Universal Mass Texture and Quark Lepton Complementarity

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Contenido:

Mixing Matrix V_{CKM}

The allowed ranges of the magnitudes of all nine *CKM* elements are[1]:

$$\begin{pmatrix} 0{,}97419 \pm 0{,}00022 & 0{,}2257 \pm 0{,}0010 & 0{,}00359 \pm 0{,}00016 \\ 0{,}2256 \pm 0{,}0010 & 0{,}97334 \pm 0{,}00023 & 0{,}0415^{+0{,}0010}_{-0{,}0011} \\ 0{,}00874^{+0{,}00026}_{-0{,}00037} & 0{,}0407 \pm 0{,}0010 & 0{,}999133^{+0{,}000044}_{-0{,}000043} \end{pmatrix}.$$

[1] A. Ceccucci, et. al. (PDG) Phys.Lett.B667:1,2008.

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The Jarlskog invariant :

$$J_q = \left(3{,}05^{+0{,}19}_{-0{,}20}\right) \times 10^{-5}.$$

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The three angles of the unitary triangle values are:

$$\alpha = (88^{+6}_{-5})^o, \quad \beta = (21{,}46 \pm 0{,}71)^o, \quad \gamma = (77^{+30}_{-32})^o.$$

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The mixing angles for quark sector at 1σ are[2]:

$$\sin\theta^q_{12} = 0{,}2257 \pm 0{,}001, \quad \sin\theta^q_{23} = 0{,}0415^{+0{,}0010}_{-0{,}0011},$$

$$\sin\theta^q_{13} = 0{,}00359 \pm 0{,}00016.$$

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Mixing Matrix U_{PMNS}

Global analysis of oscillation data.

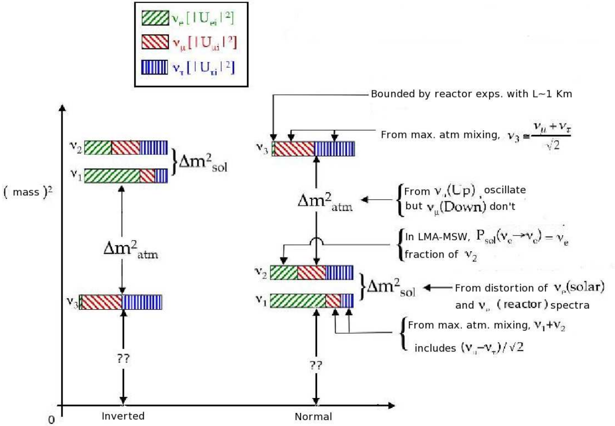
$$\Delta m^2_{21} = 7{,}67^{+0{,}67}_{-0{,}21} \times 10^{-5}\mathrm{eV}^2,$$

$$\Delta m^2_{32} = \begin{cases} -2{,}37 \pm 0{,}15 \times 10^{-3}\mathrm{eV}^2, & (m_{\nu_2} > m_{\nu_1} > m_{\nu_3}) \\ +2{,}46 \pm 0{,}15 \times 10^{-3}\mathrm{eV}^2, & (m_{\nu_3} > m_{\nu_2} > m_{\nu_1}) \end{cases}$$

$$\theta^l_{12} = 34{,}5^o \pm 1{,}4^o, \quad \theta^l_{23} = \left(42{,}3^{+5{,}1}_{-3{,}3}\right)^o, \quad \theta^l_{13} = 0{,}0^{+7{,}9^o}_{0{,}0}$$

M.C. Gonzalez-Gracia and Michele Maltoni, Phys.Rept.460:1-129,2008.

Possible hierarchies for neutrino masses



B. Kayser arXiv:hep-ph/0506165

The magnitude of the elements of the complete matrix U_{PMNS}, at 90 % CL:

$$\begin{pmatrix} 0{,}80 \to 0{,}84 & 0{,}53 \to 0{,}60 & 0{,}0 \to 0{,}17 \\ 0{,}29 \to 0{,}52 & 0{,}51 \to 0{,}69 & 0{,}61 \to 0{,}76 \\ 0{,}26 \to 0{,}50 & 0{,}46 \to 0{,}66 & 0{,}64 \to 0{,}79 \end{pmatrix}$$

M.C. Gonzalez-Gracia and Michele Maltoni, Phys.Rept.460:1-129,2008.

The mixing angles for quark:

$$\theta_{12}^{q} \approx 13^{o} \pm 0{,}05^{o}, \quad \theta_{23}^{q} \approx 2{,}37^{o} \pm 0{,}06^{o}, \quad \theta_{13}^{q} \approx 0{,}2^{o} \pm 0{,}009^{o}$$

The mixing angles for leptons:

$$\theta_{12}^{l} = 34{,}5^{o} \pm 1{,}4^{o}, \quad \theta_{23}^{l} = \left(42{,}3^{+5{,}1}_{-3{,}3}\right)^{o}, \quad \theta_{13}^{l} = 0{,}0^{+7{,}9^{o}}_{0{,}0}$$

Quark Lepton Complementarity

- The solar angle and Cabibbo angle complementarity:

$$\theta^l_{12} + \theta^q_{12} = 45^o + 1{,}5^o \pm 1{,}45^o.$$

- The atmospheric angle and θ^q_{23} angle complementarity:

$$\theta^l_{23} + \theta^q_{23} = \left(44{,}67^{+5{,}15}_{-3{,}35}\right)^o.$$

- The θ^l_{13} angle and θ^q_{13} angle complementarity:

$$\theta^l_{13} + \theta^q_{13} < 8{,}1^o.$$

Dirac o Majorana Neutrinos?

$$\mathcal{L}_m = M\bar{\Psi}_L\Psi_R + M\bar{\Psi}_L\Psi_R$$

1) The right handed component of a massive field is completely independent of the left handed component; Dirac field.
Lepton number is conserved

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1) The right handed component of a massive field is completely independent of the left handed component; Dirac field.
Lepton number is conserved
2) The right handed field component is C-conjugate to the left handed component, $\Psi = \Psi_L + e^{i\varphi}(\Psi_L)^c$; Majorana field.
Lepton number is broken into two units.

$$\mathcal{L}_m = \frac{1}{2}\bar{\Psi}_L^T CM\Psi_L + h.c.$$

where M is the symmetric mass matrix, $M = M^T$.

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The left handed Majorana neutrinos naturally acquire their small masses through an effective type I seesaw mechanism of the form

$$M_{\nu_L} = M_{\nu_D} M_{\nu_R}^{-1} M_{\nu_D}^{T},$$

where M_{ν_D} and M_{ν_R} denote the Dirac and right handed Majorana neutrino mass matrices, respectively. The mass matrix of left handed Majorana neutrino is a symmetric matrix.

Unified Treatment of Quarks and Leptons

The imposition of a flavour symmetry has been successful in reducing the number of parameters of the Standard Model. In particular, a permutational S_3 flavour symmetry and its sequential explicit breaking, allows us to represent the mass matrices as a two zeroes Fritzsch texture[3]:

$$\mathbf{M_i^{(F)}} = \begin{pmatrix} 0 & A_i & 0 \\ A_i^* & B_i & C_i \\ 0 & C_i & D_i \end{pmatrix} \quad i = u, d, l, \nu.$$

[3] A. Mondragon and E. Rodriguez Jauregui Phys. Rev. D 61 113002

Some reasons to propose the validity of the two zeros Fritzsch texture as a universal mass texture for all Dirac fermions in the theory are the following:

- The idea of S_3 flavour symmetry and its explicit breaking has been realized as a tow zeros Fritzsch texture in the quark sector to interpret the strong mass hierarchy of up and down type quarks.

Some reasons to propose the validity of the two zeros Fritzsch texture as a universal mass texture for all Dirac fermions in the theory are the following:

- The idea of S_3 flavour symmetry and its explicit breaking has been realized as a tow zeros Fritzsch texture in the quark sector to interpret the strong mass hierarchy of up and down type quarks.
- The quark mixing angles and the CP violating phase, appearing in the V_{CKM} mixing matrix, were computed as explicit, exact functions of the four quark mass ratios $(m_u/m_t, m_c/m_t, m_d/m_b, m_s/m_b)$, one symmetry breaking parameter $Z^{1/2} = \left(\frac{81}{32}\right)^{1/2}$ and one CP violating phase $\phi_{u-d} = 90^o$, in good agreement with the experimetal data.

- Since the mass spectrum of the charged leptons exhibits a hierarchy similar to the quark's one, it would be natural to consider the same S_3 symmetry and its explicit breaking to just by the use of a the same texture for the charged lepton mass matrix.

- Since the mass spectrum of the charged leptons exhibits a hierarchy similar to the quark's one, it would be natural to consider the same S_3 symmetry and its explicit breaking to just by the use of a the same texture for the charged lepton mass matrix.
- As for the Dirac neutrinos, we have no direct information about the absolute values or the relative values of the neutrino masses, but the two zeros Fritzsch texture can be incorporated in an $SO(10)$ neutrino model. Furthermore from supersymmetry arguments, it would be sensible to assume that the Dirac neutrinos have a mass hierarchy similar to that of the u-quarks and it would be natural to take for the Dirac neutrino mass matrix also a modified Fritzsch texture.

The similarity of quark and charged lepton mass hierarchies suggests the use of similar Fritzsch textures of two zeros for all Dirac fermion mass matrices in the leptonic sector

$$\mathbf{M^{(F)}} = P^{\dagger}\overline{M}^{(F)}P,$$

$$\mathbf{M^{(F)}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\phi} & 0 \\ 0 & 0 & e^{-i\phi} \end{pmatrix} \begin{pmatrix} 0 & |a| & 0 \\ |a| & b & c \\ 0 & c & d \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi} & 0 \\ 0 & 0 & e^{i\phi} \end{pmatrix}.$$

The symmetry of the mass matrix of left handed Majorana neutrinos, $M_{\nu_L} = M_{\nu_L}^T$ and the seesaw mechanism type I, fixed the form of mass matrix of the right handed Majorana neutrinos, in a symmetric matrix. In we case M_{ν_R} has the following form:

$$\mathbf{M}_{\nu_\mathbf{R}} = R\overline{M}_{\nu_R} R,$$

$$\begin{pmatrix} e^{-i\phi_c} & 0 & 0 \\ 0 & e^{i\phi_c} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & a_{\nu_R} & 0 \\ a_{\nu_R} & |b_{\nu_R}| & |c_{\nu_R}| \\ 0 & |c_{\nu_R}| & d_{\nu_R} \end{pmatrix} \begin{pmatrix} e^{-i\phi_c} & 0 & 0 \\ 0 & e^{i\phi_c} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

where $\phi_c = \arg\{c_{\nu_R}\}$.

See-saw Mechanism Type I

$$M_{\nu_L}^{(F)} = P_D^{\dagger} \bar{M}_{\nu_D}^{(F)} P_D R^{\dagger} \bar{M}_{\nu_R} R^{\dagger} P_D \bar{M}_{\nu_D}^{(F)} P_D^{\dagger},$$

Invariance of Fritzsch Texture

$$M_{\nu_L} = \begin{pmatrix} 0 & a_{\nu_L} & 0 \\ a_{\nu_L} & b_{\nu_L} & c_{\nu_L} \\ 0 & c_{\nu_L} & d_{\nu_L} \end{pmatrix},$$

$$a_{\nu_L} = \frac{|a_{\nu_D}|^2}{a_{\nu_R}}; \qquad d_{\nu_L} = \frac{d_{\nu_D}^2}{d_{\nu_R}},$$

$$b_{\nu_L} = \frac{c_{\nu_D}^2}{d_{\nu_R}} + \frac{|c_{\nu_R}|^2 - |b_{\nu_R}| d_{\nu_R}}{d_{\nu_R}} \frac{|a_{\nu_D}|^2}{a_{\nu_R}^2} e^{-i2(\phi_c + \phi_{\nu_D})} + 2 b_{\nu_D} \frac{|a_{\nu_D}|}{a_{\nu_R}} e^{-i\phi_{\nu_D}} - 2 \frac{c_{\nu_D} |c_{\nu_R}|}{d_{\nu_R}} \frac{|a_{\nu_D}|}{a_{\nu_R}} e^{-i(\phi_c + \phi_{\nu_D})},$$

$$c_{\nu_L} = \frac{c_{\nu_D} d_{\nu_D}}{d_{\nu_R}} + \frac{c_{\nu_D} |a_{\nu_D}|}{a_{\nu_R}} e^{-i\phi_{\nu_D}} - \frac{|c_{\nu_R}||a_{\nu_D}| d_{\nu_D}}{a_{\nu_R} d_{\nu_R}} e^{-i(\phi_c + \phi_{\nu_D})}.$$

In general the left handed Majorana neutrino mass matrix M_{ν_L}, is a symmetric complex matrix but not hermitian. Therefore, the two zeros texture of M_{ν_L} is not a two zeros Fritzsch texture $M^{(F)}$. However, if we extend the meaning of Fritzsch texture to include the complex symmetric but no hermitian case of the matrix M_{ν_L}, we could say that the two zeros Fritzsch texture is invariant under the action of the seesaw mechanism of type I.

Here we study the case when $M_j^{(F)} = P_j^\dagger \bar{M}_j^{(F)} P_j$ with $j = u, d, l$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\phi_j} & 0 \\ 0 & 0 & e^{-i\phi_j} \end{pmatrix} \begin{pmatrix} 0 & |a_j| & 0 \\ |a_j| & b_j & c_j \\ 0 & c_j & d_j \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_j} & 0 \\ 0 & 0 & e^{i\phi_j} \end{pmatrix},$$

and $M_{\nu_L} = Q \bar{M}_{\nu_L} Q$,

$$\begin{pmatrix} e^{-i\phi_2} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & a_{\nu_L} & 0 \\ a_{\nu_L} & |b_{\nu_L}| & |c_{\nu_L}| \\ 0 & |c_{\nu_L}| & d_{\nu_L} \end{pmatrix} \begin{pmatrix} e^{-i\phi_2} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The matrices $\bar{M}^{(F)}$ and $\bar{M}_{\nu_L}$ can be diagonalized by an orthogonal real matrix, $\mathbb{O}$, so that:

$$\mathbb{O}_j^T \bar{M}_j^{(F)} \mathbb{O}_j = \mathrm{diag}\left(m_{j1}, m_{j2}, m_{j3}\right),$$

$$\mathbb{O}_\nu^T \bar{M}_\nu^{(F)} \mathbb{O}_\nu = \mathrm{diag}\left(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}\right),$$

Then,

$$M_j^{(F)} = P_j^\dagger \bar{M}_j^{(F)} P_j = U_j^\dagger \ \mathrm{diag}\ \left(m_{j1}, m_{j2}, m_{j3}\right) U_j$$

$$M_{\nu_{\nu_L}} = Q \bar{M}_\nu Q = U_\nu \ \mathrm{diag}\ \left(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}\right) U_\nu^T$$

with the unitary matrix:

$$U_j \equiv \mathbb{O}_j^T P_j, \qquad U \equiv Q \mathbb{O}_\nu$$

In our case, with a normal hierachy in the eigenvalues, $m_2 = -|m_2|$ and $d \equiv 1 - \delta$. The orthogonal real matrix is;

$$\mathbb{O} = \begin{pmatrix} \left[\frac{\widetilde{m}_2 f_1}{D_1}\right]^{\frac{1}{2}} & -\left[\frac{\widetilde{m}_1 f_2}{D_2}\right]^{\frac{1}{2}} & \left[\frac{\widetilde{m}_1 \widetilde{m}_2 \delta}{D_3}\right]^{\frac{1}{2}} \\ \left[\frac{\widetilde{m}_1 (1-\delta) f_1}{D_1}\right]^{\frac{1}{2}} & \left[\frac{\widetilde{m}_2 (1-\delta) f_2}{D_2}\right]^{\frac{1}{2}} & \left[\frac{(1-\delta)\delta}{D_3}\right]^{\frac{1}{2}} \\ -\left[\frac{\widetilde{m}_1 f_2 \delta}{D_1}\right]^{\frac{1}{2}} & -\left[\frac{\widetilde{m}_2 f_1 \delta}{D_2}\right]^{\frac{1}{2}} & \left[\frac{f_1 f_2}{D_3}\right]^{\frac{1}{2}} \end{pmatrix},$$

where $\widetilde{m}_1 = m_1/m_3$, $\widetilde{m}_2 = m_2/m_3$, $f_1 = (1 - \widetilde{m}_1 - \delta)$, $f_2 = (1 + \widetilde{m}_2 - \delta)$.

$$D_1 = (1-\delta)(\widetilde{m}_1 + \widetilde{m}_2)(1 - \widetilde{m}_1), \quad D_2 = (1-\delta)(\widetilde{m}_1 + \widetilde{m}_2)(1 + \widetilde{m}_2),$$

$$0 < \delta < 1 - \widetilde{m}_1.$$

Mixing matrices

$$V_{CKM} = U_u U_d^\dagger = \mathbf{O_u}^T P^{(u-d)} \mathbf{O}_d,$$

where $P^{(u-d)} = \mathrm{diag}\left[1, e^{i\phi}, e^{i\phi}\right]$ with $\phi = \phi_u - \phi_d$ The Jarlskog invariant is

$$J_q = \Im m\left[V_{us} V_{cs}^* V_{ub}^* V_{cb}\right],$$

The inner angles of the unitarity triangle are

$$\alpha \equiv \arg\left(-\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*}\right), \quad \beta \equiv \arg\left(-\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*}\right), \quad \gamma \equiv \arg\left(-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}\right).$$

The quark mixing matrix computed from the theoretical expresion is

$$\left| V_{CKM}^{th} \right| = \begin{pmatrix} 0{,}97421 & 0{,}22560 & 0{,}003369 \\ 0{,}22545 & 0{,}97335 & 0{,}041736 \\ 0{,}008754 & 0{,}04094 & 0{,}99912 \end{pmatrix}.$$

with the following numerical values of quark mass ratio,

$$\widetilde{m}_u = 2{,}5469 \times 10^{-5}, \quad \widetilde{m}_c = 3{,}9918 \times 10^{-3},$$
$$\widetilde{m}_d = 1{,}5261 \times 10^{-3}, \quad \widetilde{m}_s = 3{,}2319 \times 10^{-2},$$

and the corresponding best values of the parameters δ_u and δ_d :

$$\delta_u = 3{,}829 \times 10^{-3}, \quad \delta_d = 4{,}08 \times 10^{-4}$$

and with the Dirac CP violating phase $\phi = 90^o$. The inner angles of the unitarity triangle,

$$\alpha^{th} = 91^o, \quad \beta^{th} = 20^o, \quad \gamma^{th} = 68^o.$$

The Jarlskog invariant takes the value $J_q^{th} = 2{,}9 \times 10^{-5}$, all in

The lepton mixing matrix $U_{_{PMNS}}$

$$U_{_{PMNS}}^{th} = \mathbf{O}_l^T P^{(\nu - l)} \mathbf{O}_\nu K,$$

The matrix $P^{(\nu-l)} = \mathrm{diag}\left[1, e^{i\Phi_1}, e^{i\Phi_2}\right]$ is the diagonal matrix of the Dirac phases, with $\Phi_1 = 2\phi_1 - \phi_l$ and $\Phi_2 = \phi_1 - \phi_l$.
The rephasing invariant related to the Dirac phase, analogous to the Jarlskog invariant in the quark sector, is given as:

$$J_l \equiv \Im m \left[U_{e1}^* U_{\mu 3}^* U_{e3} U_{\mu 1} \right],$$

The other two rephasing invariants associated with the two Majorana phases in the U_{PMNS} matrix, can be chosen as:

$$S_1 \equiv \Im m \left[U_{e1} U_{e3}^{*} \right], \quad S_2 \equiv \Im m \left[U_{e2} U_{e3}^{*} \right]. \qquad (1)$$

The rephasing invariants associated with the Majorana phases are not uniquely defined, but the ones shown in J_l and in (1) are relevant for the definition of the effective Majorana neutrino mass, m_{ee}, in the neutrinoless double beta decay.

The lepton mixing matrix computed from the theoretical expresion is

$$\left| U_{PMNS}^{th} \right| = \begin{pmatrix} 0{,}820421 & 0{,}568408 & 0{,}061817 \\ 0{,}385027 & 0{,}613436 & 0{,}689529 \\ 0{,}422689 & 0{,}548277 & 0{,}721615 \end{pmatrix},$$

with the charged lepton masses;

$$m_e = 0{,}5109\mathrm{MeV}, \quad m_\mu = 105{,}685\mathrm{MeV} \quad \text{and} \quad m_\tau = 1776{,}99\mathrm{GeV},$$

and taking for the masses of the left handed Majorana neutrinos a normal hierarchy with the best numerical values,

$$m_{\nu_1} = 2{,}7\times10^{-3}\mathrm{eV}, \quad m_{\nu_2} = 9{,}1\times10^{-3}\mathrm{eV}, \quad \text{and} \quad m_{\nu_3} = 4{,}7\times10^{-2}\mathrm{eV}.$$

and corresponding best values of the parameters

$$\delta_e = 0{,}06, \quad \delta_\nu = 0{,}522$$

the Dirac CP violating phases

$$\Phi_1 = 0 \quad \text{and} \quad \Phi_2 = 90^o.$$

The value of the rephasing invariant we related to the Dirac phase is $J_l^{th} = 8{,}8 \times 10^{-3}$.
The other two rephasing invariants associated with the two Majorana phases in the U_{PMNS} matrix, can not be determined numerically, since we have no information about the Majorana phases β_1 and β_2.
Therefore in order to make an estimate we maximize the rephasing invariants S_1 and S_2 thus obtaining a numerical value for the Majorana phases β_1 and β_2. Then, the maximum values of rephasing invariants are

$$S_1^{max} = -4{,}9 \times 10^{-2}, \quad S_2^{max} = 3{,}4 \times 10^{-2},$$

with $\beta_1 = -1{,}4^o$ and $\beta_2 = 77^o$.

Mixing angles for quarks sector

$$\sin^2\theta_{12}^q \approx \frac{\frac{\widetilde{m}_d}{\widetilde{m}_s} + \frac{\widetilde{m}_u}{\widetilde{m}_c} - 2\sqrt{\frac{\widetilde{m}_u}{\widetilde{m}_c}\frac{\widetilde{m}_d}{\widetilde{m}_s}}\cos\phi}{\left(1+\frac{\widetilde{m}_u}{\widetilde{m}_c}\right)\left(1+\frac{\widetilde{m}_d}{\widetilde{m}_s}\right)},$$

$$\sin^2\theta_{23}^q \approx \frac{\left(\sqrt{\delta_u}-\sqrt{\delta_d}\right)^2}{\left(1+\frac{\widetilde{m}_u}{\widetilde{m}_c}\right)}, \quad \sin^2\theta_{13}^q \approx \frac{\frac{\widetilde{m}_u}{\widetilde{m}_c}\left(\sqrt{\delta_u}-\sqrt{\delta_d}\right)^2}{\left(1+\frac{\widetilde{m}_u}{\widetilde{m}_c}\right)}.$$

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$$\sin^2\theta_{23}^q \approx \frac{\left(\sqrt{\delta_u}-\sqrt{\delta_d}\right)^2}{\left(1+\frac{\widetilde{m}_u}{\widetilde{m}_c}\right)}, \quad \sin^2\theta_{13}^q \approx \frac{\frac{\widetilde{m}_u}{\widetilde{m}_c}\left(\sqrt{\delta_u}-\sqrt{\delta_d}\right)^2}{\left(1+\frac{\widetilde{m}_u}{\widetilde{m}_c}\right)}.$$

We reproduce the numerical value of the quark mixing angles

$$\theta_{12}^q = 13^o, \quad \theta_{23}^q = 2{,}38^o, \quad \theta_{13}^q = 0{,}19^o,$$

in very good agreement with experimental data

Mixing angles for lepton sector

$$\sin^2\theta_{12}^l = \frac{f_{\nu 2}}{(1+\widetilde{m}_{\nu_2})(1-\delta_\nu)}\left\{\frac{\frac{\widetilde{m}_{\nu_1}}{\widetilde{m}_{\nu_2}} + \frac{\widetilde{m}_e}{\widetilde{m}_\mu}(1-\delta_\nu) + 2\sqrt{\frac{\widetilde{m}_{\nu_1}}{\widetilde{m}_{\nu_2}}\frac{\widetilde{m}_e}{\widetilde{m}_\mu}(1-\delta_\nu)}\cos\phi_1}{\left(1+\frac{\widetilde{m}_{\nu_1}}{\widetilde{m}_{\nu_2}}\right)\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)}\right\},$$

$$\sin^2\theta_{23}^l \approx \frac{\delta_\nu + \delta_e f_{\nu 2} - \sqrt{\delta_\nu \delta_e f_{\nu 2}}\cos(\phi_1 - \phi_2)}{\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)(1+\widetilde{m}_{\nu_2})},$$

$$\sin^2\theta_{13}^l \approx \frac{\delta_\nu}{\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)(1+\widetilde{m}_{\nu_2})}\left\{\frac{\widetilde{m}_e}{\widetilde{m}_\mu} + \frac{\widetilde{m}_{\nu_1}\widetilde{m}_{\nu_2}}{(1-\delta_\nu)} - 2\sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\frac{\widetilde{m}_{\nu_1}\widetilde{m}_{\nu_2}}{(1-\delta_\nu)}}\cos\phi_1\right\},$$

Mixing angles for lepton sector

$$\sin^2\theta^l_{12} = \frac{f_{\nu 2}}{(1+\widetilde{m}_{\nu_2})(1-\delta_\nu)} \left\{ \frac{\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu_2}} + \frac{\widetilde{m}_e}{\widetilde{m}_\mu}(1-\delta_\nu) + 2\sqrt{\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu_2}}\frac{\widetilde{m}_e}{\widetilde{m}_\mu}(1-\delta_\nu)}\cos\phi_1}{\left(1+\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu_2}}\right)\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)} \right\},$$

$$\sin^2\theta^l_{23} \approx \frac{\delta_\nu + \delta_e f_{\nu 2} - \sqrt{\delta_\nu\delta_e f_{\nu 2}}\cos(\phi_1-\phi_2)}{\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)(1+\widetilde{m}_{\nu_2})},$$

$$\sin^2\theta^l_{13} \approx \frac{\delta_\nu}{\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)(1+\widetilde{m}_{\nu_2})} \left\{ \frac{\widetilde{m}_e}{\widetilde{m}_\mu} + \frac{\widetilde{m}_{\nu_1}\widetilde{m}_{\nu_2}}{(1-\delta_\nu)} - 2\sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\frac{\widetilde{m}_{\nu_1}\widetilde{m}_{\nu_2}}{(1-\delta_\nu)}}\cos\phi_1 \right\},$$

We obtain the following numerical values

$$\theta^l_{12} = 34{,}3^o, \quad \theta^l_{23} = 43{,}6^o, \quad \theta^l_{13} = 3{,}4^o$$

in very good agreement with experimental data

Quark Lepton Complementarity

$$\tan\left(\theta_{12}^{q}+\theta_{12}^{l}\right)=1+\Delta_{12}^{th},$$

where

$$\Delta_{12}^{th}=\frac{\left(\frac{\widetilde{m}_d}{\widetilde{m}_s}+\frac{\widetilde{m}_u}{\widetilde{m}_c}\right)^{\frac{1}{2}}\left[(1-\delta_\nu)\left(1+\widetilde{m}_{\nu 2}\right)\left(1+\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}\right)\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)-f_{\nu 2}\left(\sqrt{\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}}+\sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}(1-\delta_\nu)}\right)^2\right]^{\frac{1}{2}}}{\left(1+\frac{\widetilde{m}_d}{\widetilde{m}_s}\frac{\widetilde{m}_u}{\widetilde{m}_c}\right)^{\frac{1}{2}}\left[(1-\delta_\nu)\left(1+\widetilde{m}_{\nu 2}\right)\left(1+\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}\right)\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)-f_{\nu 2}\left(\sqrt{\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}}+\sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}(1-\delta_\nu)}\right)^2\right]^{\frac{1}{2}}}\times$$

$$\times\frac{+\sqrt{f_{\nu 2}}\left(\sqrt{\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}}+\sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}(1-\delta_\nu)}\right)\left[\left(\frac{\widetilde{m}_d}{\widetilde{m}_s}+\frac{\widetilde{m}_u}{\widetilde{m}_c}\right)^{\frac{1}{2}}+\left(1+\frac{\widetilde{m}_d}{\widetilde{m}_s}\frac{\widetilde{m}_u}{\widetilde{m}_c}\right)^{\frac{1}{2}}\right]}{+\sqrt{\frac{\widetilde{m}_d}{\widetilde{m}_s}+\frac{\widetilde{m}_u}{\widetilde{m}_c}f_{\nu 2}}\left(\sqrt{\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}}+\sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}(1-\delta_\nu)}\right)}$$

$$\tan\left(\theta_{23}^{q}+\theta_{23}^{l}\right)=1+\Delta_{23}^{th},$$

where

$$\Delta_{23}^{th}=\frac{\left[\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)\left(1+\widetilde{m}_{\nu 2}\right)-\delta_\nu-\delta_e f_{\nu 2}\right]^{\frac{1}{2}}\left(\sqrt{\delta_u}-\sqrt{\delta_d}\right)+\sqrt{\delta_\nu+\delta_e f_{\nu 2}}\left(\sqrt{1+\frac{\widetilde{m}_u}{\widetilde{m}_c}-\left(\sqrt{\delta_u}-\sqrt{\delta_d}\right)^2}+\left(\sqrt{\delta_u}-\sqrt{\delta_d}\right)\right)}{\left[\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)\left(1+\widetilde{m}_{\nu 2}\right)-\delta_\nu-\delta_e f_{\nu 2}\right]^{\frac{1}{2}}\sqrt{1+\frac{\widetilde{m}_u}{\widetilde{m}_c}-\left(\sqrt{\delta_u}-\sqrt{\delta_d}\right)^2}-\left(\sqrt{\delta_u}-\sqrt{\delta_d}\right)\sqrt{\delta_\nu+\delta_e f_{\nu 2}}}+$$
$$+\frac{\left[\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)\left(1+\widetilde{m}_{\nu 2}\right)-\delta_\nu-\delta_e f_{\nu 2}\right]^{\frac{1}{2}}\sqrt{1+\frac{\widetilde{m}_u}{\widetilde{m}_c}-\left(\sqrt{\delta_u}-\sqrt{\delta_d}\right)^2}}{\left[\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)\left(1+\widetilde{m}_{\nu 2}\right)-\delta_\nu-\delta_e f_{\nu 2}\right]^{\frac{1}{2}}\sqrt{1+\frac{\widetilde{m}_u}{\widetilde{m}_c}-\left(\sqrt{\delta_u}-\sqrt{\delta_d}\right)^2}-\left(\sqrt{\delta_u}-\sqrt{\delta_d}\right)\sqrt{\delta_\nu+\delta_e f_{\nu 2}}}$$

$$\tan\left(\theta^q_{13}+\theta^l_{13}\right) = \frac{\sqrt{\frac{\widetilde{m}_u}{\widetilde{m}_c}}\left(\sqrt{\delta_u}-\sqrt{\delta_d}\right)\left[\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)\left(1+\widetilde{m}_{\nu 2}\right)-\delta_\nu\left(\sqrt{\frac{\widetilde{m}_{\nu 1}\widetilde{m}_{\nu 2}}{(1-\delta_\nu)}}-\sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}}\right)^2\right]^{\frac{1}{2}}+}{\sqrt{1+\frac{\widetilde{m}_u}{\widetilde{m}_c}-\frac{\widetilde{m}_u}{\widetilde{m}_c}\left(\sqrt{\delta_u}-\sqrt{\delta_d}\right)^2}\left[\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)\left(1+\widetilde{m}_{\nu 2}\right)-\delta_\nu\left(\sqrt{\frac{\widetilde{m}_{\nu 1}\widetilde{m}_{\nu 2}}{(1-\delta_\nu)}}-\sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}}\right)^2\right]^{\frac{1}{2}}-}\times$$

$$\times\frac{+\sqrt{\delta_\nu}\left(\sqrt{\frac{\widetilde{m}_{\nu 1}\widetilde{m}_{\nu 2}}{(1-\delta_\nu)}}-\sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}}\right)\sqrt{1+\frac{\widetilde{m}_u}{\widetilde{m}_c}-\frac{\widetilde{m}_u}{\widetilde{m}_c}\left(\sqrt{\delta_u}-\sqrt{\delta_d}\right)^2}}{-\sqrt{\frac{\widetilde{m}_u}{\widetilde{m}_c}}\left(\sqrt{\delta_u}-\sqrt{\delta_d}\right)\sqrt{\delta_\nu}\left(\sqrt{\frac{\widetilde{m}_{\nu 1}\widetilde{m}_{\nu 2}}{(1-\delta_\nu)}}-\sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}}\right)}$$

We obtain

$$\theta^q_{12}+\theta^l_{12} = 45^o + 2{,}4^o.$$

$$\theta^q_{23}+\theta^l_{23} = 45^o + 1^o,$$

$$\theta^q_{13}+\theta^l_{13} = 3{,}6^o.$$

quark-lepton complementarity arises from the combined effect of two factors:

1. The strong mass hierarchy of the Dirac fermions produces small and very small mass ratios of u and d-type quarks and charged leptons. The quark mass hierarchy is then reflected in a similar hierarchy of small and very small quark mixing angles.
2. The normal seesaw mechanism gives very small masses to the left handed Majorana neutrinos with relatively large values of the neutrino mass ratio m_{ν_1}/m_{ν_2} and allows for large θ^l_{12} and θ^l_{23} mixing angles.

The effective Majorana masses

The square of the magnitudes of the effective Majorana neutrino masses are

$$|\langle m_{ll}\rangle|^2 = \sum_{j=1}^{3} m_{\nu_j}^2 \, |U_{lj}|^4 + 2 \sum_{j<k}^{3} m_{\nu_j} m_{\nu_k} \, |U_{lj}|^2 \, |U_{lk}|^2 \cos 2 \left(w_{lj} - w_{lk} \right),$$

where $w_{lj} = \arg\{U_{lj}\}$, with $l = e, \mu, \tau$; this term includes phases of both types, Dirac and Majorana.

$$
\begin{aligned}
|\langle m_{ee}\rangle|^2 \approx & \frac{1}{\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)^2\left(1+\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}\right)^2}\left\{ m_{\nu_1}^2\left(1-4\sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}\left(1-\delta_\nu\right)}\right)\right. \\
&+\frac{m_{\nu_2}^2 f_{\nu 2}^2}{\left(1+\widetilde{m}_{\nu_2}\right)^2(1-\delta_\nu)^2}\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}\left(\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}+4\sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}\left(1-\delta_\nu\right)}+6\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\left(1-\delta_\nu\right)\right) \\
&+2\frac{m_{\nu_1} m_{\nu_3}\delta_\nu}{\left(1+\widetilde{m}_{\nu_2}\right)}\left(1+\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}\right)\left(\sqrt{\frac{\widetilde{m}_{\nu 1}\widetilde{m}_{\nu 2}}{(1-\delta_\nu)}}-\sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}}\right)^2\cos 2(w_{e1}-w_{e3}) \\
&+2\frac{m_{\nu_1} m_{\nu_2} f_{\nu 2}}{\left(1+\widetilde{m}_{\nu_2}\right)(1-\delta_\nu)}\left(\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}+2\left(1-\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}\right)\sqrt{\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\left(1-\delta_\nu\right)}\right)\cos 2(w_{e1}-w_{e2}) \\
&\left.+2\frac{m_{\nu_2} m_{\nu_3} f_{\nu 2}\delta_\nu}{\left(1+\widetilde{m}_{\nu_2}\right)^2(1-\delta_\nu)^2}\left(1+\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}\right)\left(2\widetilde{m}_{\nu_1}\widetilde{m}_{\nu_2}+\sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}\left(1-\delta_\nu\right)}\right)\cos 2(w_{e2}-w_{e3})\right\}
\end{aligned}
$$

with , $w_{e2} = \beta_1$, and

$$w_{e1} = \arctan\left\{ -\frac{\sqrt{\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu_2}}\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\delta_e\delta_\nu f_{\nu 2}}}{\sqrt{(1-\delta_\nu)}+\sqrt{\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu_2}}\frac{\widetilde{m}_e}{\widetilde{m}_\mu}}(1-\delta_\nu)} \right\},$$

$$w_{e3} = \arctan\left\{ \frac{\sqrt{\delta_\nu}\left(\sqrt{\widetilde{m}_{\nu_1}\widetilde{m}_{\nu_2}} - \sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}}(1-\delta_\nu)\right)\sin\beta_2 + \sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\delta_e f_{\nu 2}(1-\delta_\nu)}\cos\beta_2}{\sqrt{\delta_\nu}\left(\sqrt{\widetilde{m}_{\nu_1}\widetilde{m}_{\nu_2}} - \sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}}(1-\delta_\nu)\right)\cos\beta_2 - \sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\delta_e f_{\nu 2}(1-\delta_\nu)}\sin\beta_2} \right\}$$

$$|\langle m_{\mu\mu}\rangle|^2 \approx \frac{1}{\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)^2\left(1+\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}\right)^2\left(1+\widetilde{m}_{\nu_2}\right)}\left\{\frac{m_{\nu_3}^2}{\left(1+\widetilde{m}_{\nu_2}\right)}\left(1+\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu_2}}\right)^2(\delta_\nu+2\delta_e f_{\nu 2})\right.$$
$$+\frac{m_{\nu_2}^2}{\left(1+\widetilde{m}_{\nu_2}\right)(1-\delta_\nu)}\left(1-\delta_\nu-4\sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu_2}}(1-\delta_\nu)}+6\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu_2}}\right)$$
$$+2m_{\nu_1}m_{\nu_2}f_{\nu 2}\left(\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu_2}}(1-\delta_\nu)+2\sqrt{\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu_2}}\frac{\widetilde{m}_e}{\widetilde{m}_\mu}(1-\delta_\nu)}\left(1-\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu_2}}\right)\right)\cos 2(w_{\mu 1}-w_{\mu 2})$$
$$+2m_{\nu_1}m_{\nu_3}\left(1+\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu_2}}\right)\left(2\delta_\nu\sqrt{\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu_2}}\frac{\widetilde{m}_e}{\widetilde{m}_\mu}(1-\delta_\nu)}+\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu_2}}(1-\delta_\nu)(\delta_\nu+\delta_e f_{\nu 2})\right)\cos 2(w_{\mu 1}$$
$$+2\frac{m_{\nu_2}m_{\nu_3}f_{\nu 2}}{\left(1+\widetilde{m}_{\nu_2}\right)(1-\delta_\nu)}\left(1+\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu_2}}\right)\left((1-\delta_\nu)(\delta_\nu+\delta_e f_{\nu 2})-2\delta_\nu\sqrt{\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu_2}}\frac{\widetilde{m}_e}{\widetilde{m}_\mu}(1-\delta_\nu)}\right)\cos 2(w_\mu$$

with

$$w_{\mu 1} = \arctan \left\{ \frac{\sqrt{\frac{\widetilde{m}_{\nu_1}}{\widetilde{m}_{\nu_2}} \delta_e \delta_\nu f_{\nu 2}}}{\sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu} (1-\delta_\nu)} + \sqrt{\frac{\widetilde{m}_{\nu_1}}{\widetilde{m}_{\nu_2}} (1-\delta_\nu)}} \right\},$$

and

$$w_{\mu 2} = \arctan \left\{ \frac{-\sqrt{f_{\nu 2}} \sin \beta_1 - \sqrt{\delta_e \delta_\nu} \cos \beta_1}{-\sqrt{f_{\nu 2}} \cos \beta_1 + \sqrt{\delta_e \delta_\nu} \sin \beta_1} \right\}$$

$$w_{\mu 3} = \arctan \left\{ \frac{-\sin \beta_2 + \sqrt{f_{\nu 2}} \cos \beta_2}{-\cos \beta_2 - \sqrt{f_{\nu 2}} \sin \beta_2} \right\}.$$

$$|\langle m_{ee}\rangle| \approx 4{,}6 \times 10^{-3}\mathrm{eV}, \qquad |\langle m_{\mu\mu}\rangle| \approx 2{,}1 \times 10^{-2}\mathrm{eV}. \qquad (2)$$

These numerical values are consistent with the vary small experimentally determined upper bounds for the reactor neutrino mixing angle θ^{l}_{13}.

Conclusions

- The strong hierarchy in the mass spectra of the quarks and charged leptons explains the small or very small quark mixing angles, the very small charged lepton mass ratio explain the very small θ_{13}^{l} which, in our scheme, is independent of the neutrino masses.
- The see-saw mechanism type I gives very small masses to the left handed Majorana neutrinos with relatively large values of the neutrino mass ratio m_{ν_1}/m_{ν_2} and allows for large θ_{12}^{l} and θ_{23}^{l} mixing angles.

- We obtain the analytical expresion for mixing angles, mixing matrices, invariant associated at CP violation phases and for the effective Majorana masses.
- We can reproduce the numerical values of both mixing matrices, V_{CKM} and U_{PMNS}, in very good agreement with experimental data.