Holography and transport at strong coupling

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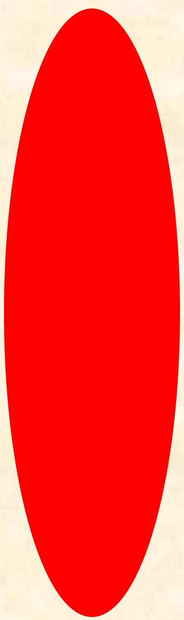
Heavy ion collision experiments at RHIC (2000-current) and LHC (2009-??) create hot and dense nuclear matter known as the “quark-gluon plasma”

(note: qualitative difference between p-p and Au-Au collisions)

Elliptic flow, jet quenching… - focus on transport in this talk

Evolution of the plasma “fireball” is described by relativistic fluid dynamics (relativistic Navier-Stokes equations)

Need to know



thermodynamics (equation of state)
kinetics (first- and second-order transport coefficients)
in the regime of intermediate coupling strength:

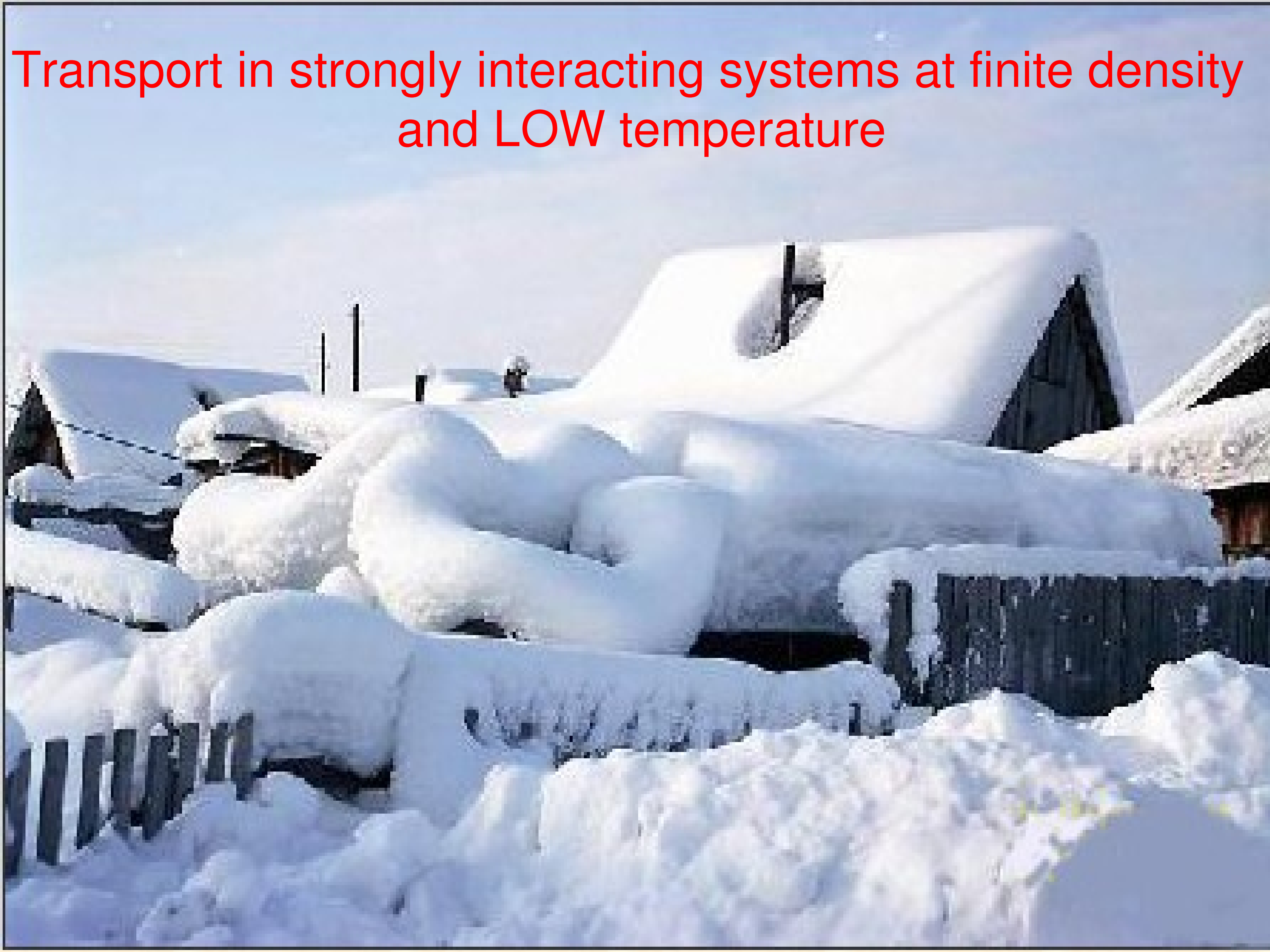
$$\alpha_s(T_{\mathrm{RHIC}}) \sim O(1)$$

initial conditions (initial energy density profile)

thermalization time (start of hydro evolution)

freeze-out conditions (end of hydro evolution)

Transport in strongly interacting systems at finite density and LOW temperature



Gauge-string duality and QCD

Approach I: use the gauge-string (gauge-gravity) duality to study N=4 SYM and similar theories, get qualitative insights into relevant aspects of QCD, look for universal quantities

(exact solutions but limited set of theories)

Approach II: bottom-up (a.k.a. AdS/QCD) – start with QCD, build gravity dual approximation

(unlimited set of theories, approximate solutions, systematic procedure unclear)

(will not consider here but see e.g. Gürsoy, Kiritsis, Mazzanti, Nitti, 0903.2859 [hep-th])

Approach III: solve QCD

Approach IIIa: pQCD (weak coupling; problems with convergence for thermal quantities)

Approach IIIb: LQCD (usual lattice problems + problems with kinetics)

$\mathcal{N} = 4$ supersymmetric YM theory

Gliozzi,Scherk,Olive'77
Brink,Schwarz,Scherk'77

- Field content:

$$A_\mu \qquad \Phi_I \qquad \Psi_\alpha^A \qquad \text{all in the adjoint of } SU(N)$$

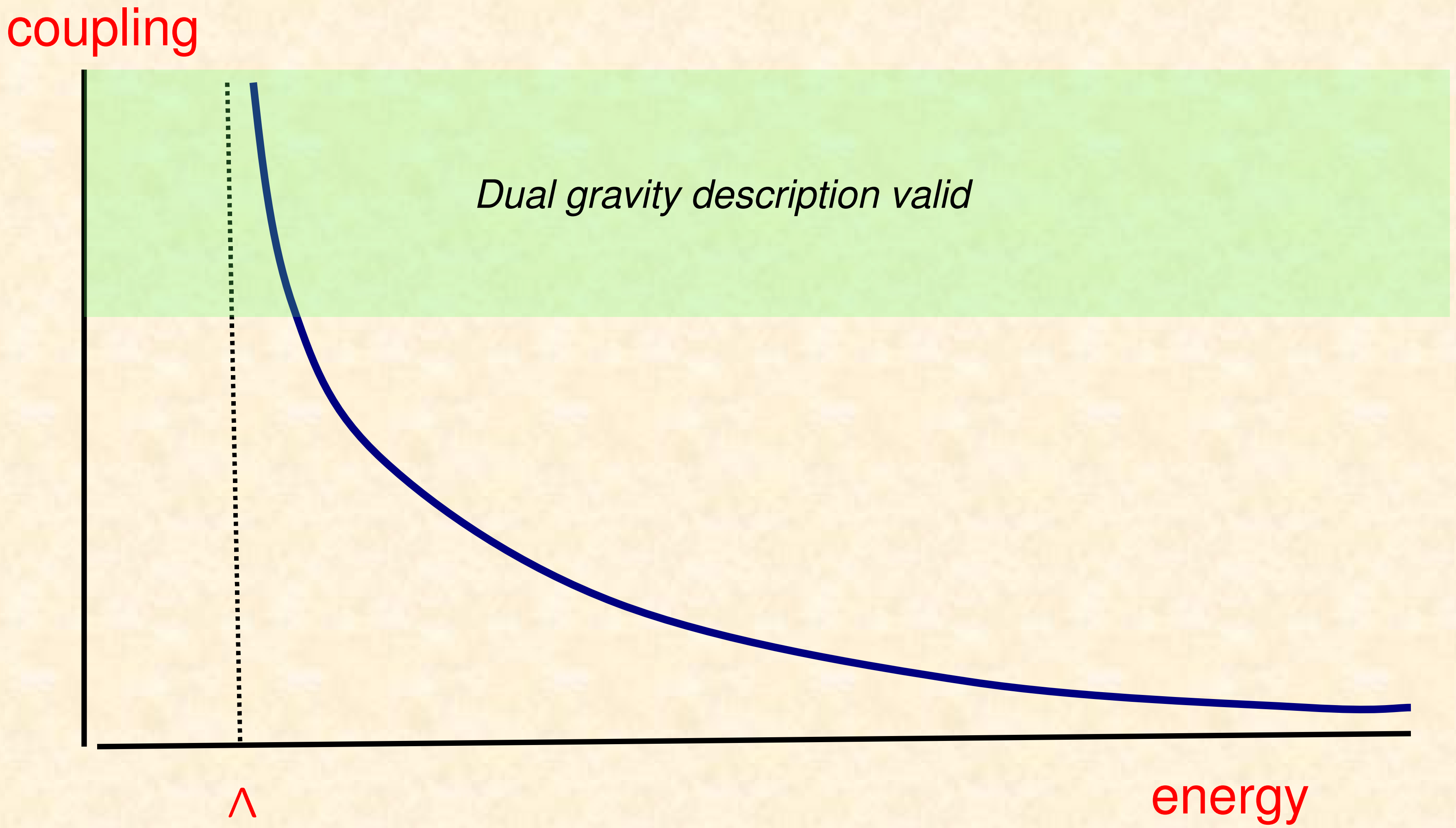
$$I = 1 \ldots 6 \qquad A = 1 \ldots 4$$

- Action:

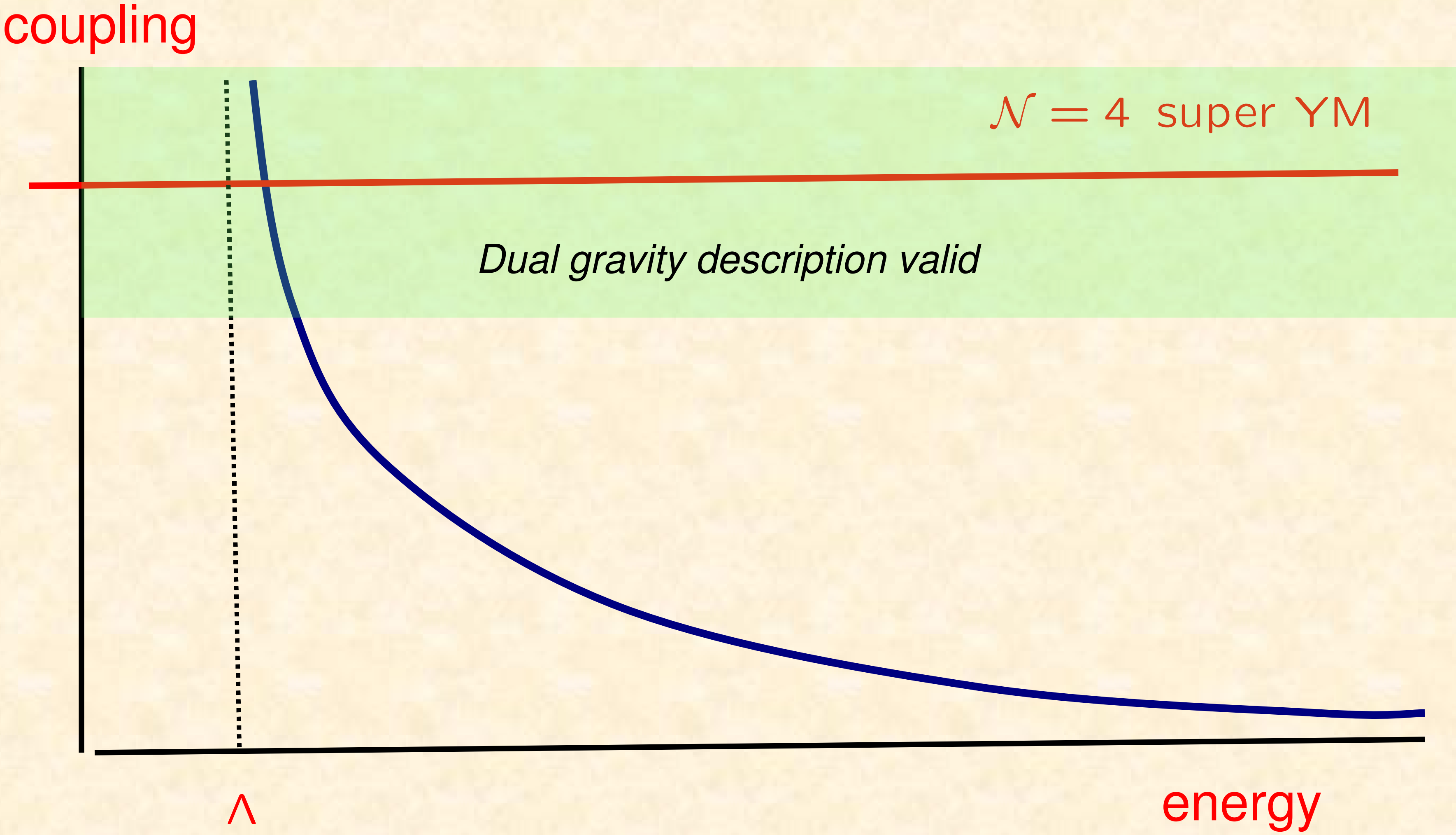
$$S = \frac{1}{g_{YM}^2} \int d^4x \ \mathrm{tr} \left\{ \frac{1}{2} F_{\mu\nu}^2 + (D_\mu \Phi_I)^2 - \frac{1}{2} [\Phi_I, \Phi_J]^2 + i\bar{\Psi}\Gamma^\mu D_\mu \Psi - \bar{\Psi}\Gamma^I [\Phi_I, \Psi] \right\}$$

(super)conformal field theory = coupling doesn't run

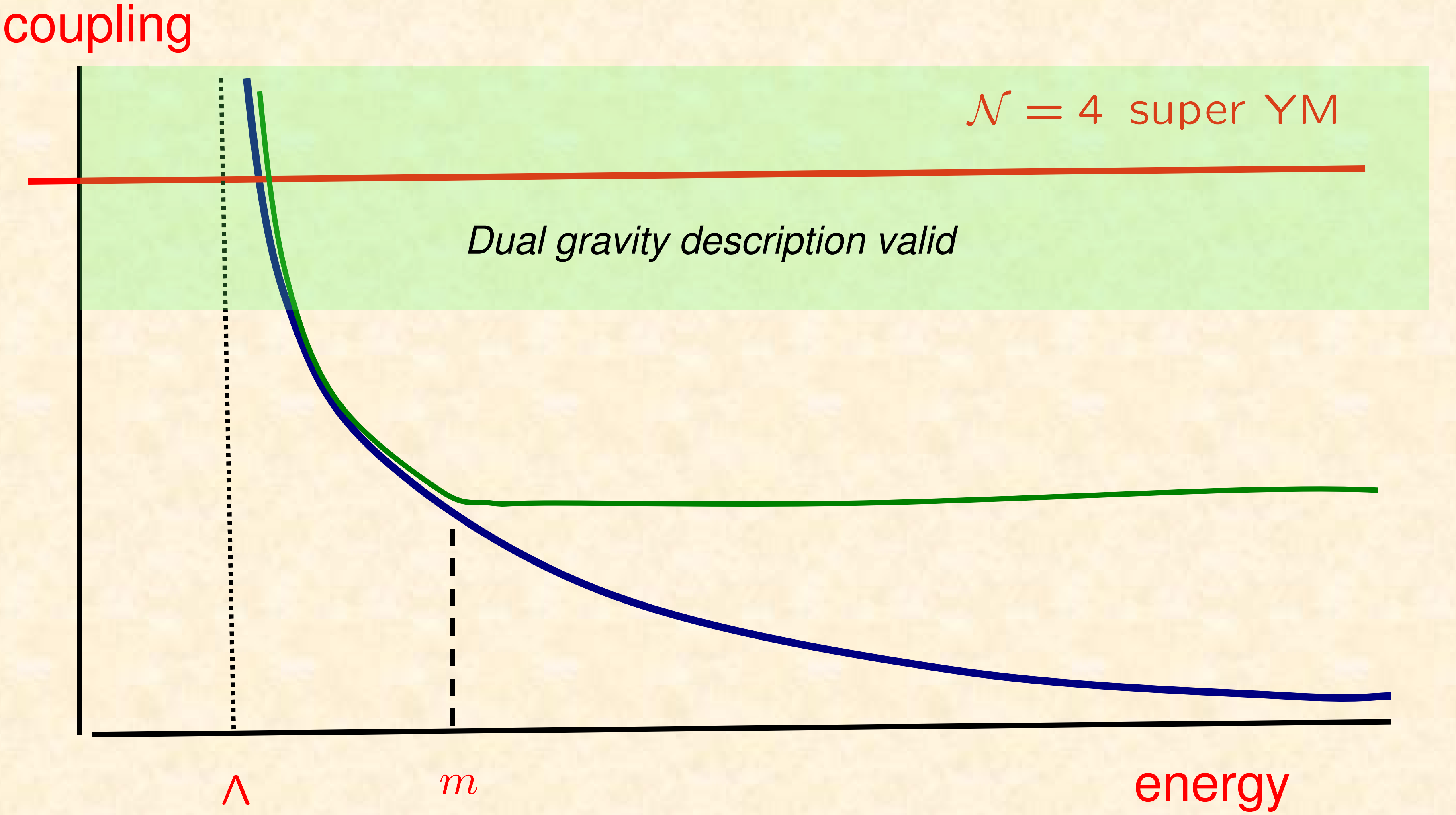
Dual to QCD? (Polchinski-Strassler)



Dual to QCD? (Polchinski-Strassler)



Dual to QCD? (Polchinski-Strassler)



At zero temperature, N=4 SYM is obviously a very bad approximation to QCD

However:

At finite temperature $T > T_c$ it is qualitatively similar to QCD

- ✓ supersymmetry broken
- ✓ non-Abelian plasma (with additional d.o.f.)
- ✓ area law for spatial Wilson loops
- ✓ Debye screening
- ✓ spontaneous breaking of Z_N symmetry at high temperature
- ✓ hydrodynamics

Energy density vs temperature for various gauge theories

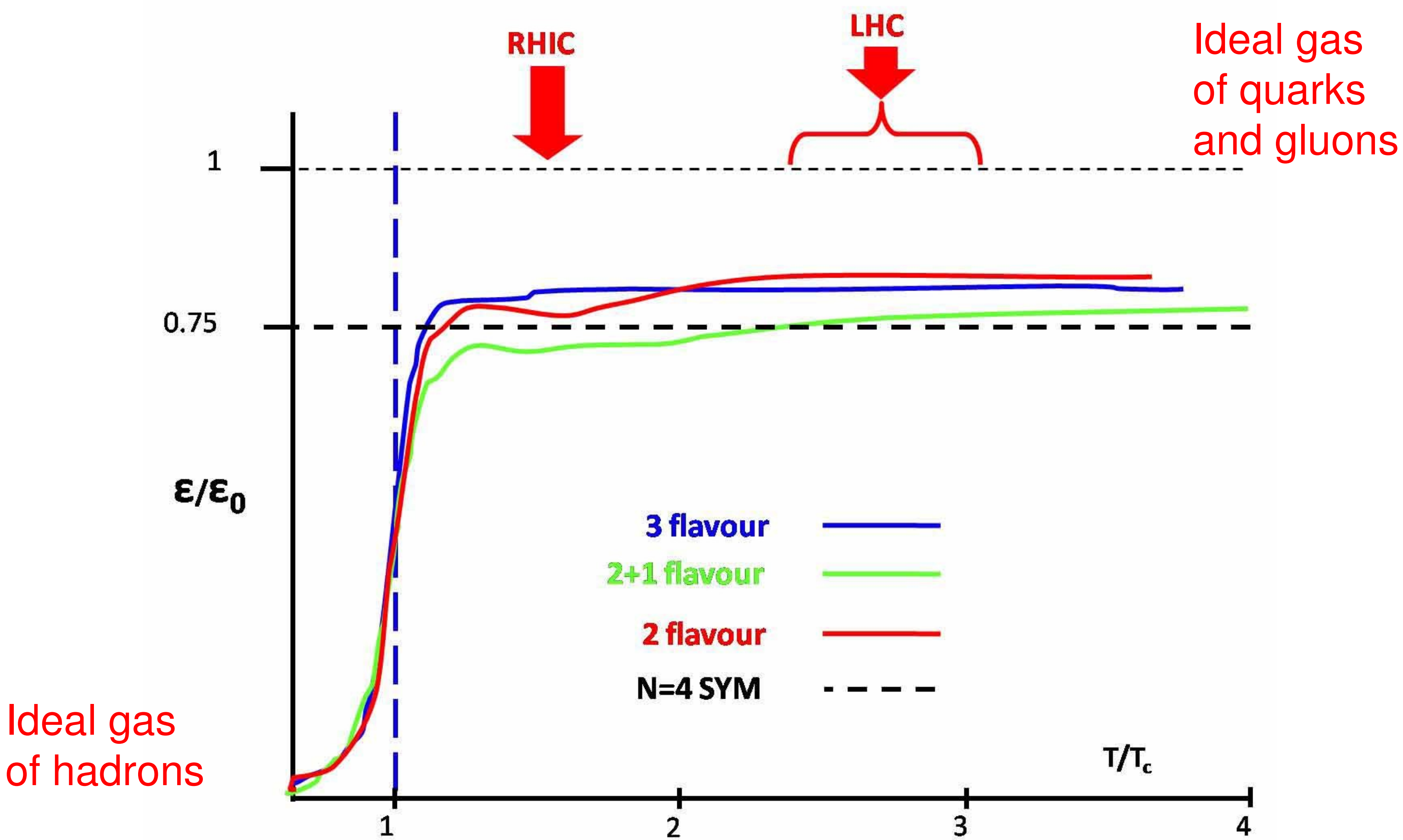


Figure: an artistic impression from Myers and Vazquez, 0804.2423 [hep-th]

Hydrodynamics: fundamental d.o.f. = densities of conserved charges

Need to add constitutive relations!

Example: charge diffusion

Conservation law $$\partial_t j^0 + \partial_i j^i = 0$$

Constitutive relation

[Fick s law (1855)] $$j_i = -D\,\partial_i j^0 + O[(\nabla j^0)^2, \nabla^2 j^0]$$

Diffusion equation $$\partial_t j^0 = D\nabla^2 j^0$$

Dispersion relation $$\omega = -i\,D\,q^2 + \cdots$$

Expansion parameters: $\omega \ll T\,,\quad q \ll T$

First-order transport (kinetic) coefficients

Shear viscosity	η
Bulk viscosity	ζ
Charge diffusion constant	D_Q
Supercharge diffusion constant	D_s
Thermal conductivity	κ_T
Electrical conductivity	σ

* Expect Einstein relations such as $\frac{\sigma}{e^2 \Xi} = D_{U(1)}$ to hold

Second-order transport (kinetic) coefficients

(for theories conformal at T=0)

Relaxation time τ_{Π}

Second order trasport coefficient λ_1

Second order trasport coefficient λ_2

Second order trasport coefficient λ_3

Second order trasport coefficient κ

In non-conformal theories such as QCD, the total number of second-order transport coefficients is quite large

AdS/CFT correspondence

$\mathcal{N}=4$ supersymmetric $SU(N_c)$ YM theory in 4 dim $\Longleftrightarrow$ type IIB superstring theory on $AdS_5 \times S^5$ backgrond

conjectured exact equivalence

Latest test: Janik'08

$$Z_{\text{SYM}}[J] = \langle e^{-\int J\,\mathcal{O}\,d^4x} \rangle_{\text{SYM}} = Z_{\text{string}}[J]$$

Generating functional for correlation functions of gauge-invariant operators $\langle \mathcal{O}\ \mathcal{O}\ \cdots \mathcal{O} \rangle$ $\Longleftrightarrow$ String partition function

In particular

$$Z_{\text{SYM}}[J] = Z_{\text{string}}[J] \simeq e^{-S_{\text{grav}}[J]}$$

$$\lambda \equiv g_{YM}^2\, N_c \gg 1$$

$$N_c \gg 1$$

Classical gravity action serves as a generating functional for the gauge theory correlators

Holography at finite temperature and density

$$\langle \mathcal{O} \rangle = \frac{\mathrm{tr}\rho\mathcal{O}}{\mathrm{tr}\rho} \qquad H \to T^{00} \to T^{\mu\nu} \to h_{\mu\nu}$$

$$\rho = e^{-\beta H + \mu Q} \qquad Q \to J^0 \to J^\mu \to A_\mu$$

Nonzero expectation values of energy and charge density translate into nontrivial background values of the metric (above extremality)=horizon and electric potential = CHARGED BLACK HOLE (with flat horizon)

$$ds^2 = -F(u)\,dt^2 + G(u)\left(dx^2 + dy^2 + dz^2\right) + H(u)\,du^2$$

$T = T_H$ temperature of the dual gauge theory

$$A_0 = P(u)$$

$\mu = P(\text{boundary}) - P(\text{horizon})$ chemical potential of the dual theory

Example: R-current correlator in $4d\ \mathcal{N}=4$ SYM

in the limit $N_c \to \infty\,, \quad g_{YM}^2 N_c \to \infty$

Zero temperature: $< J_i(x)\, J_i(y) > \sim \frac{N_c^2}{|x-y|^6}$

$$G_E(k) = \frac{N_c^2 k_E^2}{32\pi^2} \ln k_E^2$$

$$G^{\text{ret}}(k) = \frac{N_c^2 k^2}{32\pi^2} \left(\ln |k^2| - i\,\pi \theta\,(-k^2)\ \text{sgn}\,\omega \right) \qquad k^2 = -\omega^2 + q^2$$

Finite temperature: $G^{\text{ret}}(\omega, q)$

$$G^{\text{ret}}(\omega, 0) = \frac{N_c^2 T^2}{8} \left\{ \frac{i\omega}{2\pi T} + \frac{\omega^2}{4\pi^2 T^2} \left[\psi\left(\frac{(1-i)\omega}{4\pi T} \right) + \psi\left(-\frac{(1+i)\omega}{4\pi T} \right) \right] \right\}$$

Poles of G^{ret} = quasinormal spectrum of dual gravity background

(D.Son, A.S., hep-th/0205051, P.Kovtun, A.S., hep-th/0506184)

Computing transport coefficients from "first principles"

Fluctuation-dissipation theory
(Callen, Welton, Green, Kubo)

Kubo formulae allows one to calculate transport coefficients from microscopic models

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d^3x e^{i\omega t} \langle \left[T_{xy}(t, x), T_{xy}(0, 0) \right] \rangle$$

In the regime described by a gravity dual the correlator can be computed using the gauge theory/gravity duality

Example: stress-energy tensor correlator in $4d\ \mathcal{N}=4$ SYM in the limit $N_c \to \infty\,, \quad g^2_{YM} N_c \to \infty$

Zero temperature, Euclid: $$G_E(k) = \frac{N_c^2 k_E^4}{32\pi^2} \ln k_E^2$$

Finite temperature, Mink:

$$< T_{tt}(-\omega,-q), T_{tt}(\omega,q) >^{\text{ret}} = \frac{3N_c^2\pi^2T^4q^2}{2(\omega^2 - q^2/3 + i\omega q^2/3\pi T)} + \cdots$$

(in the limit $\omega/T \ll 1\,, \quad q/T \ll 1$)

The pole
(or the lowest quasinormal freq.)

$$\omega = \pm \frac{1}{\sqrt{3}} q - \frac{i}{6\pi T} q^2 + \frac{3 - 2\ln 2}{24\pi^2\sqrt{3}T^2} q^3 + \cdots$$

Compare with hydro: $$\omega = \pm v_s q - \frac{i}{2sT}\left(\zeta + \frac{4}{3}\eta\right) q^2 + \cdots$$

In CFT: $v_s = \frac{1}{\sqrt{3}}\,, \quad \zeta = 0$

$$\Rightarrow \quad \eta = \pi N_c^2 T^3/8$$

Also, $s = \pi^2 N_c^2 T^3/2$ (Gubser, Klebanov, Peet, 1996)

First-order transport coefficients in N = 4 SYM

in the limit $N_c \to \infty\,, \quad g^2_{YM} N_c \to \infty$

Shear viscosity $$\eta = \frac{\pi}{8} N_c^2 T^3 \left[1 + O\left(\frac{1}{(g^2 N_c)^{3/2}}, \frac{1}{N_c^2} \right) \right]$$

Bulk viscosity $$\zeta = 0$$

for non-conformal theories see Buchel et al; G.D.Moore et al Gubser et al.

Charge diffusion constant $$D_R = \frac{1}{2\pi T} + \cdots$$

Supercharge diffusion constant $$D_s = \frac{2\sqrt{2}}{9\pi T}$$

NEW! (G.Policastro, 2008)

Thermal conductivity $$\frac{\kappa_T \ \mu^2}{\eta \ T} = 8\pi^2 + \cdots$$

Electrical conductivity $$\sigma = e^2 \frac{N_c^2 T}{16\,\pi} + \cdots$$

Sound and supersymmetric sound in $4d \ \mathcal{N}=4$ SYM

In 4d CFT

$$\epsilon = 3\,P$$

$$\zeta = 0$$

$$\Longrightarrow$$

$$v_s = \sqrt{\frac{\partial P}{\partial \epsilon}} = \frac{1}{\sqrt{3}}$$

$$v_{SS} = \frac{P}{\epsilon} = \frac{1}{3}$$

Sound mode: $$\omega = \pm \frac{q}{\sqrt{3}} - i \frac{2\,\eta}{3sT}\, q^2 + \cdots$$

Supersound mode: $$\omega = \pm \frac{q}{3} - i D_s q^2 + \cdots$$

Quasinormal modes in dual gravity

Graviton: $$\omega = \pm \frac{q}{\sqrt{3}} - i \frac{1}{6\pi T}\, q^2 + \cdots \Longrightarrow \frac{\eta}{s} = \frac{1}{4\pi}$$

Gravitino: $$\omega = \pm \frac{q}{3} - i \frac{2\sqrt{2}}{9\pi T} q^2 + \cdots \Longrightarrow D_s = \frac{2\sqrt{2}}{9\pi T}$$

New transport coefficients in $\mathcal{N} = 4$ SYM

Sound dispersion: $$\omega = \pm\frac{1}{\sqrt{3}}q - \frac{i}{6\pi T}q^2 + \frac{3 - 2\ln 2}{24\pi^2\sqrt{3}T^2}q^3 + \cdots$$

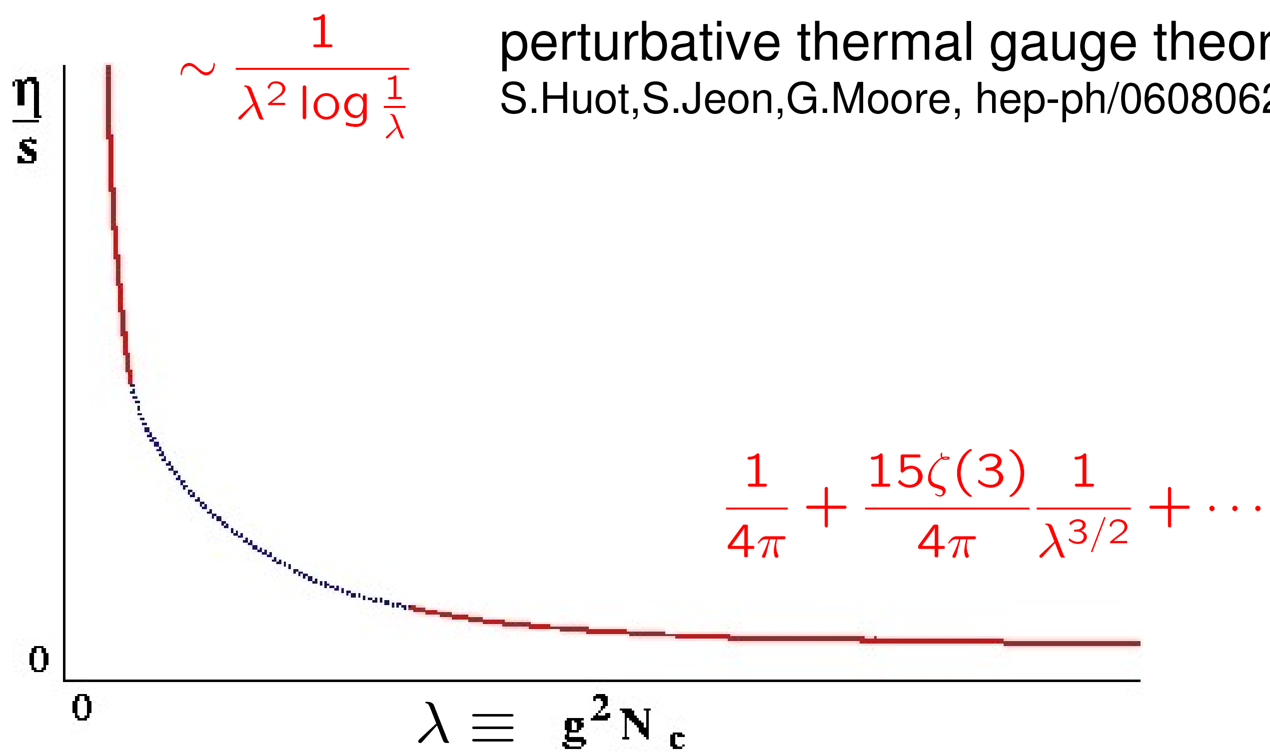
Kubo:

$$G_R^{xy,xy}(\omega, q) = -\frac{\pi^2 N_c^2 T^4}{4}\left[iw - w^2 + k^2 + w^2 \ln 2 - \frac{1}{2}\right] + O(w^3, wk^2)$$

$$w = \omega/2\pi T\,, \quad k = q/2\pi T$$

$$P = \frac{\pi^2}{8}N_c^2 T^4\,, \quad \eta = \frac{\pi}{8}N_c^2 T^3\,, \quad \tau_\Pi = \frac{2 - \ln 2}{2\pi T}\,, \quad \kappa = \frac{\eta}{\pi T}$$

Shear viscosity in $\mathcal{N} = 4$ SYM



perturbative thermal gauge theory
S.Huot,S.Jeon,G.Moore, hep-ph/0608062

Correction to $1/4\pi$: Buchel, Liu, A.S., hep-th/0406264

Buchel, 0805.2683 [hep-th]; Myers, Paulos, Sinha, 0806.2156 [hep-th]

Shear viscosity - (volume) entropy density ratio from gauge-string duality

In ALL theories (in the limit where dual gravity valid) : $\frac{1}{4\pi} + \text{corrections}$

In particular, in N=4 SYM: $\frac{1}{4\pi} + \frac{15\zeta(3)}{4\pi}\frac{1}{\lambda^{3/2}} + \cdots$

Other higher-derivative gravity actions

$$S = \int d^D x \sqrt{-g}\left(R - 2\Lambda + c_1\, R^2 + c_2\, R_{\mu\nu}R^{\mu\nu} + c_3\, R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \cdots\right)$$

Y.Kats and P.Petrov: 0712.0743 [hep-th]

M.Brigante, H.Liu, R.C.Myers, S.Shenker and S.Yaida: 0802.3318 [hep-th], 0712.0805 [hep-th].

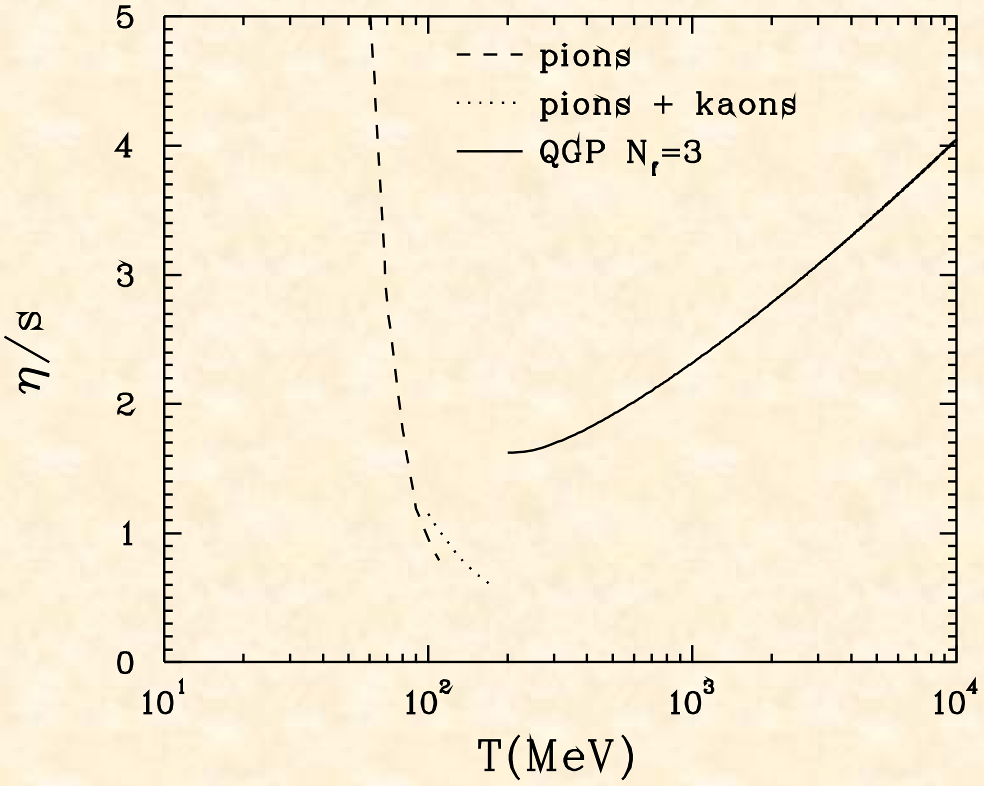
R.Myers,M.Paulos, A.Sinha: 0903.2834 [hep-th] (and ref. therein – many other papers)

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - 8c_1 + \ldots\right) \qquad \frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{1}{2N}\right)$$

for superconformal Sp(N) gauge theory in d=4

Also: The species problem: T.Cohen, hep-th/0702136; A. Dolbado, F.Llanes-Estrada: hep-th/0703132

Shear viscosity - (volume) entropy density ratio in QCD



$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B} F\Big(\frac{\Lambda_{QCD}}{T}, N_c\Big)$$

$$\frac{\eta}{s} \sim \frac{1}{\alpha_s^2 \log \alpha_s^{-1}}$$

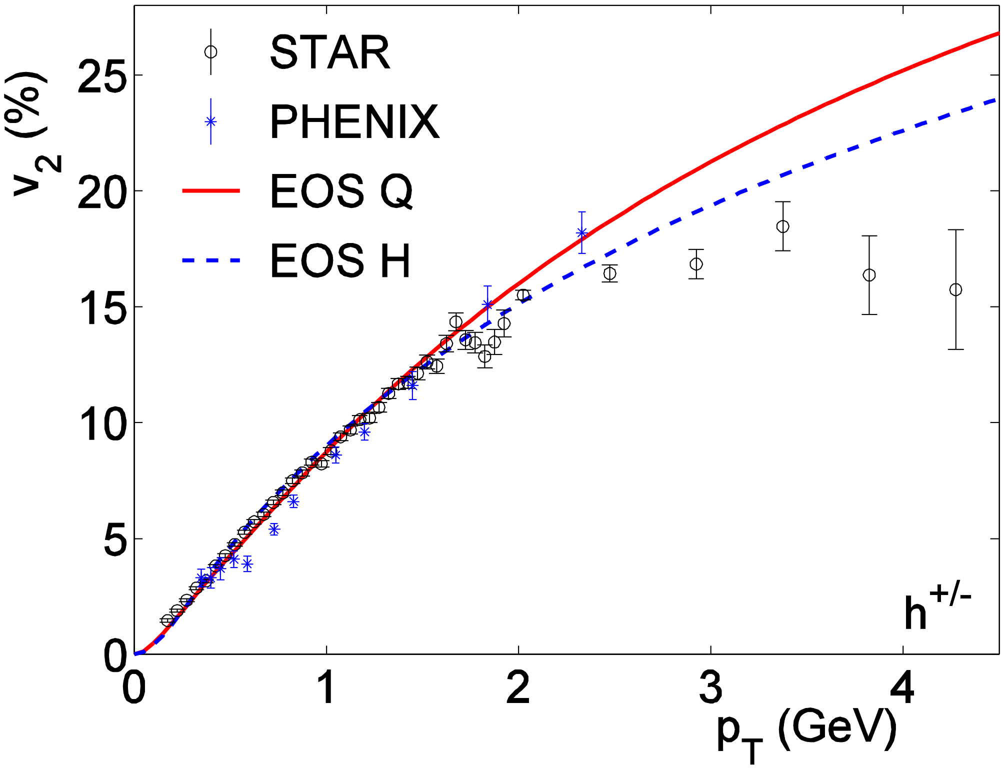
The value of this ratio strongly affects the elliptic flow in hydro models of QGP

Viscosity "measurements" at RHIC

Viscosity is ONE of the parameters used in the hydro models describing the azimuthal anisotropy of particle distribution

$$\frac{d^2N^i}{dp_T d\phi} = N_0^i \left[1 + 2v_2^i(p_T)\,\cos 2\phi + \cdots\right]$$

$v_2^i(p_T)$ -elliptic flow for particle species "i"



Elliptic flow reproduced for

$$0 < \eta/s \leq 0.5$$

e.g. Baier, Romatschke, nucl-th/0610108

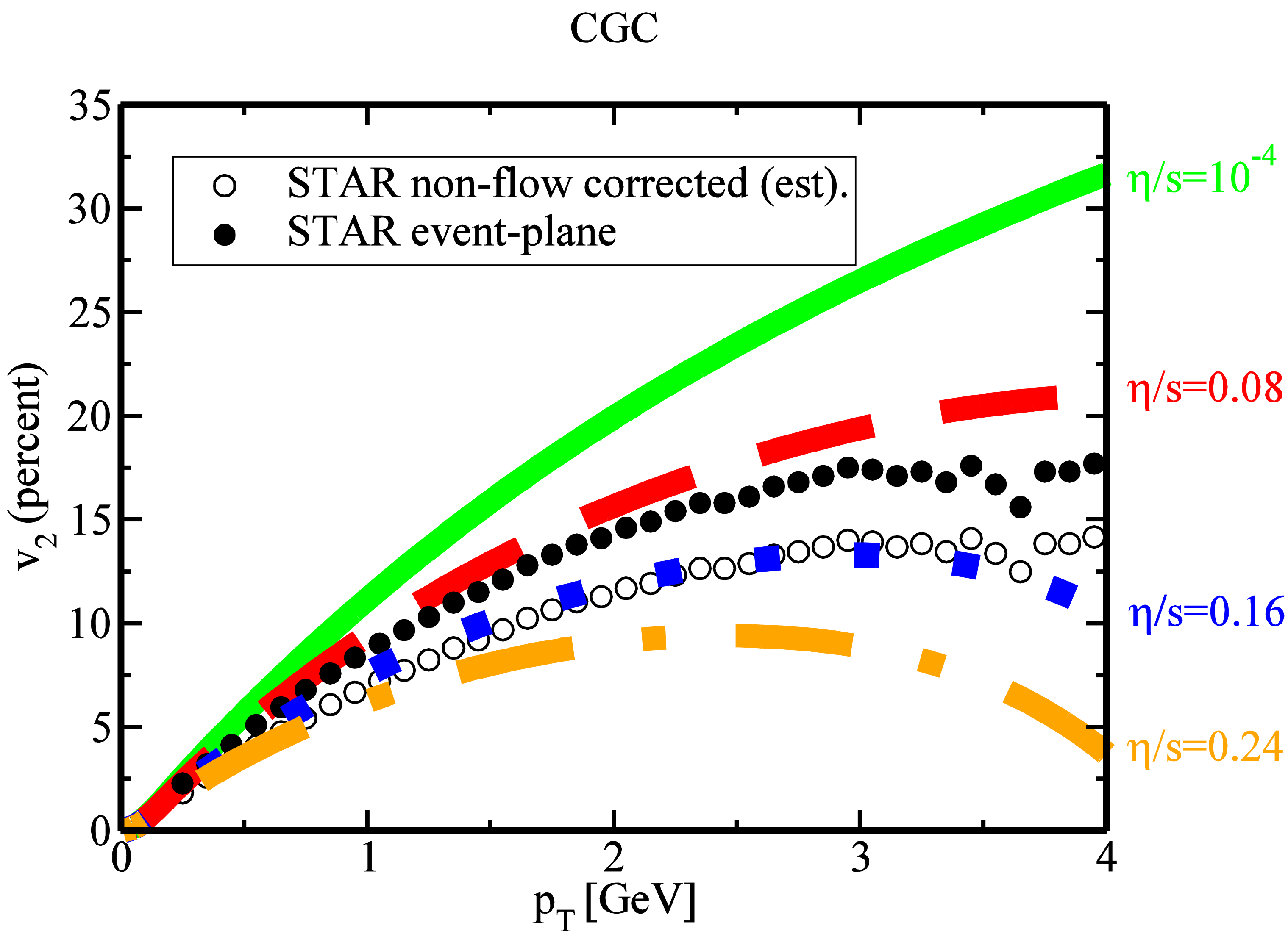
Perturbative QCD:

$$\eta/s\,(T_{\mathrm{RHIC}}) \approx 1.6 \sim 1.8$$

Chernai, Kapusta, McLerran, nucl-th/0604032

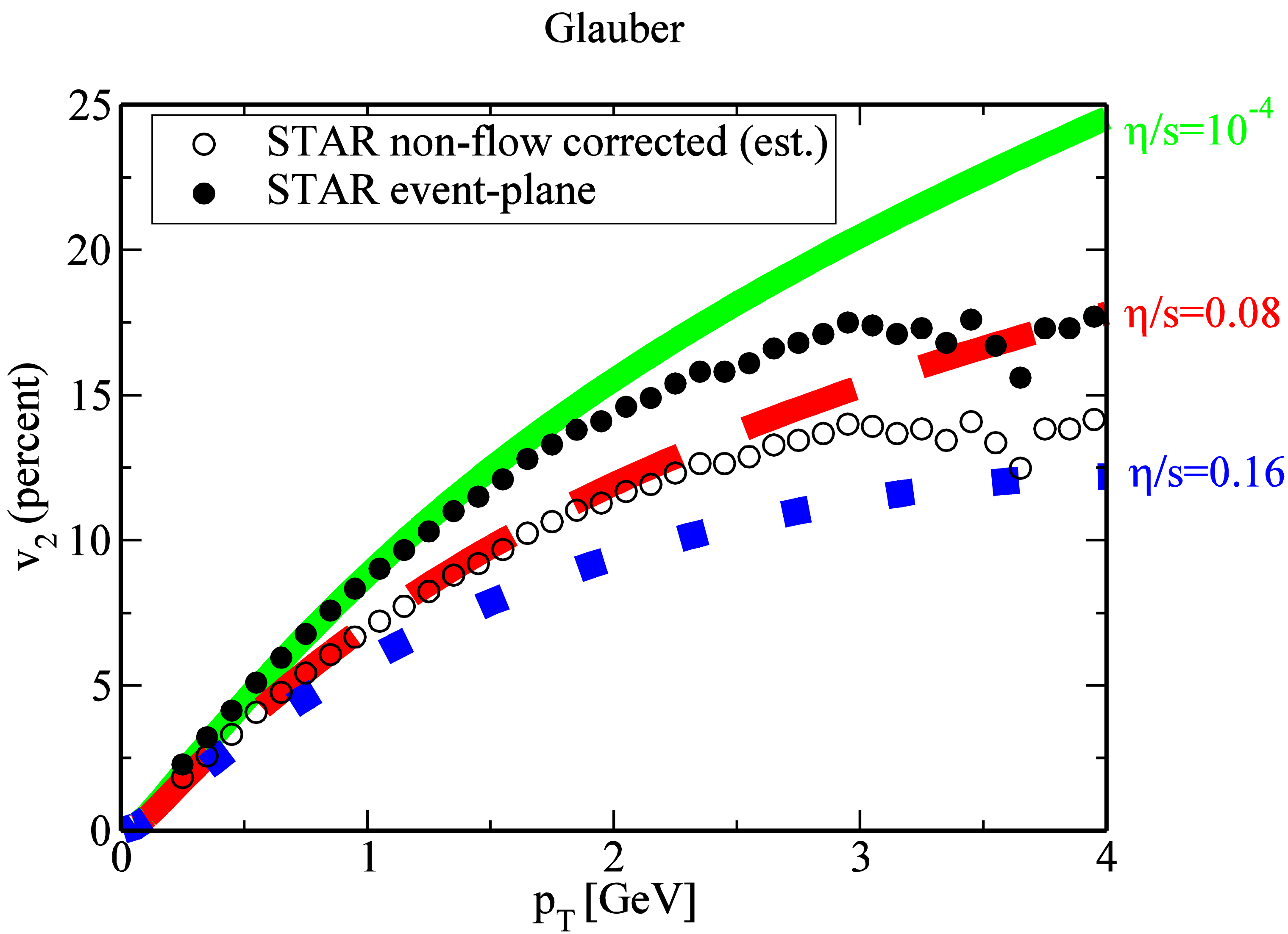
SYM: $\eta/s \approx 0.09 \sim 0.28$

Elliptic flow with color glass condensate initial conditions



Luzum and Romatschke, 0804.4015 [nuc-th]

Elliptic flow with Glauber initial conditions



Luzum and Romatschke, 0804.4015 [nuc-th]

Viscosity/entropy ratio in QCD: current status

Theories with gravity duals in the regime where the dual gravity description is valid

$$\frac{\eta}{s} = \frac{1}{4\pi} \approx 0.08$$

(universal limit)

[Kovtun, Son & A.S] [Buchel] [Buchel & Liu, A.S]

QCD: RHIC elliptic flow analysis suggests

$$0 < \frac{\eta}{s} < 0.5$$

QCD: (Indirect) LQCD simulations

$$0.08 < \frac{\eta}{s} < 0.16$$

H.Meyer, 0805.4567 [hep-th]

$$1.2\,T_c < T < 1.7\,T_c$$

Trapped strongly correlated cold alkali atoms

$$\left(\frac{\eta}{s}\right)_{\rm min} \approx 0.5$$

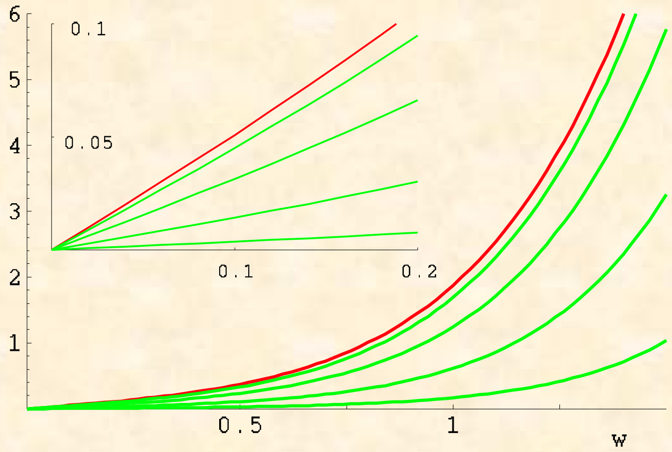
T.Schafer, 0808.0734 [nucl-th]

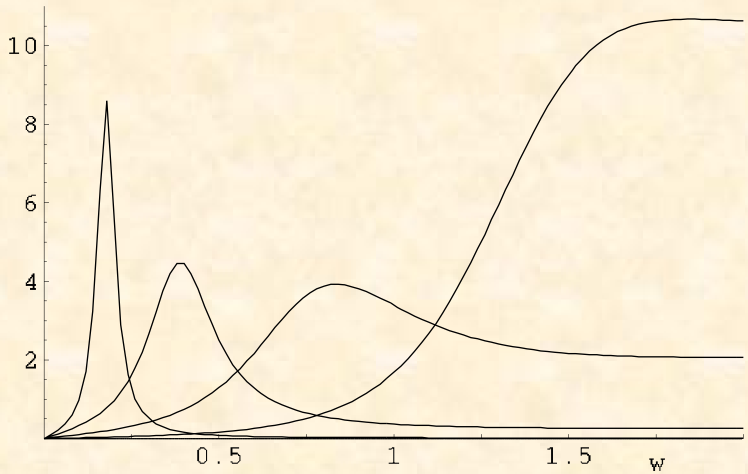
Liquid Helium-3

$$\left(\frac{\eta}{s}\right)_{\rm min} \approx 0.7$$

Spectral sum rules for the QGP

$$\chi_{\mu\nu,\alpha\beta}(k) = \int d^4x\, e^{-ikx} \langle \left[T_{\mu\nu}(x) T_{\alpha\beta}(0) \right] \rangle = -2\, \mathrm{Im}\, G^R_{\mu\nu,\alpha\beta}(\omega, q)$$





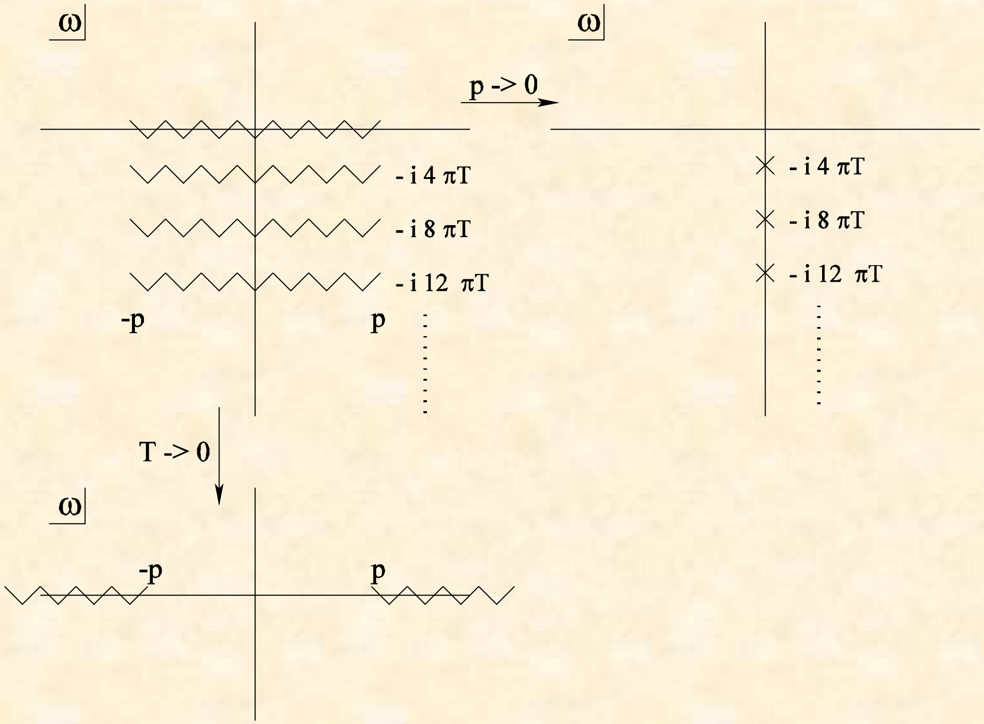
$$\frac{2}{5}\epsilon = \frac{1}{\pi} \int \frac{d\omega}{\omega} \left[\chi_{xy,xy}(\omega) - \chi^{T=0}_{xy,xy}(\omega) \right]$$

In N=4 SYM at ANY coupling

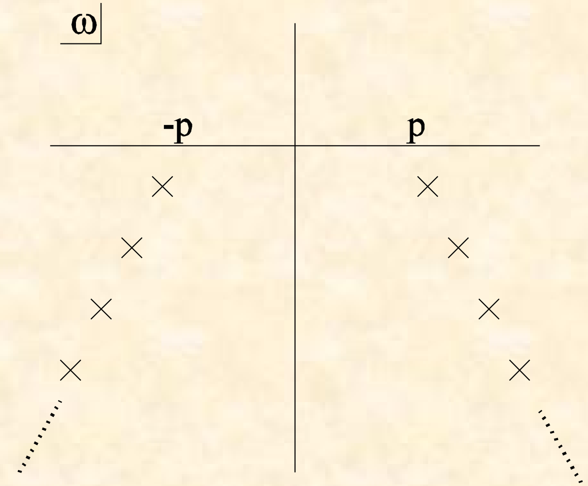
P.Romatschke, D.Son, 0903.3946 [hep-ph]

Analytic structure of the correlators

$g^2N = 0$



$g^2N = \infty$

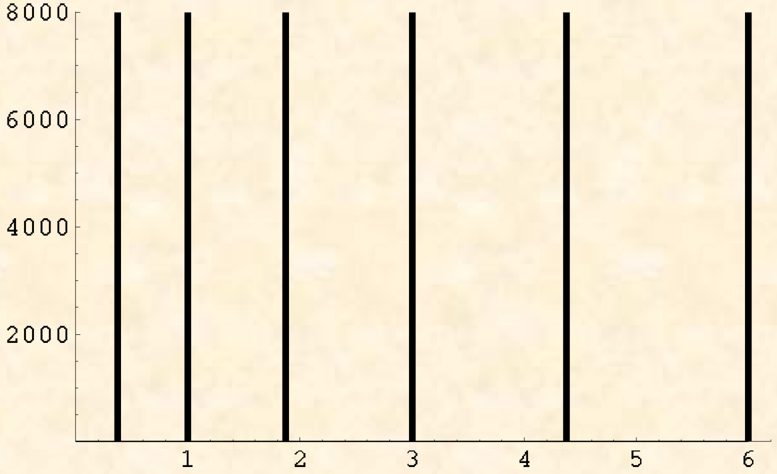


Strong coupling: A.S., hep-th/0207133

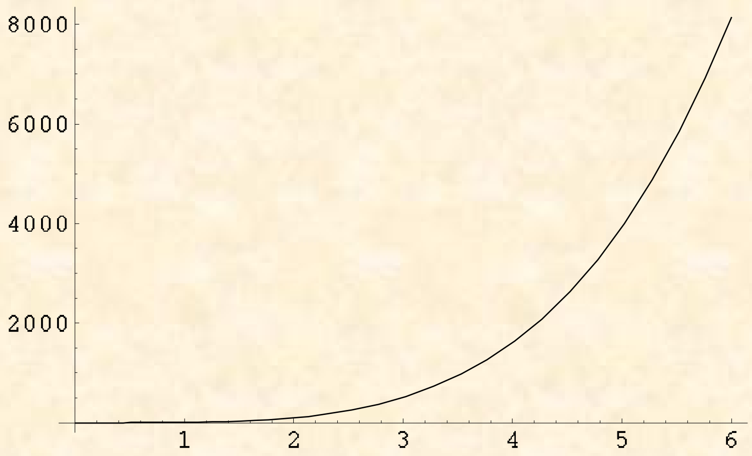
Weak coupling: S. Hartnoll and P. Kumar, hep-th/0508092

Spectral function and quasiparticles in finite-temperature "AdS + IR cutoff" model

$T < T_c$



$T > T_c$

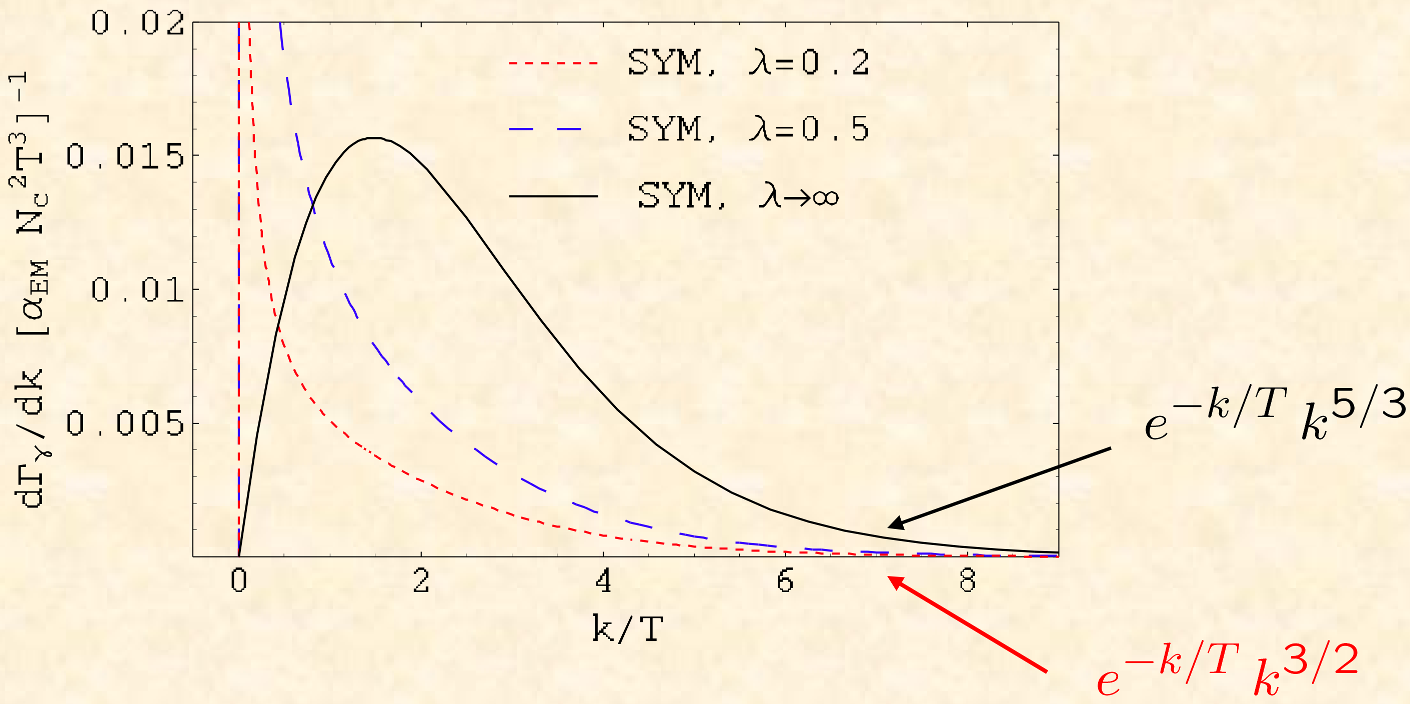


$$\chi(\omega) \sim N_c^2 \sum_{n=0}^{\infty} \omega_n^2 \rho(\omega_n) \, \delta(\omega - \omega_n)$$

$$\chi(\omega) = \frac{N_c^2}{16\pi} \frac{\omega^2 \sinh(\omega/2T)}{\cosh(\omega/2T) - \cos(\omega/2T)}$$

$\mathcal{N} = 4$ SYM

Photoproduction rate in SYM

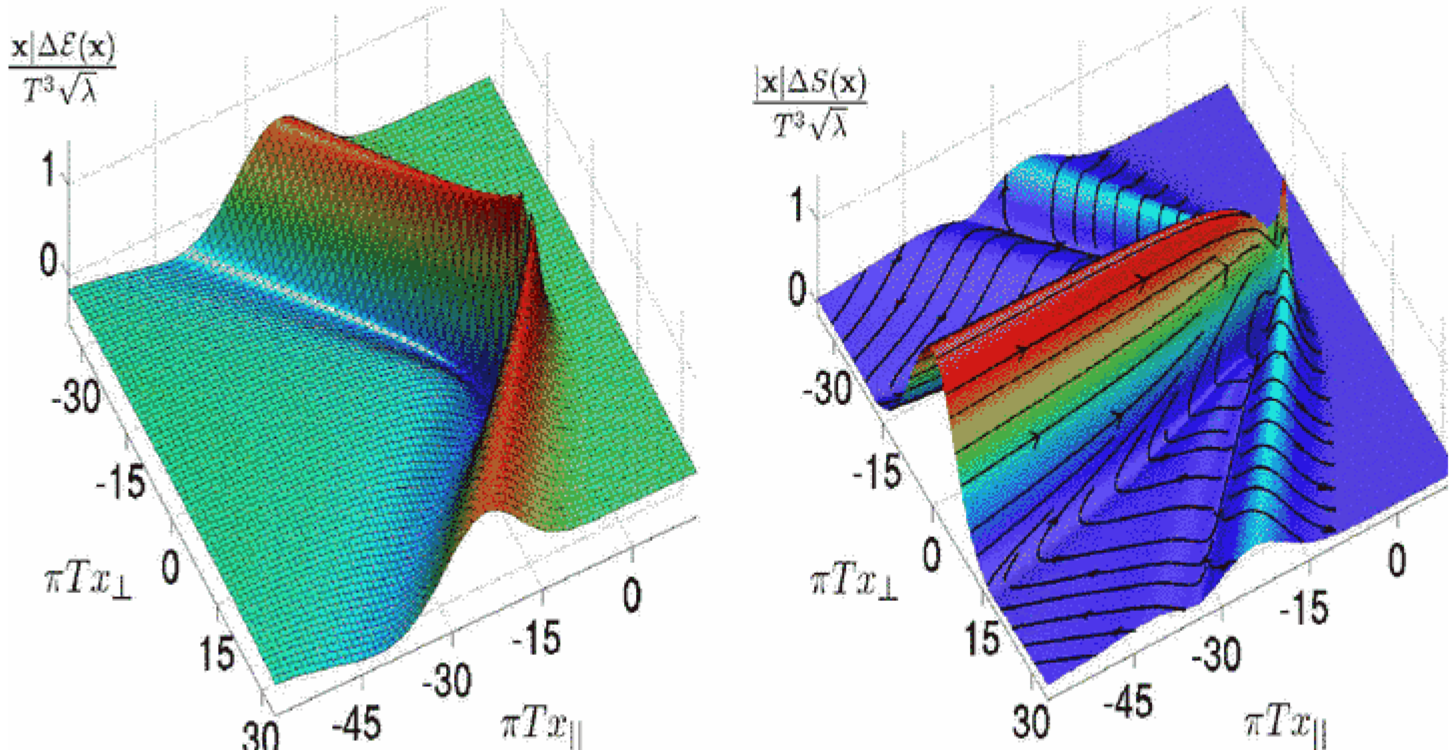


(Normalized) photon production rate in SYM for various values of ‘t Hooft coupling

$$\frac{d\Gamma_\gamma}{dk\,\alpha_{em}N_c^2T^3} = n_B(k)\left(\frac{k}{4\pi T}\right)^2 \left|\,{}_2F_1\left(1-\frac{(1+i)k}{4\pi T}, 1+\frac{(1-i)k}{4\pi T}; 1-\frac{ik}{2\pi T}; -1\right)\right|^{-2}$$

Holography beyond the near-equilibrium regime

Energy and Momentum Density



(Chesler and Yaffe)


Now consider strongly interacting systems at finite density
and LOW temperature

Probing quantum liquids with holography

Quantum liquid in p+1 dim	Low-energy elementary excitations	Specific heat at low T
Quantum Bose liquid	phonons	$\sim \; T^p$
Quantum Fermi liquid (Landau FLT)	fermionic quasiparticles + bosonic branch (zero sound)	$\sim \; T$

Departures from normal Fermi liquid occur in

- 3+1 and 2+1 –dimensional systems with strongly correlated electrons
- In 1+1 –dimensional systems for any strength of interaction (Luttinger liquid)

One can apply holography to study strongly coupled Fermi systems at low T

L.D.Landau (1908-1968)

The simplest candidate with a known holographic description is

$SU(N_c)$ $\mathcal{N}=4$ SYM coupled to N_f $\mathcal{N}=2$ fundamental hypermultiplets

at finite temperature T and nonzero chemical potential associated with the "baryon number" density of the charge $U(1)_B \subset U(N_f)$

There are two dimensionless parameters: $\frac{n_q^{1/3}}{T}$ $\frac{M}{T}$

n_q is the baryon number density

M is the hypermultiplet mass

The holographic dual description in the limit $N_c \gg 1\,,\ g_{YM}^2 N_c \gg 1\,,\ N_f$ finite is given by the D3/D7 system, with D3 branes replaced by the AdS-Schwarzschild geometry and D7 branes embedded in it as probes.

Karch & Katz, hep-th/0205236

AdS-Schwarzschild black hole (brane) background

$$ds^2 = \frac{r^2}{R^2}\left[-\left(1-\frac{r_H^4}{r^4}\right)dt^2 + d\vec{x}^2\right] + \left(1-\frac{r_H^4}{r^4}\right)^{-1}\frac{R^2}{r^2}dr^2$$

D7 probe branes

$$S_{DBI} = -N_f T_{D7}\int d^8\xi\sqrt{-\det(g_{ab}+2\pi\alpha' F_{ab})}$$

The worldvolume U(1) field A_μ couples to the flavor current J^μ at the boundary

Nontrivial background value of A_0 corresponds to nontrivial expectation value of J^0

We would like to compute

- the specific heat at low $(Tn_q^{-1/3} \ll 1)$ temperature
- the charge density correlator $G^R \sim \langle J^0(k)\, J^0(-k)\rangle$

★ The specific heat (in p+1 dimensions):

$$c_V = \mathcal{N}_q\, p \left(\frac{4\pi}{p+1}\right)^{2p+1} \frac{T^{2p}}{n_q} \left[1 + O(T n_q^{-\frac{1}{p}})\right]$$

(note the difference with Fermi $c_V \sim T$ and Bose $c_V \sim T^p$ systems)

★ The (retarded) charge density correlator $G^R \sim \langle J^0(k)\, J^0(-k)\rangle$ has a pole corresponding to a propagating mode (zero sound) - even at zero temperature

$$\omega = \pm \frac{q}{\sqrt{p}} - \frac{i\,\Gamma(\frac{1}{2})\, q^2}{n_q^{\frac{1}{p}} \Gamma(\frac{1}{2} - \frac{1}{2p}) \Gamma(\frac{1}{2p})} + O(q^3)$$

(note that this is NOT a superfluid phonon whose attenuation scales as q^{p+1})

New type of quantum liquid?

Other avenues of (related) research

Bulk viscosity for non-conformal theories (Buchel, Benincasa, Gubser, Moore…)

Non-relativistic gravity duals (Son, McGreevy,…)

Gravity duals of theories with SSB, AdS/CMT (Kovtun, Herzog, Hartnoll, Horowitz…)

Bulk from the boundary, time evolution of QGP (Janik,…)

Navier-Stokes equations and their generalization from gravity (Minwalla,…)

Quarks moving through plasma (Chesler, Yaffe, Gubser,…)

New directions

S. Hartnoll
"Lectures on holographic methods for condensed matter physics",
0903.3246 [hep-th]

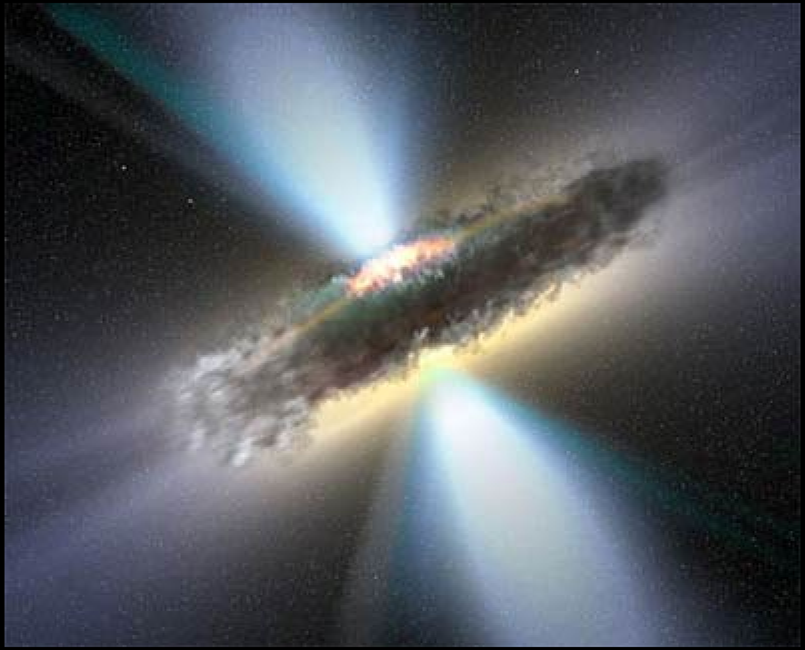
C. Herzog
"Lectures on holographic superfluidity and superconductivity",
0904.1975 [hep-th]

M. Rangamani
"Gravity and hydrodynamics: Lectures on the fluid-gravity correspondence",
0905.4352 [hep-th]

THANK YOU

Hydrodynamic properties of strongly interacting hot plasmas in 4 dimensions

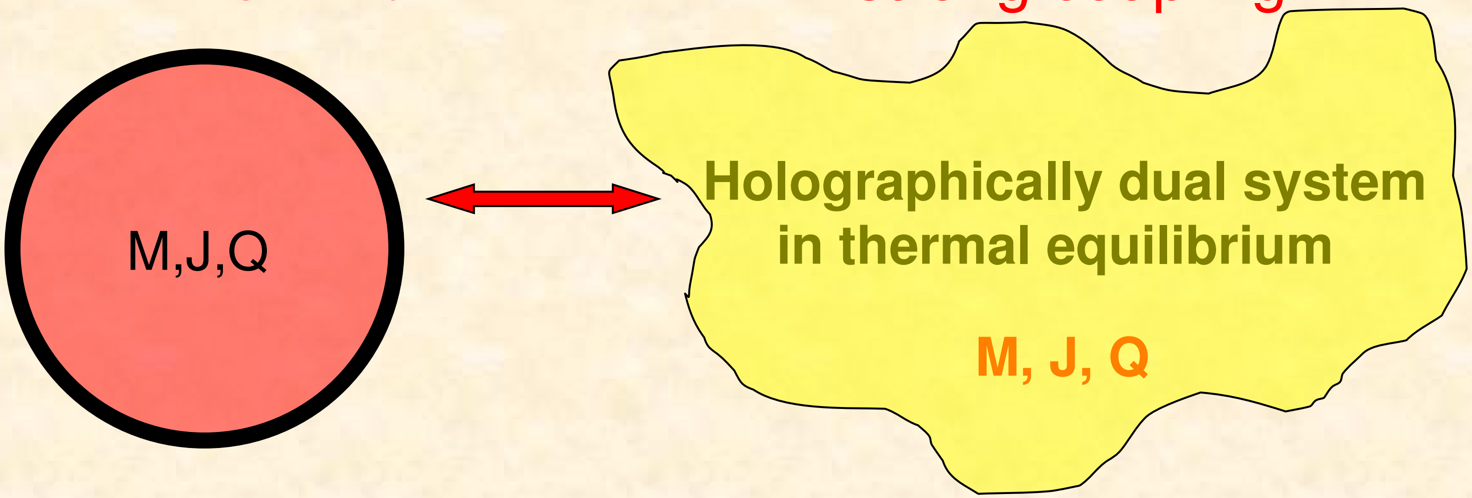
can be related (for certain models!)



to fluctuations and dynamics of 5-dimensional black holes

10-dim gravity

4-dim gauge theory – large N, strong coupling



T_{Hawking} $S_{\text{Bekenstein-Hawking}}$ $\Longleftrightarrow$ T S

Gravitational fluctuations $\Longleftrightarrow$ Deviations from equilibrium

$g_{\mu\nu}^{(0)} + h_{\mu\nu}$ $\Longleftrightarrow$????

$j_i = -D\partial_i j^0 + \cdots$

$"\Box" \; h_{\mu\nu} = 0$ and B.C. $\Longleftrightarrow$ $\partial_t j^0 + \partial_i j^i = 0$

$\partial_t j^0 = D\nabla^2 j^0$

Quasinormal spectrum $\Longleftrightarrow$ $\omega = -iDq^2 + \cdots$

AdS/CFT correspondence: the role of J

$$Z_{\text{SYM}}[J] = \langle e^{-\int J\,\mathcal{O}\,d^4x} \rangle_{\text{SYM}} \simeq e^{-S_{\text{grav}}[J]}$$

For a given operator $\mathcal{O}$, identify the source field J, e.g. $T^{\mu\nu} \Longleftrightarrow h_{\mu\nu}$

$$e^{-S_{\text{grav},M}[\phi_{\text{BG}}+\delta\phi]} = Z[J = \delta\phi\big|_{\partial M}]$$

$\delta\phi$ satisfies linearized supergravity e.o.m. with b.c. $\delta\phi \rightarrow \delta\phi_0 \equiv J$

The recipe:

To compute correlators of $\mathcal{O}$, one needs to solve the bulk supergravity e.o.m. for $\delta\phi$ and compute the on-shell action as a functional of the b.c. $\delta\phi_0 \equiv J$

Warning: e.o.m. for different bulk fields may be coupled: need self-consistent solution

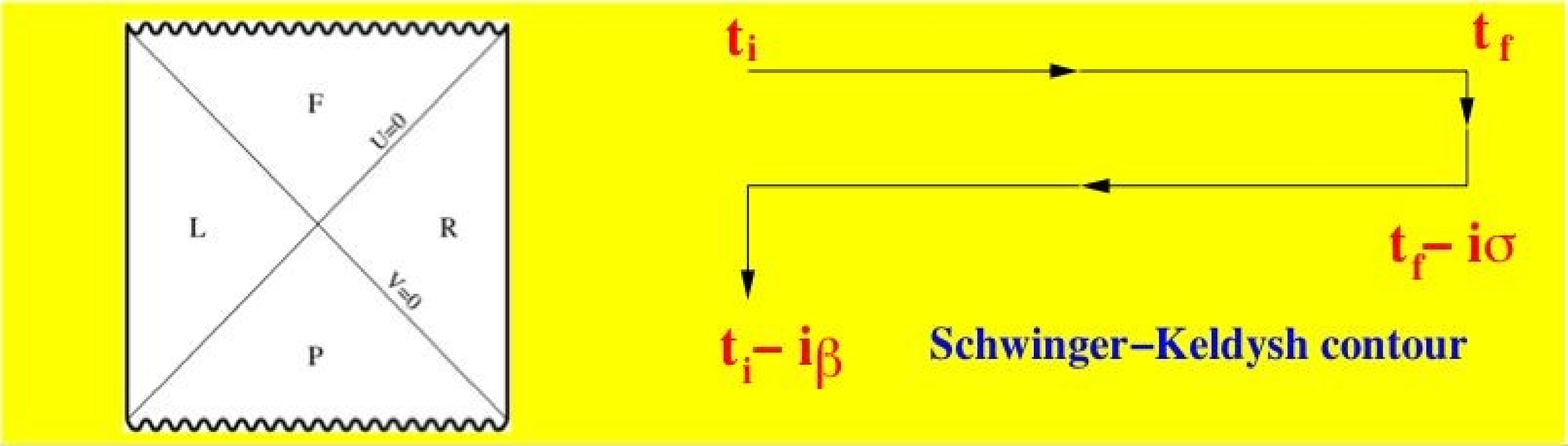
Then, taking functional derivatives of $e^{-S_{\text{grav}}[J]}$ gives $\langle \mathcal{O}\ \mathcal{O} \rangle$

Computing real-time correlation functions from gravity

To extract transport coefficients and spectral functions from dual gravity, we need a recipe for computing Minkowski space correlators in AdS/CFT

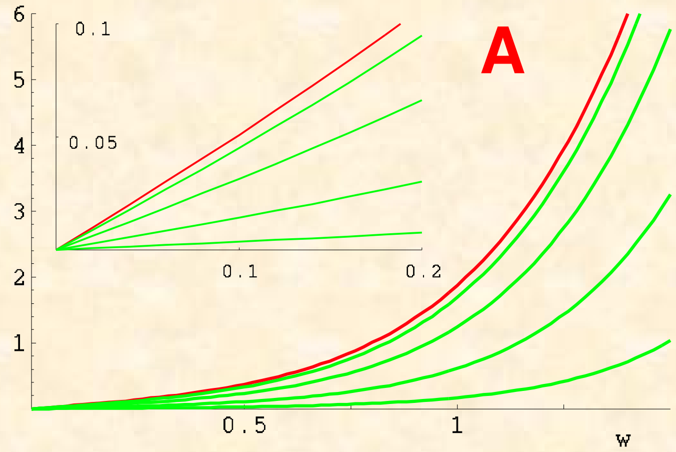
The recipe of [D.T.Son & A.S., 2001] and [C.Herzog & D.T.Son, 2002] relates real-time correlators in field theory to Penrose diagram of black hole in dual gravity

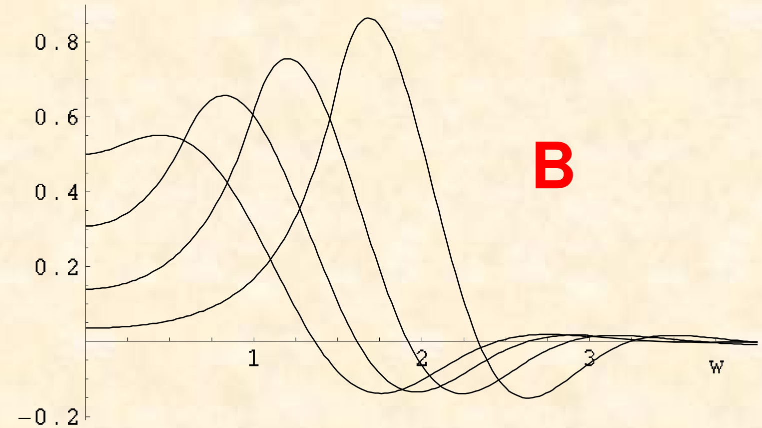
Quasinormal spectrum of dual gravity = poles of the retarded correlators in 4d theory [D.T.Son & A.S., 2001]

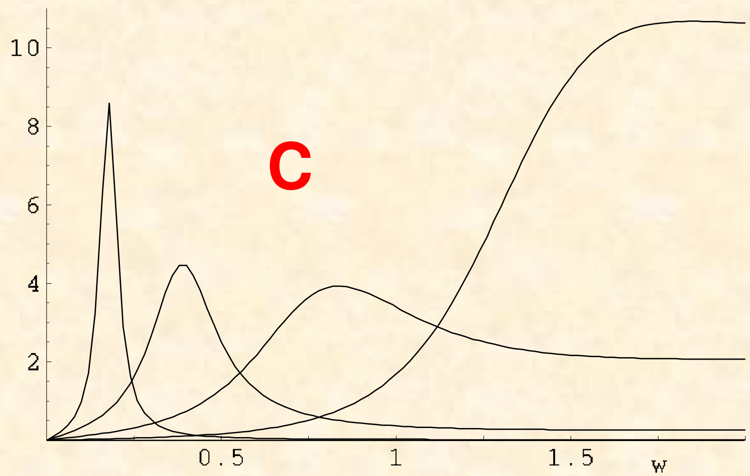


Spectral function and quasiparticles

$$\chi_{\mu\nu,\alpha\beta}(k) = \int d^4x \, e^{-ikx} \left\langle \left[T_{\mu\nu}(x) T_{\alpha\beta}(0) \right] \right\rangle = -2 \, \mathrm{Im} \, G^R_{\mu\nu,\alpha\beta}(\omega, q)$$







A: scalar channel

B: scalar channel - thermal part

C: sound channel

Is the bound dead?

- Y.Kats and P.Petrov, 0712.0743 [hep-th]
  "Effect of curvature squared corrections in AdS on the viscosity of the dual gauge theory"

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{1}{2N}\right) \qquad \mathcal{N} = 2$$ superconformal Sp(N) gauge theory in d=4

$$S = \int d^D x \sqrt{-g}\left(R - 2\Lambda + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \cdots\right)$$

$$\frac{\eta}{s} = \frac{\hbar}{4\pi} F(a,c) \quad \text{for CFT ?}$$

- M.~Brigante, H.~Liu, R.~C.~Myers, S.~Shenker and S.~Yaida,
  ``The Viscosity Bound and Causality Violation,'' 0802.3318 [hep-th],
  ``Viscosity Bound Violation in Higher Derivative Gravity,'' 0712.0805 [hep-th].

- The "species problem"
  T.Cohen, hep-th/0702136, A.Dobado, F.Llanes-Estrada, hep-th/0703132

Universality of η/s

Theorem:

For a thermal gauge theory, the ratio of shear viscosity to entropy density is equal to $1/4\pi$
in the regime described by a dual gravity theory

(e.g. at $g^2_{YM}N_c = \infty \, , N_c = \infty$ in $\mathcal{N} = 4$ SYM)

Remarks:

- Extended to non-zero chemical potential:

  Benincasa, Buchel, Naryshkin, hep-th/0610145

- Extended to models with fundamental fermions in the limit $N_f/N_c \ll 1$

  Mateos, Myers, Thomson, hep-th/0610184

- *String/Gravity dual to QCD is currently unknown*

Universality of shear viscosity in the regime described by gravity duals

$$ds^2 = f(w)\left(dx^2 + dy^2\right) + g_{\mu\nu}(w)dw^\mu dw^\nu$$

$$\eta = \lim_{\omega\to 0} \frac{1}{2\omega}\int dt\, dx e^{i\omega t}\langle\left[T_{xy}(t,x), T_{xy}(0,0)\right]\rangle$$

$$\sigma_{abs} = -\frac{16\pi G}{\omega}\,\mathrm{Im}\,G^R(\omega) = \frac{8\pi G}{\omega}\int dt\, dx e^{i\omega t}\langle\left[T_{xy}(t,x), T_{xy}(0,0)\right]\rangle$$

$$\eta = \frac{\sigma_{abs}(0)}{16\pi G}$$

Graviton's component h_y^x obeys equation for a minimally coupled massless scalar. But then $\sigma_{abs}(0) = A_H$.

Since the entropy (density) is $s = A_H/4G$ we get

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Three roads to universality of η/s

- **The absorption argument**

  D. Son, P. Kovtun, A.S., hep-th/0405231

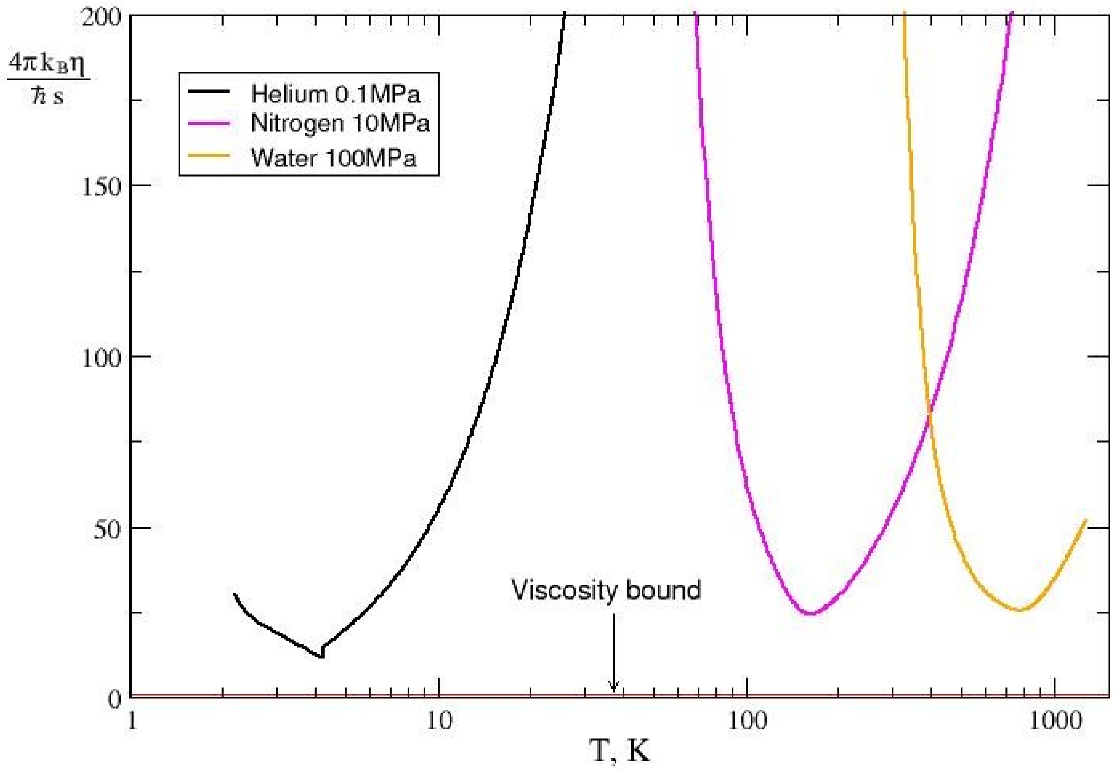
- **Direct computation of the correlator in Kubo formula from AdS/CFT** A.Buchel, hep-th/0408095

- **"Membrane paradigm" general formula for diffusion coefficient + interpretation as lowest quasinormal frequency = pole of the shear mode correlator + Buchel-Liu theorem**

  P. Kovtun, D.Son, A.S., hep-th/0309213, A.S., 0806.3797 [hep-th], P.Kovtun, A.S., hep-th/0506184, A.Buchel, J.Liu, hep-th/0311175

A viscosity bound conjecture

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \approx 6.08 \cdot 10^{-13} K \cdot s$$



Minimum of $\frac{\eta}{s}$ in units of $\frac{\hbar}{4\pi k_B}$

Xe	84
Kr	57
CO_2	32
H_2O	25
C_2H_5OH	22
Ne	17
He	8.8

P.Kovtun, D.Son, A.S., hep-th/0309213, hep-th/0405231

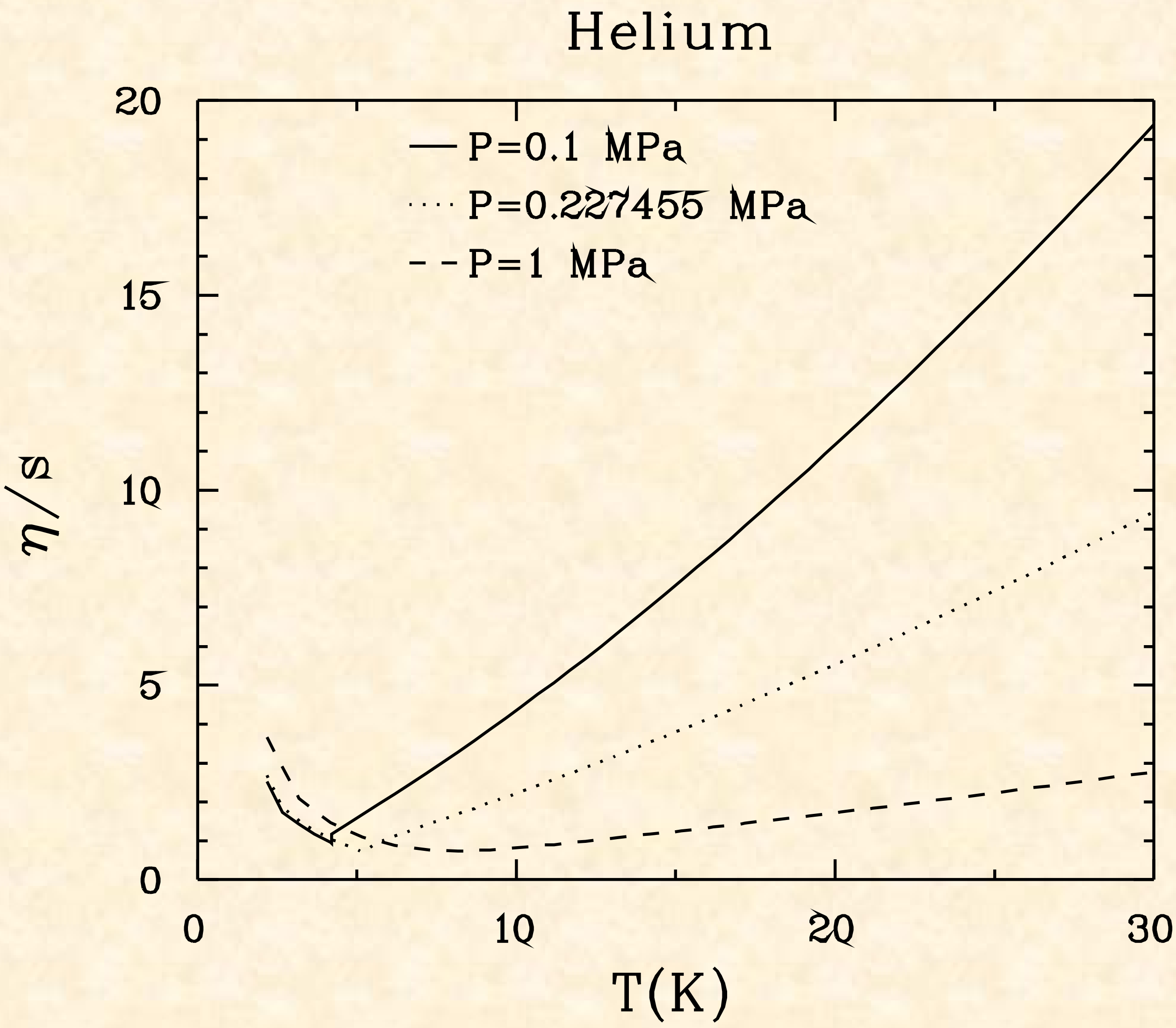
A hand-waving argument

$$\eta \sim \rho\, v\, l \;\sim \rho\, v^2 \tau \sim n\, m\, v^2 \tau \;\sim n\, \epsilon\tau$$

$$s \sim n$$

Thus $\dfrac{\eta}{s} \sim \epsilon\tau \geq \hbar$ **?**

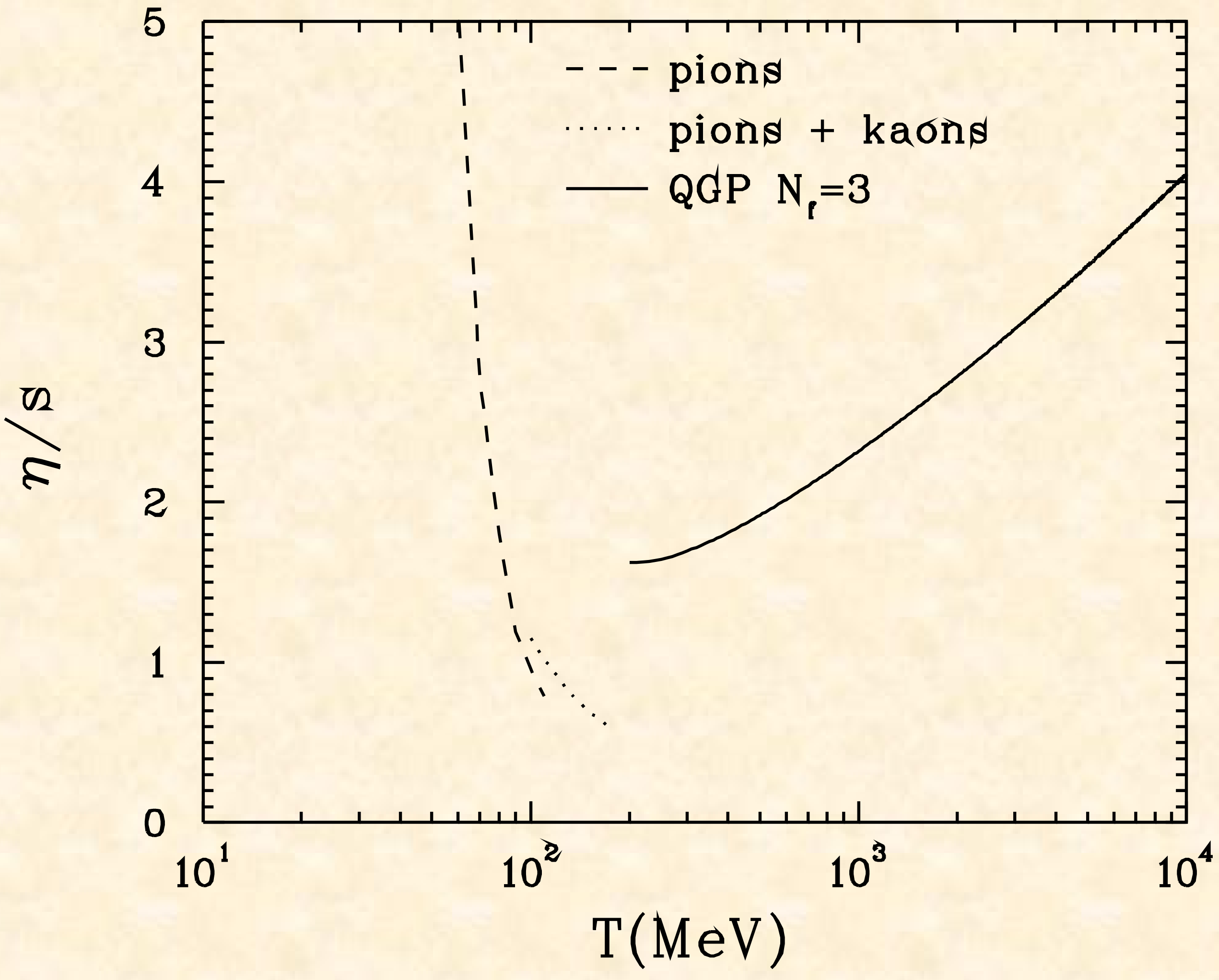
Gravity duals fix the coefficient: $\dfrac{\eta}{s} \geq \hbar/4\pi$



$$(\eta/s)_{\text{min}} \sim 8.8 \quad \text{in units of } \frac{\hbar}{4\pi k_B}$$

Chernai, Kapusta, McLerran, nucl-th/0604032

QCD



Chernai, Kapusta, McLerran, nucl-th/0604032

Shear viscosity at non-zero chemical potential

$\mathcal{N} = 4$ SYM

$q_i \in U(1)^3 \subset SO(6)_R$ $\iff$ Reissner-Nordstrom-AdS black hole with three R charges

$Z = \mathrm{tr}\, e^{-\beta H + \mu_i q_i}$

(see e.g. Yaffe, Yamada, hep-th/0602074)

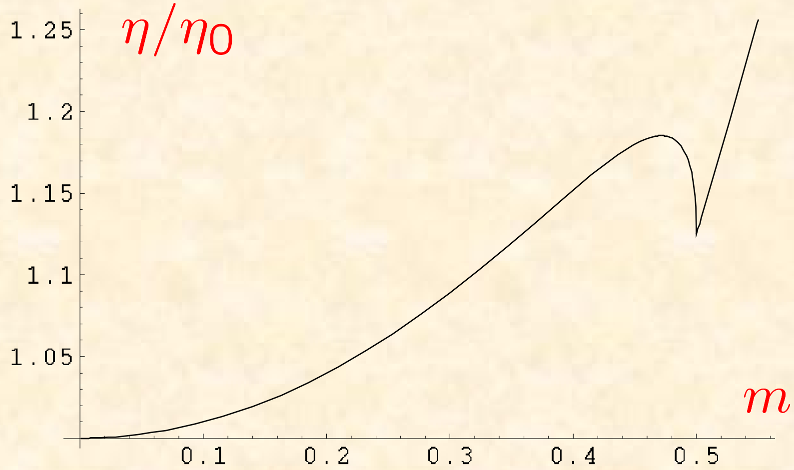
(Behrnd, Cvetic, Sabra, 1998)

We still have $\dfrac{\eta}{s} = \dfrac{1}{4\pi}$

J.Mas
D.Son, A.S.
O.Saremi
K.Maeda, M.Natsuume, T.Okamura

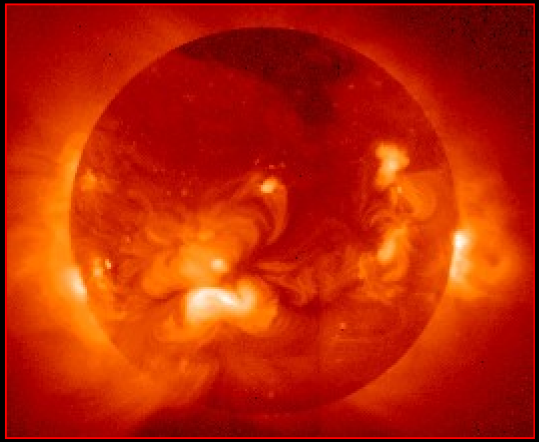
$$\eta = \pi N^2 T^3 \frac{m^2(1-\sqrt{1-4m^2}-m^2)^2}{(1-\sqrt{1-4m^2})^3}$$

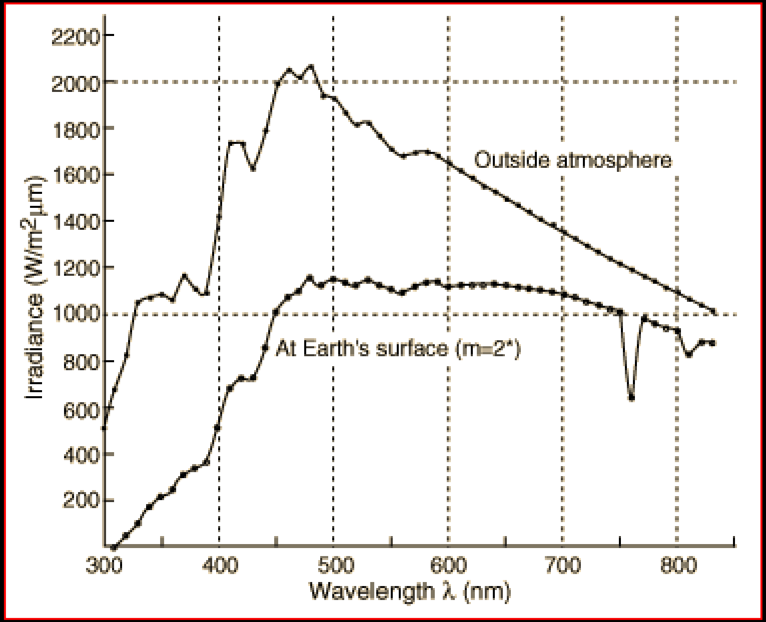
$$m \equiv \mu / 2\pi T$$

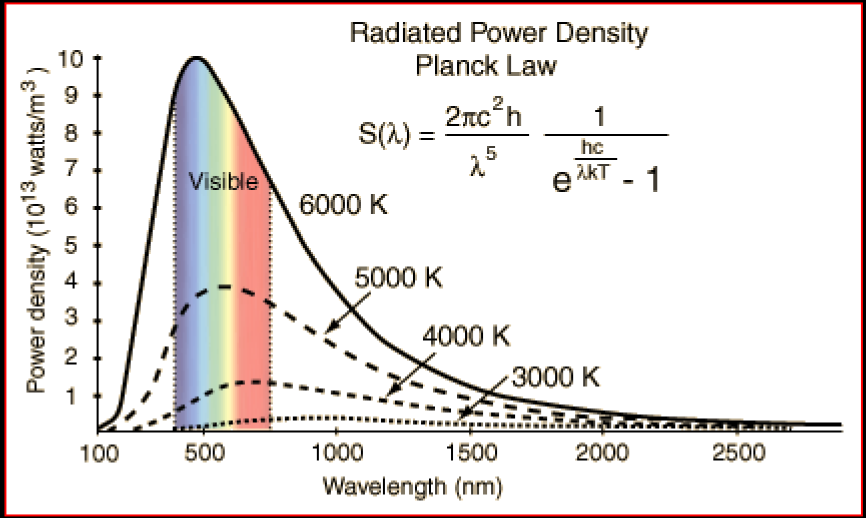


Photon and dilepton emission from supersymmetric Yang-Mills plasma

S. Caron-Huot, P. Kovtun, G. Moore, A.S., L.G. Yaffe, hep-th/0607237







Photon emission from SYM plasma

Photons interacting with matter: $e\, J_\mu^{\text{EM}}\, A^\mu$

To leading order in e $$d\Gamma_\gamma = \frac{d^3k}{(2\pi)^3}\,\frac{e^2}{2|k|}\,\eta^{\mu\nu} C^<_{\mu\nu}(k^0 = |k|)$$

$$C^<_{\mu\nu} = \int d^4X e^{-iKX} \langle J_\mu^{\text{EM}}(0) J_\nu^{\text{EM}}(X)\rangle$$

Mimic J_μ^{EM} by gauging global R-symmetry $U(1) \subset SU(4)$

$$\mathcal{L} = \mathcal{L}_{\mathcal{N}=4\,\text{SYM}} + e\, J_\mu^3\, A^\mu - \frac{1}{4} F_{\mu\nu}^2$$

Need only to compute correlators of the R-currents J_μ^3

Epilogue

- On the level of theoretical models, there exists a connection between near-equilibrium regime of certain strongly coupled thermal field theories and fluctuations of black holes
- This connection allows us to compute transport coefficients for these theories
- At the moment, this method is the only theoretical tool available to study the near-equilibrium regime of strongly coupled thermal field theories
- The result for the shear viscosity turns out to be universal for all such theories in the limit of infinitely strong coupling
- Influences other fields (heavy ion physics, condmat)

A hand-waving argument

$$\eta \sim \rho\, v\, l \;\sim \rho\, v^2 \tau \sim n\, m\, v^2 \tau \;\sim n\, \epsilon \tau$$

$$s \sim n$$

Thus $$\frac{\eta}{s} \sim \epsilon \tau \geq \hbar$$

Gravity duals fix the coefficient: $$\frac{\eta}{s} \geq \hbar / 4\pi$$

Outlook

- Gravity dual description of thermalization ?
- Gravity duals of theories with fundamental fermions:
  - phase transitions
  - heavy quark bound states in plasma
  - transport properties
- Finite ‘t Hooft coupling corrections to photon emission spectrum
- Understanding 1/N corrections
- Phonino

Equations such as $R_{\mu\nu} - 2\nabla_\mu \nabla_\nu \Phi + \frac{1}{4} H_\mu^{\lambda\rho} H_{\nu\lambda\rho} + O(\alpha') = 0$

describe the low energy $E \ll 1/l_s$ limit of string theory

As long as the dilaton is small, and thus the string interactions are suppressed, this limit corresponds to classical 10-dim Einstein gravity coupled to certain matter fields such as Maxwell field, p-forms, dilaton, fermions

Validity conditions for the classical (super)gravity approximation

- curvature invariants should be small: $\mathcal{R} \sim R_{\mu\nu\lambda\sigma} R^{\mu\nu\lambda\sigma} \ll 1/l_s^4$

- quantum loop effects (string interactions = dilaton) should be small: $g_s \ll 1$

In AdS/CFT duality, these two conditions translate into

$$\lambda \equiv g_{YM}^2 N_c \gg 1$$

and

$$N_c \gg 1$$

for $\mathcal{N} = 4$ supersymmetric $SU(N_c)$ Yang-Mills theory in 4 dim

The challenge of RHIC (continued)

Rapid thermalization ??

Large elliptic flow ★

Jet quenching ✦

Photon/dilepton emission rates ★

The bulk and the boundary in AdS/CFT correspondence

$$ds^2 = \frac{\eta_{\mu\nu}\, dx^\mu\, dx^\nu + dz^2}{z^2}$$

UV/IR: the AdS metric is invariant under $z \to \Lambda\, z \quad x \to \Lambda\, x$

z plays a role of inverse energy scale in 4D theory