Effects of brane-flux transitions on string theory compactifications

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An apology.....

You will find the words ”AdS/CFT correspondence” only in this slide

A second apology.....

This is a VERY sketchy talk....you shouldn't expect a detailed information

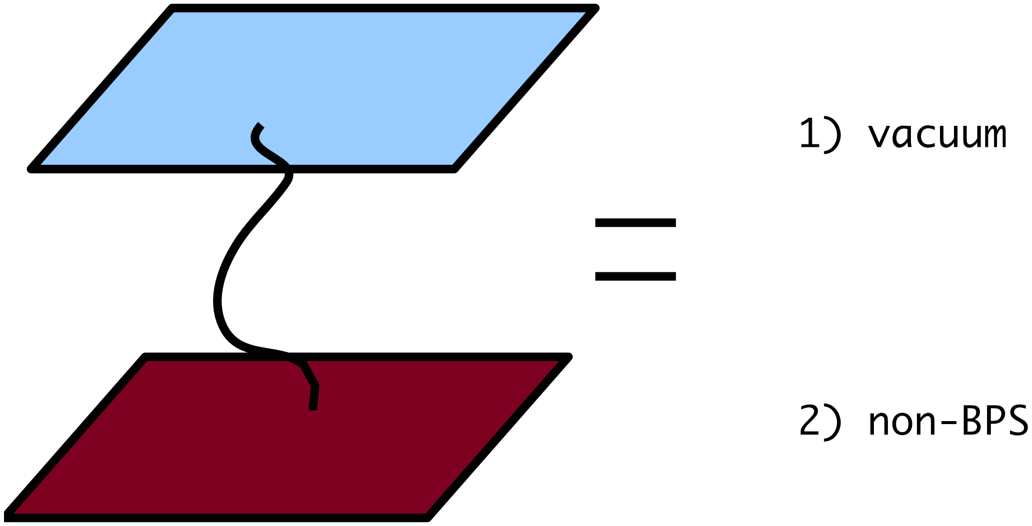
Plan

- Motivation
- Fluxes in String compactifications
- Topological inconsistencies
- Effects on supersymmetric black holes
- D-brane instabilities in geometric flux compactification
- Minkowski vacua transitions
- Final comments

Motivation

- The presence of fluxes on string compactifications has enriched our notions of geometry and increased the number of possible string vacua
- In the most conservative way, the number of vacua goes as 10^{500}.
- By considering the most general kind of flux compactifications in the context of F-theory, number of vacua increases to 10^{1000}.
- *Are some vacua connected by topological equivalences driven by the presence of fluxes?*
- A compactification threaded with fluxes leads to the existence of a transition between branes and fluxes

- *The connection appears by twisting the commoun differential operator dF into $dF + H_3 \wedge F$. This twisted version (up to the absence of orientifold and orbifold singularities)* *connects to* *K-theory*.



- K-theory equivalence classes are usually ignored in string phenomenological models. At most, models are restricted to conditions in which cohomology is enough to classify D-branes and fluxes.

- We are going to study topological transitions driven by fluxes and its consequences in string phenomenological models, which means essentially three points:

— supersymmetric black holes constructed by wrapping D3-branes on internal cycles,

— wrapping D-branes on internal cycles (to get SM like models)

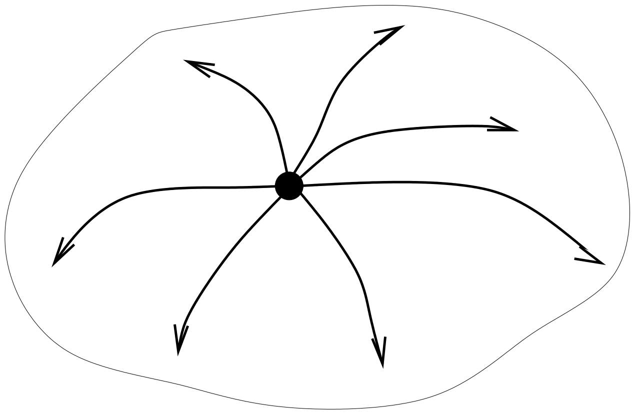
— transitions of Minkowski vacuum solutions

Flux compactification

- Typical compactifications without flux leads to a constant warping factor and a Calabi-Yau as an internal manifold [Candelas and Witten, 1995].
- There exists the problem that compactification moduli remains unfixed, pointing out the presence of large-range unobseved forces.
- In a supergravity compactification (with α' corrections neglected) to a maximal symmetric 4D spacetime , the presence of fluxes (internal or external) is prohibited (No-go theorem [Maldacena and Nunez, 2002]
- String theory avoid such theorem by including high order corrections and/or orientifold singularities.

- The price to pay is to depart from the smooth scenario of CY compactifications: fluxes backreact!
- They contribute positively to the internal energy for a maximal symmetric 4D space-time.
- and they give RR D-brane charge, e.g. through the coupling $\int_{X_6} H_3 \wedge F_3$
- Supergravity equations of motion (in type IIB) are twisted by the presence of RR and NS-NS fields

$$d * F_5 = \rho_{sources} + H_3 \wedge F_3$$



• Tadpole condition, or Bianchi identity for the self-dual F_5 requires the presence of negative charged objects: *orientifolds*.

• In the absence of NS-NS flux, *cohomology* classifies RR fields:

By SUGRA eof $\longrightarrow dF_p = 0 \Rightarrow [F_p] \in H^p(X_6; Z)$ **(quantization requires the field to be integers).**

• Which mathematical structure classifies the fields in the presence of extra fluxes, and most important, *what is the physical interpretation of having new equivalence relations of the RR fields?*

Topological inconsistencies

- The submanifold on which an stable D-brane is wrapped on is classified by ordinary homology: $[W] \longrightarrow$ D-brane world-volume (with no boundary).
- Hence, *in the fluxless case* $\partial W_p = 0$ and $dF_p = 0$.
- Turn on a NS-NS flux H_3, such that
  - Its restriction to W does not vanish,
  - all legs in the internal space X_6.

- The NS-NS flux induces a monopole charge on W ($dimW = p + 1$):,

$$S \sim \int_{W_{p+1}} H_3 \wedge A_{p-2}$$

with A the magnetic dual to the A_1 potential on W.

- N units of H induce N units of monople charge on W.
- The variation of the action wrt A_{p-2} implies $\int H = 0 \Rightarrow$ an inconsistency appears.

- This is the classical version of a more refined result known as Freed-Witten anomaly [Freed, Witten 1999].

  – The general result consider non-spin^c manifolds.

  – That means, manifolds in which spinors are not coupled with gauge fields on W.

  – I will restrict to scenarios where such coupling is possible.

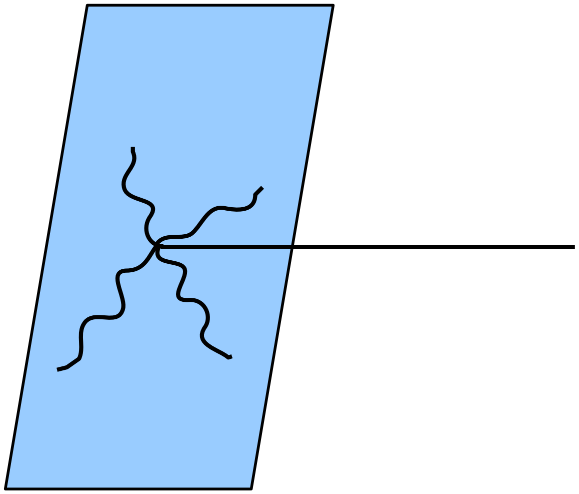
  – Hence, only the classical version of FW anomaly will be considered through this talk.

Topological inconsistencies II

- How to avoid/cancel such inconsistency?

- The traditional way is to consider a vanishing restriction (pullback) of the H_3 field to the D-brane worldvolume.

- Another one, is to provide of the magnetic sources required by a non-vanishing of H_3 in W.



- By adding a $D(p-2)$ brane ending at the world -volume of a Dp-brane, the anomaly is cured.

  – The endpoint of the Dp-brane is the magnetic monopole of the gauge theory at W

$$S \sim \int_W F_{p-1} \wedge *F_{p-1} + \int H_3 \wedge A_{p-2}$$

and the eom are fulfilled: $d * F_2 = H_3$.

What would happen for an instantonic brane?

[Maldacena, Moore, Seiberg, 2001]

Brane-flux transition

Consider a background threaded with H_3-flux

- A Dp-brane, wrapping a cycle W (identified with the worldvolume) on which H_3 **vanishes**, it is well defined (physical and mathematically).

  – The worldvolume and forms are classified by usual (co)homology

- There are cycles (submanifolds of the space background) on which the flux H_3 is supported.

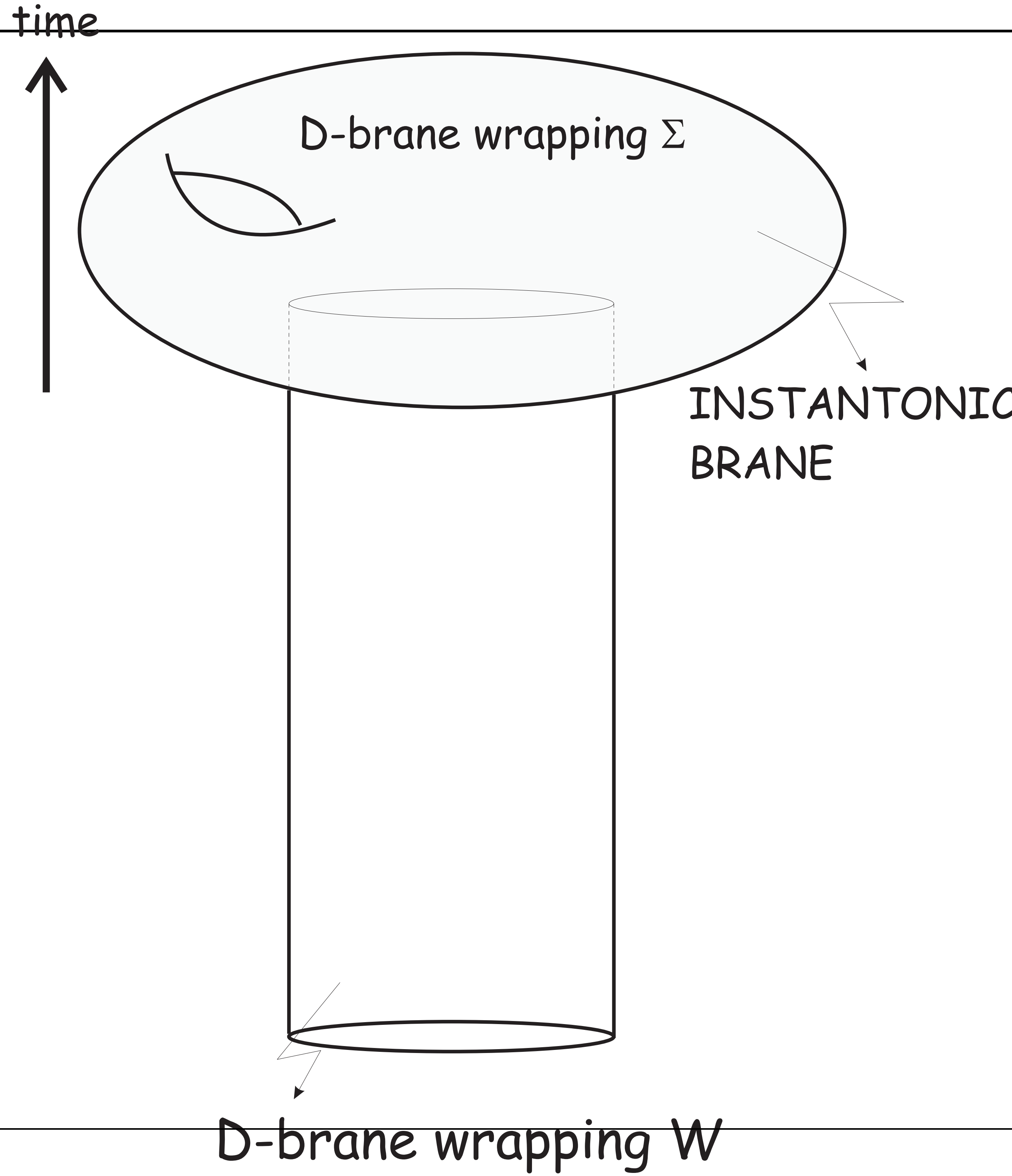
  – Dp-branes $(p \geq 3)$ wrapping them, are ill-defined. A FW anomaly appears.

– The anomaly can be cured by adding extra $D(p-2)$-branes

(Notice this is impossible in a compact manifold!)

- It could happen that, an stable Dp-brane encounters with an anomalous *instantonic* brane at some time t.

  – The (in principle) stable Dp-brane cures the anomaly.


time
D-brane wrapping Σ
INSTANTONIC
BRANE
D-brane wrapping W

- The price to pay is that such Dp-brane decays.

  — The decay is topological.

  — The same quantum numbers (charge, tension) are now carried by fluxes

  — Such fluxes are the result of the coupling between the already present NS-NS flux H_3 and the magnetic field strength of the instantonic brane.

  — E.g., A D3-brane in a background threaded by an H-flux supported on ”transversal” coordinates, will decay after an encounter with an instantonic $D5$-brane.

  — The magnetic RR field for the later is F_3. The former RR D3-brane charge is now carried by $F_3 \wedge H_3$.

- This is precisely the physical interpretation of the twisted BIanchi identities for F_5!

$$dF_5 = sources + H_3 \wedge F_3$$

- This tells us that $D3$-branes are created or annihilated by $D5$-branes supporting some units of H_3-flux

  – The number of NS-NS flux fixes de number of unstable D-branes

  – Is the total $D3$-brane charge, from physical $D3$-branes and fluxes, the quantity which is conserved.

- In a ”democratic” notation

$$d * F = H_3 \wedge F$$

where $F = \sum_n F_n$.

K-theory

All these transitions are the physical interpretation about the fact that D-branes are actually classified by *K-theory*

What is K-theory?

- Classifies pairs of D-branes- anti-D-branes modulo tachyon condensation [Witten 1995, Moore 1994].
- Classifies brane configurations preserving RR charge.
- For trivial tachyon condensation in **the fluxless case** , K-theory = cohomology
- There are backgrounds in which tachyon condensates in a ”controled” way: e.g. orientifolds
- In such cases, K-theory provides more information than cohomology (non-BPS states in type I)

In the presence of fluxes, K-theory is slightly different than cohomology,

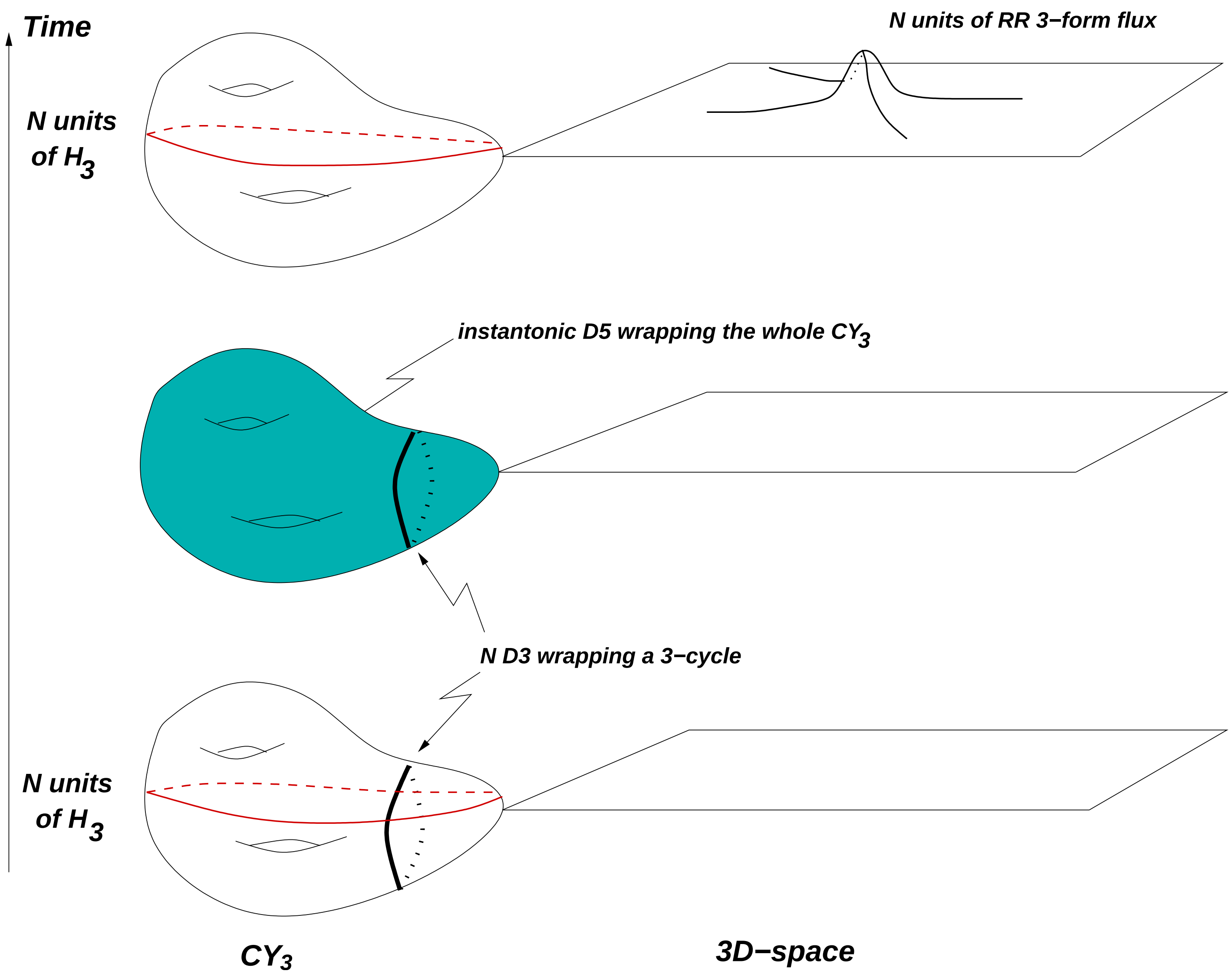
— This means there are D-brane configurations apparently stable in a cohomological classification, which turn out to be unstable in K-theory.

A way to connect cohomology with K-theory is by twisting the differential operator d to $d + H_3 \wedge$

- This is the mathematical realization of canceling FW anomaly.
- Instantonic branes do not belong to K-theory classes (they are unstable).
- D-branes transforming into flux via the instantonic brane belong to the zero class in K-theory.

An example: A black hole transition

- A Black Hole in 4D is constructed by wrapping D3-branes on internal 3-cycles [Seiberg, 1996]
- In the presence of a NS-NS flux H_3 supported on *transversal* coordinates to the D3-branes, the configuration is ”stable”.
- Stability is lost by the appearance of an instantonic D5-brane on X_6.
- D3-branes are transformed into fluxes $H_3 \wedge F_3$, with F_3 living on 4D [O. L-B., K-Y Oda, 2007].


Time
N units of RR 3−form flux
N units
of H3
instantonic D5 wrapping the whole CY3
N D3 wrapping a 3−cycle
N units
of H3
CY3
3D−space

Effects of brane/flux transition on string vacua

The presence of fluxes stabilize some moduli.

- In type IIB, 3-form fluxes does not stabilize Kähler moduli (volume)

  — Non-perturbative information is required [Kachru, Polchinski, Trivedi, 2002]

- An alternative way is to consider extra degrees of freedom at the perturbative level:

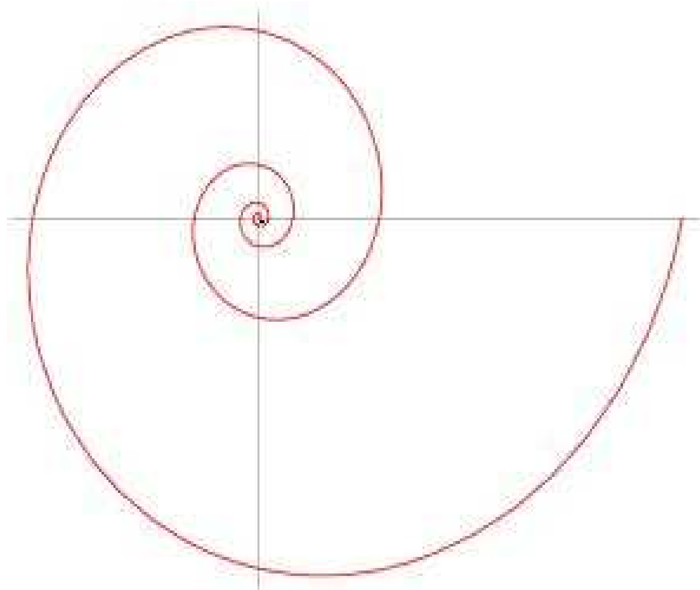
  — Under T-duality on coordinate a, a tori compactification threaded with a NS-NS flux with component H_{abc}, transforms into a *twisted* tori (metric changes) and no flux

$$H_{abc} \longrightarrow f^{a}_{bc}$$

- A subsequent T-duality on coordinate b leads to a manifold with non-global defined geometry (volume is ill-defined: there is not Kähler moduli)

$$H_{abc} \longrightarrow f^{a}_{bc} \longrightarrow Q^{ab}_{c}$$

- The tori is transformed by the first duality into a twisted tori, which is said to be threaded with ”metric fluxes” (nilpotent manifolds)
  - Flux information is mapped to geometry
- By the second duality, it is said that the tori is threaded with *non-geometric flux* [Lawrance, Wecht, Taylor, 2007]



$$ds^2 = \frac{1}{1+Q^2x_c^2}(dx_a^2 + dx_b^2) + dx_c^2$$

- A compactification on twisted torus and 3-form fluxes stabilizes all moduli.
- The superpotential : $W = \int (H_3 - SF_3 + dJ) \wedge \Omega$
- $dJ \neq 0$ because of metric fluxes.

  – Connected to compactification on generalized CY manifolds: half-flat manifolds (next slide) J. Luis, A. Micu, 2004

How is brane/flux transition mapped into this background?

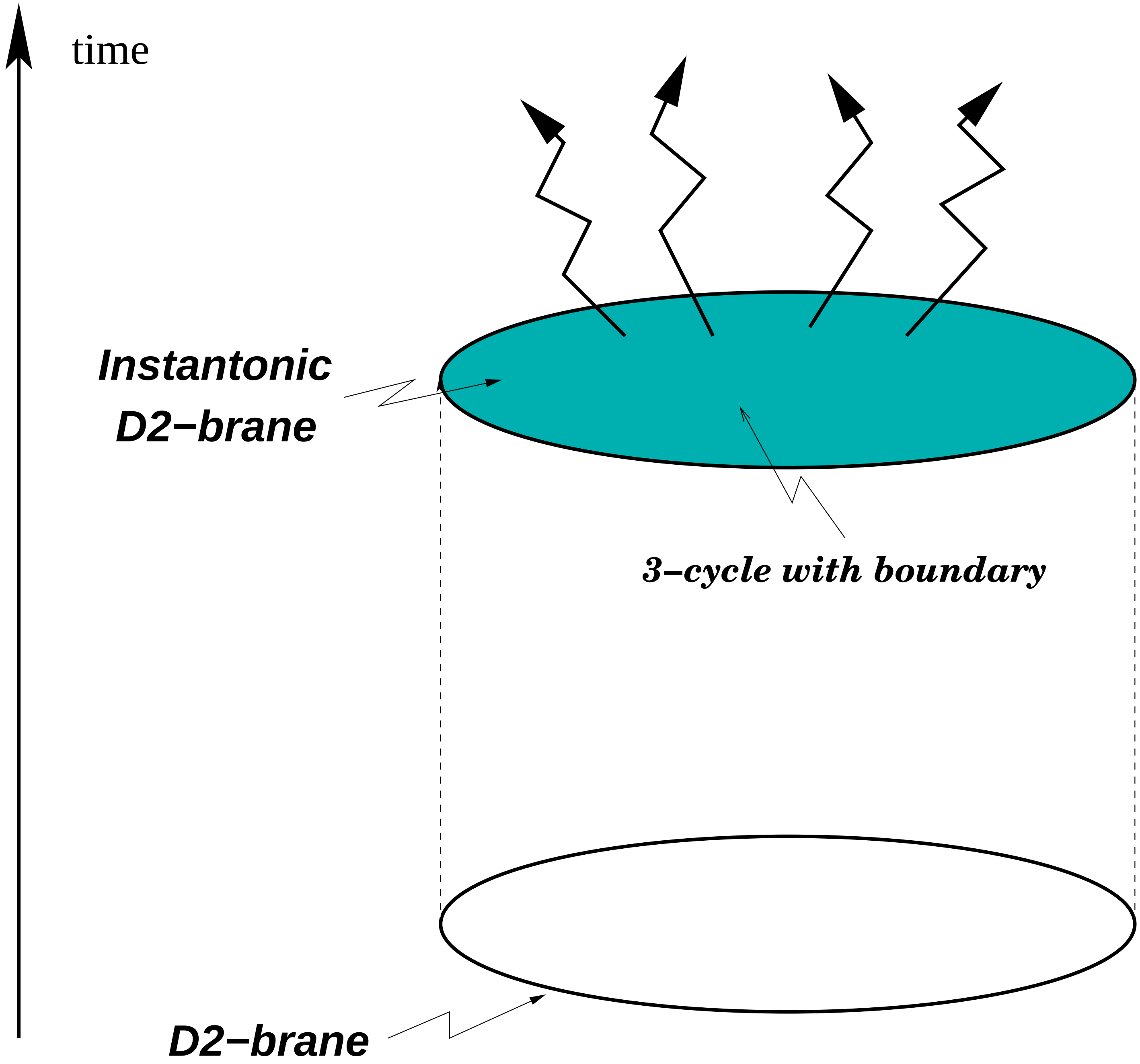
- H-Twisted cohomology is mapped into ”metric-twisted” cohomology.

  – This means in a metric flux background, flux information is contained in ordinary cohomology in the new geometry [Marchesano, 2006]

- The transition is now driven by instantonic branes with the same dimensionality as the unstable D-branes [O. L-B., 2006]

  – A Dp-brane is transformed into $f \cdot *F_{p+2}$

- Connection with K-theory is obtained by adding extra NS-NS flux


time
Instantonic
D2−brane
3−cycle with boundary
D2−brane

D6-branes	$\omega_{(x)}$	H_3
Torsion	$\mathbf{N\Sigma_{123}}$	$M\Sigma_{234}$ $M\Sigma_{135}$ $M\Sigma_{126}$ $\mathbf{M\Sigma_{123}}$
instantonic	$\mathbf{\Sigma_{456}}$	Σ_{156} Σ_{264} Σ_{345} $\mathbf{\Sigma_{456}}$

- Not all cycles are suitable for D-brane building (this changes by including orientifold and orbifold singularities) [Cvetic et. al., 2006][O.L-B. in progress]

At the level of CY compactifications, T-duality on a torus fibration corresponds to mirror symmetry Seiberg 1997

- Metric fluxes correspond to a change on the holonomy and structure group

- Topology can be written in terms of internal spinors [Graña, Tomassielo, ... 2006]

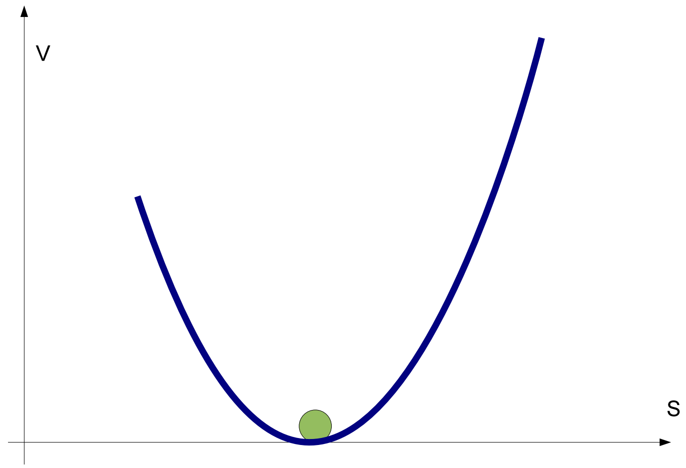
- Minkoswki Vacuum is obtained if the internal spinors satisfy $d_H\phi = 0$.

This suggests that (Minkowski) string vacua must be classified by K-theory

- This means that some vacua, which seems different from a cohomological classification of fluxes, could be related by some transition, if they belong to the same equivalence class in K-theory.
- String landscape size can be reduced.

In fact, Minkowski vacua is constructed as a solution for the SUSY equations $D_i W = 0$.

— The amount of flux we turned on, establishes the properties of the vacuum.



- But the superpotential W is not invariant under a flux-brane transition.

We expect some connections among different vacua [W. Herrera-Suarez, O. L-B., 2009]

Consider the simplest orientifold type IIB string compactification:

- A toroidal compactification with NS-NS and R-R fluxes H_3 and F_3
- $D3$-branes and $O3$-planes sitting at a point in T^6
- Total $D3$-brane charge in T^6 vanishes (tadpole condition)

The Minkowski vacuum is a solution of the supersymmetry conditions on the superpotential W: $D_iW = 0$

- In this case, $W = \int (F_3 - SH_3) \wedge \Omega = W(S, \rho)$

$$\boxed{W_H = P_1(\rho) - SP_2(\rho)}$$

- P are cubic polynomials on ρ with coefficients fixed by the amount of fluxes

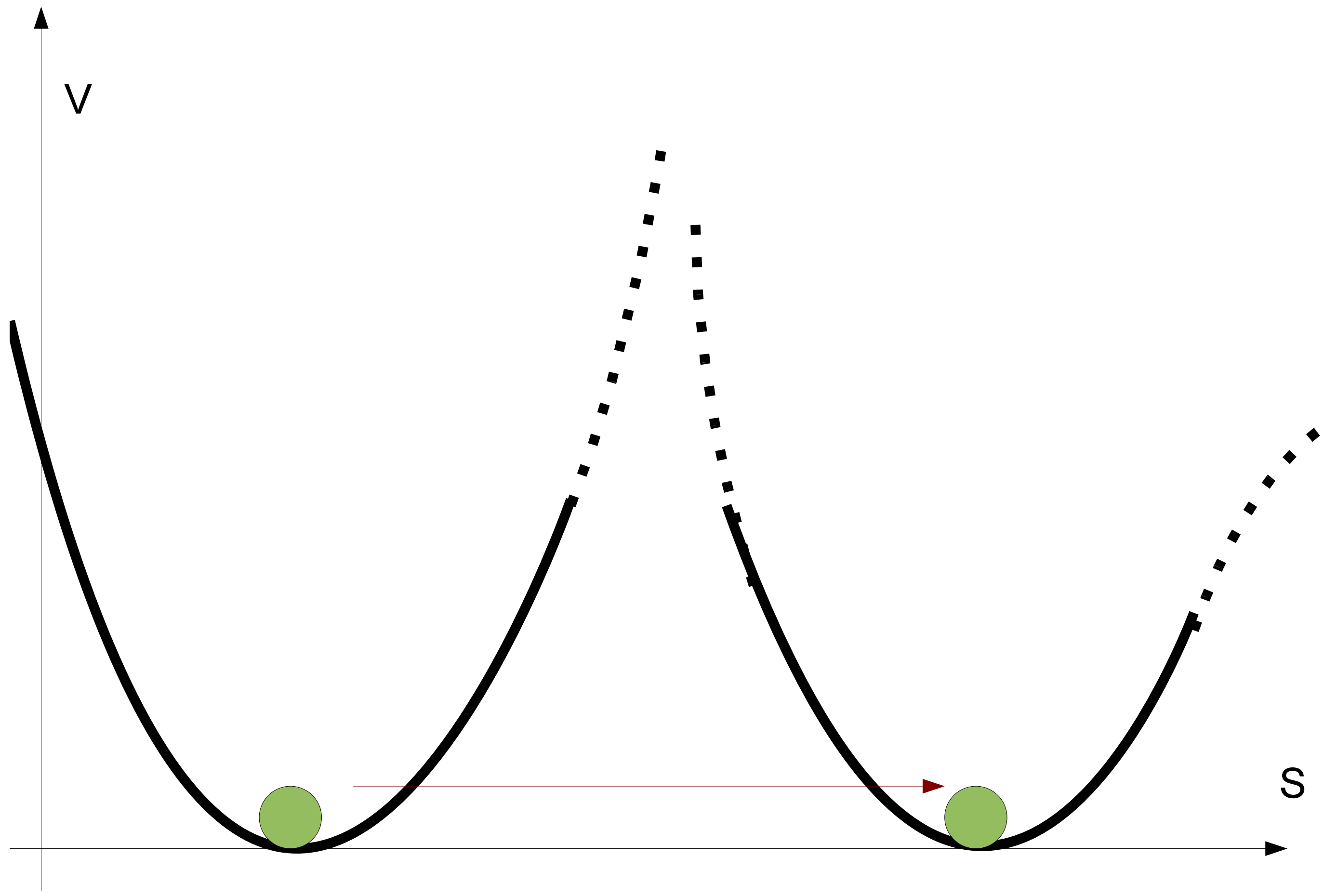
- The solution to susy equations fixed the values for S and ρ.

How those values change under a brane/flux transition?

After the appearance of an instantonic D5-brane wrapping T^6, some D3-brane become unstable and decay to fluxes.

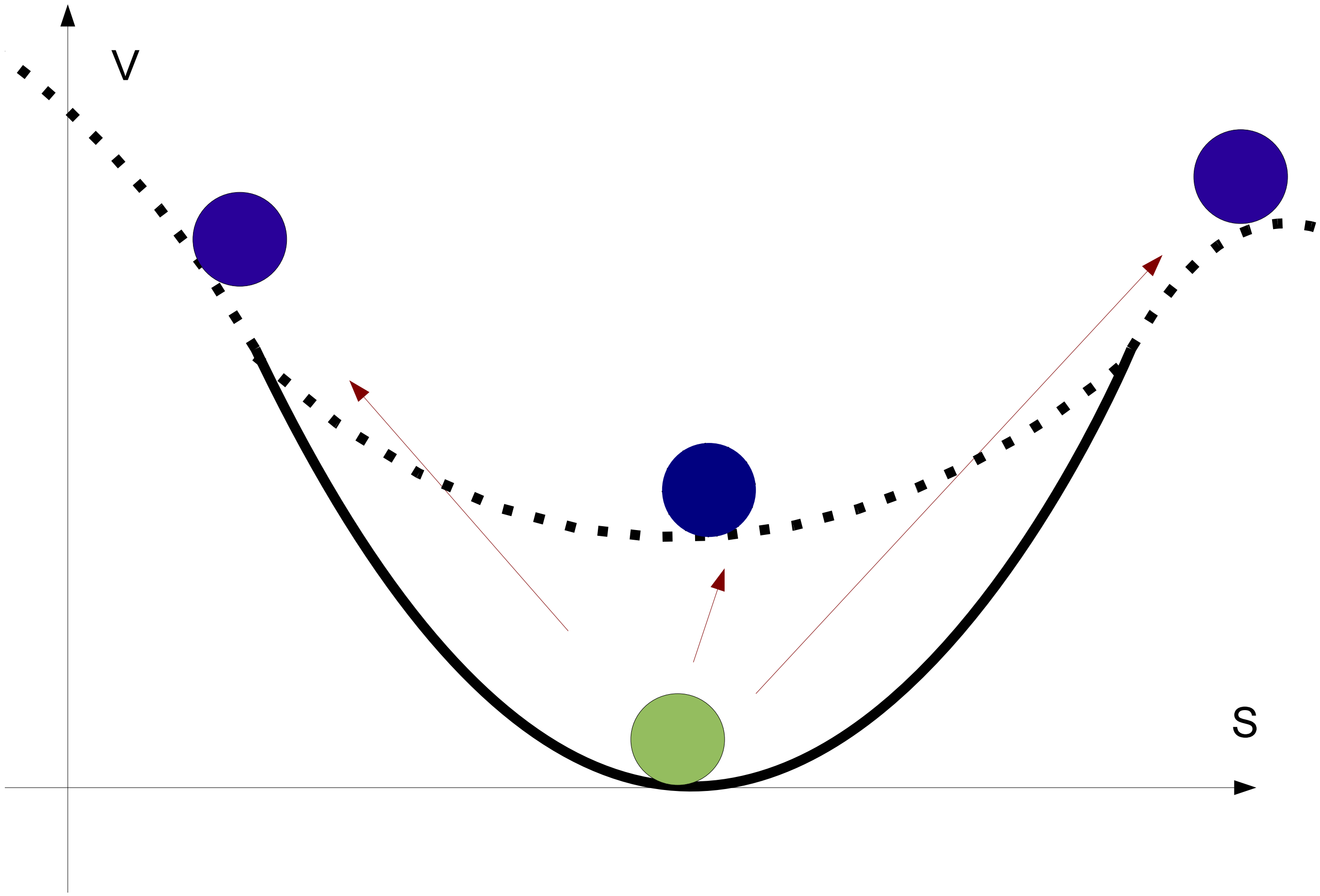
- The amount of RR flux F_3 changes.
- Then the superpotential polynomial is now different
- Solutions to susy equations suffer a change
- Both set of solutions are then connected through a brane/flux transition driven by an instantonic brane

Fora particular case (some specific instantonic branes):



— Found and studied by Kachru et. al in 2003. Different susy solutions differ by a rescaling of fluxes.

The most general case allows different type of transitions:

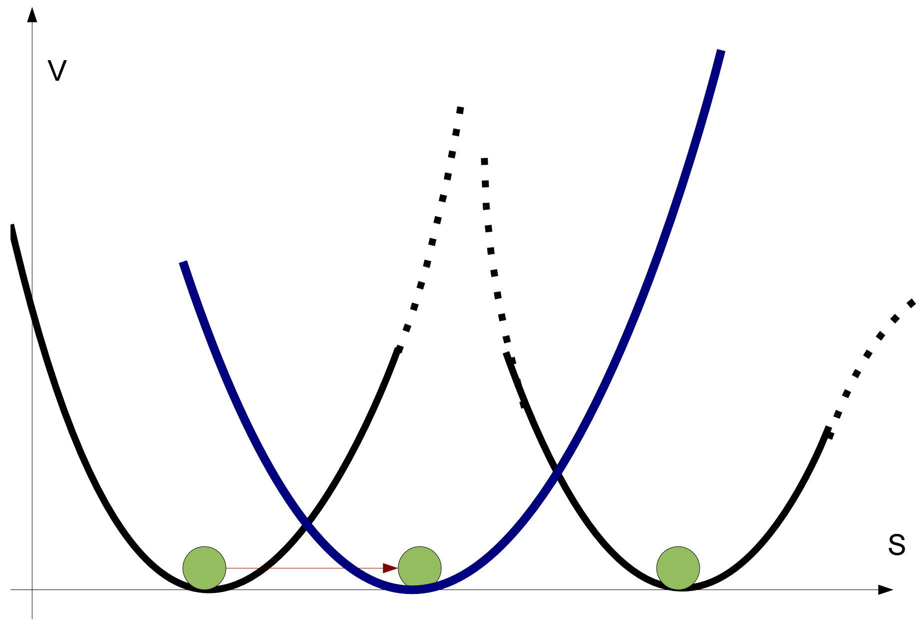


Notice the possibility of breaking susy in a minimal of the transformed scalar potential (Work in progress)

We can consider flux/brane transitions mediated by instantonic NS5-branes (S-dual version) [J. Evslin, 2003]

- The twisted differential operator reads $d_F = d + F_3\wedge$
- N D3-branes are unstable to decay into the flux configuration

$F_3 \wedge H_3$ if encounters an instantonic NS5-brane supporting N units of R-R flux F_3.



Consider a most general flux orientifold compactification with

non-geometric fluxes

- Geometric flux is projected out by the orientifold action
- S-duals of Q, denoted P-flux is also considered.
  - P-fluxes are required for S-duality for the superpotential in type IIB [Aldazabal, Camara, Font, 2006]
- The superpotential now reads

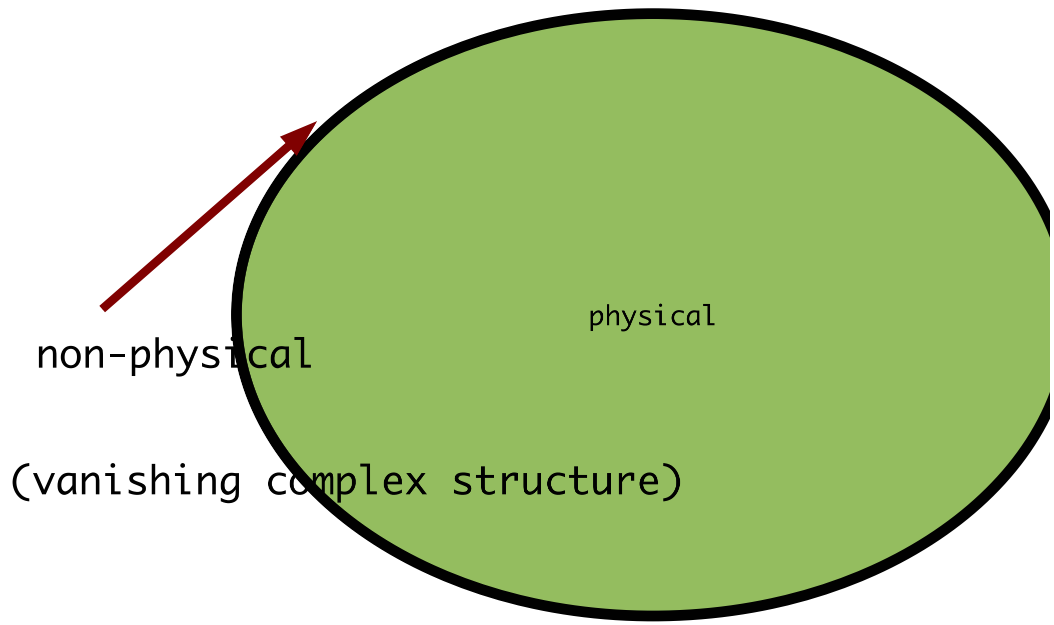
$$W(S, \rho, \tau) = \int (H_3 - SF_3 + (Q + SP) \cdot J_4) \wedge \Omega$$

There are some restrictions for the fluxes to satisfy:

- Bianchi constraints $Q \cdot H = Q \cdot Q = 0$
- Compatibility with Bianchi identities to brane/flux transitions

Hence, not all possible transitions are allowed. The number of configurations giving rise to transitions is reduced

Moduli space for complex structure



- Physical SUSY solutions (non-vanishing C.S). , flux configurations are not allowed to drive a brane/flux transition

- Unphysical SUSY solutions (vanishing C.S) allows the configuration to suffer a transitions. However the transition connects solutions also in the boundary of the moduli space.

Final comments

- We have studied topological transitions between branes and fluxes
- For the transition to actually happening, initial configuration energy must not be smaller then the final one

— A dynamical process would show this [Kachru, et, al 2003, Martelli, Maldacena, 2009]

- For the vanishing complex structure, we have degenerate tori, which for finite Kähler moduli, implies some cycles could shrink into a point. Non-perturbative corrections are needed to fully understand such case.
- It would be interesting to study under which conditions we can (or cannot) connect a susy Minkwsky vacuum to a non-susy one. A dynamical proces describing this topological transition would break susy.
- The S-duality classification of fluxes and branes by K-theory is still

an open question.