Radiative corrections in QED

in a Lorentz violating background.

Mauro Cambiaso

mauro.cambiaso@nucleares.unam.mx

ICN - UNAM

PLB**639**(2006), 586 & arXiv:0904.3557 [hep-th], in collab. with:

J. Alfaro, A. Andrianov, P. Giacconi and R. Soldati.

Outline

- Motivation & Intro:
- Fermions coupled to constant backg axial-vector b^{μ}
  - Radiatively induced corrections from ψ to γ sector
  - Different approaches in the field
  - Ambiguities or lack of knowledge of QFT in LIV scenario?
- Effective theory scale - UV physical cutoff
- Vacuum polarisation in axial-vector background, CPT-even
  - $\overline{DR}$-regularisation
  - Physical UV cutoff regularisation
  - Finite-temperature
- Conclusions and outlook
  - Role of physical UV cutoff
  - $\overline{DR}$ and anomalous induced photon (Proca) mass
  - Subtleties

Motivation & Intro:

- For example: Classical electrodynamics extended by a Chern-Simons like term ($\mathcal{L}_{CS} = -\zeta\frac{1}{2}\eta_\mu A_\nu \widetilde{F}^{\mu\nu}$) would imply Lorentz Invariance Violation (LIV) if gauge symmetry is required.
  - Under the gauge transformation $\delta A^\mu = \partial^\mu\phi$ the Lagrangian transforms as: (up to a divergence)

$$\delta\mathcal{L}_{CS} = \zeta\frac{1}{4}\phi\widetilde{F}^{\nu\lambda}(\partial_\nu\eta_\lambda - \partial_\lambda\eta_\nu).$$

Gauge invariance (for arbitrary ϕ), requires $\partial_{[\mu}\eta_{\nu]} = 0$. Thus η_μ "picks a preferred direction" and therefore Lorentz symmetry would be violated, CFJ.

$\Rightarrow$ Can such a term be generated by quantum corrections from other sector (of the SME), and if so what are the implications?

Motivation & Intro:

- For example: Classical electrodynamics extended by a Chern-Simons like term ($\mathcal{L}_{CS} = -\zeta \frac{1}{2} \eta_\mu A_\nu \widetilde{F}^{\mu\nu}$) would imply Lorentz Invariance Violation (LIV) if gauge symmetry is required.
- Also, LIV may hamper the consistency or validity of the effective theory even at the free theory level - Kostelecký & Lehnert.

Motivation & Intro:

- For example: Classical electrodynamics extended by a Chern-Simons like term ($\mathcal{L}_{CS} = -\zeta \frac{1}{2} \eta_\mu A_\nu \widetilde{F}^{\mu\nu}$) would imply Lorentz Invariance Violation (LIV) if gauge symmetry is required.
- Also, LIV may hamper the consistency or validity of the effective theory even at the free theory level - Kostelecký & Lehnert.
- Though LIV is expected to be relevant at very high-energies, the SME allows to parametrise its effects at low-energies, regardless of the underlying theory. But, are there any implications on the low-energy regime from what happens at very high-energies? Specially in loop corrections!

  Which is the domain of validity of the effective theory.

Motivation & Intro:

- For example: Classical electrodynamics extended by a Chern-Simons like term ($\mathcal{L}_{CS} = -\zeta \frac{1}{2} \eta_\mu A_\nu \widetilde{F}^{\mu\nu}$) would imply Lorentz Invariance Violation (LIV) if gauge symmetry is required.
- Also, LIV may hamper the consistency or validity of the effective theory even at the free theory level - Kostelecký & Lehnert.
- Though LIV is expected to be relevant at very high-energies, the SME allows to parametrise its effects at low-energies, regardless of the underlying theory. But, are there any implications on the low-energy regime from what happens at very high-energies? Specially in loop corrections!

  Which is the domain of validity of the effective theory.
- Does this information help to disambiguate the issue of radiative corrections?

Motivation & Intro:

- For example: Classical electrodynamics extended by a Chern-Simons like term ($\mathcal{L}_{CS} = -\zeta \frac{1}{2} \eta_\mu A_\nu \widetilde{F}^{\mu\nu}$) would imply Lorentz Invariance Violation (LIV) if gauge symmetry is required.
- Also, LIV may hamper the consistency or validity of the effective theory even at the free theory level - Kostelecký & Lehnert.
- Though LIV is expected to be relevant at very high-energies, the SME allows to parametrise its effects at low-energies, regardless of the underlying theory. But, are there any implications on the low-energy regime from what happens at very high-energies? Specially in loop corrections!

  Which is the domain of validity of the effective theory.
- Does this information help to disambiguate the issue of radiative corrections?
- If we are shaking the foundations no wonder if a disaster occurs. Care must be taken. Anomalies.

Fermions coupled to a constant axial vector

- The CPT-odd nature of the CS term in CFJ's model, suggests that such a term could be induced at one-loop level from the fermion to the photon by sector means of an axial coupling, and that fostered a lot of efforts into the area.

Fermions coupled to a constant axial vector

- The CPT-odd nature of the CS term in CFJ's model, suggests that such a term could be induced at one-loop level from the fermion to the photon by sector means of an axial coupling, and that fostered a lot of efforts into the area.
- The model: Consider only one species of massive fermions with an extra coupling to a constant background axial-vector

$$\mathcal{L}_{\text{spinor}} = \bar{\psi}(x)\left(\mathrm{i}\partial\!\!\!/ - eA\!\!\!/(x) - m - b\!\!\!/\gamma^5\right)\psi(x)\,,$$

wherefrom the exact propagator is obtained:

Fermions coupled to a constant axial vector

- The CPT-odd nature of the CS term in CFJ's model, suggests that such a term could be induced at one-loop level from the fermion to the photon by sector means of an axial coupling, and that fostered a lot of efforts into the area.
- The model: Consider only one species of massive fermions with an extra coupling to a constant background axial-vector

$$\mathcal{L}_{\text{spinor}} = \bar{\psi}(x)\left(\mathrm{i}\partial\!\!\!/ - eA\!\!\!/(x) - m - b\!\!\!/\gamma^5\right)\psi(x)\,,$$

wherefrom the exact propagator is obtained:

$$S_F(p;b) = \frac{i}{p\!\!\!/ - m - b\!\!\!/\gamma_5} \qquad \text{rationalizing}\ldots$$

Fermions coupled to a constant axial vector

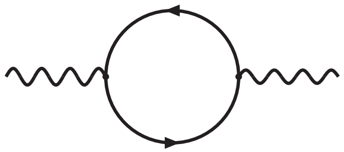
- The CPT-odd nature of the CS term in CFJ's model, suggests that such a term could be induced at one-loop level from the fermion to the photon by sector means of an axial coupling, and that fostered a lot of efforts into the area.
- The model: Consider only one species of massive fermions with an extra coupling to a constant background axial-vector

$$\mathcal{L}_{\text{spinor}} = \bar{\psi}(x)\left(\mathrm{i}\partial\!\!\!/ - eA\!\!\!/(x) - m - b\!\!\!/\gamma^5\right)\psi(x)\,,$$

wherefrom the exact propagator is obtained:

$$S_F(p;b) = i\Big(\gamma^\nu p_\nu + m + b^\nu\gamma_\nu\gamma_5\Big)\frac{p^2 + b^2 - m^2 + 2\left(p\cdot b + m b^\lambda\gamma_\lambda\right)\gamma_5}{\left(p^2 + b^2 - m^2 + i\varepsilon\right)^2 - 4\left[\,(p\cdot b)^2 - m^2 b^2\right]}\,.$$

- 1-loop contribution to vacuum polarisation:



$$= \Pi_2^{\mu\nu}(k;b;m) = -i(eq)^2\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}[\gamma^\mu\, S_F(p,b)\,\gamma^\nu\, S_F(p-k;b)].$$

Which is UV-divergent, and needs to be regularised!

Different approaches - ambiguous results

- There have been numerous points of view regarding how to address this issue. Most of them trying to answer whether there is an induced CS term in the photondynamics by fermions coupled to an axial-vector or not.

Different approaches - ambiguous results

- There have been numerous points of view regarding how to address this issue. Most of them trying to answer whether there is an induced CS term in the photondynamics by fermions coupled to an axial-vector or not.
- To this end, different regularisation methods have been employed and different results (for the induced CS vector) have been obtained.

Different approaches - ambiguous results

- There have been numerous points of view regarding how to address this issue. Most of them trying to answer whether there is an induced CS term in the photondynamics by fermions coupled to an axial-vector or not.
- To this end, different regularisation methods have been employed and different results (for the induced CS vector) have been obtained.
- From our point of view, most of these points of view rely on arguments which are valid in Lorentz invariant theory! Do not forget that a full quantum field theory in a Lorentz violating background does not exist. Only then could we answer such questions with "formal" arguments.

Different approaches - ambiguous results

- There have been numerous points of view regarding how to address this issue. Most of them trying to answer whether there is an induced CS term in the photondynamics by fermions coupled to an axial-vector or not.
- To this end, different regularisation methods have been employed and different results (for the induced CS vector) have been obtained.
- From our point of view, most of these points of view rely on arguments which are valid in Lorentz invariant theory! Do not forget that a full quantum field theory in a Lorentz violating background does not exist. Only then could we answer such questions with "formal" arguments.
- However, none of the above take into consideration the effective character of this model. There is a scale above which the model ceases to be valid due to instability of the basic constituents of the theory, and therefore constrain the domain of integration in pert. theory.

Different approaches - ambiguous results

- There have been numerous points of view regarding how to address this issue. Most of them trying to answer whether there is an induced CS term in the photondynamics by fermions coupled to an axial-vector or not.
- To this end, different regularisation methods have been employed and different results (for the induced CS vector) have been obtained.
- From our point of view, most of these points of view rely on arguments which are valid in Lorentz invariant theory! Do not forget that a full quantum field theory in a Lorentz violating background does not exist. Only then could we answer such questions with "formal" arguments.
- However, none of the above take into consideration the effective character of this model. There is a scale above which the model ceases to be valid due to instability of the basic constituents of the theory, and therefore constrain the domain of integration in pert. theory.
- As mentioned before, most of the phenomenology of LIV deals with low-energy experiments, understanding they "originate" at a scale much larger, and "percolate" down to scales of experimental reach. However, at very high energies, eg., decays that are kinematically forbidden in conventional QED, are triggered by the LIV background and must be taken into account for our model's validity.

Effective theory scale - UV physical cutoff

- As will be shown afterwards, the QED Lagrangian will contain:

$$\begin{aligned}
\mathcal{L} &\supset \mathcal{L}_{\rm INV} + \mathcal{L}_{\rm LIV} \\
\mathcal{L}_{\rm INV} &= -\frac{1}{4} F^{\alpha\beta}(x) F_{\alpha\beta}(x) + \bar{\psi}(x)[\not{\partial} - e\not{A}(x) - m_e]\,\psi(x) \\
\mathcal{L}_{\rm LIV} &= \frac{1}{2} m_\gamma^2\, A_\nu(x) A^\nu(x) + \frac{1}{2}\,\eta_\alpha A_\beta(x) \tilde{F}^{\,\alpha\beta}(x) + b_\mu\, \bar{\psi}(x)\,\gamma_5\,\gamma^\mu\,\psi(x)
\end{aligned}$$

- From the above we can see that the mass-shell conditions for fermions and photons are modified (for simplicity of the argument, consider timelike LIV and (induced) CS vectors, *i.e.* $\eta_\mu = (\eta_0, \vec{0})$ and $b_\mu = (b_0, \vec{0})$) and the dispersion relations read:

$$\begin{aligned}
k_0 &= \pm\sqrt{(k \pm \frac{1}{2}\eta)^2 + m_\gamma^2 - \frac{1}{4}\eta^2} \equiv \pm\,\omega_{\mathbf{k}\,,\,\pm}\,, \\
p_0 &= \pm\sqrt{(p \pm b)^2 + m_e^2} \equiv \pm\,\omega_{\,\mathbf{p}\,,\,\pm}
\end{aligned}$$

- Negative-chirality 1-particle states and positive-chirality 1-antiparticle states turn acausal $(p_\mu p^\mu < 0)$ for:

$$|\mathbf{p}| > \frac{m_e^2}{2b} \equiv \Lambda_e\,.$$

Physical UV cutoff — cont'd

- Focusing on "Vacuum Čerenkov" radiation (or LIV fermion decay) $\widetilde{e}^{\pm} \to \widetilde{e}^{\pm}\,\widetilde{\gamma}$, and in electron-positron creation by photon annihilation, which are, of course, forbidden in ordinary QED, one can find similar $\mathcal{O}(3)$ symmetric bounds, $|\mathbf{p}| < \Lambda_e$ and $|\mathbf{k}| < \Lambda_\gamma$ to render the theory stable against such "decays".
- From our point view, those states which leave the causality region or originate instabilities in its constituents, should be removed from the intermediate states in loop calculations.
- In conventional physics, this may not be the usual practice, but has been adopted in another cases where, eg, a canonical quantisation was performed for a LIV model, namely the Myers-Pospelov model, (Reyes, Urrutia, Vergara).
- In so doing, for example, it has been found by Andrianov, Giacconi and Soldati, that:
  - The CPT-odd part of the induced CS term was the same as that with dimensional regularisation,
  - The result was non-vanishing, in accordance with AGS and Ebers-Zhukovsky-Razumovsky.
  - For the first time, the same result for η_μ was obtained with different regulators, hence claimed as unambiguous. OK, this is not exhaustive and no proof, however points to the importance of the high-energy behaviour of the effective theory, previously dismissed.

$\Pi^{\mu\nu}$ in axial-vector background

- Up to order b^2 one has:

$$\begin{aligned}
\mathrm{reg}\,\Pi^{\mu\nu}(b,k,m) &= \mathrm{reg}\,\Pi^{\mu\nu}_{\mathrm{cov}}(b,k,m) + \Delta\Pi^{\mu\nu}_{\mathrm{even}}(b,k,m) + \Delta\Pi^{\mu\nu}_{\mathrm{odd}}(b,k,m) \\
\mathrm{reg}\,\Pi^{\mu\nu}_{\mathrm{cov}}(b,k,m) &= (\,k^2\,g^{\mu\nu} - k^\mu k^\nu\,)\,\mathrm{reg}\,\Pi(k^2) \\
\Delta\Pi^{\mu\nu}_{\mathrm{even}}(b,k,m) &= \frac{\alpha}{\pi}\left\{\frac{2}{3}\,b^2\,g^{\mu\nu} - A(b,k,m)\,S^{\mu\nu}\right\} \\
\Delta\Pi^{\mu\nu}_{\mathrm{odd}}(b,k,m) &= 2\mathrm{i}\,\frac{\alpha}{\pi}\,\varepsilon^{\mu\nu\rho\sigma}\,b_\rho\,k_\sigma
\end{aligned}$$

where

$$S^{\mu\nu} = [\,(b\cdot k)^2 - b^2\,k^2\,]\,\bar{g}^{\mu\nu} - (b\cdot k)\,(b^\mu\,k^\nu + b^\nu\,k^\mu) + b^2\,k^\mu k^\nu + k^2\,b^\mu b^\nu$$

and $A(b,k,m)$ has been determined too.

- Each piece has been computed with $\overline{DR}$ and physical UV cutoff.
- The effective modified photon Lagrangian reads:

$$\mathcal{L}_{\mathrm{eff}} = -\frac{1}{4}(1+\varepsilon)\,F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\varepsilon\,m^2\,A^\mu A_\mu + \varepsilon\,\frac{b^\lambda\,b^\nu}{2b^2}\,F_{\lambda\rho}F_\nu{}^\rho - \frac{\alpha}{\pi}b_\mu\,A_\nu\,\widetilde{F}^{\mu\nu} \qquad \left(\varepsilon = \frac{2\alpha b^2}{3\pi m^2}\right)$$

$\Pi^{\mu\nu}$ CPT-even - 'tHVBM Dim. reg.

- Some terms with an even # of γ_5 will contribute $\therefore$ 'tHVBM algebraically consistent rules for dealing with the Dirac matrices in $d = 2\omega$ dimensions, ($\Rightarrow$ correct expression for the axial anomaly).
- In loop integrals:

$$\int d^4 p \to \int \mu^{2\epsilon} (2\pi)^{-2\omega} d^{2\omega} p \,, \quad \epsilon \equiv 2 - \omega \,.$$

- The Lor. covariant piece yields the usual modification of the Coulomb potential and the ensuing wave function renormalization.
- The LIV piece is that presented before. A possible non-vanishing photon mass may be radiatively induced to lowest order in b^μ if: $\mathrm{reg}\Delta\Pi^{\mu\nu}_{\mathrm{even}}(k=0,b,m,\mu) \equiv \mathrm{reg}\Delta\Pi^{\mu\nu}_{\mathrm{even}}(b,m,\mu) \neq 0$.
- It turns out that there is such a term, namely:

$$\mathrm{reg}\Delta\Pi^{\mu\nu}_{\mathrm{even}} = \frac{2\alpha}{3\pi} b^2 g^{\mu\nu} \,.$$

i.e. , in presence of the explicit LIV in the fermion sector, there is an anomalous loss of gauge symmetry due to radiative corrections, since this term produces a radiative Proca mass term.

Induced photon mass - UV cutoff

- We saw that for modified fermions, the causality/stability bound implied: $|\mathbf{p}| < \Lambda_e \approx \frac{m^2}{2b}$. In the 1-loop calculation this entails:

$$\int d^4p \to \int d^4p \; \theta(\Lambda^2 - \mathbf{p}^2)$$

- The covariant piece of the vac. pol. tensor picks up a divergent piece, as usual, and is removed by a Proca mass Lorentz invariant counterterm in the Lagrangian. Comparing with the result from the anomalous $\overline{DR}$ photon mass allows to fix the finite part of the counterterm.
- The LIV piece, though, yields:

$$\lim_{k \to 0} \; \mathrm{reg}\Delta\Pi^{\mu\nu}_{\mathrm{even}}(k, b, m, \Lambda) = \frac{2\alpha}{\pi} \cdot \frac{m^2}{\Lambda^2} \left(1 + \frac{m^2}{\Lambda^2}\right)^{-\frac{5}{2}} (b^2 g^{\mu\nu} - b^\mu b^\nu)$$

- Note the calculation reveals that with the cutoff, the LIV part of the amplitude does indeed vanish when the cutoff is removed. One example where cutoffs do not lead to finely tuned results!

IPM - Finite Temperature

- To this end we consider:

$$\frac{1}{(2\pi)^4}\int d^4p\theta(\Lambda^2-\mathbf{p}^2) \quad\rightarrow\quad \frac{i}{\beta}\sum_{m=-\infty}^{\infty}\int\frac{d\mathbf{p}}{(2\pi)^3}\theta(\Lambda^2-\mathbf{p}^2)\,,$$

$$p^0 \quad\rightarrow\quad i\omega_n=\frac{i\;pi}{\beta}(2n+1), n\in\mathbf{Z}\,.$$

- In so doing we first consider the CPT-odd case as a check. We find the same result as AGS (with UV cutoff and $\overline{DR}$) and also the same result as Ebers-Zhukovsky-Razumovsky.
- The form factors contributing to the photon mass of the CPT-even piece yields:

$$\lim_{\beta\rightarrow 0} A_\beta=0\,, \qquad \lim_{\beta\rightarrow\infty} A_\beta=\frac{2\alpha}{\pi}\cdot\frac{m^2}{\Lambda^2}\left(1+\frac{m^2}{\Lambda^2}\right)^{-\frac{5}{2}}\,.$$

Conclusions and Outlook

- The UV energy scale above which the model lacks meaning has been taken into account for the computation of radiative corrections in LIV QED.
  - The results thus obtained for the induced photondynamics modification agree with the calculations using $\overline{DR}$ and finite temperature regularisation.
- Ultimately, deriving the values of induced coupling constants by formal arguments valid in Lorentz invariant scenarios may be important BUT the UV physical cutoff here presented can not be avoided, and therefore should be considered also.
- A point to stress is that all regulators fall into 2 categories. Those which are consistent with the physical UV fermion cutoff and those ones which are not. If the energy scale of the effective theory is taken seriously into account, then there is no ambiguity whatsoever.