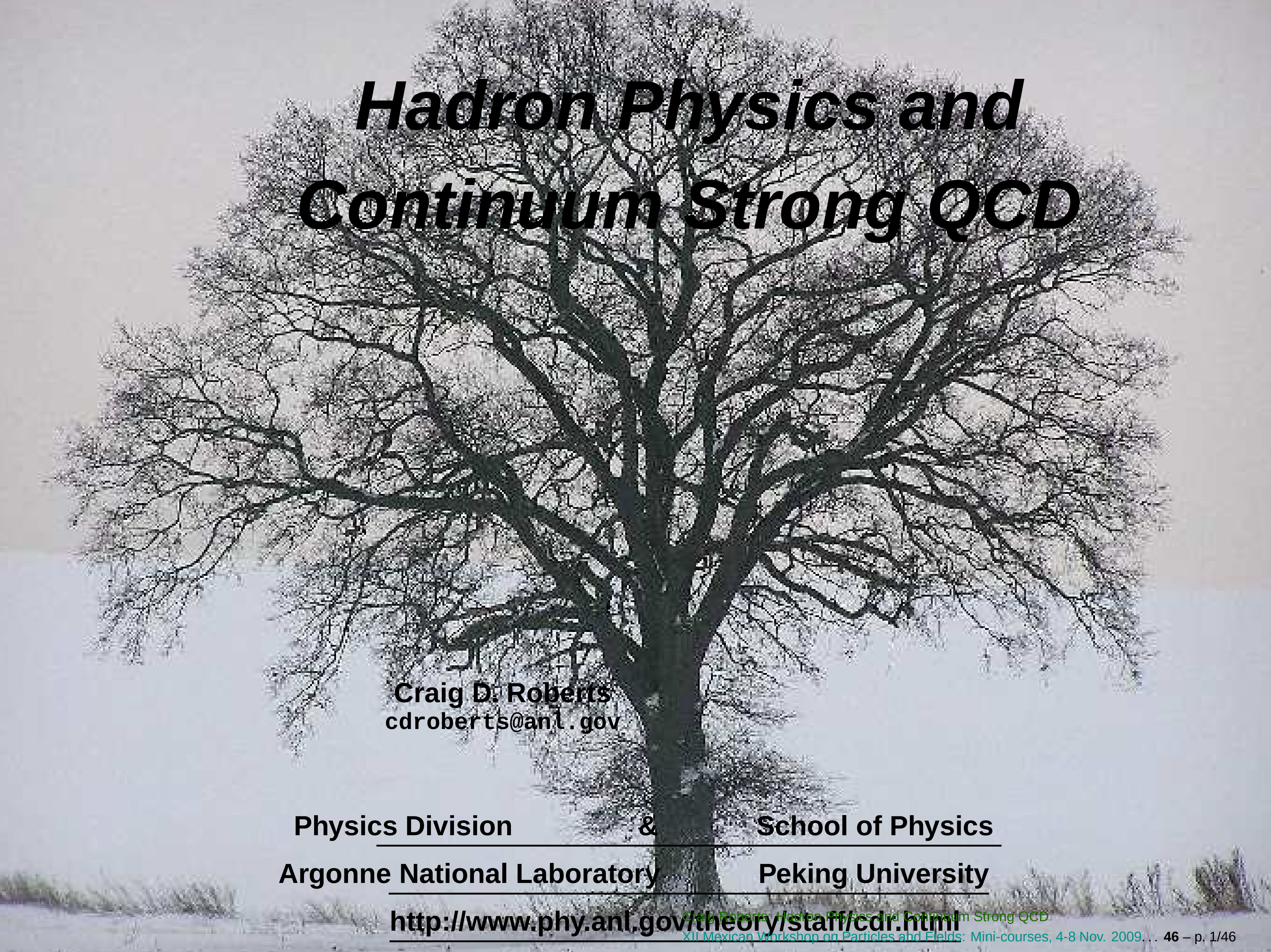


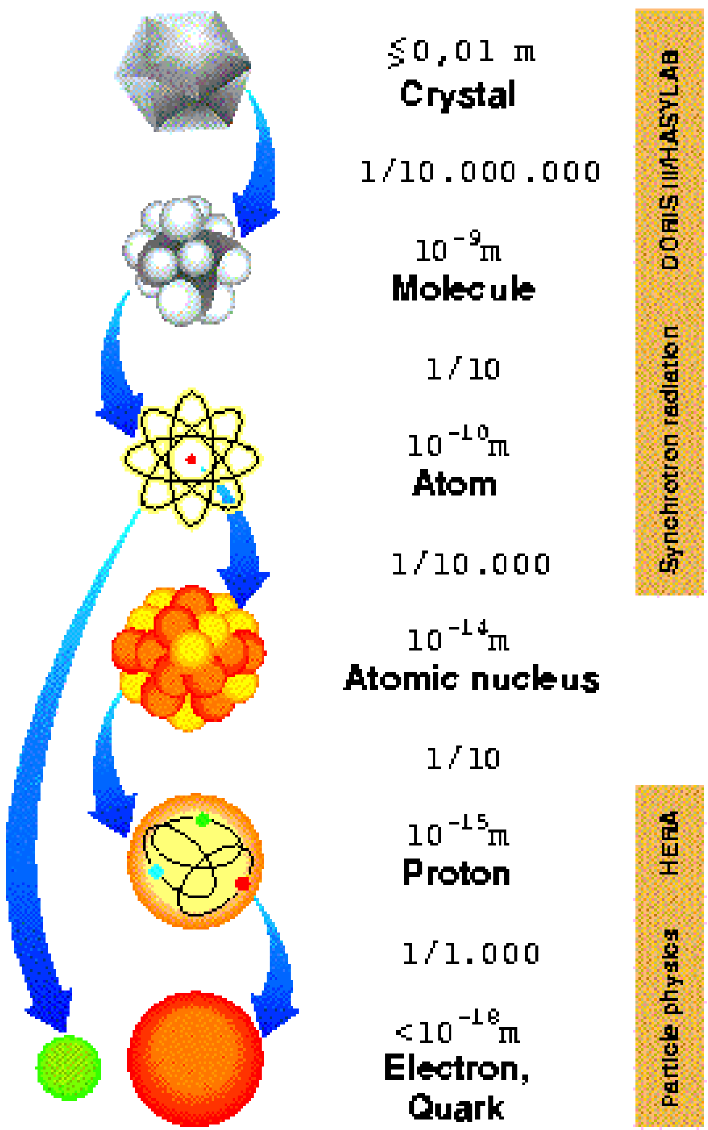
Hadron Physics and Continuum Strong QCD

Craig D. Roberts
cdroberts@anl.gov

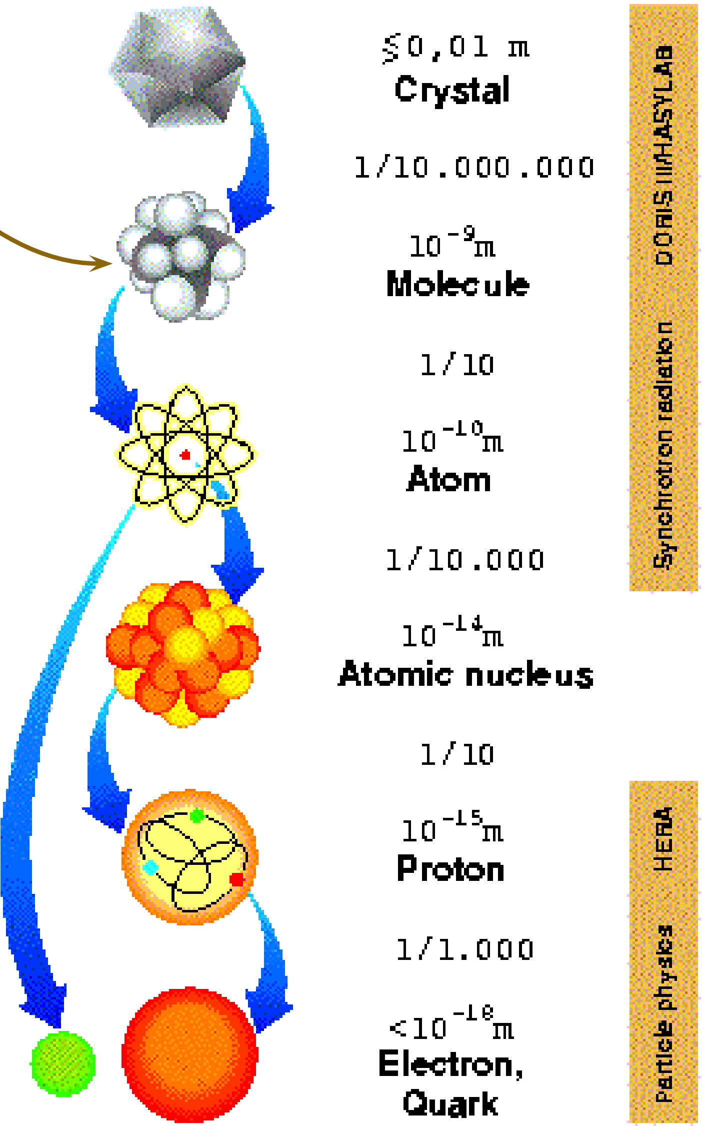
Physics Division & School of Physics
Argonne National Laboratory Peking University

http://www.phy.anl.gov/theory/staff/cdr.html

Hadron Physics

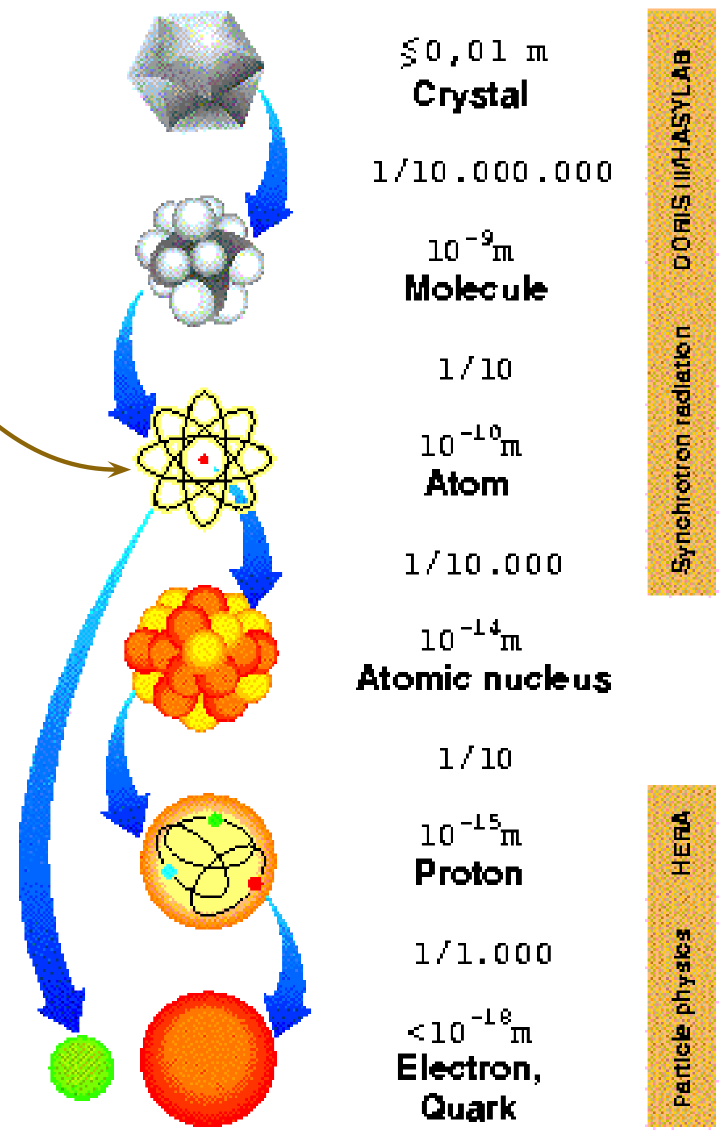


Molecular Physics
Scale = nm

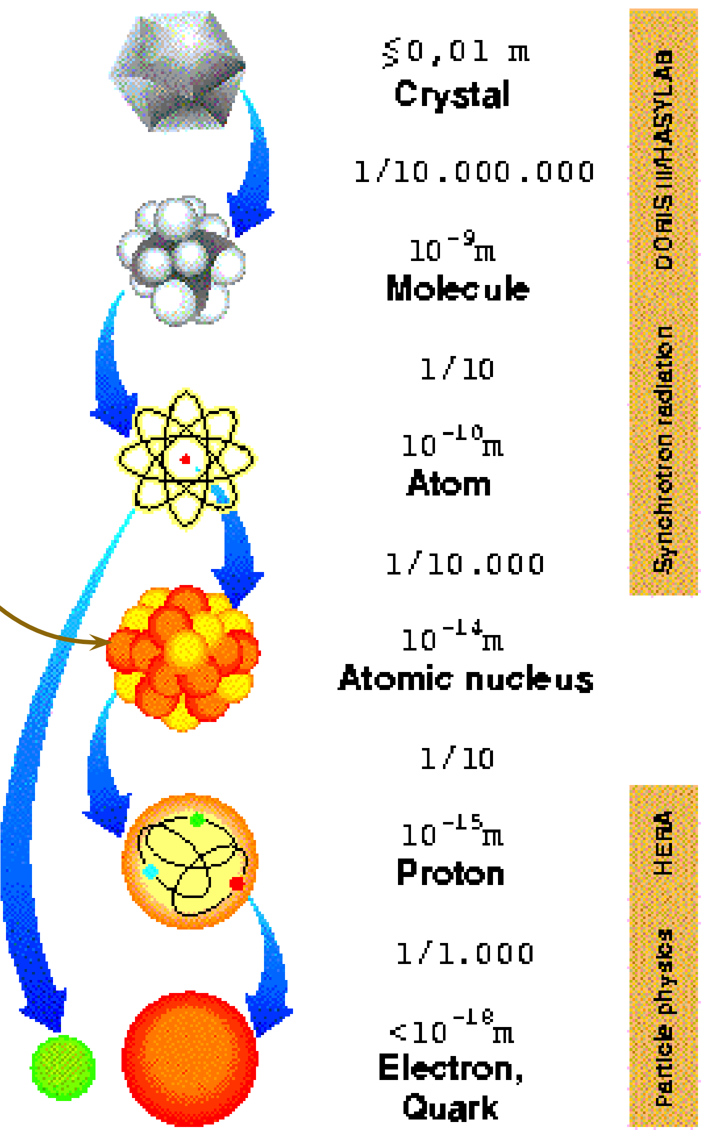


Atomic Physics

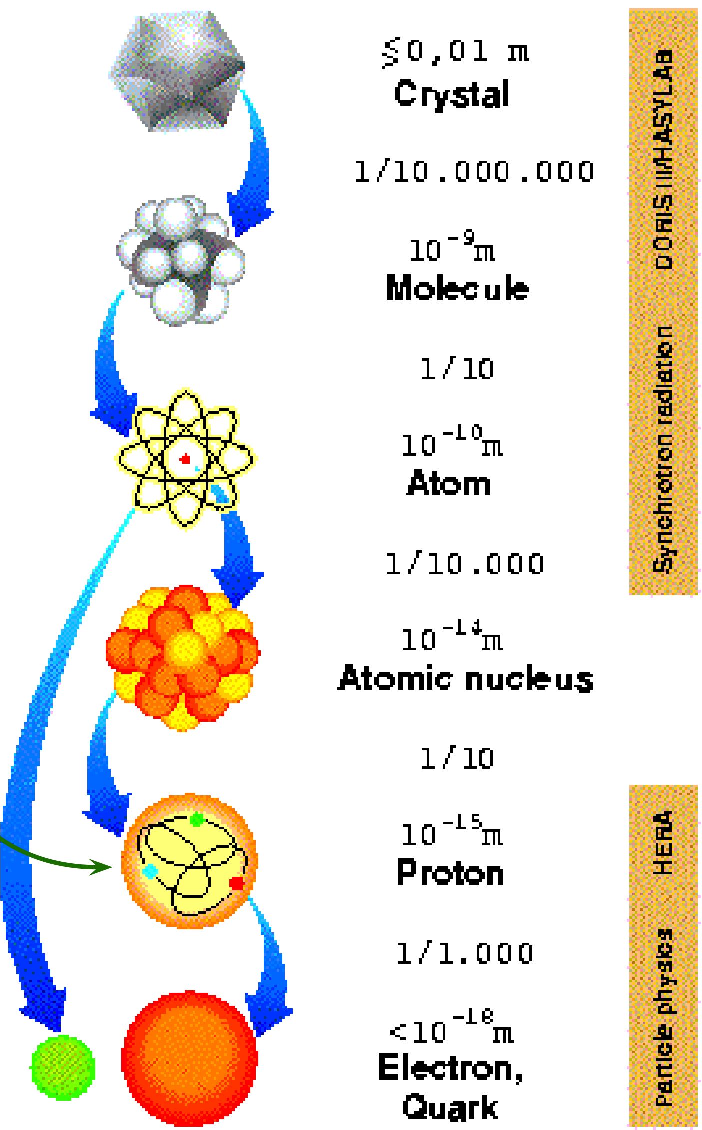
Scale = Å



Nuclear Physics
Scale = 10 fm



Hadron Physics
Scale = 1 fm



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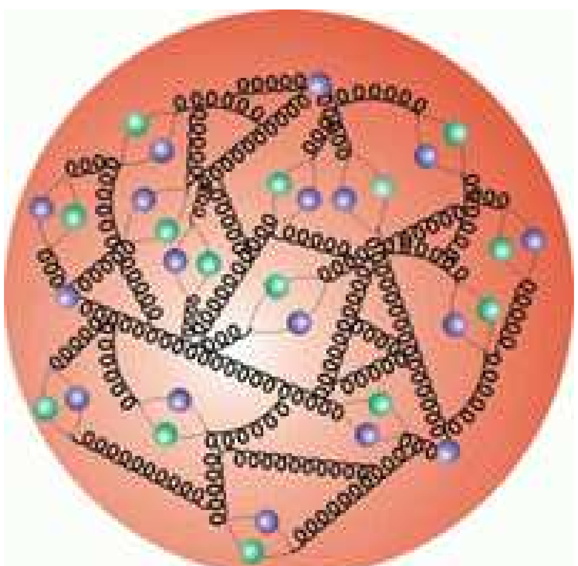
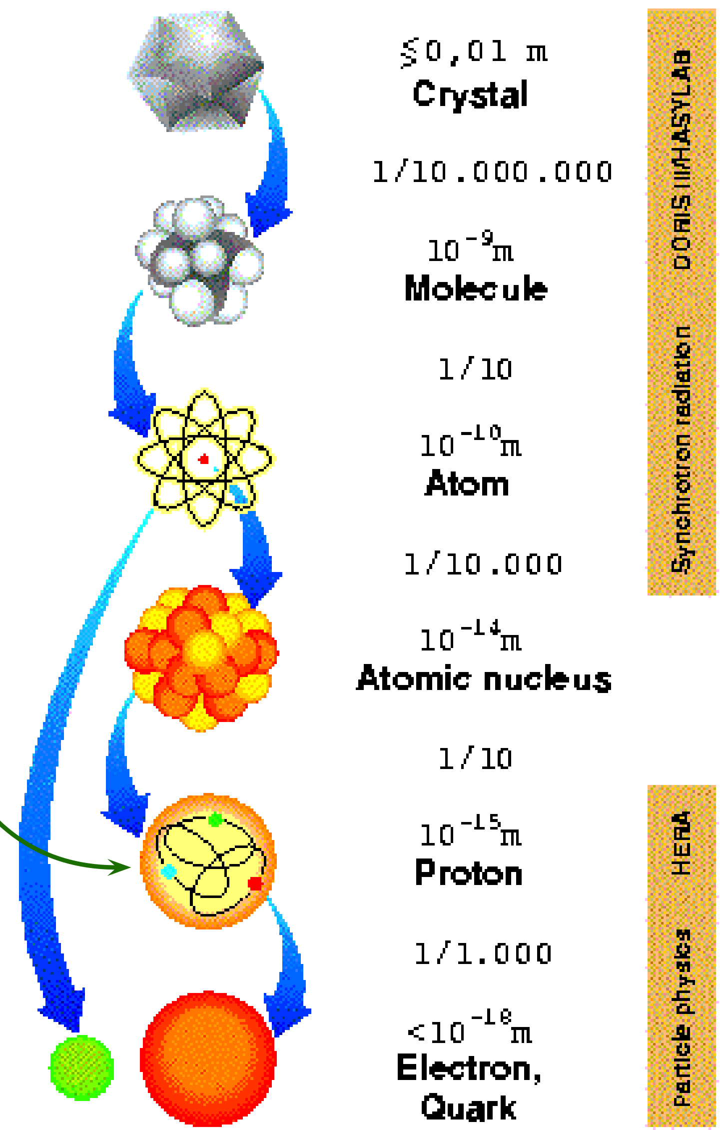
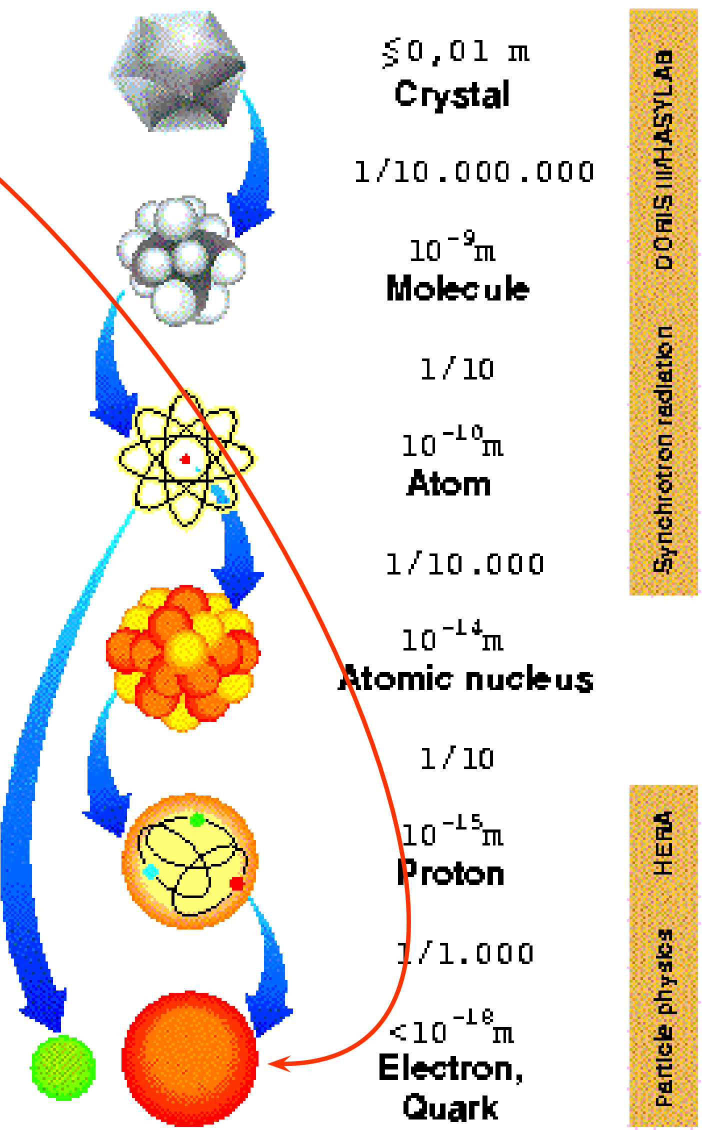
Hadron Physics
Scale = 1 fm





Hadron Physics

Meta-Physics
Scale = Limited only
by Theorists
Imagination



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Nucleon . . . 2 Key Hadrons
= Proton and Neutron

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- Fermions – two static properties:
  proton electric charge $= +1$; and magnetic moment, μ_p

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- Stern (1933) – $\mu_p = (1 + 1.79)\dfrac{e\hbar}{2M}$
  - Big Hint that Proton is not a point particle
  - Proton has constituents
  - These are Quarks and Gluons
    Quark discovery via $e^- p$-scattering at SLAC in 1968
    – the elementary quanta of Quantum Chromo-dynamics

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- Action, in terms of local Lagrangian density:

$$S[A^a_\mu,\bar q,q] = \int d^4x\,\{\frac{1}{4}F^a_{\mu\nu}(x)F^a_{\mu\nu}(x) + \frac{1}{2\xi}\partial_\mu A^a_\mu(x)\,\partial_\nu A^a_\nu(x) + \bar q(x)[\gamma_\mu D_\mu + M]q(x)\} \quad (1)$$

  - Chromomagnetic Field Strength Tensor – $\partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) + g f^{abc} A^b_\mu(x) A^c_\nu(x)$
  - Covariant Derivative – $D_\mu = \partial_\mu - ig\frac{\lambda^a}{2}A^a_\mu(x)$
  - Current-quark Mass matrix: $\begin{pmatrix} m_u & 0 & 0 & \ldots \\ 0 & m_d & 0 & \ldots \\ 0 & 0 & m_s & \ldots \\ \vdots & \vdots & \vdots & \end{pmatrix}$
- Understanding JLab Observables means knowing all that this Action predicts.
- Perturbation Theory (asymptotic freedom) is not enough!
  - Bound states are not perturbative
  - Confinement is not perturbative
  - DCSB is not perturbative

Euclidean Metric

- Almost all nonperturbative studies in relativistic quantum field theory employ a Euclidean Metric. (NB. Remember the Wick Rotation?)
- It is possible to view the Euclidean formulation of a quantum field theory as definitive; e.g.,
  - Symanzik, K. (1963) in *Local Quantum Theory* (Academic, New York) edited by R. Jost.
  - Streater, R.F. and Wightman, A.S. (1980), *PCT, Spin and Statistics, and All That* (Addison-Wesley, Reading, Mass, 3rd edition).
  - Glimm, J. and Jaffee, A. (1981), *Quantum Physics. A Functional Point of View* (Springer-Verlag, New York).
  - Seiler, E. (1982), *Gauge Theories as a Problem of Constructive Quantum Theory and Statistical Mechanics* (Springer-Verlag, New York).
- That decision is crucial when a consideration of nonperturbative effects becomes important. In addition, the discrete lattice formulation in Euclidean space has allowed some progress to be made in attempting to answer existence questions for interacting gauge field theories.
  - A lattice formulation is impossible in Minkowski space – the integrand is not non-negative and hence does not provide a probability measure.

Euclidean Metric: Transcription Formulae

- To make clear our conventions: for 4-vectors a, b:
  $a \cdot b := a_\mu \, b_\nu \, \delta_{\mu\nu} := \sum_{i=1}^{4} a_i \, b_i$, Hence, a spacelike vector, Q_μ, has $Q^2 > 0$.
- Dirac matrices:
  - Hermitian and defined by the algebra $\{\gamma_\mu, \gamma_\nu\} = 2\,\delta_{\mu\nu}$;
  - we use $\gamma_5 := -\,\gamma_1\gamma_2\gamma_3\gamma_4$, so that $\mathrm{tr}\,[\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma] = -4\,\varepsilon_{\mu\nu\rho\sigma}$, $\varepsilon_{1234} = 1$.
  - The Dirac-like representation of these matrices is:

$$\vec{\gamma} = \begin{pmatrix} 0 & -i\vec{\tau} \\ i\vec{\tau} & 0 \end{pmatrix}, \; \gamma_4 = \begin{pmatrix} \tau^0 & 0 \\ 0 & -\tau^0 \end{pmatrix}, \tag{2}$$

where the 2×2 Pauli matrices are:

$$\tau^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \; \tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \; \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \; \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3}$$

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Euclidean Metric: Transcription Formulae

- It is possible to derive every equation introduced above assuming certain analytic properties of the integrands. However, the derivations can be sidestepped using the following *transcription rules*:

Configuration Space

1. $\int^M d^4x^M \to -i \int^E d^4x^E$
2. $\partial\!\!\!/ \to i\gamma^E \cdot \partial^E$
3. $A\!\!\!/ \to -i\gamma^E \cdot A^E$
4. $A_\mu B^\mu \to -A^E \cdot B^E$
5. $x^\mu \partial_\mu \to x^E \cdot \partial^E$

Momentum Space

1. $\int^M d^4k^M \to i \int^E d^4k^E$
2. $k\!\!\!/ \to -i\gamma^E \cdot k^E$
3. $A\!\!\!/ \to -i\gamma^E \cdot A^E$
4. $k_\mu q^\mu \to -k^E \cdot q^E$
5. $k_\mu x^\mu \to -k^E \cdot x^E$

- These rules are valid in perturbation theory; i.e., the correct Minkowski space integral for a given diagram will be obtained by applying these rules to the Euclidean integral: they take account of the change of variables and rotation of the contour. However, for diagrams that represent DSEs which involve dressed n-point functions, whose analytic structure is not known *a priori*, the Minkowski space equation obtained using this prescription will have the right appearance but it's solutions may bear no relation to the analytic continuation of the solution of the Euclidean equation. Any such differences will be nonperturbative in origin.

What is QCD?

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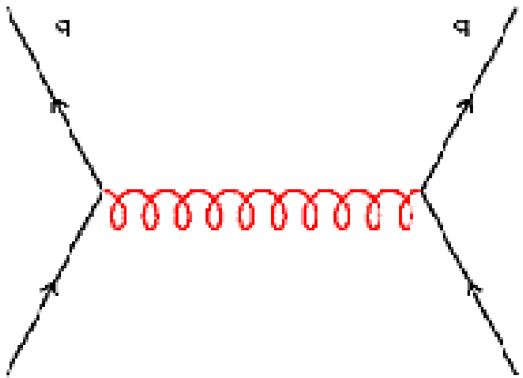
- Gauge Theory:

  Interactions Mediated by massless vector bosons

Feynman Diagram of Quark—Quark Scattering



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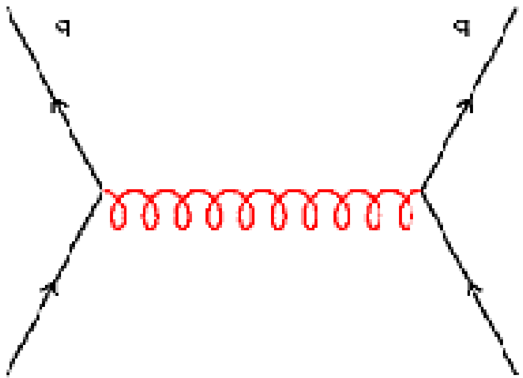

- Similar interaction in QED

What is QCD?

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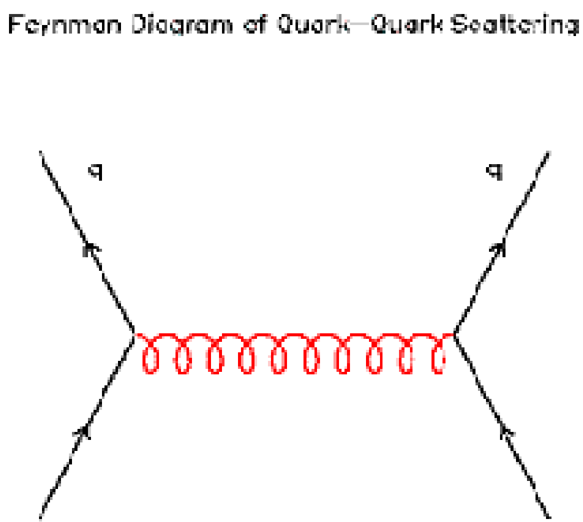
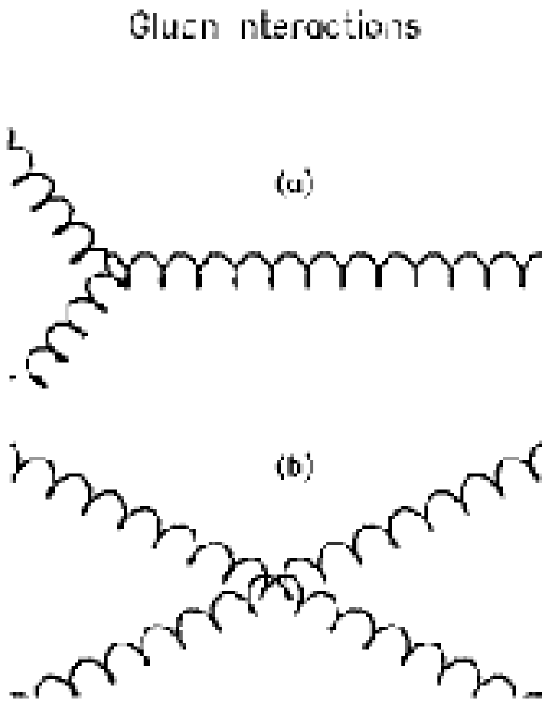




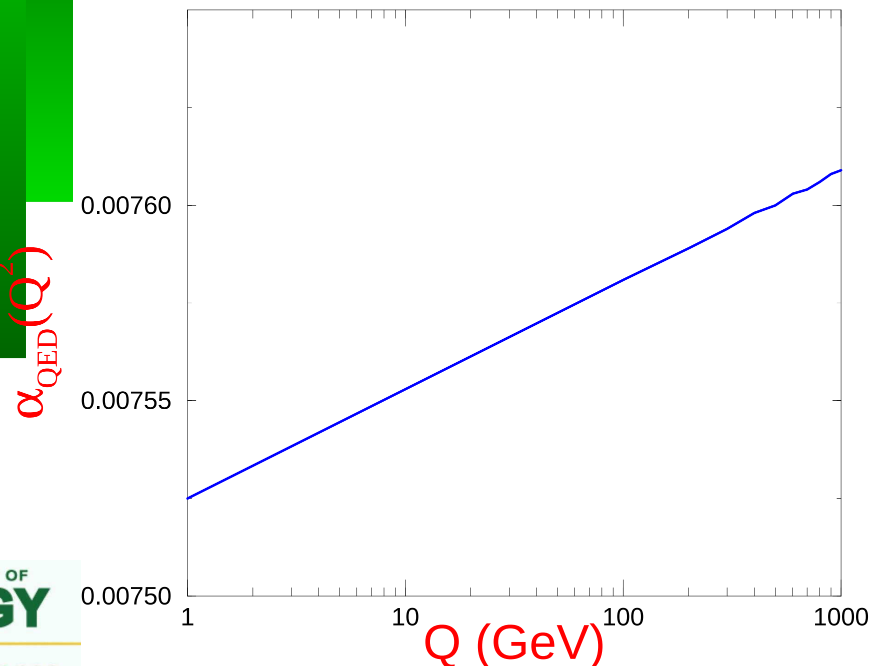
- Similar interaction in QED

- Special Feature of QCD – gluon self-interactions

  Completely Change the Character of the Theory

QED cf. QCD

QED cf. QCD



$$\alpha_{\rm QED} = \frac{\alpha}{1 - \alpha/3\pi \ln\left(Q^2/m_e^2\right)}$$

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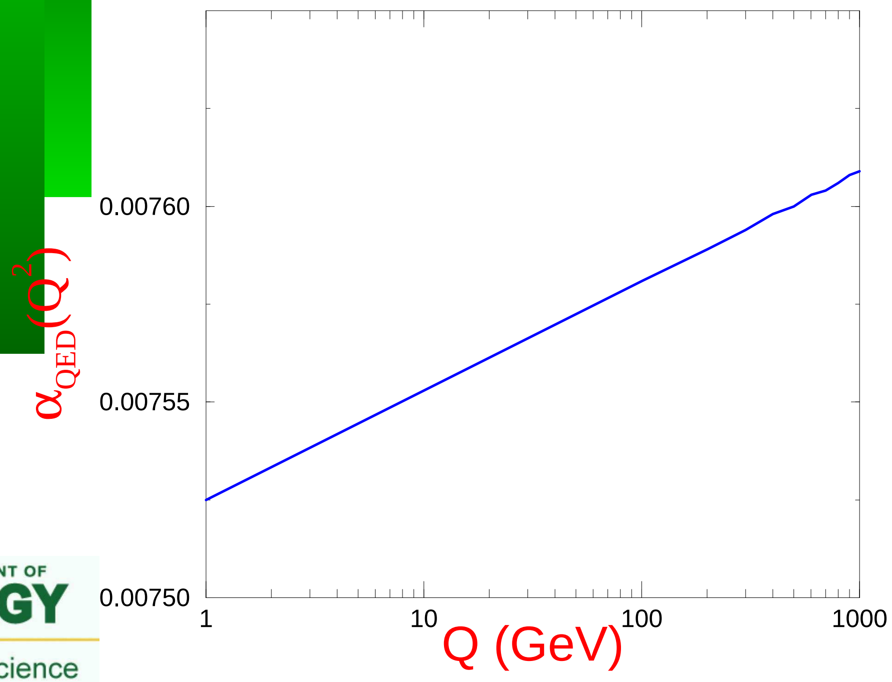
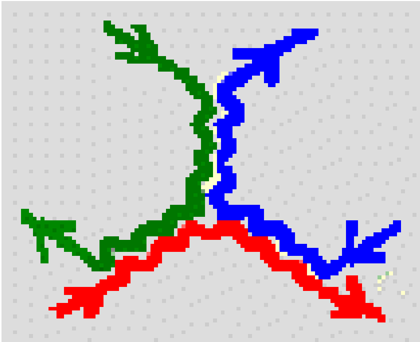
QED cf. QCD

Add three-gluon interaction





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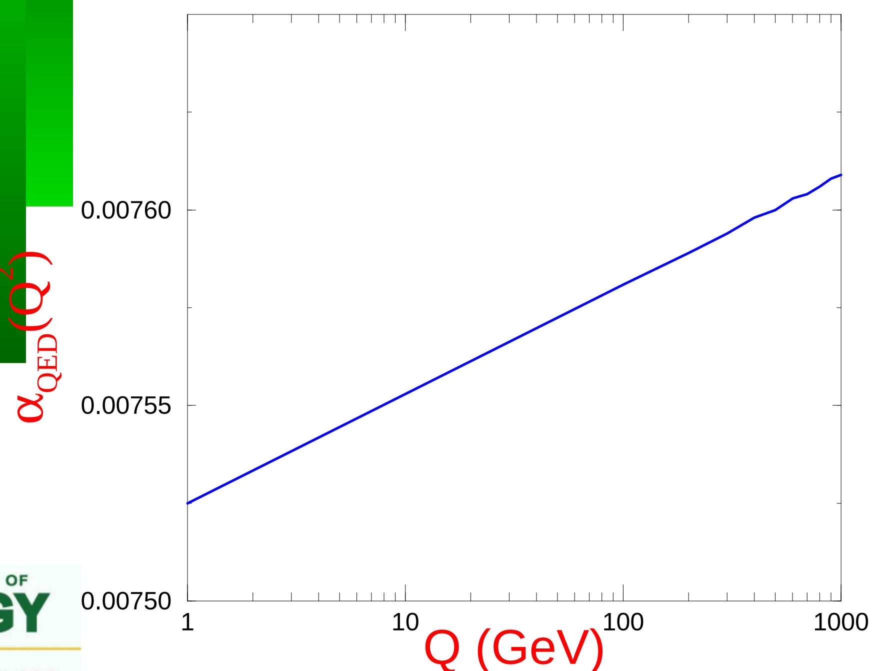
QED cf. QCD



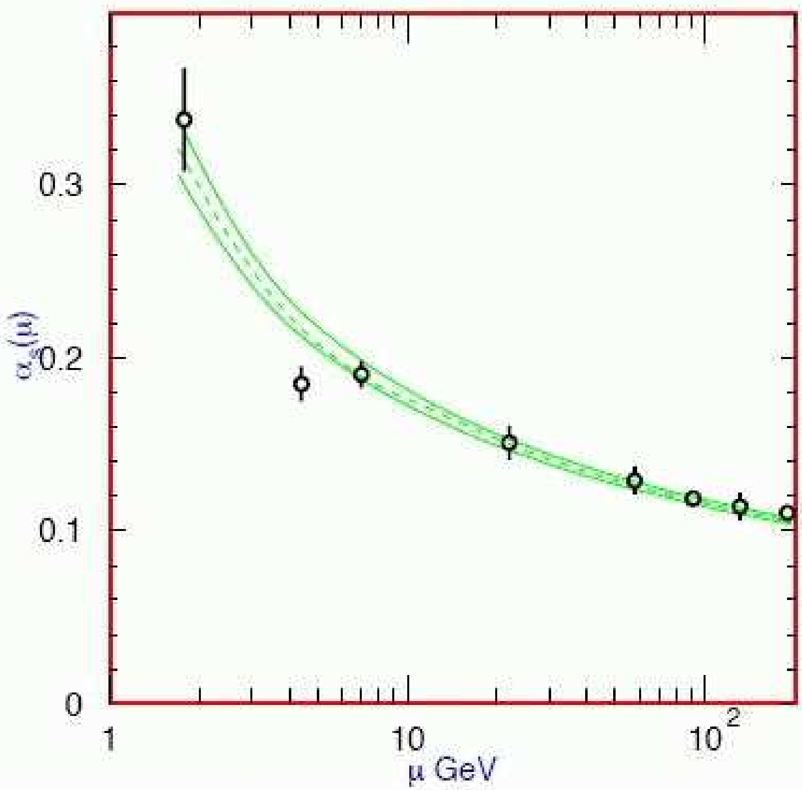


Figure 9.2: Summary of the values of $\alpha_s(\mu)$ at the values of μ where they are measured. The lines show the central values and the $\pm 1\sigma$ limits of our average. The figure clearly shows the decrease in $\alpha_s(\mu)$ with increasing μ. The data are, in increasing order of μ, τ width, Υ decays, deep inelastic scattering, e^+e^- event shapes at 22 GeV from the JADE data, shapes at TRISTAN at 58 GeV, Z width, and e^+e^- event shapes at 135 and 189 GeV.

$$\alpha_{\rm QED} = \frac{\alpha}{1 - \alpha/3\pi \ln\left(Q^2/m_e^2\right)}$$

$$\alpha_{\rm QCD} = \frac{12\pi}{(33 - 2N_f)\ln\left(Q^2/\Lambda^2\right)}$$

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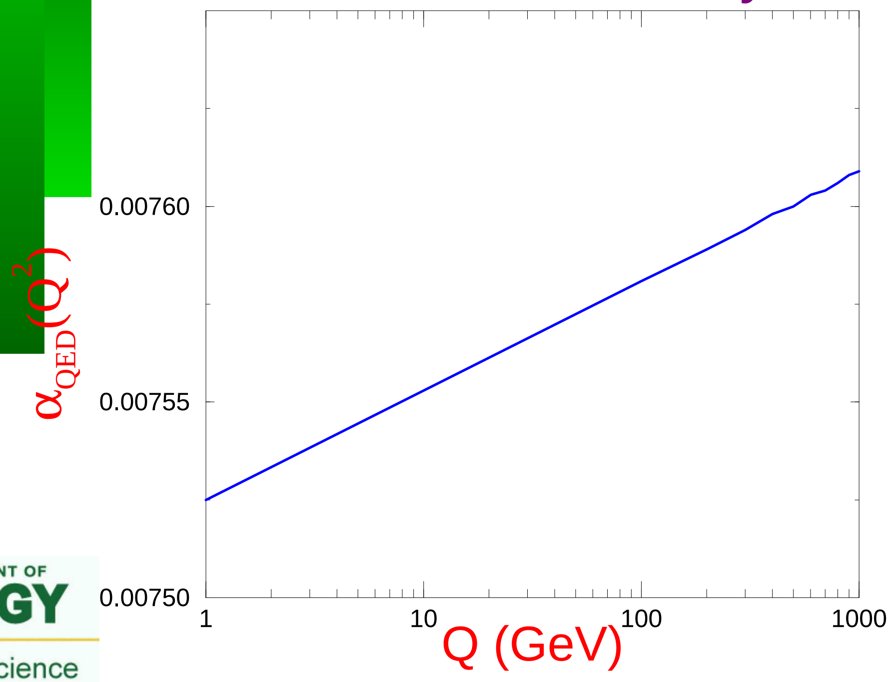
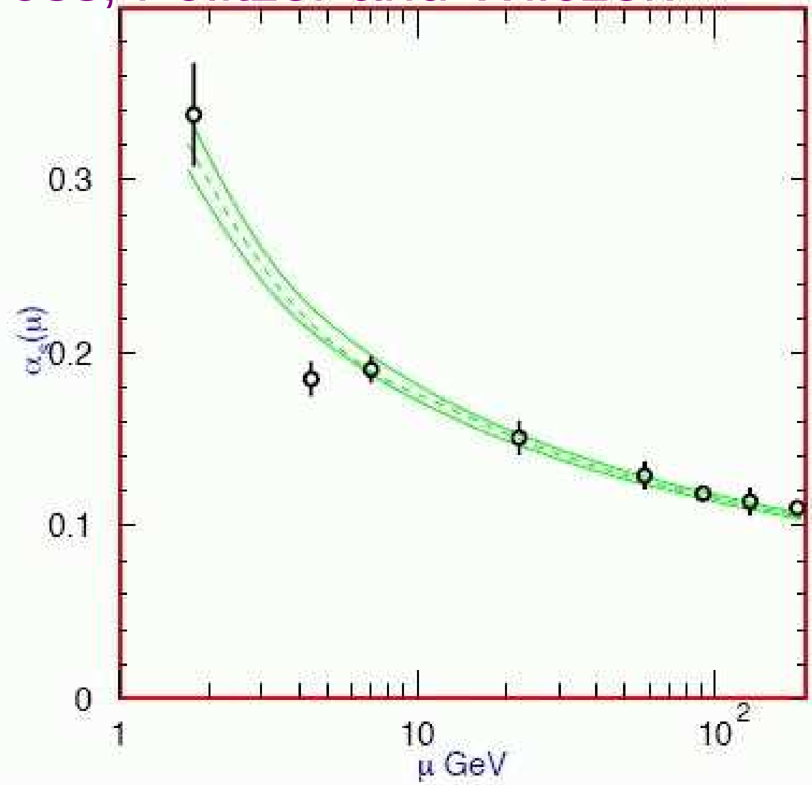
QED cf. QCD

2004 Nobel Prize in Physics: Gross, Politzer and Wilczek





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Quarks and Nuclear Physics

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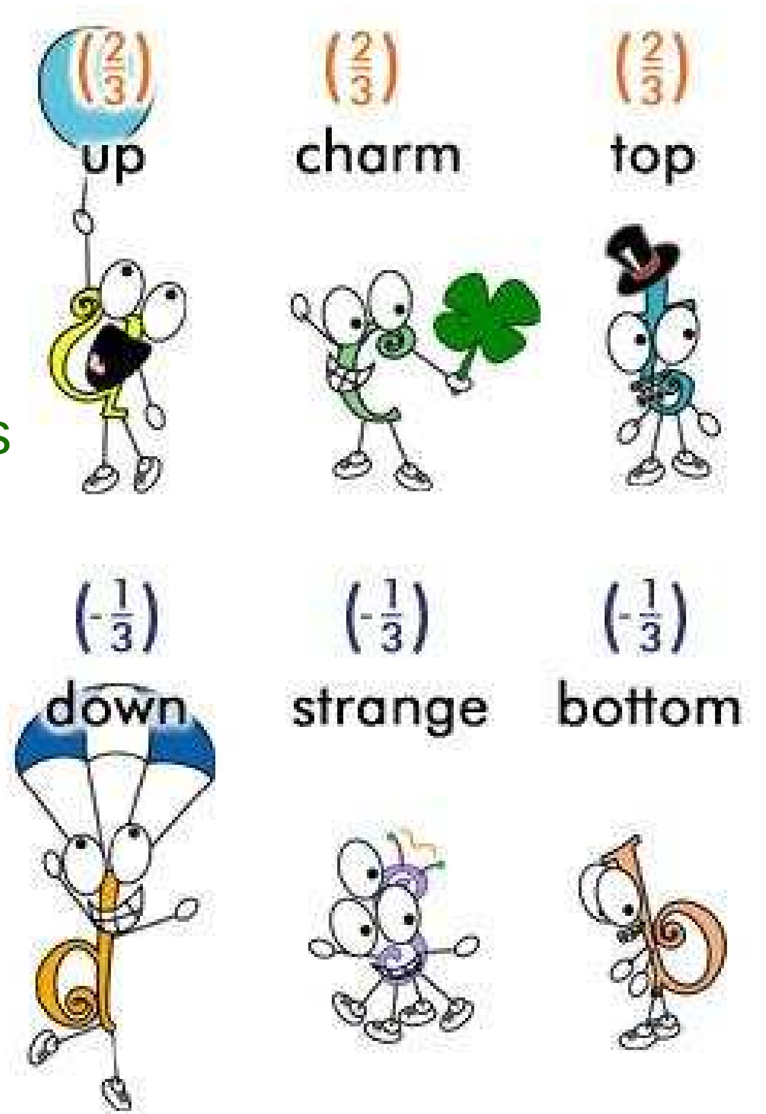
Quarks and Nuclear Physics

Standard Model

of Particle Physics

Six Flavours



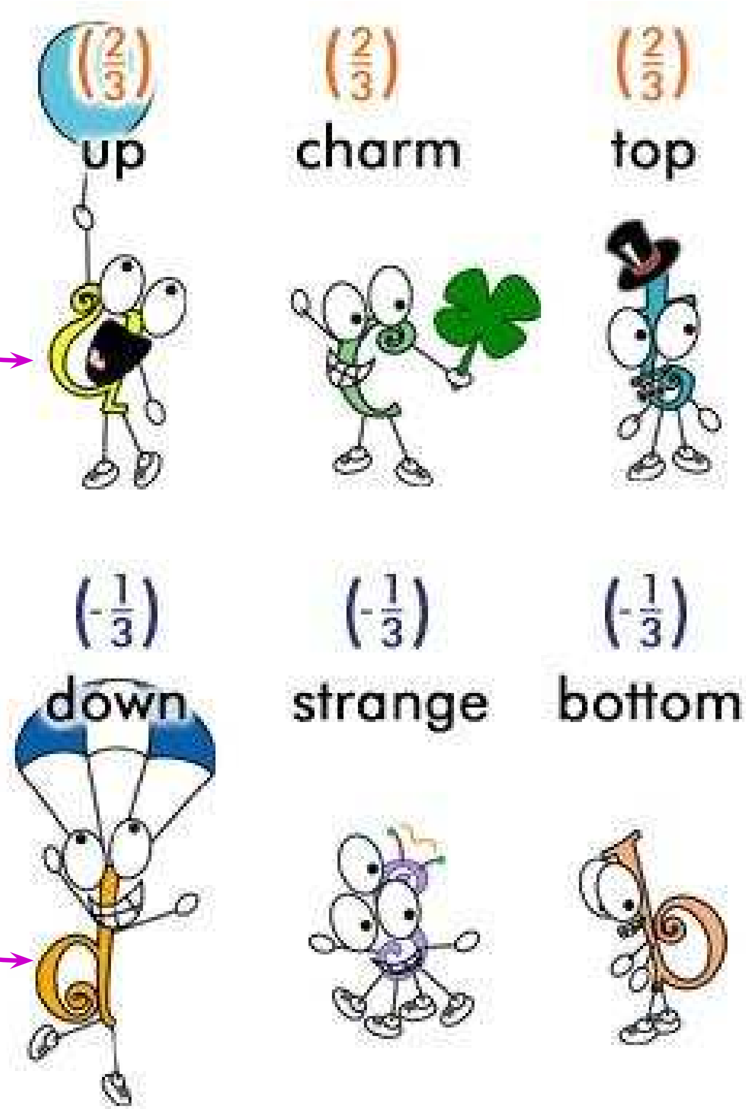
Quarks and Nuclear Physics

Real World

Normal Matter . . .

Only Two Light

Flavours Active



Quarks and Nuclear Physics

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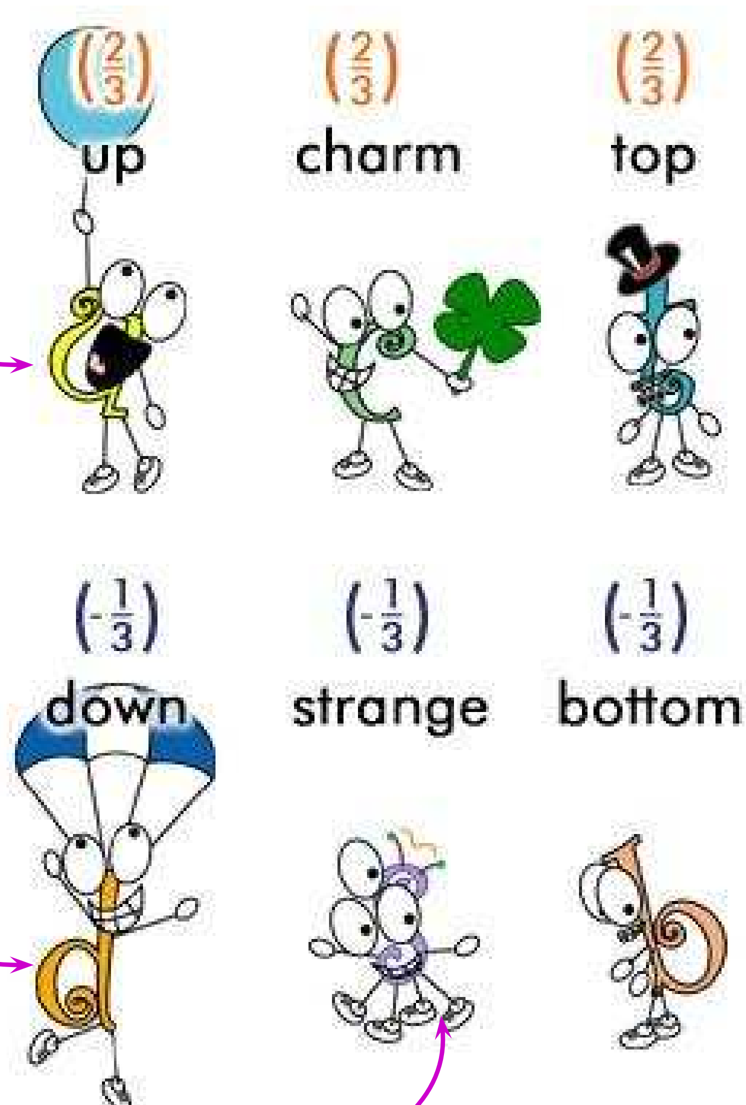
Flavours Active

or, perhaps, three



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Quarks and Nuclear Physics

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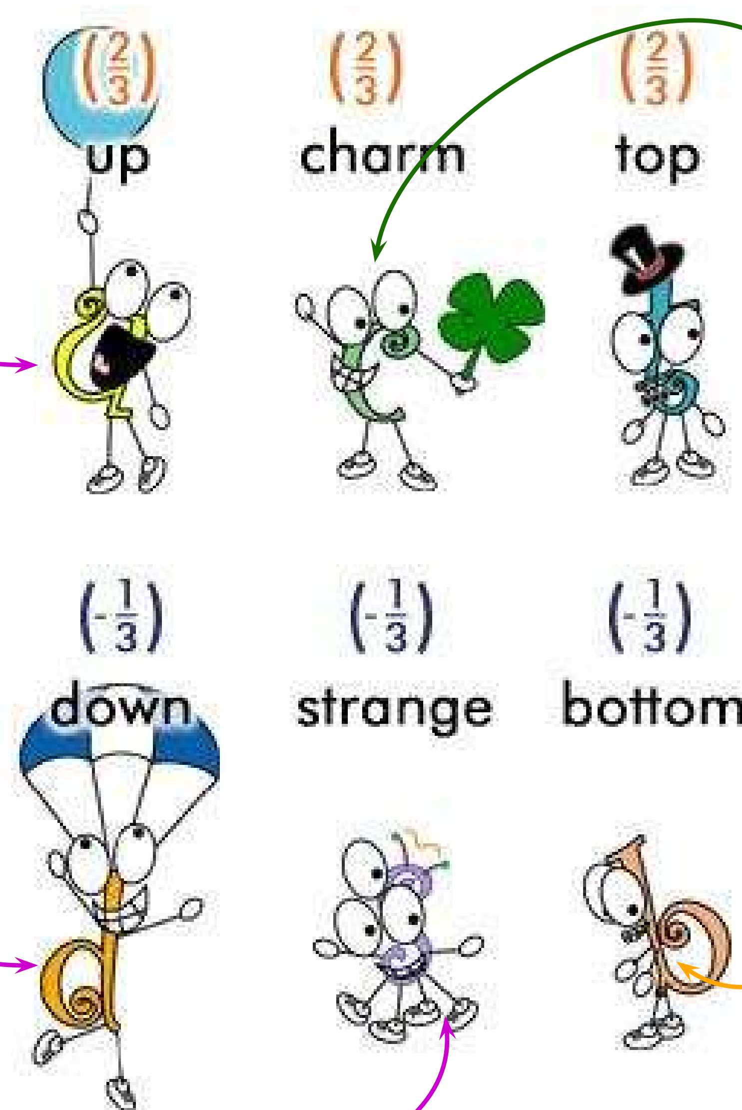

For numerous good reasons, much research also focuses on accessible heavy-quarks

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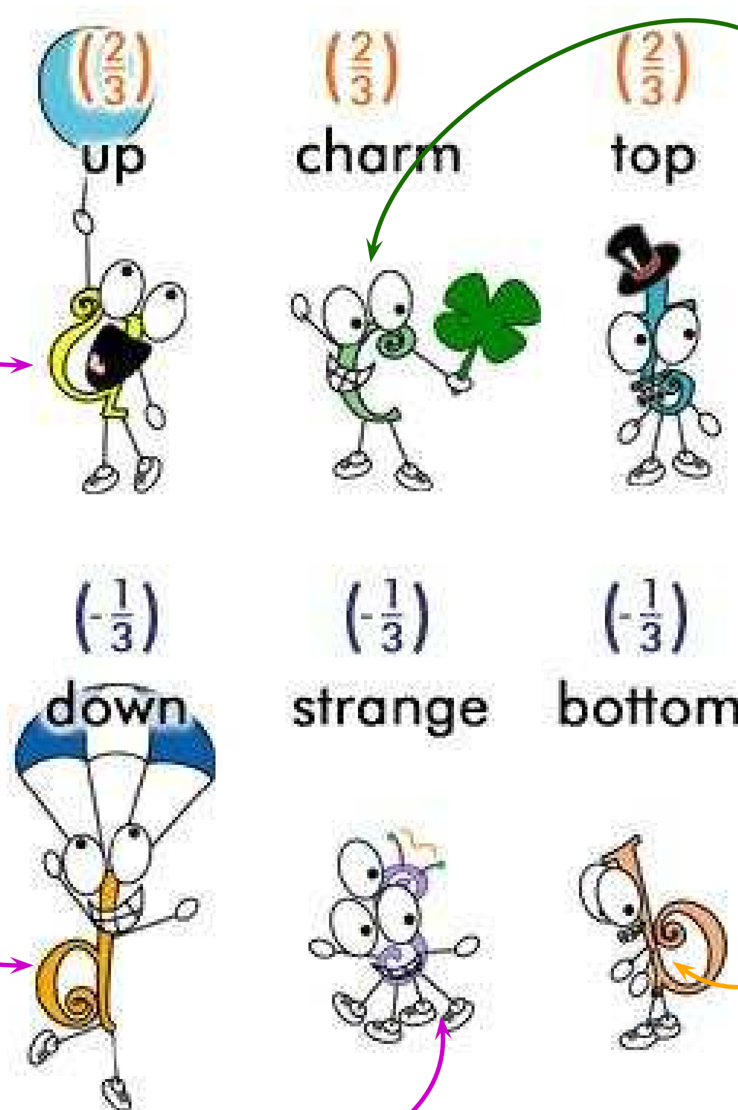
Quarks and Nuclear Physics

Nevertheless, I will focus primarily on the light-quarks.



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Simple Picture

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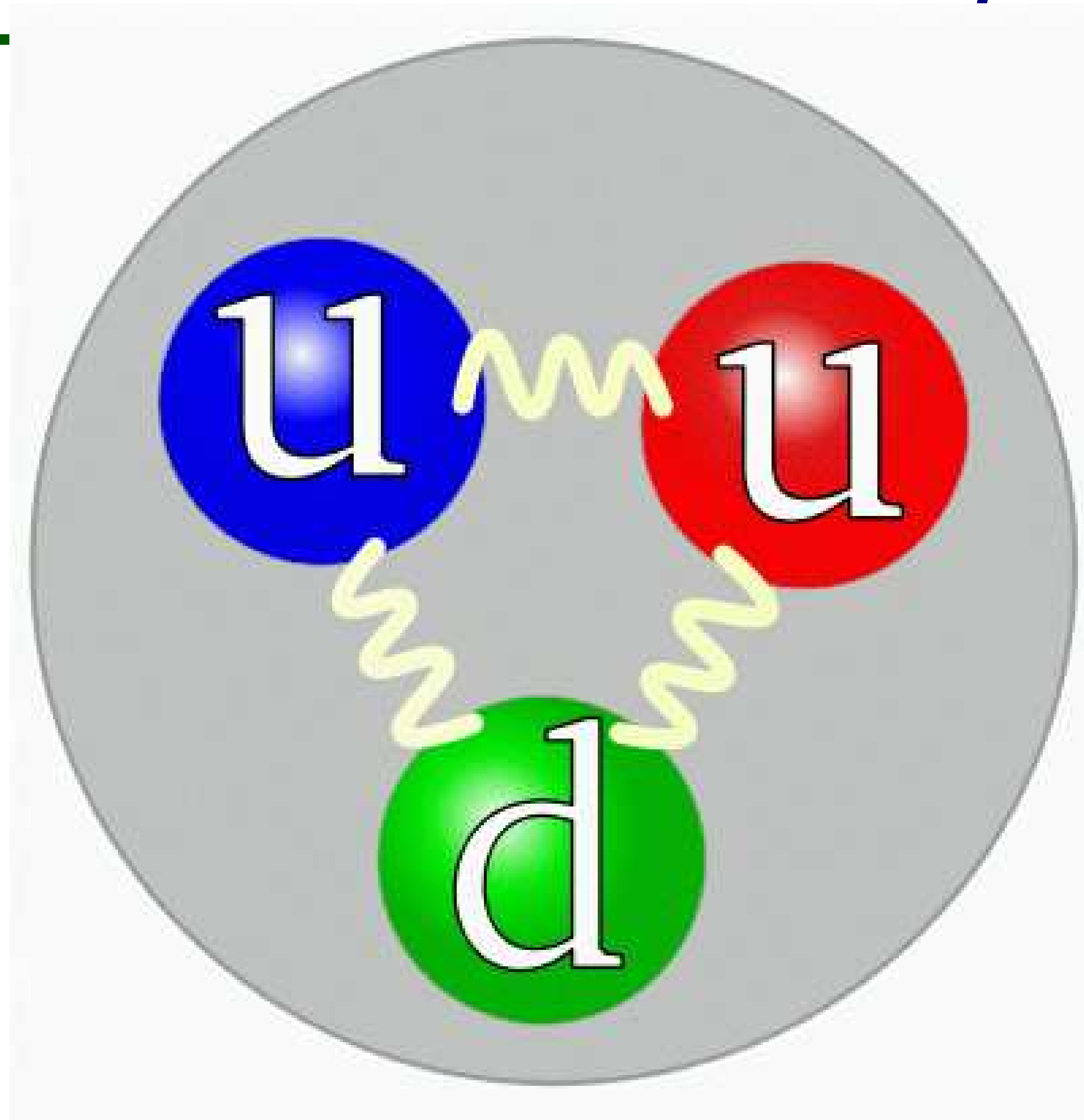
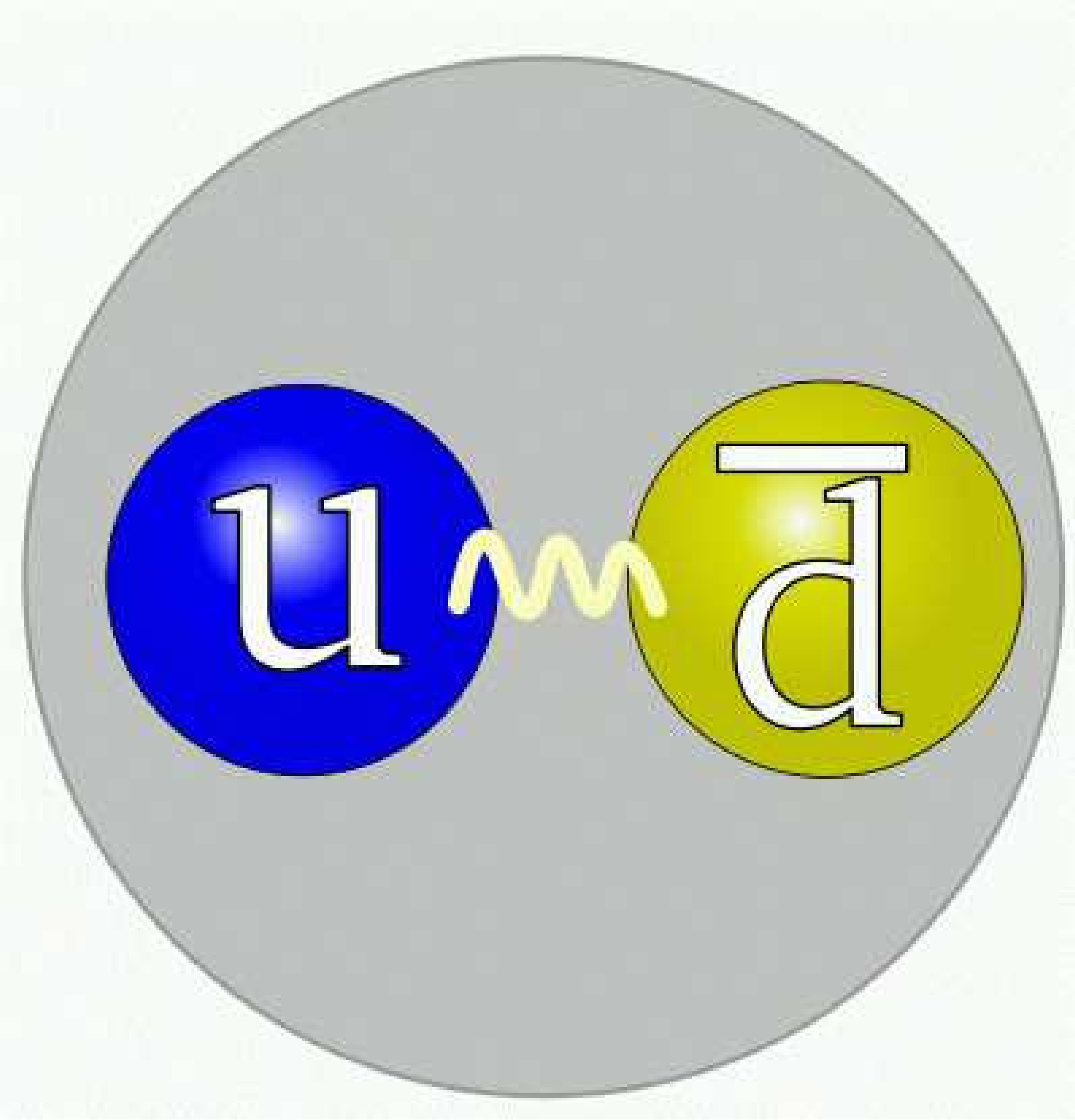
Simple Picture



PROTON

Simple Picture



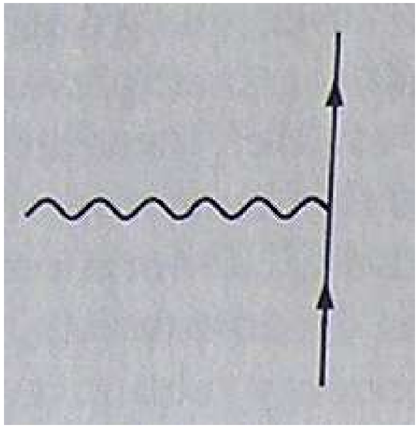
PION

Study Structure via
Nucleon Form Factors

Study Structure via Nucleon Form Factors

- Electron's relativistic electromagnetic current:

$$j_\mu(P',P) = ie\,\bar{u}_e(P')\,\Lambda_\mu(Q,P)\,u_e(P)\,,\; Q = P' - P$$
$$= ie\,\bar{u}_e(P')\,\gamma_\mu(-1)\,u_e(P)$$



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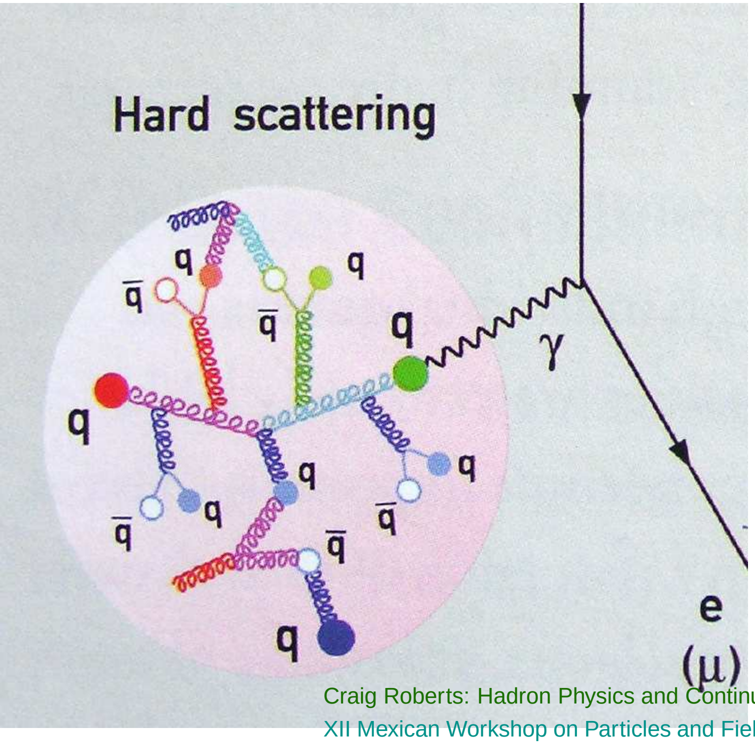

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Study Structure via Nucleon Form Factors

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$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)\,,\;\; G_M(Q^2) = F_1(Q^2) + F_2(Q^2)\,.$$

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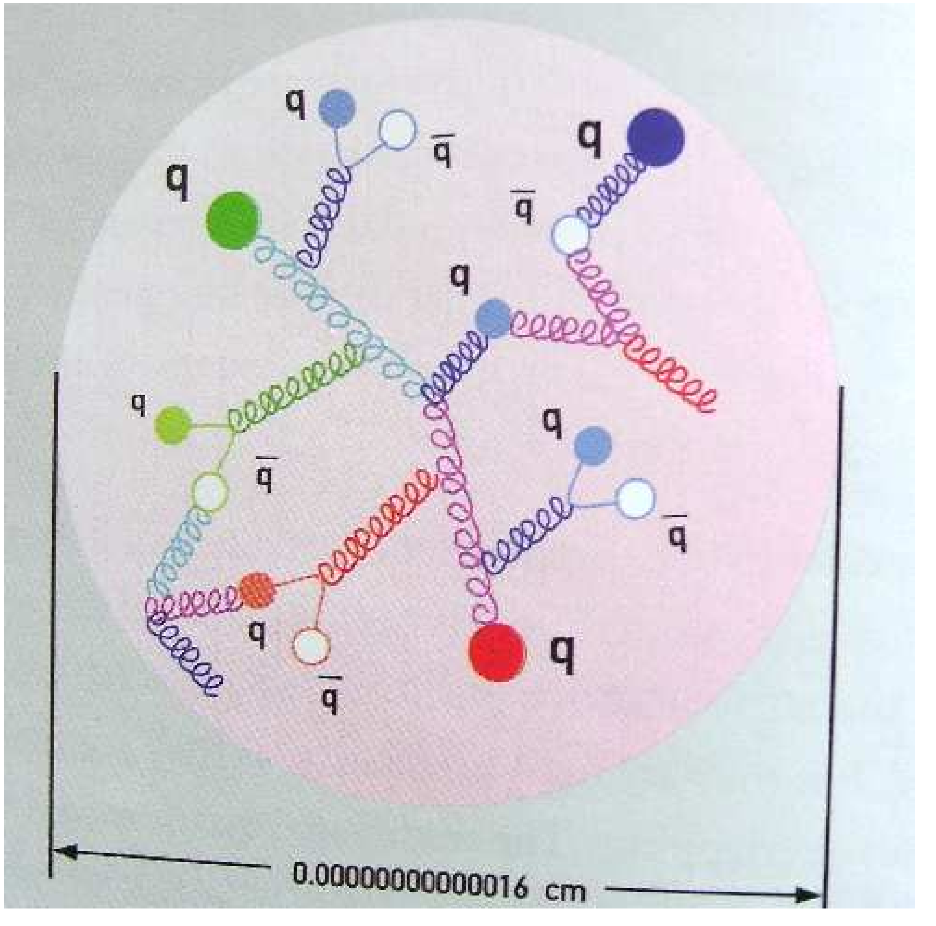
Point-particle: $\boldsymbol{F_2 \equiv 0 \Rightarrow G_E \equiv G_M}$

NSAC Long Range Plan

A central goal of nuclear physics is to understand the structure and properties of protons and neutrons, and ultimately atomic nuclei, in terms of the quarks and gluons of QCD



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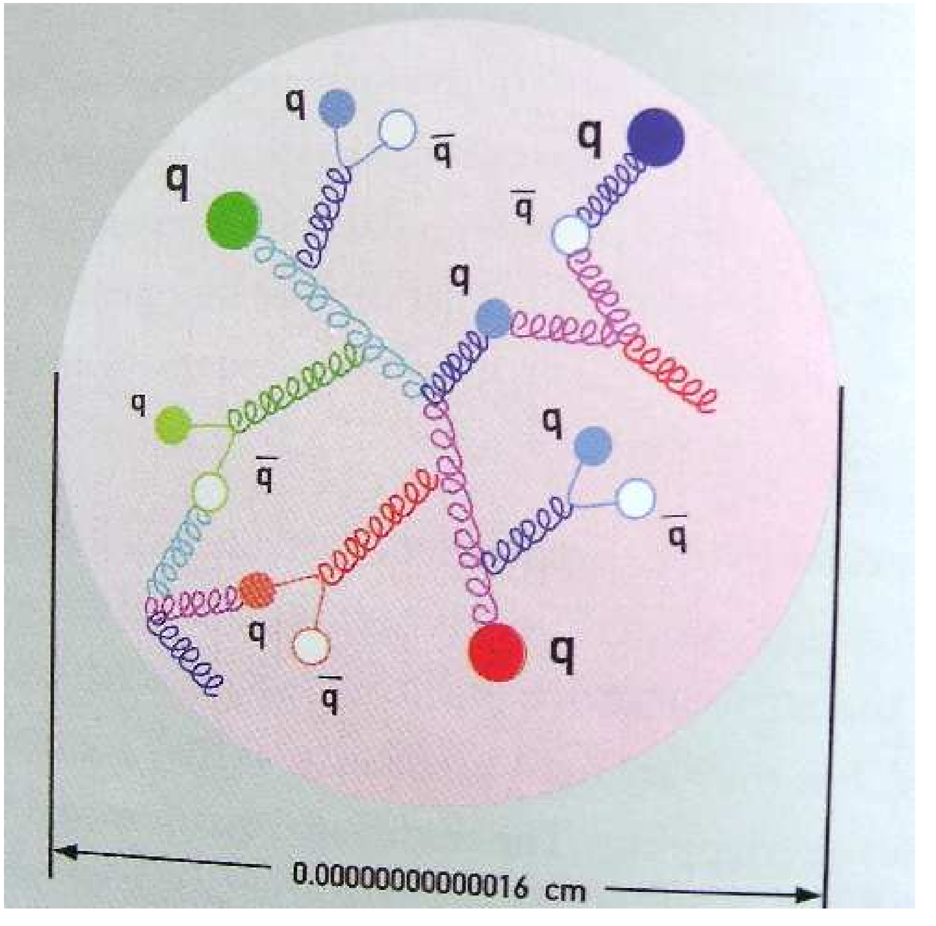

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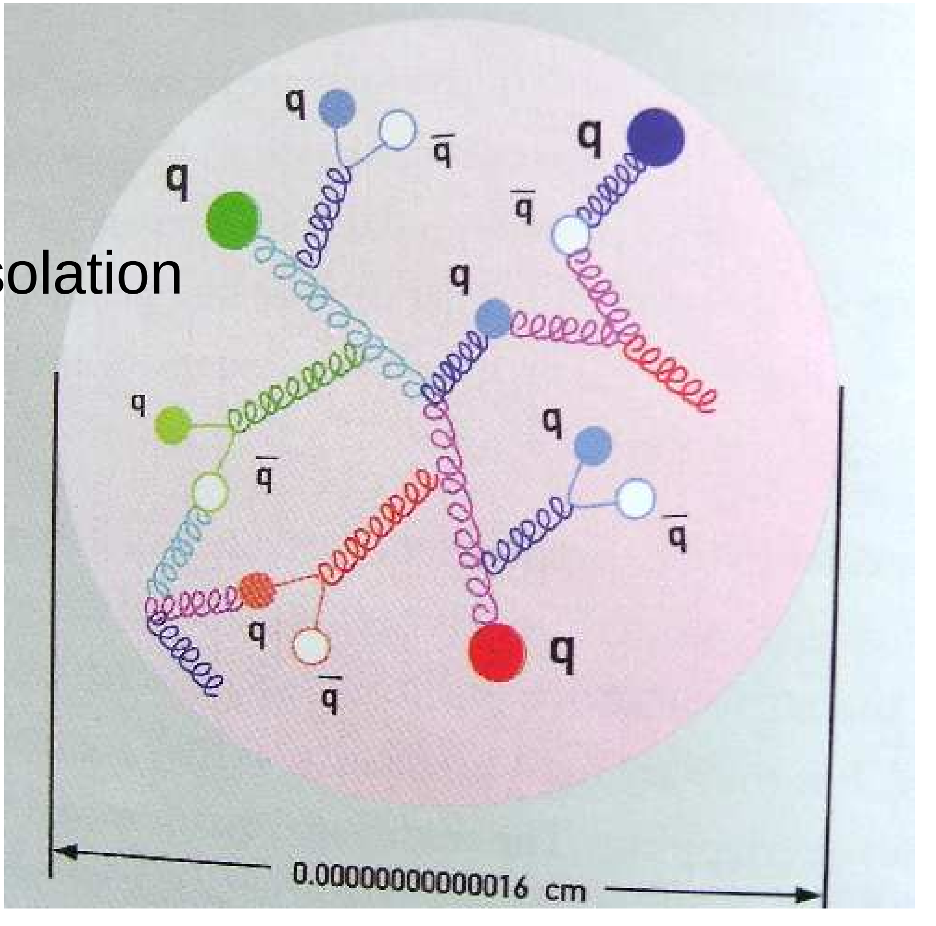
- Confinement
  – No quark ever seen in isolation



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NSAC Long Range Plan

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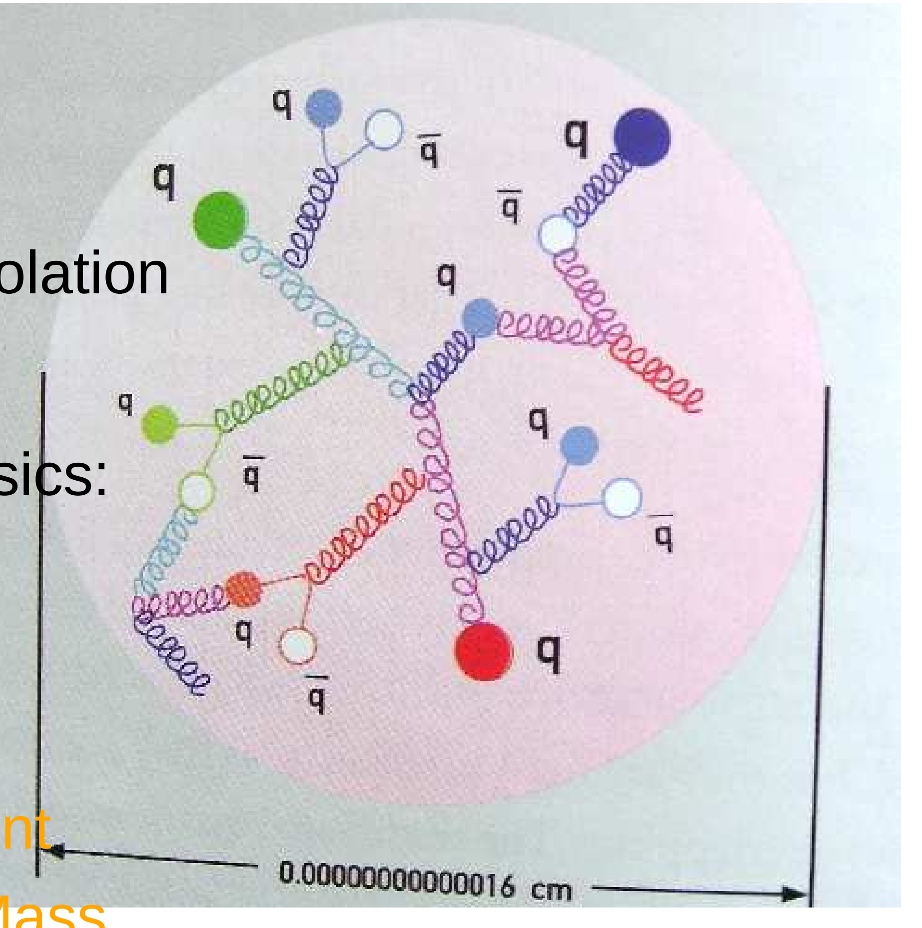
So, what's the problem?

- Confinement
  – No quark ever seen in isolation
- Weightlessness
  – 2004 Nobel Prize in Physics:
  Mass of $u-$ & $d-$quarks,
  each just 5 MeV;
  Proton Mass is 940 MeV
  $\Rightarrow$ No Explanation Apparent
  for 98.4 % of Mass



Meson Spectrum

LIGHT UNFLAVORED $(S = C = B = 0)$	$I^G(J^{PC})$		$I^G(J^{PC})$	STRANGE $(S = \pm1, C = B = 0)$	$I(J^P)$
• $\pi^\pm$	$1^-(0^-)$	• $\pi_2(1670)$	$1^-(2^{-+})$	• $K^\pm$	$1/2(0^-)$
• π^0 140 MeV	$1^-(0^{-+})$	• $\phi(1680)$	$0^-(1^{--})$	• K^0	$1/2(0^-)$
• η	$0^+(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$	• K^0_S	$1/2(0^-)$
• $f_0(600)$	$0^+(0^{++})$	• $\rho(1700)$	$1^+(1^{--})$	• K^0_L	$1/2(0^-)$
• $\rho(770)$ 770	$1^+(1^{--})$	$a_2(1700)$	$1^-(2^{++})$	$K^*_0(800)$	$1/2(0^+)$
• $\omega(782)$	$0^-(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$	• $K^*(892)$	$1/2(1^-)$
• $\eta'(958)$	$0^+(0^{-+})$	$\eta(1760)$	$0^+(0^{-+})$	• $K_1(1270)$	$1/2(1^+)$
• $f_0(980)$	$0^+(0^{++})$	• $\pi(1800)$	$1^-(0^{-+})$	• $K_1(1400)$	$1/2(1^+)$
• $a_0(980)$	$1^-(0^{++})$	$f_2(1810)$	$0^+(2^{++})$	• $K^*(1410)$	$1/2(1^-)$
• $\phi(1020)$	$0^-(1^{--})$	$X(1835)$	$?^?(?^{-+})$	• $K^*_0(1430)$	$1/2(0^+)$
• $h_1(1170)$	$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$	• $K^*_2(1430)$	$1/2(2^+)$
• $b_1(1235)$	$1^+(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$	$K(1460)$	$1/2(0^-)$
• $a_1(1260)$	$1^-(1^{++})$	$\rho(1900)$	$1^+(1^{--})$	$K_2(1580)$	$1/2(2^-)$
• $f_2(1270)$	$0^+(2^{++})$	$f_2(1910)$	$0^+(2^{++})$	$K(1630)$	$1/2(?^?)$
• $f_1(1285)$	$0^+(1^{++})$	• $f_2(1950)$	$0^+(2^{++})$	$K_1(1650)$	$1/2(1^+)$
• $\eta(1295)$	$0^+(0^{-+})$	$\rho_3(1990)$	$1^+(3^{--})$	• $K^*(1680)$	$1/2(1^-)$
• $\pi(1300)$	$1^-(0^{-+})$	• $f_2(2010)$	$0^+(2^{++})$	• $K_2(1770)$	$1/2(2^-)$

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Modern Miracles in Hadron Physics

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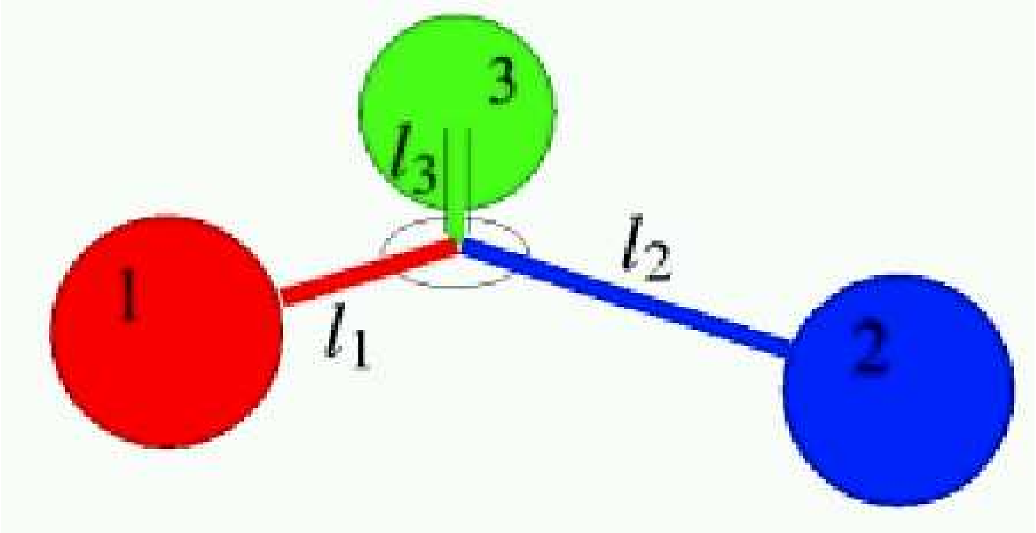
- proton = three constituent quarks



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- $M_{\mathbf{proton}} \approx \mathbf{1\,GeV}$

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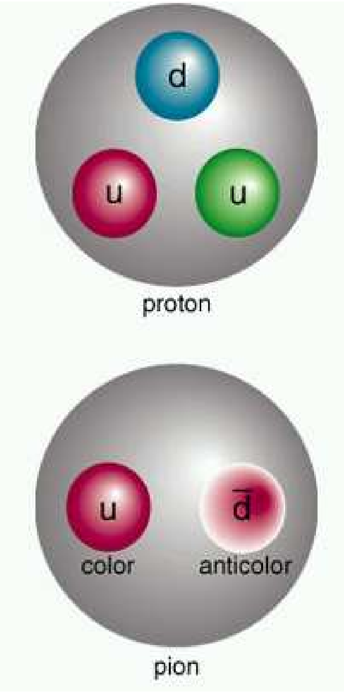

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- Another meson:
  . . . . $M_\rho = 770\,\text{MeV}$ . . . . No Surprises Here

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- WRONG .... $M_{\rm pion} = 140\,{\rm MeV}$
- What is “wrong” with the pion?

Closer look at Spectrum

- Features of the Spectrum:
  - $\dfrac{m_\rho^2}{m_\pi^2} = 30$
  - $\dfrac{m_{a_1}^2}{m_\sigma^2} = 4.4$

? Hyperfine Splitting

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- $\dfrac{m_N}{m_\pi} \approx 7$ - $\dfrac{m_N}{m_\rho} = \dfrac{5}{4} \approx \dfrac{3}{2}$? Quark Counting

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Dichotomy of Pion
– Goldstone Mode and *Bound state*

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The correct understanding of pion observables;
e.g. mass, decay constant and form factors,
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Highly Nontrivial

What's the Problem?

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Relativistic QFT!

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- Differences!
  - Here relativistic effects are crucial – *virtual particles*, quintessence of Relativistic Quantum Field Theory – must be included
  - Interaction between quarks – the Interquark "Potential" – unknown throughout $>$ 98% of a hadron's volume

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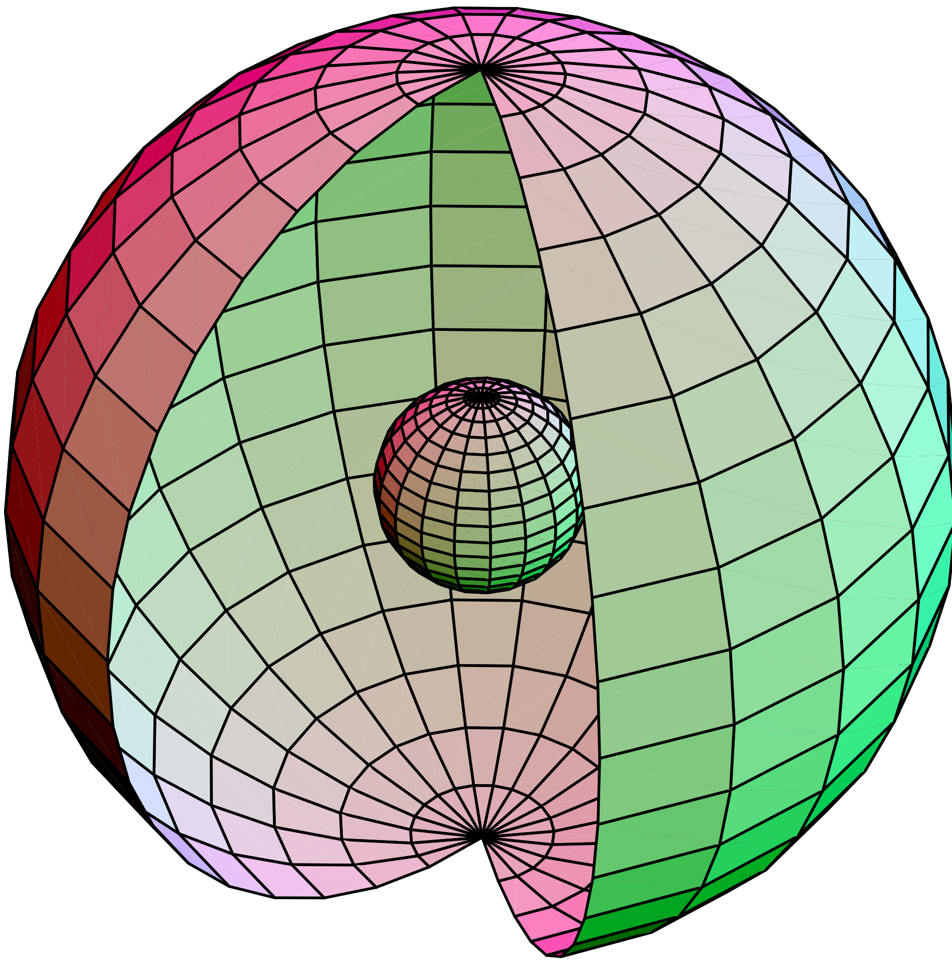
Intranucleon Interaction

Intranucleon Interaction



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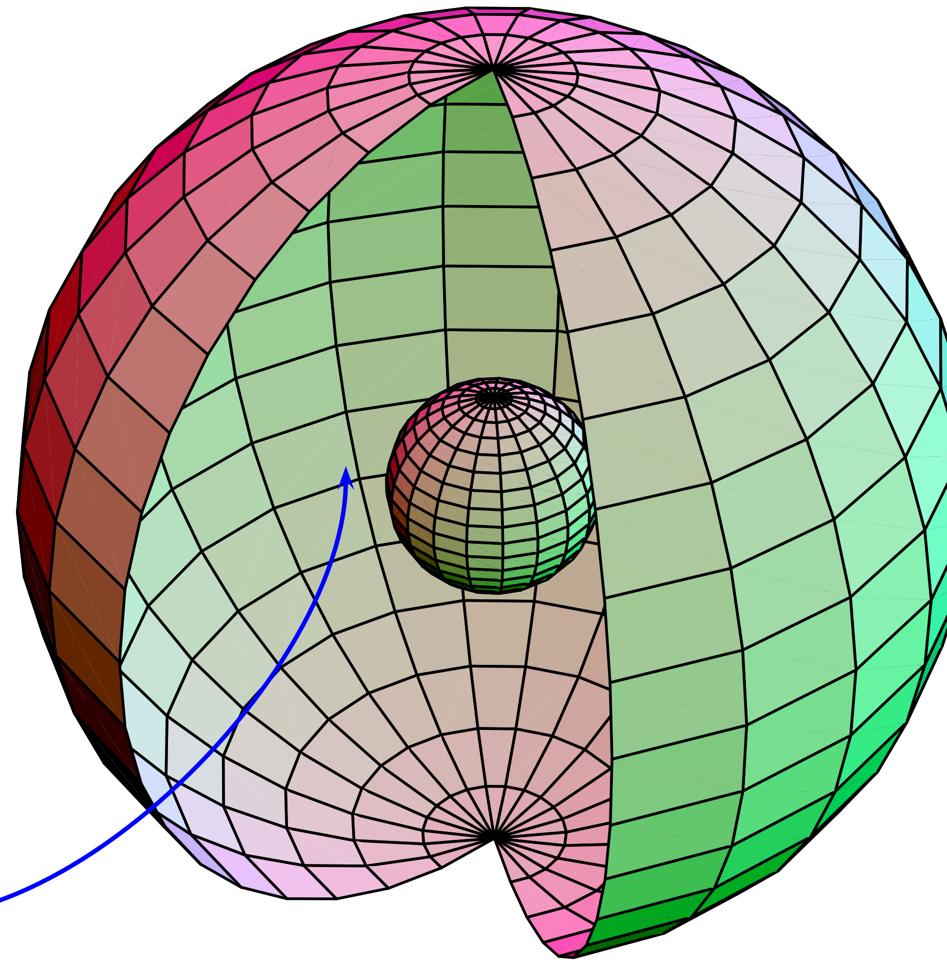
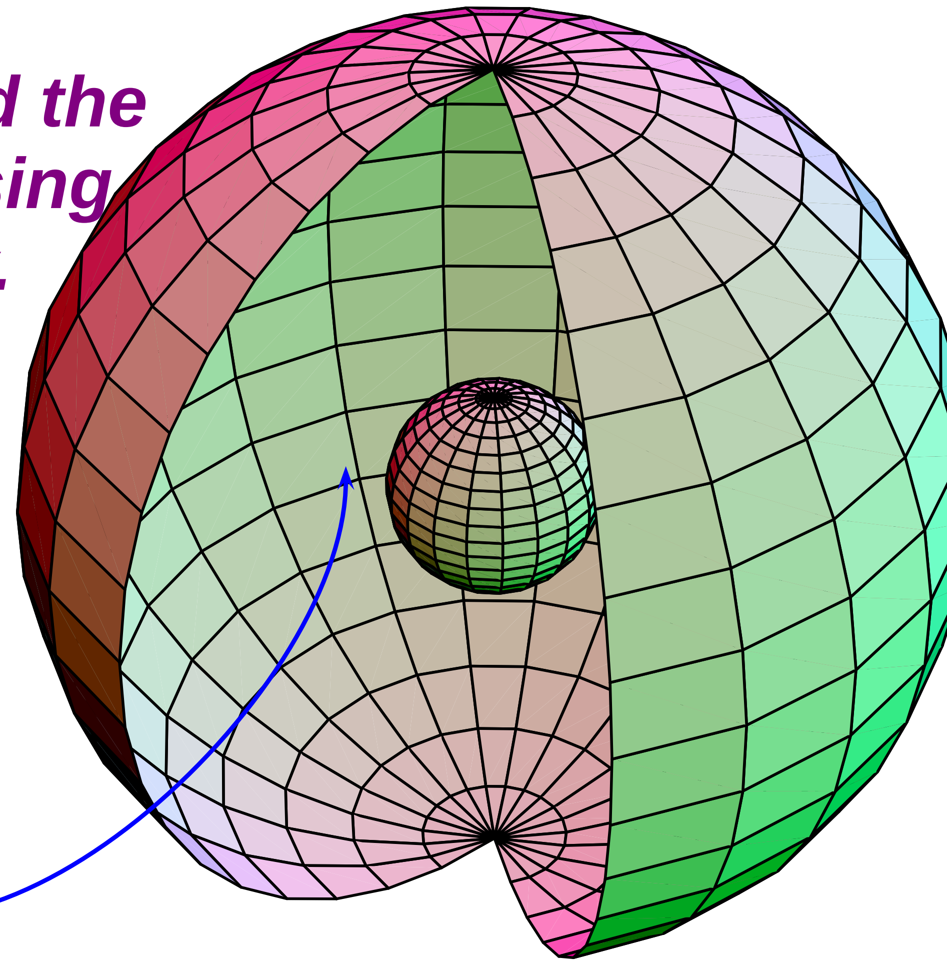


98% of the volume

What is the Intranucleon Interaction?

The question must be rigorously defined, and the answer mapped out using experiment and theory.



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QCD’s Challenges

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- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon

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- Dynamical Chiral Symmetry Breaking
  - Very unnatural pattern of bound state masses
    - e.g., Lagrangian (pQCD) quark mass is small but . . . no degeneracy between $J^{P=+}$ and $J^{P=-}$

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QCD's Challenges

Understand Emergent Phenomena

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- QCD – Complex behaviour

  arises from apparently simple rules

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Why should You care?

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Absent DCSB: $m_\pi = m_\rho \Rightarrow$ repulsive and attractive forces in nucleon-nucleon interaction both have SAME range and there is No intermediate range attraction!

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- Is ^{12}C stable?
  - Probably not, if range range $\sim \dfrac{1}{2\,M_Q}$

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Is a unique long-range interaction between light-quarks responsible for all this or are there an uncountable infinity of qualitatively equivalent interactions?

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Gauge Theories with Massless Fermions have

CHIRAL SYMMETRY

Chiral Symmetry

- Helicity $\lambda \propto J \cdot p$
  - Projection of Spin onto Direction of Motion
  - For massless particles, helicity is a Lorentz invariant *Spin Observable*.
  - $\lambda = \pm$ ($\|$ or anti-$\|$ to p_μ)

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Chiral Symmetry

- Chirality Operator: γ_5
  - Chiral Transformation $q(x) \to \mathrm{e}^{i\gamma_5\theta}\, q(x)$

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- Chirality Operator: γ_5
  - Chiral Transformation $q(x) \to \mathrm{e}^{i\gamma_5\theta}\, q(x)$
  - Chiral Rotation $\theta = \dfrac{\pi}{2}$
    - $q_{\lambda=+} \to q_{\lambda=+},\; q_{\lambda=-} \to -\,q_{\lambda=-}$
    - Hence, a theory invariant under chiral transformations can only contain interactions that are insensitive to a particle's helicity.

Chiral Symmetry

- Chirality Operator: γ_5
  - Chiral Transformation $q(x) \to \mathrm{e}^{i\gamma_5\theta}\, q(x)$
  - Chiral Rotation $\theta = \dfrac{\pi}{4}$
    - Composite Particles: $J^{P=+} \leftrightarrow J^{P=-}$
    - Equivalent to "Parity Conjugation" Operation

Chiral Symmetry

- A Prediction of Chiral Symmetry
  - Degeneracy between Parity Partners

$$N(\tfrac{1}{2}^+, 938) = N(\tfrac{1}{2}^-, 1535),$$
$$\pi(0^-, 140) = \sigma(0^+, 600),$$
$$\rho(1^-, 770) = a_1(1^+, 1260)$$

- Doesn't Look too good

  Predictions *not* Valid – Violations *too* Large.

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How can pion mass be so small

If quarks are so heavy?!

Propagators

- Extraordinary Effects in QCD Tied to Properties of *Dressed*-Quark and -Gluon Propagators

Quark | Gluon

$$S_f(x-y) \equiv \langle q_f(x)\bar{q}_f(y)\rangle \quad D_{\mu\nu}(x-y) \equiv \langle A_\mu(x)A_\nu(y)\rangle$$

- Describe in-Medium *Propagation Characteristics* of Elementary Particles

- Example: Solid-State Physics
  - γ propagating in a Dense e$^-$ Gas
  - Acquires a Debye Mass

$$m_{\rm D}^2 \propto k_F^2 : \; \frac{1}{Q^2} \to \frac{1}{Q^2 + m_{\rm D}^2}$$

  - γ develops an Effective-mass

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  - γ develops an Effective-mass
  - Leads to Screening of the Interaction: $r \propto \dfrac{1}{m_D}$
- Quark and Gluon Propagators:

  Modified in a similar way -

  Momentum Dependent Effective Masses
- The Effect of this is Observable in QCD

Explicit Chiral Symmetry Breaking

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- Chiral Symmetry

  Can be discussed in terms of Quark Propagator

  - Free Quark Propagator $S_0(p) = \dfrac{-i\gamma \cdot p + m}{p^2 + m^2}$

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- Chiral Transformation

$$\begin{aligned}\mathbf{S}_0(p) &\rightarrow \mathrm{e}^{i\gamma_5\theta} S_0(p)\mathrm{e}^{i\gamma_5\theta} \\ &= \frac{-i\gamma\cdot p}{p^2+m^2} + \mathrm{e}^{2i\gamma_5\theta}\frac{m}{p^2+m^2}\end{aligned}$$

  - Symmetry Violation $\propto m$
  - $\mathbf{m} = 0$: $S_0(p) \rightarrow S_0(p)$

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- Quark Condensate

  $$\langle \bar{q}q \rangle_\mu \equiv \int_\mu^\Lambda \frac{d^4p}{(2\pi)^4} \, \mathrm{tr}\,[S(p)] \propto \int_\mu^\Lambda \frac{d^4p}{(2\pi)^4} \, \frac{m}{p^2 + m^2}$$

  - A Measure of the Chiral Symmetry Violating Term

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  - A Measure of the Chiral Symmetry Violating Term
  - Perturbative QCD: Vanishes if $m = 0$

Dynamical **Symmetry Breaking**

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$$V(x,y) = (\sigma^2 + \pi^2 - 1)^2$$

Hamiltonian: $T + V$, is Rotationally Invariant

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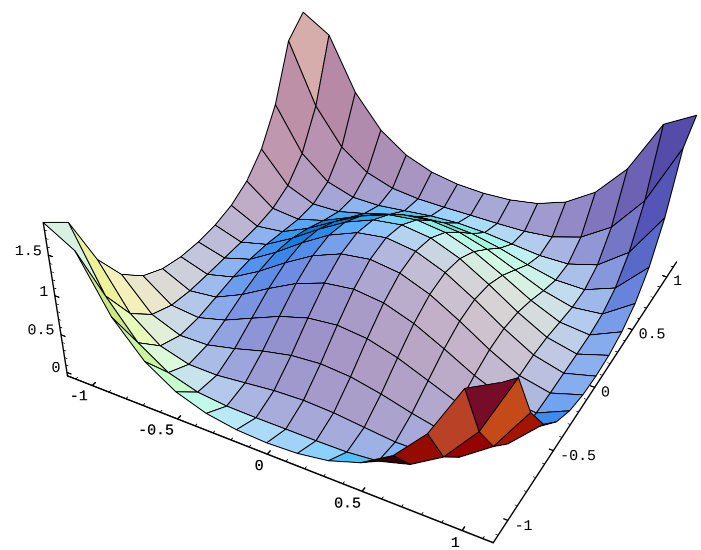
Hamiltonian: $T + V$, is Rotationally Invariant



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Dynamical Symmetry Breaking

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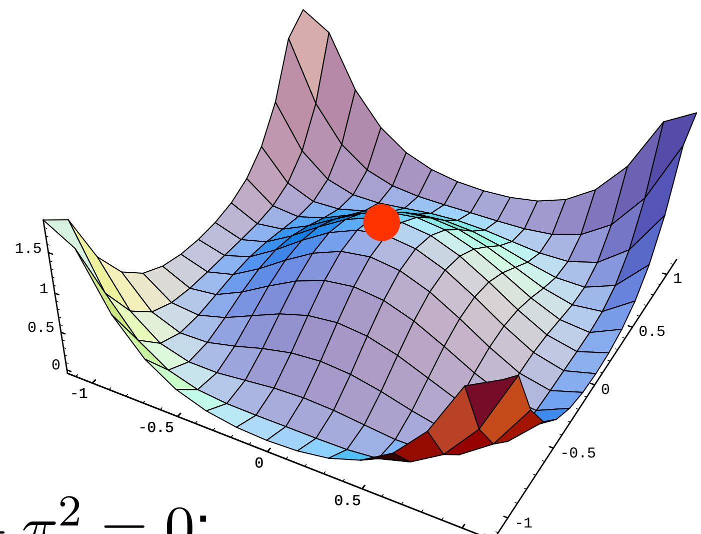

Ground State?

- Ball at (σ, π) for which $\sigma^2 + \pi^2 = 0$:
  - Rotationally Invariant

UNSTABLE

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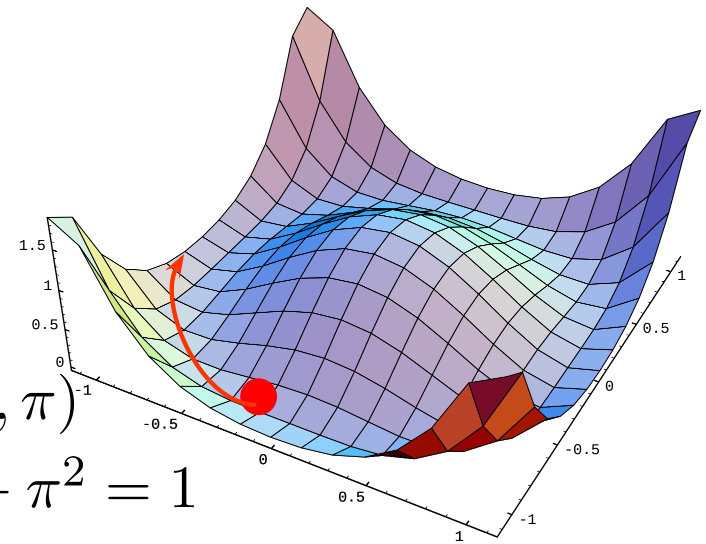

Ground State

- Ball at any (σ, π) for which $\sigma^2 + \pi^2 = 1$
  - All Positions have Same (Minimum) Energy
  - But not invariant under rotations

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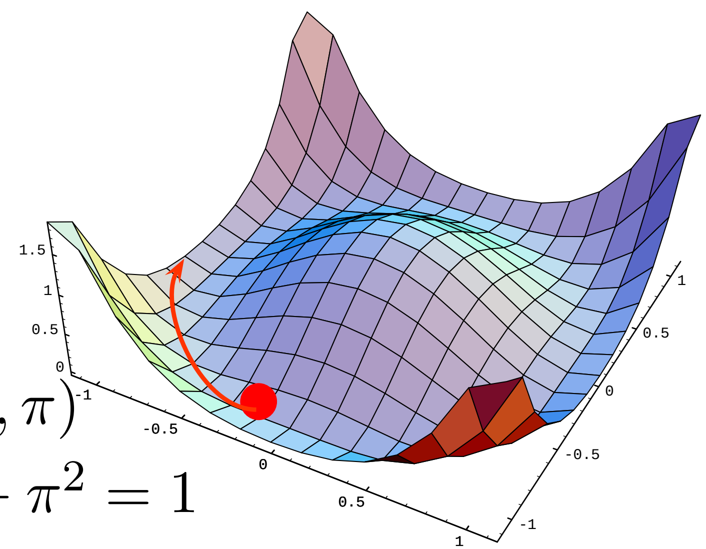
Symmetry of Ground State $\neq$ Symmetry of Hamiltonian



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Dynamics and Symmetries

Dynamics and Symmetries

- Confinement:

  NO quarks or gluons have ever reached a detector alone

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- Plainly, nonperturbative method is necessary.

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NB. Hadron Physics Milestone, 2012: Measure the electromagnetic excitations of low-lying hadrons and their transition form factors.

Model QCD

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Traditional approach to strong force problem

Model QCD

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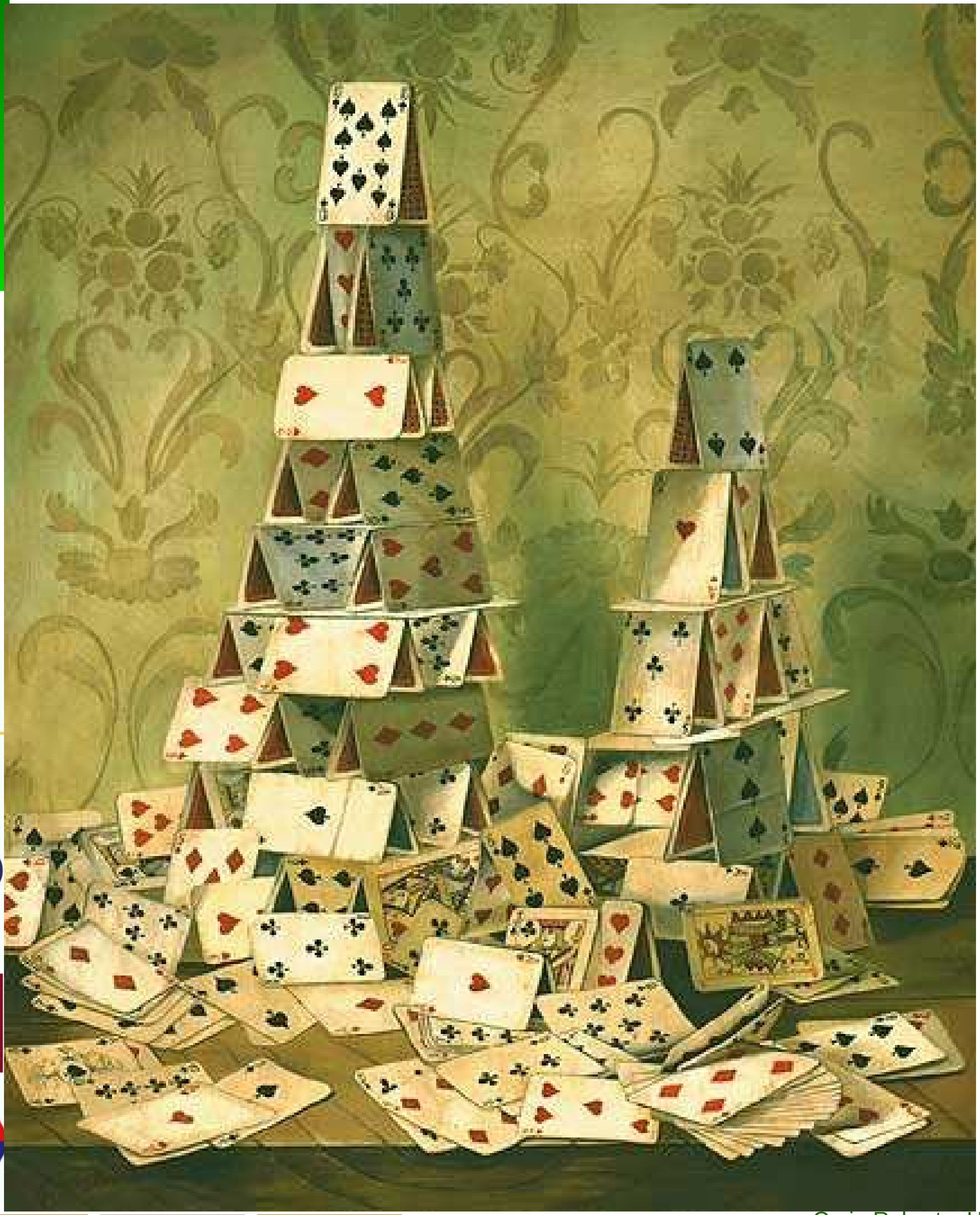
Model QCD



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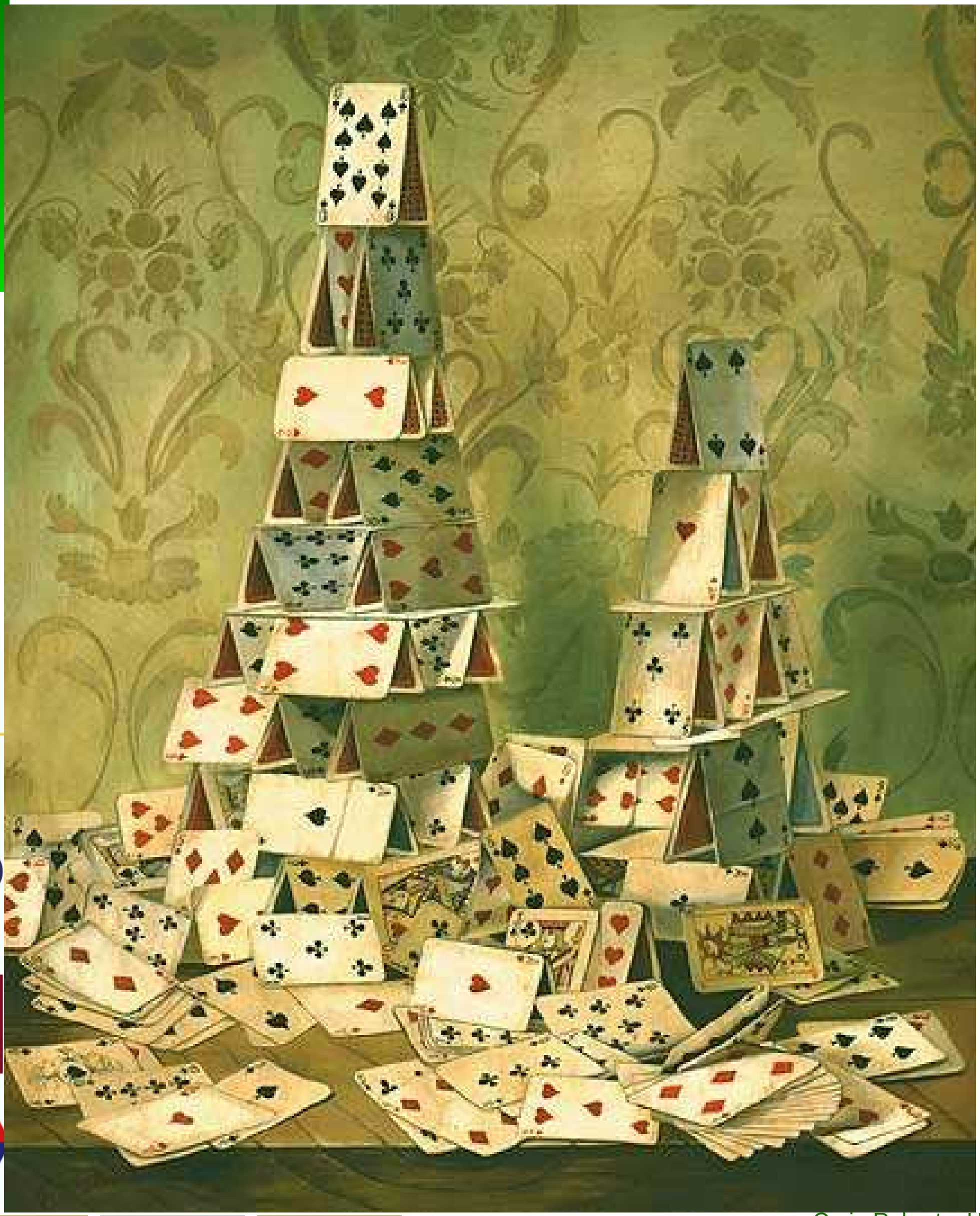

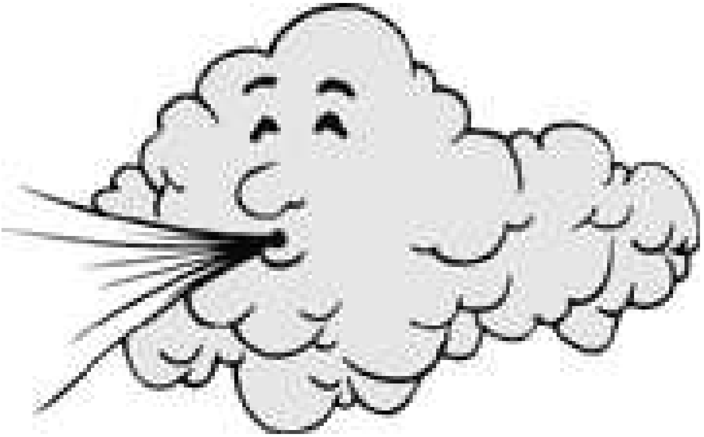

Lattice QCD

One modern nonperturbative approach ***Lattice QCD***

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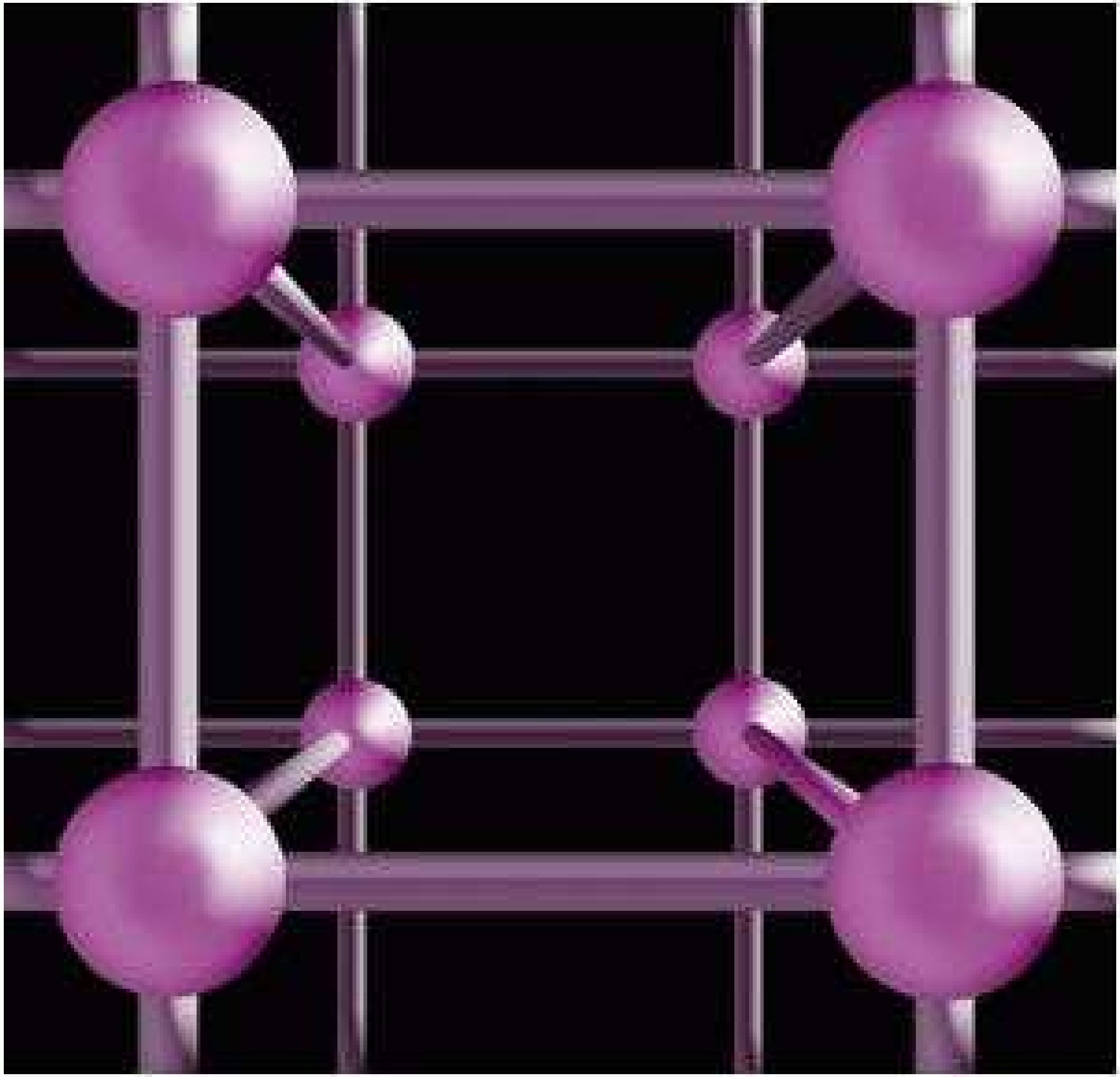

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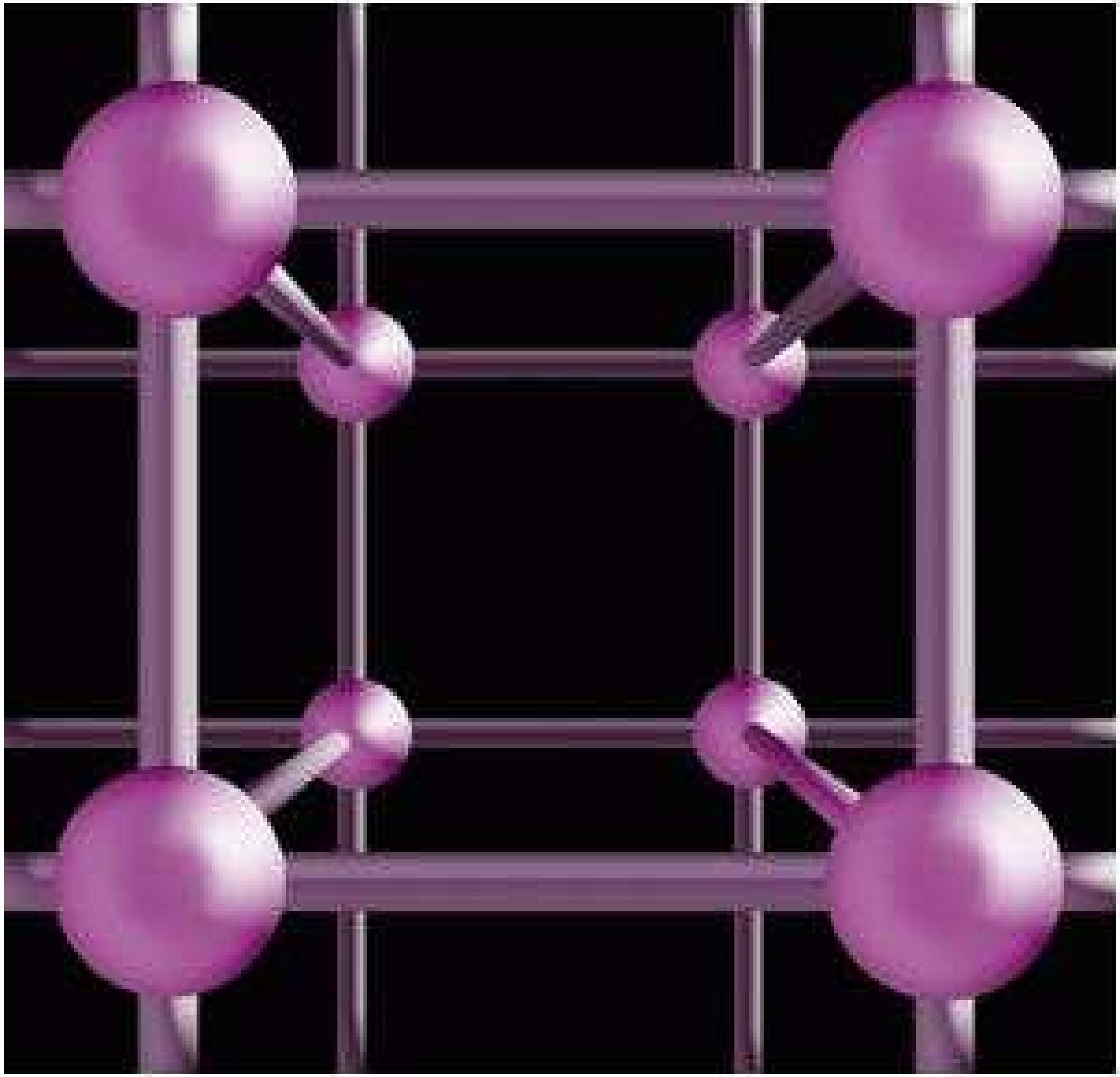

$\sim$ 500 people worldwide.

Collaborations $\sim$ 20 people

One modern nonperturbative approach *Lattice QCD*

A Compromise?
Dyson-Schwinger Equations

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A Compromise?
Dyson-Schwinger Equations

- 1994 . . . "*As computer technology continues to improve, lattice gauge theory [LGT] will become an increasingly useful means of studying hadronic physics through investigations of discretised quantum chromodynamics [QCD]. . . . .*"

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- 1994 . . . "*However, it is equally important to develop other complementary nonperturbative methods based on continuum descriptions. In particular, with the advent of new accelerators such as CEBAF and RHIC, there is a need for the development of approximation techniques and models which bridge the gap between short-distance, perturbative QCD and the extensive amount of low- and intermediate-energy phenomenology in a single covariant framework. . . .*"

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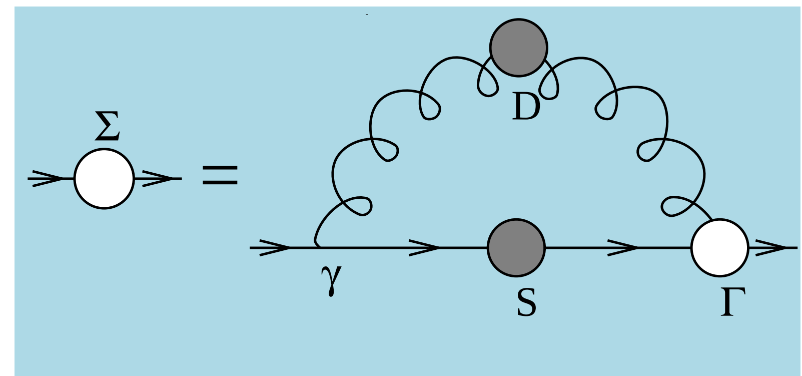

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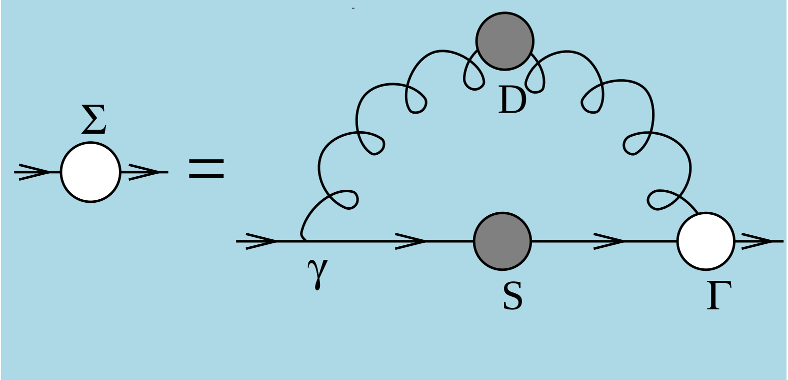

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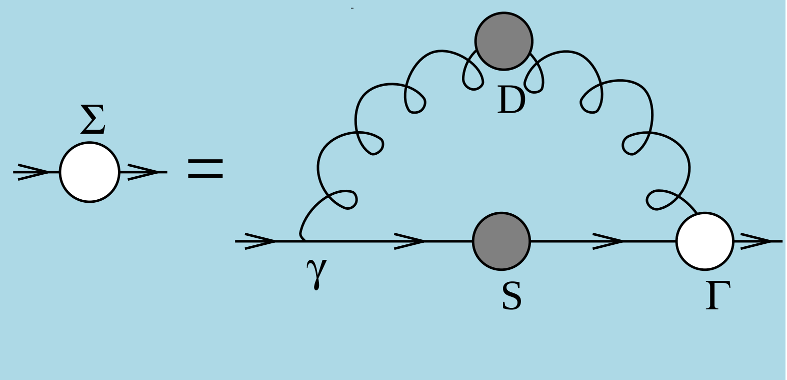

- These are nonperturbative equivalents in quantum field theory to the Lagrange equations of motion.
- Essential in simplifying the general proof of renormalisability of gauge field theories.

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Euler-Lagrange equations for quantum field theory

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- Understanding $\Rightarrow$ InfraRed behaviour of $\alpha_s(Q^2)$
- Method yields Schwinger Functions $\equiv$ Propagators

Cross-Sections built from Schwinger Functions

Perturbative Dressed-quark Propagator

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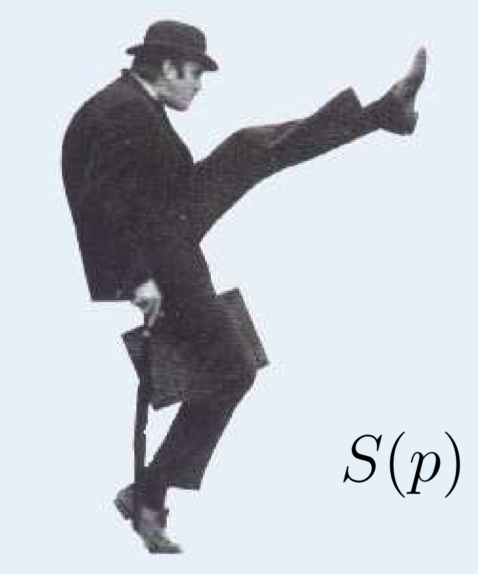
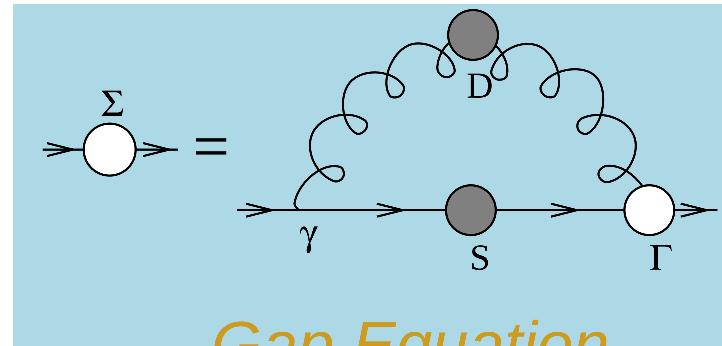
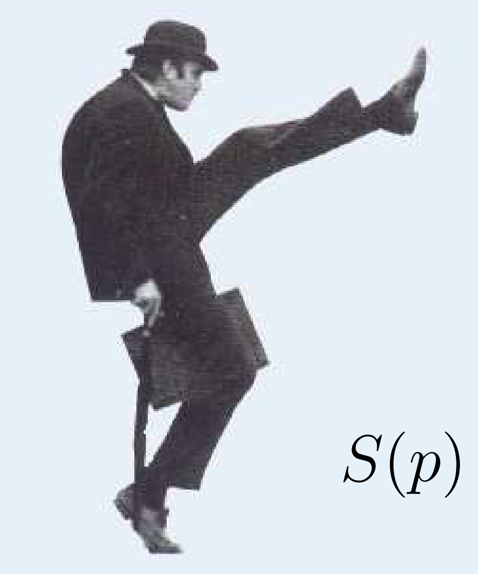


Perturbative Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$



Gap Equation



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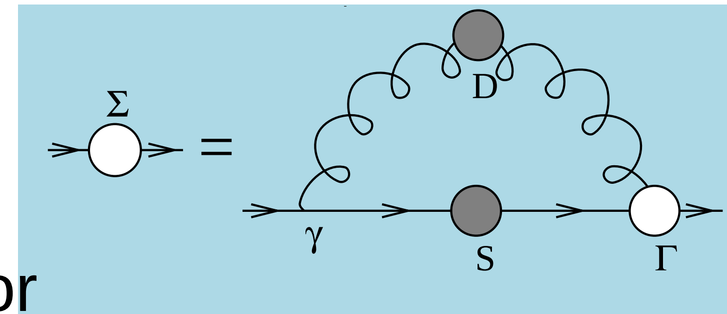

Gap Equation

$$S(p) = \frac{1}{i\gamma \cdot p\, A(p^2) + B(p^2)}$$

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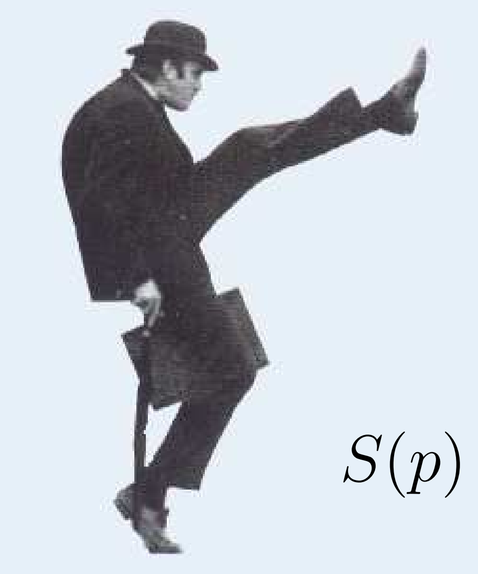

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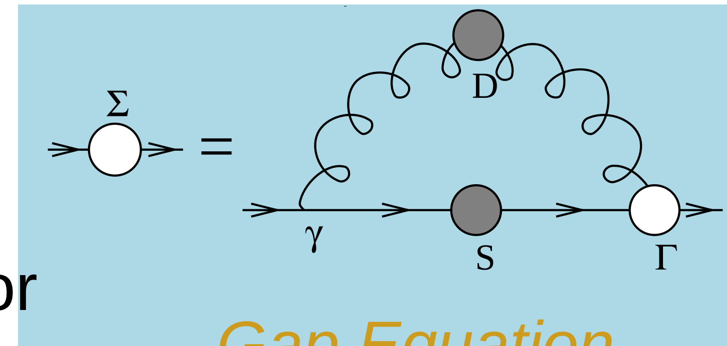

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- Weak Coupling Expansion

  Reproduces Every Diagram in Perturbation Theory

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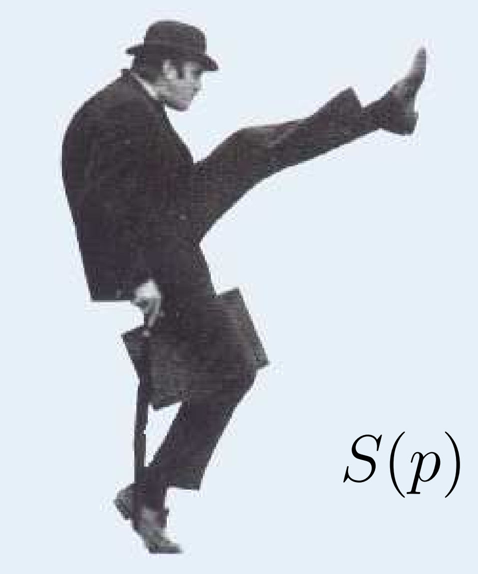

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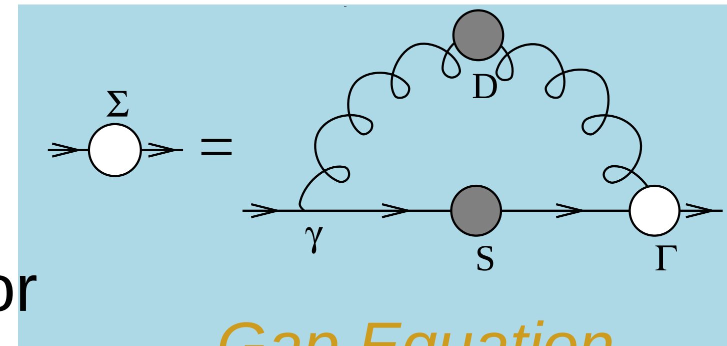

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- But in Perturbation Theory

$$B(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \stackrel{m\to 0}{\to} 0$$

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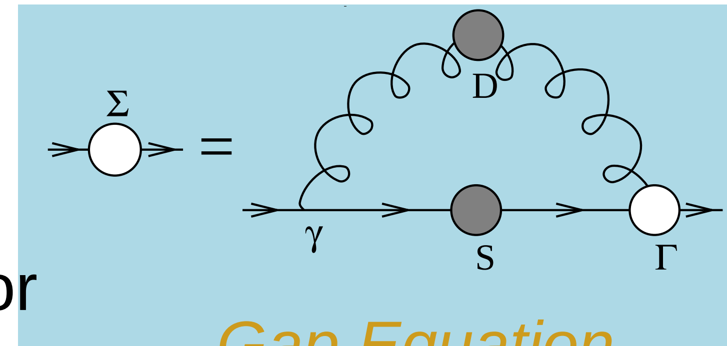

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No DCSB Here!

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Nambu–Jona-Lasinio Model

- Recall the Gap Equation:

$$S^{-1}(p) = i\gamma\cdot p\, A(p^2) + B(p^2) = i\gamma\cdot p + m + \int^{\Lambda} \frac{d^4\ell}{(2\pi)^4}\, g^2\, D_{\mu\nu}(p-\ell)\, \gamma_\mu \frac{\lambda^a}{2} \frac{1}{i\gamma\cdot\ell A(\ell^2) + B(\ell^2)} \Gamma^a_\nu(\ell,p) \quad (4)$$

- NJL: $\Gamma^a_\mu(k,p)_{\rm bare} = \gamma_\mu \dfrac{\lambda^a}{2}$;

$$g^2 D_{\mu\nu}(p-\ell) \to \delta_{\mu\nu}\, \frac{1}{m_G^2}\, \theta(\Lambda^2 - \ell^2) \quad (5)$$

  - Model is not renormalisable
    $\Rightarrow$ regularisation parameter (Λ) plays a dynamical role.

- NJL Gap Equation

$$i\gamma\cdot p\, A(p^2) + B(p^2) = i\gamma\cdot p + m + \frac{4}{3}\frac{1}{m_G^2} \int \frac{d^4\ell}{(2\pi)^4}\, \theta(\Lambda^2-\ell^2)\, \gamma_\mu \frac{-i\gamma\cdot\ell A(\ell^2) + B(\ell^2)}{\ell^2 A^2(\ell^2) + B^2(\ell^2)}\, \gamma_\mu \quad (6)$$

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Solving NJL Gap Equation

- Multiply Eq. (6) by $(-i\gamma \cdot p)$; trace over Dirac indices:

$$p^2\, A(p^2) = p^2 + \frac{8}{3}\,\frac{1}{m_G^2}\int \frac{d^4\ell}{(2\pi)^4}\,\theta(\Lambda^2 - \ell^2)\, p\cdot\ell\, \frac{A(\ell^2)}{\ell^2 A^2(\ell^2) + B^2(\ell^2)} \qquad (7)$$

  - Angular integral vanishes, therefore

$$A(p^2) \equiv 1\,. \qquad (8)$$

This owes to the the fact that NJL model is defined by four-fermion contact interaction in configuration space, entails momentum-independence of interaction in momentum space.

- Tracing over Dirac indices; use Eq. (8):

$$B(p^2) = m + \frac{16}{3}\,\frac{1}{m_G^2}\int \frac{d^4\ell}{(2\pi)^4}\,\theta(\Lambda^2 - \ell^2)\,\frac{B(\ell^2)}{\ell^2 + B^2(\ell^2)}\,, \qquad (9)$$

  - Integral is p^2-independent.
  - Therefore $B(p^2) = \text{constant} = M$ is the only solution.

NJL Mass Gap

- Evaluate integrals; Eq. (9) becomes

$$M = m + M \frac{1}{3\pi^2} \frac{1}{m_G^2} \mathcal{C}(M^2, \Lambda^2)\,, \qquad (10)$$

$$\mathcal{C}(M^2, \Lambda^2) = \Lambda^2 - M^2 \ln \; 1 + \Lambda^2/M^2 \; . \qquad (11)$$

- Λ defines model's mass-scale. Henceforth set $\boxed{\Lambda = 1}$. Then all other dimensioned quantities are given in units of this scale, in which case the gap equation can be written

$$M = m + M \frac{1}{3\pi^2} \frac{1}{m_G^2} \mathcal{C}(M^2, 1)\,. \qquad (12)$$

- Chiral limit: $m = 0$, $\boxed{M = M \frac{1}{3\pi^2} \frac{1}{m_G^2} \mathcal{C}(M^2, 1)}$
  - Solved if $M \equiv 0$
    ... This is the **perturbative result**: start with no mass, end up with no mass.
  - Suppose $M \neq 0$
    - Solved *iff* $\boxed{1 = \frac{1}{3\pi^2} \frac{1}{m_G^2} \mathcal{C}(M^2, 1)}$.

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NJL Dynamical Mass

- Can one satisfy $1 = \frac{1}{3\pi^2}\,\frac{1}{m_G^2}\,\mathcal{C}(M^2,1)$?
- $\mathcal{C}(M^2,1) = 1 - M^2 \ln\left(1 + 1/M^2\right)$
  - Monotonically decreasing function of M
  - Maximum value at $M = 0$: $\mathcal{C}(0,1) = 1$.
- Consequently $\exists\ M \neq 0$ solution *iff* $\frac{1}{3\pi^2}\,\frac{1}{m_G^2} > 1$
  - Typical scale for hadron physics $\Lambda \sim 1$ GeV.
    - $M \neq 0$ solution *iff* $m_G^2 < \frac{\Lambda^2}{3\pi^2} \simeq (0.2\,GeV)^2$
- Interaction Strength is proportional to $\frac{1}{m_G^2}$
  - When interaction is strong enough,
    one can start with no mass but end up with a massive quark.

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Dynamical Chiral Symmetry Breaking

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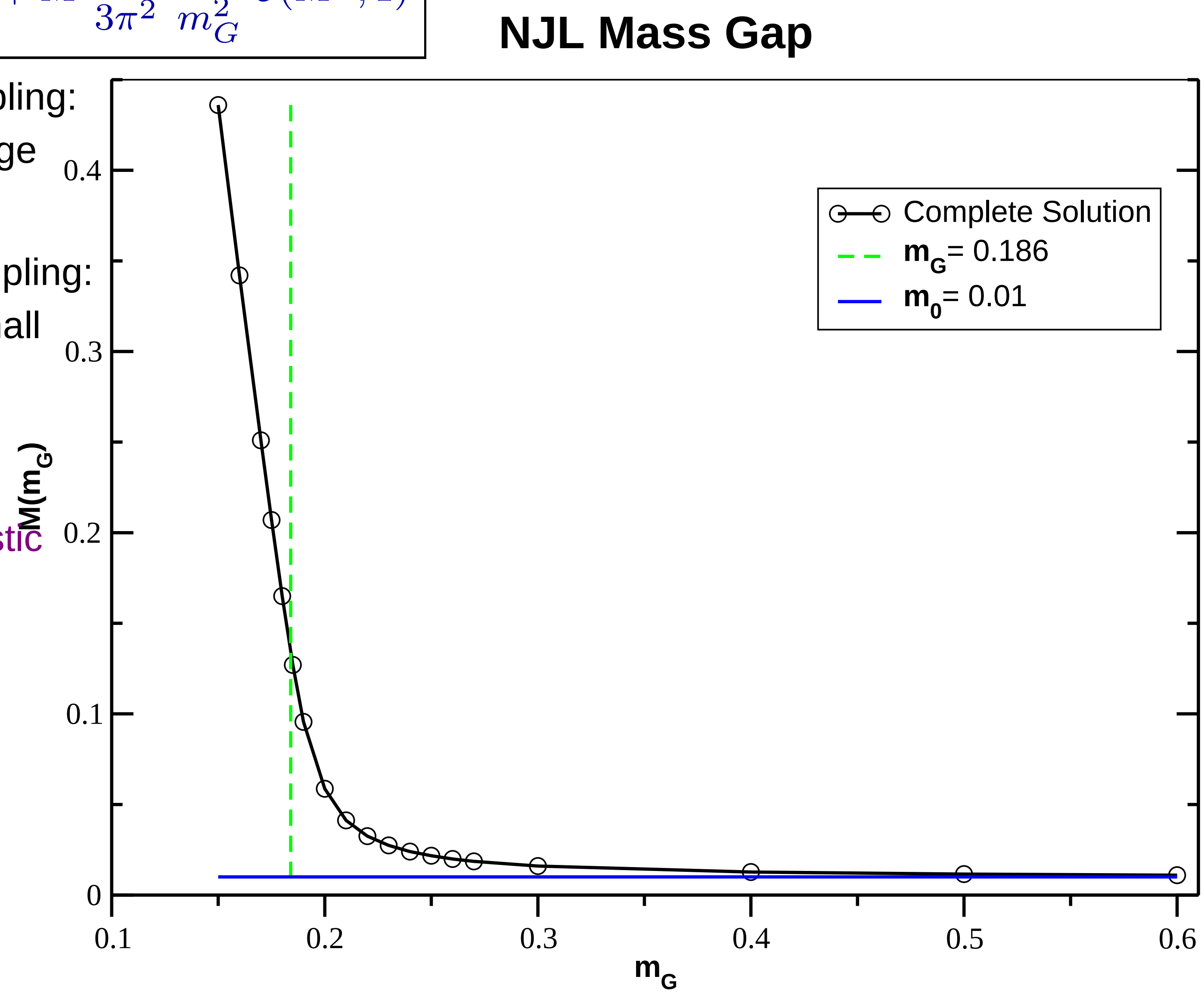
NJL Dynamical Mass

Solve $$M = m_0 + M \frac{1}{3\pi^2} \frac{1}{m_G^2} \mathcal{C}(M^2, 1)$$

- Weak coupling:
  $\Leftrightarrow m_G$ large
  $M \sim m_0$
- Strong coupling:
  $\Leftrightarrow m_G$ small
  $M \gg m_0$
  This is the essential characteristic of DCSB



NJL Model and Confinement?

- **Confinement** – no free-particle-like quarks

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NJL Model and Confinement?

- **Confinement** – no free-particle-like quarks
- Fully-dressed NJL propagator

$$S(p)^{\rm NJL} = \frac{1}{i\gamma\cdot p[A(p^2)=1]+[B(p^2)=M]} = \frac{-i\gamma\cdot p+M}{p^2+M^2} \qquad (15)$$

NJL Model and Confinement?

- **Confinement** – no free-particle-like quarks
- Fully-dressed NJL propagator

$$S(p)^{\rm NJL} = \frac{1}{i\gamma\cdot p[A(p^2)=1] + [B(p^2)=M]} = \frac{-i\gamma\cdot p + M}{p^2+M^2} \qquad (17)$$

- This is merely a free-particle-like propagator with a shifted mass:

$$p^2 + M^2 = 0 \Rightarrow \text{ Minkowski-space mass} = M. \qquad (18)$$

NJL Model and Confinement?

- **Confinement** – no free-particle-like quarks
- Fully-dressed NJL propagator

$$S(p)^{\rm NJL} = \frac{1}{i\gamma\cdot p[A(p^2)=1]+[B(p^2)=M]} = \frac{-i\gamma\cdot p + M}{p^2+M^2} \quad (19)$$

- This is merely a free-particle-like propagator with a shifted mass:

$$p^2+M^2=0 \Rightarrow \text{ Minkowski-space mass} = M. \quad (20)$$

- Hence, while **NJL Model** certainly contains DCSB,
  it **does not exhibit confinement**.

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Munczek-Nemirovsky Model

- Munczek, H.J. and Nemirovsky, A.M. (1983), "The Ground State $q\bar{q}$ Mass Spectrum In QCD," *Phys. Rev*. **D 28**, 181.
- $\Gamma^a_\mu(k,p)_{\rm bare} = \gamma_\mu \, \frac{\lambda^a}{2}$;

$$g^2 D_{\mu\nu}(k) \to (2\pi)^4 \, G \, \delta^4(k) \left[\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right] \quad (21)$$

Here G defines the model's mass-scale.

  - δ-function in momentum space
    cf. NJL, which has δ-function in configuration space.
- Gap equation

$$i\gamma\cdot p\, A(p^2) + B(p^2) = i\gamma\cdot p + m + G\,\gamma_\mu \, \frac{-i\gamma\cdot p\, A(p^2) + B(p^2)}{p^2 A^2(p^2) + B^2(p^2)}\, \gamma_\mu \quad (22)$$

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MN Model's Gap Equation

- The gap equation yields the following two coupled equations (set the mass-scale $G = 1$):

$$A(p^2) = 1 + 2\,\frac{A(p^2)}{p^2 A^2(p^2) + B^2(p^2)} \tag{23}$$

$$B(p^2) = m + 4\,\frac{B(p^2)}{p^2 A^2(p^2) + B^2(p^2)}\,, \tag{24}$$

- Consider the chiral limit equation for $B(p^2)$:

$$B(p^2) = 4\,\frac{B(p^2)}{p^2 A^2(p^2) + B^2(p^2)}\,. \tag{25}$$

  - Obviously, $B \equiv 0$ is a solution.
  - Is there another?

DCSB in MN Model

- The existence of a $B \not\equiv 0$ solution; i.e., a solution that dynamically breaks chiral symmetry, requires (in units of G)

$$p^2 A^2(p^2) + B^2(p^2) = 4\,. \tag{26}$$

- Substituting this identity into equation Eq. (23), one finds

$$A(p^2) - 1 = \frac{1}{2} A(p^2) \;\Rightarrow\; A(p^2) \equiv 2\,, \tag{27}$$

which in turn entails

$$B(p^2) = 2\sqrt{1-p^2}\,. \tag{28}$$

- Physical requirement: quark self energy is real on the spacelike domain $\Rightarrow$ complete chiral-limit solution –

$$A(p^2) = \begin{cases} 2\,; & p^2 \leq 1 \\ \frac{1}{2}\left(1+\sqrt{1+8/p^2}\right)\,; & p^2 > 1 \end{cases} \tag{29}$$

$$B(p^2) = \begin{cases} \sqrt{1-p^2}\,; & p^2 \leq 1 \\ 0\,; & p^2 > 1\,. \end{cases} \tag{30}$$

- NB. Dressed-quark self-energy is momentum dependent, as is the case in QCD.

Confinement in MN Model

- Solution is continuous and defined for all p^2, even $p^2 < 0$; namely, timelike momenta.
- Examine the propagator's denominator:

$$p^2 A^2(p^2) + B^2(p^2) > 0 \, , \; \forall \, p^2 \, . \tag{31}$$

This is positive definite ... there are *no zeros*

- This is nothing like a free-particle propagator. It can be interpreted as describing a **confined** degree-of-freedom
- Note that, in addition there is no critical coupling: the nontrivial solution exists so long as **G** > 0.
- Conjecture: **All confining theories exhibit DCSB**.
  - NJL model demonstrates that converse is not true.

Massive Solution in MN Model

- In the chirally asymmetric case the gap equation yields

$$A(p^2) = \frac{2\,B(p^2)}{m + B(p^2)}\,, \tag{32}$$

$$B(p^2) = m + \frac{4\,[m + B(p^2)]^2}{B(p^2)([m + B(p^2)]^2 + 4p^2)}\,. \tag{33}$$

  - Second is a quartic equation for $B(p^2)$.
  - Can be solved algebraically with four solutions, available in a closed form.
  - Only one has the correct $p^2 \to \infty$ limit: $B(p^2) \to m$.
  - NB. The equations and their solutions always have a smooth $m \to 0$ limit, a result owing to the persistence of the DCSB solution.

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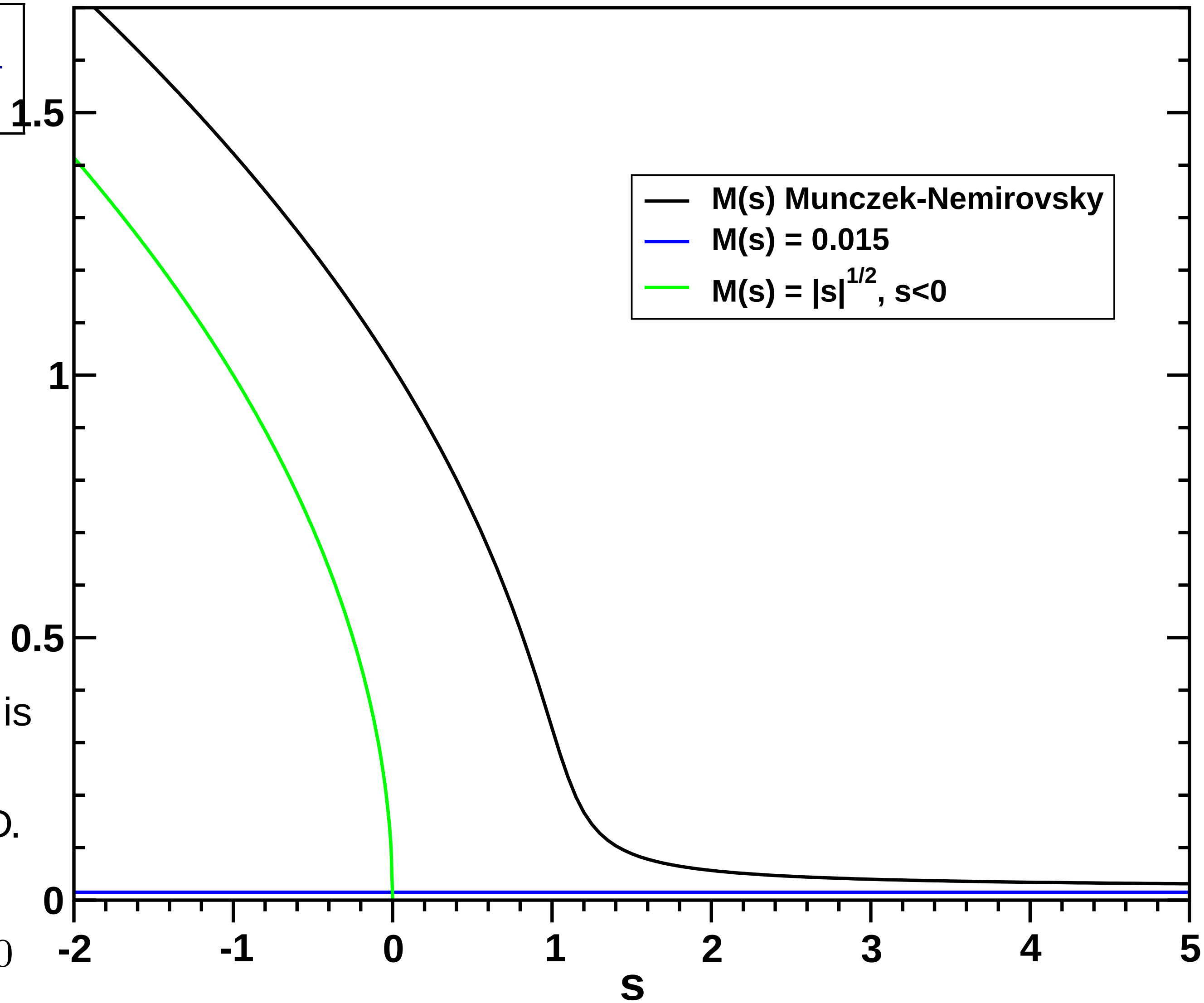
MN Dynamical Mass

$$M(s=p^2) = \frac{B(s)}{A(s)}$$

- Large s:
  $M(s) \sim m_0$
- Small s
  $M \gg m_0$
  This is the essential characteristic of DCSB
- p^2-dependent mass function is quintessential feature of QCD.
- No solution of
  $s + M(s)^2 = 0$
  confinement.



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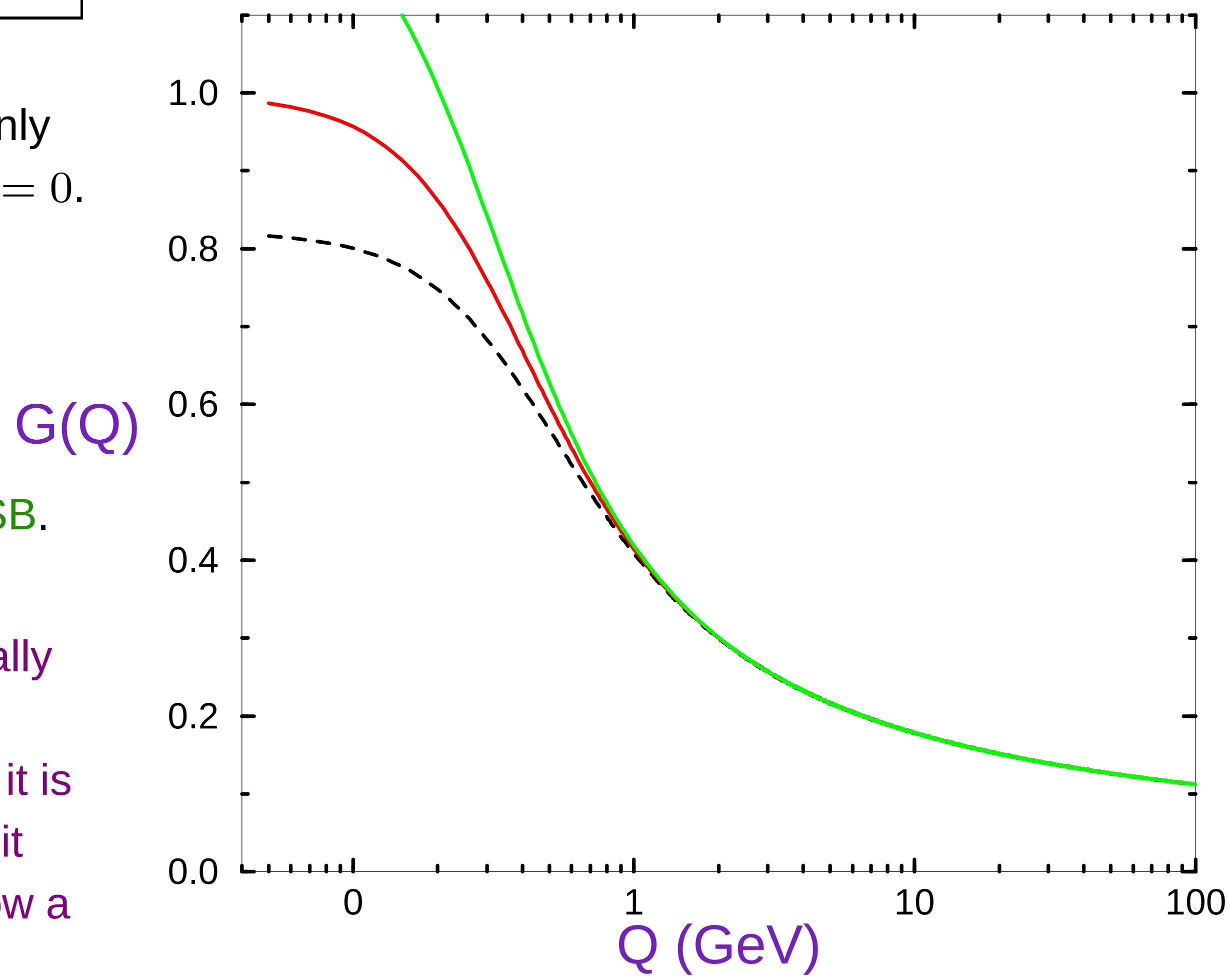
Real World Alternatives

$$g^2 D(Q^2) = 4\pi \frac{G(Q^2)}{Q^2}$$

- $G(0) < 1$: $M(s) \equiv 0$ is only solution for $m = 0$.
- $G(0) \geq 1$ $M(s) \neq 0$ is possible and energetically favoured: DCSB.
- $M(0) \neq 0$ is a new, dynamically generated mass-scale. If it is large enough, it can explain how a theory that is apparently massless (in the Lagrangian) possesses the spectrum of a massive theory.



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- Confinement and Dynamical Chiral Symmetry Breaking are Key Emergent Phenomena in QCD
- Understanding requires Nonperturbative Solution of Fully-Fledged Relativistic Quantum Field Theory
  - Mathematics and Physics still far from being able to accomplish that

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- What's the story in QCD?