Chaos in the Gauge/Gravity Correspondence

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The main question

- Semiclassical Physics Makes Sense
- Classical Trajectories are Important
- What is the meaning of it all? What is the meaning on the full Phase Space?
- A window into non-equilibrium physics

Outline

- The Gauge/Gravity Correpondence: Regge, 't Hooft, Maldacena
- Operator/State correspondence in the AdS/CFT
- More Operator/State: Regge trajectories, BMN, GKP
- From Trajectory to Phase Space
- Beyond Integrability.
- What is chaos?
- What is the meaning of Chaos in the meaning of Chaos in the Gauge/Gravity Correspondence
- outlook

Regge Trajectories

- Hadronic Physics was a string theory already!
- Regge trajectories are best explain by spectrum of a rotating string: $J \sim M^2$.

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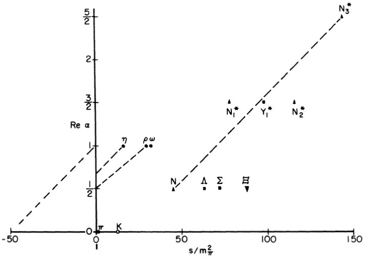
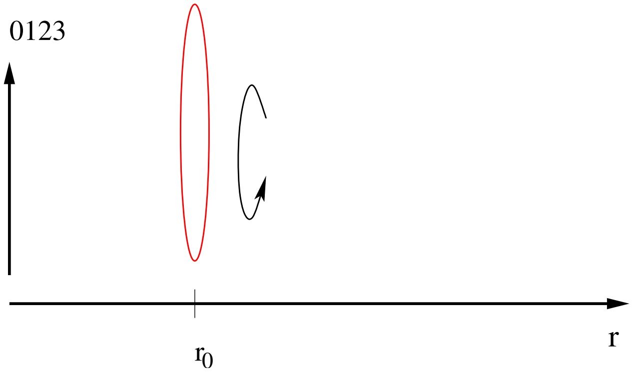


FIG. 1. The spin of particles of baryon number less than two, plotted against the square of their mass in units of m_π^2. In order to give a rough indication of slopes, the dashed lines connect pairs of points supposedly on the same trajectories, as explained in the text, but a strict linear behavior of the trajectories is not to be inferred.


0123
r
r_0

AdS/CFT Correspondence

$$Z_{FT}[J] = Z_{String}[\phi]. \quad (1)$$

• Parameters:

$g_{YM}^2 = 4\pi\, g_{string}$

$N = \int_{S^5} F_5$

$R_{S^5} = R_{AdS_5} = (g_{YM}^2 N)^{1/4} l_s$

• Same superconformal symmetry

$SO(2,4) \times SO(6) \subset SU(2,2|4)$

• States on AdS $=$ Operators in the CFT

Single Particle state $\leftrightarrow$ single Trace $TrFX...$

Multi-Particle $\leftrightarrow$ Multitrace $Tr(FX...X)Tr(F...X..X)$

Chiral primaries (protected, BPS) $\leftrightarrow$ Supergravity modes

Berenstein-Maldacena-Nastase

Almost BPS operators

$$\mathcal{O}^J = \frac{1}{\sqrt{JN^J}} \mathrm{Tr}\ Z^J. \tag{2}$$

$$Z = \phi_1 + i\phi_2. \tag{3}$$

$$\Delta - J = \sum_{-\infty}^{\infty} N_n \sqrt{1 + \frac{\lambda}{J^2} n^2}. \tag{4}$$

$$\mathcal{O}^J_{n,-n} = \frac{1}{\sqrt{JN^{J+2}}} \sum_{l=0}^{J} e^{\frac{2\pi i n}{J} l} \mathrm{Tr} \left(\phi Z^l \psi Z^{J-l} \right). \tag{5}$$

S^5
ω

Twist-two Operators Past and Present

- Twist-two operators Gross-Wilczek (DIS)

$$\mathrm{Tr}\bar{\Psi}\nabla_{(a_1}\ldots\nabla_{a_n)}\Psi, \Delta - S = f(\lambda)\ln n. \tag{6}$$

- Twist-two operators in AdS/CFT: GKP, Kruczenski, Makeenko

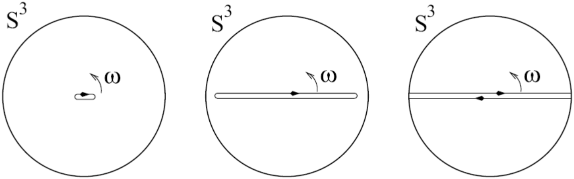
$$\mathrm{Tr}\Phi\nabla_{(a_1}\ldots\nabla_{a_n)}\Phi, \tag{7}$$

$$\Delta - S = \frac{\sqrt{\lambda}}{\pi}\ln S. \tag{8}$$

- A long production about the precise spectrum of these operators.

$$
\begin{aligned}
L &= -4\frac{R^2}{2\pi\alpha'}\int_0^{\rho_0} d\rho\sqrt{\cosh^2\rho - (\dot{\phi})^2\sinh^2\rho}, \quad \coth^2\rho_0 = \dot{\phi} = \omega^2, \\
E &= 4\frac{R^2}{2\pi\alpha'}\int_0^{\rho_0} d\rho\frac{\cosh^2\rho}{\sqrt{\cosh^2\rho - \omega^2\sinh^2\rho}} \\
S &= 4\frac{R^2}{2\pi\alpha'}\int_0^{\rho_0} d\rho\frac{\omega\sinh^2\rho}{\sqrt{\cosh^2\rho - \omega^2\sinh^2\rho}}
\end{aligned} \tag{9}
$$

$$
\Delta - S = \frac{\sqrt{\lambda}}{\pi}\ln S. \tag{10}
$$



General Properties

- Classical conserved quantities $\equiv$ Quantum Numbers
- What matters are the conserved quantities not the trajectories
- Q: What is the meaning of the trajectory more generally, is it just to give the quantum numbers?
- It is natural to study the whole phase space, the space of all possible trajectories

Setup

$$\mathcal{L} = \frac{1}{2}\sqrt{g}g^{ab}G_{\mu\nu}\partial_a X^\mu \partial_b X^\nu .$$

$$ds^2 = -f dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta d\psi^2 + \cos^2\theta d\phi^2\right),$$

$$f(r) = 1 + \frac{r^2}{b^2}, \quad f(r) = 1 + \frac{r^2}{b^2} - \frac{w_4 M}{r^2}$$

$$t = t(\tau),\ r = r(\tau),\ \theta = \theta(\tau),\ \phi = \phi(\tau),\ \psi = \alpha\sigma,$$

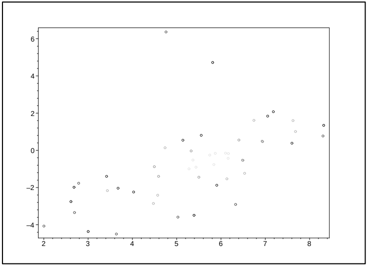
$$\mathcal{L} = \frac{1}{2}f\dot{t}^2 - \frac{\dot{r}^2}{2f} - \frac{r^2}{2}\left(\dot{\theta}^2 + \cos^2\theta\dot{\phi}^2\right) + \frac{r^2}{2}\alpha^2\sin^2\theta$$

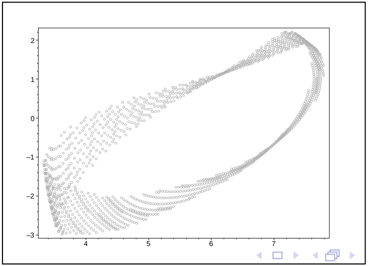
$$
\begin{aligned}
H_r &= \frac{f}{2}p_r^2 + \frac{1}{2r^2}p_\theta^2 + \frac{l^2}{2r^2\cos^2\theta} + \frac{\alpha^2}{2}r^2\sin^2\theta - \frac{E^2}{2f}, \\
\dot{r} &= -f p_r, \\
\dot{p_r} &= \frac{E^2}{2f^2}f' + \frac{f'}{2}p_r^2 - \frac{1}{r^3}p_\theta^2 - \frac{l^2}{r^3\cos^2\theta} + \alpha^2 r\sin^2\theta, \\
\dot{\theta} &= -\frac{1}{r^2}p_\theta, \\
\dot{p}_\theta &= \frac{l^2}{r^2}\frac{\sin\theta}{\cos^3\theta} + \alpha^2 r^2 \sin\theta\cos\theta, \\
H_r &= 0, \qquad \text{Constraint}
\end{aligned}
\tag{11}
$$

What is chaos?

- No water-tight definition: Sensitibity to the initial conditions
- Largest Lyapunov Exponent.
- Poincaré sections: Breaking of the Kolmogorov-Arnold-Moser tori.
- Power Spectrum

- Poincaré sections and the destruction of the KAM torus



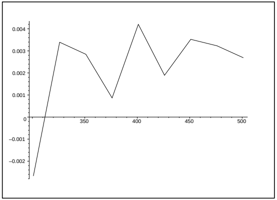


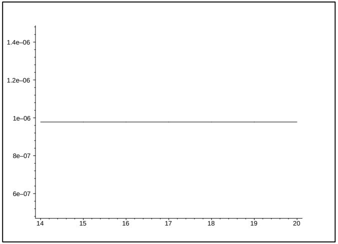
Lyapunov Exponent and the Poincaré Recurrence Time

$$\delta\vec{X}(t) = e^{\vec{\lambda}t}\delta\vec{X}(0). \tag{12}$$

$$\boxed{t_{PR} = \frac{1}{\lambda.}} \tag{13}$$

Largest Lyapunov Exponet: Precision



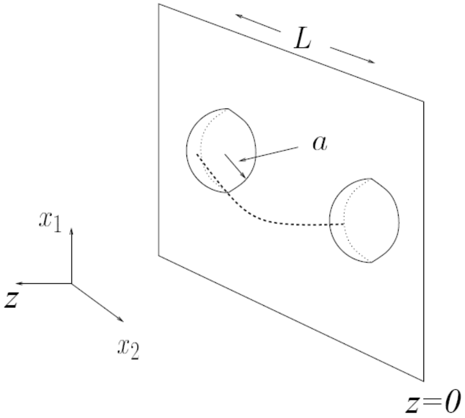


The distance between two strings

- Berenstein-Corrado-Fischler-Maldacena: Correlations of two circular Wilson Loops $\equiv$ exchange of light states

$$\frac{\langle W(\mathcal{C},L)W(\mathcal{C},0)\rangle}{\langle W(\mathcal{C},L)\rangle\langle W(\mathcal{C},0)\rangle} = \sum_{i,j;m,n} c_i^{(m)} c_j^{(n)}\, a^{\Delta_i^{(m)}+\Delta_j^{(n)}} \langle \mathcal{O}_i^{(m)}(L)\mathcal{O}_j^{(n)}(0)\rangle$$

$$= \sum_i \left(c_i^{(0)}\right)^2 \frac{a^{2\Delta_i^{(0)}}}{L^{2\Delta_i^{(0)}}} + \sum_{i,\{m,n\}\neq\{0,0\}} c_i^{(m)} c_i^{(n)}\, a^{\Delta_i^{(m)}+\Delta_i^{(n)}} \langle \mathcal{O}_i^{(m)}(L)\mathcal{O}_i^{(n)}(0)\rangle.$$



L
a
x_1
z
x_2
z=0

Poincare recurrences and Unitarity

- Initial perturbation a thermal system will be damped by thermal dissipation as long as the time scale is too short to resolve possible gaps in the spectrum (Heisenberg times).
- Heisenberg time $t_H = 1/\omega$- discreteness. For $t \ll t_H$ the spectrum is approximately continuous.

$$
\begin{aligned}
A(t) &= e^{itH} A(0) e^{-itH}, \\
G_E(t) &= e^{-S(E)} \sum_{E_i, E_j \leq E} |A_{ij}|^2 e^{i(E_i - E_j)t}.
\end{aligned}
$$

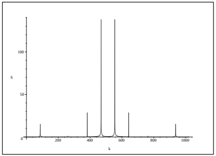
Poincare recurrences

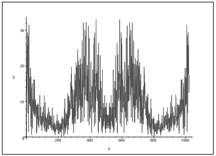
- If the matrix elements of some operator A in the energy basis have frequency with Γ, the correlator will decay with characteristic lifetime of order Γ^{-1}: $G_E(t)$ Standard dissipative behavior in $\Gamma^{-1} \ll t \ll t_H$
- For $t > t_H$ most phases in $G_E(t)$ would have completed a period and the function $G_E(t)$ starts showing irregularities; it is a quasiperiodic function of time. Despite thermal damping, it returns arbitrarily close to the initial value over periods of the order of the recurrence time.
- The new conditions: We look at strings and observe the enhancing effect of some other charge.

$$e^{-N^2} \longrightarrow e^{-N^2/J^2} \sim \mathcal{O}(1). \tag{14}$$

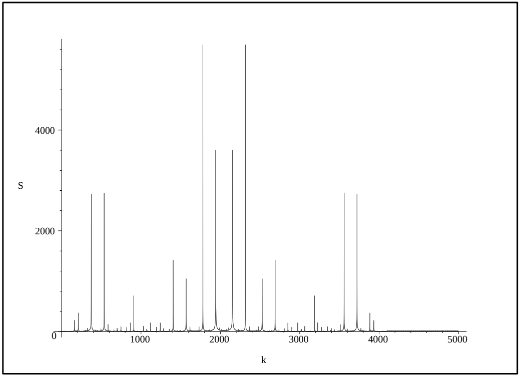
The power spectrum of Henon-Heiles

$$H = \frac{1}{2m}\left(p_x^2 + p_y^2\right) + \frac{k}{2}\left(x^2 + y^2\right) + \lambda\left(x^2\,y - \frac{1}{3}y^3\right) \quad (15)$$





Power Spectrum: Ring string in Schwarzschild-AdS



Summary

- Following the trajectories of nearby strings $\longrightarrow$ Positive Lyapunov Exponent.
- Distance between two strings $\equiv$ Correlation of function of the corresponding operators
- Late-time behavior of correlation functions $\equiv$ Poincaré Recurrences
- Lyapunov Exponent $\leftrightarrow$ Poincaré recurrence time
- Poincaré recurrences $\equiv$ Unitarity
- Unitarity: The anti-information loss.

Outlook

- A concrete **Quantitative** argument involving unitarity in black hole physics.

$$\boxed{t_{PR} = \frac{1}{\lambda.}} \qquad (16)$$

- What about other configurations? Branes moving in AdS/CFT. The $1/N$ exactness of Wilson Loops.

$$e^{-N^2} \longrightarrow e^{-N^2/S^2} \approx \mathcal{O}(1). \qquad (17)$$

- The integrability/chaos balance for point particles and for strings: KAM-theorem, Anosov story.