Neutrino mass matrices

Virendra Gupta

Departamento de Física Aplicada

CINVESTAV-IPN, Unidad Mérida

History

- 1932 neutron $n \to p + e^-$.
- Pauli *little neutron* $n \to p + e^- \bar{\nu}_e$.
- 1956 $\bar{\nu}_e$.
- 1962 ν_e ν_μ.
- 1975 τ ν_τ.

History

Nobel Prize.

- 1935: n.
- 1988: $\nu_e \neq \nu_\mu$.
- 1995: n_e and ν_τ.

ν oscillations

- Paucity of ν_e from the Sun, suggested flavor mixing between the two neutrinos and there were speculation that there were maybe three neutrino flavors.

- We will discuss the 3 flavor mixing matrix suggested by Maki-Nakagawa-Sakata and Pontecorvo (MNSP) which we denote by U.

- Majorana neutrinos key test

$$N(A,Z) \to N(A,Z+2) + e^- + e^- \tag{1}$$

- Many speculations on more ν's.

Lepton-Quark symmetry

- SM: Adler-Bell-Jackiw anomaly cancellation in each generation.

- Are ν and q mixing similar?

The moduli of he CKM matrix V are very accurately known. Its diagonal elements are close to unity and the off-diagonal elements are small implying small mixing angles. In fact, in degrees, $\theta_{12} = 13{,}044$, $\theta_{23} = 2{,}378$ and $\theta_{13} = 0{,}2057$. The CP-violating phase δ is about 60 degrees. The matrix is nearly moduli symmetric with respect to the main diagonal. The magnitude of this asymmetry is about 0.00006 which is of the same order as the magnitude of J, the Jarlskog invariant which determines CP violation in the quark sector.

Neutrino sector

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\, e^{-i\,\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\, e^{i\,\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\, e^{i\,\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\, e^{i\,\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\, e^{i\,\delta} & c_{23}c_{13} \end{pmatrix} \times \mathrm{diag}\begin{bmatrix} e^{i\,\alpha_1/2} \\ e^{i\,\alpha_2/2} \\ 1 \end{bmatrix}$$

($c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$). The data here is not so accurate and is patchy since neutrinos are hard to detect.

Neutrino data

- Summary of the present data on the angles is

$$\sin^2\theta_{12} = 0{,}30 \pm 0{,}02,\ \ \sin^2\theta_{23} = 0{,}50^{+0{,}07}_{-0{,}06},\ \ \sin^2\theta_{13} < 0{,}040.$$

As one can see the mixing angles here are quite different from the quark case, In degrees, $\theta_{12} = 33{,}2$, $\theta_{23} = 45{,}0$, and $\theta_{13} < 11{,}5$.

- The value of the CP-violating phase δ is unconstrained because the CP-violating term present in neutrino oscillations has not been accessible experimentally.

Neutrino data

The oscillation data can only determine $\Delta m_{ij}^2 = m_i^2 - m_j^2$, where $ij = 1, 2, 3$. The present data gives

$$\Delta m_{21}^2 = (7{,}65^{+0{,}23}_{-0{,}20}) \times 10^{-5}\,\mathrm{eV}^2, \quad |\Delta m_{31}^2| = (2{,}40^{+0{,}12}_{-0{,}11}) \times 10^{-3}\,\mathrm{eV}^2.$$

Neutrino mass matrices

What kind of mass matrix can explain the above sparse data in the basis in which the charged leptons mass matrix is diagonal? There is extensive earlier work. It is clear that a 3 texture matrix does not work. In this note, we confront the data with a CGS-type matrix which was used for the CKM matrix. In the quark case, it gave a better fit to the data than the 3 texture Fritzsch type matrix, with $M_{11} = M_{22} = M_{13} = 0$, and the 2 texture Gupta-Rajpoot type matrix, with $M_{11} = M_{13} = 0$. The two texture CGS-type matrix has $M_{11} = M_{22} = 0$ and has the virtue that it gives CP-violation in all the 3 bases(physical or one mass matrix diagonal). In contrast, the F-type and GR-type matrices can give CP-violation only in the physical basis because in their case $\mathrm{Im}(M_{12}M_{23}M^*_{13}) = 0$.

Basic formulas

The 3×3 hermitian mass matrix $M_{\rm q}$ is diagonalized by $V_{\rm q}$ so that $M_{\rm q} = V_{\rm q}^{\dagger} \hat{M}_{\rm q} V_{\rm q}$, q=u,d. The eigenvalues are denoted by ($\lambda_{\rm u}$,$\lambda_{\rm c}$,$\lambda_{\rm t}$) and ($\lambda_{\rm d}$,$\lambda_{\rm s}$,$\lambda_{\rm b}$) for the up and down mass matrices, respectively. Note that the eigenvalues are real but not necessarily positive. Each mass matrix can be expressed in terms of its projectors.

$$M_{\rm u} = \sum_{\alpha={\rm u,c,t}} \lambda_\alpha N_\alpha^{\rm u} \qquad \text{and} \qquad M_{\rm d} = \sum_{j={\rm d,s,b}} \lambda_j N_j^{\rm d}. \qquad (2)$$

Basic formulas

Since $V = V_{\rm u} V_{\rm d}^{\dagger}$, it follows that

$$|V_{\alpha\, j}|^2 = \mathrm{Tr}[N_{\alpha}^{\rm u} N_{j}^{\rm d}], \tag{3}$$

$$N_{\alpha}^{\rm u} = \frac{(\lambda_\beta - M_{\rm u})(\lambda_\gamma - M_{\rm u})}{(\lambda_\beta - \lambda_\alpha)(\lambda_\gamma - \lambda_\alpha)} \tag{4}$$

$$N_{j}^{\rm d} = \frac{(\lambda_k - M_{\rm d})(\lambda_l - M_{\rm d})}{(\lambda_k - \lambda_j)(\lambda_l - \lambda_j)}, \tag{5}$$

with (α,β,γ) and (j,k,l) any permutation of (u,c,t) and (d,s,b), respectively.

CGS type matrices

We now consider a basis in which the down-lepton (charged leptons) mass matrix is diagonal and the up-lepton (neutrinos) mass matrix is hermitian of the CGS-type, namely,

$$M = \begin{pmatrix} 0 & a & d \\ a^* & 0 & b \\ d^* & b^* & c \end{pmatrix}$$

For $d = 0$ this reduces to the Fritzsch-type mass matrix. In the leptonic case, working in the charged-lepton (down) mass matrix diagonal basis, the neutrino (up) mass matrix $M = M_{\rm u}$ is diagonalized by $V_{\rm u}$. So the MNSP-matrix $U = V_{\rm u}$ since $V_{\rm d} = I$.

Fit to data

• On the experimental side, along with the experimental values of Δm^2_{21} and $|\Delta m^2_{31}|$, assume $\theta_{13} = 0$, so that we can get the "experimental"values of $|U_{e\,1}|^2$ and $|U_{\mu\,1}|^2$.
• On the theoretical side, taking a, b, c, and d to be real. From the characteristic equations we can solve for a, b, and c as functions of the three eigenvalues λ_1, λ_2, λ_3 and d. With this information and the theoretical expressions for Δm^2_{21} and $|\Delta m^2_{31}|$ and for $|U_{e\,1}|^2$ and $|U_{\mu\,1}|^2$.
• A possible solution in the normal hierarchy ($0 < m_1 < m_2 < m_3$) is given by (in eV).

$$m_1 = 0.00809 \;, \quad m_2 = 0.01191 \;, \quad m_3 = 0.04965.$$

Fit to data

In this particular solution the relative phases between the eigenvalues (not necesarily positive) and the physical masses is given by $\lambda_1 = -m_1$, $\lambda_2 = -m_2$, $\lambda_3 = m_3$. The corresponding CGS-mass matrix elements are (in eV),

$$a = -0.00829, \qquad b = -0.02144, \qquad c = 0.02965.$$

$$d = 0.01919 \; .$$

Conclusion

- It is interesting that the CGS type matrix can fit both quark and neutrino mixing data.