A theory of Gravity from spontaneous Lorentz violation

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Work in collaboration with V. Alan Kostelecký



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Outline

Symmetry vs. Broken Symmetry

Bootstrap









Masslessness from symmetry or broken symmetry

Gauge Symmetries

Generator of unbroken gauge symmetry \Rightarrow massless vector boson



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General Relativity

Diffeomorphism invariance \Rightarrow massless gravitons



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General RelativityDiffeomorphism invariance \Rightarrow massless gravitons

Spontaneously Broken Global Symmetry

Spontaneously broken global symmetry \Rightarrow massless Nambu-Goldstone boson



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$$L=-rac{1}{4}(\partial_\mu B_
u-\partial_
u B_\mu)^2+V(B_\mu B^\mu\pm b^2)$$

photons as Nambu-Goldstone modes

$$L = -\frac{1}{4}(\partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu})^2 + V(B_{\mu}B^{\mu} \pm b^2)$$

• potential V breaks U(1) gauge invariance

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- non-linear dynamics of massive modes at high energies/temperatures
- model can be coupled consistently to gravity Bluhm, Kostelecky (2004)

gravitons as Nambu-Goldstone modes

$$L = rac{1}{2} C^{\mu
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• $K_{\mu\nu\alpha\beta}$: ghost-free quadratic kinetic operator for spin 2

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- fluctuations around vev: $C^{\mu
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At low energy, assume V can be approximated by sum of delta-functions that fix the 4 independent scalars: $V = \sum_{n=1}^{4} \frac{\lambda_n}{n} X_n$



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equations of motion:

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Low-energy dynamics of $\tilde{C}_{\mu\nu}$ -fluctuations around vev equal to linearized general relativity (in axial-type gauge)!



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Counting degrees of freedom

Propagating massless degrees of freedom

• Can be considered Nambu-Goldstone modes of spontanously broken Lorentz generators $\mathcal{E}_{\mu}{}^{\alpha}$:

$$\tilde{C}_{\mu\nu} = \mathcal{E}_{\mu}{}^{\alpha} c_{\alpha\nu} + \mathcal{E}_{\nu}{}^{\alpha} c_{\mu\alpha}$$



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- Equations of motion imply masslessness $\partial^2 \tilde{C}_{\mu\nu} = 0$ and Lorenz conditions $\partial^\mu \tilde{C}_{\mu\nu} = 0$
- Number of propagating massless degrees of freedom: 6 4 = 2



Write metric as Minkowski + fluctuations (gravitons):

 $g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$



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Quadratic action for (free) gravitons: $\mathcal{L}_{GR}^{L} = \frac{1}{2} h^{\mu\nu} K_{\mu\nu\alpha\beta} h^{\alpha\beta}$



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Linear coupling to matter EM

 ${\cal L} \supset h^{\mu
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- $\tau_{\mu\nu}$: trace-inversed energy-momentum tensor
- linear coupling to EM-tensor gives rise to linearized Einstein equation

$$K_{\mu\nu\alpha\beta}h^{\alpha\beta}\equiv R^{L}_{\mu\nu}=\tau_{\mu\nu}$$

consistent coupling

consistent coupling to total EM tensor



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After resumming all terms one obtains Einstein-Hilbert action!

Cardinal bootstrap (V.A. Kostelecky and R.P., Phys. Rev. D (2009))

Deser's procedure

 bootstrap can be done in one step using procedure developed by Deser for GR



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- bootstrap can be done in one step using procedure developed by Deser for GR
- use trace-reverted field: $\mathfrak{L}^{\mu\nu} = -C^{\mu\nu} + \frac{1}{2}\eta^{\mu\nu}C^{\alpha}_{\alpha}$



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- can rewrite original linearized cardinal dynamics using Palatini formalism with auxiliary field $\Gamma^{\alpha}_{\mu\nu}$:

$$\mathcal{L}^{L} = \mathfrak{L}^{\mu\nu} (\Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma_{\mu,\nu}) + \eta^{\mu\nu} (\Gamma^{\alpha}_{\mu\nu} \Gamma_{\alpha} - \Gamma^{\alpha}_{\beta\mu} \Gamma^{\beta}_{\alpha\nu})$$



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1 step coupling

 \bullet Have to include coupling to energy-momentum tensor of \mathcal{L}^L in self-consistent manner

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1 step coupling

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- Use Rosenfeld method, promoting $\eta^{\mu\nu}$ to variable metric density and partial derivatives to $\eta^{\mu\nu}\text{-}\mathrm{covariant}$ ones.

Cardinal bootstrap

It follows:
$$-\frac{1}{2}\tau_{\mu\nu} = \frac{\delta \mathcal{L}^{L}}{\delta \eta^{\mu\nu}} = \Gamma^{\alpha}_{\mu\nu}\Gamma_{\alpha} - \Gamma^{\alpha}_{\beta\mu}\Gamma^{\beta}_{\alpha\nu} + \text{total derivative}$$



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Full nonlinear action obtained by coupling nonderivative part of $\tau_{\mu\nu}$ as source for $\mathfrak{L}^{\mu\nu}$:

$$\mathcal{L} = \mathcal{L}^{L} + \mathfrak{L}^{\mu\nu} (\Gamma^{\alpha}_{\mu\nu} \Gamma_{\alpha} - \Gamma^{\alpha}_{\beta\mu} \Gamma^{\beta}_{\alpha\nu})$$



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final result for kinetic term

recursive process yields nonlinear "bootstrapped" action

$$\int d^4 x \left(\mathfrak{L}^{\mu\nu} (\Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma_{\mu,\nu}) + (\eta + \mathfrak{L})^{\mu\nu} (\Gamma^{\alpha}_{\mu\nu} \Gamma_{\alpha} - \Gamma^{\alpha}_{\beta\mu} \Gamma^{\beta}_{\alpha\nu}) \right)$$
$$\equiv \int d^4 x (\eta + \mathfrak{L})^{\mu\nu} R_{\mu\nu} (\Gamma)$$

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final result for kinetic term

recursive process yields nonlinear "bootstrapped" action

$$\int d^4 x \left(\mathfrak{L}^{\mu\nu} (\Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma_{\mu,\nu}) + (\eta + \mathfrak{L})^{\mu\nu} (\Gamma^{\alpha}_{\mu\nu} \Gamma_{\alpha} - \Gamma^{\alpha}_{\beta\mu} \Gamma^{\beta}_{\alpha\nu}) \right)$$
$$\equiv \int d^4 x (\eta + \mathfrak{L})^{\mu\nu} R_{\mu\nu} (\Gamma)$$

Thus $(\eta + \mathfrak{L})^{\mu\nu}$ is naturally interpreted as curved-space metric density!

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Bootstrap of matter tensor and scalar potential

Bootstrap can also be applied to scalar potential and matter EM tensor

• flat-space matter EM-tensor yields curved-space matter lagrangian with metric density $(\eta + \mathfrak{C})^{\mu\nu}$: $\mathcal{L}_{M,\mathfrak{C}} = \sqrt{|\eta + \mathfrak{C}|} \mathcal{L}_{M,\mathfrak{C}}^{L}|_{\eta \to \eta + \mathfrak{C}}$



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Particular linear combination yields cosmological constant $\sqrt{|\eta + \mathfrak{L}|}$
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Bootstrap of scalar potential

integrable scalar potentials

Particularly interesting: Scalar potentials of the form

$$V(\{\mathfrak{X}_i\}) = \frac{1}{2} \sum_{i,j} m_{ij} (\mathfrak{X}_i - \mathfrak{x}_i) (\mathfrak{X}_j - \mathfrak{x}_j) + \mathcal{O}(\mathfrak{X}_i - \mathfrak{x}_i)^3$$

with local minimum at $\mathfrak{X}_i = \mathfrak{x}_i \ (i = 1 \dots 4)$

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- Integrability and stability highly nontrivial conditions
- Limit $m_{ij}
 ightarrow \infty$ corresponds to bootstrap of linearized limit

$$V^L = \lambda_1(\mathfrak{X}_1 - \mathfrak{x}_1) + \lambda_2(\mathfrak{X}_2 - \frac{\mathfrak{X}_1^2}{2} - \mathfrak{x}_2 + \frac{\mathfrak{x}_1^2}{2}) + \dots$$

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Bootstrapped Lagrangian

 $(\eta + \mathfrak{L})^{\mu\nu} R_{\mu\nu}(\Gamma) - \sqrt{-|\eta + \mathfrak{L}|} V(\mathfrak{X}_1, \mathfrak{X}_2, \mathfrak{X}_3, \mathfrak{X}_4) + L_{matter}(\mathfrak{L}, \eta, \phi_i, \partial_\mu \phi_i)$



R. Potting (Algarve)

A theory of Gravity from spontaneous Lo

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Linearized equations of motion

$$K_{\mu\nu\alpha\beta}h^{\alpha\beta} = (\eta_{\mu\nu}\partial_1 + 2\eta_{\mu\alpha}c^{\alpha\beta}\eta_{\beta\nu}\partial_2 + ...)V + \tau^{(m)}_{\mu\nu}(\eta,\phi_i,\partial_\mu\phi_i)$$
$$\partial_n \equiv \frac{\partial}{\partial X_n} \qquad X_n = (C \cdot \eta)^n \qquad (n = 1...4)$$



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"vacuum energy-momentum tensor"

$$T^{(\mathsf{vac})}_{\mu
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Choosing $T_{\mu\nu}^{(vac)}$ to be zero at suitable initial timelike/spacelike spacetime slices ensures it is zero at all spacetime

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Quantum effective action

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- In general the 4 auxiliary modes can become propagating
- Yields Lorentz-violating corrections to Lagrangian (Carroll et.al. (2009))


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Fixed points of Renormalization Group

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- Similar analysis for cardinal model very interesting but challenging

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• Construction of alternative theory of gravity possible



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- Quantum effective action can turn auxiliary modes propagating at low energy



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• Classification of all integrable and bootstrapped potentials



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- Extension of cardinal model: extra massless modes (ex. combination with bumblebee)?

