Noncommutative Gravity from Gauge Theory

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Motivation

space-time deformation: $[x^\mu, x^\nu] = \theta^{\mu\nu}$

This deformation induce a deformation of the algebra of fields

$$\phi\psi \rightarrow \phi \star \psi$$

where

$$(\Phi \star \Psi)(x) = \Phi(x) e^{\frac{i}{2}\theta^{\mu\nu}\overleftarrow{\partial_\mu}\overrightarrow{\partial_\nu}} \Psi(x)$$

so, for instance

$$S^\theta = \int d^4x \Big[\frac{1}{2}\partial^\mu\Phi \star \partial_\mu\Phi - \frac{1}{2}m^2\Phi \star \Phi - \frac{\lambda}{4!}\Phi \star \Phi \star \Phi \star \Phi \star \Phi\Big]$$

Context of Noncommutative deformations of gravity

- Use Moyal quantization and replace the usual Algebra of functions with a star Algebra
- Deform de co-product of the bi-Algebra by an abelian twist. Recover tensor calculus. Wess, et. al., Chaichian, Alvarez-Gaume.....
- Gauging the noncommutative version of SO(4,1). Use SW map and contract to ISO(3,1) Chamseddine, Calmet, Kobakhidze....
- Restrict diffeomorphism to a subgroup. Unimodular Gravity. Symplectomorphism. Volume preserving transformations Calmet, Kobakhidze....

Star products are NOT invariant under difeomorphism transformations

Context of Noncommutative deformations of gravity

- Teleparallel version of Gravity

Sazbo, Landsmann...

- Dimensional reduction of noncommutative YM theory
- Gauging the twist-deformed Poincaré symmetry

Wess, Chaichian, Calmet......

- Position dependent noncommutative parameter

Harikumar, Rivelles, Aschieri ......

- Extract General Relativity from the dynamics of gauge fields

Steinacker, Grosse,..

- Matrix Models, emergent Gravity

Central Idea

Two deformations:

A) Consistent deformation of a Gauge Theory κ

B) Noncommutative deformation θ

Gauge Theory → A def → DGT → Gravity

Gauge Theory → B def → NCFT → A def → NCG

NCFT are trivial A def (SW map)

Promote the global translation symmetry to a local symmetry (teleparallel gravity) Cho, Hehl...

New deformation of Gravity

- Consistent deformation of YM theory: non polynomial in the gauge fields F. Brandt

  merging properties that comes from the original gauge symmetries and space-time symmetries

  related with the idea of emergent gravity

  general relativity itself will emerge from our deformation as a gauge theory of space-time translations: teleparallel gravity

  deformation constructed using a killing vector of the flat space background metric

- NC deformation enters here through the Seiberg-Witten map of the standard Yang-Mills action.

New deformation of Gravity

- The deformed action can be cast in the form of a Yang-Mills action in a curved space-time whose metric depends in a given way on the gauge fields and the killing vectors of the background metric.

- Minimal construction: gauge the translation symmetry of the background

- Constructed from a stardard noncommutative deformation of a Gauge Theory

- It is not clear that the resulting action can be reorganized in the form of a Moyal star product deformation

Toy Model: Maxwell Case

FIRST DEFORMATION

Action: $$S^{(0)} = -\frac{1}{4}\int d^4x F_{\mu\nu}F^{\mu\nu},$$

$$\delta_\lambda^{(0)} A_\mu = \partial\lambda, \qquad \delta A_\mu = \xi^\nu F_{\nu\mu},$$

Local   Global

Killing eq. : $$\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = \frac{1}{2}\eta_{\mu\nu}\partial_\rho \xi^\rho,$$

Start deformation from:

$$S^{(1)} = \int d^4x A_\mu \xi^\nu \left(-\frac{1}{4}\delta_\nu^\mu F_{\rho\sigma}F^{\rho\sigma} + F_{\nu\rho}F^{\mu\rho}\right).$$

Toy Model: Maxwell Case

FIRST DEFORMATION

Consistent deformation condition:

$$\delta_\lambda^{(0)} S^{(1)} + \delta_\lambda^{(1)} S^{(0)} = 0$$

Solution: $\delta_\lambda^{(1)} A_\mu = \lambda \xi^\nu F_{\nu\mu},$

Deformation can be constructed to any order in the deformation parameter!!

$$L = -\frac{1}{4}(1 + \xi^\rho A_\rho)\hat{F}_{\mu\nu}\hat{F}^{\mu\nu}, \qquad \delta_\lambda A_\mu = \partial_\mu \lambda + \lambda \xi^\nu \hat{F}_{\nu\mu},$$

where: $\hat{F}_{\mu\nu} = E_\mu^\rho E_\nu^\sigma F_{\rho\sigma},$ $\quad E_\mu^\rho = \delta_\mu^\rho - \dfrac{\xi^\rho A_\mu}{(1 + \xi \cdot A)}.$

Toy Model: Maxwell Case

FIRST DEFORMATION

"Geometric version":

$$L = -\frac{1}{4}\sqrt{-g}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}$$

where: $g_{\mu\nu} = \eta_{\alpha\beta}e^{\alpha}_{\mu}e^{\beta}_{\nu}, \quad e^{\alpha}_{\mu} = \delta^{\alpha}_{\mu} + \xi^{\alpha}A_{\mu},$

Symmetries: $(\varepsilon^{\mu} = \omega\xi^{\mu})$

$$\delta_{\omega}A_{\mu} = \partial_{\mu}\omega + \mathcal{L}_{\varepsilon}A_{\mu} = \partial_{\mu}\omega + \varepsilon^{\nu}\partial_{\nu}A_{\mu} + A_{\nu}\partial_{\mu}\varepsilon^{\nu},$$

$$\Rightarrow \quad \delta_{\omega}g_{\mu\nu} = \mathcal{L}_{\varepsilon}g_{\mu\nu} = \varepsilon^{\rho}\partial_{\rho}g_{\mu\nu} + g_{\rho\nu}\partial_{\mu}\varepsilon^{\rho} + g_{\mu\rho}\partial_{\nu}\varepsilon^{\rho} \quad !!!!$$

$$\delta_{\omega}e^{\alpha}_{\mu} = \varepsilon^{\rho}\partial_{\rho}e^{\alpha}_{\mu} + (\partial_{\mu}\varepsilon^{\rho})e^{\alpha}_{\rho} + \frac{1}{2}\omega\Lambda^{\alpha}{}_{\rho}e^{\rho}_{\mu}$$

Deformation of YM

$$S_{YM} = \int d^4 x F^A_{\mu\nu} F^{\mu\nu}_A.$$

$$\delta A^A_\mu = D_\mu \omega^A + \varepsilon^\nu \partial_\nu A^A_\mu + A^A_\nu \partial_\mu \varepsilon^\nu = D_\mu \omega^A + \mathcal{L}_\varepsilon A^A_\mu,$$

$$\varepsilon^\mu = \omega^A \xi^\mu_A$$

$$\boxed{L = -\frac{1}{4}\sqrt{-g} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu}{}^A F_{\rho\sigma\,A} = -\frac{1}{4}(1 + \xi^\rho_A A^A_\rho) \hat{F}^A_{\mu\nu} \hat{F}^{\mu\nu}_A,}$$

$$\mathcal{A}_\mu{}^\nu = \xi^\nu_A A^A_\mu \qquad \hat{F}^A_{\mu\nu} = E^\rho_\mu E^\sigma_\nu F^A_{\rho\sigma} \qquad e_\mu{}^\nu = \delta_\mu{}^\nu + \mathcal{A}_\mu{}^\nu$$

$$g_{\mu\nu} = \eta_{\alpha\beta} e^\alpha_\mu e^\beta_\nu = \eta_{\mu\nu} + \mathcal{A}_{\mu\nu} + \mathcal{A}_{\nu\mu} + \mathcal{A}_{\mu\rho}\mathcal{A}_\nu{}^\rho,$$

$$\delta g_{\mu\nu} = \mathcal{L}_\varepsilon g_{\mu\nu} \;\; !!!!$$

Relation with standard Gravity

Consider only the translation subgroup T4

Define:

$$\mathcal{F}_{\mu\nu}{}^{\rho} = \xi^{\rho}_{A} F_{\mu\nu}{}^{A} = \partial_{\mu}\mathcal{A}^{\rho}_{\nu} - \partial_{\nu}\mathcal{A}^{\rho}_{\mu} = \partial_{\mu}e^{\rho}_{\nu} - \partial_{\nu}e^{\rho}_{\mu}.$$

Relate them with the Ricci Rotation coefficients

$$T_{\mu\nu}{}^{\rho} \sim \Omega_{\mu\nu}{}^{\rho} \qquad\qquad \Omega_{\sigma\kappa}{}^{\rho} = \hat{\mathcal{F}}_{\sigma\kappa}{}^{\rho} = E^{\mu}_{\sigma}E^{\nu}_{\kappa}\mathcal{F}_{\mu\nu}{}^{\rho}.$$

$$L = -\frac{1}{4}\underbrace{\left(1+\xi^{\rho}_{A}A^{A}_{\rho}\right)}_{e}\underbrace{\hat{F}^{A}_{\mu\nu}\hat{F}^{\mu\nu}_{A}}_{\Omega_{\mu\nu}{}^{\rho}\Omega^{\mu\nu}{}_{\rho},}, \quad \Longrightarrow \quad L = -\frac{1}{4}e\,\Omega_{\mu\nu}{}^{\rho}\Omega^{\mu\nu}{}_{\rho},$$

Relation with standard Gravity

Invariants:

$$I_1 = \Omega_{\mu\nu\rho}\Omega^{\mu\nu\rho}, \quad I_2 = \Omega_{\mu\nu\rho}\Omega^{\rho\mu\nu}, \quad I_3 = \Omega_{\mu\rho}{}^{\rho}\Omega^{\mu}{}_{\sigma}{}^{\sigma}.$$

Pellegrini-Plebanski Lagrangian

$$L = ec^i I_i$$

Recover full Einstein Gravity for:

$$c_1 = 1, c_2 = 2, c_3 = -4$$

$$\Rightarrow \quad \boxed{L = e(\Omega_{\mu\nu\rho}\Omega^{\mu\nu\rho} + 2\Omega_{\mu\nu\rho}\Omega^{\rho\mu\nu} - 4\Omega_{\mu\rho}{}^{\rho}\Omega^{\mu}{}_{\sigma}{}^{\sigma})}$$

NC deformed gauge theory

NC-YM Lagrangian

$$L = -\frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma}F_{\mu\nu}{}^{A} \star F_{\rho\sigma}{}_{A}$$

$$F^{A}_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu - [A_\mu, A_\nu]^A_\star$$

Seiberg-Witten map:

Lie Algebra

$$F^C_{\mu\nu} \to F^C_{\mu\nu} + \frac{1}{2}\theta^{\alpha\beta}d^{ABC}\left(F^A_{\mu\alpha}F^B_{\nu\beta} - A^A_\alpha\partial_\beta F^B_{\mu\nu} + \frac{1}{2}f^{BDE}A^A_\alpha A^E_\beta F^D_{\mu\nu}\right)$$

Enveloping Algebra

Seiberg-Witten Lagrangian up to first order:

$$L = -\frac{1}{4}\left(\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + \theta^{\alpha\beta}d_{ABC}F^{A\mu\nu}(\frac{1}{4}F^B_{\beta\alpha}F^C_{\mu\nu} + F^B_{\mu\alpha}F^C_{\nu\beta})\right)$$

NC deformed gauge theory

NC action ansatz:

$$L = -\frac{1}{4}(1 + \xi^\rho_A A^A_\rho)\left(\mathrm{Tr}(\hat{F}_{\mu\nu}\hat{F}^{\mu\nu}) + \theta^{\alpha\beta} d_{ABC}\hat{F}^{A\mu\nu}(\frac{1}{4}\hat{F}^B_{\beta\alpha}\hat{F}^C_{\mu\nu} + \hat{F}^B_{\mu\alpha}\hat{F}^C_{\nu\beta})\right),$$

Symmetries: $\delta_w A^A_\mu = D_\mu \omega^A + \omega^B \xi^\nu_B \hat{F}^A_{\nu\mu}.$

$$\delta A^A_\mu = D_\mu \omega^A + \varepsilon^\nu \partial_\nu A^A_\mu + A^A_\nu \partial_\mu \varepsilon^\nu = D_\mu \omega^A + \mathcal{L}_\varepsilon A^A_\mu$$

where $\varepsilon^\mu = \omega^A \xi^\mu_A$

condition: $\omega^A \mathcal{L}_{\xi^A} \theta^{\mu\nu} = 0$

Rigid frames $\delta e_\mu{}^\nu = \varepsilon^\rho \partial_\rho e_\mu{}^\nu + e_\rho{}^\nu \partial_\mu \varepsilon^\rho$

NC Gravity from NC deformed Gauge Theory

NC action ansatz:

$$L=-\frac{1}{4}(1+\xi^{\rho}_{A}A^{A}_{\rho})\left(\ \mathrm{Tr}(\hat{F}_{\mu\nu}\hat{F}^{\mu\nu})+\theta^{\alpha\beta}d_{ABC}\hat{F}^{A\mu\nu}(\frac{1}{4}\hat{F}^{B}_{\beta\alpha}\hat{F}^{C}_{\mu\nu}+\hat{F}^{B}_{\mu\alpha}\hat{F}^{C}_{\nu\beta})\right),$$

Dictionary:

$$\mathcal{F}_{\mu\nu}{}^{\rho}=\xi^{\rho}_{A}F_{\mu\nu}{}^{A}=\partial_{\mu}\mathcal{A}^{\rho}_{\nu}-\partial_{\nu}\mathcal{A}^{\rho}_{\mu}=\partial_{\mu}e^{\rho}_{\nu}-\partial_{\nu}e^{\rho}_{\mu}$$

$$\Omega_{\sigma\kappa}{}^{\rho}=\hat{\mathcal{F}}_{\sigma\kappa}{}^{\rho}=E^{\mu}_{\sigma}E^{\nu}_{\kappa}\mathcal{F}_{\mu\nu}{}^{\rho}$$

Lagrangian (invariant I_1)

$$L_1=e\left(\Omega_{\mu\nu}{}^{\rho}\Omega^{\mu\nu}{}_{\rho}+\theta^{\alpha\beta}d_{\rho\sigma\delta}\Omega^{\rho\mu\nu}(\frac{1}{4}\Omega^{\sigma}_{\beta\alpha}\Omega^{\delta}_{\mu\nu}+\Omega^{\sigma}_{\mu\alpha}\Omega^{\delta}_{\nu\beta})\right),$$

NC Gravity from NC deformed Gauge Theory

To construct the other invariants:

$$\Omega_{\mu\nu}{}^{\rho} \rightarrow \Omega_{\mu\nu}{}^{\rho} + (\Omega_{NC})_{\mu\nu}{}^{\rho}$$

$$(\Omega_{NC})_{\mu\nu}{}^{\rho} = \theta^{\alpha\beta} d_{\sigma\delta}{}^{\rho} (\frac{1}{4}\Omega^{\sigma}_{\beta\alpha}\Omega^{\delta}_{\mu\nu} + \Omega^{\sigma}_{\mu\alpha}\Omega^{\delta}_{\nu\beta})$$

Then: $L_2 = e\,(\Omega_{\mu\nu\rho}\Omega^{\rho\mu\nu} + \Omega^{\rho\mu\nu}(\Omega_{NC})_{\mu\nu\rho} + \Omega^{\mu\nu\rho}(\Omega_{NC})_{\rho\mu\nu})\,.$

and the trace: $L_3 = e\,(\Omega_{\mu}\Omega^{\mu} + 2\Omega^{\mu}(\Omega_{NC})_{\mu})\,.$

$$L_{NCG} = L +$$

$$2e\,(\Omega^{\rho\mu\nu}(\Omega_{NC})_{\rho\mu\nu} + \Omega^{\rho\mu\nu}(\Omega_{NC})_{\mu\nu\rho} + \Omega^{\mu\nu\rho}(\Omega_{NC})_{\rho\mu\nu} - 4\Omega^{\mu}(\Omega_{NC})_{\mu})\,.$$

CONCLUSIONS

- NC Gravity from a Gauge theory
- NC come from standard SW map
- Deformed NC: formal series in $\theta^{\mu\nu}$
- The result could not be recast as the know star product

Future work:

- Linearized theory, FP Lagrangian
- Phenomenological implications???
- Are the vertices of this NCG the same that come from String Theory???