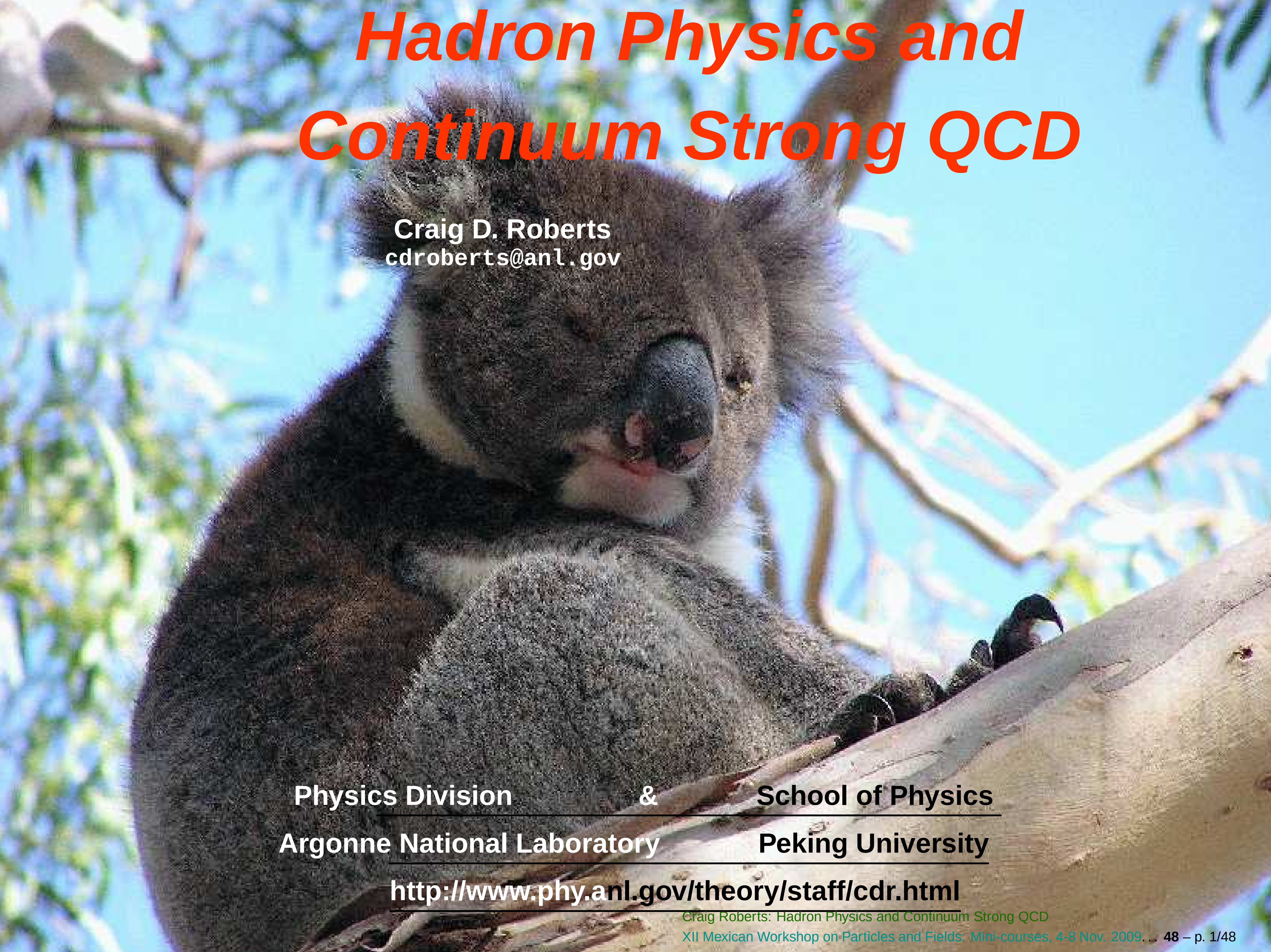

Hadron Physics and Continuum Strong QCD

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Physics Division & **School of Physics**

Argonne National Laboratory **Peking University**

http://www.phy.anl.gov/theory/staff/cdr.html

Form Factors: Why?

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- Experimental and theoretical studies of nucleon electromagnetic form factors have made rapid and significant progress during the last several years, including new data in the time like region, and material gains have been made in studying the pion form factor.
- Despite this, many urgent questions remain unanswered.

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- World's Premier Hadron Physics Facility

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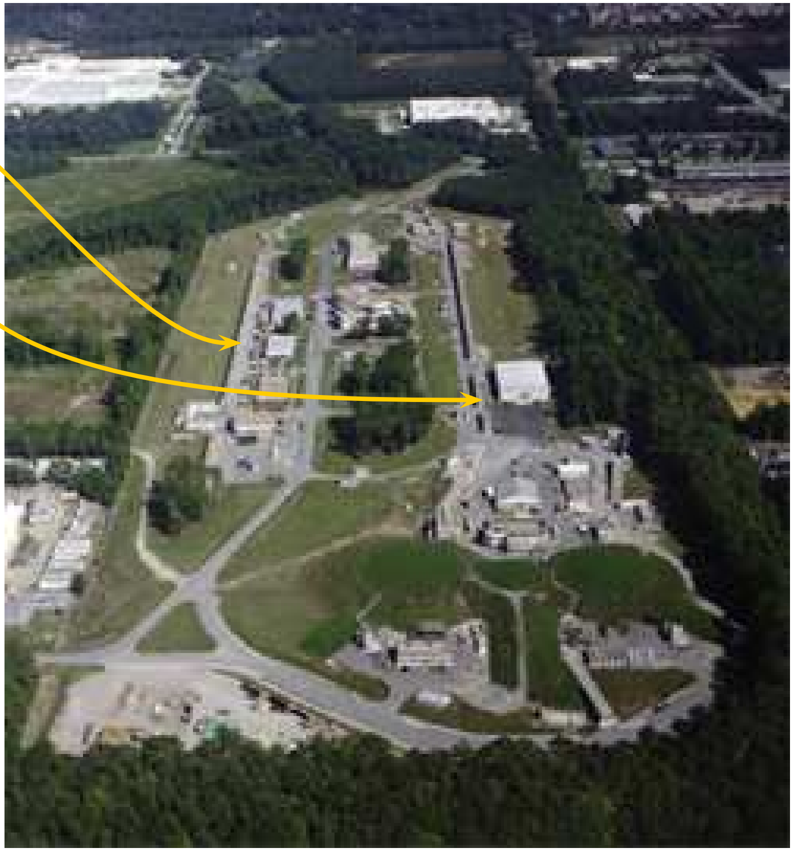

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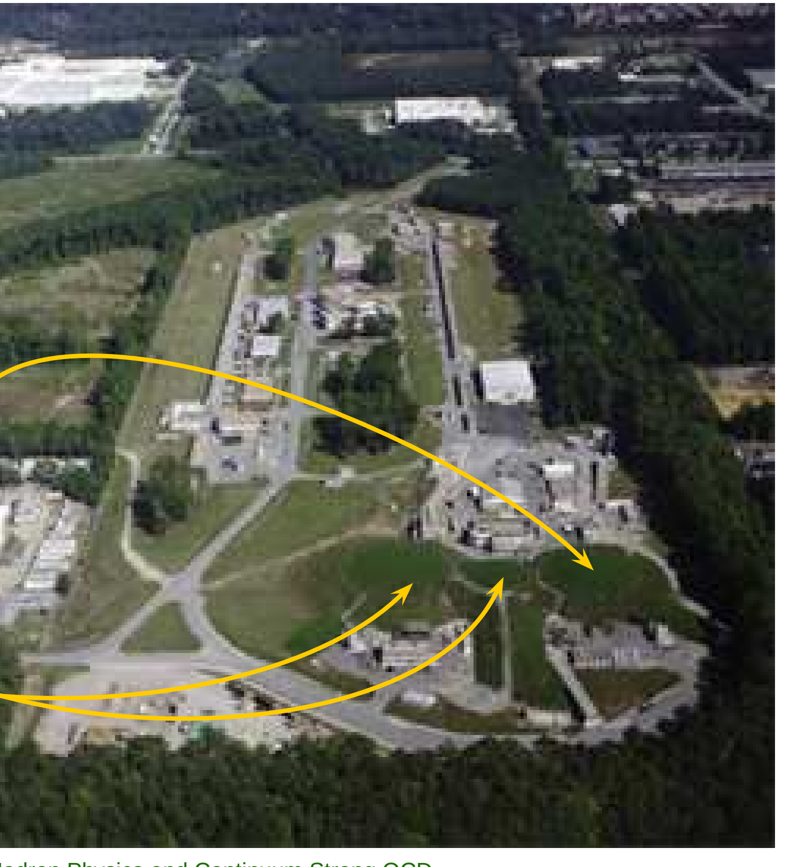
- Once desired energy is reached, Beam is directed into Experimental Halls A, B and C



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- Electrons accelerated by repeated journeys along *linacs*
- Once desired energy is reached, Beam is directed into Experimental Halls A, B and C
- Current Peak Electron Beam Energy Nearly 6 GeV

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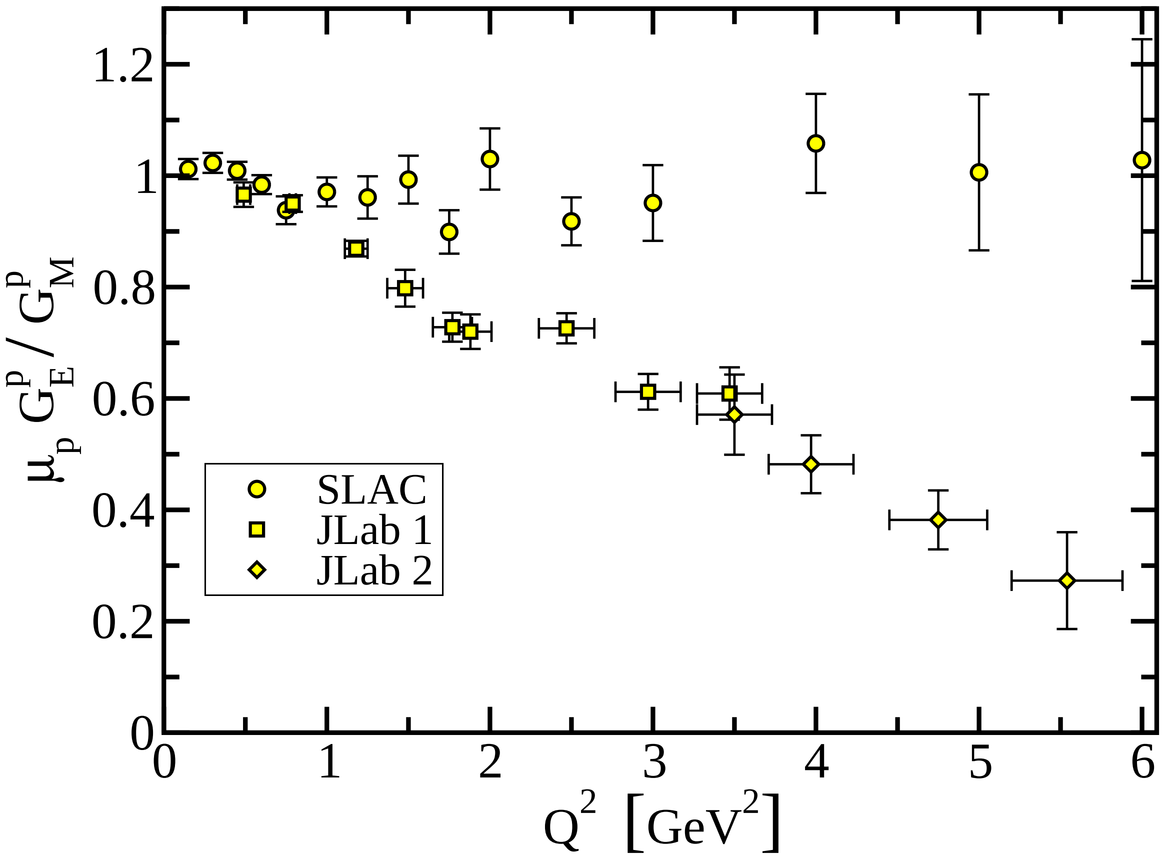
JLab Hall-A

JLab Hall-A

- Measured Ratio of

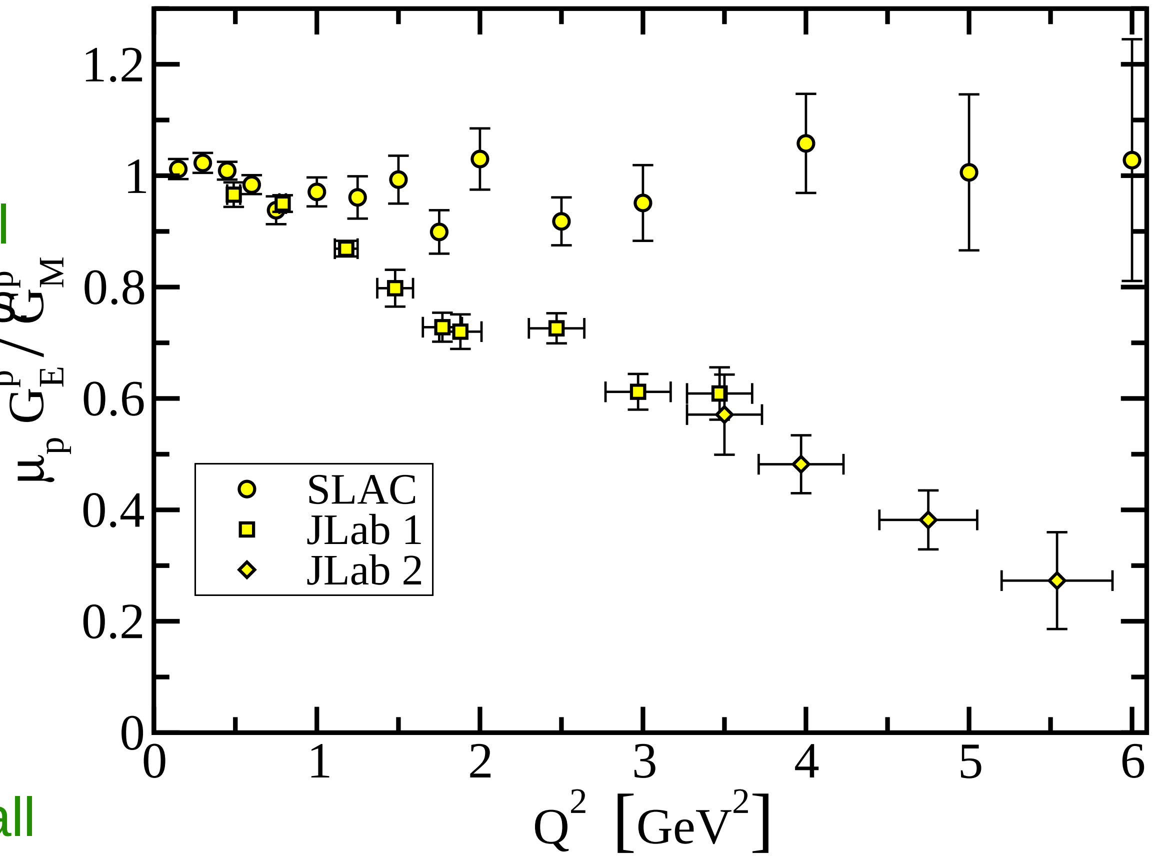
  Proton's Electric and Magnetic Form Factors

JLab Hall-A



JLab Hall-A

- Walker *et al.*, Phys. Rev. **D 49**, 5671 (1994). (SLAC)
- Jones *et al.*, JLab Hall A Collaboration, Phys. Rev. Lett. **84**, 1398 (2000)
- Gayou, *et al.*, Phys. Rev. **C 64**, 038202 (2001)
- Gayou, *et al.*, JLab Hall A Collaboration, Phys. Rev. Lett. **88** 092301 (2002)



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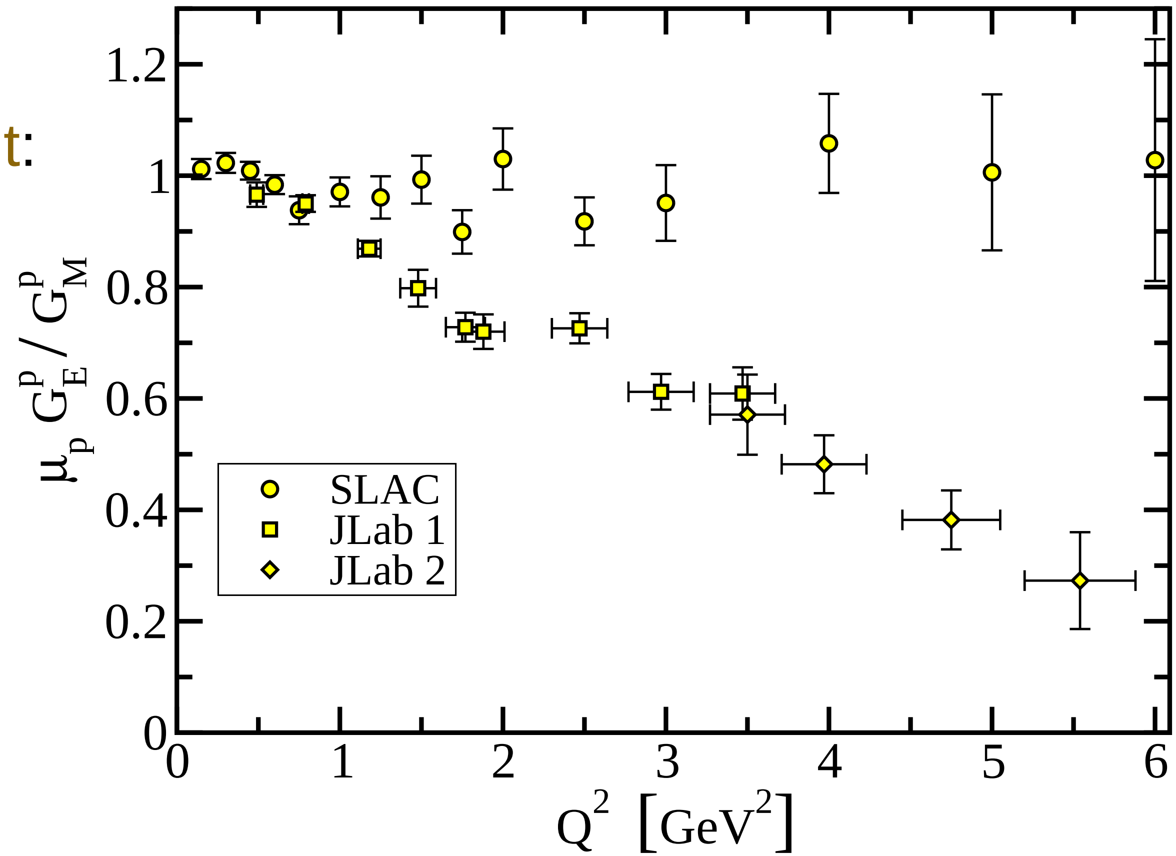
- If JLab Correct, then
  Completely
  Unexpected Result:
  In the Proton
  – On Relativistic
  Domain
  – Distribution of
  Quark-Charge
  Not Equal
  Distribution of
  Quark-Current!



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Some Questions

- What is the role of pion cloud in nucleon electromagnetic structure?
- Can we understand the pion cloud in a more quantitative and, perhaps, model-independent way?

Some Questions

- Where is the transition from non-pQCD to pQCD in the pion and nucleon electromagnetic form factors?

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Some Questions

- Do we understand the high Q^2 behavior of the proton form factor ratio in the space-like region?
- Can we make model-independent statements about the role of relativity or orbital angular momentum in the nucleon?

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Some Questions

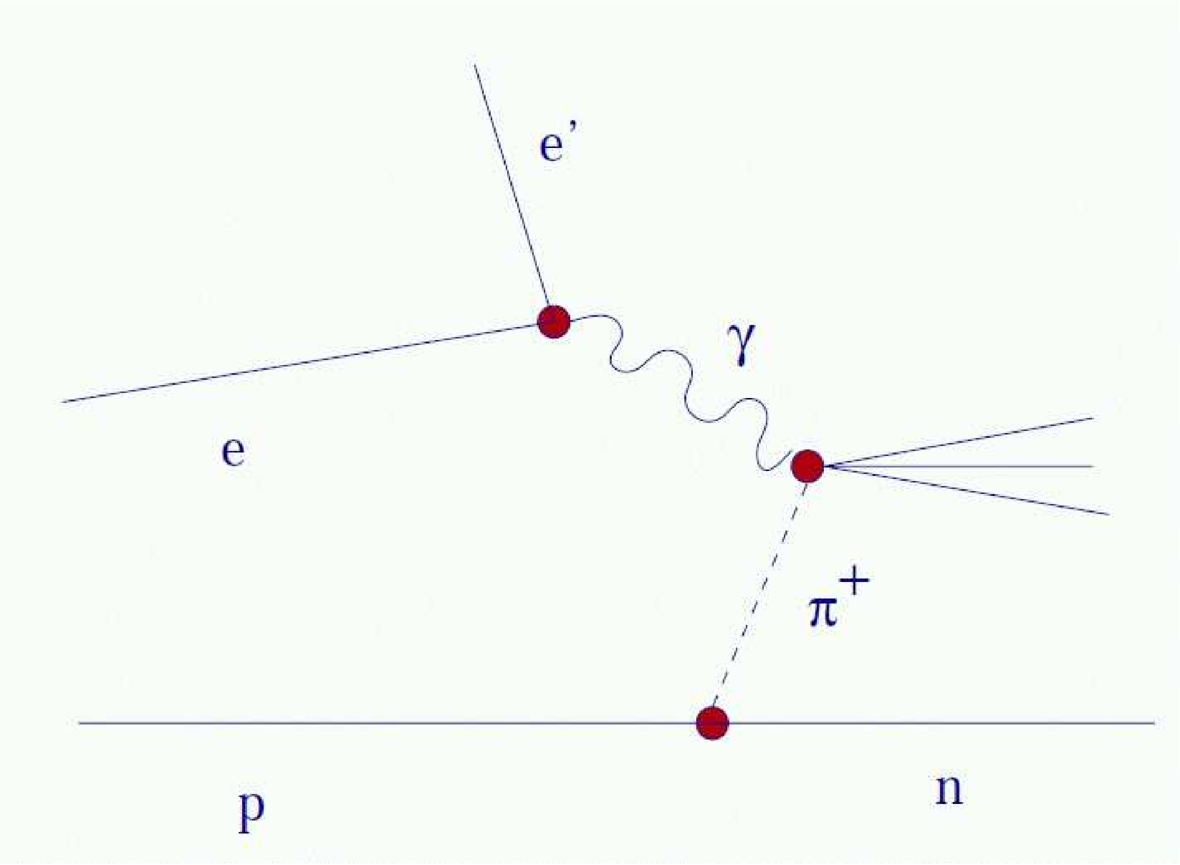
- Can we understand the rich structure of the time-like proton form factors in terms of resonances?
- What do we expect for the proton form factor ratio in the time-like region?
- What is the relation between proton and neutron form factor in the time-like region?
- How do we understand the ratio between time-like and space-like form factors?

Some Questions

- What is the role of two-photon exchange contributions in understanding the discrepancy between the polarization and Rosenbluth measurements of the proton form factor ratio?
- What is the impact of these contributions on other form factor measurements?

Some Questions

- How accurately can the pion form factor be extracted from the $ep \to e' n \pi^+$ reaction?



Status

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Status

- Current status is described in
  - J. Arrington, C. D. Roberts and J. M. Zanotti "Nucleon electromagnetic form factors," J. Phys. G **34**, S23 (2007); [arXiv:nucl-th/0611050].
  - C. F. Perdrisat, V. Punjabi and M. Vanderhaeghen, "Nucleon electromagnetic form factors," Prog. Part. Nucl. Phys. **59**, 694 (2007); [arXiv:hep-ph/0612014].

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    Prog. Part. Nucl. Phys. **59**, 694 (2007);
    [arXiv:hep-ph/0612014].
- Most recently:
  "ECT* Workshop on Hadron Electromagnetic Form Factors"
  Organisers: Alexandrou, Arrington, Friedrich, Maas, Roberts
  Presentations, etc., available on-line
  http://ect08.phy.anl.gov/

QCD's Challenges

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QCD’s Challenges

- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon

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- Dynamical Chiral Symmetry Breaking
  - Very unnatural pattern of bound state masses
    - e.g., Lagrangian (pQCD) quark mass is small but . . . no degeneracy between $J^{P=+}$ and $J^{P=-}$

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- Neither of these phenomena is apparent in QCD's Lagrangian ***yet*** they are the dominant determining characteristics of real-world QCD.

Understand Emergent Phenomena

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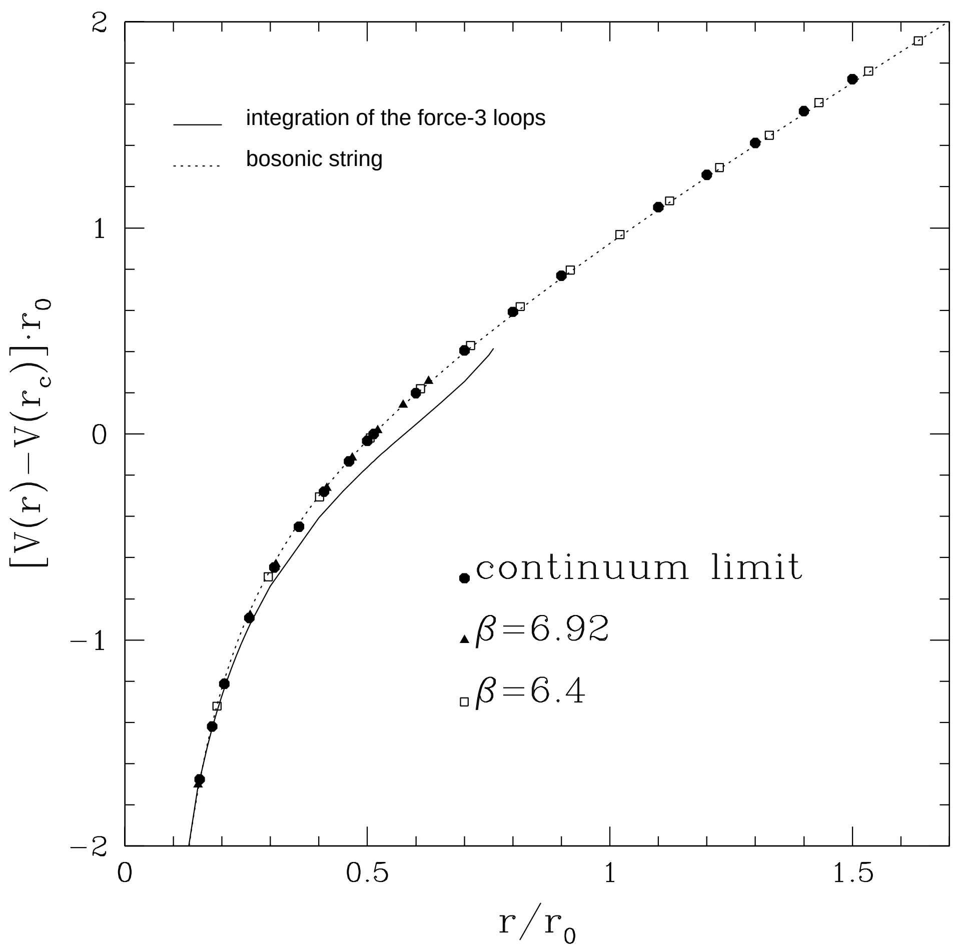
- Neither of these phenomena is apparent in QCD's Lagrangian ***yet*** they are the dominant determining characteristics of real-world QCD.
- QCD – Complex behaviour

  arises from apparently simple rules

Confinement

Confinement

- Infinitely Heavy Quarks . . . Picture in Quantum Mechanics



$$V(r) = \sigma\, r - \frac{\pi}{12}\frac{1}{r}$$

$\sqrt{\sigma} \sim 470$ MeV

Necco & Sommer
he-la/0108008

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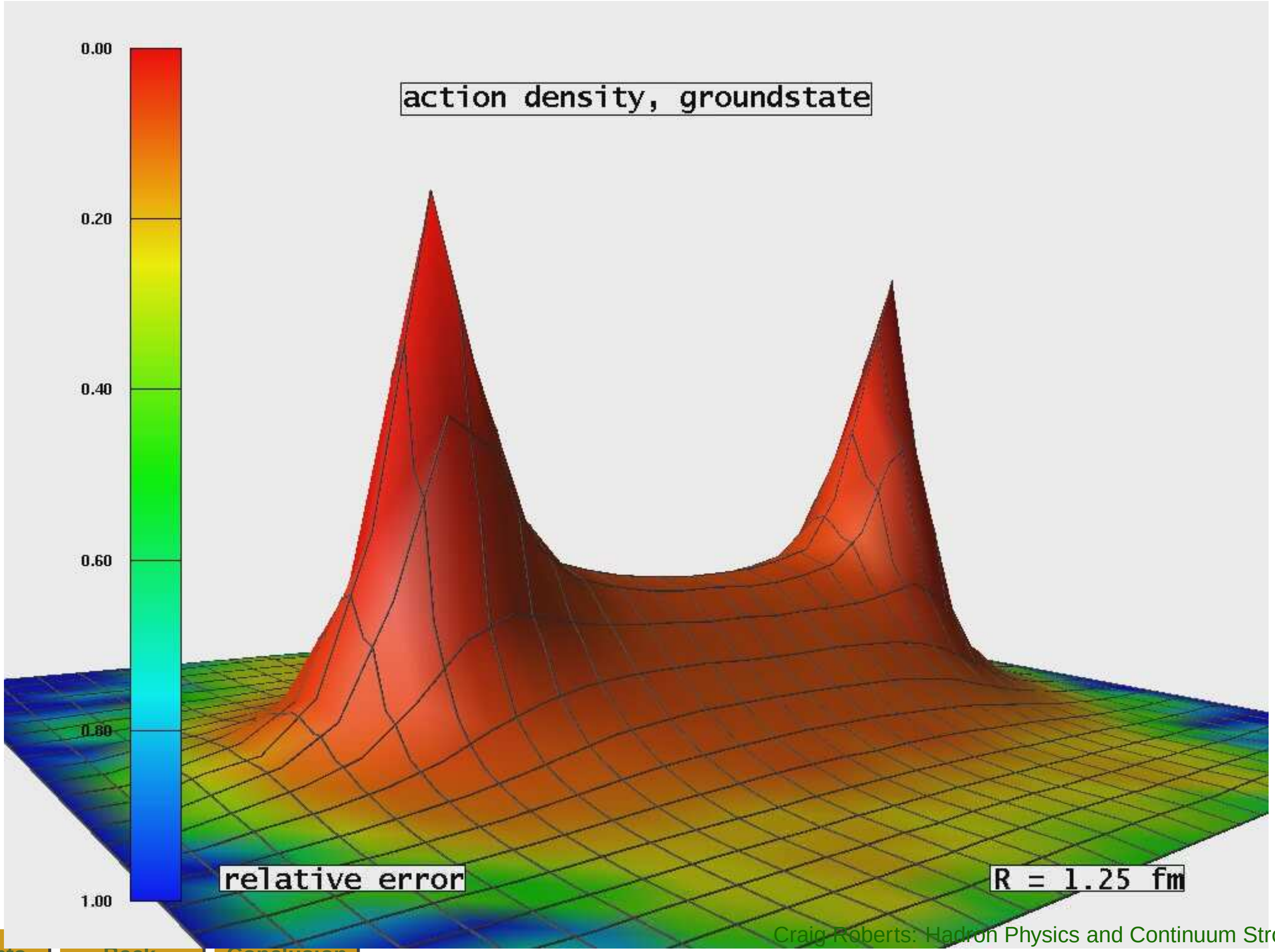
- Illustrate this in terms of the action density … analogous to plotting the Force $= F_{\bar{Q}Q}(r) = \sigma + \frac{\pi}{12}\frac{1}{r^2}$



Confinement

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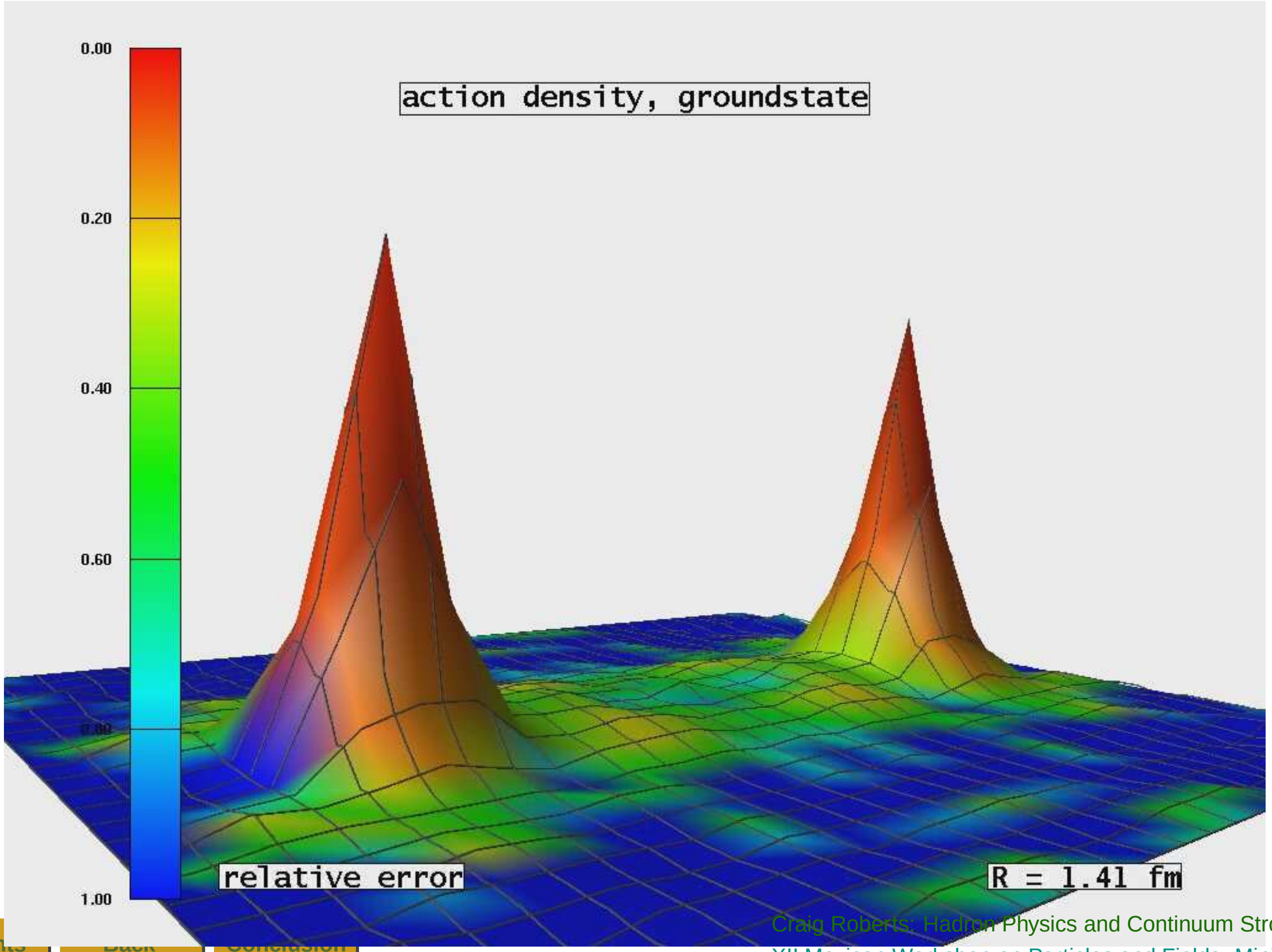

Bali, *et al*. he-la/0512018

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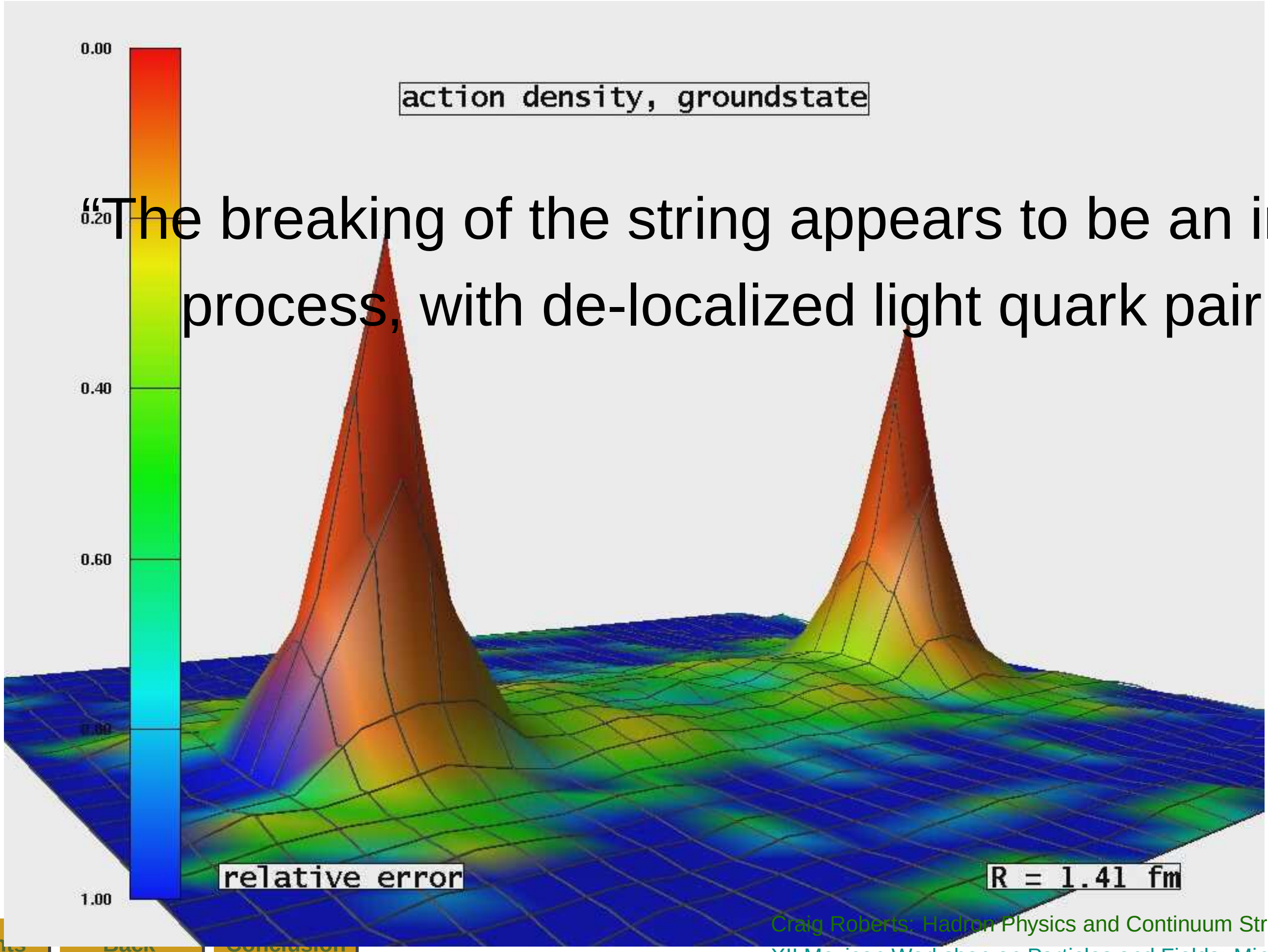

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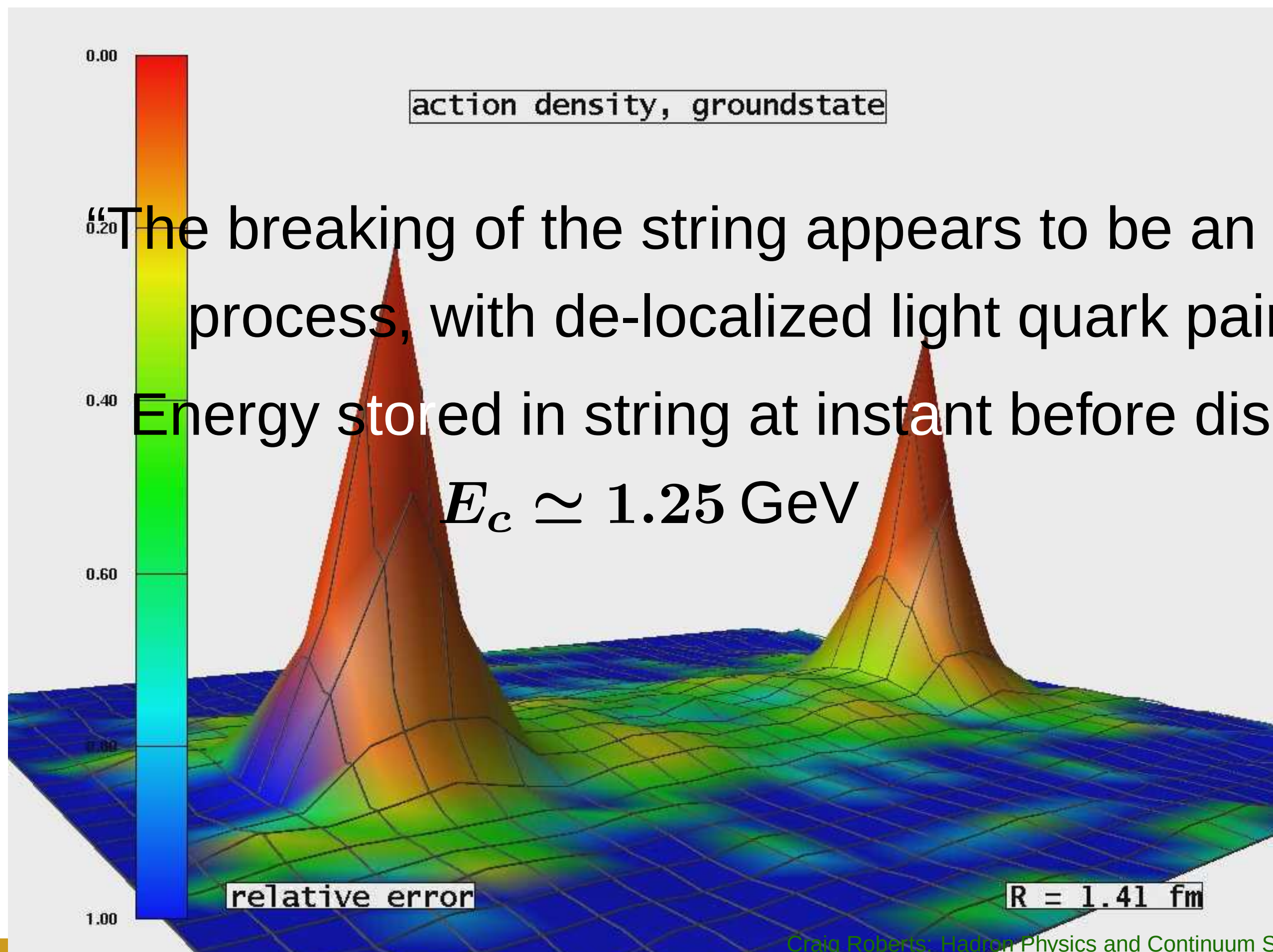

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Energy stored in string at instant before disappearance:

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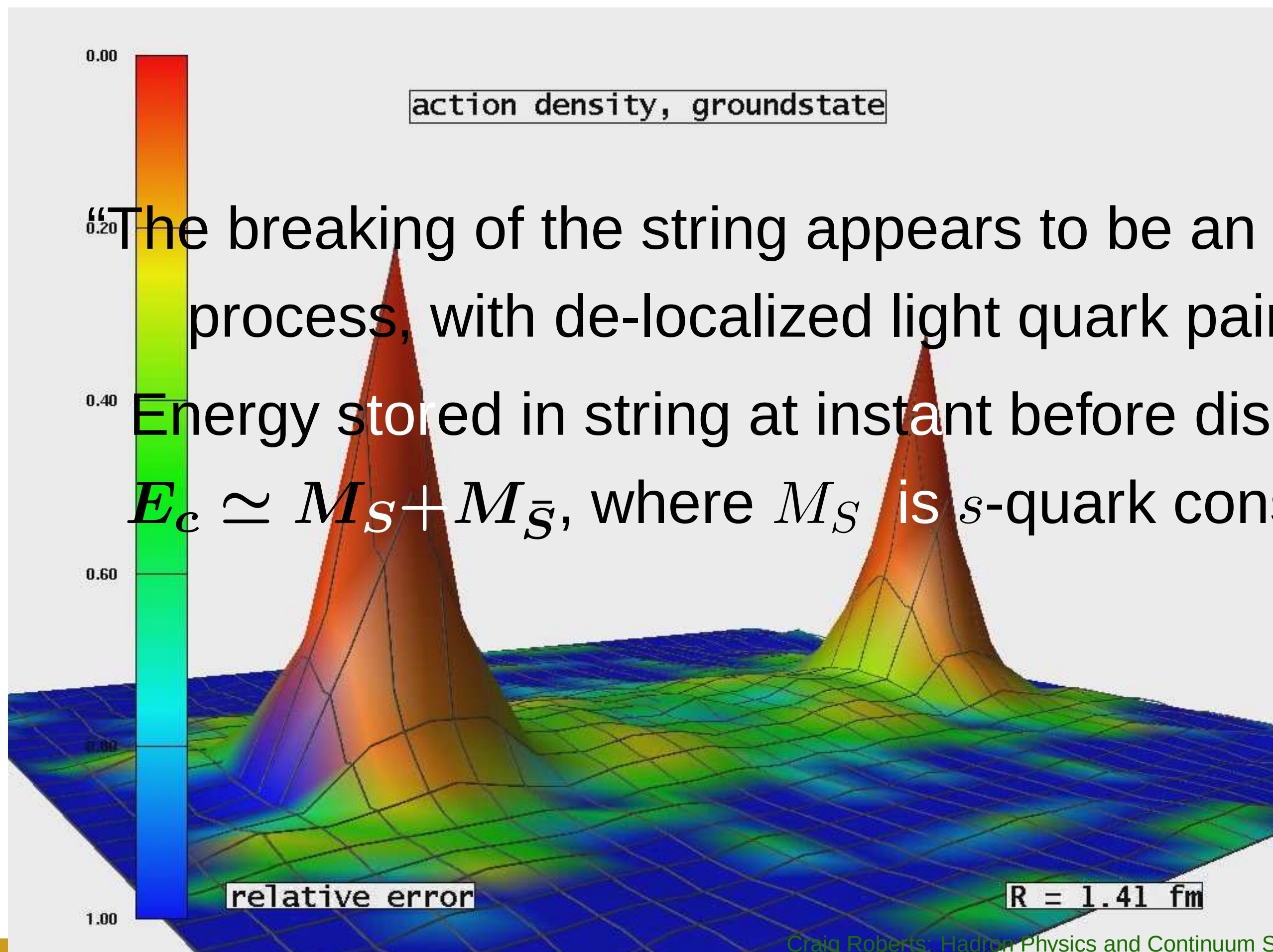

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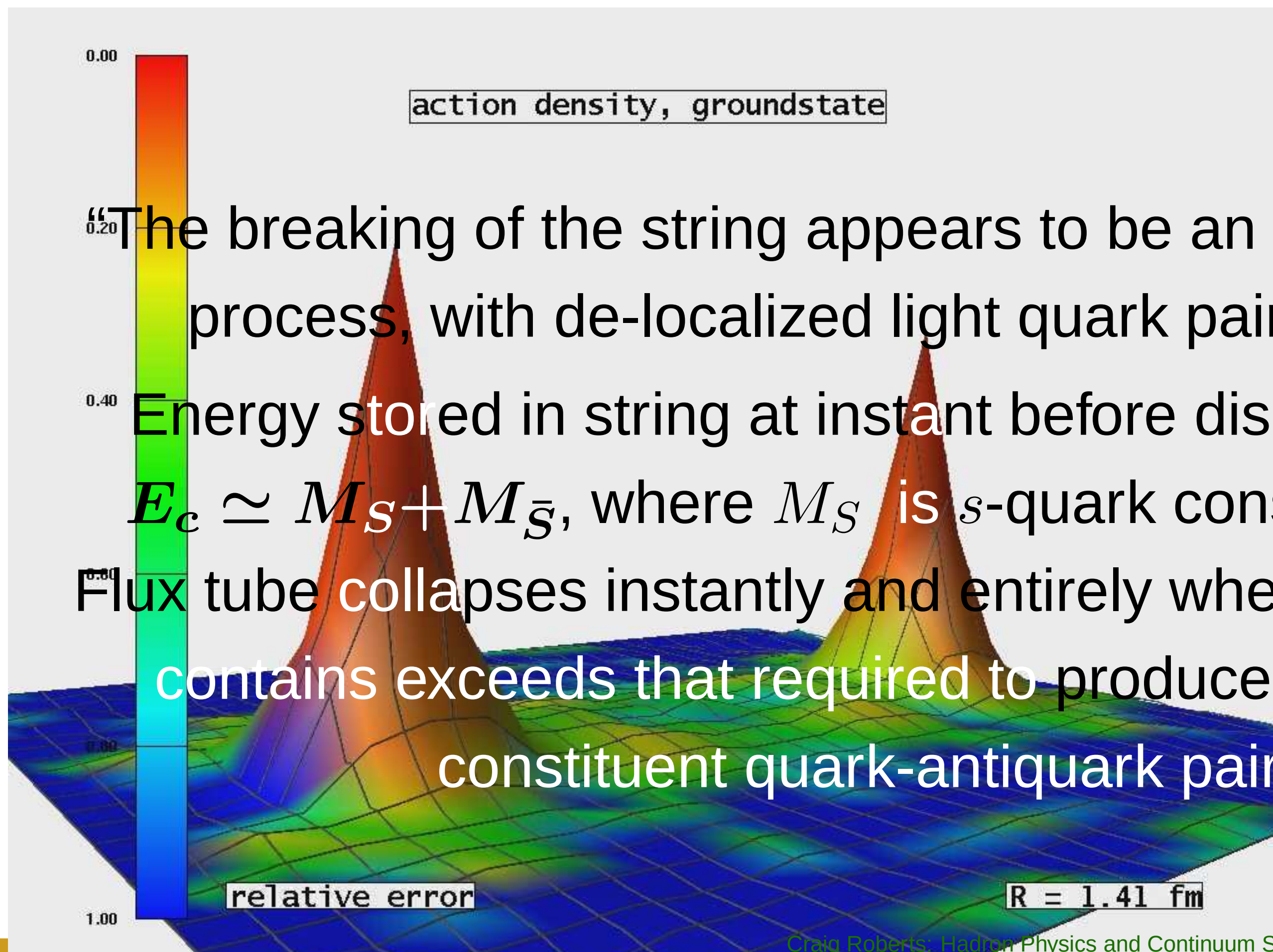

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Therefore . . . No information on *potential* between light-quarks. **Confinement**

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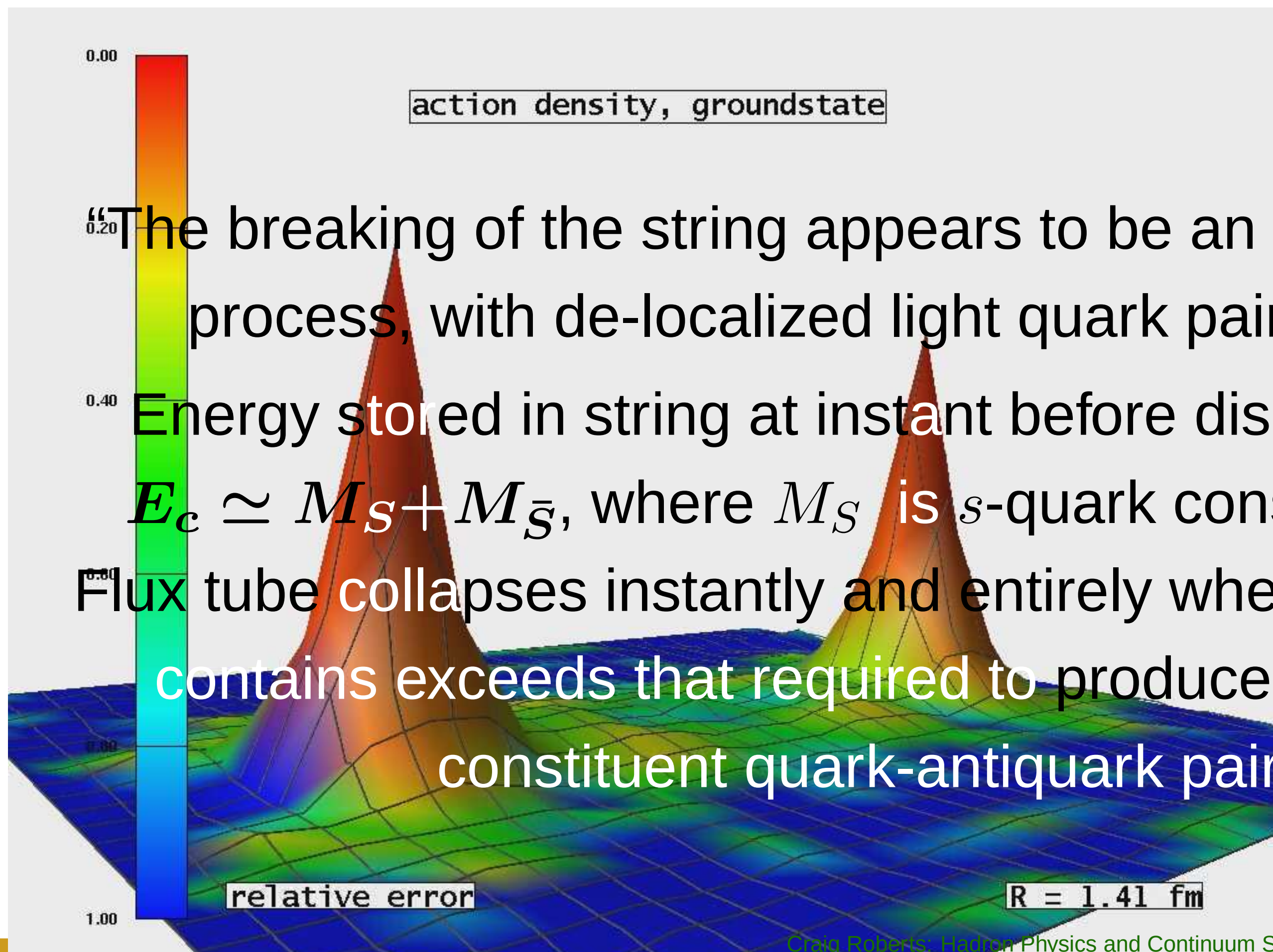

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- Method yields Schwinger Functions $\equiv$ Propagators

Cross-Sections built from Schwinger Functions

Schwinger Functions

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Schwinger Functions

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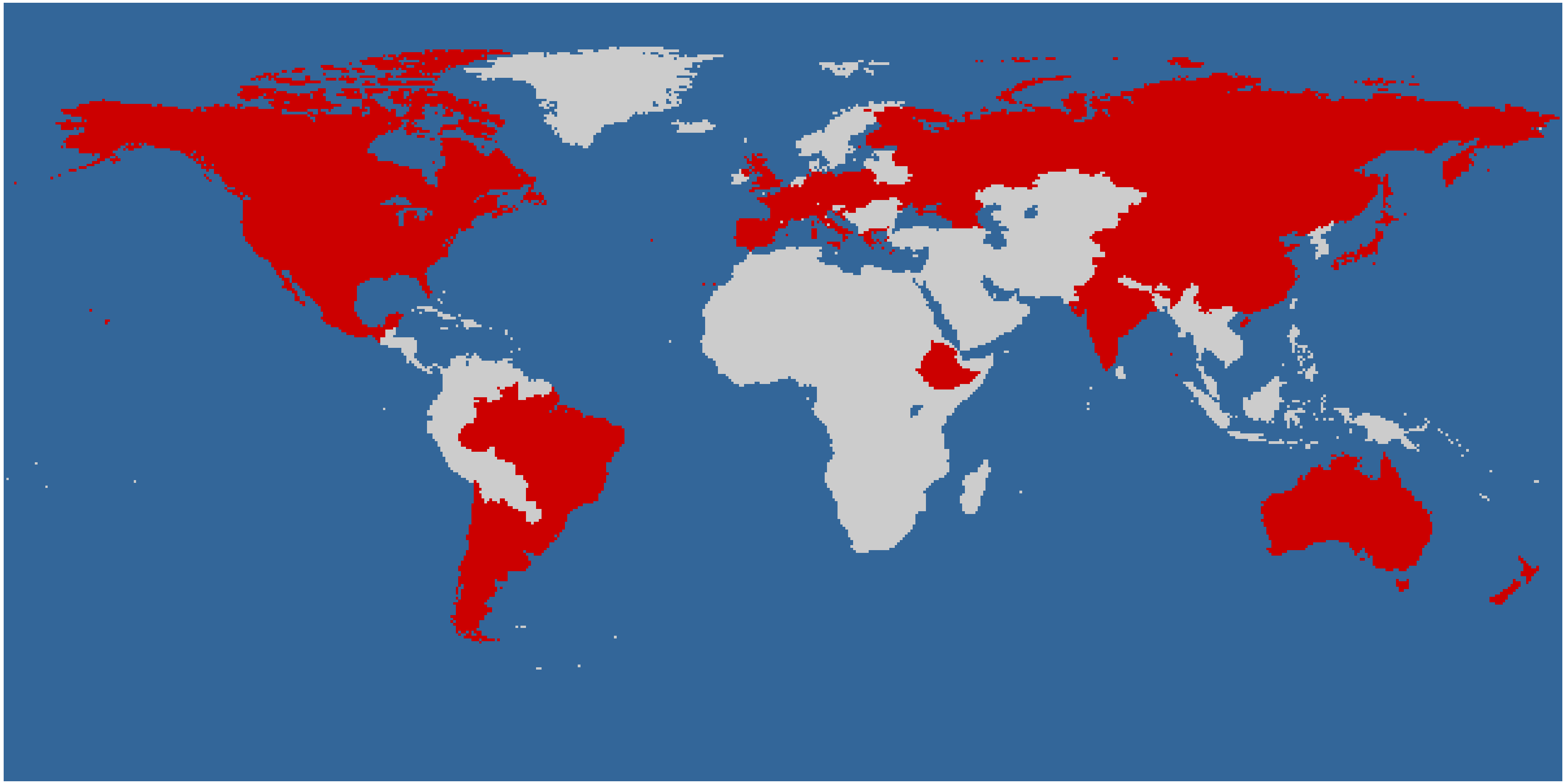
- Proving fruitful.

World . . .



Persistent Challenge

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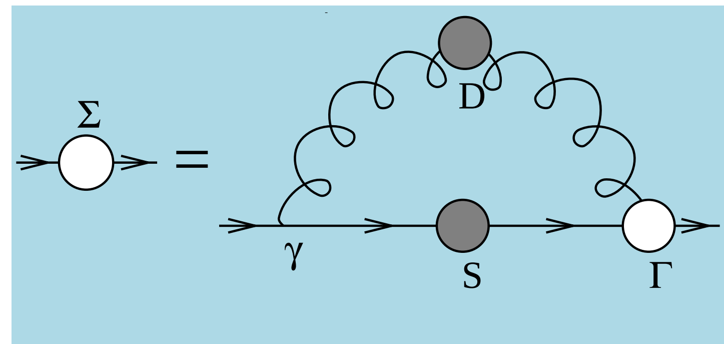
Persistent Challenge

- Infinitely Many Coupled Equations



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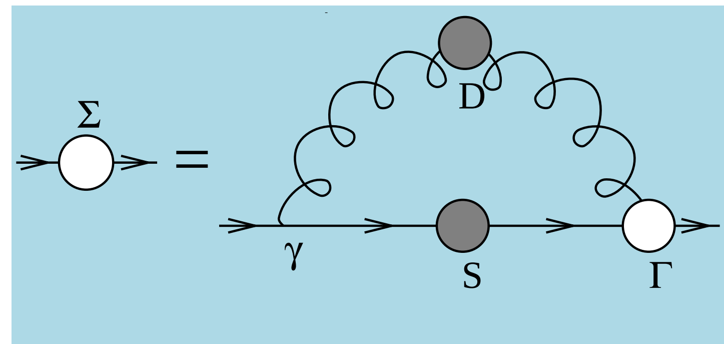

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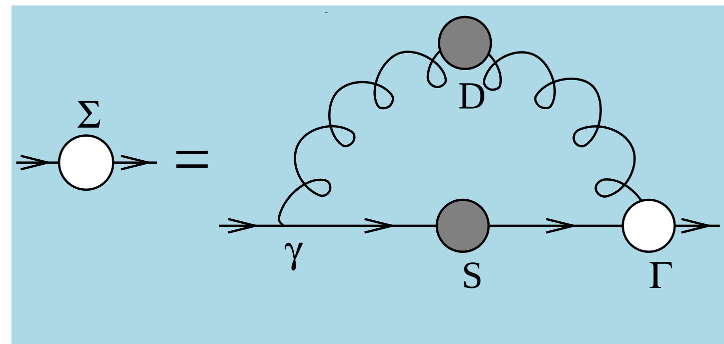

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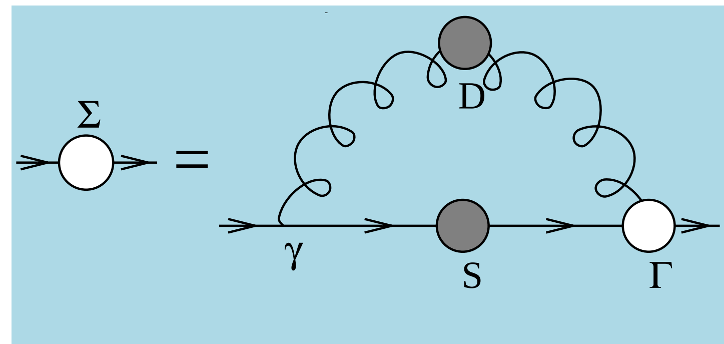

- Coupling between equations necessitates truncation
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    Not useful for the nonperturbative problems
    in which we're interested

Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme

  H.J. Munczek Phys. Rev. D **52** (1995) 4736

  Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations

  A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7

  Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation

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- Examples:

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Perturbative Dressed-quark Propagator

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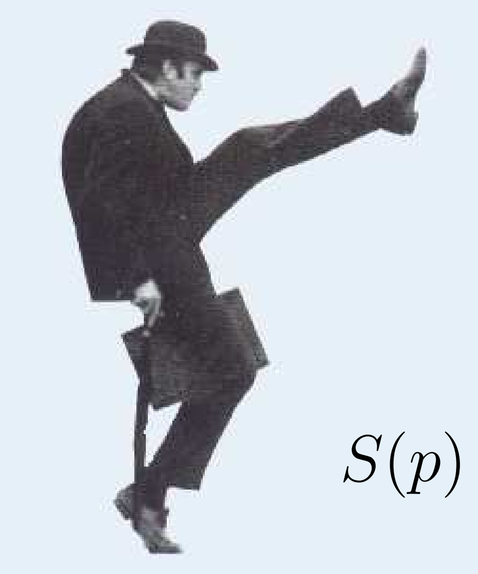
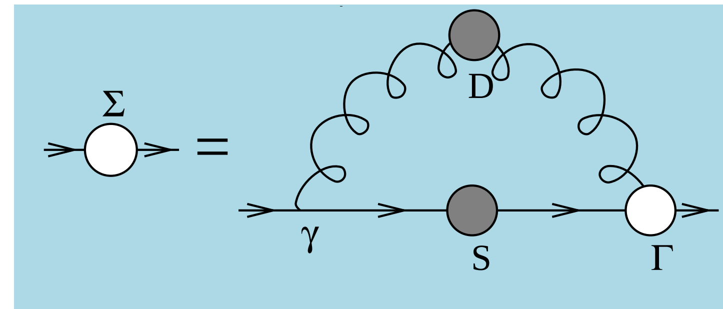
Perturbative Dressed-quark Propagator



$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

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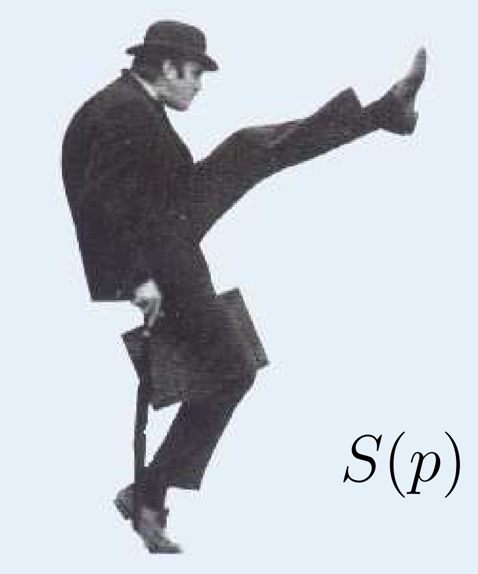

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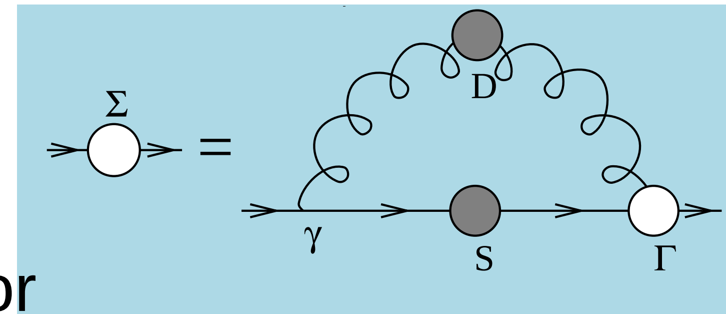

- dressed-quark propagator

Gap Equation

$$S(p) = \frac{1}{i\gamma \cdot p \, A(p^2) + B(p^2)}$$

Perturbative Dressed-quark Propagator

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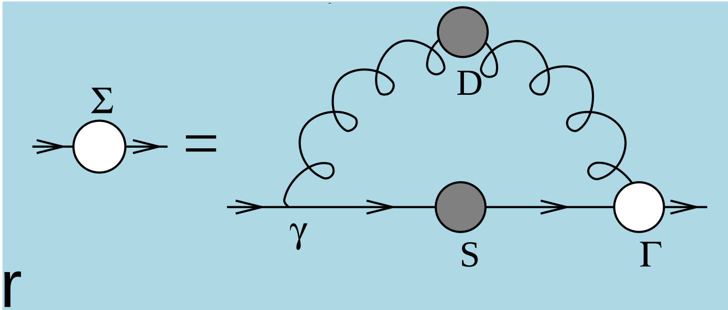

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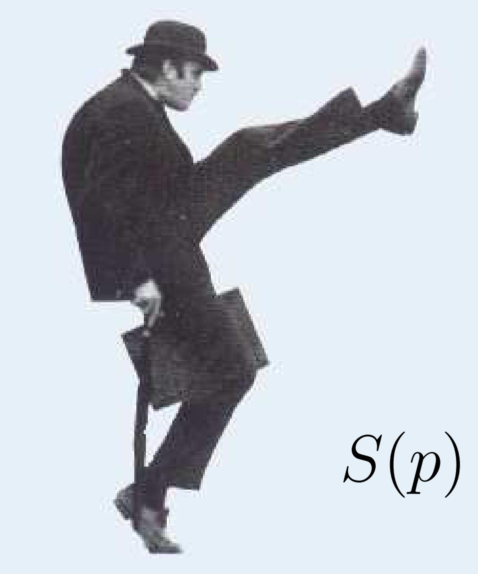
- Weak Coupling Expansion

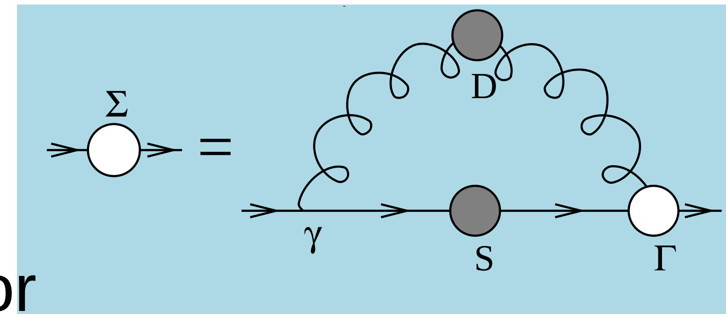
  Reproduces Every Diagram in Perturbation Theory



Perturbative Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$

- dressed-quark propagator



Gap Equation

$$S(p) = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$$

- Weak Coupling Expansion

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- But in Perturbation Theory

$$B(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \stackrel{m\to 0}{\to} 0$$

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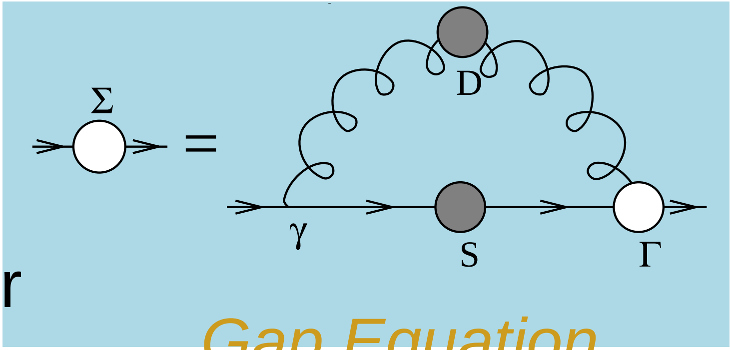

Gap Equation

- dressed-quark propagator

$$S(p) = \frac{1}{i\gamma \cdot p\, A(p^2) + B(p^2)}$$

No DCSB Here!

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Explanation?

QCD & Interaction Between Light-Quarks

- Kernel of Gap Equation: $D_{\mu\nu}(p-q)\,\Gamma_\nu(q)$
  Dressed-gluon propagator and dressed-quark-gluon vertex
- Reliable DSE studies of Dressed-gluon propagator:
  - R. Alkofer and L. von Smekal, *The infrared behavior of QCD Green's functions* . . . , Phys. Rept. **353**, 281 (2001).

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- Dressed-gluon propagator – lattice-QCD simulations confirm that behaviour:
  - D. B. Leinweber, J. I. Skullerud, A. G. Williams and C. Parrinello [UKQCD Collaboration], *Asymptotic scaling and infrared behavior of the gluon propagator*, Phys. Rev. D **60**, 094507 (1999) [Erratum-ibid. D **61**, 079901 (2000)].
- Exploratory DSE and lattice-QCD studies of dressed-quark-gluon vertex

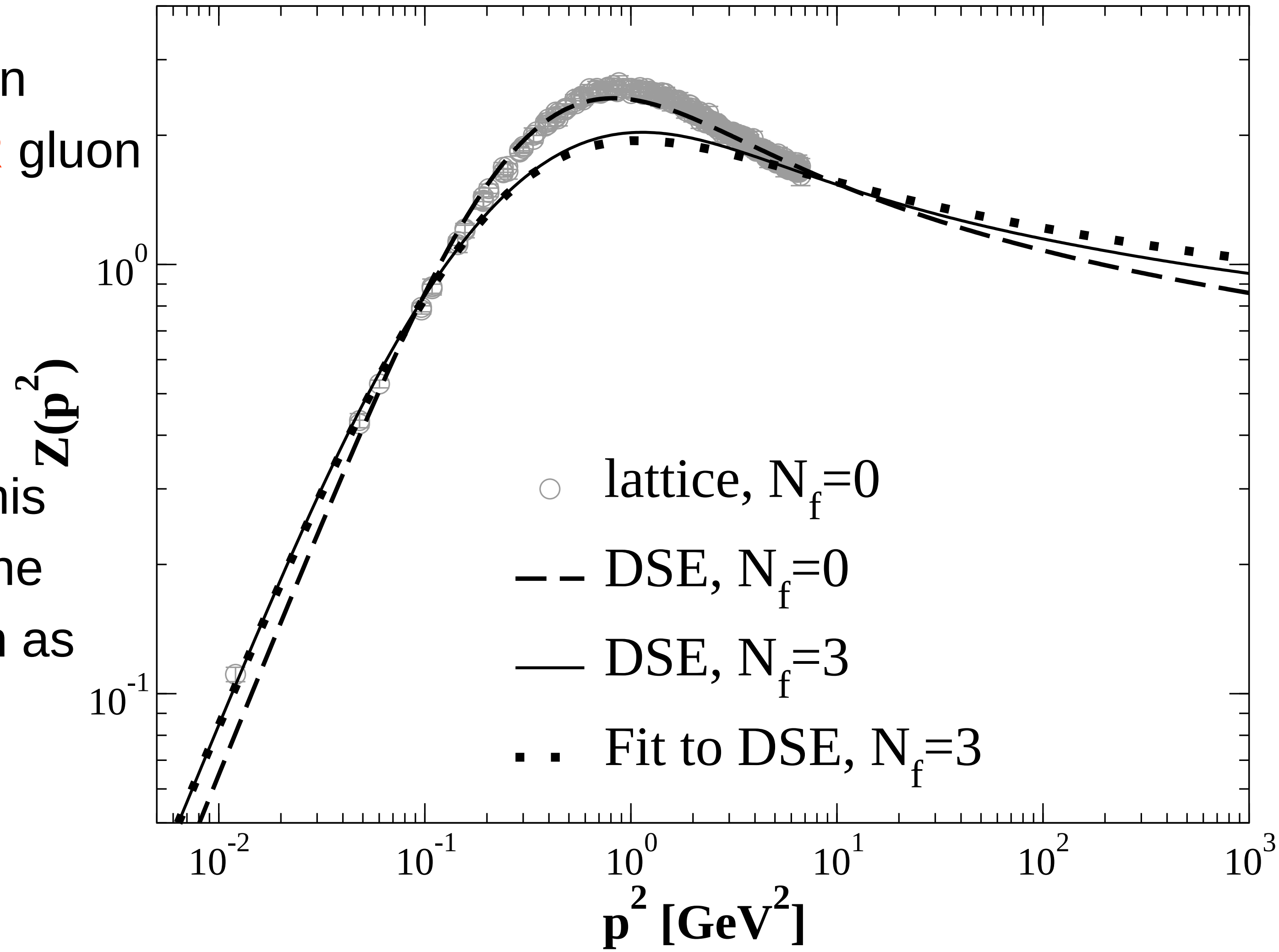
Alkofer, Detmold, Fischer, Maris: he-ph/0309078

Dressed-gluon Propagator

- $D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \frac{Z(k^2)}{k^2}$

- Suppression means $\exists$ IR gluon mass-scale $\approx 1\,\text{GeV}$

- Naturally, this scale has the same origin as Λ_{QCD}



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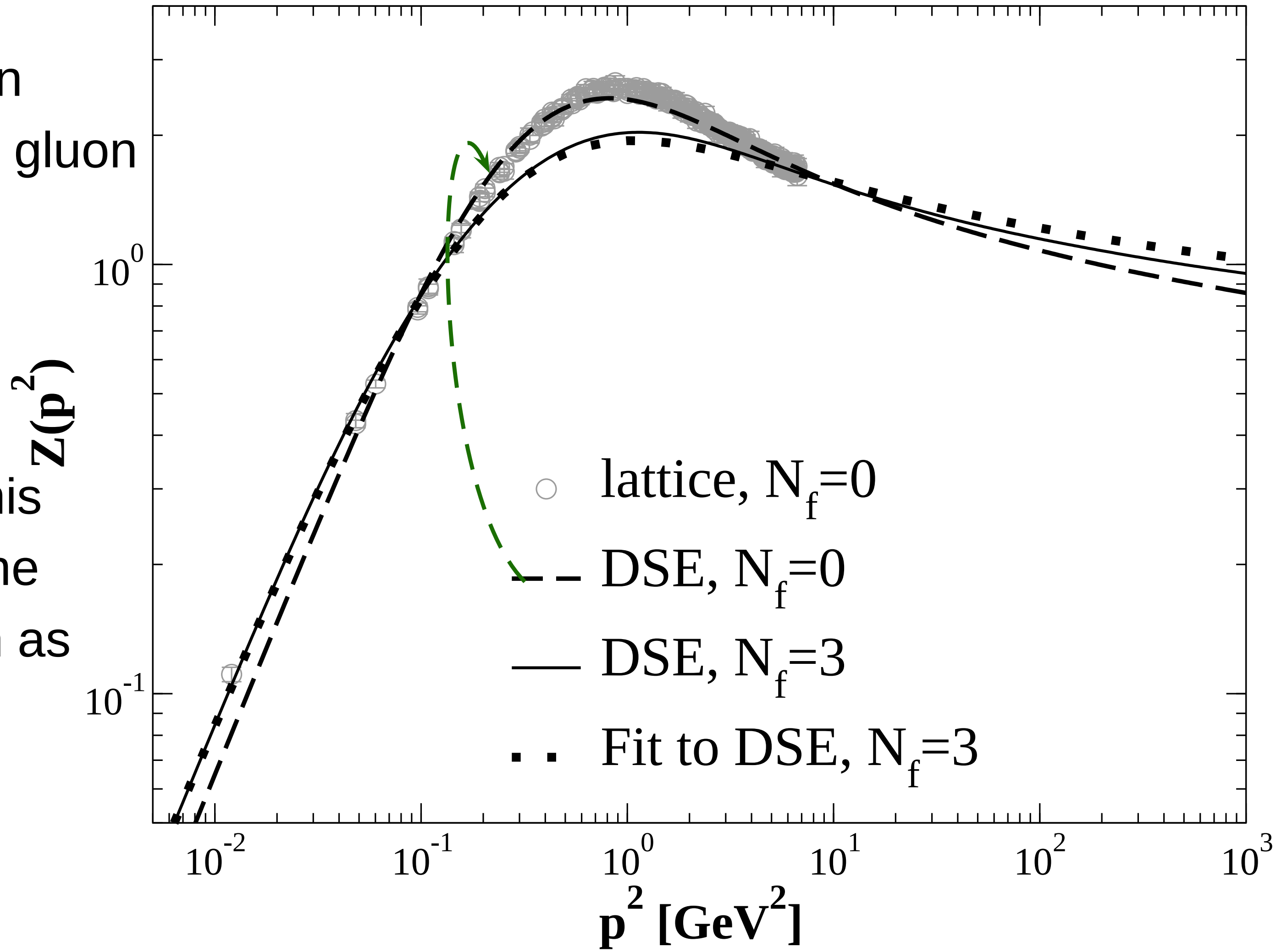

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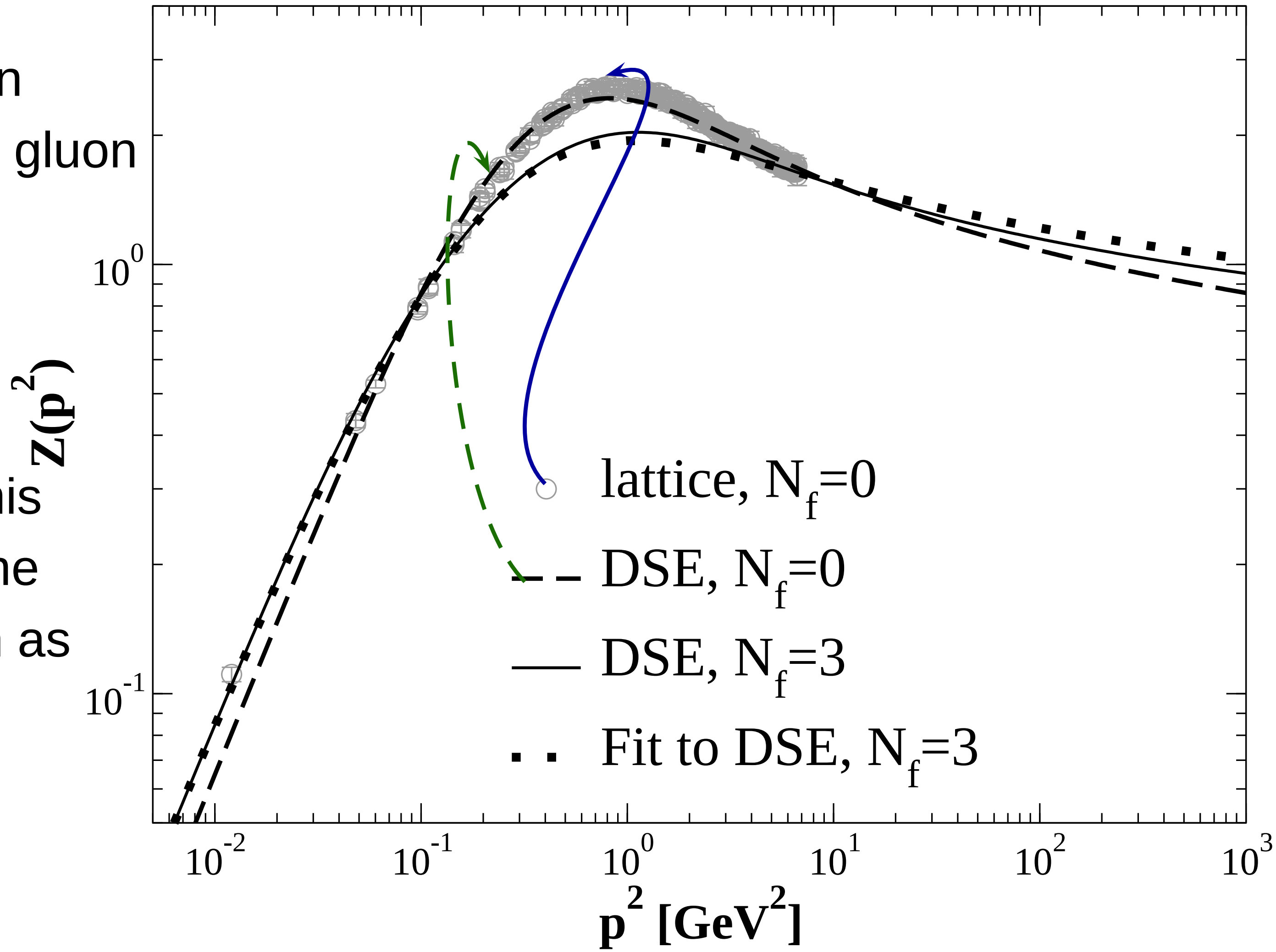

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Dressed-Quark Propagator

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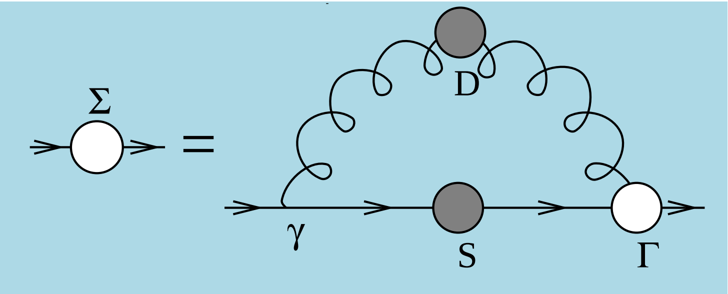

Gap Equation

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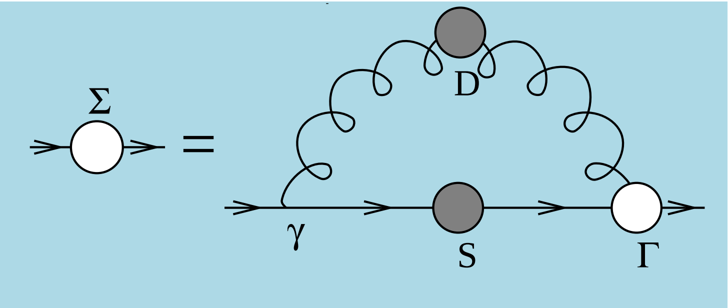
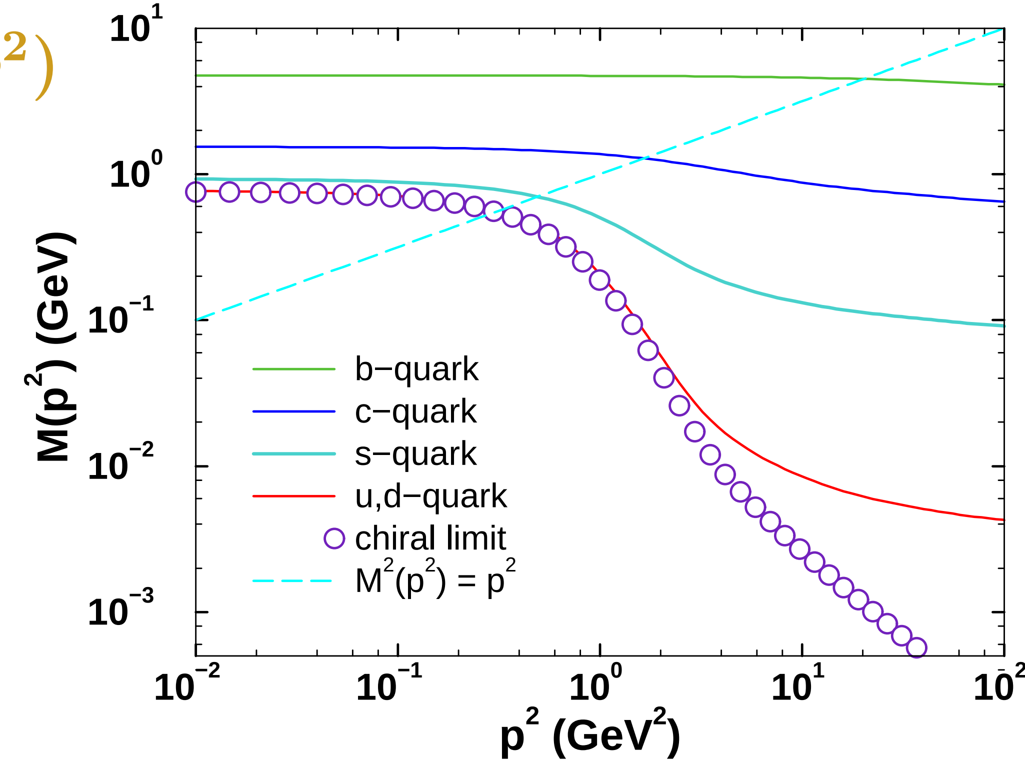


Gap Equation

- Gap Equation's Kernel Enhanced on IR domain

$\Rightarrow$ IR Enhancement of $M(p^2)$



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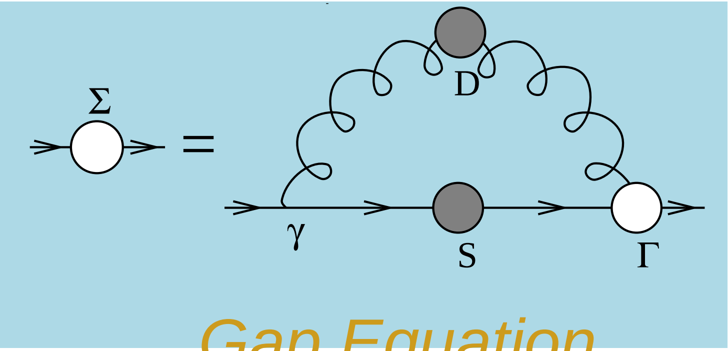

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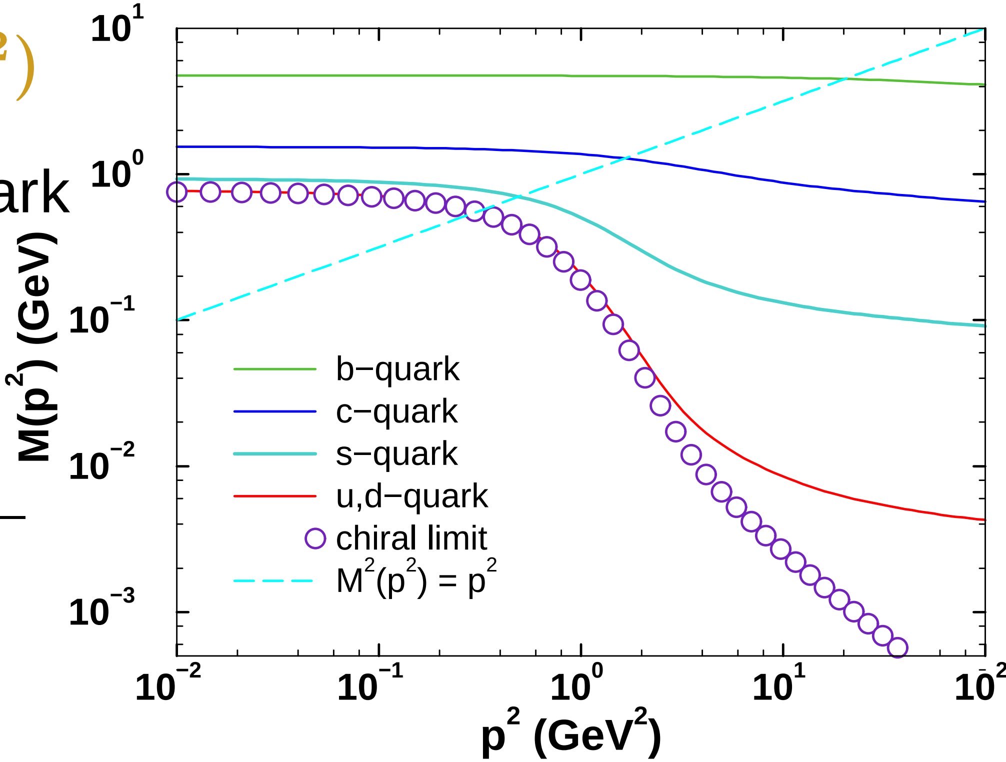
- Euclidean Constituent–Quark Mass: M_f^E: $p^2 = M(p^2)^2$



flavour	u/d	s	c	b
$\frac{M^E}{m_\zeta}$	$\sim 10^2$	~ 10	~ 1.5	~ 1.1

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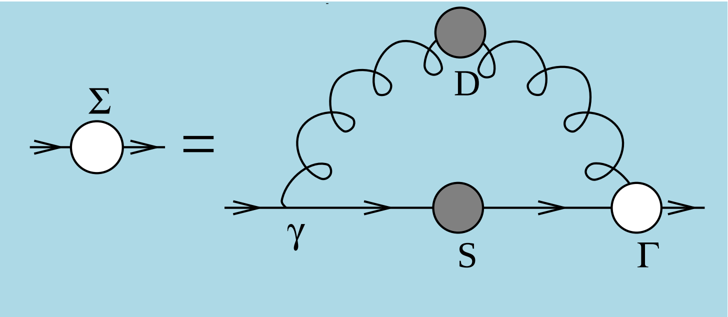

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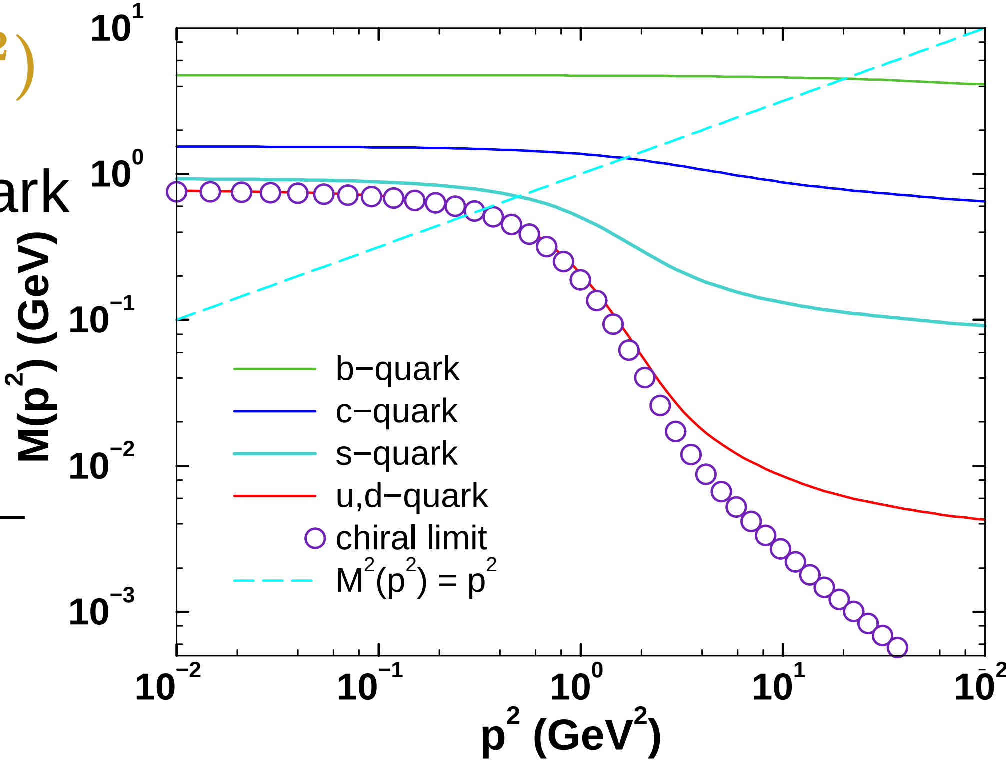

Predictions confirmed in numerical simulations of lattice-QCD

Dressed-Quark Propagator

Dressed-Quark Propagator

- Longstanding Prediction of Dyson-Schwinger Equation Studies

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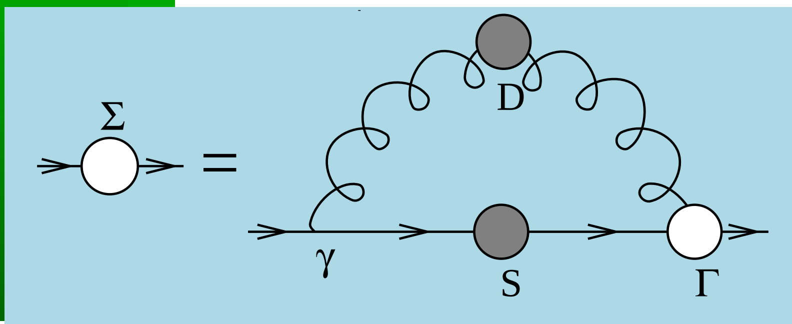
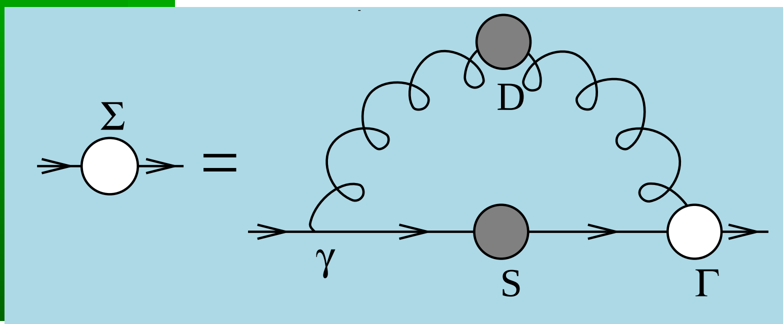
Frontiers of Nuclear Science: A Long Range Plan (2007)



Gap Equation

Frontiers of Nuclear Science: Theoretical Advances



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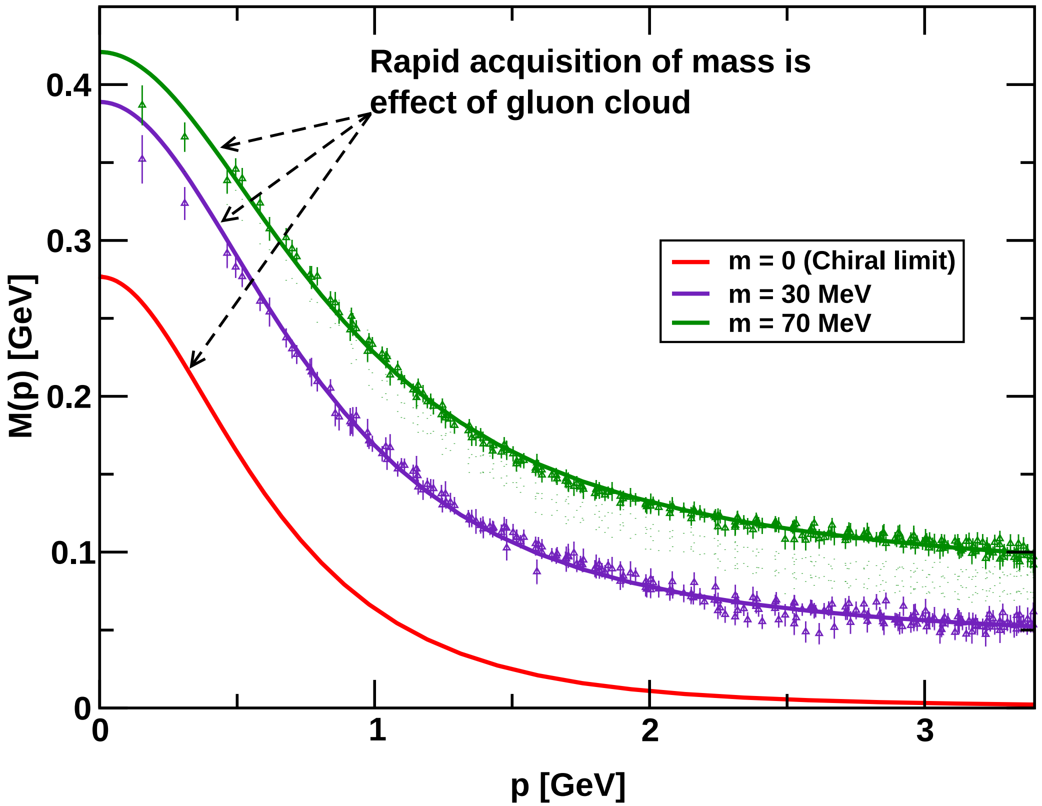
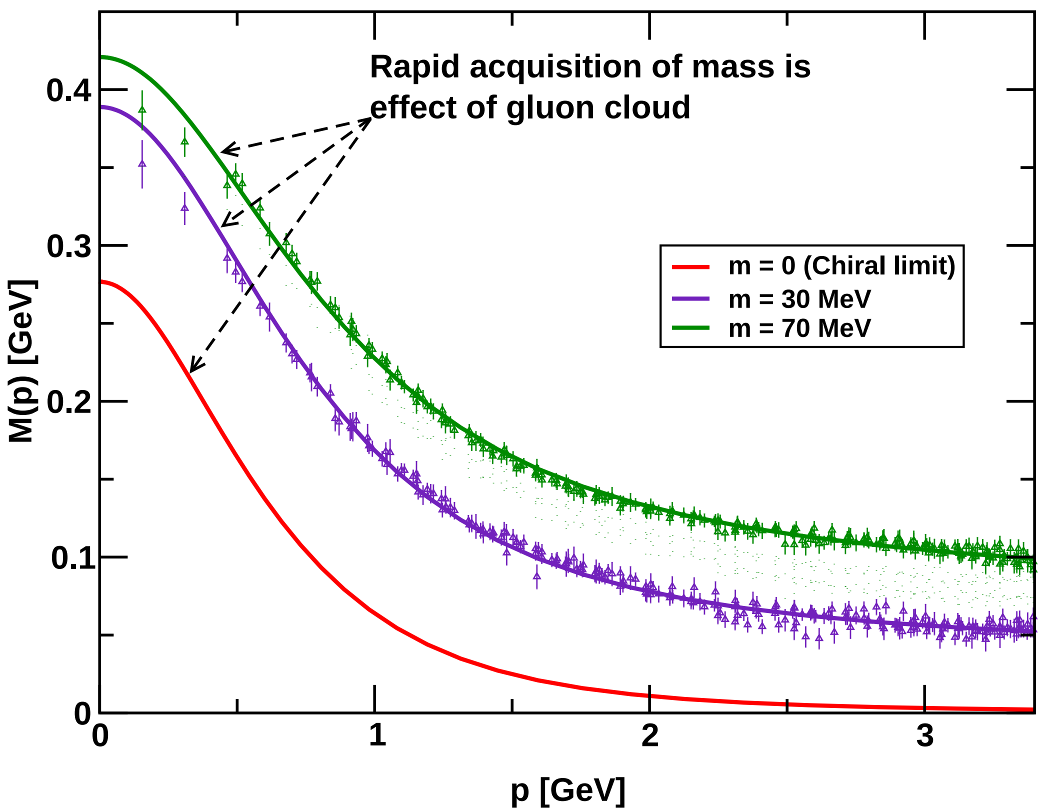


Mass from nothing.

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.



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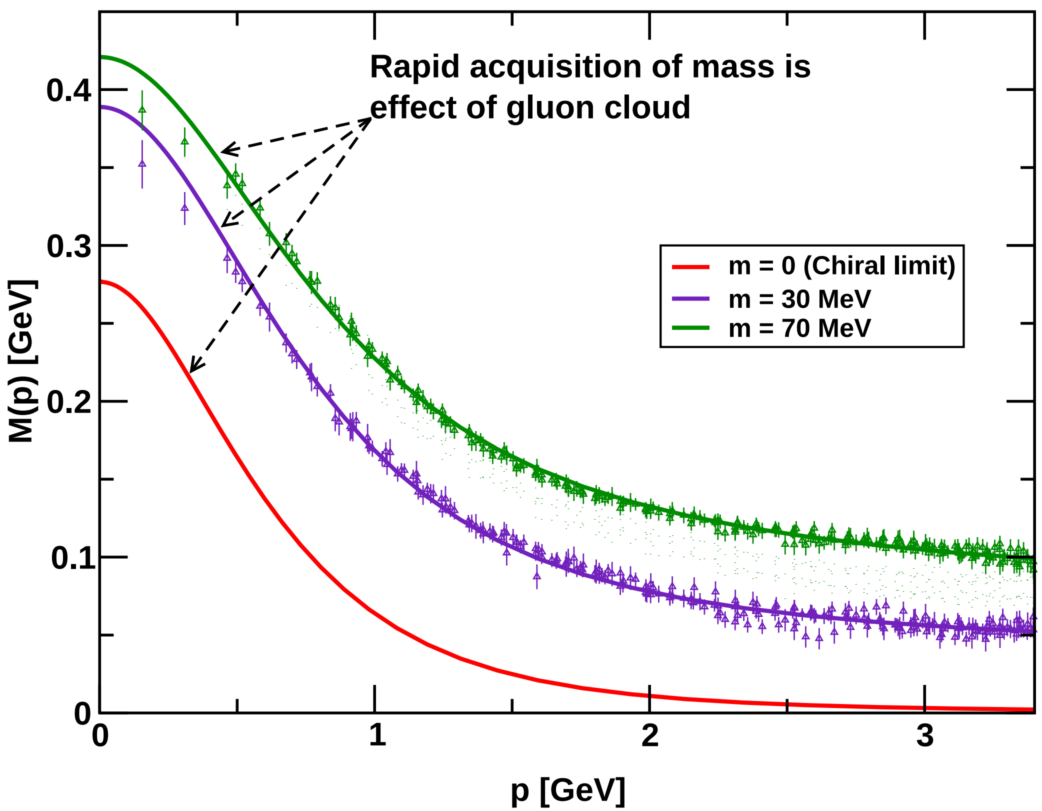

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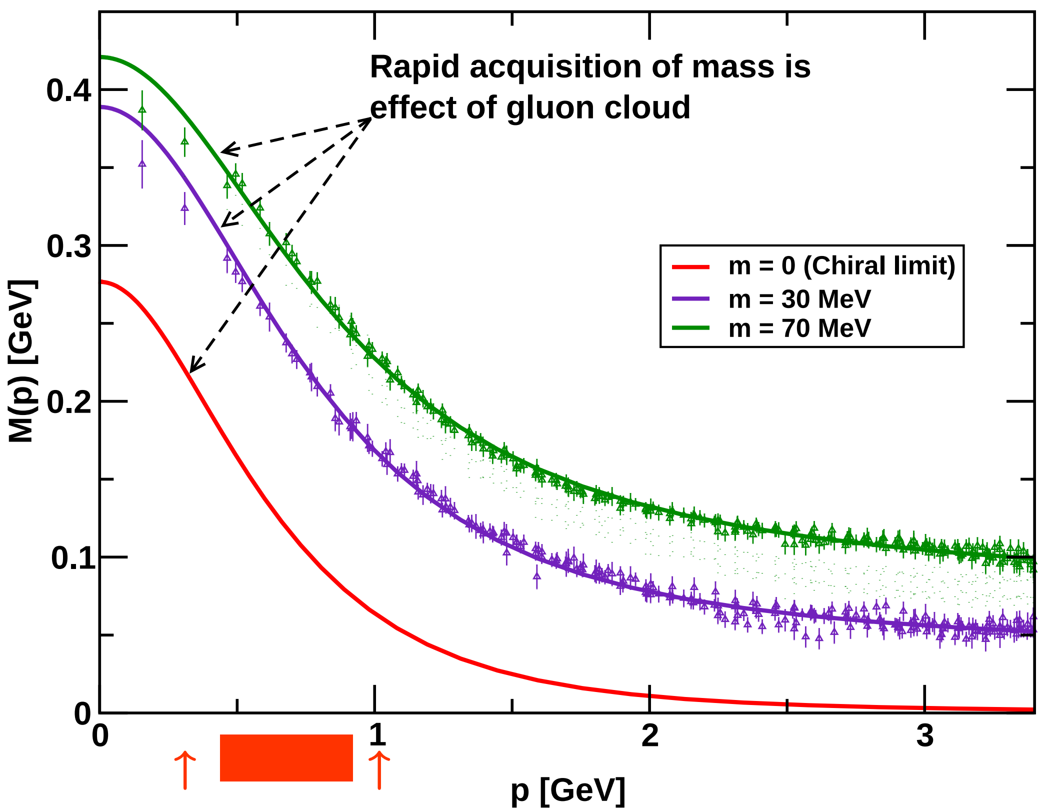

Scanned by $Q^2 \in [2, 9]\ \mathrm{GeV}^2$ Baryon Form Factors

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In QCD

a quark’s mass must depend on

its momentum

俳句


ALWAYS LOOK
ON THE
BRIGHT SIDE

- Established understanding of two- and three-point functions

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- Established understanding of two- and three-point functions
- What about bound states?

- Without bound states, Comparison with experiment is impossible

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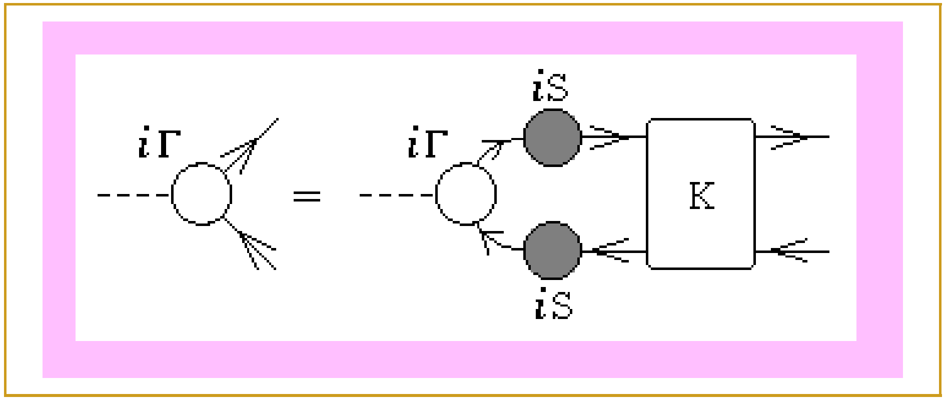
- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions

- Without bound states, Comparison with experiment is impossible
- Bethe-Salpeter Equation



QFT Generalisation of Lippmann-Schwinger Equation.

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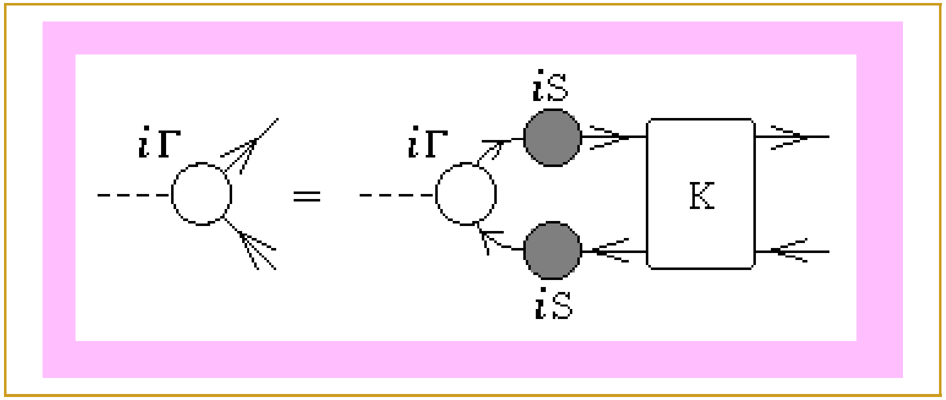

QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel, K?

or What is the long-range potential in QCD?

What is the light-quark Long-Range Potential?

What is the light-quark Long-Range Potential?

Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD ***is not related*** in any simple way to the light-quark interaction.

Bethe-Salpeter Kernel

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \, \Gamma^l_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \, \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta \, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

QFT Statement of Chiral Symmetry

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- Nontrivial constraint

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Kernels very different

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- Failure ⇒ Explicit Violation of QCD's Chiral Symmetry

Goldstone's Theorem

- In the chiral limit the QCD Action possesses chiral symmetry
- The chiral limit is a good approximation in QCD for u- and d-quarks
- If this $SU(N_f = 2)$ chiral symmetry is dynamically broken, then there is a massless composite particle associated with each generator of chiral transformations; i.e., three Goldstone Bosons
- These three Goldstone Bosons have long been identified with the pions: $\boldsymbol{\pi}^+ , \boldsymbol{\pi}^0 , \boldsymbol{\pi}^-$

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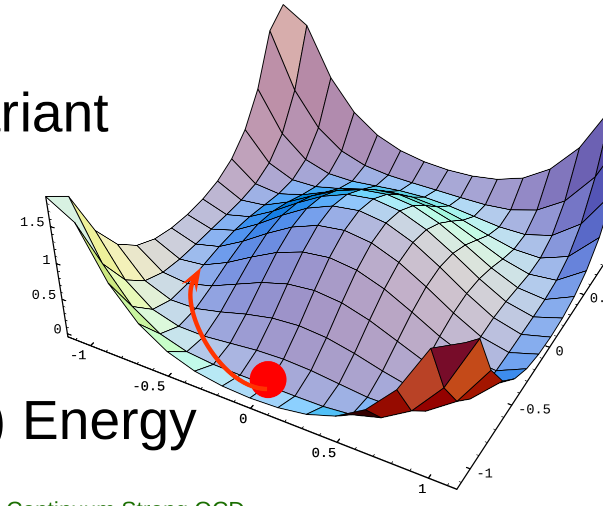
  E.g., $V(x, y) = (\sigma^2 + \pi^2 - 1)^2$

  – Hamiltonian: $T + V$, is Rotationally Invariant

  Ground State

  - Ball at any (σ, π) for which $\sigma^2 + \pi^2 = 1$
    - All Positions have Same (Minimum) Energy
    - But not invariant under rotations



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- If one assumes the s-quark is also light; namely, assumes that $SU(N_f = 3)$ chiral symmetry is a good approximation, then the kaons are four more Goldstone Bosons

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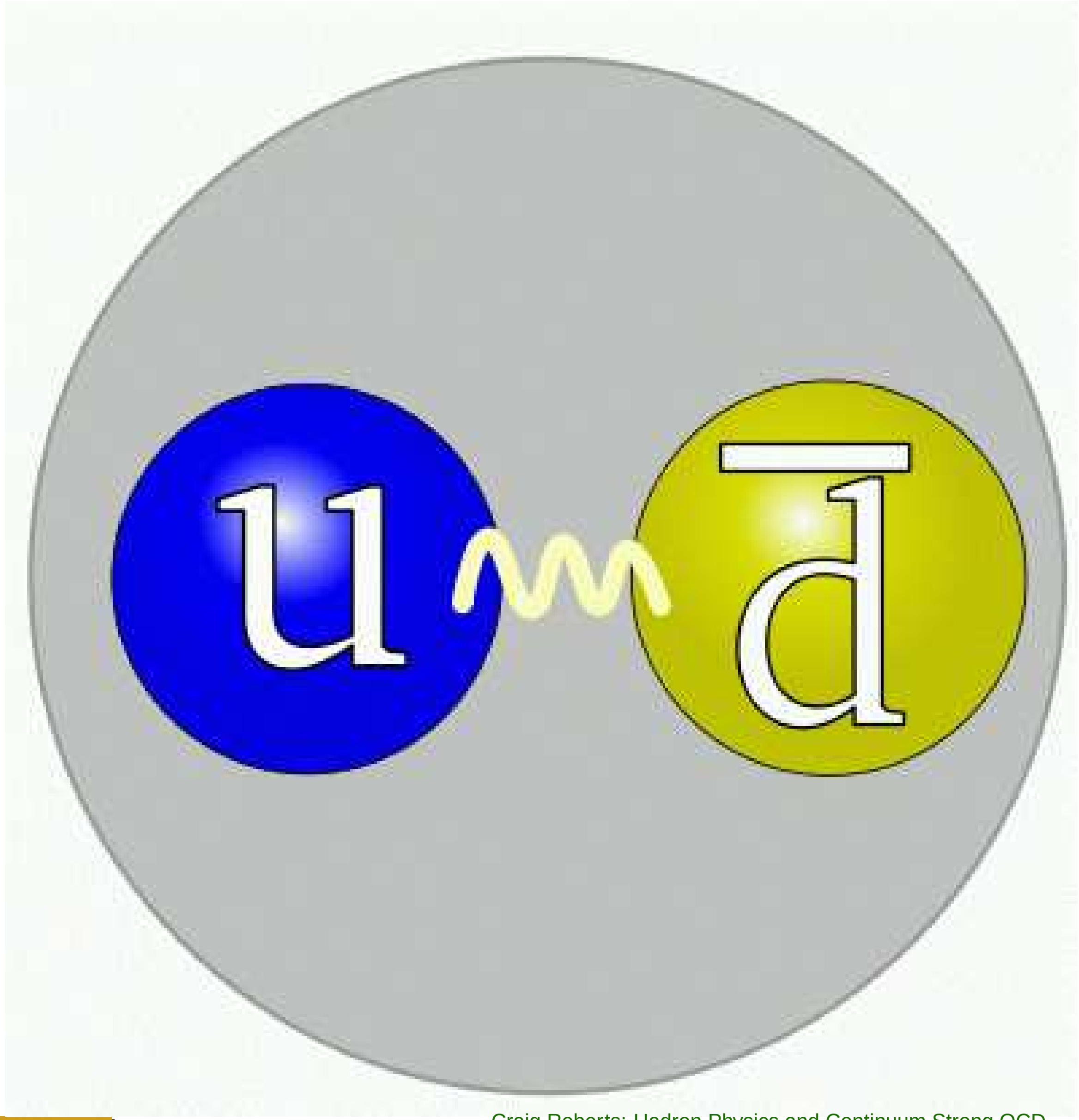

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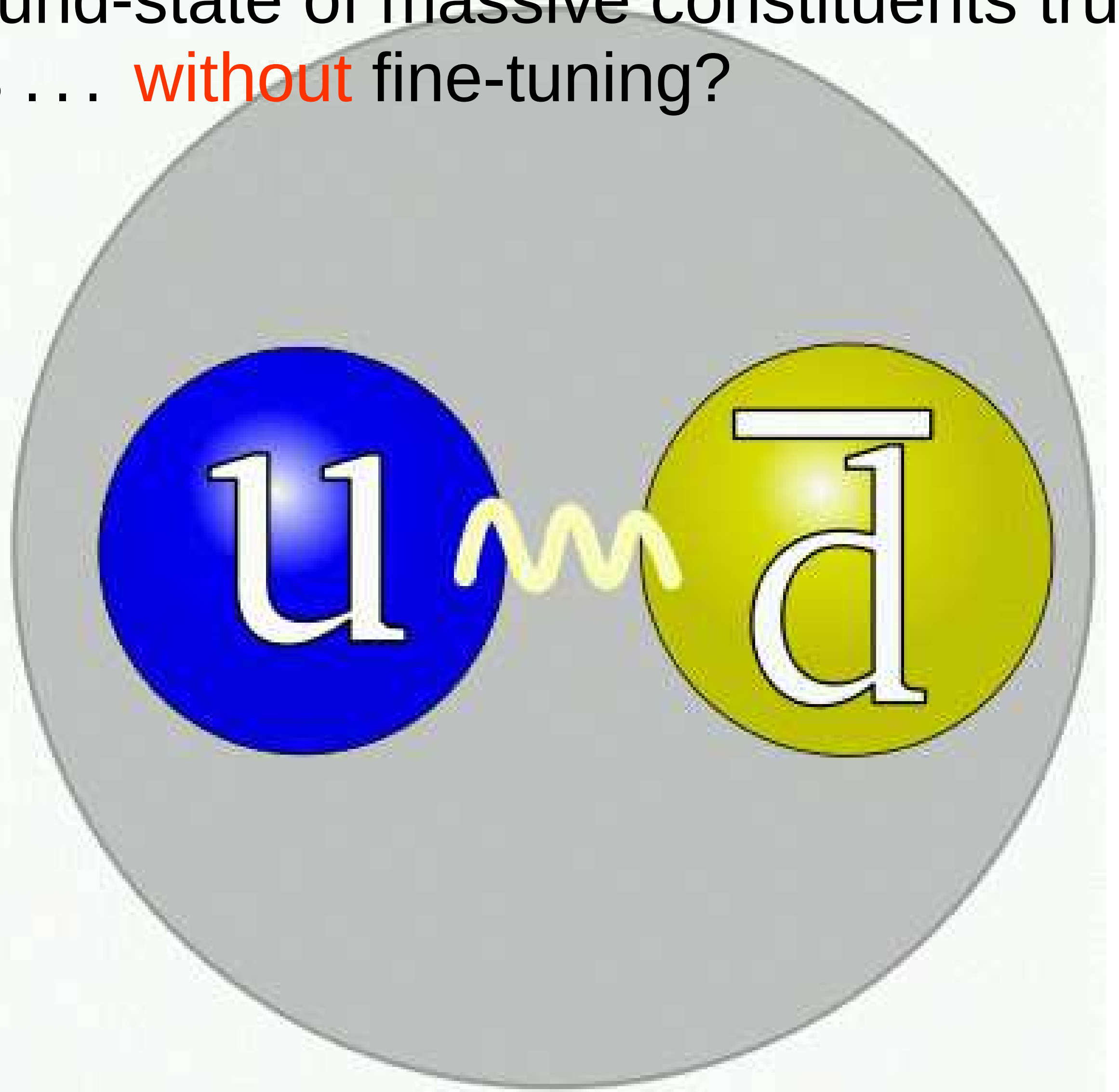
Pseudoscalar Mesons?

Can a bound-state of massive constituents truly be massless . . . without fine-tuning?



Dichotomy of Pion
– Goldstone Mode and *Bound state*

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Dichotomy of Pion
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- How does one make an almost massless particle
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  Must exhibit $m_\pi^2 \propto m_q$

Current Algebra . . . 1968

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Current Algebra . . . 1968

The correct understanding of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a

- well-defined and valid chiral limit;
- and an accurate realisation of dynamical chiral symmetry breaking.

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Highly Nontrivial

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Resolving the Dichotomy

- Minimal requirements
  - detailed understanding of connection between Current-quark and Constituent-quark masses;
  - and systematic, symmetry preserving means of realising this connection in bound-states.

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Resolving the Dichotomy

- Minimal requirements
  - detailed understanding of connection between Current-quark and Constituent-quark masses;
  - and systematic, symmetry preserving means of realising this connection in bound-states.
- Satisfying these requirements enables
  - Proof of numerous exact results for pseudoscalar mesons
  - Formulation of reliable models
    - To illustrate those results
    - Make predictions of observables with quantifiable errors

Maris, Roberts, Tandy
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Goldberger-Treiman for pion

Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

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$\Rightarrow$ $$f_\pi E_\pi(k;P=0) = B(p^2)$$

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Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

$$\begin{aligned}\Gamma_{\pi^j}(k;P) &= \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P)+\gamma\cdot P F_\pi(k;P)\\&\quad+\gamma\cdot k\, k\cdot P\, G_\pi(k;P)+\sigma_{\mu\nu}\,k_\mu P_\nu\, H_\pi(k;P)\Big]\end{aligned}$$

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$\Rightarrow$

$$\begin{aligned}f_\pi E_\pi(k;P=0) &= B(p^2)\\F_R(k;0)+2\,f_\pi F_\pi(k;0) &= A(k^2)\\G_R(k;0)+2\,f_\pi G_\pi(k;0) &= 2A'(k^2)\\H_R(k;0)+2\,f_\pi H_\pi(k;0) &= 0\end{aligned}$$

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Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

Pseudovector components necessarily nonzero

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5 \left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P) \right]$$

- Dressed-quark Propagator: $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$
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$\Rightarrow$ Exact in Chiral QCD

$$\begin{aligned} f_\pi E_\pi(k;P=0) &= B(p^2) \\ F_R(k;0) + 2\, f_\pi F_\pi(k;0) &= A(k^2) \\ G_R(k;0) + 2\, f_\pi G_\pi(k;0) &= 2A'(k^2) \\ H_R(k;0) + 2\, f_\pi H_\pi(k;0) &= 0 \end{aligned}$$

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

Radial Excitations & Chiral Symmetry

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Mass^2 of pseudoscalar hadron

Radial Excitations & Chiral Symmetry

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

$$\mathcal{M}_H := \mathrm{tr}_{\mathrm{flavour}} \left[M_{(\mu)} \left\{ T^H, \left(T^H\right)^{\mathrm{t}} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2} \left(\lambda^4 + i \lambda^5 \right)$

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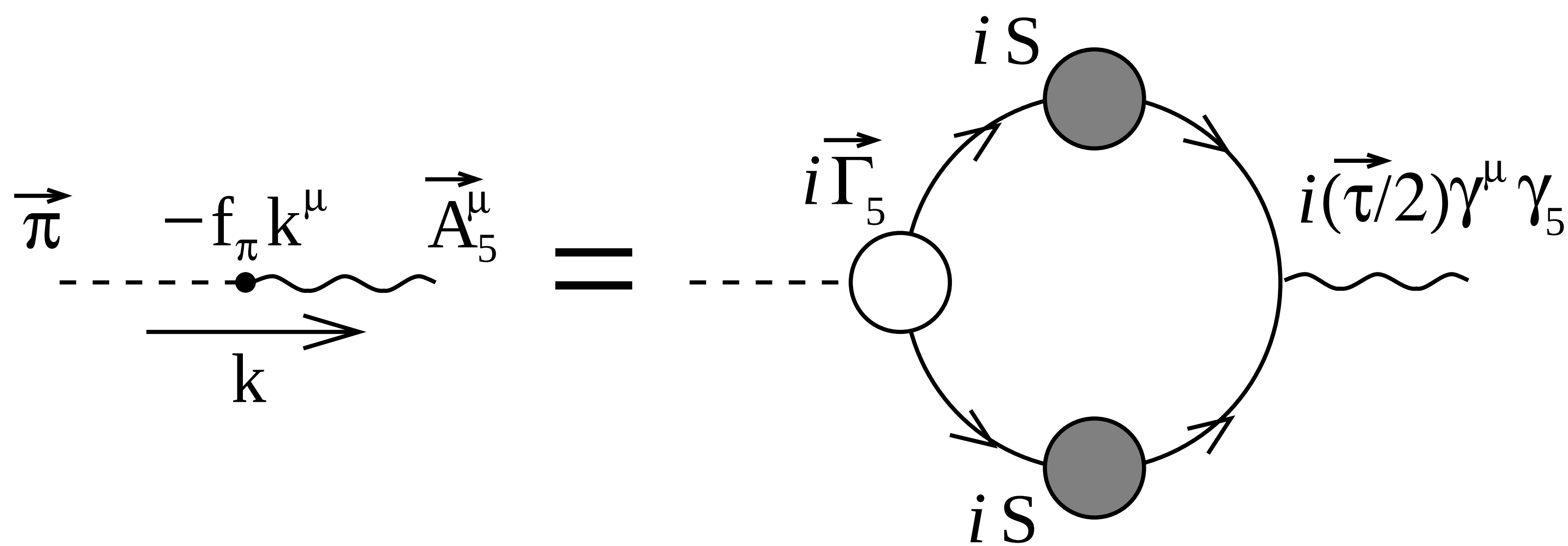
$\langle 0|\bar{q}\gamma_5\gamma_\mu q|\pi\rangle$

$$f_H\, m_H^2 = -\;\rho_\zeta^H\;\;\mathcal{M}_H$$

$$f_H\, p_\mu = Z_2 \int_q^\Lambda \tfrac{1}{2}\mathrm{tr}\left\{\left(T^H\right)^{\mathrm{t}}\gamma_5\gamma_\mu\, \mathcal{S}(q_+)\Gamma_H(q;P)\mathcal{S}(q_-)\right\}$$

- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

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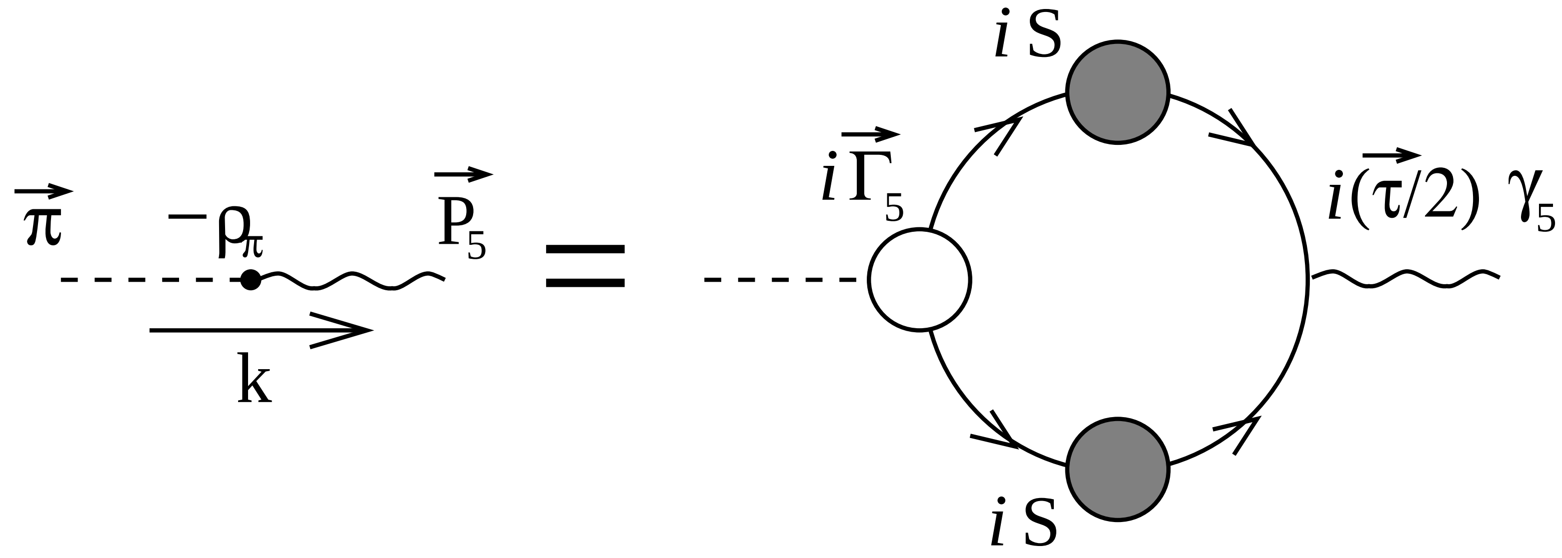

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H$$

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  - $f_H \to f_H^0$ & $\rho_\zeta^H \to \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$, Independent of m_q

Hence $m_H^2 = \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q$ . . . GMOR relation, a corollary

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Hence $m_H^2 = \dfrac{-\langle \bar{q}q\rangle_\zeta^0}{(f_H^0)^2}\, m_q$ …GMOR relation, a corollary

- Heavy-quark + light-quark

$\Rightarrow f_H \propto \dfrac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

Hence, $m_H \propto m_q$

… QCD Proof of Potential Model result

Radial Excitations

Radial Excitations

- Spectrum contains 3 pseudoscalars [$I^G(J^P)L = 1^-(0^-)S$]

  masses below 2 GeV: $\pi(140)$; $\pi(1300)$; and $\pi(1800)$

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- Consituent-Q Model: 1^{st} three members of $n\,^1S_0$ trajectory; i.e., ground state plus radial excitations?

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- But $\pi(1800)$ is narrow ($\Gamma = 207 \pm 13$) & decay pattern might indicate some "flux tube angular momentum" content:
  $S_{\bar{Q}Q} = 1 \oplus L_F = 1 \Rightarrow J = 0$
  & $L_F = 1 \Rightarrow\ {}^3S_1 \oplus\ {}^3S_1$ ($\bar{Q}Q$) decays suppressed?

Radial Excitations

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- NSAC Long-Range Plan, 2002: ... an understanding of confinement "remains one of the greatest intellectual challenges in physics"

Radial Excitations & Chiral Symmetry

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- "radial" excitation of π-meson, not the ground state, so $\boldsymbol{m^2_{\pi_{n\neq 0}} > m^2_{\pi_{n=0}} = 0}$, in chiral limit

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Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

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Radial Excitations & Chiral Symmetry

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- $\Rightarrow \boldsymbol{f_H = 0}$
  ALL pseudoscalar mesons except $\boldsymbol{\pi(140)}$ in chiral limit
- Dynamical Chiral Symmetry Breaking
  – Goldstone's Theorem –
  impacts upon *every* pseudoscalar meson

Charge Neutral Pseudoscalar Mesons

non-Abelian Anomaly and η-η' mixing

Charge Neutral Pseudoscalar Mesons

non-Abelian Anomaly and η-η' mixing

- Mesons containing $\bar{s}$-s are special: η & η'

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non-Abelian Anomaly and η-η' mixing

- Mesons containing $\bar{s}$-s are special: η & η'
  Problem: η' is a pseudoscalar meson but it's much more massive than the other eight constituted from light-quarks.

non-Abelian Anomaly and η-η' mixing

- Mesons containing $\bar{s}$-s are special: η & η'
  Problem: η' is a pseudoscalar meson but it's much more massive than the other eight constituted from light-quarks.
  Origin: While the classical action associated with QCD is invariant under $U_A(1)$ (Abelian axial transformations generated by $\lambda^0\gamma_5$), the quantum field theory is not!

non-Abelian Anomaly and η-η' mixing

- Mesons containing $\bar{s}$-s are special: η & η'
- Flavour mixing takes place in singlet channel: $\lambda^0 \Leftrightarrow \lambda^8$

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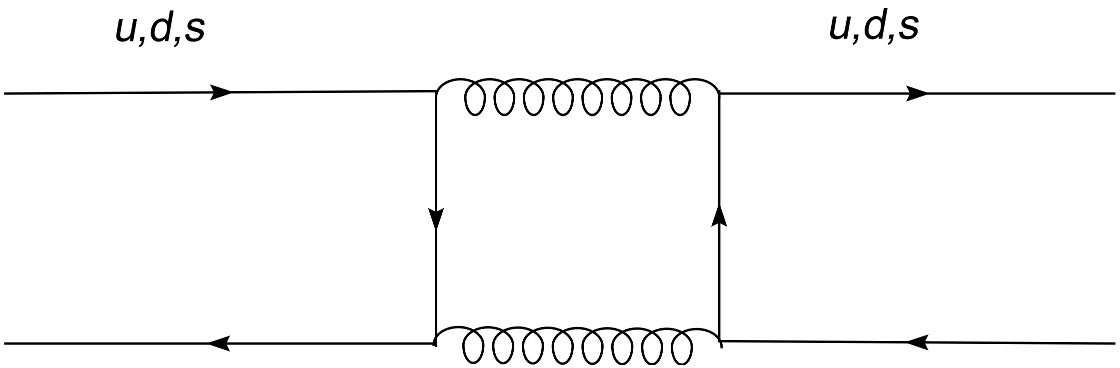

non-Abelian Anomaly and η-η' mixing

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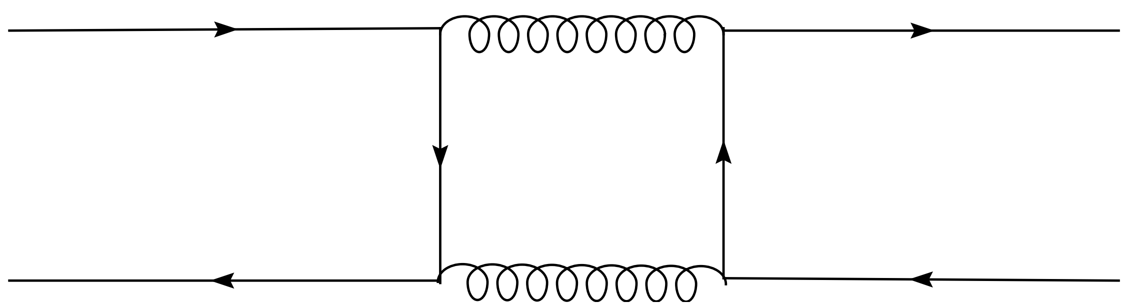

- This is a perturbative diagram.
  It has almost nothing to do with $\eta \Leftrightarrow \eta'$ mixing.

non-Abelian Anomaly and η-η' mixing

- Mesons containing $\bar{s}$-s are special: η & η'
- Driver is the non-Abelian anomaly

Charge Neutral Pseudoscalar Mesons

non-Abelian Anomaly and η-η' mixing

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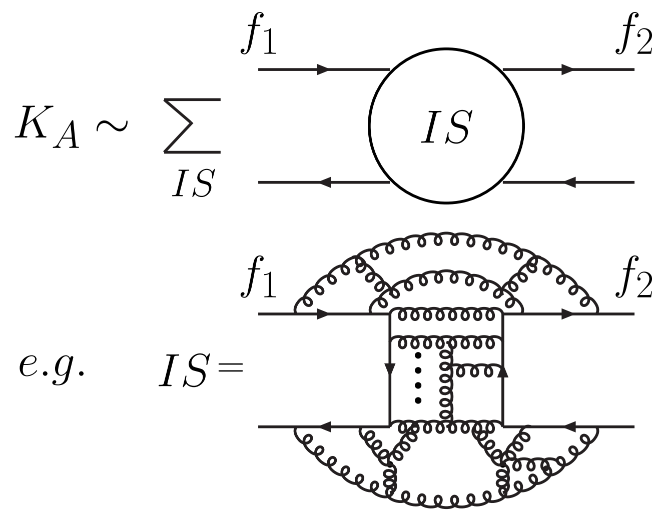
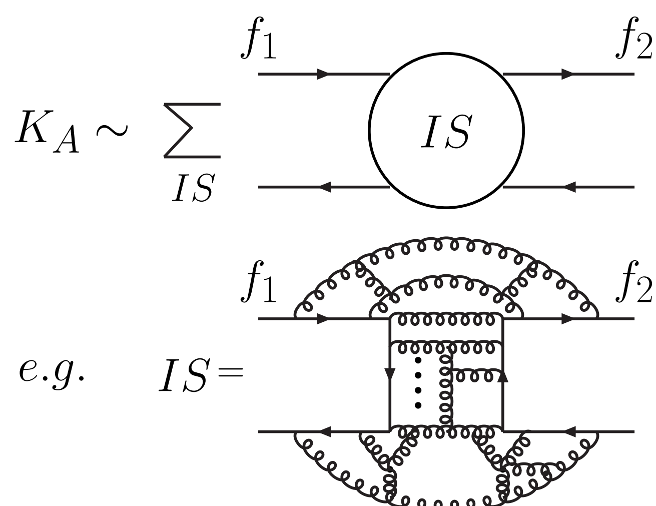


non-Abelian Anomaly and η-η' mixing

- Mesons containing $\bar{s}$-s are special: η & η'
- Driver is the non-Abelian anomaly
- Contribution to the Bethe-Salpeter kernel associated with the non-Abelian anomaly. All terms have the "hairpin" structure.



No finite sum of such intermediate states is sufficient to veraciously represent the anomaly.

Charge Neutral Pseudoscalar Mesons

$$
\begin{aligned}
P_\mu \Gamma^a_{5\mu}(k;P) &= \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) \\
& -2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)
\end{aligned}
$$

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Charge Neutral Pseudoscalar Mesons

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

- $\{\mathcal{F}^a \,|\, a = 0, \ldots, N_f^2 - 1\}$ are the generators of $U(N_f)$
- $\mathcal{S} =$ diag$[S_u, S_d, S_s, S_c, S_b, \ldots]$
- $\mathcal{M}^{ab} = \mathrm{tr}_F \left[\{\mathcal{F}^a, \mathcal{M}\} \mathcal{F}^b \right]$,
  $\mathcal{M} = \mathrm{diag}[m_u, m_d, m_s, m_c, m_b, \ldots] =$ matrix of current-quark bare masses

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Charge Neutral Pseudoscalar Mesons

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

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- The final term in the second line expresses the non-Abelian axial anomaly.

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Charge Neutral Pseudoscalar Mesons

$$P_\mu \Gamma^a_{5\mu}(k;P) \;=\; \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) \\ -2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

- $\mathcal{A}^a(k;P) = \mathcal{S}^{-1}(k_+)\,\delta^{a0}\,\mathcal{A}_U(k;P)\mathcal{S}^{-1}(k_-)$
  $\mathcal{A}_U(k;P) = \int d^4x d^4y\, e^{i(k_+\cdot x - k_-\cdot y)} N_f \langle \mathcal{F}^0 q(x)\, \mathcal{Q}(0)\, \bar{q}(y)\rangle$

Charge Neutral Pseudoscalar Mesons

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  ... The topological charge density operator.

Charge Neutral Pseudoscalar Mesons

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. . . The topological charge density operator.
(Trace is over colour indices & $F_{\mu\nu} = \frac{1}{2}\lambda^a F^a_{\mu\nu}$.)

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$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

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  … The topological charge density operator.
- Important that only $\mathcal{A}^{a=0}$ is nonzero.

Charge Neutral Pseudoscalar Mesons

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

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  … The topological charge density operator.
- NB. While $\mathcal{Q}(x)$ is gauge invariant, the associated Chern-Simons current, K_μ, is not $\Rightarrow$ in QCD *no physical* boson can couple to K_μ and hence *no physical* states can contribute to resolution of $U_A(1)$ problem.

Charge Neutral Pseudoscalar Mesons

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

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Charge Neutral Pseudoscalar Mesons

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

- Only $\mathcal{A}^0 \not\equiv 0$ is interesting

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Charge Neutral Pseudoscalar Mesons

- Only $\mathcal{A}^0 \not\equiv 0$ is interesting ... otherwise all pseudoscalar mesons are Goldstone Modes!

Charge Neutral Pseudoscalar Mesons

- Anomaly term has structure

$$\mathcal{A}^0(k;P) = \mathcal{F}^0\gamma_5\left[i\mathcal{E}_\mathcal{A}(k;P) + \gamma\cdot P\mathcal{F}_\mathcal{A}(k;P)\right.$$
$$\left. +\gamma\cdot k k\cdot P\mathcal{G}_\mathcal{A}(k;P) + \sigma_{\mu\nu}k_\mu P_\nu\mathcal{H}_\mathcal{A}(k;P)\right]$$

Bhagwat, Chang, Liu, Roberts, Tandy
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Charge Neutral Pseudoscalar Mesons

- AVWTI gives generalised Goldberger-Treiman relations

$$
\begin{aligned}
2 f^0_{\eta'} E_{BS}(k;0) &= 2 B_0(k^2) - \mathcal{E}_{\mathcal{A}}(k;0), \\
F^0_R(k;0) + 2 f^0_{\eta'} F_{BS}(k;0) &= A_0(k^2) - \mathcal{F}_{\mathcal{A}}(k;0), \\
G^0_R(k;0) + 2 f^0_{\eta'} G_{BS}(k;0) &= 2 A_0'(k^2) - \mathcal{G}_{\mathcal{A}}(k;0), \\
H^0_R(k;0) + 2 f^0_{\eta'} H_{BS}(k;0) &= -\mathcal{H}_{\mathcal{A}}(k;0),
\end{aligned}
$$

A_0, B_0 characterise gap equation's chiral limit solution.

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Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

Charge Neutral Pseudoscalar Mesons

- AVWTI gives generalised Goldberger-Treiman relations

$$\begin{aligned} 2f^0_{\eta'} E_{BS}(k;0) &= 2B_0(k^2) - \mathcal{E}_{\mathcal{A}}(k;0), \\ F^0_R(k;0) + 2f^0_{\eta'} F_{BS}(k;0) &= A_0(k^2) - \mathcal{F}_{\mathcal{A}}(k;0), \\ G^0_R(k;0) + 2f^0_{\eta'} G_{BS}(k;0) &= 2A'_0(k^2) - \mathcal{G}_{\mathcal{A}}(k;0), \\ H^0_R(k;0) + 2f^0_{\eta'} H_{BS}(k;0) &= -\mathcal{H}_{\mathcal{A}}(k;0), \end{aligned}$$

  A_0, B_0 characterise gap equation's chiral limit solution.

- Follows that $\mathcal{E}_{\mathcal{A}}(k;0) = 2B_0(k^2)$ is necessary and sufficient condition for absence of massless η' bound-state.

Bhagwat, Chang, Liu, Roberts, Tandy
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Charge Neutral Pseudoscalar Mesons

- $\mathcal{E}_{\mathcal{A}}(k;0) = 2B_0(k^2)$
  Discussing the chiral limit
  - $B_0(k^2) \neq 0$ if, and only if, chiral symmetry is dynamically broken.
  - Hence, absence of massless η' bound-state is only assured through existence of intimate connection between DCSB and an expectation value of the topological charge density.

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- Further highlighted … proved

$$\langle \bar{q}q \rangle_\zeta^0 = -\lim_{\Lambda\to\infty} Z_4(\zeta^2,\Lambda^2)\,\mathrm{tr}_{\rm CD}\int_q^\Lambda S^0(q,\zeta)$$

$$= N_f \int d^4x\, \langle \bar{q}(x) i\gamma_5 q(x) \mathcal{Q}(0)\rangle^0 \,.$$

Bhagwat, Chang, Liu, Roberts, Tandy
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Charge Neutral Pseudoscalar Mesons

- AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons

Charge Neutral Pseudoscalar Mesons

Bhagwat, Chang, Liu, Roberts, Tandy
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Bhagwat, Chang, Liu, Roberts, Tandy
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- Employed in an analysis of pseudoscalar- and vector-meson bound-states

Bhagwat, Chang, Liu, Roberts, Tandy
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Charge Neutral Pseudoscalar Mesons

- AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons
- Implications of mass formulae illustrated using elementary dynamical model, which includes *Ansatz* for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly
- Despite its simplicity, model is elucidative and phenomenologically efficacious; e.g., it predicts
  - η–η' mixing angles of $\sim -15^\circ$ (Expt.: $-13.3^\circ \pm 1.0^\circ$)
  - π^0–η angles of $\sim 1.2^\circ$ (Expt. $p\,d \to {}^3$He π^0: $0.6^\circ \pm 0.3^\circ$)
  - Strong neutron-proton mass difference . . .
    $\lesssim 75\,\%$ current-quark mass-difference

Ab-Initio **Calculations**

Ab-Initio **Calculations**

Pieter Maris

Peter Tandy

Ab-Initio Calculations

Maris & Tandy, Series of Five Articles: 1999 – Present

Perfected a Renormalisation-Group Improved Rainbow-Ladder Model of Quark-Quark Interaction

Ab-Initio **Calculations**

Maris & Tandy, Series of Five Articles: 1999 – Present

Perfected a Renormalisation-Group Improved Rainbow-Ladder Model of Quark-Quark Interaction

- Rainbow-Ladder $=$ First Order
  in Truncation Described Above

- Anticipate Accurate for 0^- & 1^- Mesons

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Ab-Initio **Calculations**

Maris & Tandy, Series of Five Articles: 1999 – Present

Perfected a Renormalisation-Group Improved Rainbow-Ladder Model of Quark-Quark Interaction

- One Parameter $=$ Interaction Energy:

  $$\mathcal{E} \approx 700\,\text{MeV}$$

- Dressed-Glue Mass scale:

  Characterises DCSB and light-quark Confinement

- Both Phenomena Disappear for $\mathcal{E} \lesssim 200\,\text{MeV}$

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- *Dyson-Schwinger equations:*
  A Tool for Hadron Physics
  P. Maris and C.D. Roberts, nu-th/0301049

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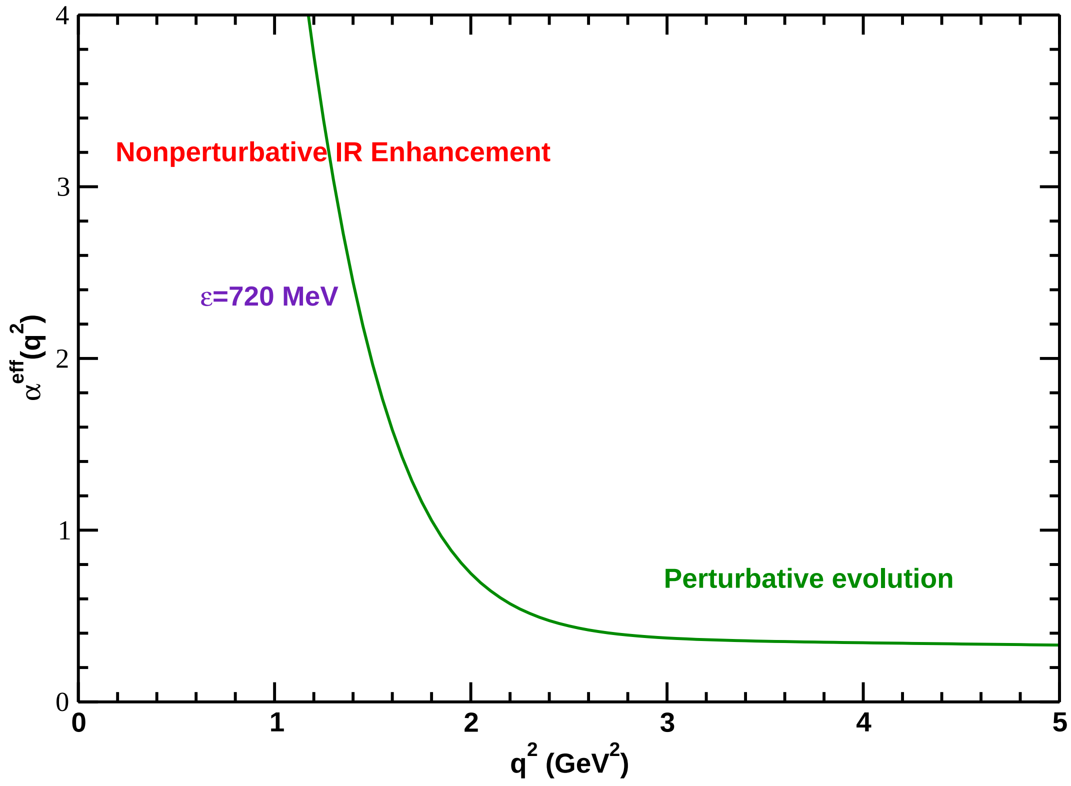
Kernel of Bethe-Salpeter Equation

$$K(p,k;P) \approx \frac{\alpha^{\rm eff}((p-k)^2)}{(p-k)^2}$$



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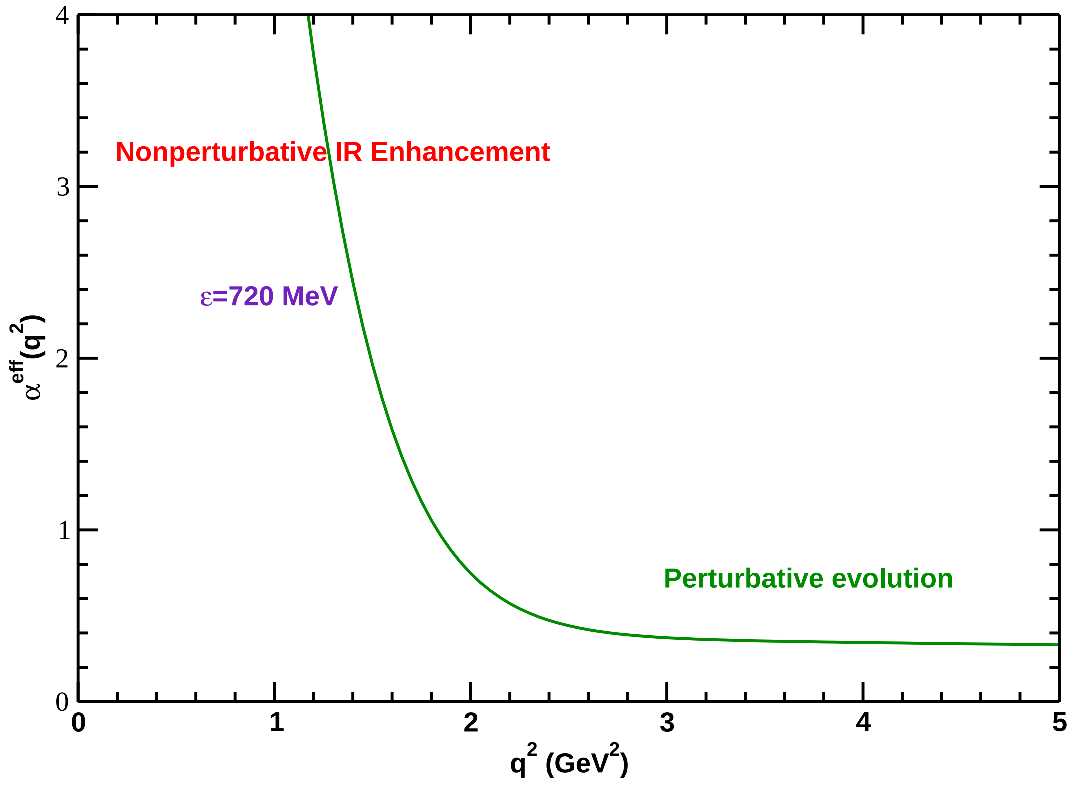
Prescribes Gap Equation’s Kernel



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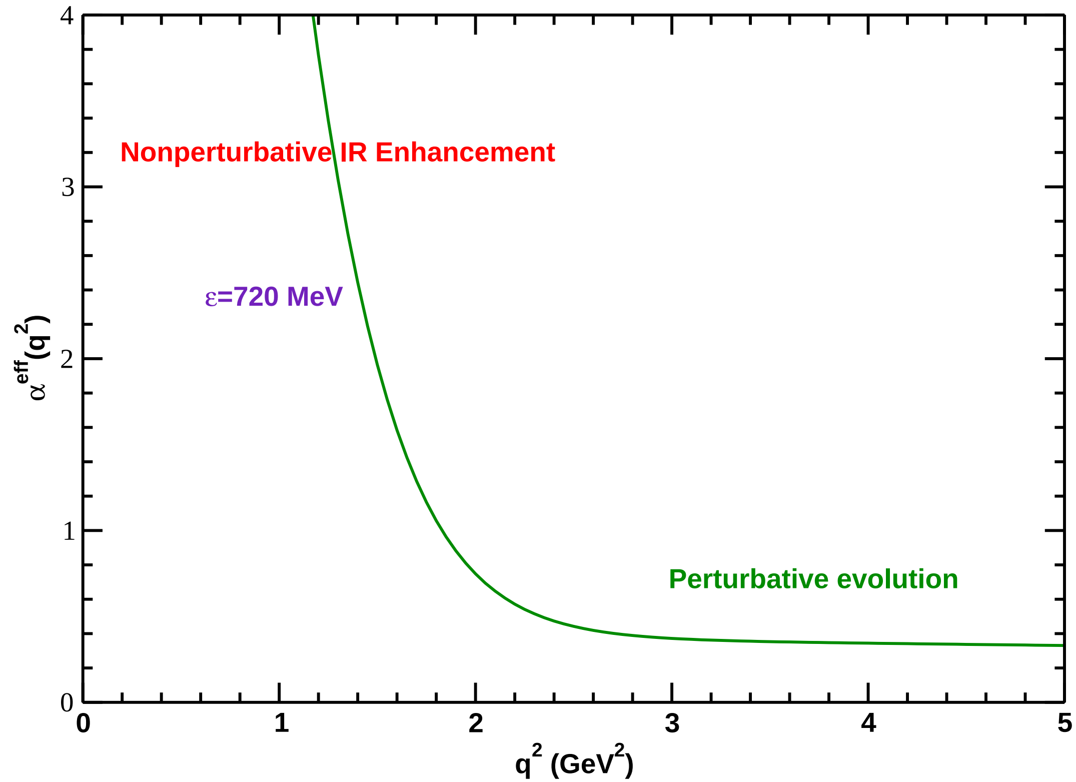

Prescribes Gap Equation's Kernel

Connects *Ansatz* for long-range part of QCD's interaction with Observables.

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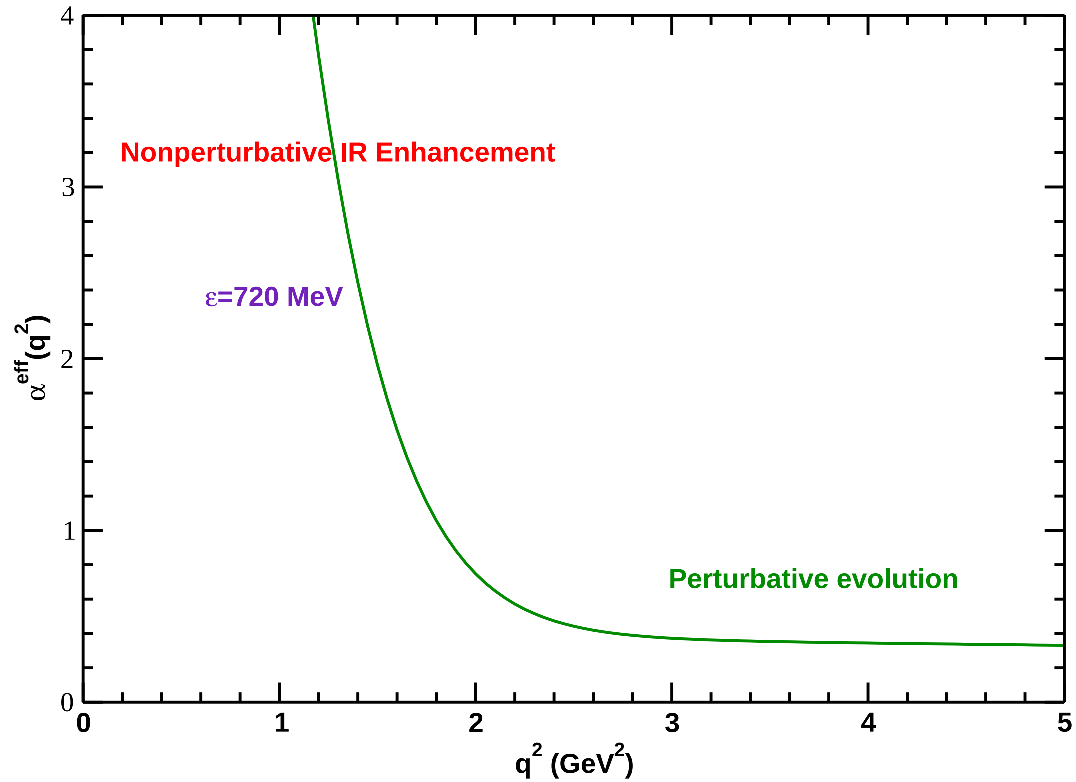

Prescribes Gap Equation's Kernel

IR-*Enhancement* at long-range agrees semi-quantitatively with Bhagwat, *et al*.

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Pion Form Factor

Procedure Now Straightforward

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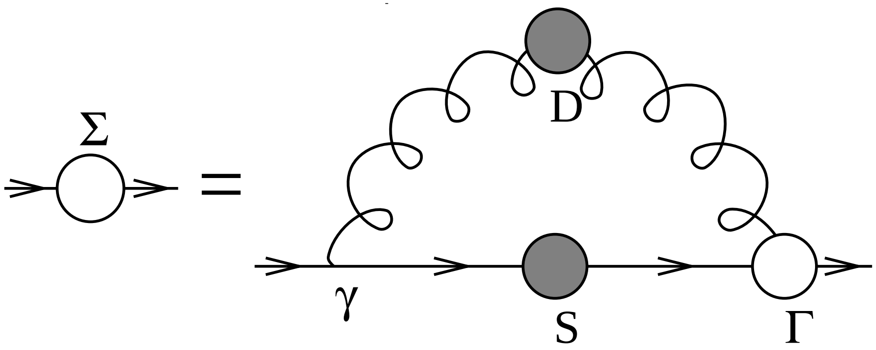
Pion Form Factor

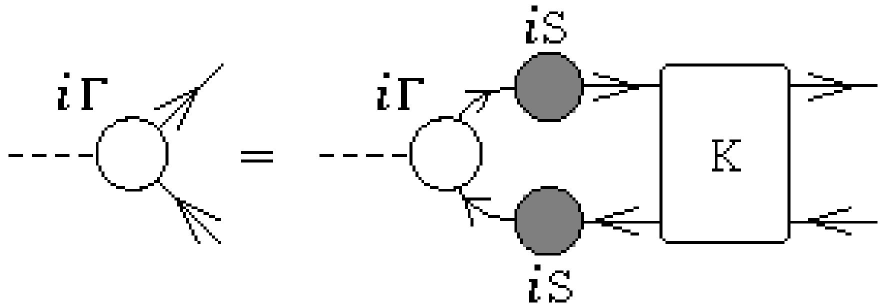
- Solve Gap Equation

  $\Rightarrow$ Dressed-Quark Propagator, $S(p)$



Pion Form Factor

- Use that to Complete Bethe Salpeter Kernel, K
- Solve Homogeneous Bethe-Salpeter Equation for Pion Bethe-Salpeter Amplitude, Γ_π



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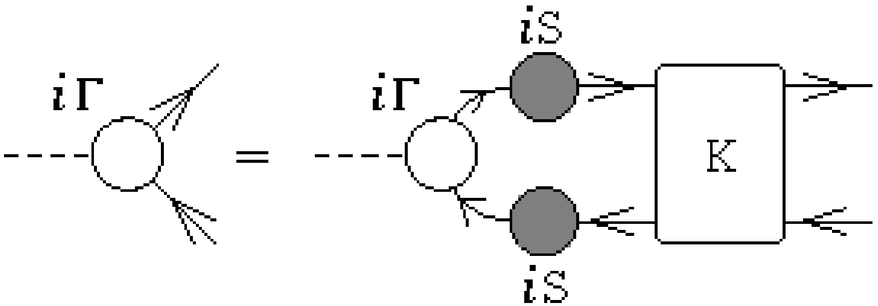
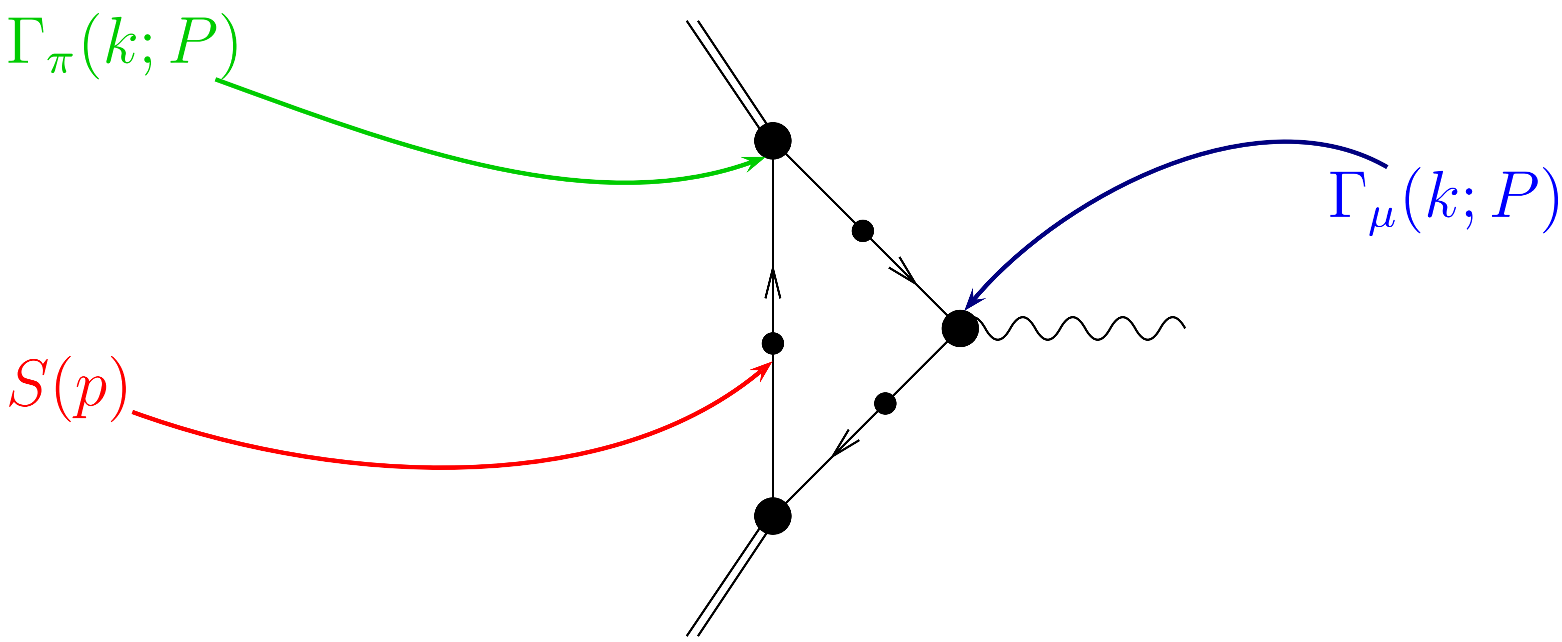


- Solve Inhomogeneous Bethe-Salpeter Equation for Dressed-Quark-Photon Vertex, Γ_μ

Pion Form Factor

- Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor



Pion Form Factor

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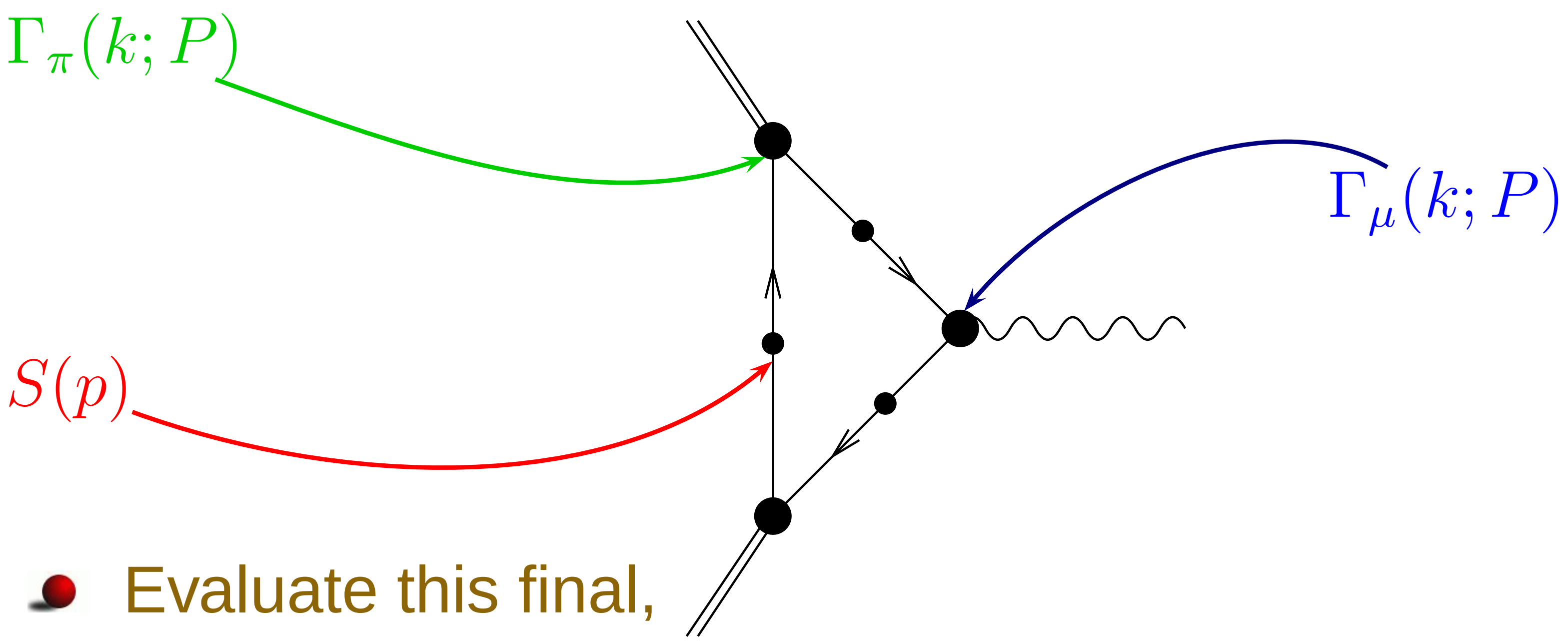
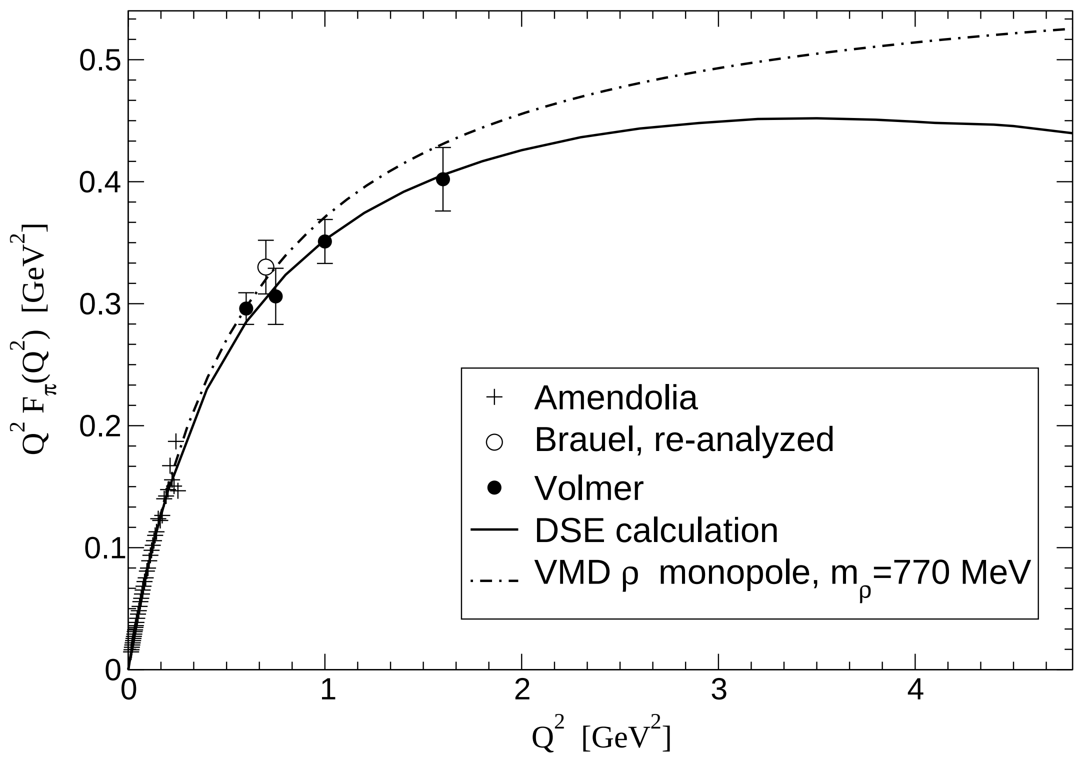


- Evaluate this final, three-dimensional integral

Calculated Pion Form Factor

Calculation published in 1999; No Parameters Varied



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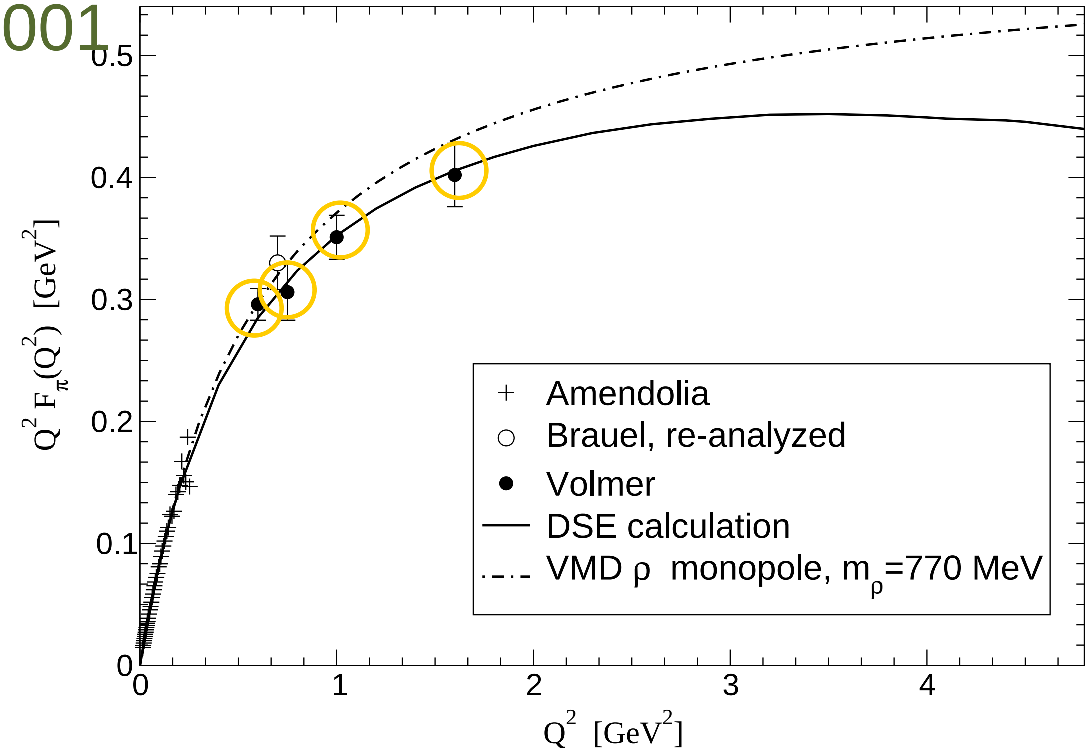
Calculated Pion Form Factor

Calculation published in 1999; No Parameters Varied

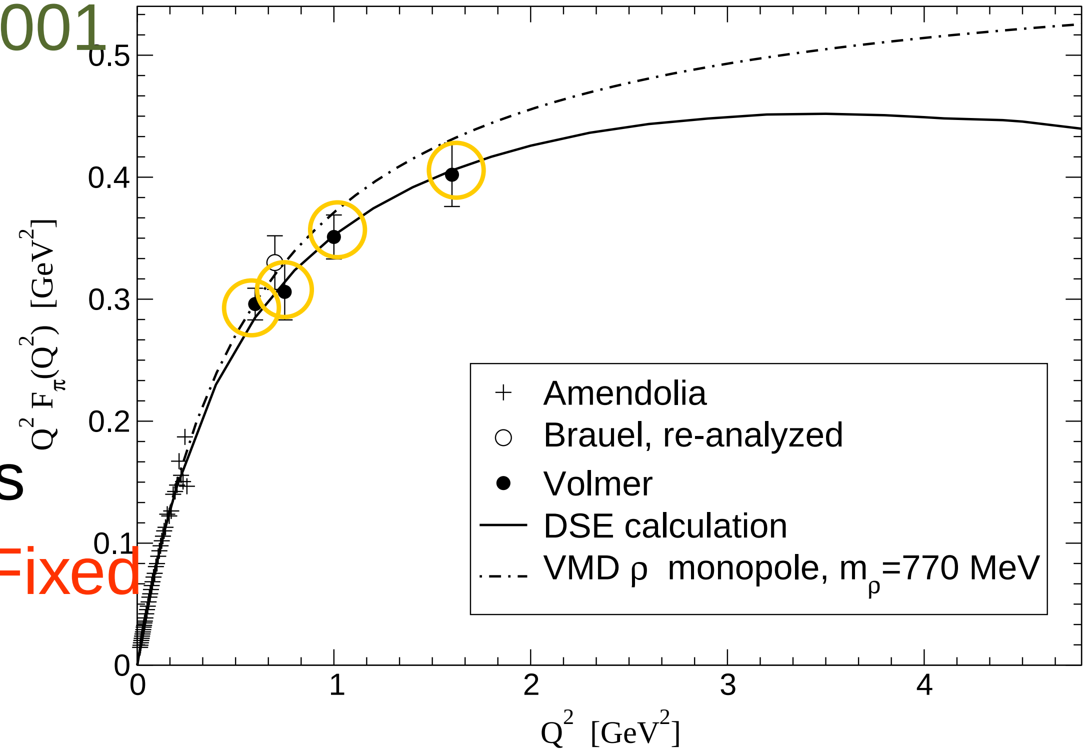
Data published in 2001



Calculated Pion Form Factor

Calculation published in 1999; No Parameters Varied

Data published in 2001



Many subsequent successful applications

. . Again, parameters Fixed

Notably π π Scattering

Maris, et al., Phys. Rev. D 65, 076008

Bicudo, Phys. Rev. C 67, 035201

Pseudoscalar meson Bethe-Salpeter amplitude

$$\begin{aligned}\chi_\pi(k;P) &= \gamma_5 \left[i\mathcal{E}_{\pi_n}(k;P) + \gamma\cdot P\mathcal{F}_{\pi_n}(k;P) \right.\\ &\quad \left. \gamma\cdot k\, k\cdot P\, \mathcal{G}_{\pi_n}(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, \mathcal{H}_{\pi_n}(k;P) \right]\end{aligned}$$

Pion . . . $J = 0$ but . . .

Pseudoscalar meson Bethe-Salpeter amplitude

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- Orbital angular momentum is not a Poincaré invariant. However, if absent in a particular frame, it will appear in another frame related via a Poincaré transformation.

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Pion . . . $J = 0$ but . . .

Pseudoscalar meson Bethe-Salpeter amplitude

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- Nonzero quark orbital angular momentum is thus a necessary outcome of a Poincaré covariant description.

Pseudoscalar meson Bethe-Salpeter amplitude

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- In QCD, a Poincaré invariant theory with interactions, $\mathcal{E} \neq 0$ forces nonzero results for $\mathcal{F}$, $\mathcal{G}$ and $\mathcal{H}$

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Introduce mixing angle θ_π such that

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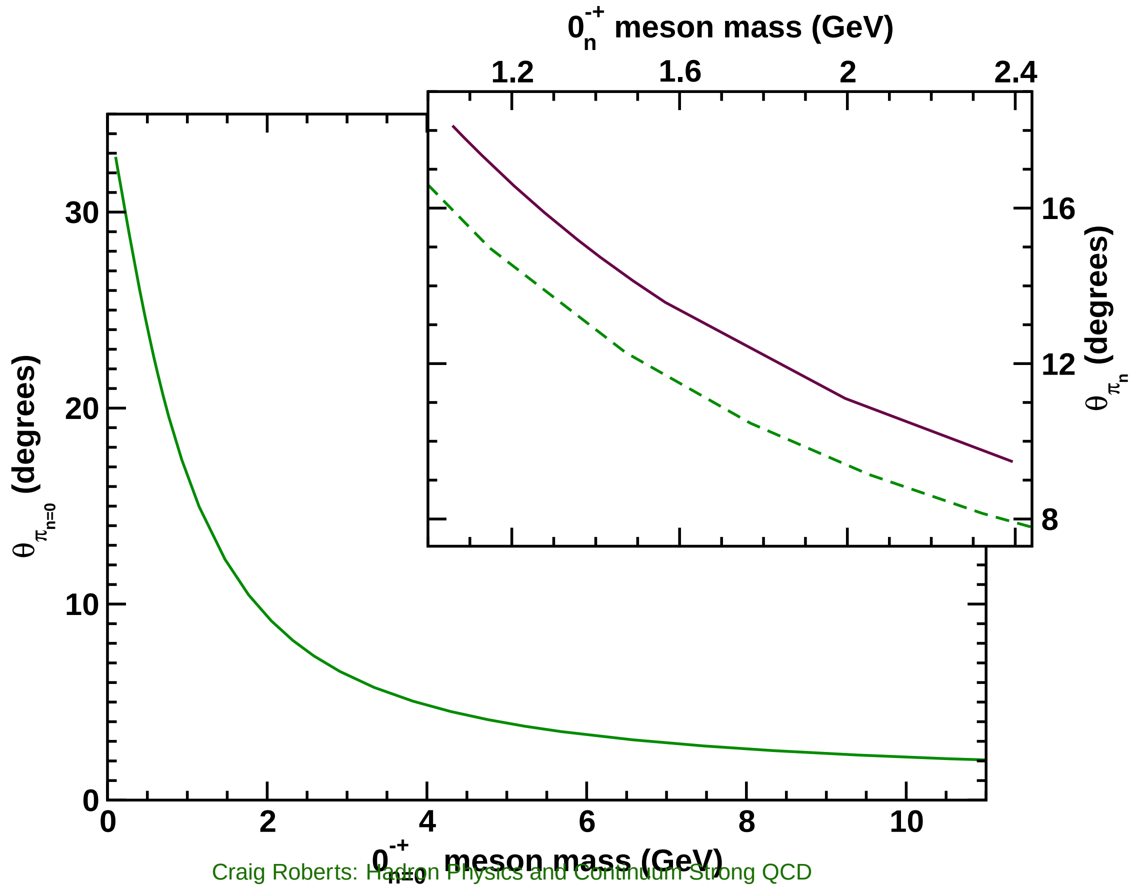

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Introduce mixing angle θ_π such that

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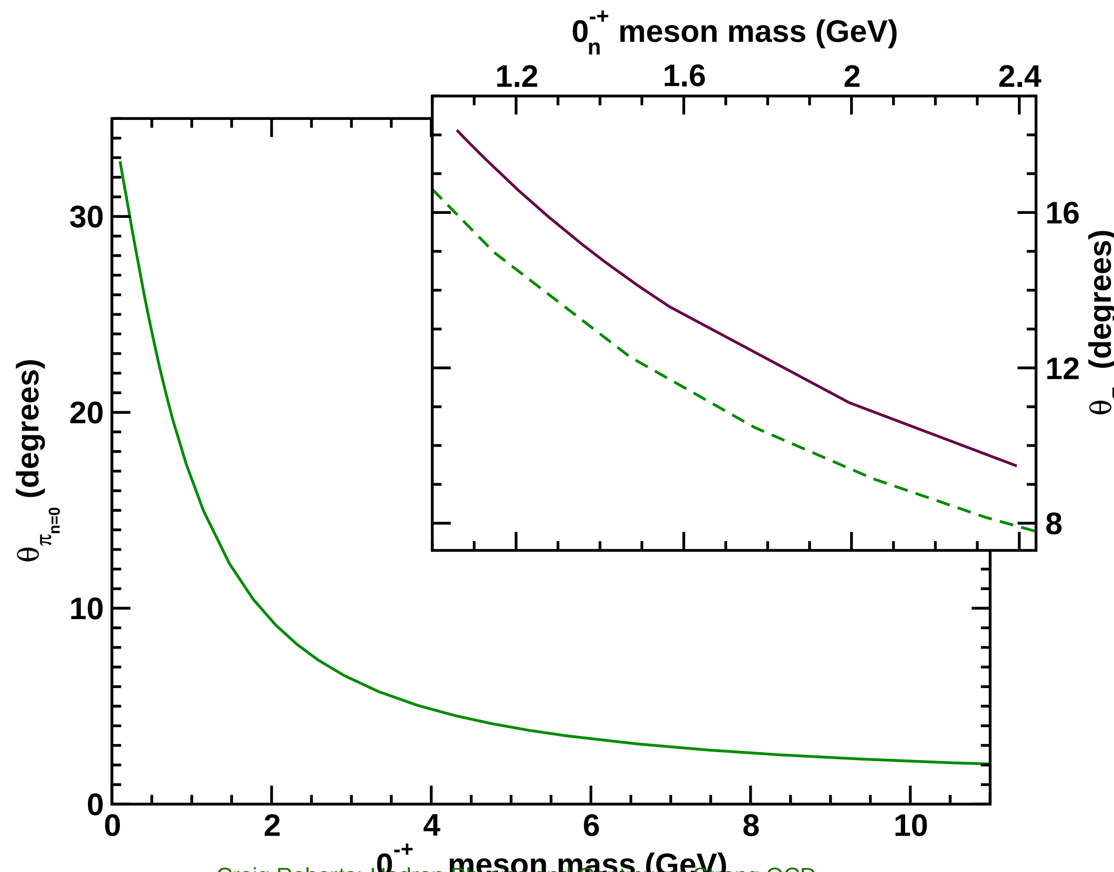
L is significant in the neighbourhood of the chiral limit, and decreases with increasing current-quark mass.



Deep-inelastic scattering

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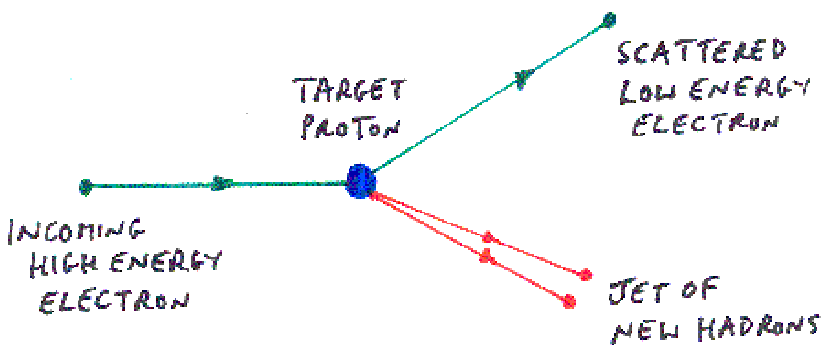
Deep-inelastic scattering

- Looking for Quarks

Deep-inelastic scattering



- Looking for Quarks

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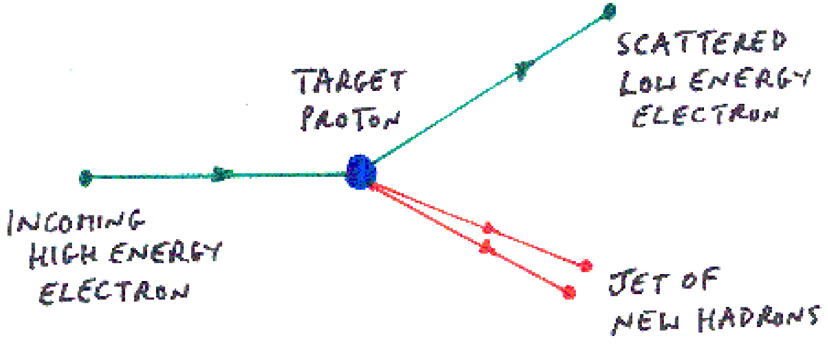
Deep-inelastic scattering



- Looking for Quarks

- **Signature Experiment** for QCD:

Discovery of Quarks at SLAC

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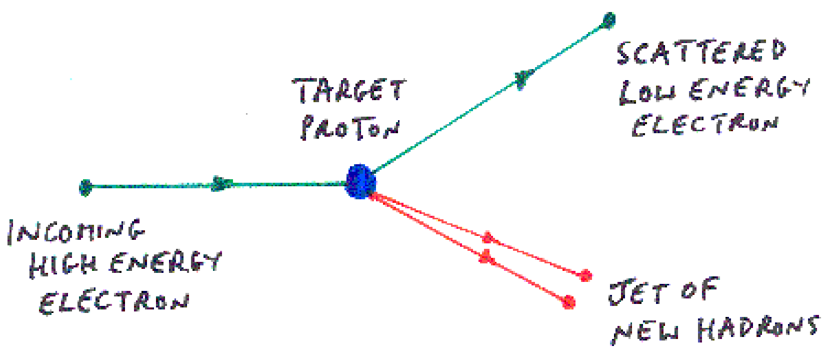
Deep-inelastic scattering



- Looking for Quarks

- Signature Experiment for QCD:
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- Cross-section: Interpreted as Measurement of Momentum-Fraction Prob. Distribution: $q(x)$, $g(x)$

Pion's valence quark distn

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Pion's valence quark distn

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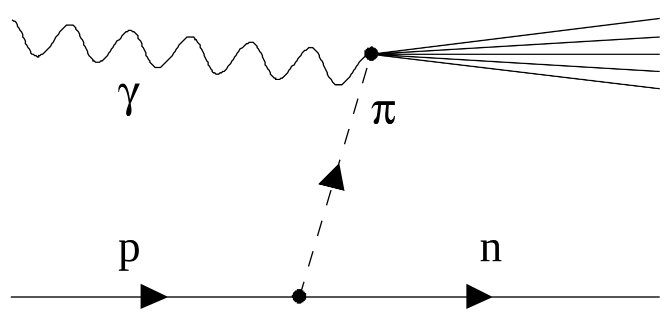
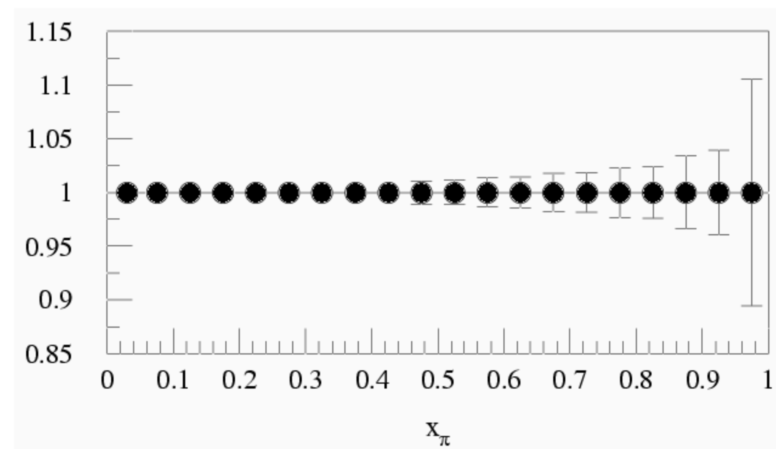
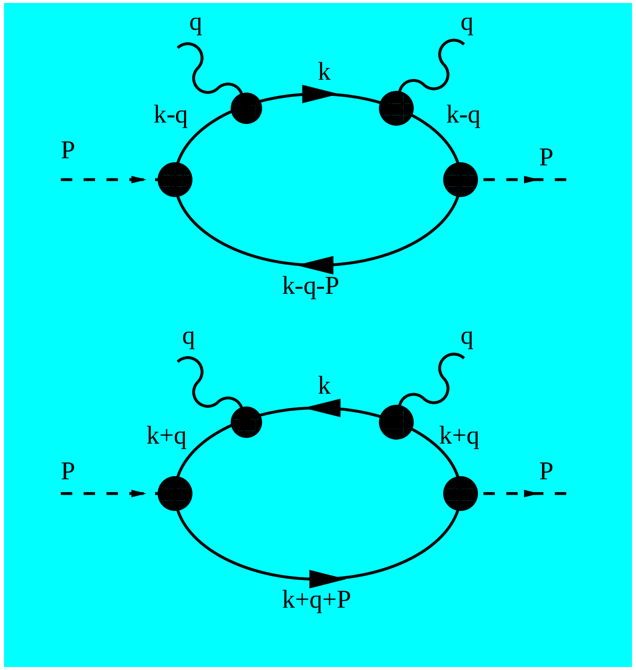
- Proposal (Holt & Reimer, ANL, nu-ex/0010004)

$e^-_{5\,\mathrm{GeV}} - p_{25\,\mathrm{GeV}}$ Collider $\to$ Accurate “Measurement”





Handbag diagrams



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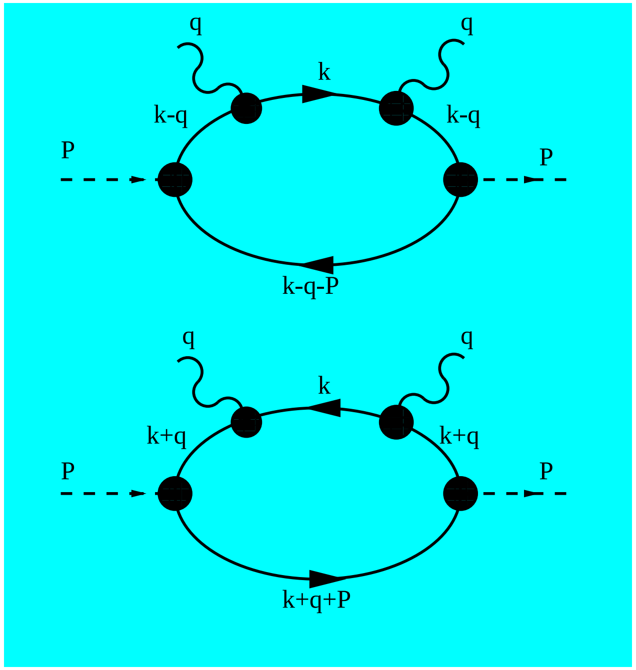

$$W_{\mu\nu}(q;P) = \frac{1}{2\pi}\mathrm{Im}\left[T^{+}_{\mu\nu}(q;P) + T^{-}_{\mu\nu}(q;P)\right]$$

$$T^{+}_{\mu\nu}(q,P) = \mathrm{tr}\int \frac{d^4k}{(2\pi)^4}\, \tau_- \bar{\Gamma}_\pi(k_{-\frac{1}{2}};-P)\, S(k_{-0})\, ieQ\Gamma_\nu(k_{-0},k)$$
$$\times\, S(k)\, ieQ\Gamma_\mu(k,k_{-0})\, S(k_{-0})\tau_+\Gamma_\pi(k_{-\frac{1}{2}};P)\, S(k_{--})$$

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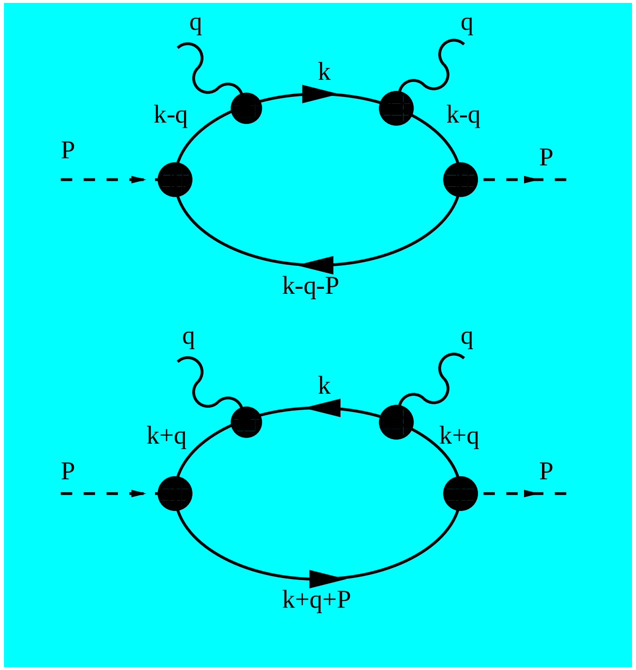
Handbag diagrams

Bjorken Limit: $q^2 \to \infty\,,\quad P\cdot q \to -\infty$

but $x := -\dfrac{q^2}{2P\cdot q}$ fixed.

Numerous algebraic simplifications



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Hecht, Roberts, Schmidt
nucl-th/0008049

Calc. $u_V(x)$ cf. Drell-Yan data

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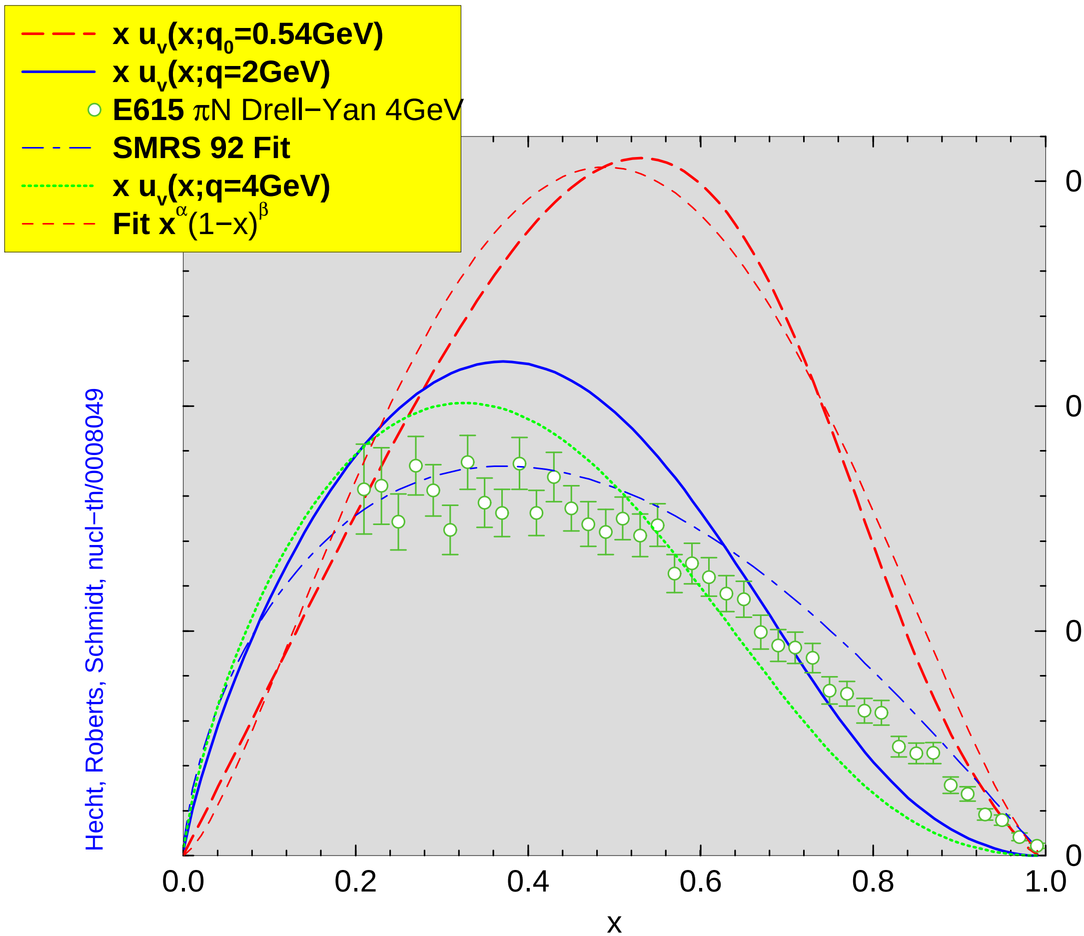
Hecht, Roberts, Schmidt
nucl-th/0008049

Calc. $u_V(x)$ cf. Drell-Yan data



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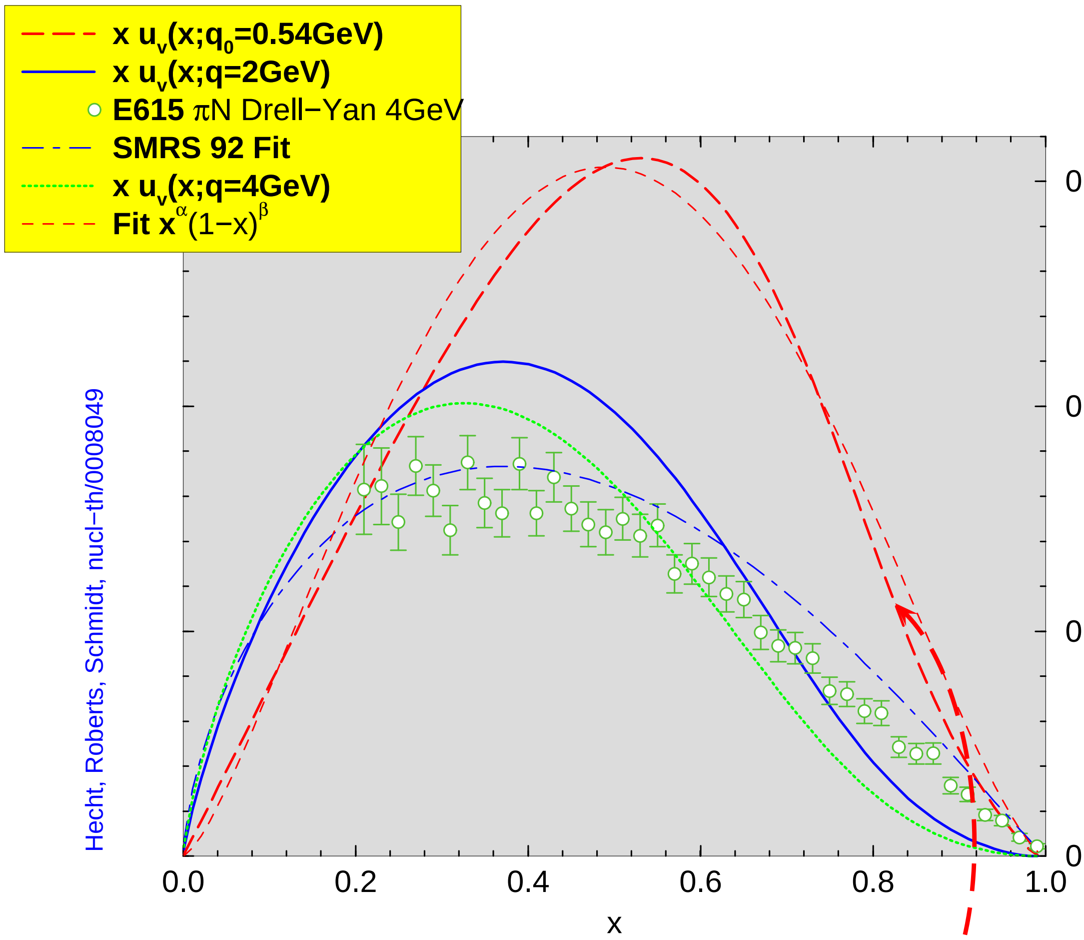

Resolving Scale: $q_0 = 0.54\,\mathrm{GeV} = 1/(0.37\,\mathrm{fm})$

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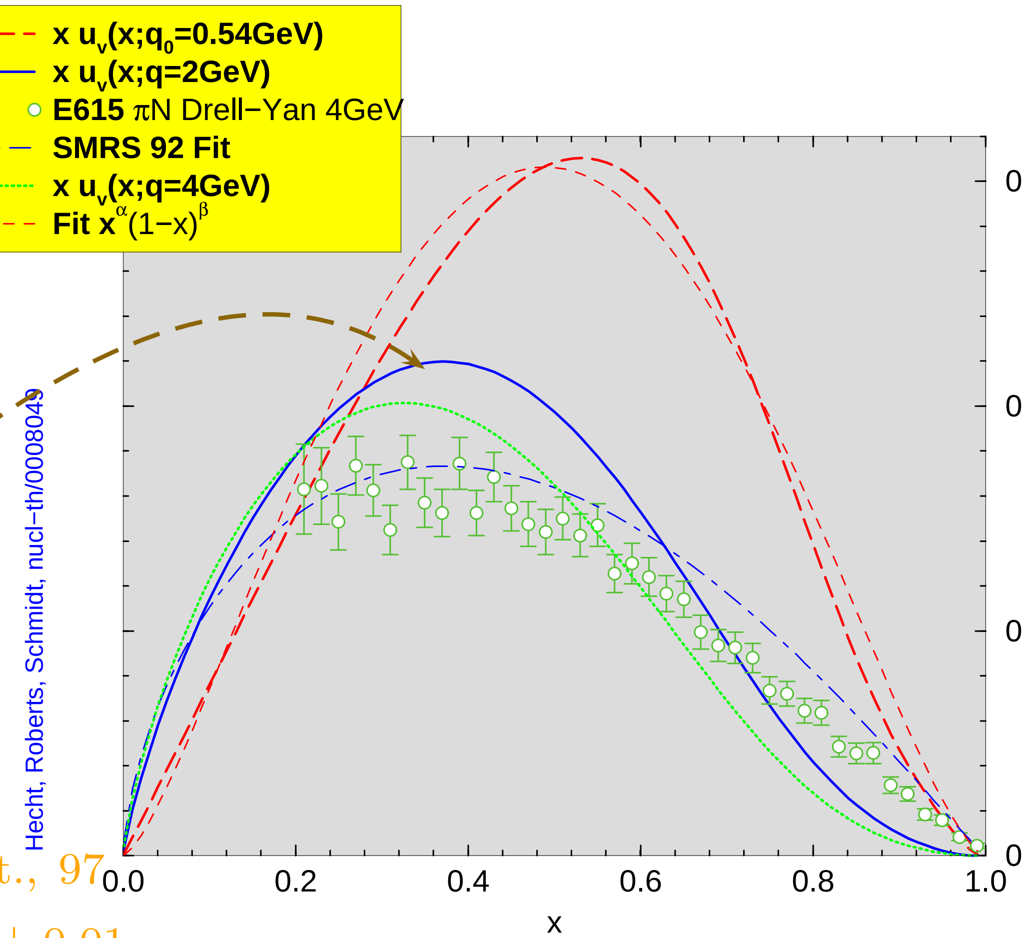
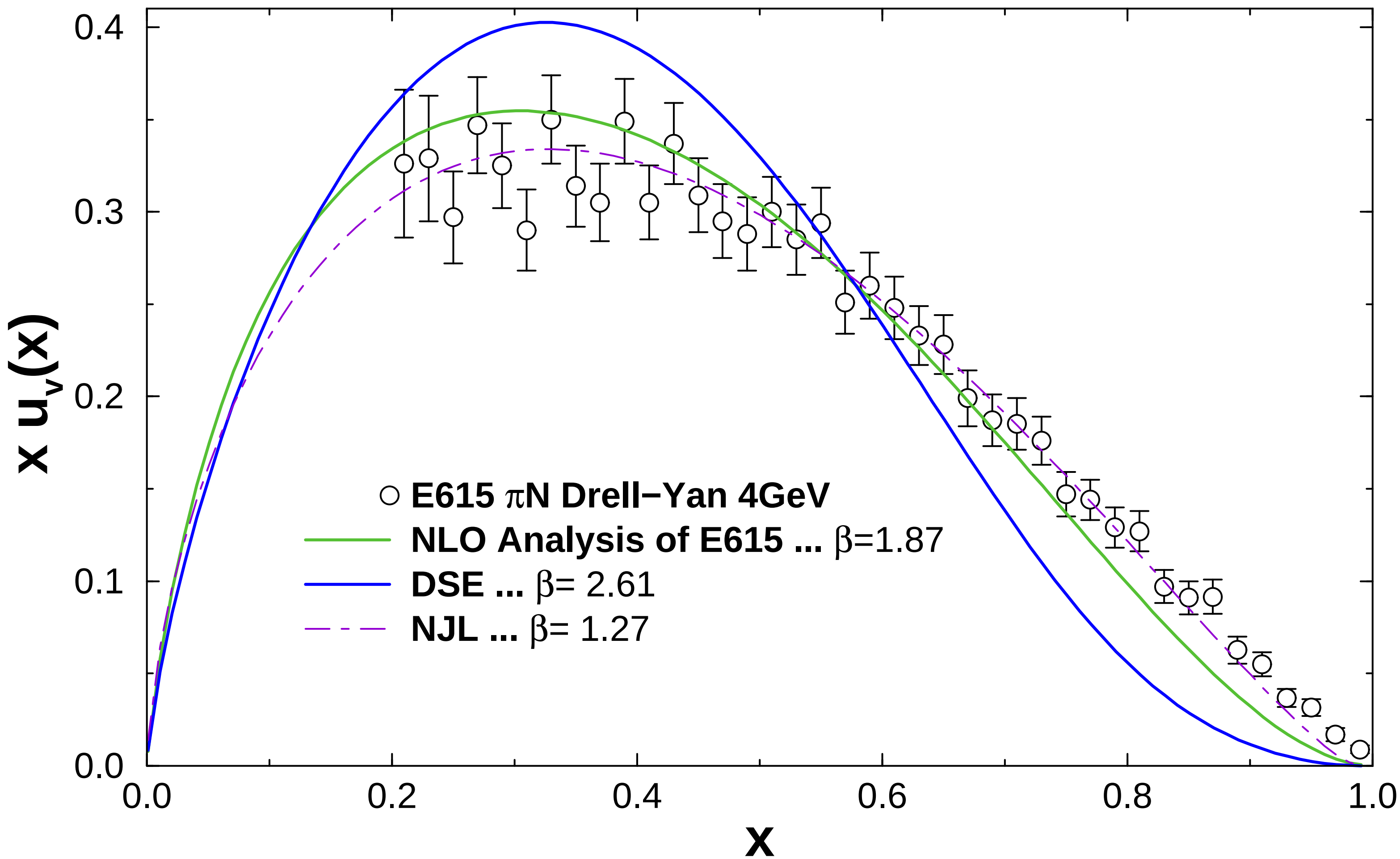
Calc. $u_V(x)$ cf. Drell-Yan data



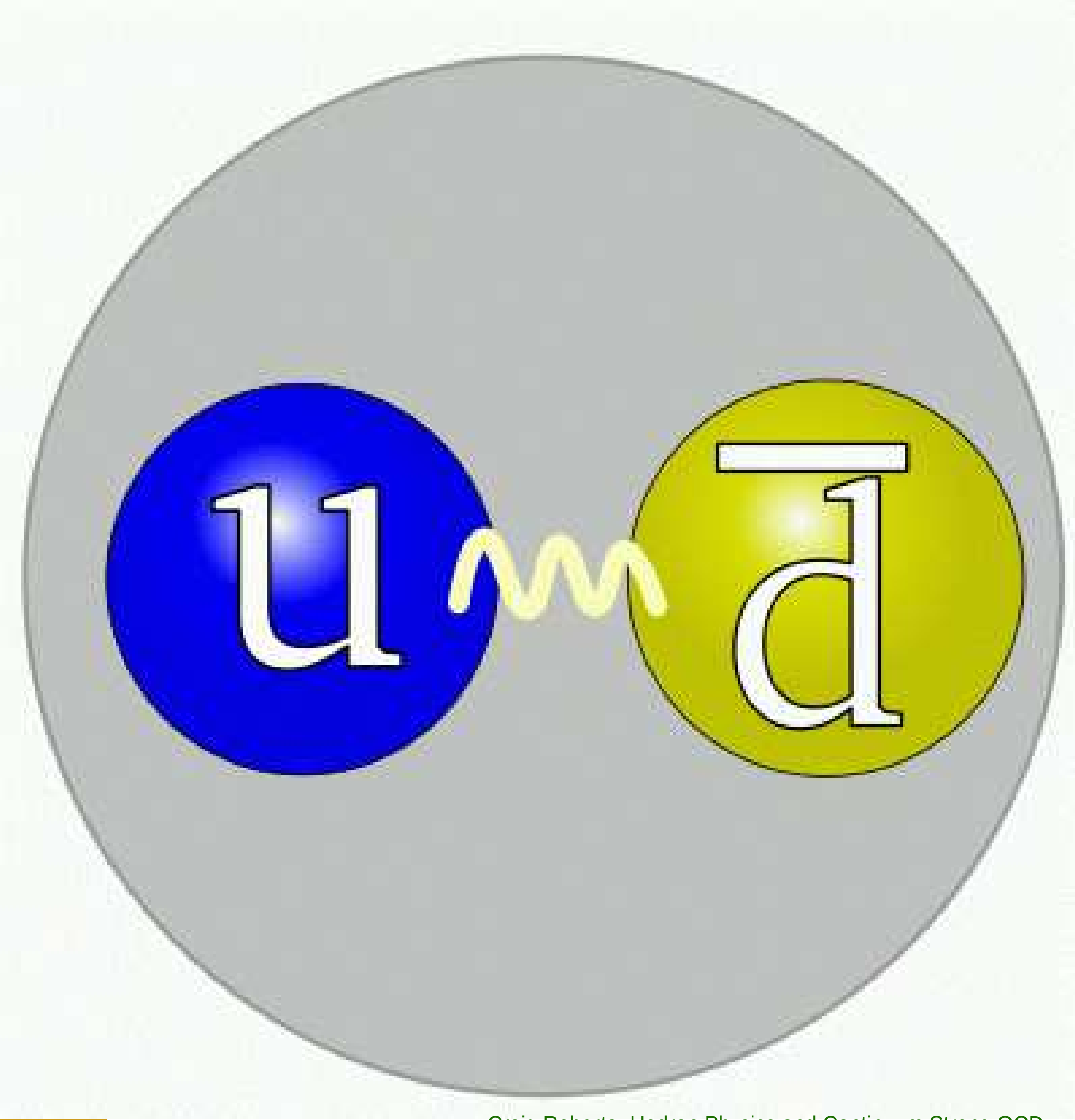
$q = 2\,\mathrm{GeV}$	Calc.	Fit, 92	Latt., 97
$\langle x \rangle_q$	0.24	0.24 ± 0.01	0.27 ± 0.01
$\langle x^2 \rangle_q$	0.10	0.10 ± 0.01	0.11 ± 0.3
$\langle x^3 \rangle_q$	0.050	0.058 ± 0.004	0.048 ± 0.020

Extant theory vs. experiment

K. Wijersooriya, P. Reimer and R. Holt,
nu-ex/0509012 ... Phys. Rev. C (Rapid)



Answer for the pion



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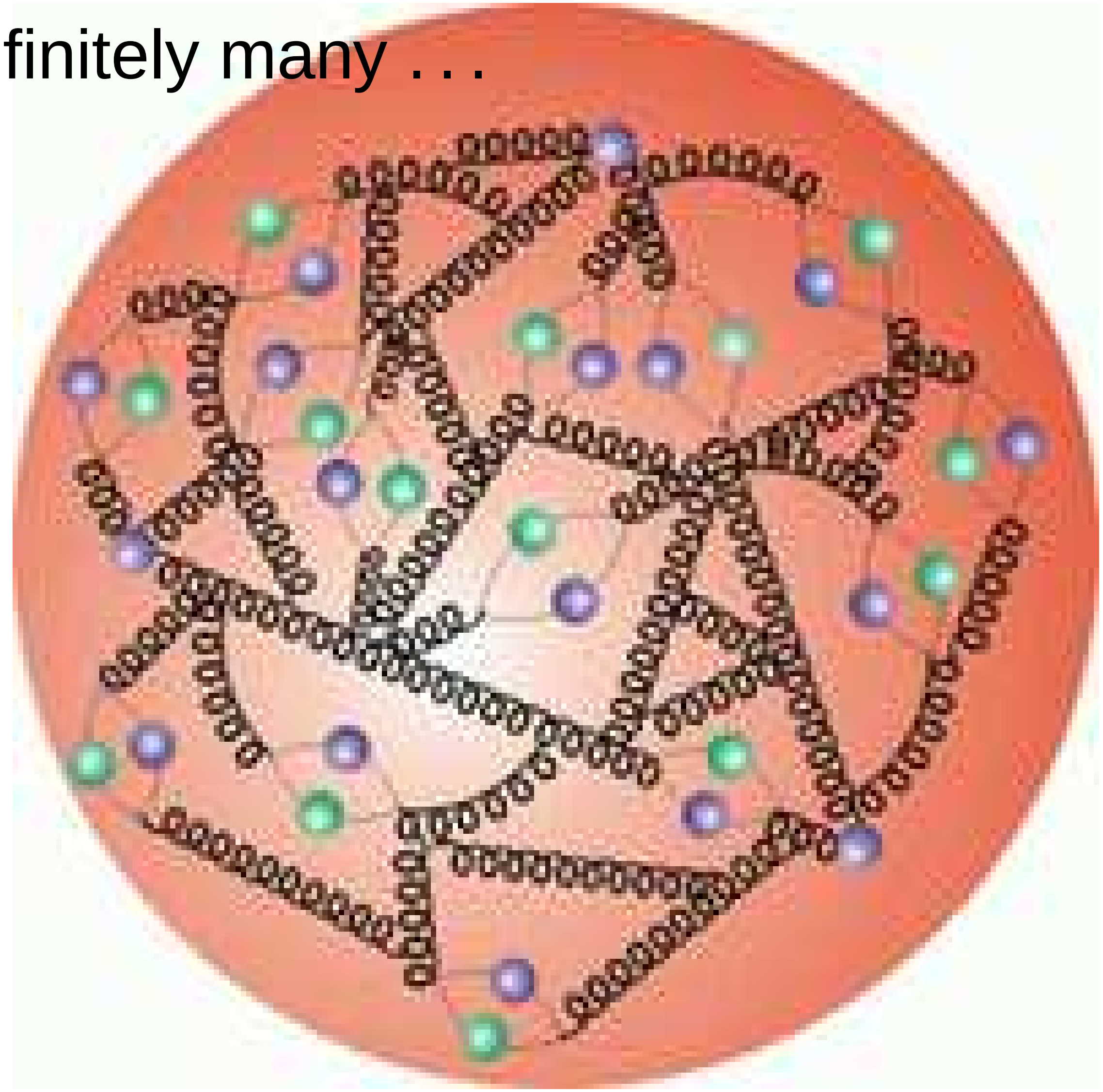
Answer for the pion

Two → Infinitely many . . .



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Exploring Nuclear Matter - Quarks to Stars
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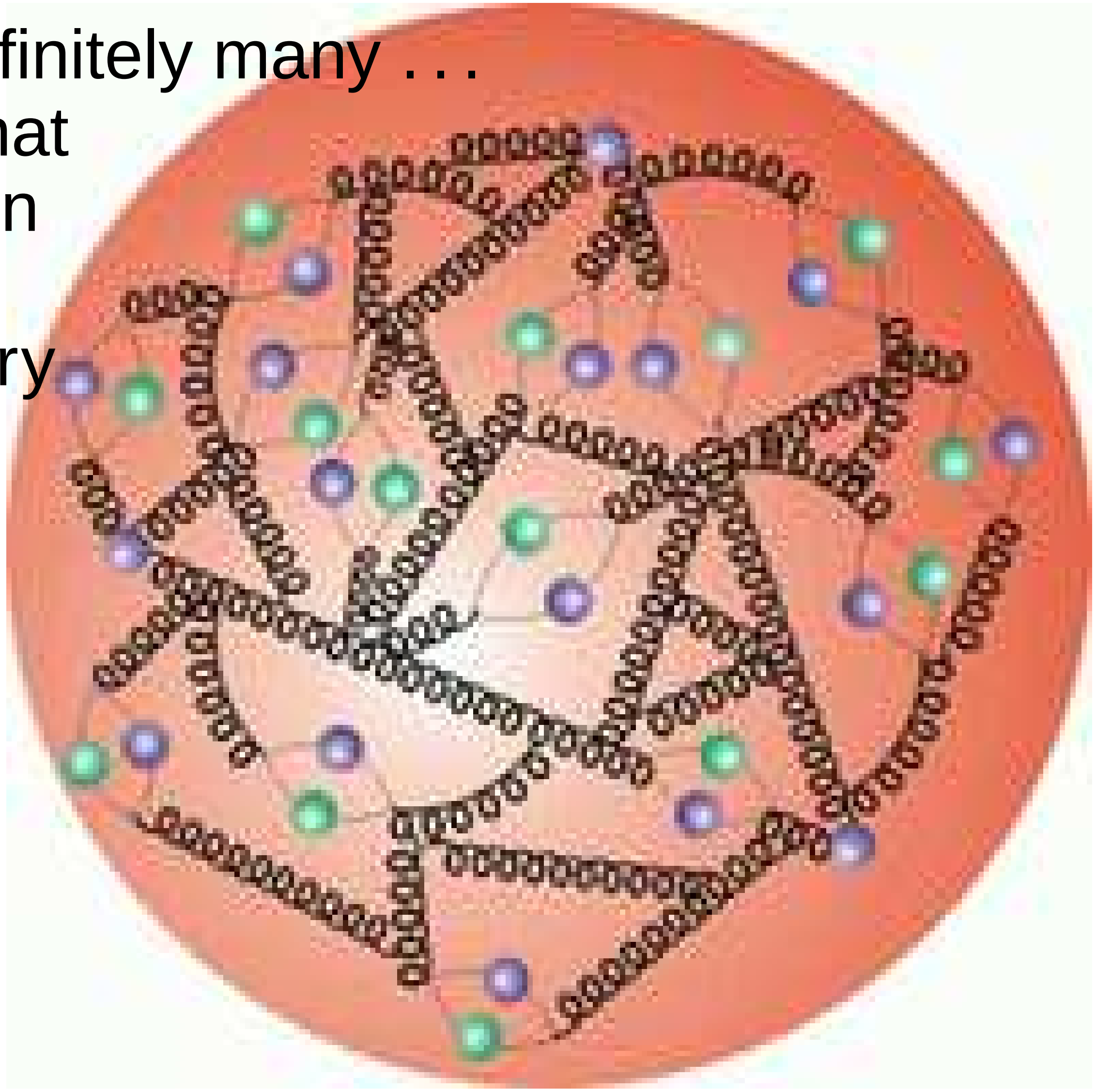
Handle that properly in quantum field theory



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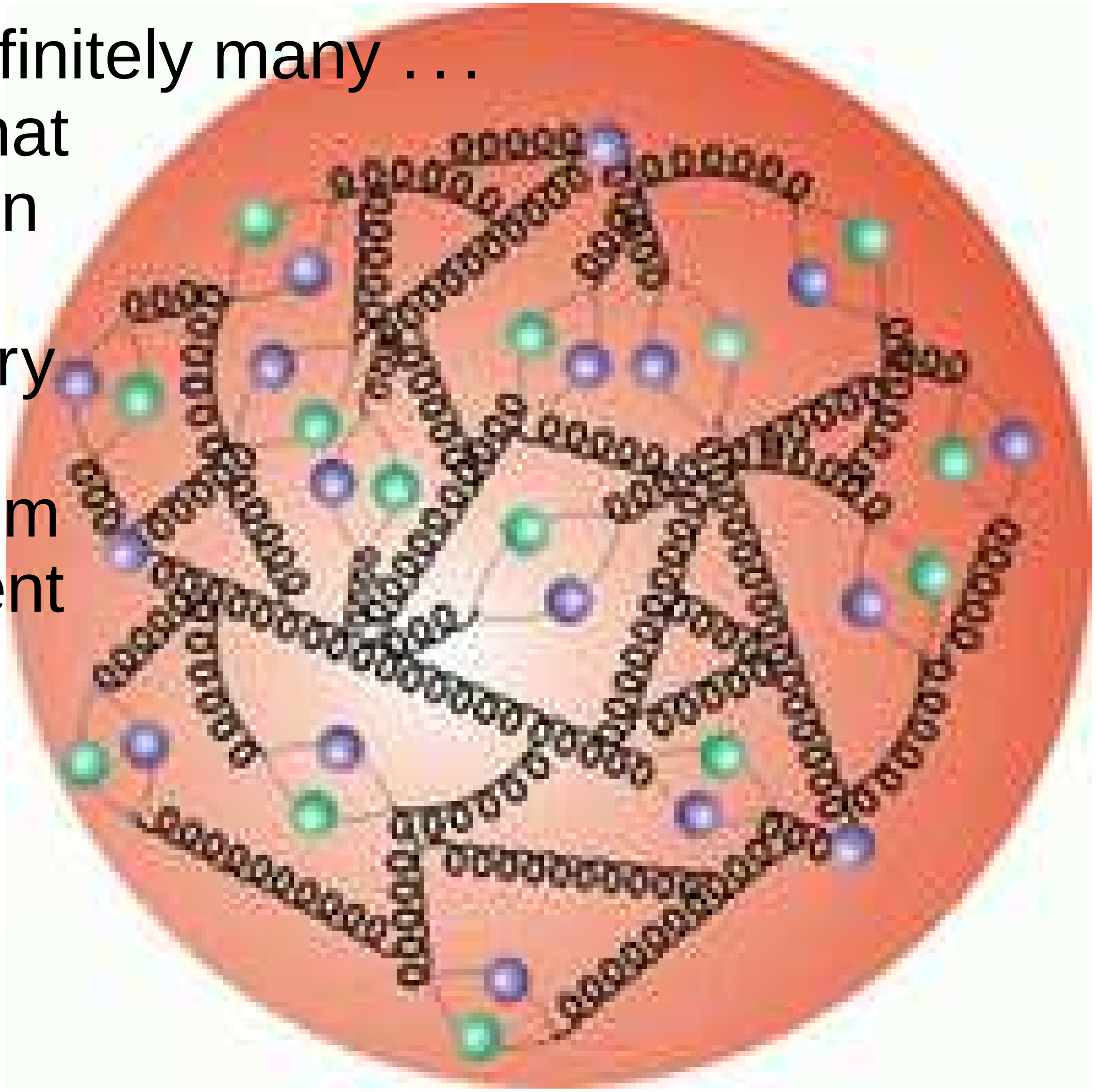
momentum-dependent dressing



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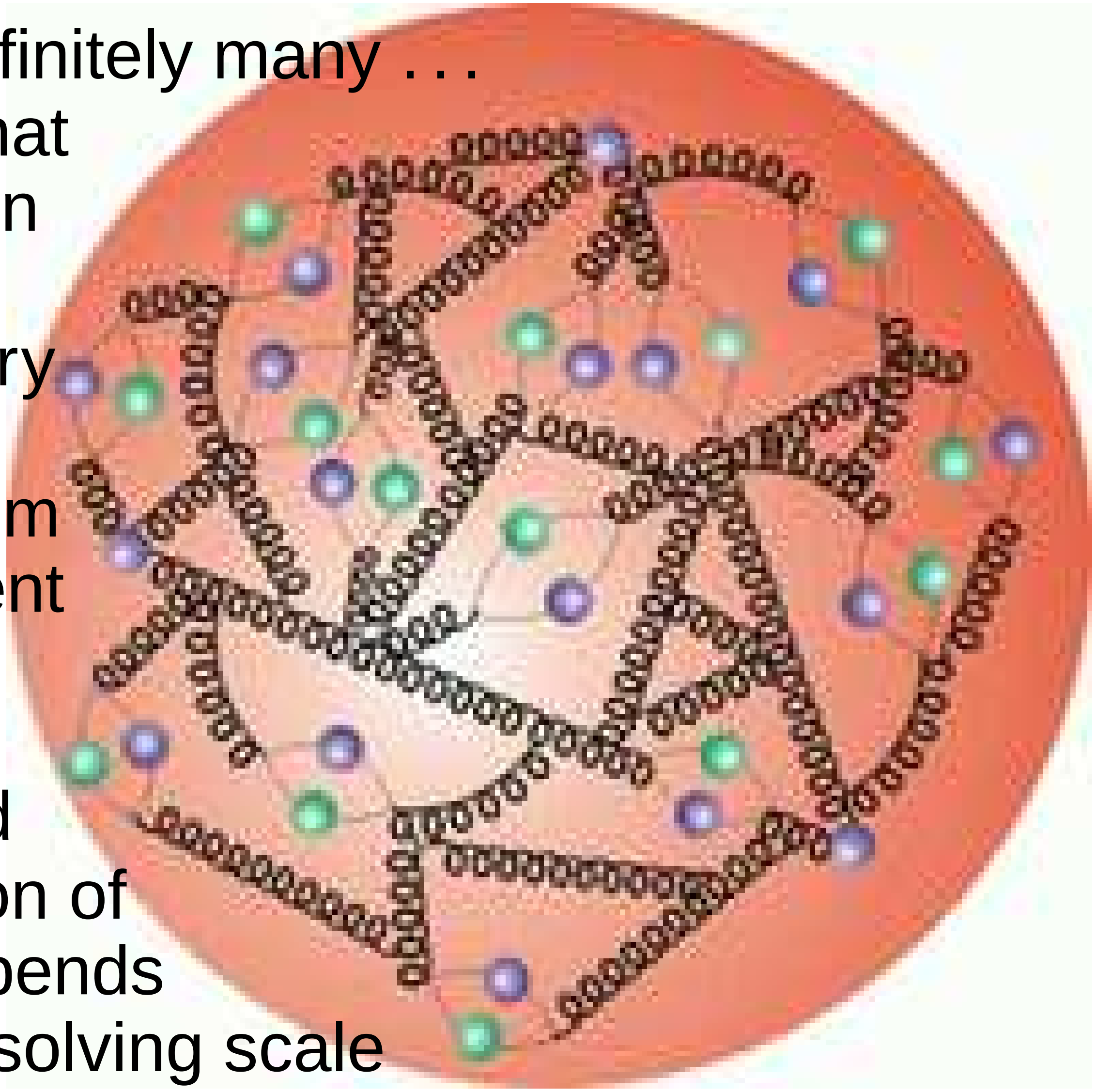

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perceived distribution of mass depends on the resolving scale

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  - Elastic electromagnetic pion form factor
  - Nature of Baryons