XII MEXICAN WORKSHOP ON PARTICLES AND FIELDS

CP violation of the neutral Higgs bosons in a THDM and MSSM

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Motivation

The Higgs sector with CP violation has very particular and interesting issues:

- Studying mechanisms and new sources of CP violation
  $\longrightarrow$ is a required condition for **baryogenesis**.

- The heavy Higgs bosons are unstable particles they occur as resonances of the propagator, related to self-energies.
  $\longrightarrow$ The masses as poles of the propagator are of Breit-Wigner's form: $[s - M_\phi^2 + iM_\phi\Gamma_\phi]^{-1}$

- Degeneracy of the heavy neutral Higgses in the CP invariant case is manifested with H^0 and A^0 as incoherent states.
  $\longrightarrow$ in the CPV, degeneracy will be revealed with two mix states which are coherent and interesting features will appear.

- In a CP non-invariant Higgs sector the physical states are no longer CP defined
  $\longrightarrow$ which implies differences in amplitudes where neutral Higgses are involved.

Outline

- **I.** CP invariant Higgs sector
- **II.** Extended Higgs sector with CP violation
- **III.** Self-energy neutral Higgs mass corrections
- **IV.** Degeneracy of heavy neutral CP non-invariant Higgs bosons.
- Conclusions

I. CP invariant Higgs sector

- Two SU(2) Higgs doublets:

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + H_1 + iA_1) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + H_2 + iA_2) \end{pmatrix}$$

with

H_i , $CP = 1 \longrightarrow$ scalar fields,

and

A_i , $CP = -1 \longrightarrow$ pseudoscalar fields.

- SSB: Assuming the scalar fields to develop nonzero vacuum expectation values that break $SU(2)_L$

$$< \Phi_1 >= \frac{v_1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad < \Phi_2 >= \frac{v_2}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1}$$

Defining: $\tan\beta = \frac{v_2}{v_1}$ and $v = (v_1^2 + v_2^2)^{1/2} \approx 246\ GeV.$

I.1 THDM and MSSM Higgs potential

In terms of the two $SU(2)$ Higgs doublet model potential (2HDM)

$$\begin{aligned}\mathcal{L}_V^{2HDM} &= \mu_1^2(\Phi_1^\dagger\Phi_1) + \mu_2^2(\Phi_2^\dagger\Phi_2) + m_{12}^2(\Phi_1^\dagger\Phi_2) + \lambda_1(\Phi_1^\dagger\Phi_1)^2 \\ &+\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\end{aligned} \tag{2}$$

And in the minimal supersymmetric case

$$\begin{aligned}\mu_1^2 = -m_1^2 - |\mu|^2, \qquad \mu_2^2 = -m_2^2 - |\mu|^2, \quad m_{12}^2 = B\mu \\ \lambda_1 = \lambda_2 = -\frac{1}{4}(g_w^2 + g'^2), \qquad \lambda_3 = -\tfrac{1}{4}(g_w^2 - g'^2), \quad \lambda_4 = \tfrac{1}{2}g_w^2.\end{aligned} \tag{3}$$

At **tree level** we have the CP-even neutral Higgs masses releted using m_{A^0} as free parameter:

$$
\begin{aligned}
m^2_{h,H} &= \frac{1}{2}(m^2_A + m^2_Z) \\
&\mp \frac{1}{2}\sqrt{(m^2_A + m^2_Z)^2 - 4m^2_A m^2_Z \cos^2 2\beta} \\
m^{\pm}_H &= m^2_A + \cos^2\theta_w m^2_Z
\end{aligned} \tag{4}
$$

- The relations within MSSM parameters imposse, at tree level, a strong hierarchical structure on mass spectrum:

  $m_h < m_Z$, $m_A < m_H$ and $m_W < m_{H^\pm}$, $\longrightarrow$ which is broken by radiative corrections.

- Including **radiative corrections** the upper limit on light Higgs mass, m_h will lead on the limits for the parameters $m_A \gg m_Z$ and $\tan\beta \gg 1$.

- In this limit the heavier CP-even, charged and CP-odd Higgs bosons become almost degenerate in mass:

$$m_H \simeq m_{H^\pm} \simeq m_A.$$

Considering **CP violation** through radiative corrections the situation will change.

II. Extended Higgs sector with CP $violation$

In terms of the two $SU(2)$ Higgs doublet model potential (2HDM) with CP violation

$$
\begin{aligned}
\mathcal{L}_V^{2HDM} \quad = \quad & \mu_1^2(\Phi_1^\dagger\Phi_1) + \mu_2^2(\Phi_2^\dagger\Phi_2) + \mathbf{m_{12}^2}(\Phi_1^\dagger\Phi_2) + \lambda_1(\Phi_1^\dagger\Phi_1)^2 \\
& + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
& + \boldsymbol{\lambda_5}(\Phi_1^\dagger\Phi_2)^2 + [\boldsymbol{\lambda_6}(\Phi_1^\dagger\Phi_1) + \boldsymbol{\lambda_7}(\Phi_2^\dagger\Phi_2)](\Phi_1^\dagger\Phi_2) + h.c. \qquad (5)
\end{aligned}
$$

In the MSSM the complex couplings emerge at **one-loop level** couplings

$$
\mathbf{(1)} \;\; m_{12}^2 = m_{12}^{2R} + i m_{12}^{2I} \qquad\qquad \mathbf{(2)} \;\; \lambda_{5,6,7} = \lambda_{5,6,7}^R + i\lambda_{5,6,7}^I \qquad (6)
$$

and necessarily generates the mixing of heavy neutral $H - A$ Higgses.

[Grzadkowski,Gunion,Kalinowski 99],[Choi,Kalinowski,Liao,Zerwas 05]

II.1 MSSM CP violation sources

MSSM Higgs sector CP violation sources are:

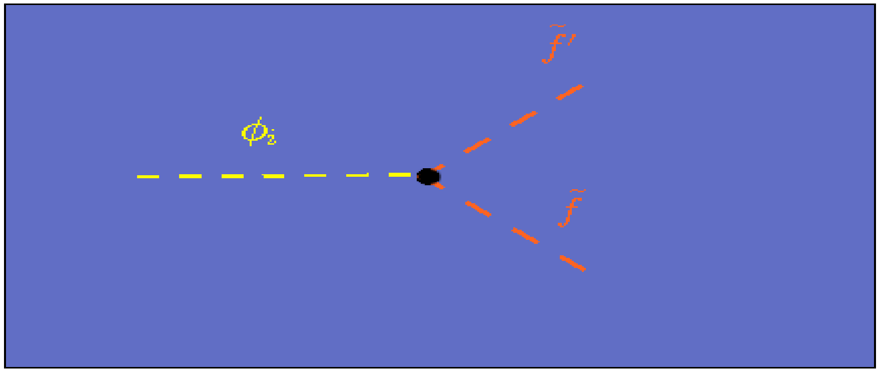
① a relative phase between doublets:

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + H_1 + iA_1) \end{pmatrix}, \quad \Phi_2 = e^{i\xi} \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + H_2 + iA_2) \end{pmatrix}$$

② on the Soft SUSY Breaking $\mathcal{L}$

$$\mathcal{L}^{\tilde{q}}_{soft} = -(A_{u,ij} \bar{\tilde{U}}^i \tilde{Q}^j H_1 + A_{d,ij} \bar{\tilde{D}}^i \tilde{Q}^j H_2 + h.c) \quad (7)$$

complex trilinear couplings, $\phi \tilde{f} \tilde{f}$:



II.2 Neutral Higgs mass matrix

The complete 4×4 renormalized neutral Higgs mass matrix can be written as

$$\mathcal{M}^2_{\phi^0} = m^{(0)2}_{\phi^0} + \delta\mathcal{M}^2_{\phi^0} = M^2_{\phi^0} - iM_{\phi^0}\Gamma_{\phi^0} \tag{8}$$

The radiative correction mass, $\delta\mathcal{M}_{\phi^0}$ written in the basis $H_1 H_2 A_1 A_2$, may be expressed as

$$\delta\mathcal{M}_{HHAA} = \begin{pmatrix} \delta M^S & \delta M^{SP} \\ \delta M^{SP\dagger} & \delta M^P \end{pmatrix} \tag{9}$$

with

$$\delta M^S = \begin{pmatrix} M_{H_1H_1} & M_{H_1H_2} \\ M^\dagger_{H_2H_1} & M_{H_2H_2} \end{pmatrix}; \quad \delta M^P = \begin{pmatrix} M_{A_1A_1} & M_{A_1A_2} \\ M^\dagger_{A_2A_1} & M_{A_2A_2} \end{pmatrix}; \tag{10}$$

(11)

and the mixing part

$$\delta M^{SP} = \begin{pmatrix} M_{H_1A_1} & M_{H_1A_2} \\ M^\dagger_{H_2A_1} & M_{H_2A_2} \end{pmatrix} \tag{12}$$

[Frank,Hahn,Heinemeyer,Hollik,Rzehak and Weiglein 07]

II.3 Neutral Higgs mass matrix for 2HDM

The 3×3 squared mass matrix $\mathcal{M}_0^2$ of neutral Higgs fields, is obtained after the rotation that decouples the Goldstone boson, and has the following form, derived from $\mathcal{L}_V^{2HDM}$

$$\mathcal{M}_0^2 = v^2 \begin{pmatrix} \lambda & -\hat{\lambda} & -\hat{\lambda}_p \\ -\hat{\lambda} & \lambda - \lambda_A + \frac{1}{v^2} M_A^2 & -\lambda_p \\ -\hat{\lambda}_p & -\lambda_p & \frac{1}{v^2} M_A^2 . \end{pmatrix} \tag{13}$$

The λ, $\hat{\lambda}$ and λ_A are functions of $\Re e \lambda_i$ while λ_p and $\hat{\lambda}_p$ are functions of $\Im m \lambda_i$ in $\mathcal{L}_V^{2HDM}$.

To be supplemented by the anti-hermitian decay matrix $-iM\Gamma(s)$ which is a function of s. This includes the widths of the Higgs states in the diagonal elements, as well as the transition matrix elements for any combination of pairs of states in the off diagonal elements.

$$\mathcal{M}^2(s) = \mathcal{M}_0^2 - iM\ \Gamma(s)\ . \tag{14}$$

II.4 CP non-invariant neutral Higgs bosons mass matrix $\mathcal{M}^2(s)$

In the decoupling limit, defined by the inequality [Gunion, Haber 03]

$$m_A^2 >> |\lambda_i| v^2, \qquad (15)$$

the, mixing between the light state, $H_1(\rightarrow\ h^0)$, and the heavy states, H_2 and H_3, is small, compared with the mixing of the nearly degenerate heavy Higgs states H_2 and H_3.
[Félix-Beltrán,Gómez-Bock,Hernández, Mondragón,Mondragón *to appear*]

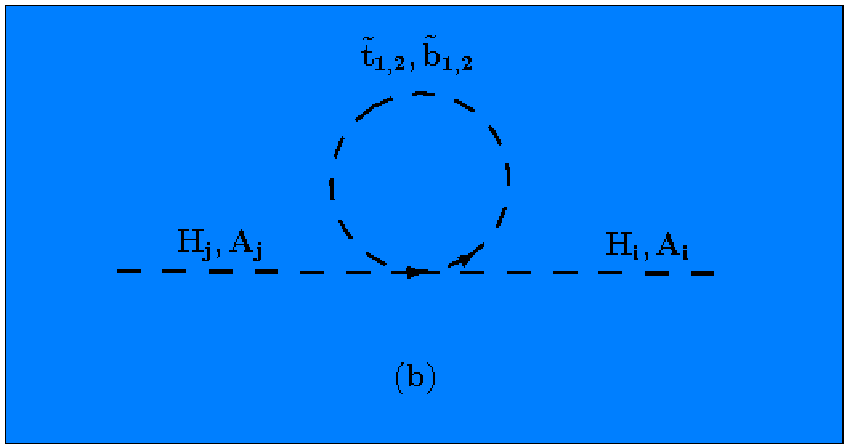
$$\mathcal{M}^2_{H_2-H_3}(s) = \begin{pmatrix} M_H^2(s) - iM_H\Gamma_H(s) & \Delta^2_{HA}(s) \\ \Delta^2_{HA}(s) & M_A^2(s) - i\Gamma_A M_A(s) \end{pmatrix} \qquad (16)$$

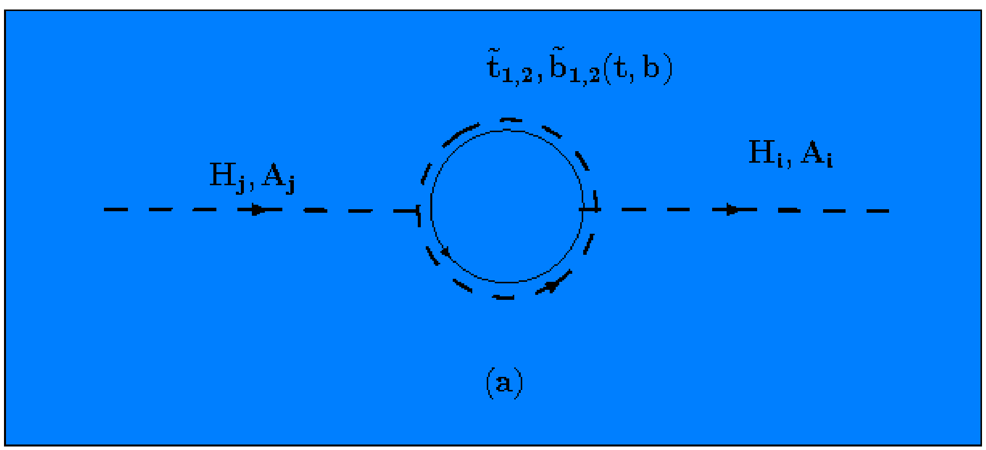
[Pilaftsis 98],[Demir 99]

The mass matrix of the heavy neutral higgs CPV system is not Hermitian.

III. Self-energy neutral Higgs mass corrections

The elements of the mass matrix $M^2_{H_2-H_3}(s)$ is constructed explicitly from dominant self-energies contributions diagrams





III.1 Renormalization schemes

Various approaches have been applied: [Dabelstein 94]

① **The effective potential method** . The tree level mass matrix $\mathcal{M}^0$ of the neutral scalar system is diagonalized as usual with angles α, β. Loop contributions to the quadratic part of the potential (neglecting the $q^2 - dependence$ of the diagrams) modify the mass matrix $M^0 \rightarrow M^0 + \delta M = \mathcal{M}$. Re-diagonalizing the one-loop Matrix $\mathcal{M}$ yields the corrected mass eigenvalues m_{H^0,h^0}, an effective mixing angle α_{eff} instead of α

② **The renormalization group method**. Solving the renormalization group equations for the parameters of a general 2- doublet model and imposing the SUSY constraints at the scale $\mu = M_{SUSY}$ yields the effective parameters of the Higgs potential at the electroweak scale. Large $\log$r terms are resummed, but effects from realistic mass spectra are not covered by this approximation.

③ **Complete one-loop calculation**. A complete one-loop calculation to masses and couplings accommodates all SUSY particles and mass parameters (or soft breaking parameters, respectively) in the radiatively corrected version of masses (2-point functions) and mixing angles and, in addition, provides the 3-point functions required for Higgs boson production and decay processes. They are necessary to check the quality of the other two approximations and allow a detailed study of the full parameter dependence of production cross sections and decay rates

In the literature this issue has been treted from different approaches

- SM contributions resummed in the *Pinch Technique* framework in a general [Papavassiliou and Pilaftsis 96]
- In MSSM CPV considering that this diagrams are dominated by the large CP-violationg trilineal A-terms couplings to the stop and sbottom.[Carena, Ellis, Pilaftsis and Wagner 02]
- All MSSM contributions calculated numerically by [Frank,Hahn,Heinemeyer,Hollik,Rzehak and Weiglein 07]

At one-loop level,the renormalized self-energies, $\hat{\Pi}(s)$ can be expressed through a sum of

① the unrenormalized self-energies: $\Pi(s)$,

② the field renolrmalization constants: $\Phi_i \longrightarrow (1 + \frac{1}{2}\delta Z_{\Phi_i})$, and

③ counter terms arising from the 4×4 mass matrix: $\delta\mathcal{M}_{hHAG} = U_{(0)}\mathcal{M}_{H_1H_2A_1A_2}U^{\dagger}_{(0)}$

obtaining that each element of the renormalized self-energy matrix will have have the form:

$$\hat{\Pi}_{ij}(s) = \Pi_{ij}(s) + \delta Z_{ij}(s - m^2_{i,j}) - \delta m^2_{ij} \tag{17}$$

III.2. Neutral heavy Higgs bosons as s-channel resonances

- Masses and mixings of the H-A system may be detected as two relatively closely spaced or even overlapping resonances in the s-channel reaction.

$$\mu^+\mu^- \to A^*/H^* \to f\bar{f} \tag{18}$$

[Pilaftsis 97],[Bernabeu, Binosi, Papavasiliu 06]

- The line shape of this process would indicate the presence (or absence) of CPV in the heavy Higgs system.
- In the resonant region, the t-channel amplitude is relatively small and may be ignored
- Then, in the electroweak basis, the transition amplitude matrix between states with CP-violation via resonant Higgs exchange is

$$\mathcal{T}^{res}(s) = V^P \hat{\Delta}^{-1}_{H_2-H_3}(s) V^D \tag{19}$$

where we identify the propagator as

$$\hat{\Delta}^{-1}_{H_2-H_3}(s) = s\mathbf{1}_{2\times 2} - \mathcal{M}^2_{H_2-H_3}(s) = \begin{pmatrix} s - (M_H^2 - \hat{\Pi}_{HH}(s)) & -\hat{\Pi}_{HA}(s) \\ -\hat{\Pi}_{HA}(s) & s - (M_A^2 - \hat{\Pi}_{AA}(s)) \end{pmatrix} \tag{20}$$

At the one loop level, the production and decay vertices, $V_i^P(s)$ and $V_j^D(s)$ coincide with the corresponding tree-level vertices.

III.3. Physical masses as the poles of the propagator

The physical masses of the neutral heavy Higgs bosons are identified with the poles of the propagator matrix $\hat{\Delta}_{H_2-H_3}(s)$. Hence, the masses of the neutral, heavy Higgs bosons are defined as the solutions of the implicit equation

$$det\left[\hat{\Delta}_{H_2-H_3}(s^\star)\right] = det\left[(s^\star)\mathbf{1}_{2\times 2} - \mathcal{M}^2_{H_2-H_3}(s^\star)\right] = 0. \tag{21}$$

where

$$\mathcal{M}^2_{H_2-H_3}(s^\star) = \begin{pmatrix} M^2_H(s^\star) - iM_H\Gamma_H(s^\star) & \Delta^2_{HA}(s^\star) \\ \Delta^2_{HA}(s^\star) & M^2_A(s^\star) - i\Gamma_A M_A(s^\star) \end{pmatrix} \tag{22}$$

is the mass matrix of the neutral and heavy Higgs bosons in the CP-invariant basis. In the physical basis, $\mathcal{M}^2_{H_2-H_3}(s)$ is diagonal, then eq. (24) becomes

$$(s_2 - \mu^2_{H_2}(s_2))(s_3 - \mu^2_{H_3}(s_3)) = 0\,, \tag{23}$$

where $\mu_{H_i}(s)$ are the complex eigenvalues of $\mathcal{M}^2_{H_2-H_3}(s)$.

As the determinant is an invariant quantity, we may write eq.(24)

$$det\left[\hat{\Delta}_{H_2-H_3}(s^\star)\right] = det\left[(s^\star)\mathbf{1}_{2\times 2} - \mathcal{M}^2_{H_2-H_3}(s^\star)\right] = 0; \quad (24)$$

Which in terms of the eigenvalues of the squared mass matrix $\mathcal{M}^2_{H_2-H_3}(s^\star)$ at the pole, we obtain a system of two equations for $s_2^\star \neq s_3^\star$:

$$\mu^2_{H_i}(s_i^\star) - s_i^\star = 0, \quad (25)$$

with $i = 2, 3$. where $s_i^\star(x_1, x_2)$ is a function of two or more free parameters in $\mathcal{L}$. Now, we may identify $s_i^\star$ with **the pole mass of the neutral, heavy Higgs bosons**,

$$M^2_{H_i}(s^\star) - iM_i\Gamma_i(s^\star) := \mu^2_{H_i}(s_i^\star). \quad (26)$$

III.3 The **propagator** *of the neutral heavy Higgs system*

Another way of writing the mass matrix in terms of the Pauli matrices

$$\mathcal{M}^2_{H_2-H_3}(s) = \frac{1}{2}T\mathbf{1}_{2\times 2} + (\vec{R} - i\vec{\Gamma})\cdot\vec{\sigma} \tag{27}$$

Then the propagator of the neutral heavy Higgs bosons takes the form

$$\begin{aligned} \hat{\Delta}_{H_2-H_3}(s) &= \left[s - \mathcal{M}^2_{H_2-H_3}(s)\right]^{-1} \\ &= \frac{1}{(s-\frac{1}{2}T)^2 - (\vec{R}-i\vec{\Gamma})^2}\left[(s-\frac{1}{2}T)^2\mathbf{1} + (\vec{R}-i\vec{\Gamma})\cdot\vec{\sigma}\right] \end{aligned} \tag{28}$$

where

$$T = \frac{1}{2}\left[(M_H^2 + M_A^2) - i(M_H\Gamma_H + M_A\Gamma_A)\right] \tag{29}$$

is the trace of the mass matrix and

$$\vec{R} = \left(\frac{1}{2}(M_H^2 - M_A^2), 0, Re\Delta^2_{HA}\right), \quad \vec{\Gamma} = \left(\tfrac{1}{2}(M_H\Gamma_H - M_A\Gamma_A), 0, Im\Delta^2_{HA}\right) \tag{30}$$

III.4 Eigenvalues of the mass matrix

The eigenvalues of the mass matrix $\mathcal{M}^2_{H_2,H_3}(s)$ are

$$\mu^2_{H_i}(s) = \frac{1}{2}T \pm \sqrt{(\vec{R} - i\vec{\Gamma})^2} \tag{31}$$

Hence, there is a pole on a degenerated masses eigenvalues

$$\begin{aligned}
\hat{\Delta}_{H_2-H_3}(s) &= \frac{1}{(s-\mu^2_{H_2}(s))(s-\mu^2_{H_3}(s))}\left[(s-\frac{1}{2}T)^2\mathbf{1} + (\vec{R}-i\vec{\Gamma})\cdot\vec{\sigma}\right] \\
&= \frac{1}{(\mu^2_{H_3}(s)-\mu^2_{H_2}(s))}\left[\frac{1}{(s-\mu^2_{H_2}(s))} - \frac{1}{(s-\mu^2_{H_3}(s))}\right] \\
&\quad \times \left[(s-\frac{1}{2}T)^2\mathbf{1} + (\vec{R}-i\vec{\Gamma})\cdot\vec{\sigma}\right]
\end{aligned} \tag{32}$$

IV. Degeneracy of neutral heavy CP non-invariant Higgs bosons

Considering the full s-dependance, the true physical masses are identified with the poles of the propagator. Therefore, the heavy Higgs bosons masses should be defined by the solutions of the implicit equations [Stuart 95],[Bohm,Kaldass and Wickramasekara 02]

$$\mu^2_{H_i}(s^*_{H_i}) - s^*_{H_1} = 0; \qquad i = 2,3 \tag{33}$$

We say that the two heavy neutral Higgs bosons are mass degenerate if there exist an s^* such that

$$\left.\begin{array}{l} s^* - \mu^2_{H_2}(s^*) = 0 \\ \\ s^* - \mu^2_{H_3}(s^*) = 0 \end{array}\right. \Rightarrow \mu^2_{H_2}(s^*) = \mu^2_{H_3}(s^*),$$

From the previous result

$$\mu^2_{H_3}(s^*) - \mu^2_{H_2}(s^*) = \sqrt{(\vec{R}_d - i\vec{\Gamma}_d)^2} = 0 \tag{34}$$

that is $R_d^2(s^\star) = \Gamma_d^2(s^\star)$ and $\vec{R}_d(s^\star) \cdot \vec{\Gamma}_d(s^\star) = 0$ for $\vec{R}_d, \vec{\Gamma}_d \neq 0$.

From this conditions for degeneration we found that at degeneracy

$$(\vec{R}_d - i\vec{\Gamma}_d) \cdot \vec{\sigma} = M_d\Gamma_d \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix}, \tag{35}$$

The propagator can be written as

$$\Delta^{(d)}_{H_2-H_3}(s) = \begin{pmatrix} \frac{1}{(s-M_d^2+iM_d\Gamma_d)} & \frac{1}{(s-M_d^2+iM_d\Gamma_d)^2} \\ 0 & \frac{1}{(s-M_d^2+iM_d\Gamma_d)} \end{pmatrix} \tag{36}$$

We then can write the resonant transition matrix in the mass representation as

$$\begin{aligned} \mathcal{T}^{res(d)}(s) &= (\tilde{V}_1^P, \tilde{V}_2^P)\Delta^{(d)}_{H_2,H_3}(s) \begin{pmatrix} \tilde{V}_1^D \\ \tilde{V}_2^D \end{pmatrix} \\ &= \tilde{V}_1^P \frac{1}{s-m^2} \tilde{V}_1^D + \tilde{V}_2^P \frac{1}{s-m^2} \tilde{V}_2^D + \tilde{V}_1^P \frac{1}{(s-m^2)^2} \tilde{V}_2^D \end{aligned} \tag{37}$$

with

$$\tilde{V}_i^{P,D} = \frac{1}{2} \begin{pmatrix} 1 & +i \\ 1 & -1 \end{pmatrix} \begin{pmatrix} V_1^{P,D} \\ V_2^{P,D} \end{pmatrix} \tag{38}$$

IV.1 An approach for neutral Higgs boson mass matrix $\mathcal{M}^2(s)$ *in the 2HDM*

The matrix elements are expressed as functions of the model parameters. In the decoupling limit $M_A^2 >> |\lambda_i|v^2$, we may find a simplifying approach for the relations of the mass matrix elements as [Choi,Kalinowski,Liao and Zerwas 05]

$$\begin{aligned} M_H^2 - M_A^2 &\approx \lambda v^2 \cos\phi & (39)\\ 32\pi[M_H\Gamma_H - M_A\Gamma_A] &\approx [\Delta_t + 9\lambda^2 v^2 \cos 2\phi] & (40)\\ Re\Delta_{HA}^2 &\approx -\frac{1}{2}\lambda v^2 \sin\phi & (41)\\ 32\pi\ Im\Delta_{HA}^2 &\approx -\frac{9}{2}\lambda^2 v^2 \sin 2\phi & (42) \end{aligned}$$

We have taken the magnitudes of all λ_i as same order, and ϕ is the CP violating common phase of the complex couplings. And

$$\Delta_t = -12M_{H/A}^2(m_t/v)^2(1-\beta_t^2)\beta_t, \quad (43)$$

is the one loop contribution of the top quark.

IV.2 Exceptional point in the mass complex surfaces

With this relations of the mass matrix elements we are able to write explicity the masses of the heavy neutral Higgs bosons as functions of the parameters λ and ϕ and if further more we neglect the weak s dependence of the elements of $\mathcal{M}^2_{HA}$, we found an approximation for pole position mass. The term under the square root is a regular function of its arguments and may admit a Puiseux expansion series around the exceptional point [Hernández,Jáuregui and Mondragón 06]

$$\mu^2_{2,3}(\lambda,\phi) = \frac{1}{2}\sqrt{c_1^{(1)}(\lambda-\lambda^\star) + c_2^{(1)}(\phi-\phi^\star) + \ldots} \tag{44}$$

where the degeneracy conditions we get the exceptional point as: $\lambda^\star = 0.1075, \;\; \phi^\star = \pi/2$ and $c_k^{(1)}$ are the derivatives of $\mu^2_{2,3}$ with respect to the parameters λ and ϕ.

$$\Re e\mu^2_{2,3} = \pm\frac{1}{2\sqrt{2}}|\vec{\zeta}|^{1/2}\left[\sqrt{(\vec{\mathcal{R}}\cdot\hat{\zeta})^2 + (\vec{\mathcal{I}}\cdot\hat{\zeta})^2} + (\vec{\mathcal{R}}\cdot\hat{\zeta})\right]^{1/2} \tag{45}$$

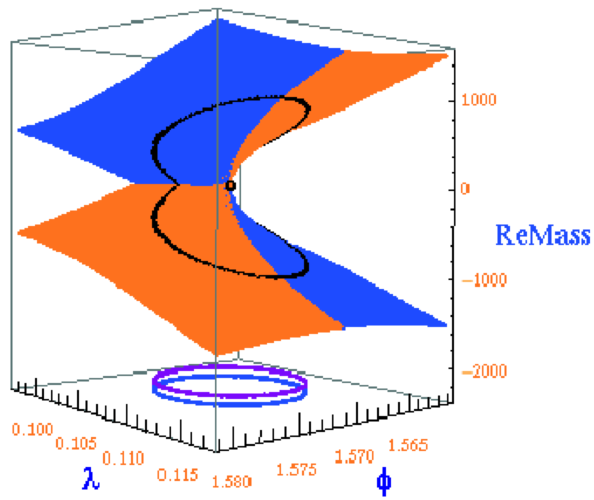
$$\Im m\mu^2_{2,3} = \pm\frac{1}{2\sqrt{2}}|\vec{\zeta}|^{1/2}\left[\sqrt{(\vec{\mathcal{R}}\cdot\hat{\zeta})^2 + (\vec{\mathcal{I}}\cdot\hat{\zeta})^2} - (\vec{\mathcal{R}}\cdot\hat{\zeta})\right]^{1/2} \tag{46}$$

with

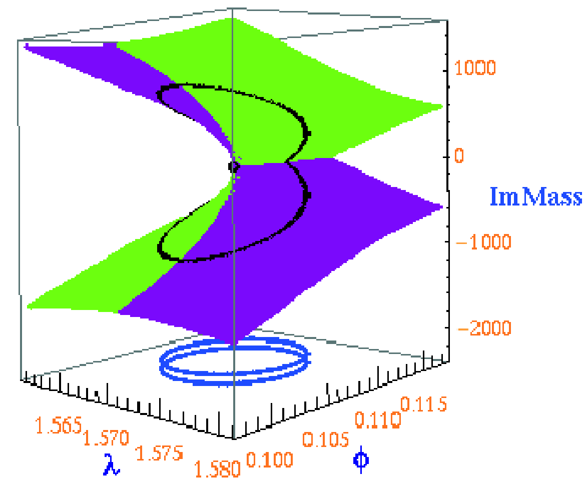
$$\vec{\mathcal{R}} = \left(\Re e\, c_1^{(1)}, \Re e\, c_2^{(1)}\right), \quad \vec{\mathcal{I}} = \left(\Im m\, c_1^{(1)}, \Im m\, c_2^{(1)}\right), \quad \vec{\zeta} = \begin{pmatrix} \lambda-\lambda^\star \\ \phi-\phi^\star \end{pmatrix}. \tag{47}$$

IV.3 Unfolding of the exceptional point

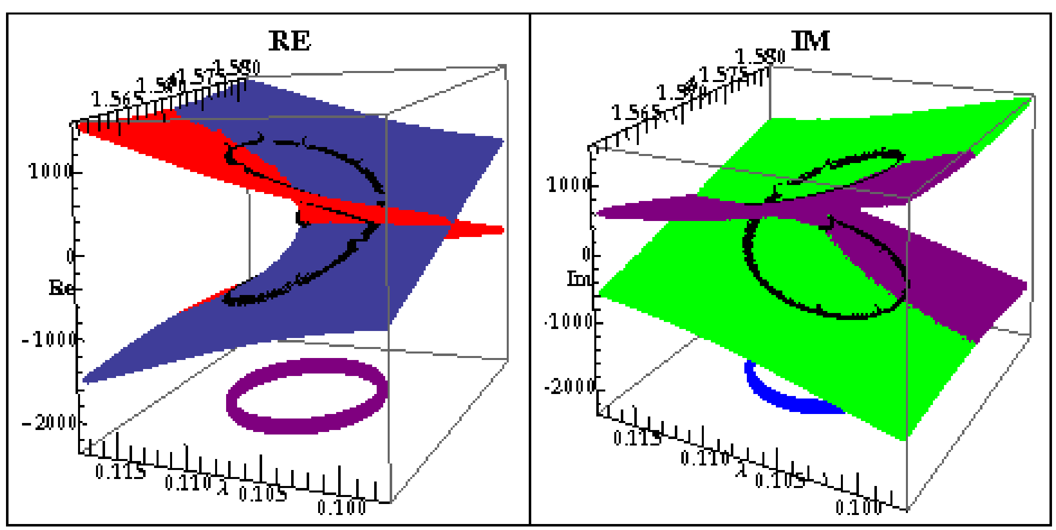
The figures show the mass hypersurface representing the imaginary parts of $\mu^2_{2,3}$ as function of the Lagrangian parameters in the neighbourhood of the exceptional point



(a)

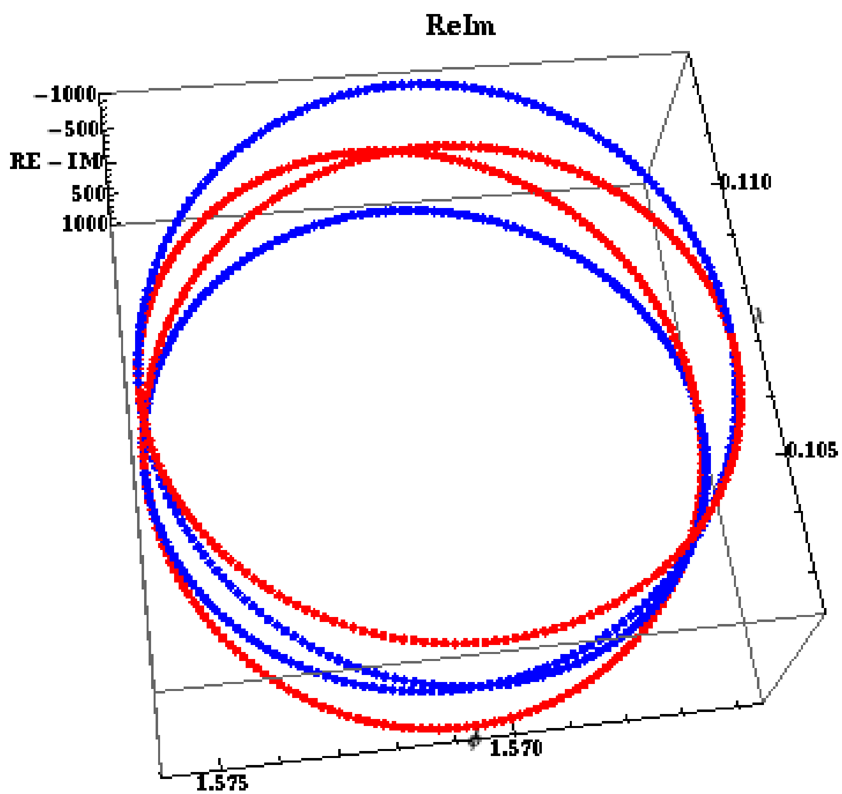


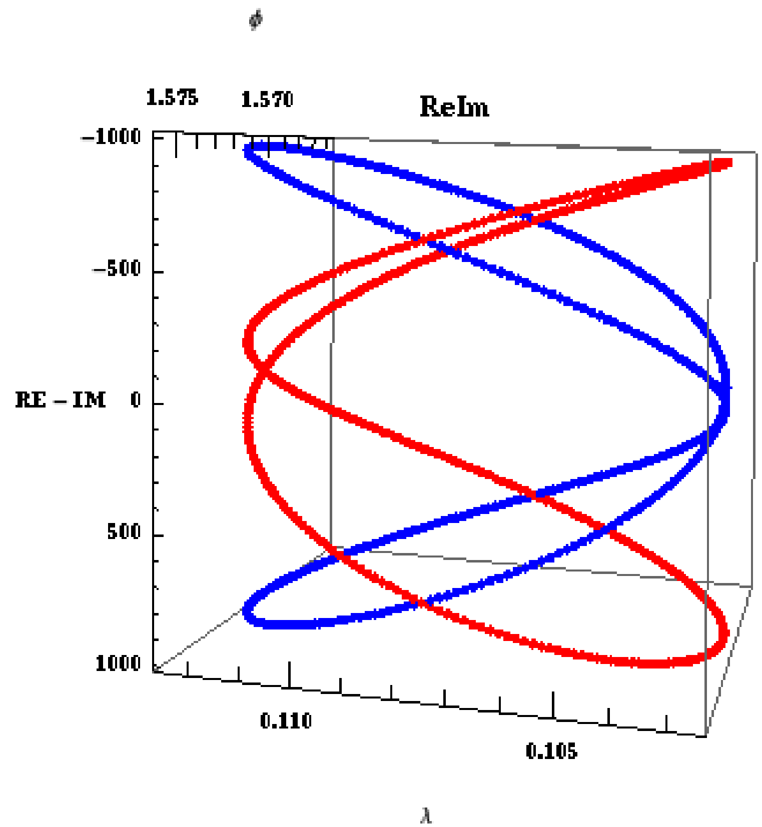
(b)



IV.4 Trajectory of the physical system on the mass surfaces

Real and imaginary trajectories of the physical system on the mass surfaces when the system makes a circular excursion around the exceptional point :





The system requires 4π in order to return to its original position, giving rise to a Berry phase.

Conclusions

- CP-violating complex couplings allow for the possibility of mixing and degeneracy of the $H_2,\ H_3$ system.
- The mass matrix of the system is no longer Hermitian.
- At exact mass degeneracy, the propagator of the system has one double and one single pole in the complex energy s-plane.
- In parameter space the mass surfaces have one branch point of rank one where exact degeneracy occurs.
- Real and imaginary parts have branch cuts starting at the same branch point and extending in opposite directions in parameter space.
- At degeneracy, the identification of the two particles will depend strongly on the values of the parameters.
- These features would make a CP-violating Higgs sector of the MSSM easily discernible from a CP-preserving one.

thank you