Abrikosov Gluon Vortices in Color Superconductors

Efraín J. Ferrer

The University of Texas at El Paso

EJF & de la Incera, PRL 97 (2006) 122301
EJF & de la Incera, PRD 76 (2007) 045011
EJF & de la Incera, PRD 76 (2007) 114012

OUTLINE

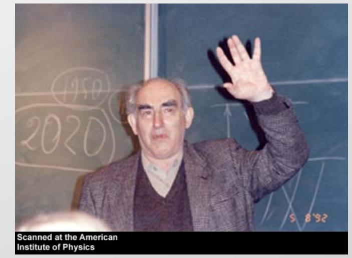
- ***Magnetized CS***
- ***Gluon Vortices and Magnetic Antiscreening in MCS***
- ***Chromomagnetic Instabilities &*** $G - \tilde{B}$ ***Condensates***
- ***Conclusions and Future Directions***

XII-MWPF 2009.

Cooper Pair Condensation

L. D. Landau

$$F[\Psi] = \int d^3x \left\{ \frac{\hbar^2}{2\hat{m}} \vec{\nabla}\Psi^* \vec{\nabla}\Psi + \alpha(T - T_C)\Psi^*\Psi + \frac{\beta}{2}(\Psi^*\Psi)^2 \right\}$$



V. L. Ginzburg

Electric Superconductivity

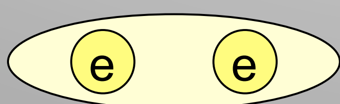
J. Bardeen, L. N. Cooper and J. R. Schrieffer

Color Superconductivity

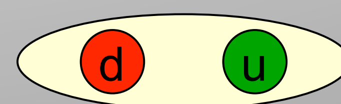
Barrois '77; Frautschi '78;
Bailin and Love'84;
Alford, Rajagopal and Wilczek '98;
Rapp, Schafer, Shuryak and Velkovsky '98

✓ *Boson: Zero Spin and opposite momenta*

✓ Electric Charge



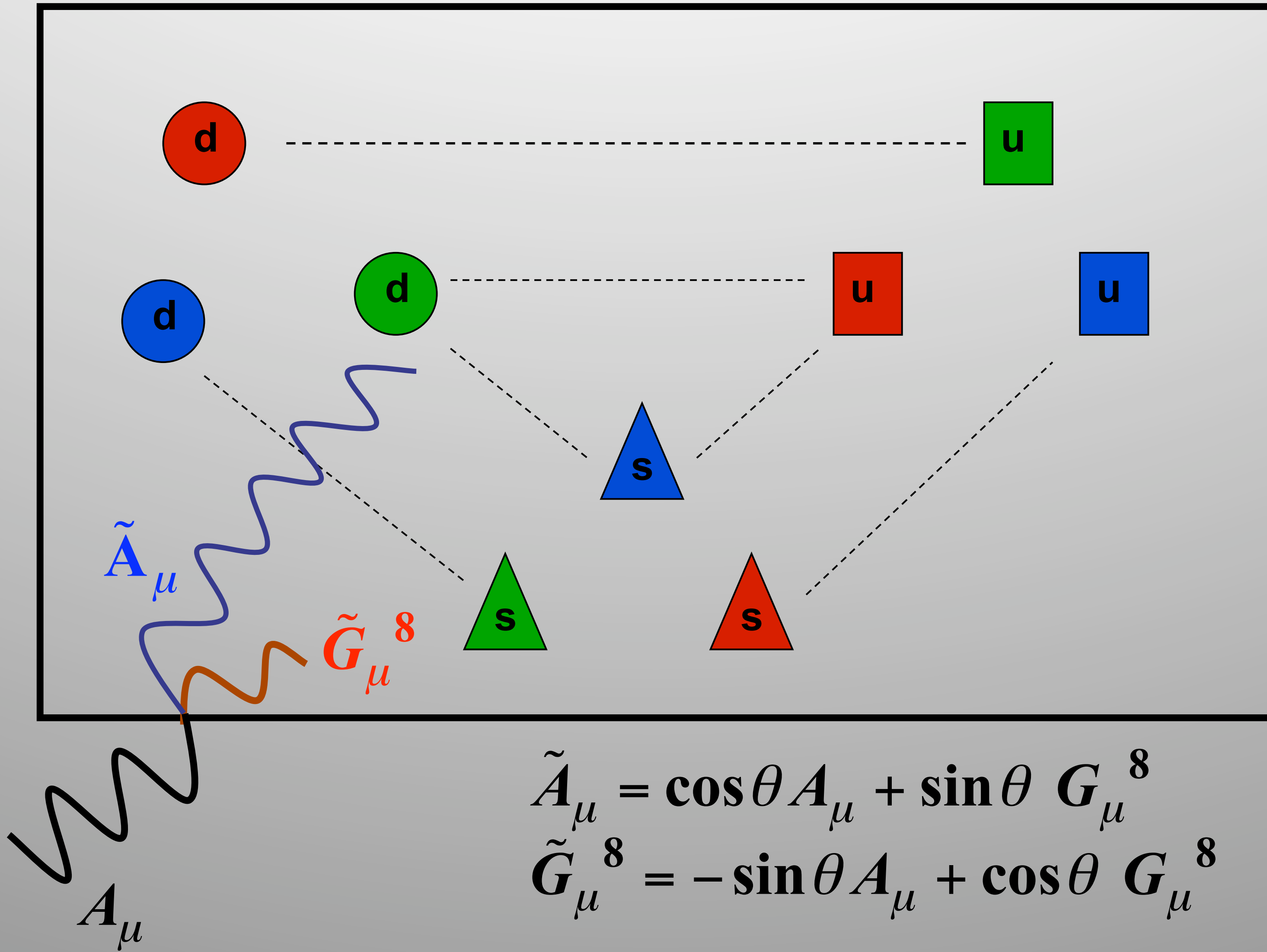
✓ Color Charge



✓ Broken Symmetry : $U(1)_{em}$

✓ Broken Symmetry : $SU(3)_{C}$, $U(1)_{em}$

In-Medium Magnetic Field



$$\tilde{A}_\mu = \cos\theta\, A_\mu + \sin\theta\ G_\mu{}^8$$
$$\tilde{G}_\mu{}^8 = -\sin\theta\, A_\mu + \cos\theta\ G_\mu{}^8$$

Neutron Stars

Diameter:

$R \approx 10\ km$

Mass:

$1.25M_{\odot} \lesssim M \lesssim 2M_{\odot}$

Temperature:

$10\ keV \lesssim T \lesssim 10MeV$

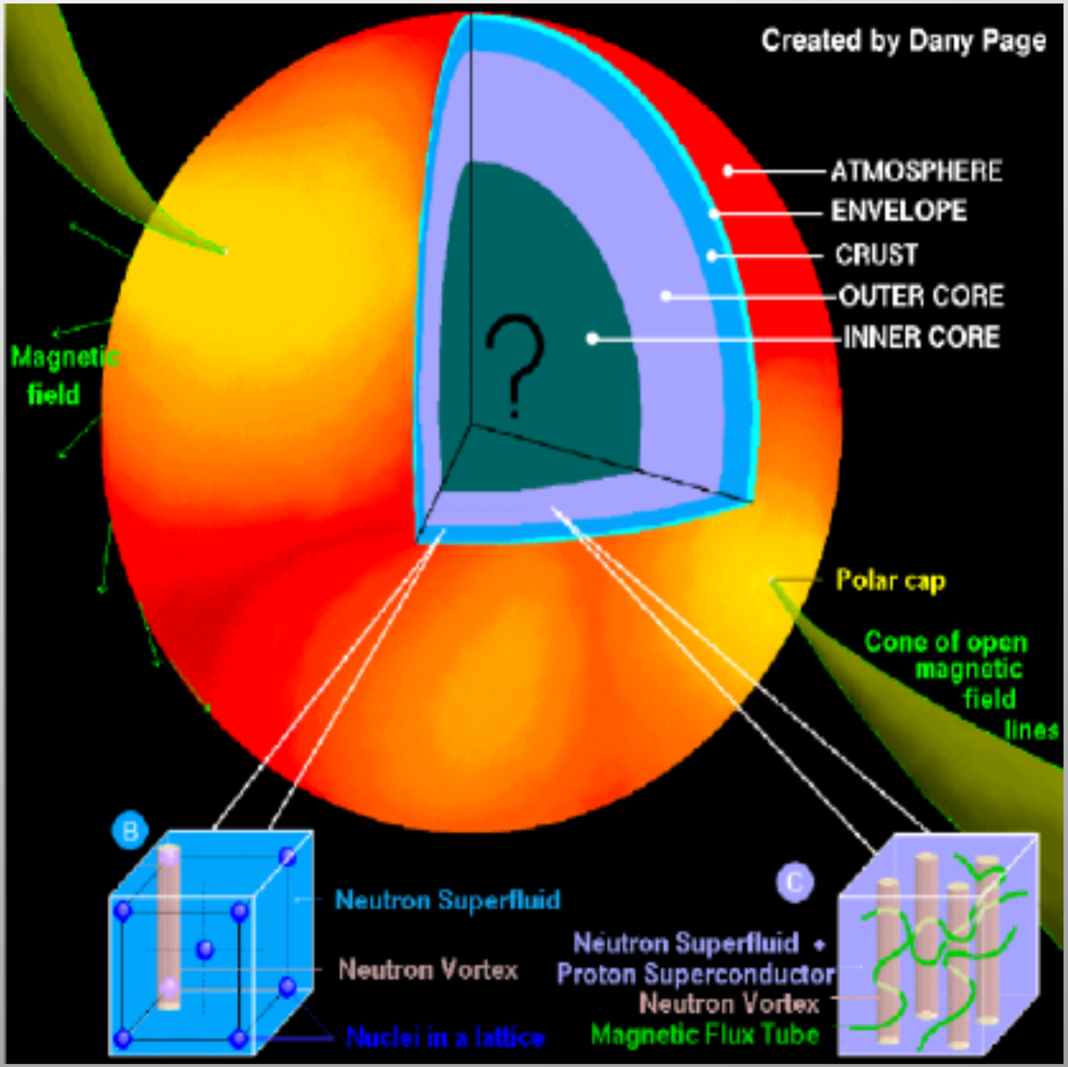
Magnetic fields:

pulsar's surface:

$B \sim 10^{12}$–$10^{14}G$

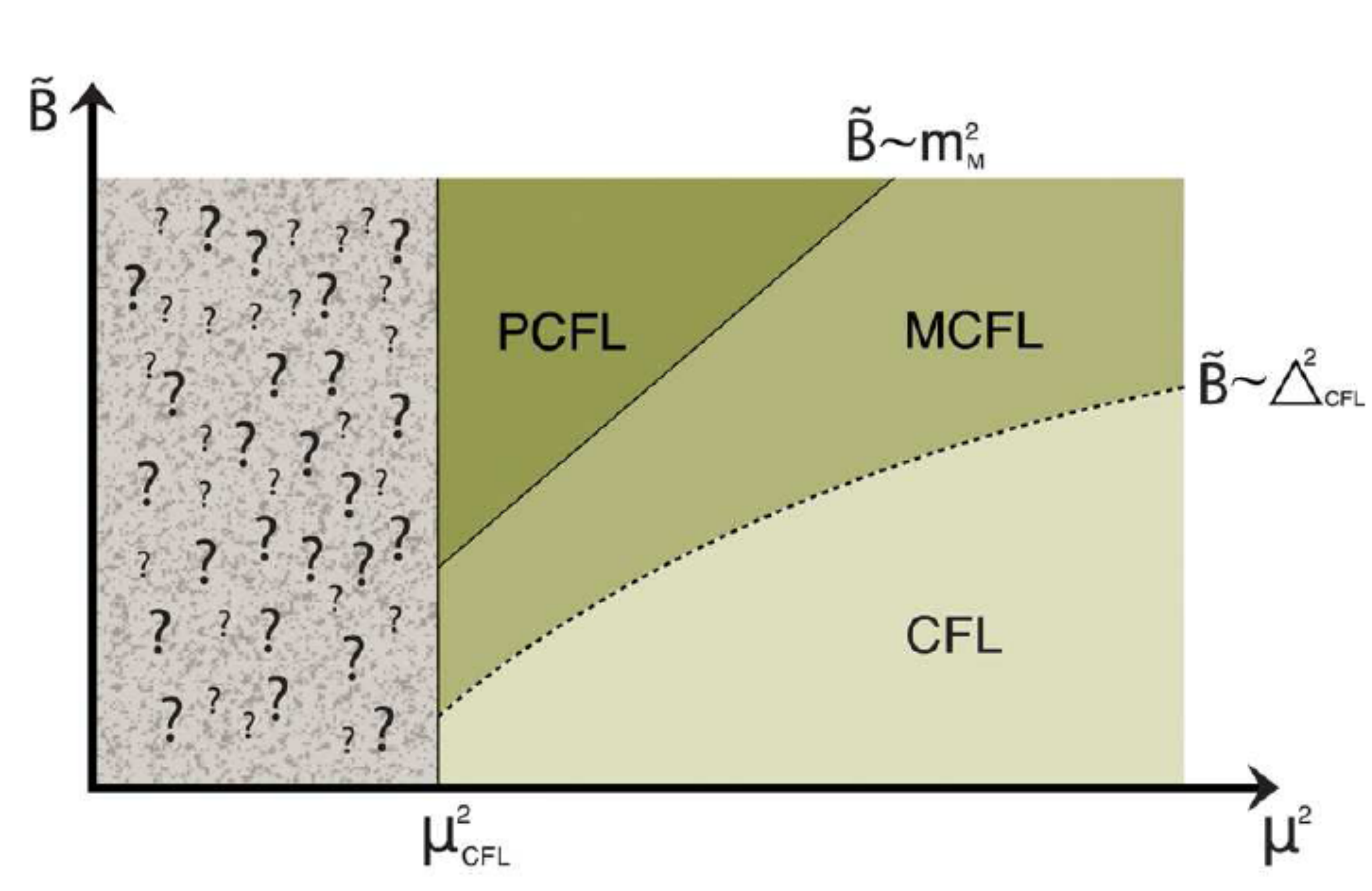
magnetar's surface:

$B \sim 10^{15}$–$10^{16}G$



Sketch of $\tilde{B}$ vs μ phases of a Color Superconductor with Three-Quark Flavors

EJF & Incera, PRD 76 (2007)045011



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Magnetic Effects on the Gluon Sector

EJF & de la Incera, PRL 97 (2006) 122301

Because of the modified electromagnetism, gluons are charged in the color superconductor

G^1_μ	G^2_μ	G^3_μ	G^+_μ	G^-_μ	I^+_μ	I^-_μ	$\tilde{G}^8_\mu$
0	0	0	1	-1	1	-1	0

Gluon Mean-Field-Effective Action in the CFL Phase:

$$\begin{aligned}\Gamma_{eff} &= -\frac{1}{4}\int d^4x[(G^a_{\mu\nu})^2+(f_{\mu\nu})^2]+\int d^4x L_g(x)\\ &- \frac{1}{2}\int d^4x d^4y G^a_\mu(x)\Pi^{ab}_{\mu\nu}(x,y)G^b_\nu(y)\\ &- \frac{1}{2}\int d^4x d^4y A_\mu(x)\Pi_{\mu\nu}(x,y)A_\nu(y)\end{aligned}$$

Charged Gluons Effective Action

G^1_μ	G^2_μ	G^3_μ	G^+_μ	G^-_μ	I^+_μ	I^-_μ	$\tilde{G}^8_\mu$
0	0	0	1	-1	1	-1	0

Effective action for the charged gluons within CFL at asymptotic densities

$$\Gamma^c_{eff} = \int d^4x \{ -\frac{1}{4}(\widetilde{f}_{\mu\nu})^2 - \frac{1}{2}|\widetilde{\Pi}_\mu G^-_\nu - \widetilde{\Pi}_\nu G^-_\mu|^2$$
$$-[(m^2_D \delta_{\mu 0}\delta_{\nu 0} + m^2_M \delta_{\mu i}\delta_{\nu i}) + i\widetilde{e}\widetilde{f}_{\mu\nu}]G^+_\mu G^-_\nu$$
$$+\frac{g^2}{2}[(G^+_\mu)^2(G^-_\nu)^2 - (G^+_\mu G^-_\mu)^2] + \frac{1}{\lambda}G^+_\mu \widetilde{\Pi}_\mu \widetilde{\Pi}_\nu G^-_\nu \},$$

where

$$\widetilde{\Pi}_\mu = \partial_\mu - i\widetilde{e}\widetilde{A}_\mu$$

Magnetic Instability for Charged Spin-1 Fields

Assuming that there is an external magnetic field $\tilde{H}$ in the z-direction, one mode becomes unstable when $\tilde{H} > m^2{}_M$

$$\begin{pmatrix} m_M^2 & i\tilde{e}\tilde{H} \\ -i\tilde{e}\tilde{H} & m_M^2 \end{pmatrix} \rightarrow \begin{pmatrix} m_M^2 + \tilde{e}\tilde{H} & 0 \\ 0 & m_M^2 - \tilde{e}\tilde{H} \end{pmatrix}$$

with corresponding eigenvector: $(G_1{}^+, G_2{}^+) \rightarrow G(1,i)$

"Zero-mode problem" for non-Abelian gauge fields whose solution is the formation of a vortex condensate of charged spin-1 fields.

Nielsen & Olesen NPB 144 (1978)

Skalozub, Sov.JNP23 (1978);ibid 43 (1986)

Ambjorn & Olesen, NPB315 (1989)

EJF & Incera, IJMP 11 (1996)

Gibbs Free-Energy:

$$\mathcal{G}_c = \mathcal{F}_{n0} - 2G^\dagger \widetilde{\Pi}^2 G - 2(2\widetilde{e}\widetilde{B} - m_M^2)|G|^2 + 2g^2|G|^4 + \frac{1}{2}\widetilde{B}^2 - \widetilde{H}\widetilde{B}$$

Minimum Equations:

$$-\widetilde{\Pi}^2 G - (2\widetilde{e}\widetilde{B} - m_M^2)G + 2g^2|G|^2 G = 0,$$

In the approximation: $\widetilde{e}\widetilde{H} \approx m_M^2 \gg |G|^2, \widetilde{e}(\widetilde{B} - \widetilde{H}_c)$

$$\widetilde{\Pi}^2 G + \widetilde{e}\widetilde{H}_c G \approx 0$$

$$2\widetilde{e}|G|^2 - \widetilde{B} + \widetilde{H} \approx 0$$

Minimum Equations:

$$\widetilde{\Pi}^2 G + \widetilde{e}\widetilde{H}_c G \approx 0$$

$$|\widetilde{\Pi}_\mu K_\nu - \widetilde{\Pi}_\nu K_\mu|^2 = 0$$

$$\widetilde{H} + 2\widetilde{q}|G|^2 - \widetilde{B} \approx 0$$

Magnetic Antiscreening

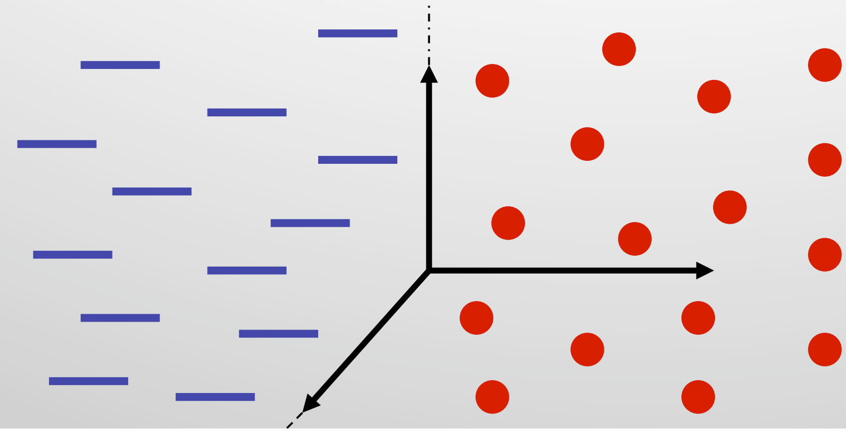
Abrikosov's Equation

$$[\partial_j^2 - \frac{4\pi i}{\widetilde{\Phi}_0}\widetilde{H}_C x \partial_y - 4\pi^2 \frac{\widetilde{H}_C^2}{\widetilde{\Phi}_0^2} x^2 + \frac{1}{\xi^2}]G = 0, \quad j = x, y$$

$$\widetilde{\Phi}_0 \equiv 2\pi/\widetilde{e},$$

$$\xi^2 \equiv 1/(2\widetilde{e}\widetilde{H}_C - m_M^2) = 1/m_M^2$$

Surface Energy Density and Vortex Nucleation



$$\alpha_{nc} = \int_{-\infty}^{\infty} [\mathcal{G} - \mathcal{G}_n] dx = \int_{-\infty}^{\infty} \{-2G^\dagger \widetilde{\Pi}^2 G + \frac{1}{2}(\widetilde{B} - \widetilde{H})^2 - 2(2\widetilde{e}\widetilde{B} - m_M^2)|G|^2 + 2g^2|G|^4\} dx$$

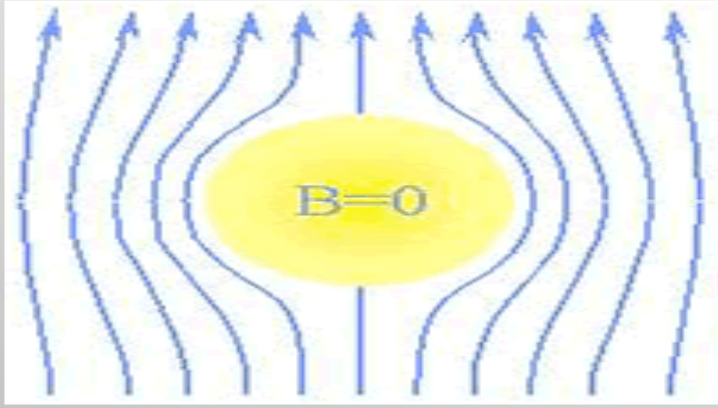
$$\mathcal{G}_n = \mathcal{F}_{n0} - \widetilde{H}^2/2$$

$$\alpha_{nc} = \int_{-\infty}^{\infty} (1 - \frac{g^2}{\widetilde{e}^2}) \frac{(\widetilde{B} - \widetilde{H})^2}{2} dx$$

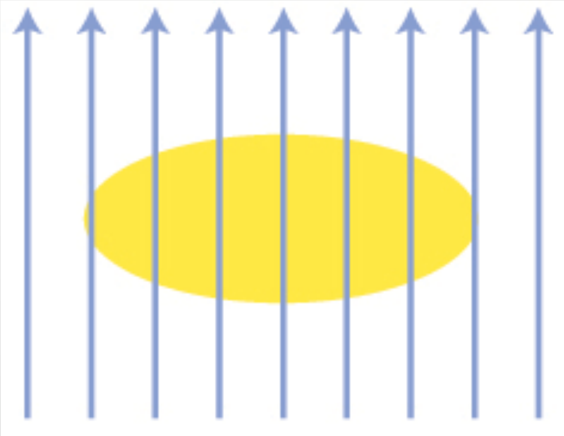
Since $g^2/\tilde{e}^2 > 1$ we have that $\alpha_{nc} < 0$ which is the required condition for vortex nucleation.

Conventional Superconductor *vs* Color Superconductor

Conventional Superconductor

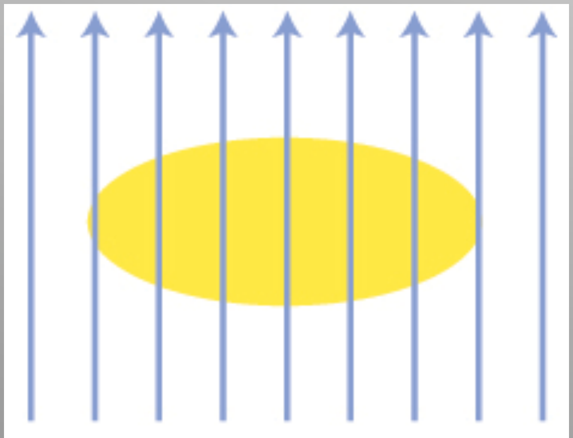


H < Hc

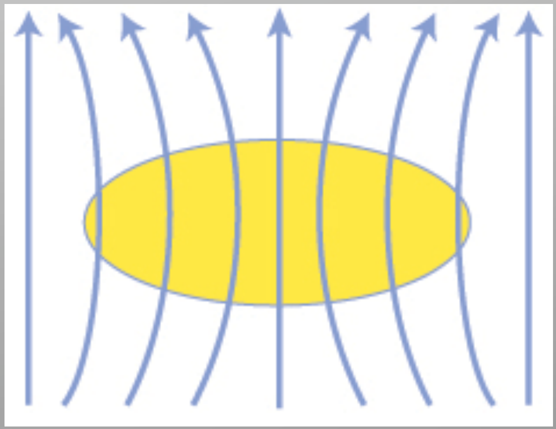


H ≥ Hc

Color Superconductor

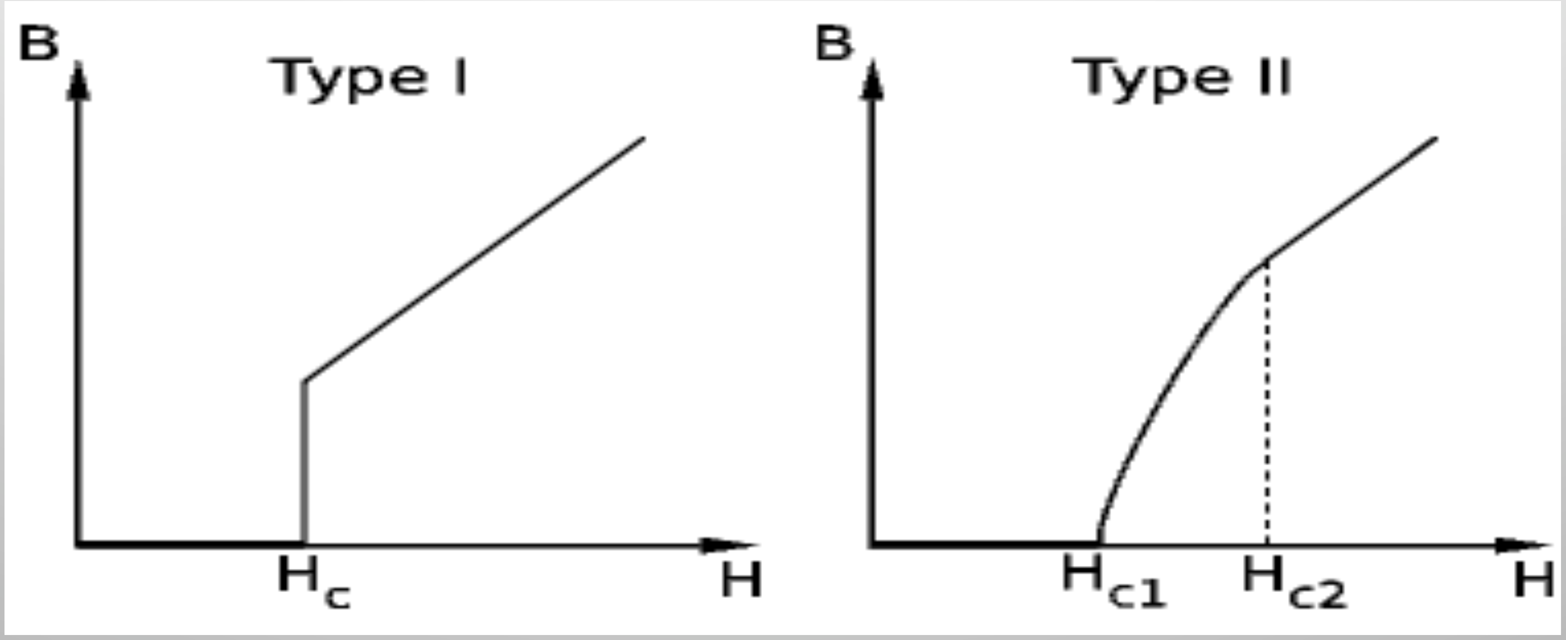


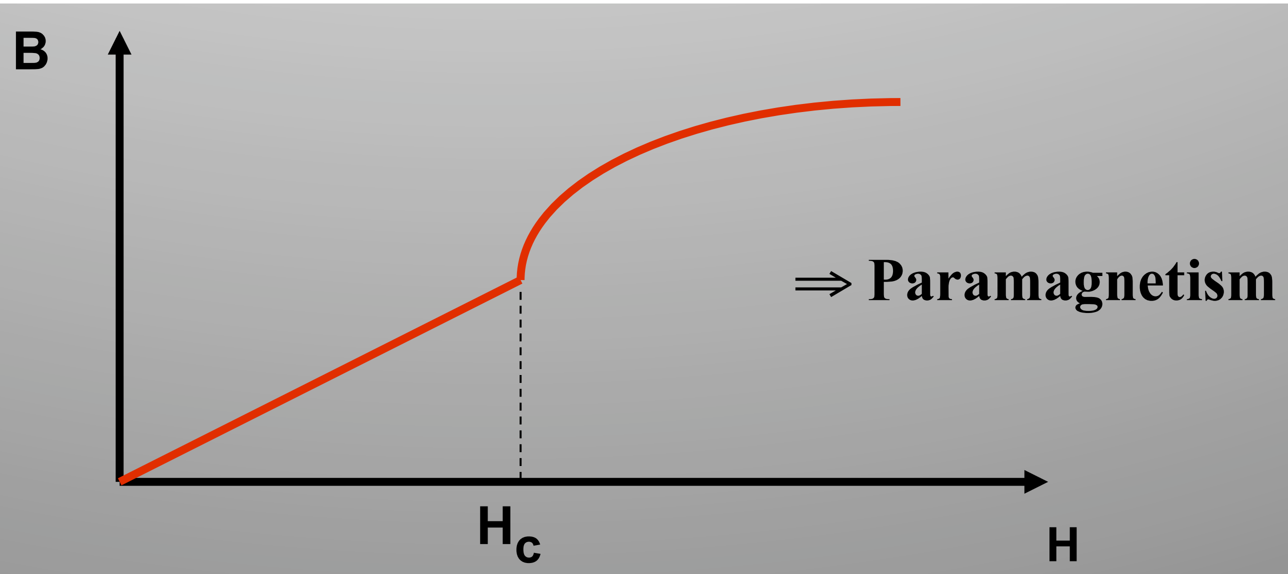
H < Hc



H ≥ Hc

Variation of internal magnetic field (B) with applied magnetic field (H) for Type I, Tipe II and Color Superconductors





Linearizing the minimum equation for G* around the critical field

$$[\partial_j^2 - \frac{4\pi i}{\widetilde{\Phi}_0}\widetilde{H}_C x\partial_y - 4\pi^2\frac{\widetilde{H}_C^2}{\widetilde{\Phi}_0^2}x^2 + \frac{1}{\xi^2}]G = 0, \quad j = x, y$$

where

$$\tilde{\Phi}_0 = \frac{2\pi}{\tilde{e}}, \qquad \xi^2 = 1/(2\tilde{e}\tilde{H} - m^2_M) = \frac{1}{m^2_M}$$

to find the solution

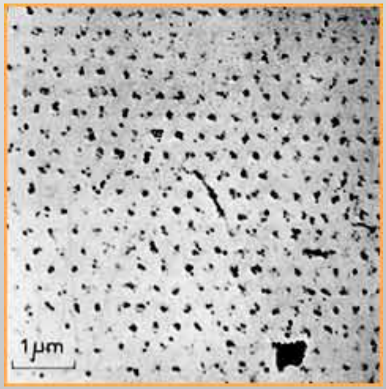
$$G_k = \exp[-iky]\exp[-\frac{(x-x_k)^2}{2\xi^2}]$$

$$x_k = \frac{k\Phi_0}{2\pi\tilde{H}}$$

Vortex Solution

From the experience with conventional type II superconductivity, it is known that the inhomogeneous condensate solutions prefer periodic lattice domains to minimize the energy. Then, putting on periodicity in the *y*-direction:

$$\Delta y = b \quad \Rightarrow \quad k = 2\pi n / b, \quad n = 0, \pm 1, \pm 2, \ldots$$



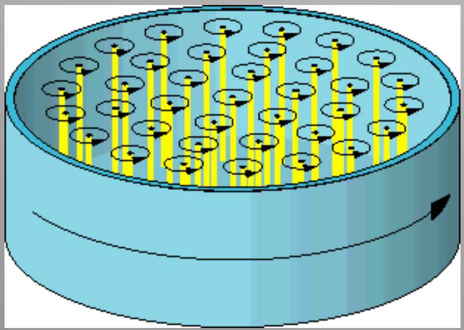
Vortex lattice,
First image 1967

The periodicity is also transferred to the *x*-direction:

$$x_n = k_n \xi^2 = 2\pi n \xi^2 / b$$

Then, the general solution is given by the superposition:

$$\overline{G}(x, y) = [1/\sqrt{2}\tilde{e}\xi] e^{-\frac{1}{2\xi^2}x^2} \vartheta_3(u, q)$$



The vortex lattice induces a magnetic field that forms a fluxoid along the z-direction. The magnetic flux through each periodicity cell in the vortex lattice is quantized

$$\widetilde{B} \Delta x \Delta y = 2\pi / \tilde{e}$$

Neutrality Conditions

$$\frac{\partial \Omega}{\partial \mu_B} = J_0^{(B)}$$

$$\frac{\partial \Omega}{\partial \mu_e} = 0$$

$$\frac{\partial \Omega}{\partial \mu_8} = 0 \qquad \frac{\partial \Omega}{\partial \mu_3} = 0$$

$$\mu_8 = (\sqrt{3}g/2)\left\langle G_0^{(8)} \right\rangle \qquad \mu_3 = (g/2)\left\langle G_0^{(3)} \right\rangle$$

Cooper Pairing and Neutrality Conditions

- The optimal Cooper pairing occurs when

$$\mu_d \simeq \mu_n \Rightarrow n_d \simeq \frac{\mu_d^3}{\pi^2} \simeq \frac{\mu_u^3}{\pi^2} \simeq n_u$$

- Matter inside a star should be electrically neutral to guarantee the star stability and to minimize the system energy

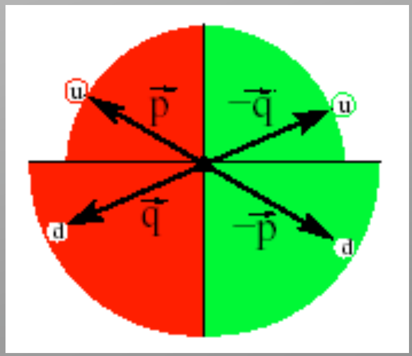
$$\frac{2}{3}n_u - \frac{1}{3}n_d - n_e = 0 \Rightarrow n_d \simeq 2n_u$$

- There are no enough electrons in β equilibrium to balance the charge deficit

$$\mu_e = \mu_d - \mu_u \simeq \frac{1}{4}\mu_u \Rightarrow n_e \simeq \frac{1}{192}n_u \ll n_u$$

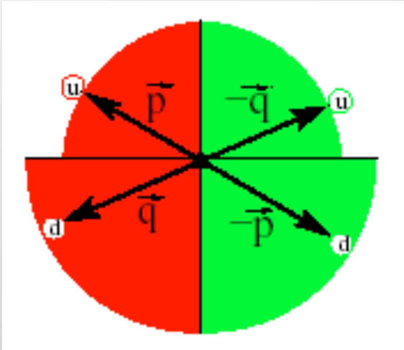
Cooper pairing is consequently distorted by the Fermi sphere mismatch.

$$\delta\mu \equiv \frac{\mu_d - \mu_u}{2} = \frac{\mu_e}{2} \neq 0$$



Chromomagnetic Instabilities in 2SC

Color Neutrality and beta equilibrium



Gluons Masses

Stable Gapped 2SC

a=1,2,3 *massless*
a=4,5,6,7 positive
a=8 positive

$$\Delta > \sqrt{2}\delta\mu$$

Unstable Gapped 2SC

a=1,2,3 *massless*
a=4,5,6,7 *negative*
a=8 positive

$$1 < \Delta/\delta\mu < \sqrt{2}$$

Gapless 2SC

a=1,2,3 *massless*
a=4,5,6,7 *negative*
a=8 *negative*

$$\delta\mu > \Delta$$

Huang/Shovkovy, PRD 70 (2004) 051501

Some Suggestions

- Crystalline Superconductivity
  Alford, Bowers & Rajagopal, PRD 63 (2001) 074016

- Phases with Additional Bose Condensates
  Bedaque & Schäfer, NPA 697 (2002) 802

- Homogeneous Gluon Condensate
  Gorbar, Hashimoto & Miransky, PLB 632 (2006) 305

- Inhomogeneous Gluon Condensate with an Induced Magnetic Field
  Ferrer& Incera, PRD 76 (2007) 114012

Chromomagnetic Instabilities & G-B Condensates in 2SC

EJF & de la Incera, PRD 76 (2007) 114012

$$\{G_\mu^{(1)}, G_\mu^{(2)}, G_\mu^{(3)}, K_\mu, K_\mu^\dagger, \widetilde{G}_\mu^8, \widetilde{A}_\mu\}$$

$$K_\mu \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} G_\mu^{(4)} - iG_\mu^{(5)} \\ G_\mu^{(6)} - iG_\mu^{(7)} \end{pmatrix}$$

G_μ^1	G_μ^2	G_μ^3	K_μ	$K_\mu^\dagger$	$\widetilde{G}_\mu^8$
0	0	0	1/2	-1/2	0

$$\widetilde{e} = e\cos\theta$$

Neutrality Conditions

$$\frac{\partial \Omega}{\partial \mu_B} = J_0^{(B)}$$

$$\frac{\partial \Omega}{\partial \mu_e} = 0$$

$$\frac{\partial \Omega}{\partial \mu_8} = 0 \qquad \frac{\partial \Omega}{\partial \mu_3} = 0$$

$$\mu_8 = (\sqrt{3}g/2)\left\langle G_0^{(8)} \right\rangle \qquad \mu_3 = (g/2)\left\langle G_0^{(3)} \right\rangle$$

Stable Phase:

$$\mu_3 = 0, \qquad \mu_8 << \mu_e < \mu, \qquad \mu_8 \sim \Delta / \mu$$

Huang/Shovkovy, PLB 564 (2003) 205

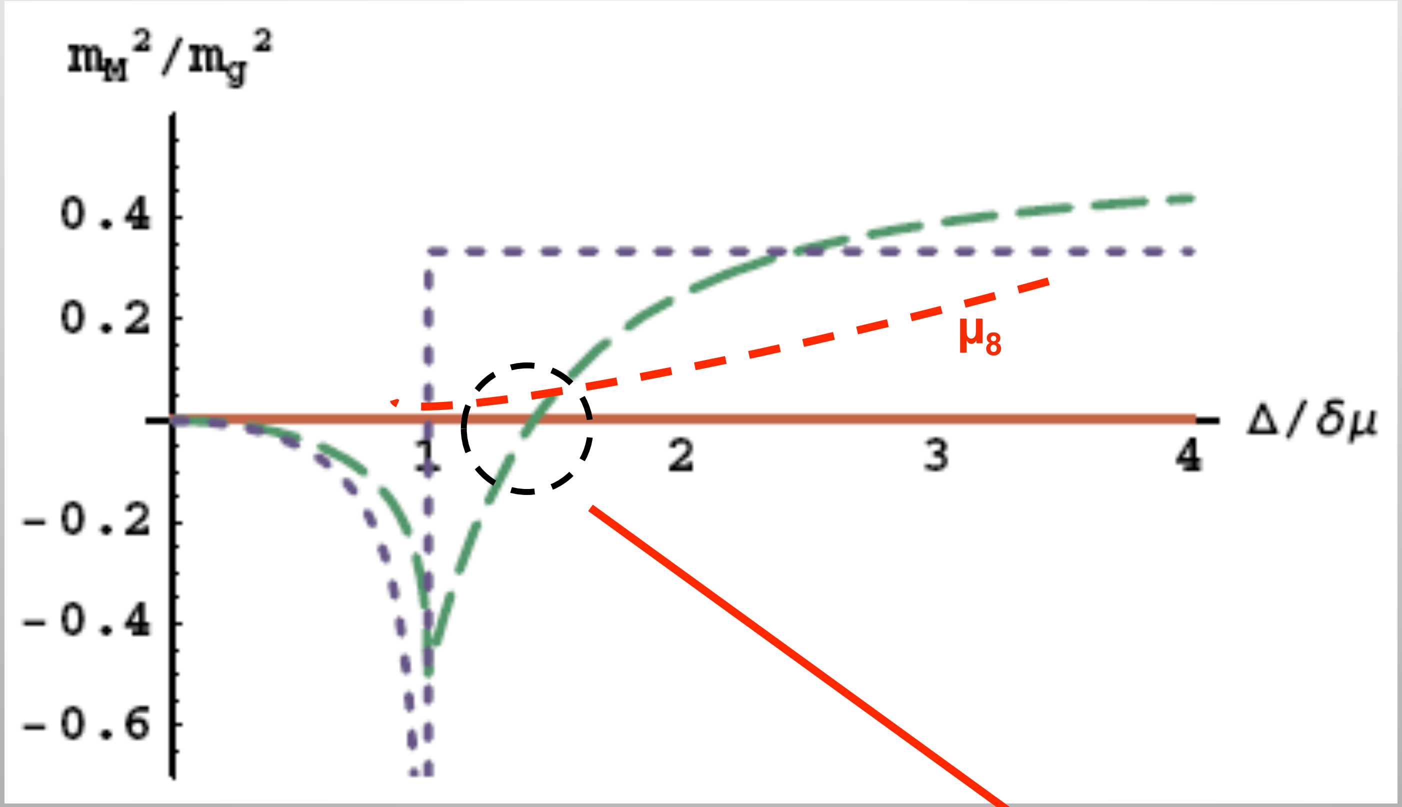
Effective Action

$$\Gamma^g_{eff} = \int d^4x\{-\frac{1}{4}(\widetilde{f}_{\mu\nu})^2 - \frac{1}{2}|\widetilde{\Pi}_\mu K_\nu - \widetilde{\Pi}_\nu K_\mu|^2$$
$$- [m_M^2 \delta_{\mu i}\delta_{\nu i} - (\quad \mu_8 \quad)^2 \delta_{\mu\nu} + i\widetilde{q}\widetilde{f}_{\mu\nu}] K_\mu K_\nu^\dagger$$
$$+ \frac{g^2}{2}[(K_\mu)^2 (K_\nu^\dagger)^2 - (K_\mu K_\mu^\dagger)^2] + \frac{1}{\lambda} K_\mu^\dagger \widetilde{\Pi}_\mu \widetilde{\Pi}_\nu K_\nu\}$$

$$\langle K_\mu \rangle \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \overline{G}_\mu \\ 0 \end{pmatrix} ,$$

$$\overline{G}_\mu \equiv G(x,y)(1,-i,0,0), \qquad \langle \widetilde{f}_{12} \rangle = \widetilde{B}$$

Weakly First-Order Phase transition



$$m_M^2 = \frac{2\alpha_s\overline{\mu}^2}{3\pi}[1 - \frac{2\delta\mu^2}{\Delta^2}],$$

At $\Delta > \delta\mu > \Delta/\sqrt{2}$ → ***Tachyonic Mode of Charged Gluons***

Free-Energy:

$$\mathcal{F}_g = \frac{\widetilde{B}^2}{2} - 2\overline{G}^*\widetilde{\Pi}^2\overline{G} + 2g^2|\overline{G}|^4$$
$$-2[2\widetilde{q}\widetilde{B} + (\mu_8 - \mu_3)^2 - m_M^2]|\overline{G}|^2$$

Minimum Equations:

$$\frac{\partial \mathcal{F}_g}{\partial \mu_3} = 0 \quad \Rightarrow \quad \mu_3 = \mu_8$$

$$\frac{\partial \mathcal{F}_g}{\partial \overline{G}^*} = 0 \quad \Rightarrow$$

$$-\widetilde{\Pi}^2\overline{G} - (2\widetilde{q}\widetilde{B} + |m_M^2|)\overline{G} + 2g^2|\overline{G}|^2\overline{G} = 0$$

Linear Equations

$$\delta\mu \simeq \delta\mu_c$$

$$|\tilde{\Pi}_\mu K_\nu - \tilde{\Pi}_\nu K_\mu|^2 \approx 0$$

$$-\tilde{\Pi}^2\overline{G} - (2\tilde{q}\tilde{B} + |m_M^2|)\overline{G} + 2g^2|\overline{G}|^2\overline{G} = 0$$

$$\tilde{\Pi}^2\overline{G} + \tilde{q}\tilde{B}\overline{G} \simeq 0$$

$$2\tilde{q}|\overline{G}|^2 - \tilde{B} \simeq 0$$

$$\overline{\mathcal{F}}_g \simeq -2(g^2 - \tilde{q}^2)|\overline{G}|^4$$

$$|\overline{G}|^2 \simeq \Lambda_{g/\tilde{q}}|m_M^2|/2\tilde{q}^2 + \mathcal{O}(m_M^4)f(x,y)$$

$$\tilde{q}\tilde{B} \simeq \Lambda_{g/\tilde{q}}|m_M^2| + \mathcal{O}(m_M^4)g(x,y)$$

$$\Lambda_{g/\tilde{e}} = (1 - g^2/\tilde{e}^2)^{-1}$$

Gluon-Condensate Solution

$$\widetilde{\Pi}^2\overline{G} + \widetilde{q}\widetilde{B}\overline{G} \simeq 0$$

$$[\frac{1}{r}\partial_r(r\partial_r) + \frac{1}{r^2}\partial_\theta^2 + \frac{1}{\xi^2}(1 - i\partial_\theta) - \frac{r^2}{4\xi^4}]G(r,\theta) = 0$$

$$\widetilde{A}_i = -(\widetilde{B}/2)\epsilon_{ij}x_j$$

$$\xi^2 \;\equiv\; 1/\Lambda_{g/\widetilde{q}}|m_M^2|$$

$$r \ll \xi$$

$$\frac{r^2}{4\xi^4} \ll \frac{1}{\xi^2}$$

$$G(r,\theta) \sim R(r)e^{i\chi}$$

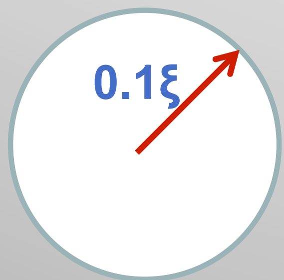
$$[r\partial_r(r\partial_r) + \frac{r^2}{\xi^2}]R(r) = 0$$

$$G(r) = (1/\sqrt{2}\widetilde{q}\xi)J_0(r/\xi)\exp i\chi$$

$$|\overline{G}|^2 \simeq \frac{1}{2\widetilde{q}^2\xi^2} - \frac{r^2}{4\widetilde{q}^2\xi^4}$$

Condensate Free-Energy

Inhomogeneous Condensate: *EJF & de la Incera, PRD 76 (2007) 114012*

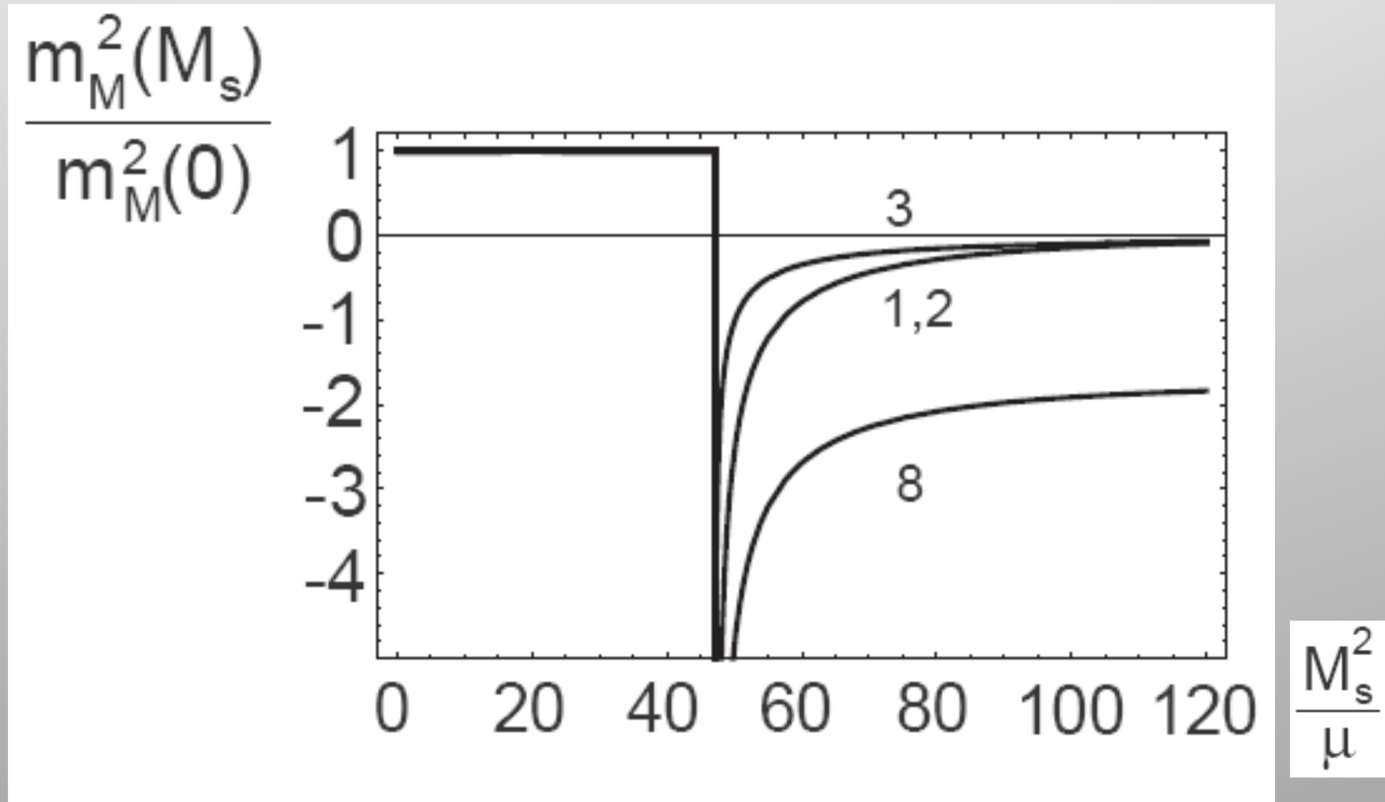


$$\overline{F}_g \approx -\frac{\pi\left|m_M{}^2\right|}{200\tilde{q}^2}$$

Homogeneous Condensate: *Gorbar/Hashimoto/Miransky, PLB 632 (2006) 305*

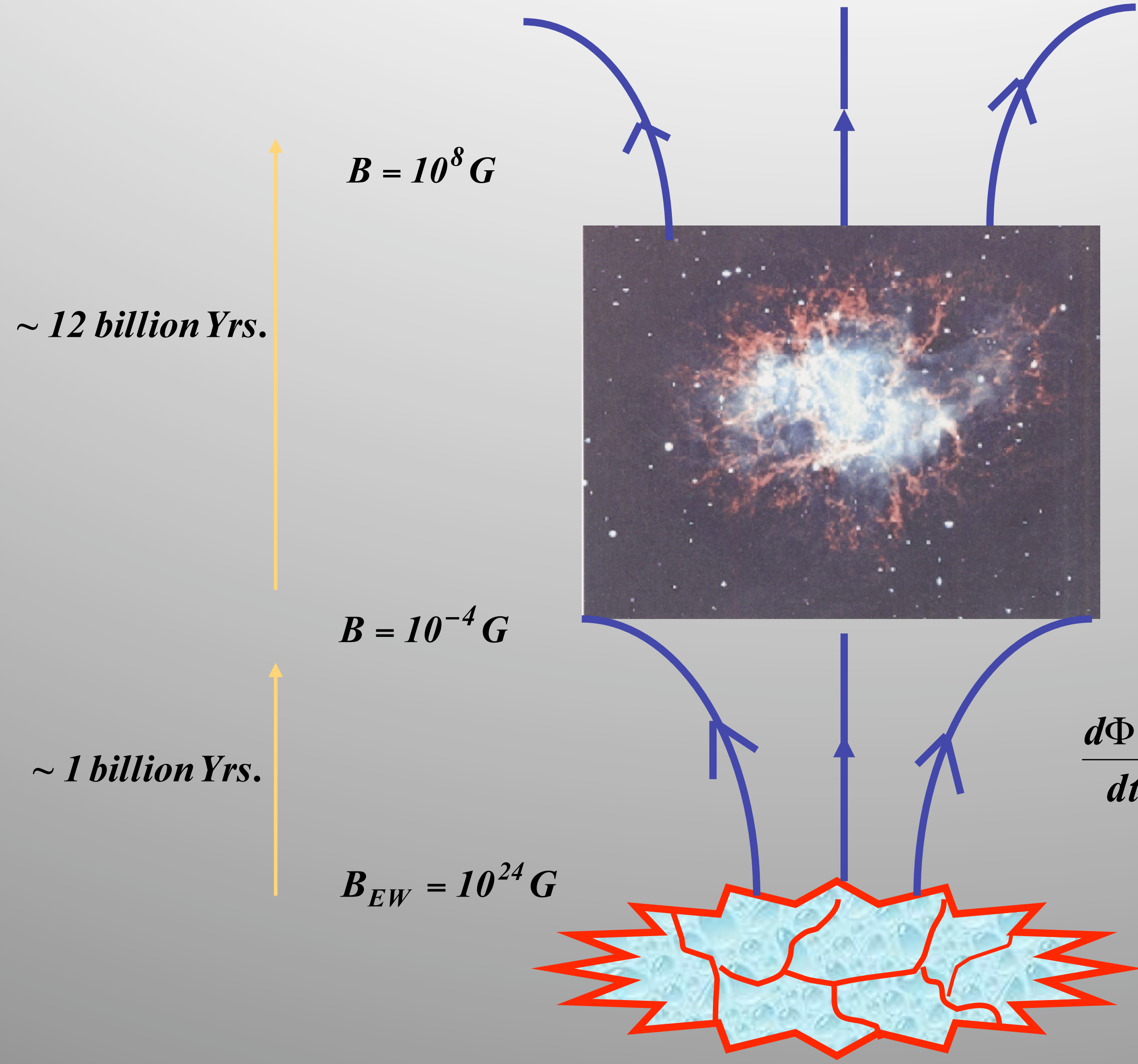
$$\overline{F}_g \approx -\frac{\left(\frac{g^2}{\tilde{q}^2}-1\right)\pi^2\left|m_M{}^2\right|^2}{200\,\alpha_s{}^3 m_g{}^2}$$

Meissner Masses & Chromomagnetic Instabilities in Gapless-Three Flavor Quark Matter

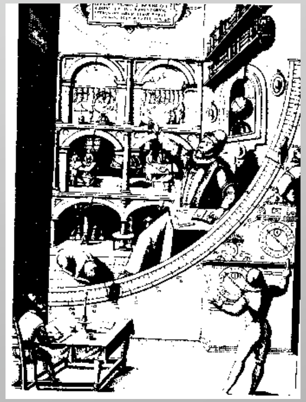


Fukushima, PRD 72 (2005) 074002; Casalbouni et al, PLB 605 (2005) 362; Alford/Wang JPG 31 (2005) 719.

Origin of Stellar Magnetic Fields



$$\frac{d\vec{B}}{dt} = \nabla \times (\vec{\mathbf{v}} \times \vec{\mathbf{B}})$$



$$\frac{d\Phi_S}{dt} = -\frac{1}{4\pi\sigma}\int_S \nabla \times (\nabla \times \vec{B}) \cdot d\vec{S} \approx 0$$

$$B \propto R^{-2} \propto T^2$$

Conclusions and Future Directions

- ***Magnetism in Color Superconductivity is totally different from magnetism in Conventional Superconductivity***
- ***Magnetism is reinforced in Color Superconductivity***

Magnetars ⟷ CS Cores (?)

- ***Numerically solving the nonlinear equation, looking for the realization of the vortex state***
- ***Exploring the possibility to induce a magnetic field in a three-flavor system: vortex state in gCFL***