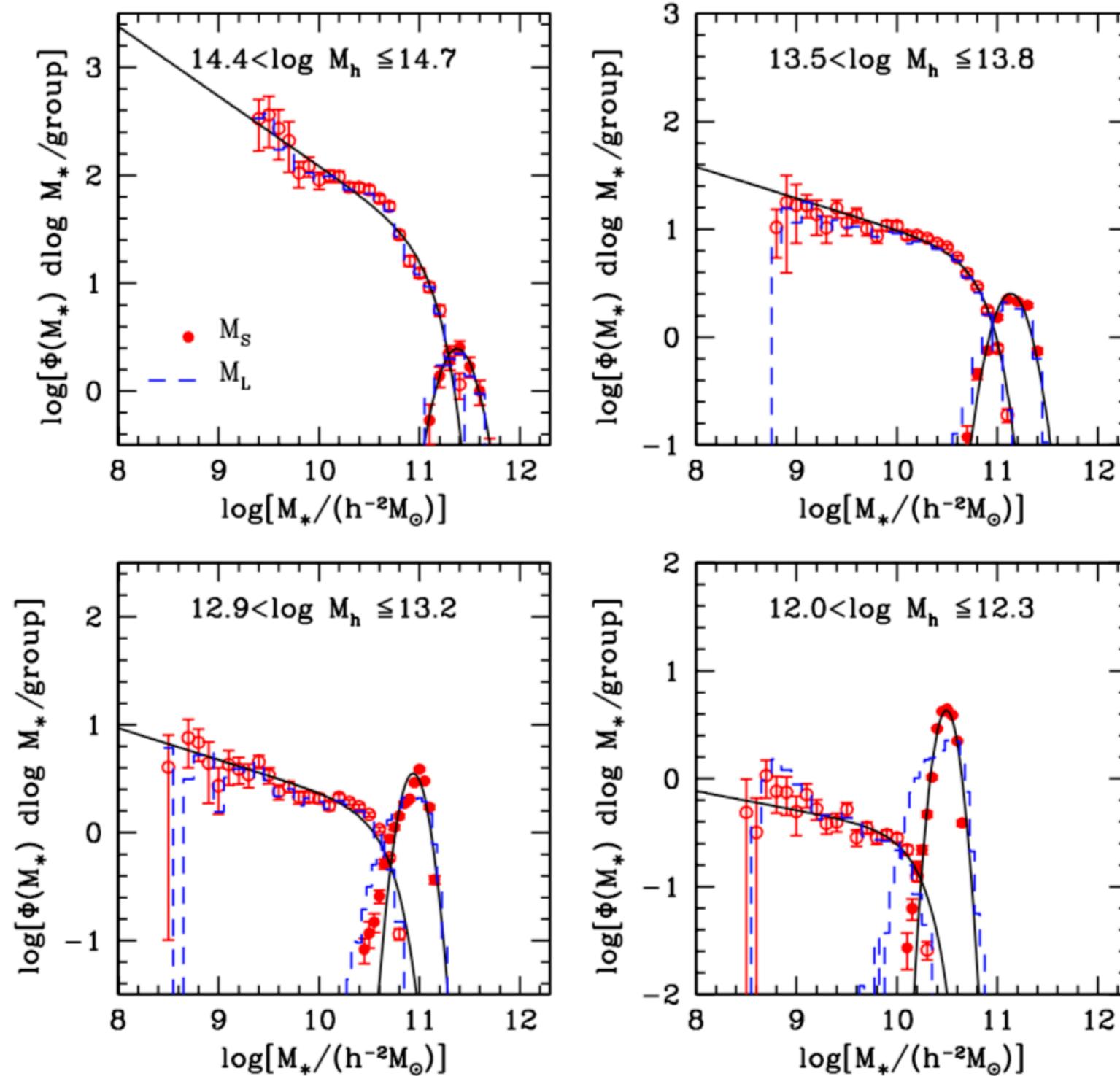


Dark Matter  
Halos

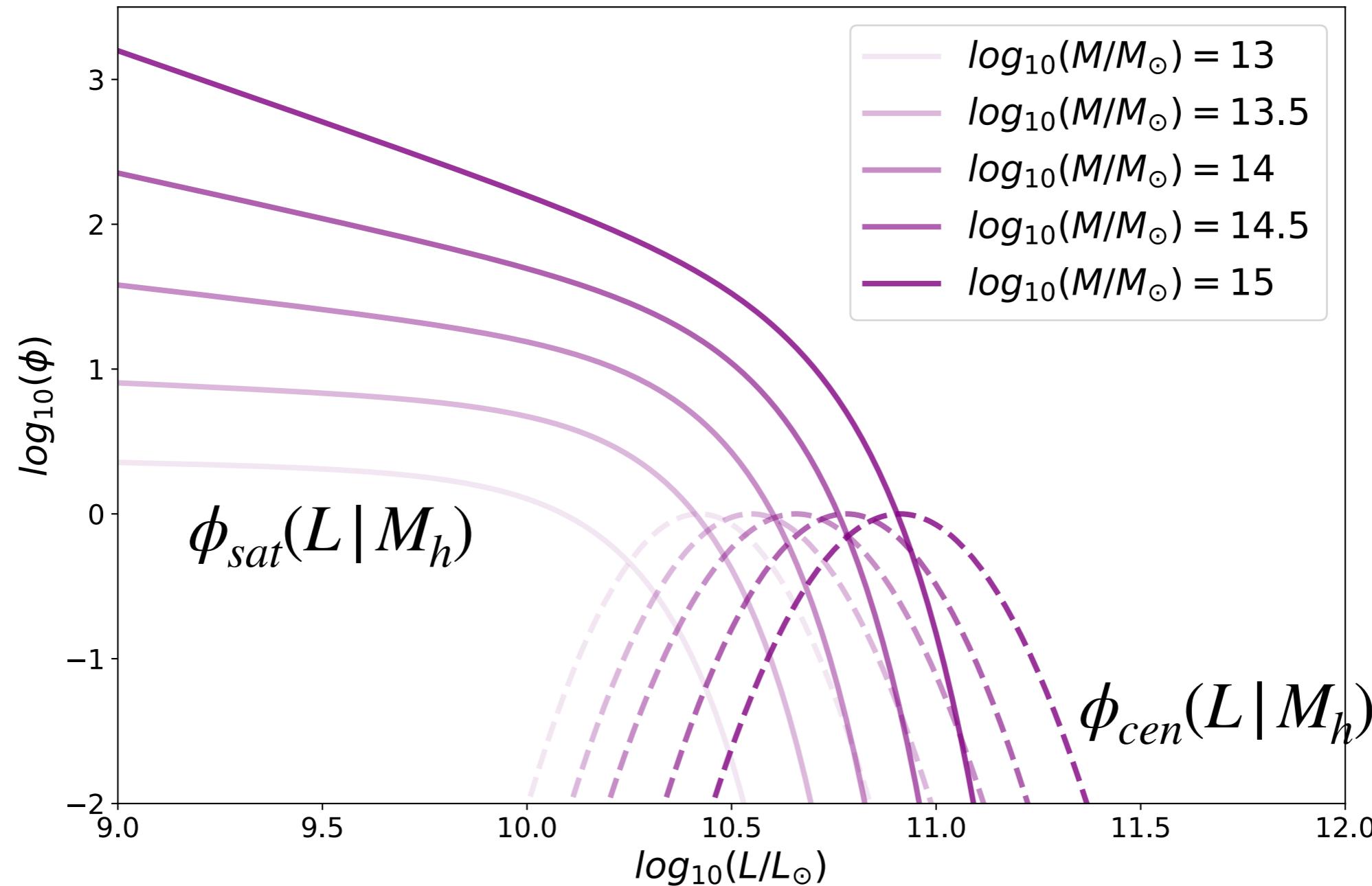


Galaxies with  
Luminosity

# Conditional Luminosity Function (Yang, Mo, Van den Bosch 2009)



# Conditional Luminosity Function



# Conditional Luminosity Function

Satellite density as modified Schechter function:

$$\phi_{sat}(L | M_h) = \phi_s^* \left( \frac{L}{L_s^*} \right)^{(\alpha_s^*)} \exp \left\{ - \left( \frac{L}{L_s^*} \right)^2 \right\}$$

Follow the sub-halo mass function

Central density as modified log-normal function:

$$\phi_{cen}(L | M_h) = \frac{1}{\sqrt{2\pi}\sigma_c} \exp \left\{ -\frac{(\log L - \log L_c)^2}{2\sigma_c^2} \right\}$$

Canibal evol.  
Can be at the  
most one.

# Reverting the information

---

$$\mathcal{P}(M_h | L) = \frac{\mathcal{P}(L | M_h) \cdot \mathcal{P}(M_h)}{\mathcal{P}(L)}$$

**POSTERIOR WE WANT TO EVALUATE**

**CONDITIONAL LUMINOSITY FUNCTION**

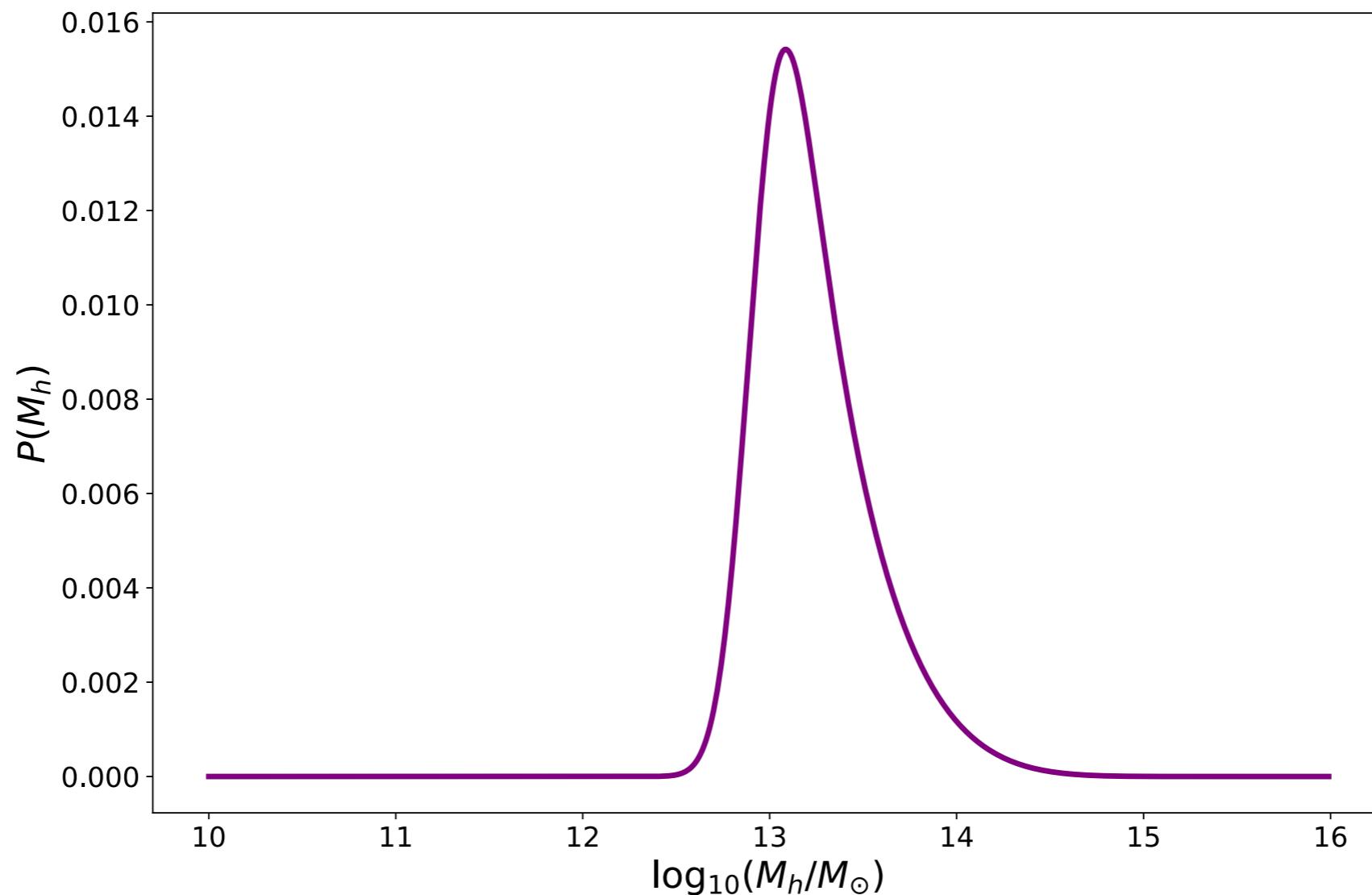
**CONVOLUTION OF HOD AND HMF**

**NORMALIZATION WHICH HAVE NOT IMPACT HERE**

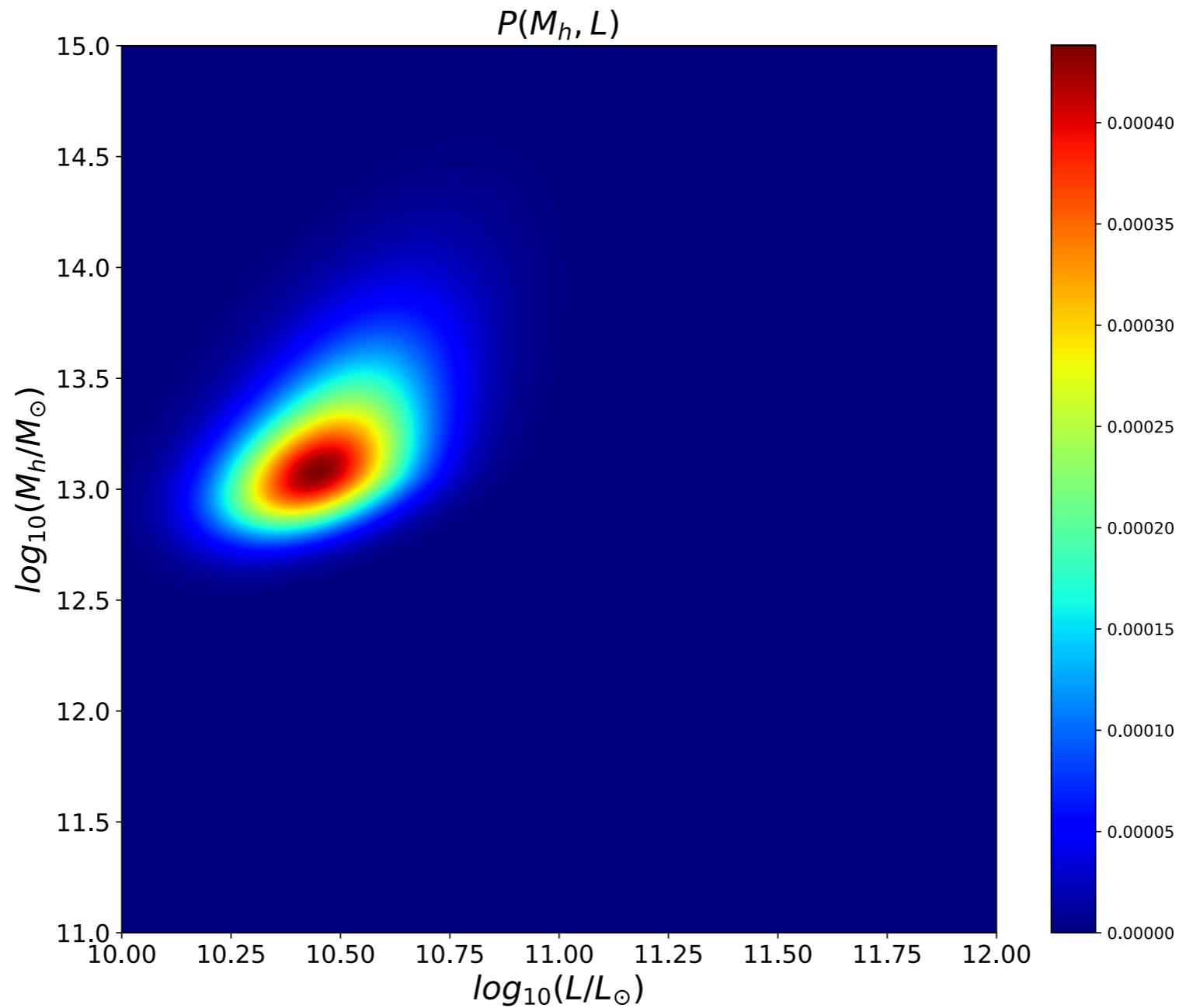
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# Prior estimation

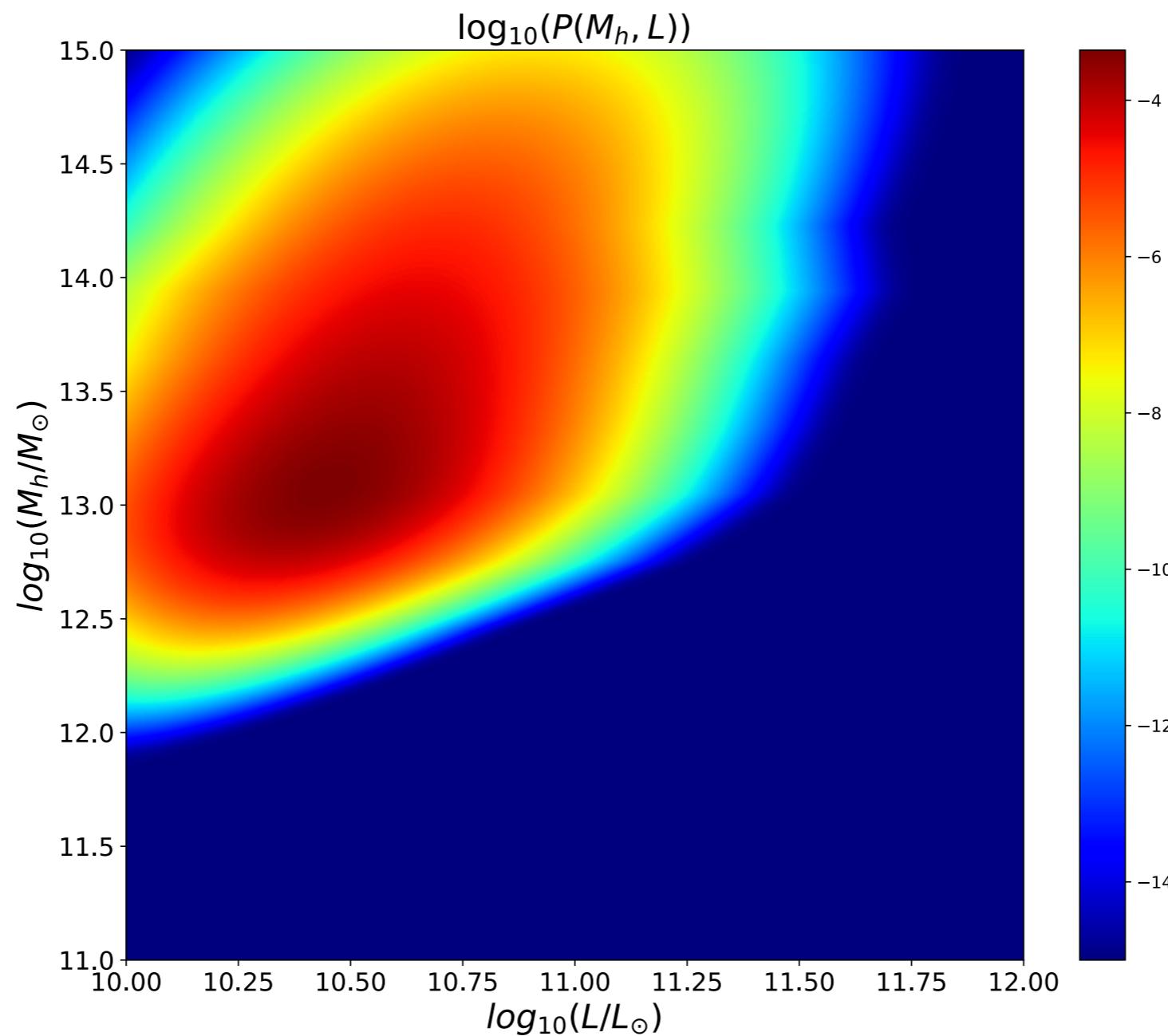
$$\mathcal{P}(M_h) = \frac{\mathcal{P}(N_{gal} \geq 1 | M_h) \times \frac{dn(M_h)}{d \log M}}{\int_0^{\infty} d \log M \mathcal{P}(N_{gal} \geq 1 | M_h) \times \frac{dn(M)}{d \log M}},$$



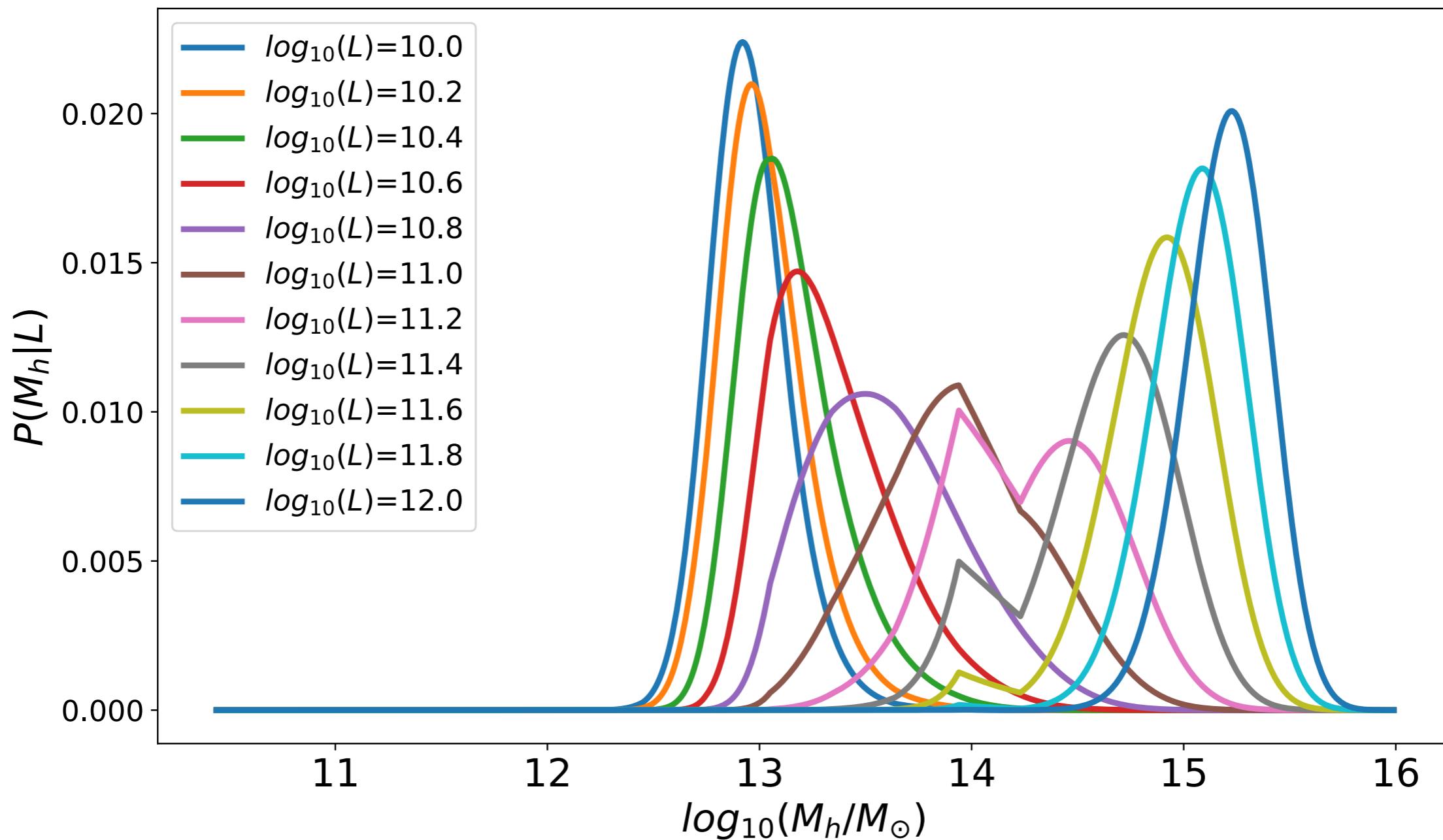
# Joint Probability



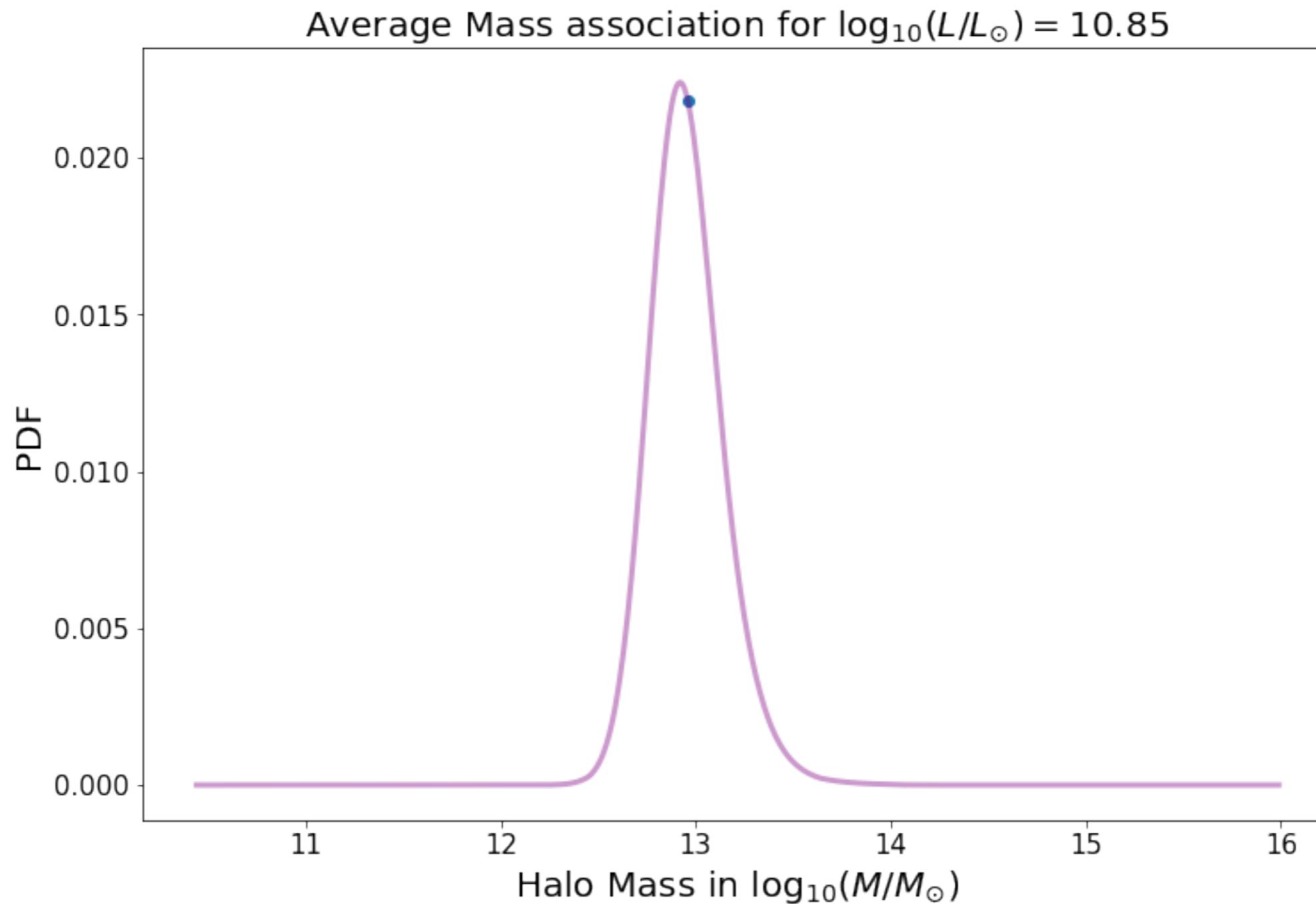
# Joint Probability



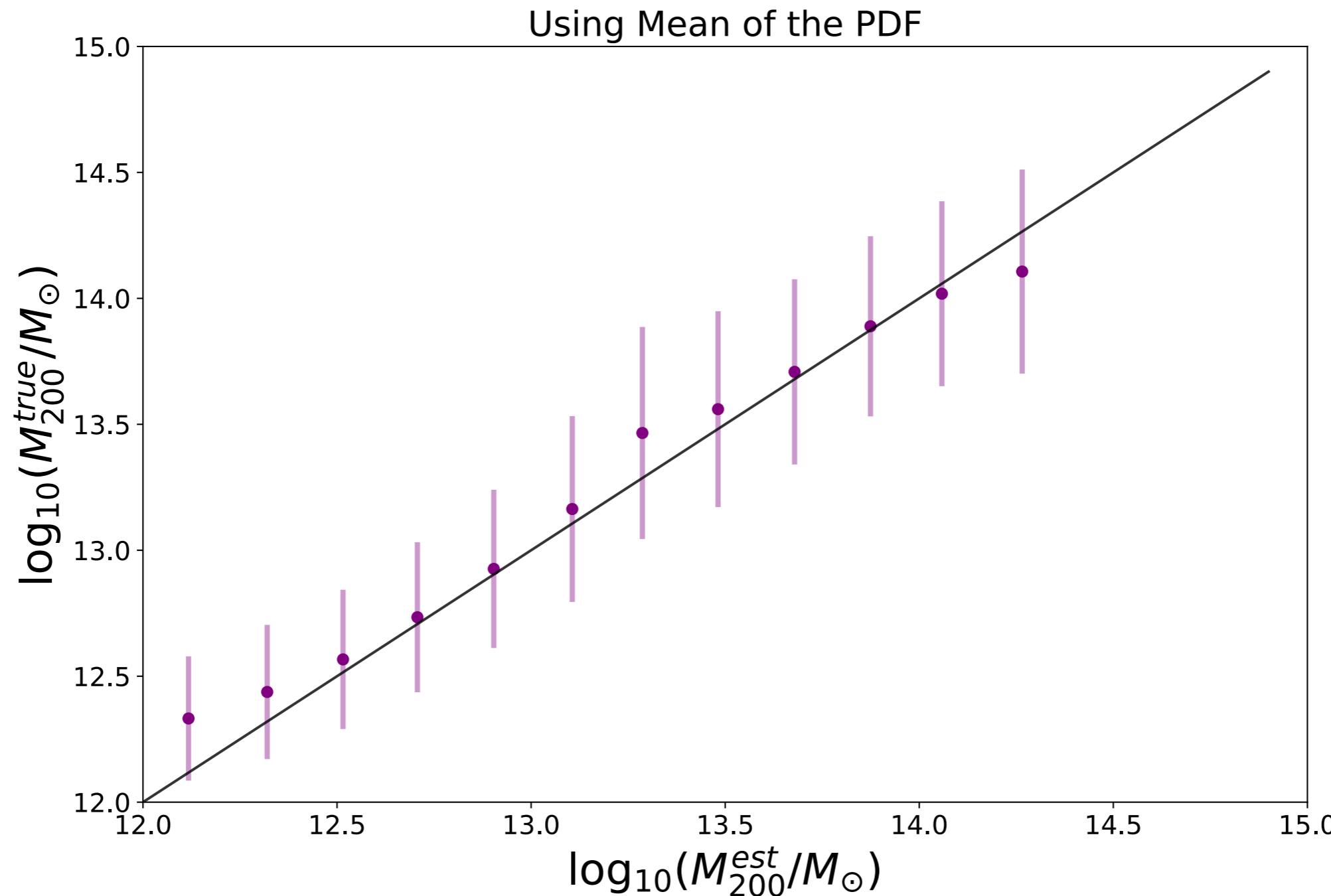
# Posterior Results



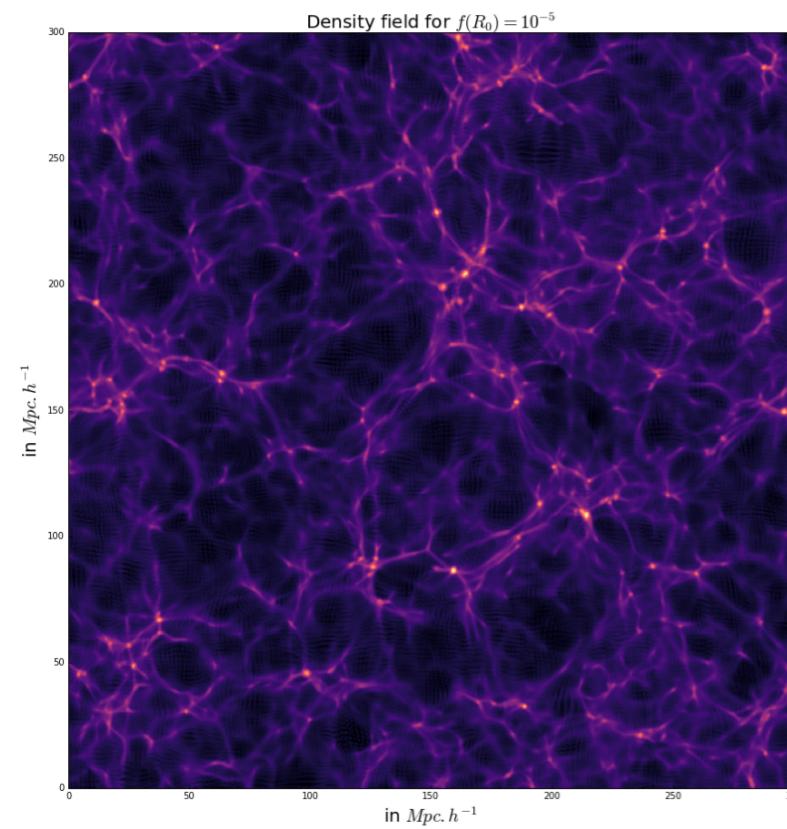
# Average Mass



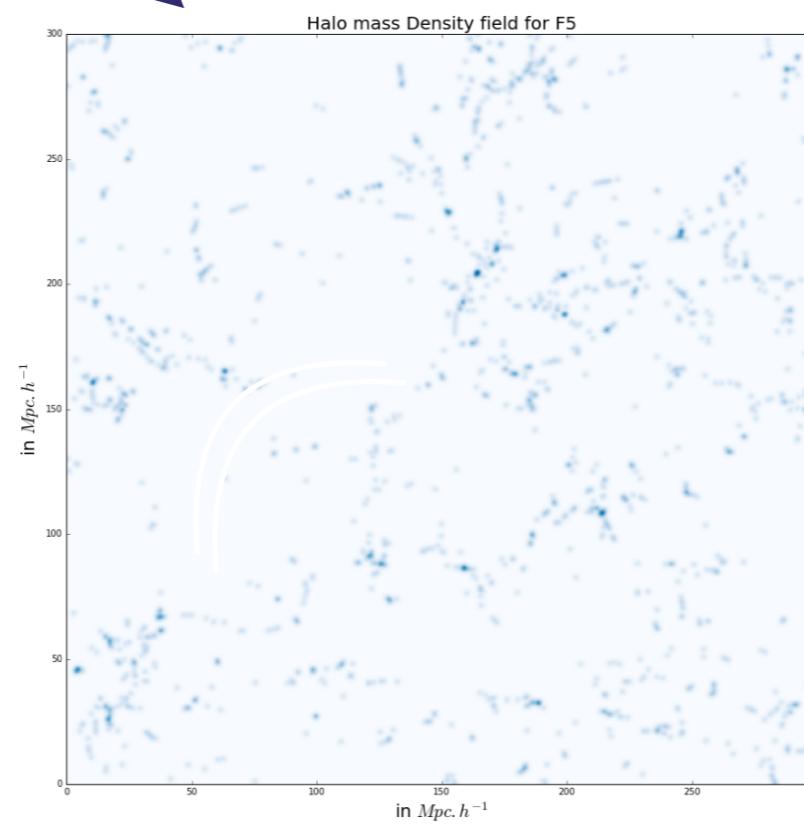
# Estimation of the mass using L In simulation



**ROCKSTAR**

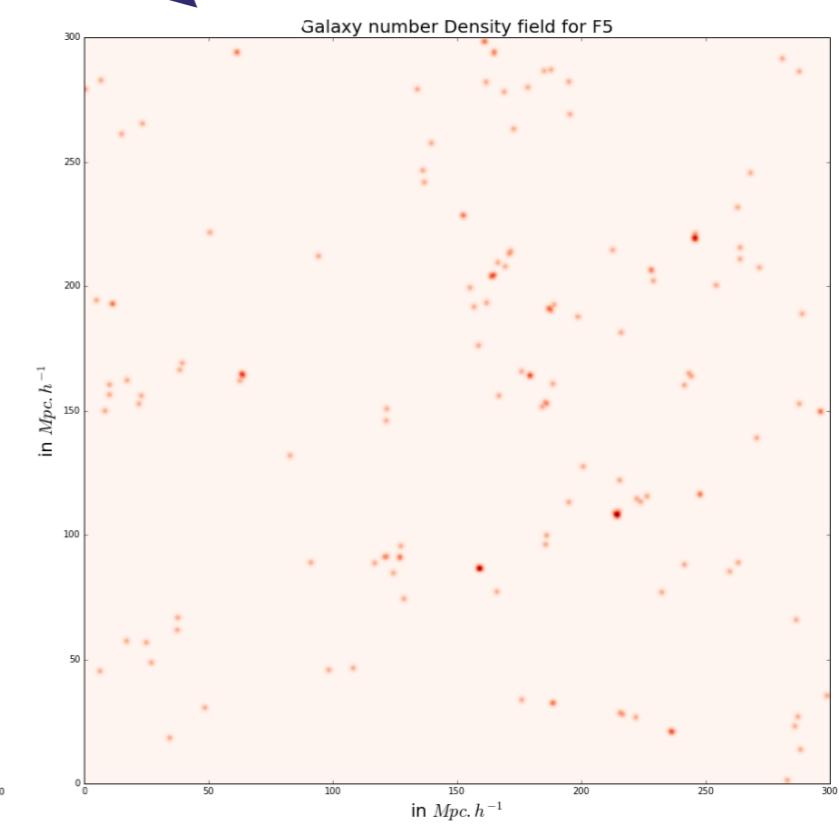


Dark Matter  
particles

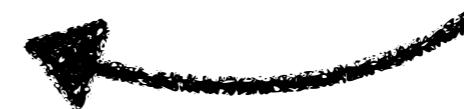


Dark Matter  
Halos

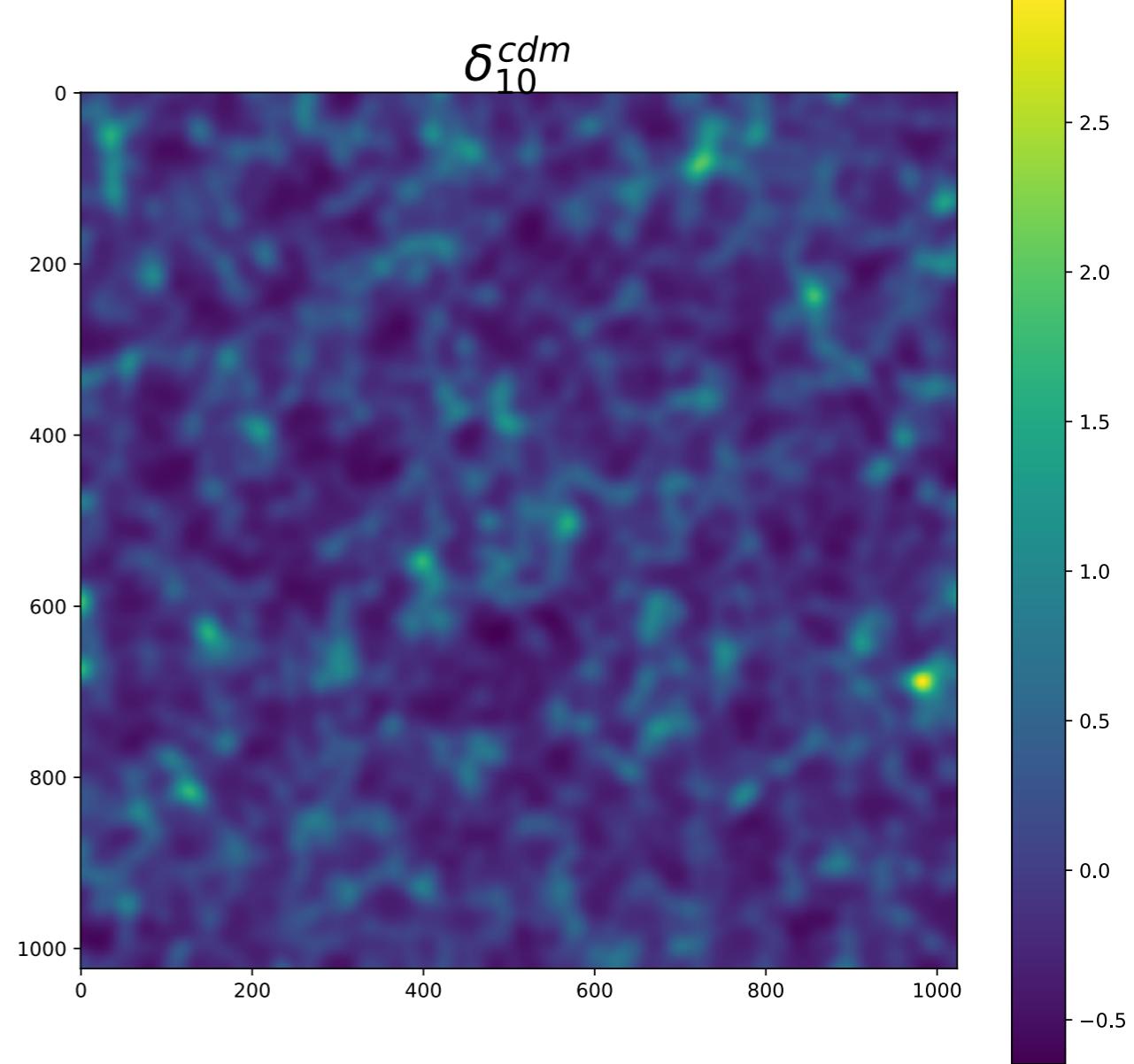
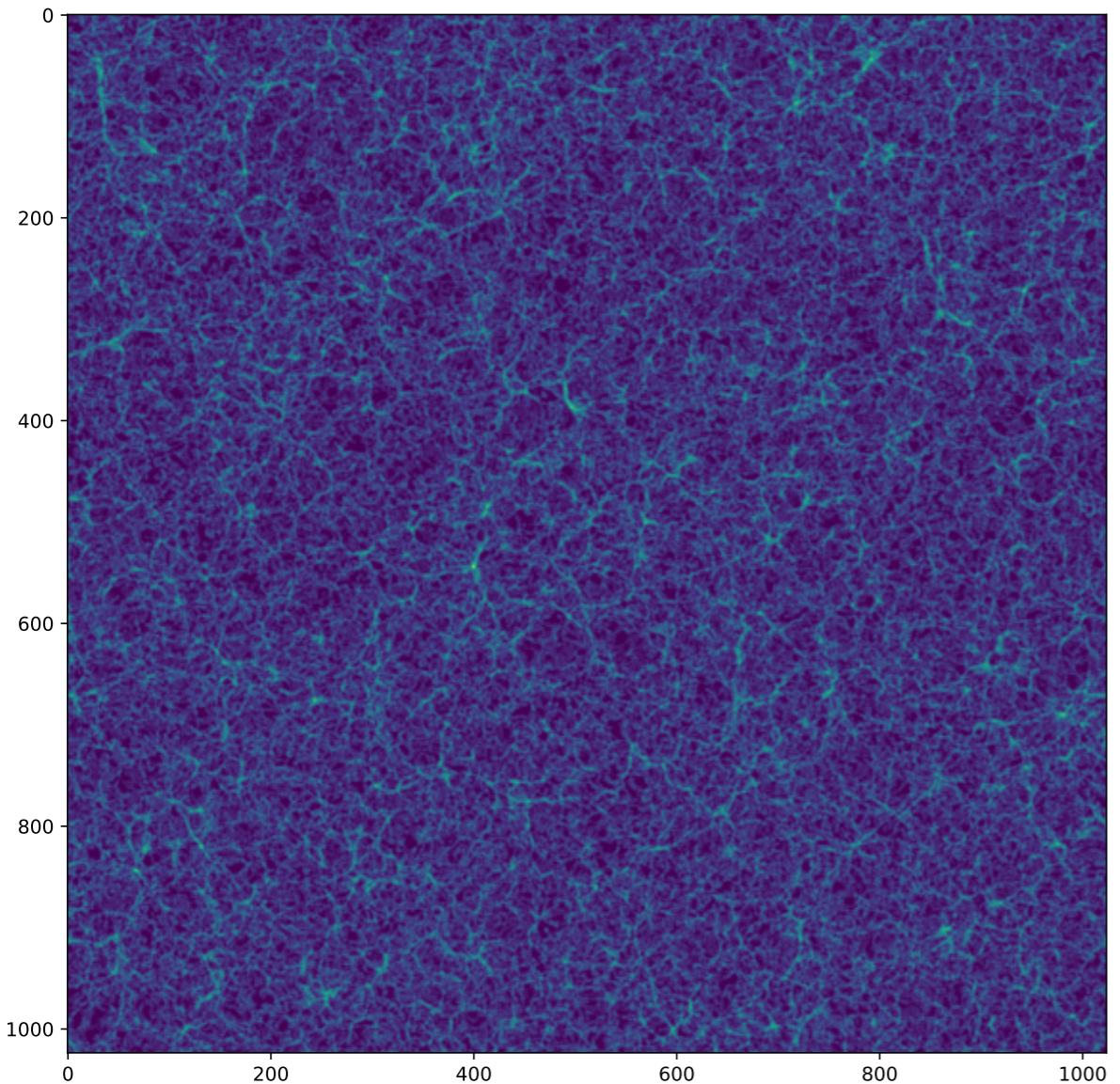
**HOD + CLF**



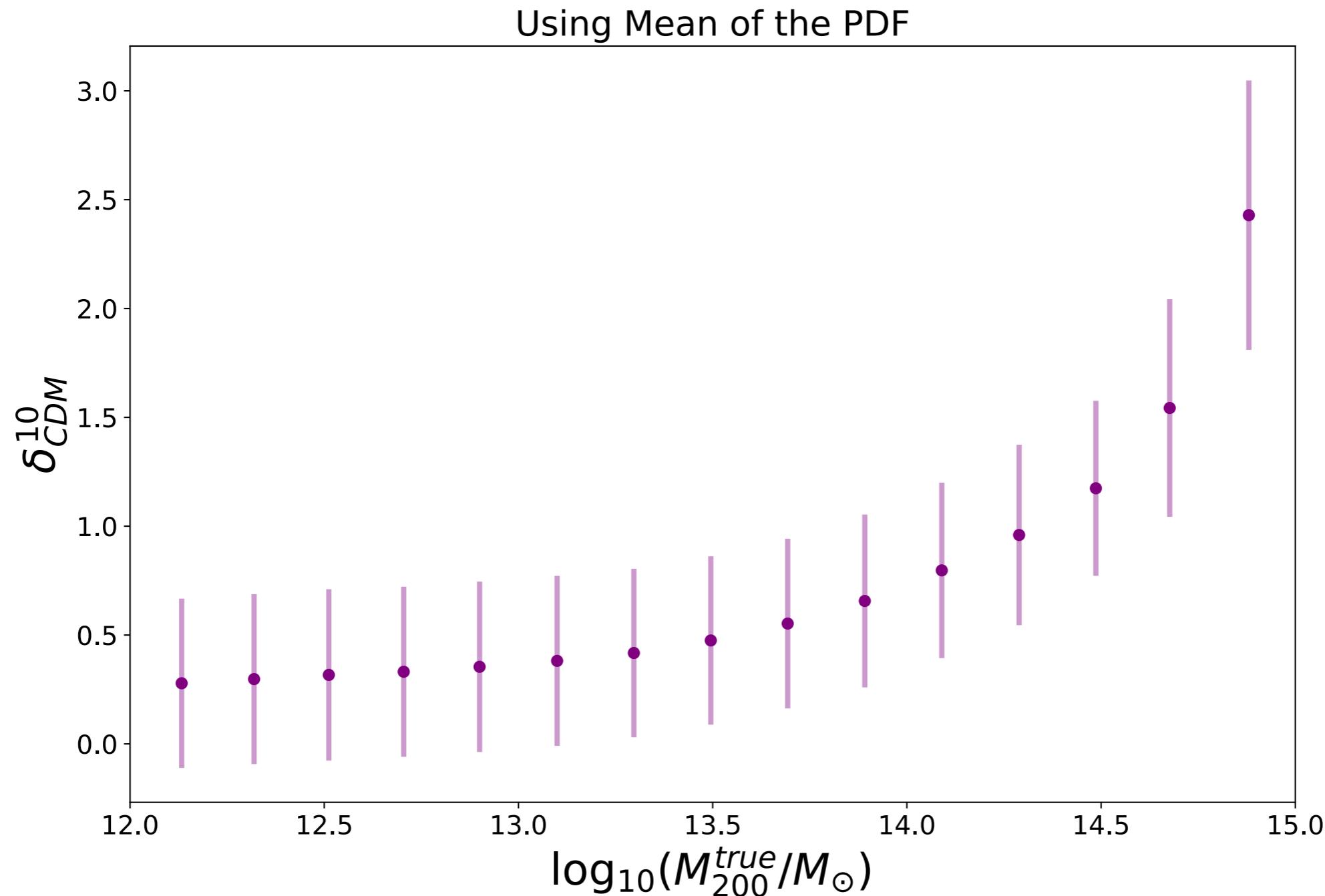
Galaxies with  
Luminosity



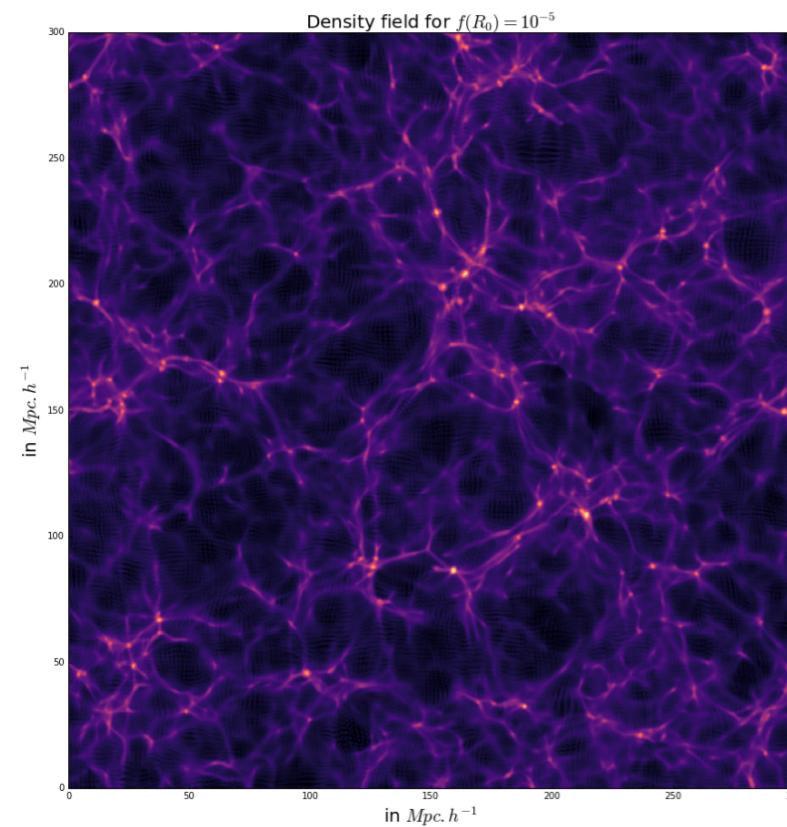
# Gaussian filter at 10 Mpc



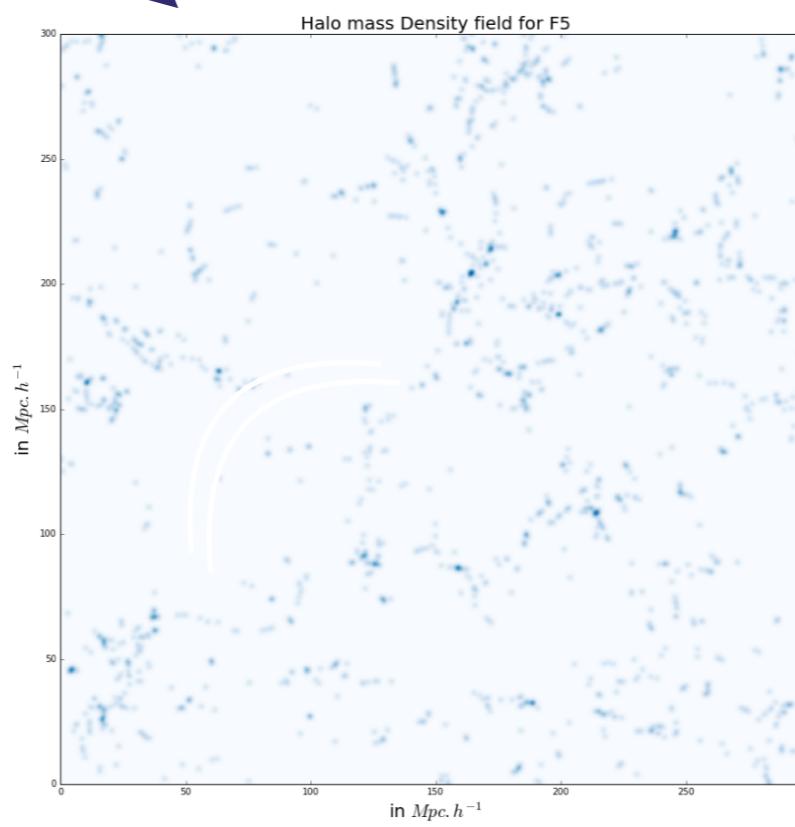
# Gaussian filter at 10 Mpc



**ROCKSTAR**

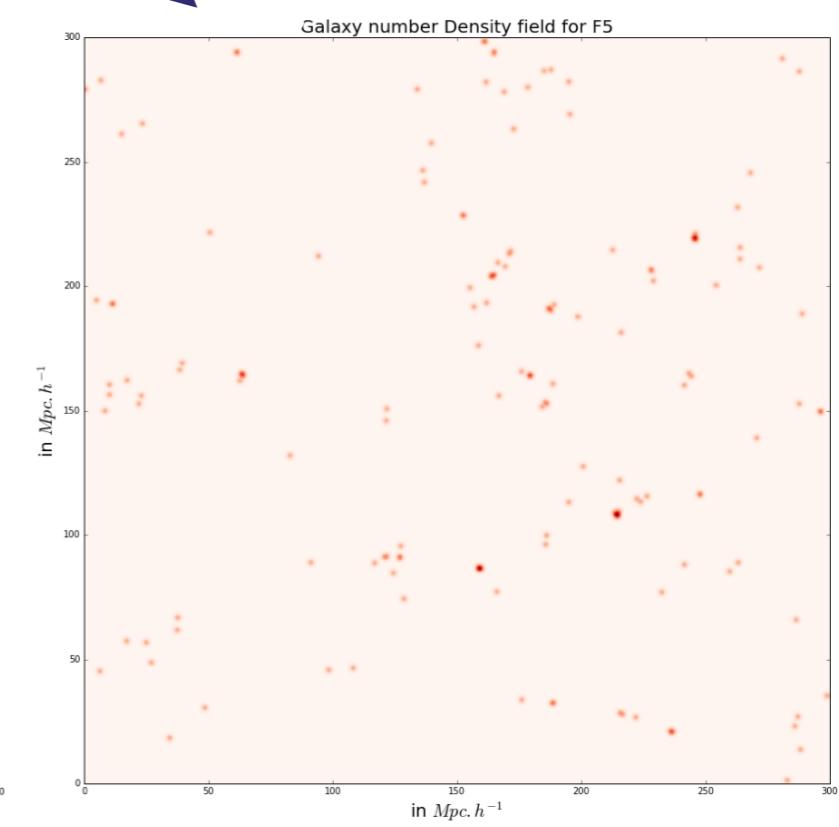


Dark Matter  
particles

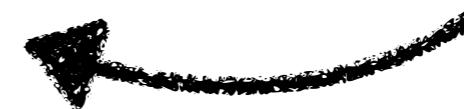


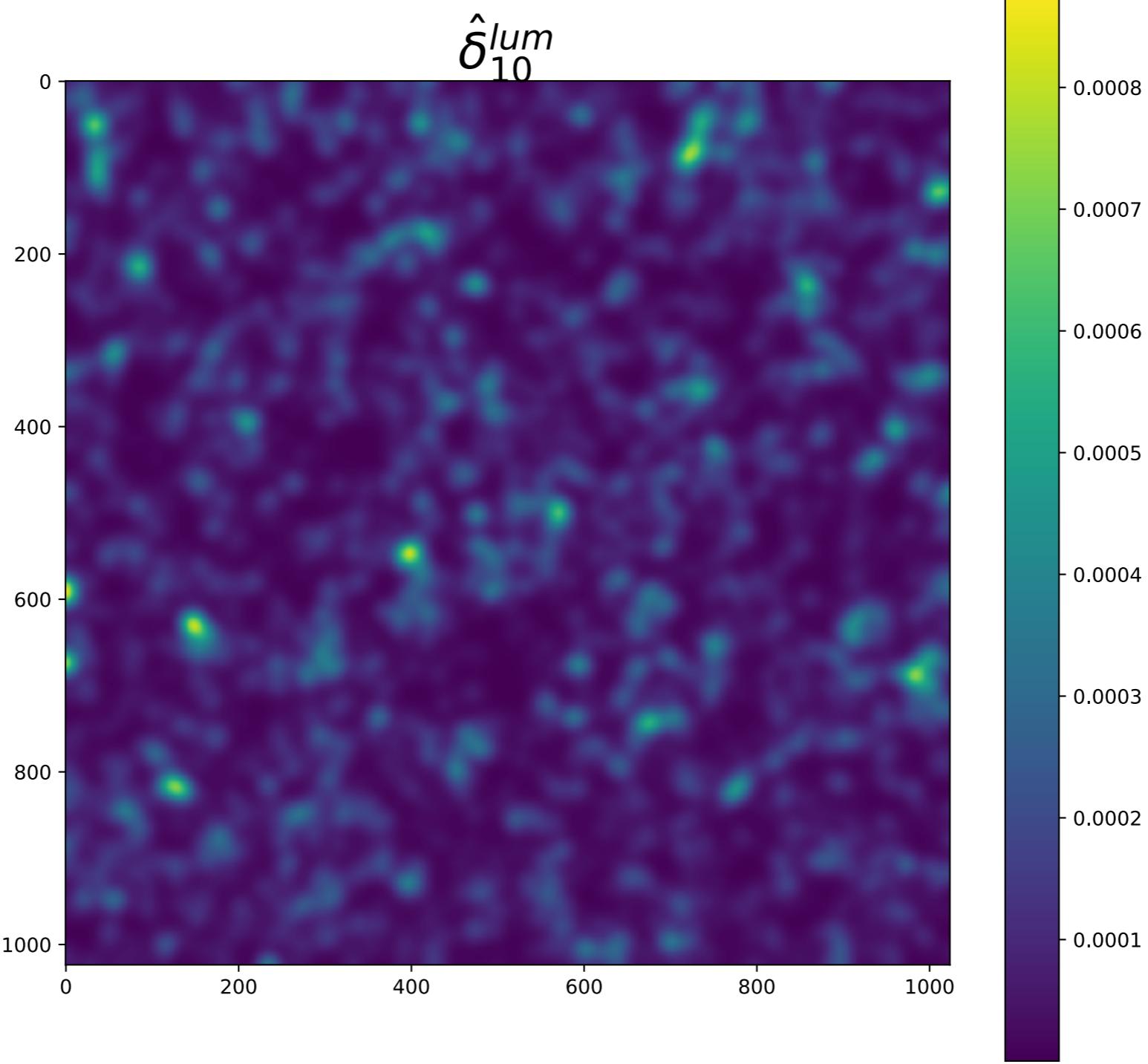
Dark Matter  
Halos

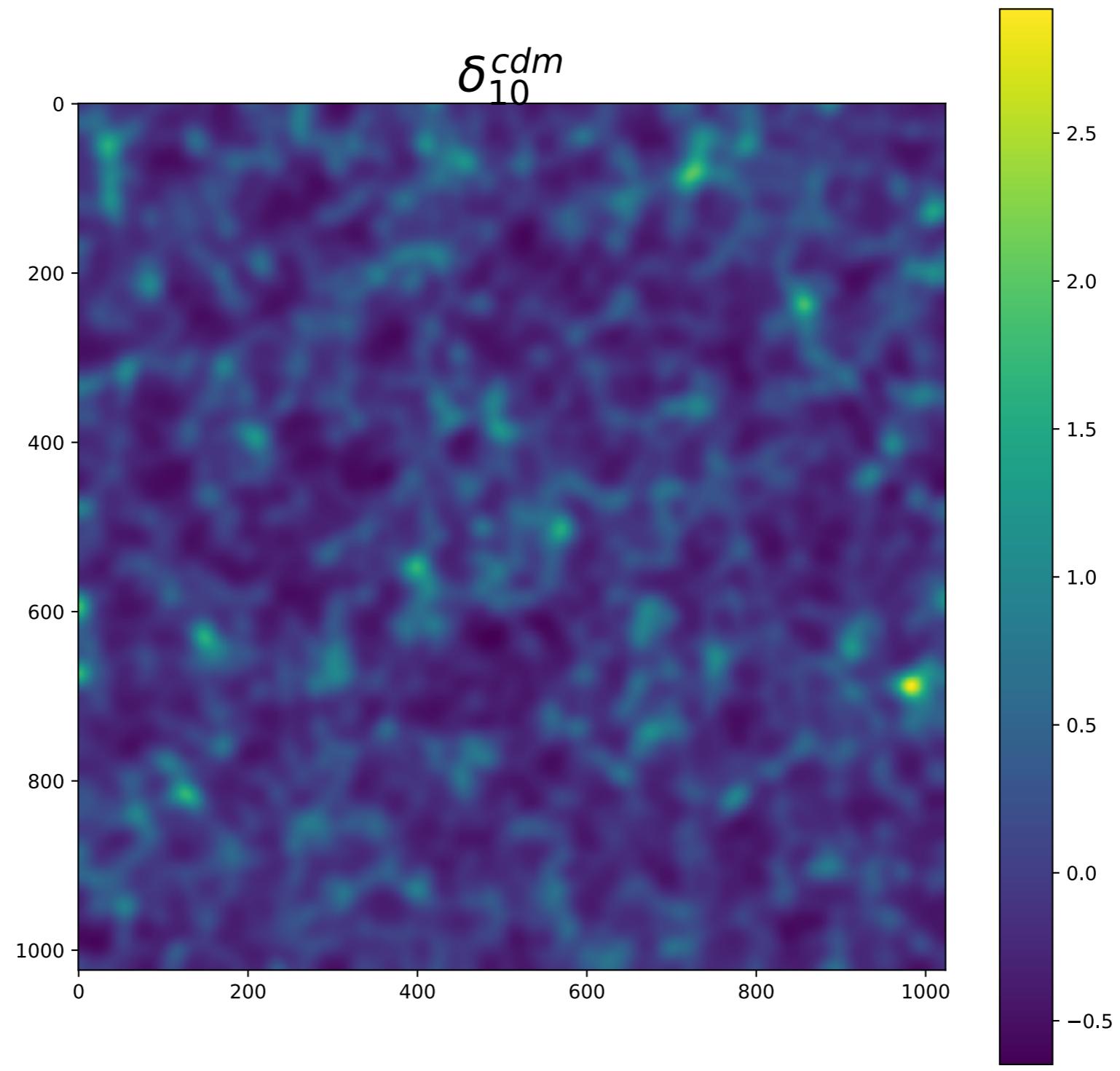
**HOD + CLF**



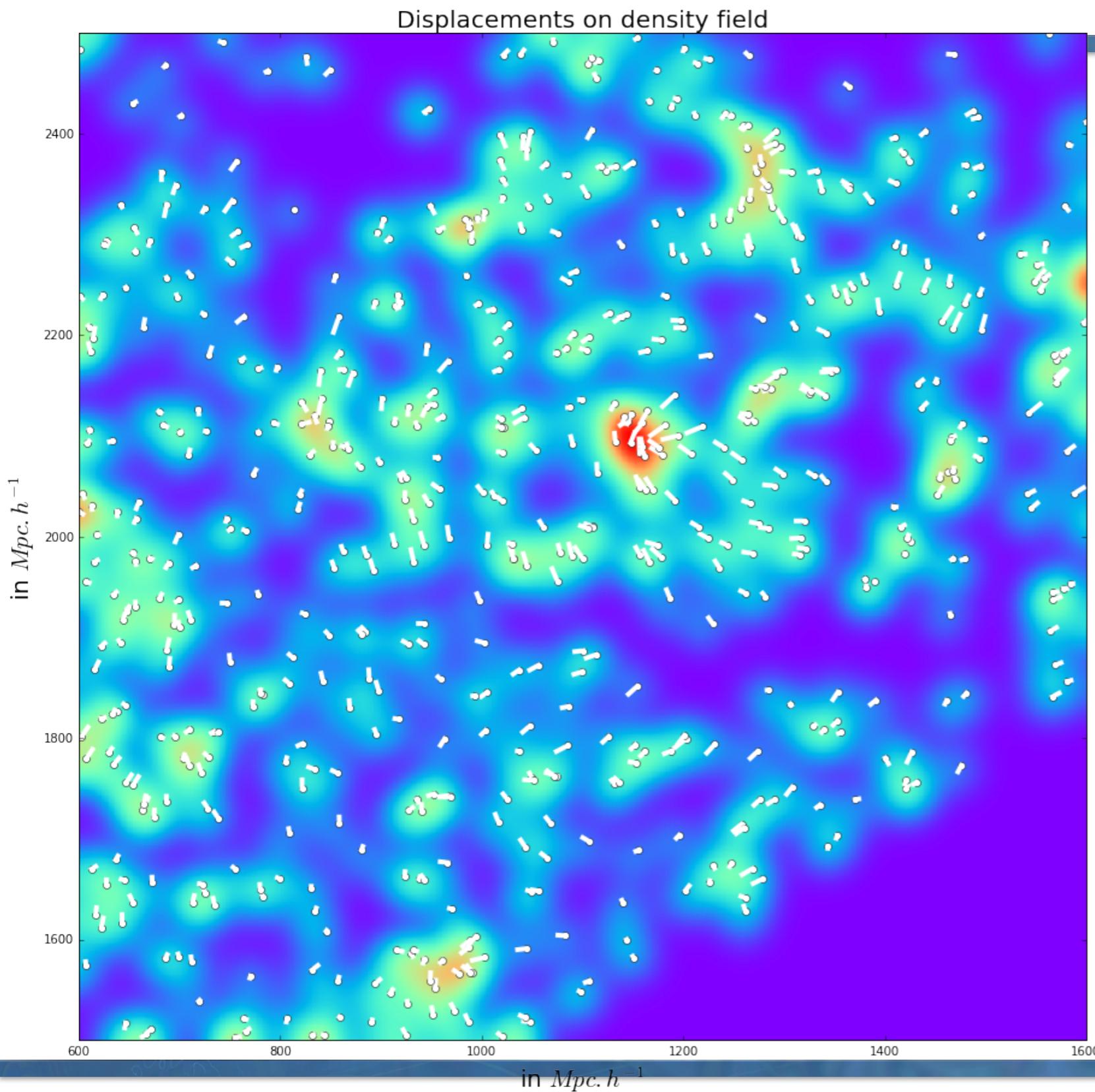
Galaxies with  
Luminosity







# Reconstruction to enhance BAO peak



Vargas, Ho, Fromenteau, Cuesta (2017)

We estimate the smoothed galaxy density field



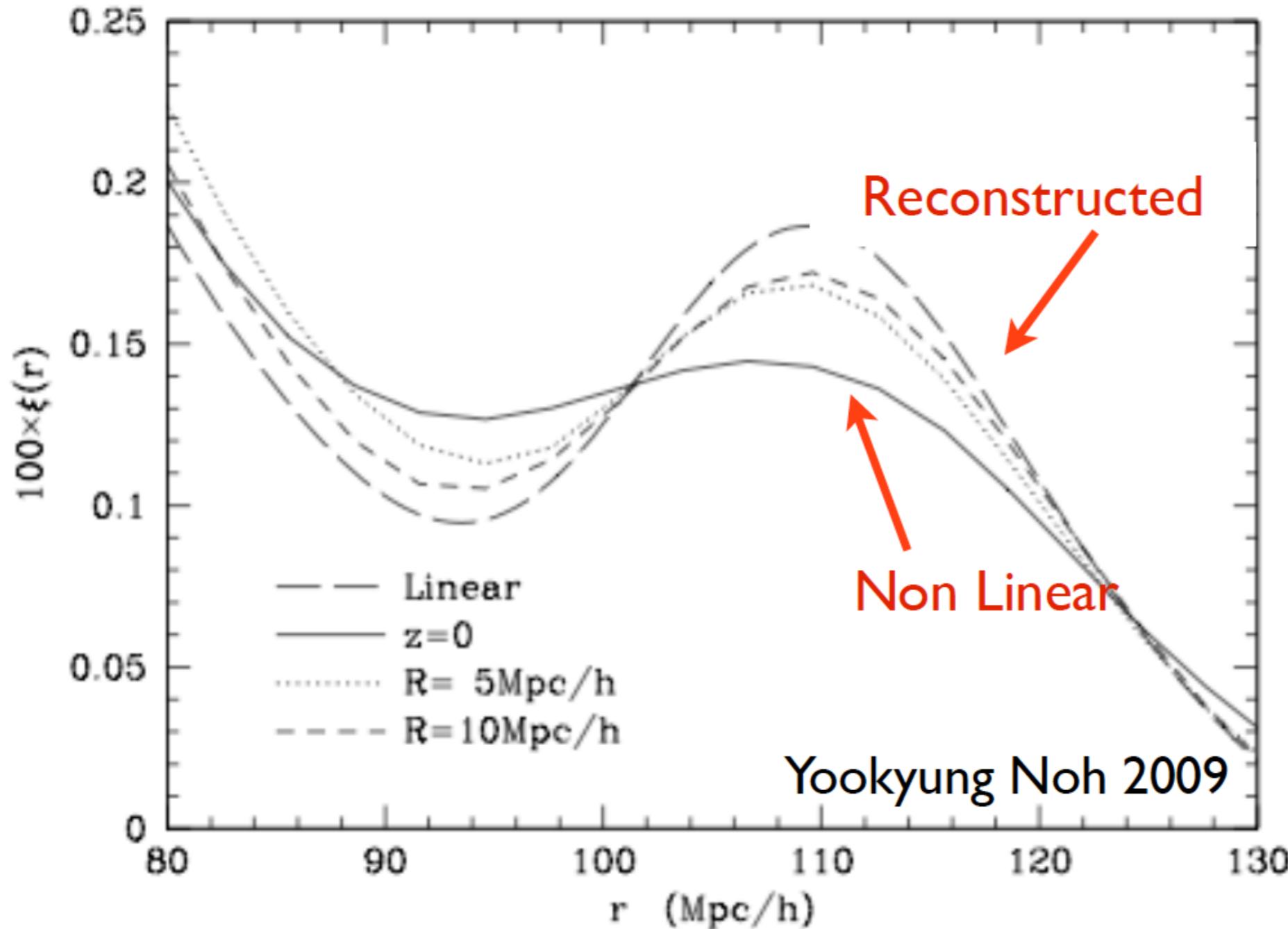
Using linear bias, we derive the matter density field



We move back the galaxies to their original position

$$\vec{\psi}(\vec{k}) = \frac{-i\vec{k}}{k^2} \frac{\delta_g(\vec{k})}{b_g}$$

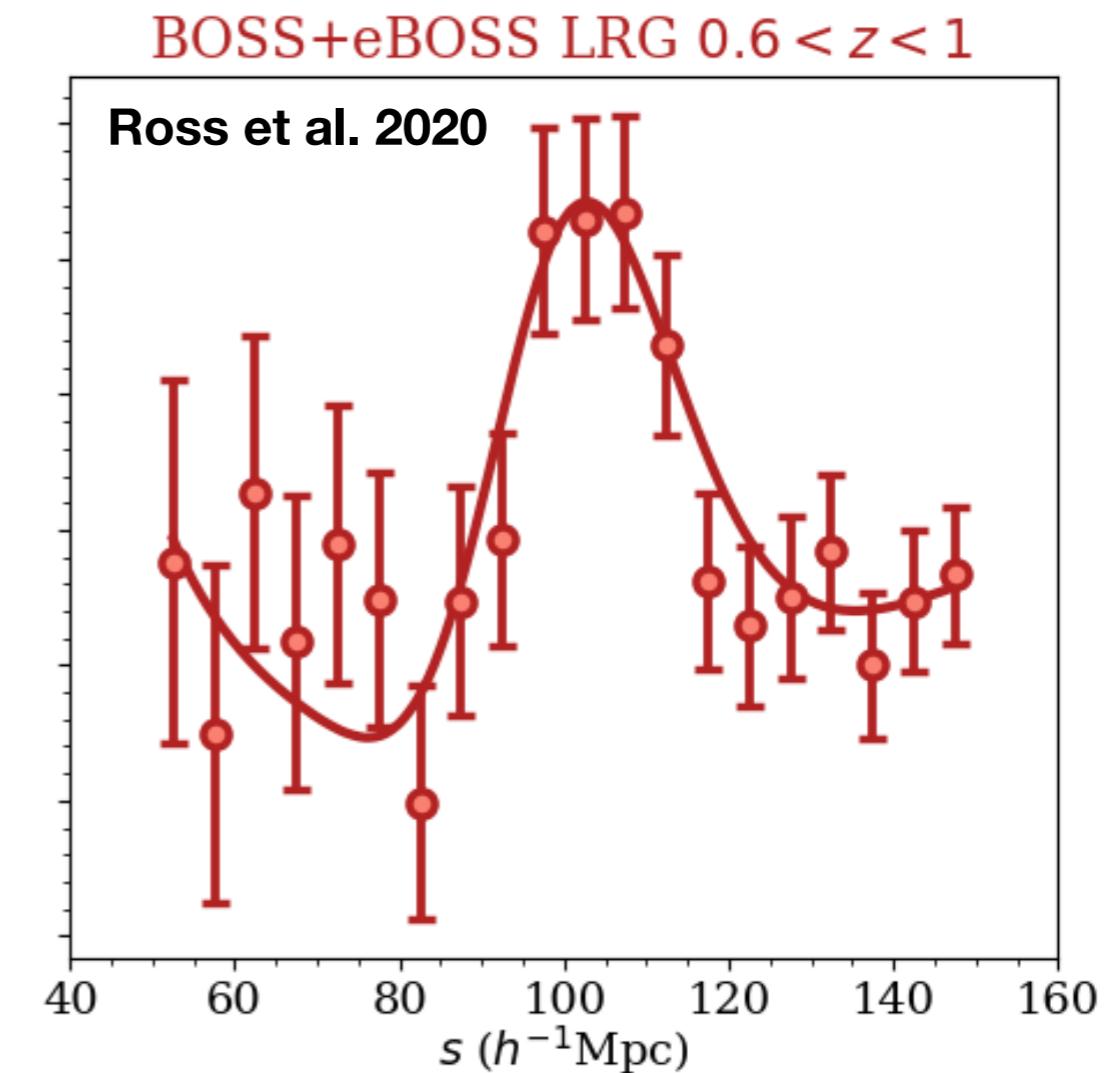
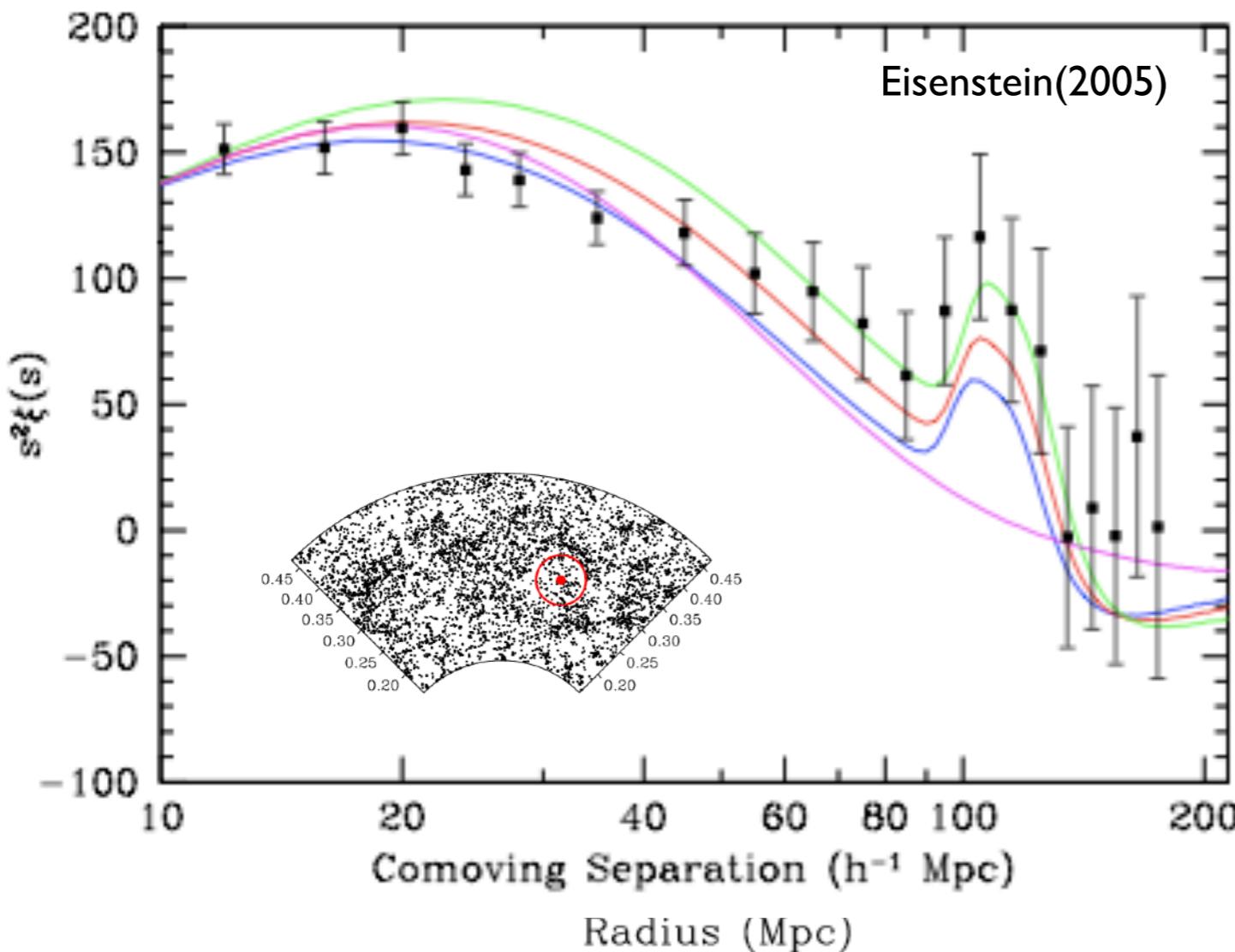
# Reconstruction to enhance BAO peak



Is part of the standard  
BAO analysis

# Baryonic Acoustic Oscillations Peak

BAO Detection in the correlation function of Luminous Red Galaxies



Mariana's talk on Saturday

# Conclusions

**Obtain non-linear local bias information using luminosity information in a probabilistic way**

**This method can be generalized to any statistical information connecting the tracer to the halo mass (for example density)**

**We can use this method in order to implement a redshift dependent reconstruction inside the large surveys**

# Different tracers



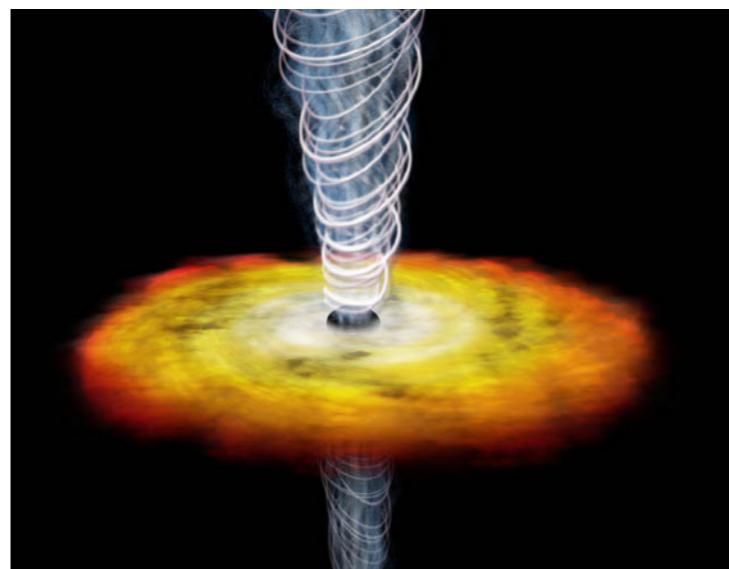
**Luminous Red  
Galaxy**

**LRG**



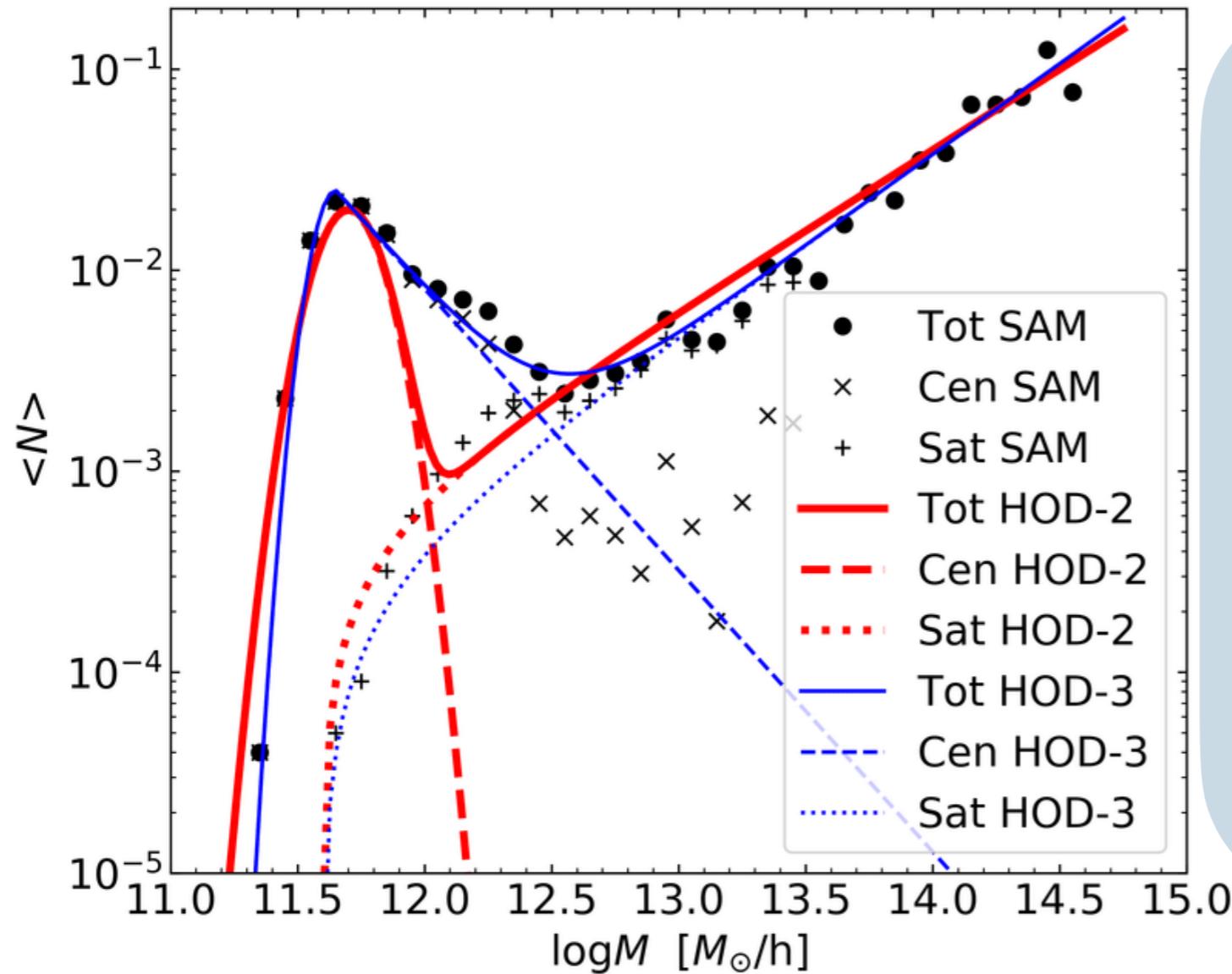
**Emission Line  
Galaxy**

**ELG**



**Quasar**  
**QSO/AGN**

# Emission Line galaxies



Avila et al 2020

**Important new tracer for high redshift**

**The galaxy Halo connection is very complex**

**So galaxy bias is highly non-trivial**



**THANKS !**

# Halo Occupation Distribution

## 1 HALO TERM

$$\xi_{gg}(r) \propto \int d \log(M) \frac{dn}{d \log(M)} \langle N(M) \rangle (\langle N(M) \rangle - 1) P_{NFW}(r | M)$$

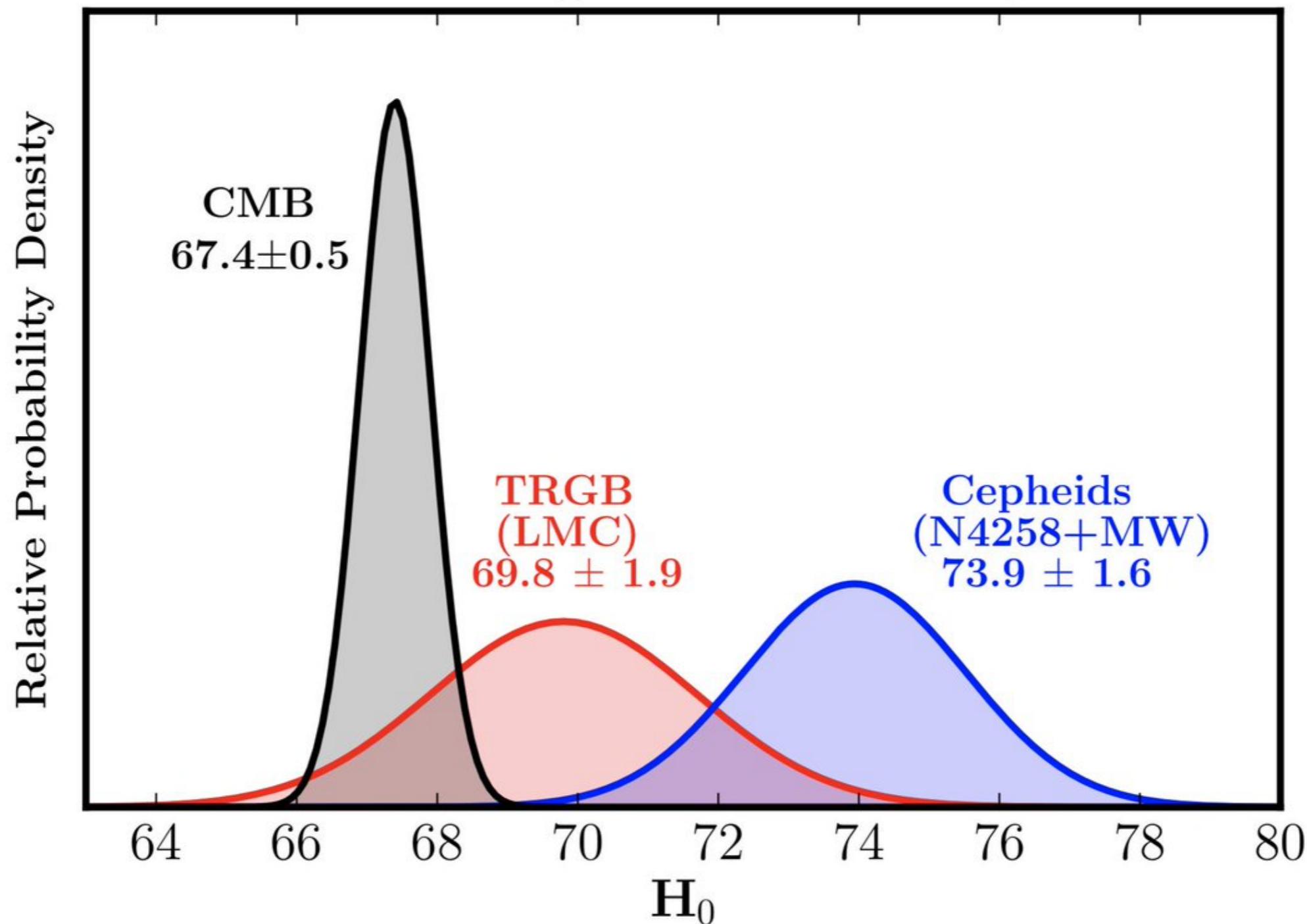
$$+ \int d \log(M_1) \int d \log(M_2) \langle N(M_1) \rangle . \langle N(M_2) \rangle b^2 \xi_{mm}(r) \times \\ \times \frac{P_{NFW}(r_1 | M_1) dn}{d \log(M_1)} \frac{P_{NFW}(r_2 | M_2) dn}{d \log(M_2)}$$

## 2 HALO TERM

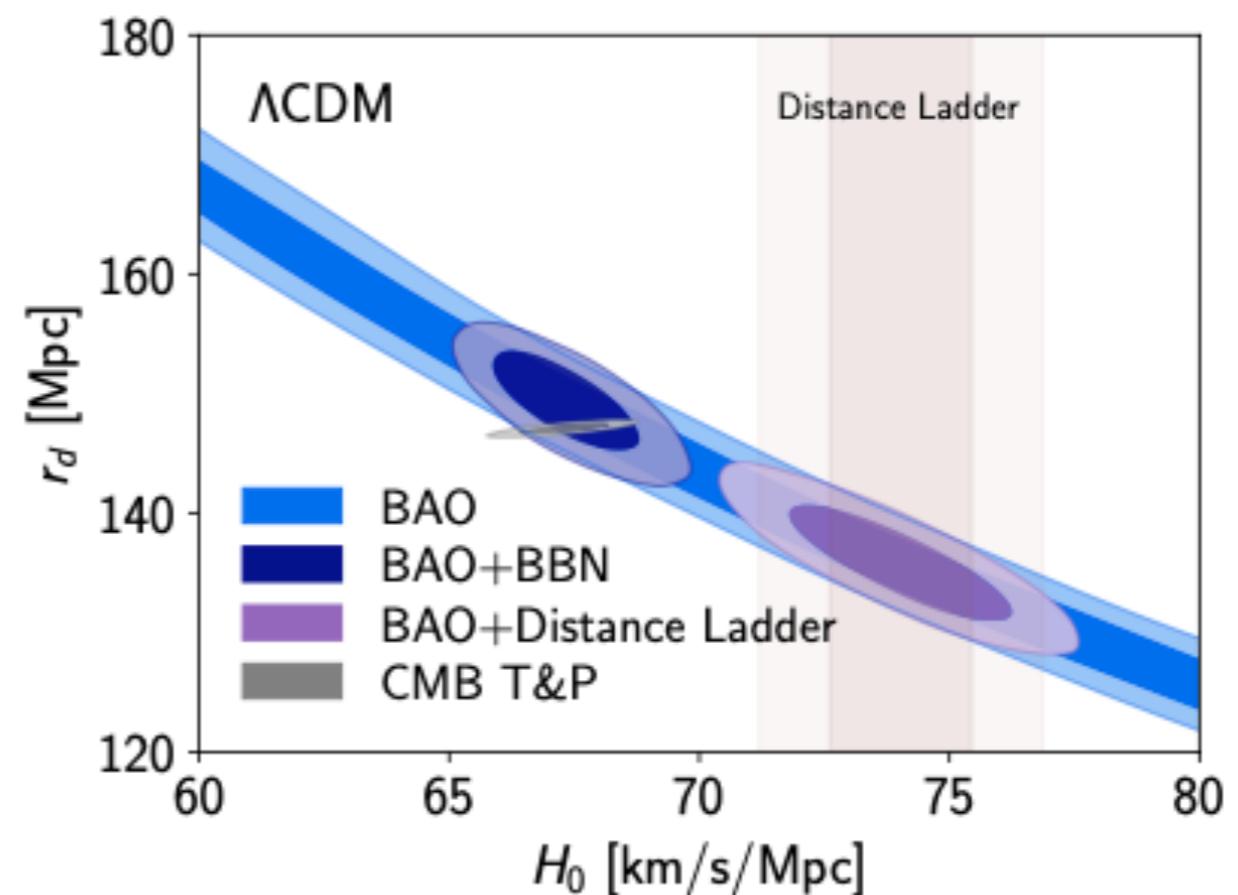
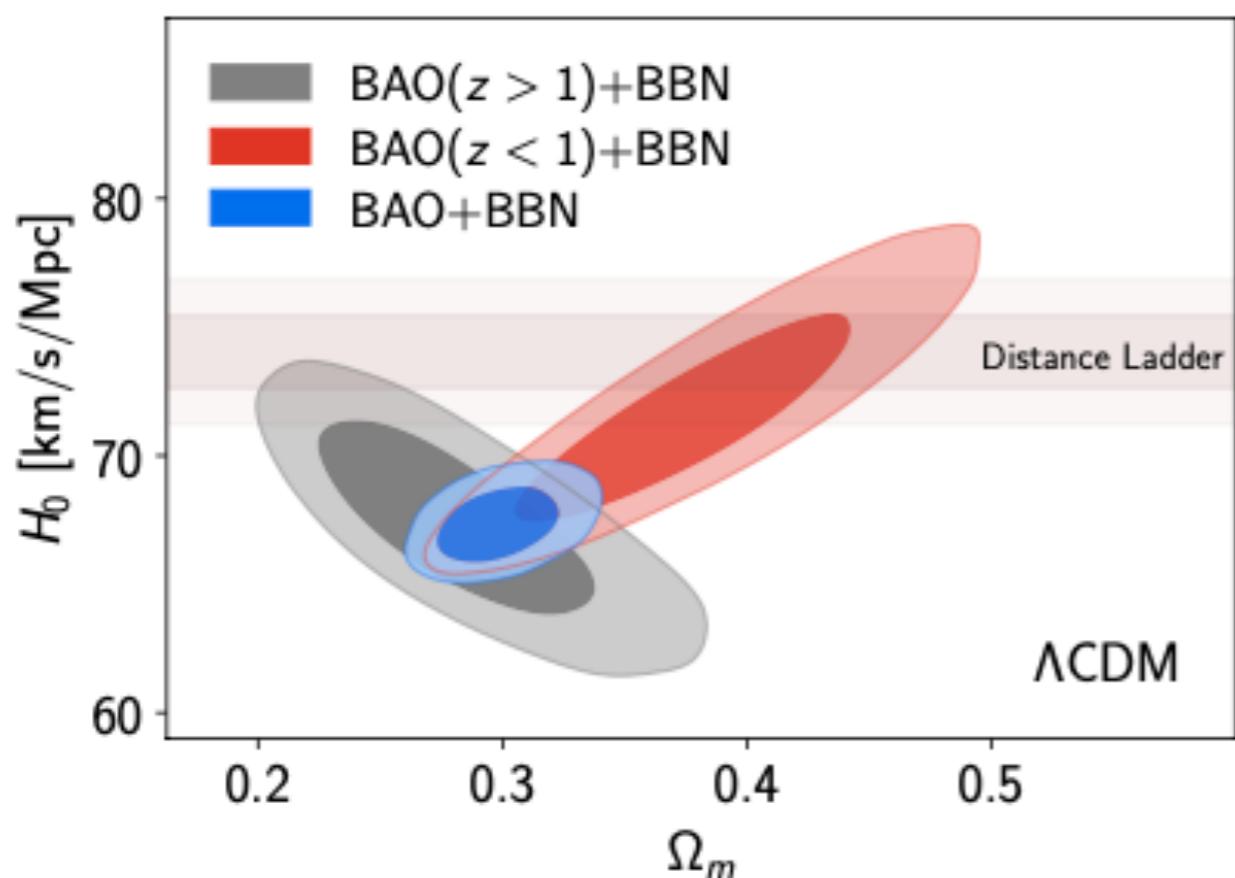
$$P_{NFW}(r | M) = \frac{\rho_{NFW}(r | M) \times 4\pi r^2 dr}{M}$$

# Increase $H_0$ crisis

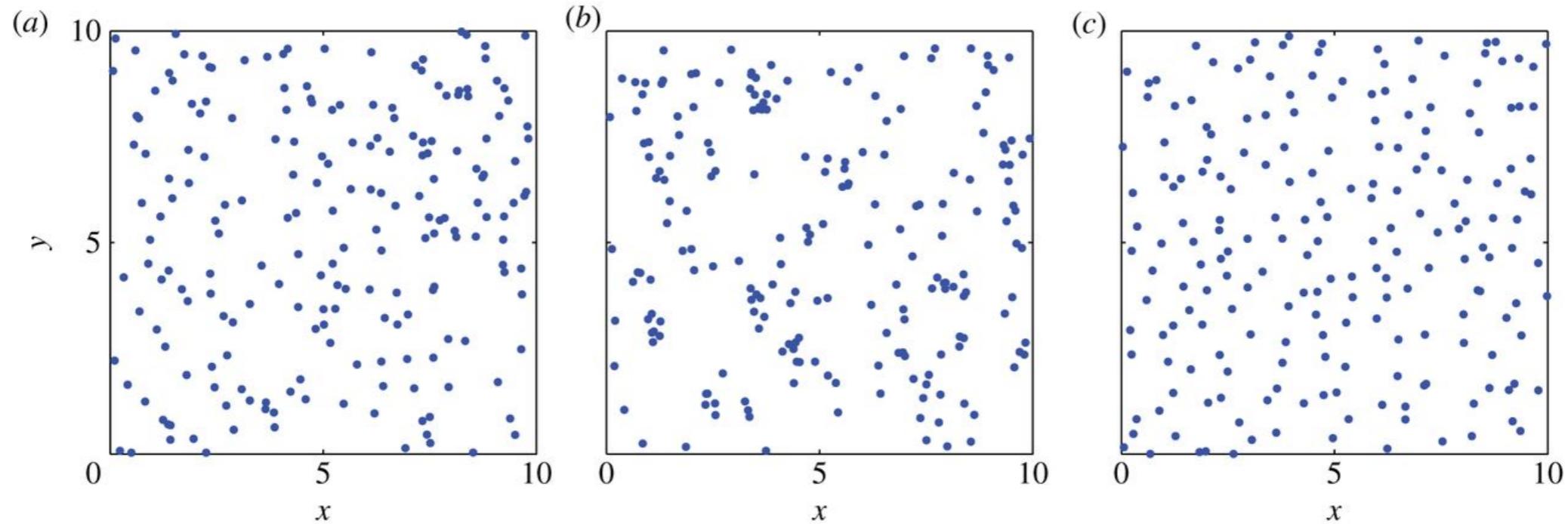
CMB and Independent Local  $H_0$  values



# Increase $H_0$ crisis



# 2-point correlation function



We count the number of pairs of galaxies for each distance  $r$

We compare with the number obtain for random distribution

Estimator

$$\xi(r) = \frac{N_{pairs}(r) - N_{pairs_{rand}}(r)}{N_{pairs_{rand}}(r)}$$

Quantity

$$P(r) = 4\pi r^2 dr \cdot \bar{n} [1 + \xi(r)]$$

# 2-point correlation function

