Anomalies, Beta Functions and Supersymmetric Unification with Multi-Dimensional Higgs Representations

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- Supersymmetric Grand Unified Theories (SGUT)have achieved some degree of success.
- Although this degree of success is already present in the minimal models there are open problems that suggest the need to incorporate more elaborate constructions specifically the use of higher dimensional representations in the Higgs sector.
- When one adds these higher-dimensional Higgs representation within the context of $\mathcal{N} = 1$ SUSY GUTs, one must verify the cancellation of anomalies associated to their fermionic partners.
- Straightforward solution is obtained by including vector-like representations.

- Pourpose: find alternatives to this option, namely to create an anomaly free Higgs sector, including some representation ψ and a set of other representations of lower-dimension $\{\phi_1, \phi_2, ...\}$.
- The unification condition imposes some restrictions on the GUT-scale masses of the gauge bosons, gauginos, Higgses, and Higgsinos.
- The perturbative validity of the model is affected by the inclusion of additional multiplets.
- We also study the effect of those multiplets on the evolution of the gauge coupling up to the Planck scale, for models that involve different sets of fields and representations that satisfy the anomaly-free conditions [1].

[1] Work done in collaboration with A. Aranda and Diaz-Cruz.

- The need to require anomaly cancellation in any gauge theory stems from the fact that their presence destroys the quantum consistency of the theory. It turns out that all one needs to calculate or identify the anomaly is the triangle diagrams.
- For a given fermionic representation of a gauge group G, the anomaly can then be written as

$$A(D)d^{abc} \equiv Tr\left[\left\{T_a^{D_i}, T_b^{D_i}\right\} T_c^{D_i}\right], \tag{1}$$

where $T_a^{D_i}$ denotes the generators of the gauge group G in the representation D_i, and d^{abc} denotes the anomaly associated with the fundamental representation.

- The anomaly coefficients $a_D \equiv A(D)$ for the most common representations for SU(N) groups are known in the literature [2]

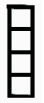
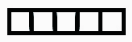
Table: Dimensions, Dynkin indexes and anomaly coefficients for some representations of SU(N)

$Irrep$	$dim(r)$	$2T(r)$	$A(r)$
$\Box$	N	1	1
Ad	N^2-1	$2N$	0
$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	$\frac{N(N-1)}{2}$	$N-2$	$N-4$
$\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix}$	$\frac{N(N-1)(N-2)}{6}$	$\frac{(N-3)(N-2)}{2}$	$\frac{(N-3)(N-6)}{2}$
$\Box\Box\Box$	$\frac{N(N+1)(N+2)}{6}$	$\frac{(N+2)(N+3)}{2}$	$\frac{(N+3)(N+6)}{2}$

[2] J. Terning, *Modern Supersymmetry, Dinamics and Duality*, Oxford Science Publications,(2006).

- We have extended these results to include higher-dimensional representations.

Table: Dimensions and anomaly coefficients for higger representations of SU(N)

Irrep	$dim(r)$	$A(r)$
	$\frac{N(N-1)(N-2)(N-3)}{24}$	$\frac{(N-4)(N-3)(N-8)}{6}$
	$\frac{N(N+1)(N+2)(N+3)(N+4)}{120}$	$\frac{(N+3)(N+4)(N+5)(N+10)}{24}$
	$\frac{N^2(N+1)(N+2)(N-1)}{24}$	$\frac{N(N+5)(5N^2-3N-50)}{24}$

To obtain these results one can use the following relations:

1. For a representation R that its a direct sum of two representations, $R = R^1 \oplus R^2$, the anomaly is given by

$$A_R = A(R_1 \oplus R_2) = A(R_1) + A(R_2). \tag{2}$$

2. For a representation R that is the tensor product of two representations, the anomaly is given by:

$$A_R = A(R) = A(R_1 \otimes R_2) = D(R_1)A(R_2) + D(R_2)A(R_1), \tag{3}$$

with $D(R_{1,2})$ denoting the dimensions of representations $R_{1,2}$.

- We use too the following expresions [3] for the completely symmetric tensorial product of m fundamental N-dimensional representations of SU(N) (a Young diagram with only m boxes in a row)

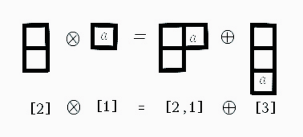
$$D(m) = \frac{(m+N-1)!}{m!(N-1)!}; \quad A = \frac{(N+m)!(N+2m)}{(N+2)!(m-1)!}. \tag{4}$$

- For the completely antisymmetric product of p fundamental representations (a column with p boxes) we have

$$D(p) = \frac{(N)!}{p!(N-p)!}; \quad A = \frac{(N-3)!(N-2p)}{(N-p-1)!(p-1)!}. \tag{5}$$

[3] Jay Banks, H. Georgi, *Comment On Gauge Theories Without Anomalies*, Phys. Rev. D, **14**, 1159 (1976)

Example: Calculate the anomaly of [2, 1] representation. For this, we first take the tensorial product



Then, from (2): $A([2] \otimes [1]) = A([2,1]) \oplus A([3])$,

but from (3)

$$A([2] \otimes [1]) = D([2])A([1]) \oplus D([1])A([2]),$$

so,

$$A([2,1]) = D([2])A([1]) \oplus D([1])A([2]) - A([3]).$$

Then, using (4) and (5) we obtain

$$
\begin{aligned}
D([1]) &= N, & A([1]) &= 1, \\
D([2]) &= \frac{N(N-1)}{2}, & A([2]) &= N-4, \\
D([3]) &= \frac{N(N-1)(N-2))}{6}, & A([3]) &= \frac{(N-3)(N-6)}{2}.
\end{aligned}
$$

therefore

$$A([2,1]) = N^2 - 9.$$

It is known that anomaly free theories arise when:

i) The gauge group itself is safe, i.e. it is always free of anomalies.

ii) The gauge group is a subgroup of an anomaly free group, and the representations form a complete representation of the anomaly free group. For instance, this happens in the SU(5) case for the $\mathbf{5} + \bar{\mathbf{10}}$ representations, which together are anomaly free.

iii) The fermionic representations appear in conjugate pairs, i.e., they are vector-like. This is the most common choice when the Higgs sector of SUSY GUT is extended.

- The results of this work have been published in Phys. Rev. D 80, 085027 (2009).
- Let us consider an SU(5) SUSY GUT model. There are three copies of $\bar{\mathbf{5}}$ and $\mathbf{10}$ representations to accommodate the three families of quarks and leptons.
- Breaking of the GUT group to the SM: $SU(5) \rightarrow SU(3)_C \times SU(2)_L \times U(1)$, is achieved by including a (chiral) Higgs supermultiplet in the adjoint representation ($\mathbf{24}$).
- The minimal Higgs sector needed to break the SM gauge group can be accommodated with a pair of $\mathbf{5}$ and $\bar{\mathbf{5}}$ representations, which is indeed vectorial and therefore anomaly free (case iii above).

- Within this minimal model one obtains the mass relations $m_{d_i} = m_{e_i}$, which are predicted by the $\mathbf{5}+\bar{\mathbf{5}}$ Higgs sector.
- One way to solve this problem is to add a **45** representation, which couples to the d-type quarks, but not to the up-type, then one obtains the Georgi-Jarlskog factor needed for the correct mass relations.
- Most models that obtain this relations within an extended Higgs sector, include the conjugate representation $\mathbf{45} + \overline{\mathbf{45}}$ to cancel anomalies [4].

[4] P. Fileviez Perez, arXiv:0710.1321 [hep-ph]; P. Fileviez Perez, Phys. Rev. D **76**, 071701 (2007) [arXiv:0705.3589 [hep-ph]].

- Consider the representations of SU(5) (and their conjugates) shown in table

Irrep	*Multiplet*	$dim(r)$	$A(r)$	$2T(r)$
[5]	(0, 0, 0, 0)	1	0	0
[1]	(1, 0, 0, 0)	5	1	1
[2]	(0, 1, 0, 0)	10	1	3
[1, 1]	(2, 0, 0, 0)	15	9	7
Ad		24	0	10
[4, 1]	(1, 0, 0, 1)	24	0	10
[1, 1, 1]	(3, 0, 0, 0)	35	44	28
[2, 1]	(1, 1, 0, 0)	40	16	22
[3, 1]	(1, 0, 1, 0)	45	6	24

- We can write down the following anomaly-free combinations

$$\begin{aligned} A(\mathbf{45}) + A(\overline{\mathbf{45}}) &= 0 \\ A(\mathbf{45}) + 6A(\bar{\mathbf{5}}) &= 0 \\ A(\mathbf{45}) + 6A(\overline{\mathbf{10}}) &= 0 \\ A(\mathbf{45}) + fA(\bar{\mathbf{5}}) + f'A(\overline{\mathbf{10}}) &= 0, \quad \text{with } f + f' = 6 \end{aligned} \tag{6}$$

- One could also invoke a $\overline{\mathbf{15}}$ representation, which has $A = -9$, therefore the following combination is also anomally-free,

$$A(\mathbf{45}) + A(\overline{\mathbf{15}}) + 3A(\mathbf{5}) = 0 \tag{7}$$

- These are clearly non-equivalent models.

- The 1-loop β functions for a general SUSY theory with gauge group G and matter field appearing in chiral supermultiplets are given by:

$$\beta_1 = \sum_R T_R - 3C_A, \tag{8}$$

where T_R denotes the Dynkin index for the representation R, and C_A the cuadratic Casimir invariant for the adjoint representation. For SU(N) type gauge groups $C_A = N$.

- From MSSM β-functions, the RGE's and one-loop β functions for the gauge couplings are

$$\frac{d\alpha_i}{dt} = \beta_i \alpha_i^2, \quad \beta_i = \begin{pmatrix} 33/5 \\ 1 \\ -3 \end{pmatrix} + \beta^X \tag{9}$$

where $t = (2\pi)^{-1} \ln(\text{mass scale})$, $i = 1, 2, 3$ refers to the U(1), SU(2) and SU(3) gauge group, and $\beta^X = \sum_\Phi T(\Phi)$ are the contributions of the extensions of the MSSM (the sum is over all SU(5) additional multiplets Φ).

- Assuming $M_{SUSY} \approx M_t$, one obtains that the gauge coupling is aproximately $\alpha_5(M_{GUT}) = 0.0416$, and unification occurs at [5] $M_{GUT} = 2 \times 10^{16}$ GeV

[5] R. Hempfling, Phys. Lett. B **351**, 206 (1995) [arXiv:hep-ph/9502201].

- This simple 1-loop results can be improved by using the 2-loop equations, in such case we solve numerically the corresponding RGE and we find that at the GUT scale $M_{GUT} = 1.28 \times 10^{16}$ the unified gauge coupling is $\alpha_5(M_{GUT}) = 0.040$ and $h^t(M_{GUT}) = 0.6572$.

- Now we are interested in evaluating the effect of the different representations in the running from M_{GUT} up to the Planck scale. Besides evaluating the effect of the different anomaly free combinations, we are also interested in finding which representations are perturbatively valid up to the Planck scale. The unified gauge coupling obeys the 1-loop RGE

$$\mu \frac{d\alpha_5^{-1}}{d\mu} = \frac{-\beta_1}{2\pi} = \frac{3-\beta^X}{2\pi} \tag{10}$$

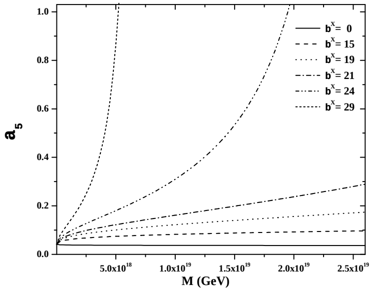
where $-\beta_1 = \beta_{MIN} - \beta^X$, with $\beta_{MIN} = 3$ denoting the MSSM contribution to the SU(5) beta function, including the contribution of the gauge sector.

- The 1-loop β functions for some interesting anomaly-free combinations are found to be:

$$\begin{aligned} \beta^X(\mathbf{45}+\overline{\mathbf{45}}) &= 24 \\ \beta^X(\mathbf{45}+6(\overline{\mathbf{5}})) &= 15 \\ \beta^X(\mathbf{45}+6(\overline{\mathbf{10}})) &= 21 \; . \\ \beta^X(\mathbf{45}+\overline{\mathbf{15}}+2(\mathbf{10})+\mathbf{5}) &= 19 \\ \beta^X(\mathbf{50}+\overline{\mathbf{40}}+\mathbf{5}) &= 29 \end{aligned}$$

- Solving the RGE equation from the GUT to Planck scale, assuming unification, we obtain

$$\alpha_5^{-1}(m) = \frac{(3-\beta^X)}{2\pi} \ln\left(\frac{m}{M_{GUT}}\right) + 24 \qquad (11)$$



- As it is shown in the figure, the model with $\beta^X = 29$ induces a running of the gauge coupling that blows at the scale $M = 6.61 \times 10^{18}$, while for $\beta^X = 24$ this happens at $M = 2.63 \times 10^{19}$. The models with $\beta^X = 15, 19, 21$ are found to evolve safely even up to the Planck scale.

- It is also interesting to consider the RGE effect associated with the Yukawa coupling that involve the additional Higgs representations. In order to do this we shall consider the 2-loop beta functions for the gauge coupling, but will keep only the one-loop RGE for the new Yukawa couplings.

- The gauge coupling at 2-loops for a general supersymmetric model is given by [6]

$$\frac{d}{dt}g = \frac{1}{16\pi^2}\beta_g^{(1)} + \frac{1}{(16\pi^2)^2}\beta_g^{(2)} \tag{12}$$

$$\beta_g^{(1)} = g^3[T(R) - 3C(G)], \tag{13}$$

$$\beta_g^{(2)} = g^5\left\{-6[C(G)]^2 + 2C(G)T(R) + 4T(R)C(R)\right\} - g^3 Y^{ijk} Y_{ijk} C(k)/d(G). \tag{14}$$

where $Y_{ijk} = (Y^{ijk})^*$, $T(R)$ is the Dynkin index summed over all quiral multipletes, and $T(R)C(R)$ is the sum of the Dynkin indices weighted by the cuadratic Casimir invariant

[6] S. P. Martin and M. T. Vaughn, Phys. Rev. D **50**, 2282 (1994) [Erratum-ibid. D 78, 039903 (2008)] [arXiv:hep-ph/9311340].

The 2-loop β functions for the Yukawa couplings are

$$\frac{d}{dt}Y^{ijk} = Y^{ijp}\left[\frac{1}{16\pi^2}\gamma_p^{(1)k} + \frac{1}{(16\pi^2)^2}\gamma_p^{(2)k}\right] + (k \leftrightarrow i) + (k \leftrightarrow j), \tag{15}$$

where

$$\gamma_i^{(1)j} = \frac{1}{2}Y_{ipq}Y^{jpq} - 2\delta_i^j g^2 C(i), \tag{16}$$

and

$$\begin{aligned}\gamma_j^{(2)i} &= -\tfrac{1}{2}Y_{imn}Y^{npq}Y_{pqr}Y^{mrj} + g^2 Y_{ipq}Y^{jpq}[2C(p) - C(i)] \\ &\quad +2\delta_i^j g^4[C(i)T(R) + 2C(i)^2 - 3C(G)C(i)].\end{aligned} \tag{17}$$

- Thus, we shall consider the following superpotential for the SUSY SU(5) GUT model. This superpotential involves the Higgs representations **5**, $\bar{\mathbf{5}}$ y **24**, namely:

$$\begin{aligned} W &= \tfrac{f}{3}Tr\Sigma^3 + \tfrac{1}{2}fVTr\Sigma^2 + \lambda \bar{H}_\alpha(\Sigma^\alpha_\beta + 3V\delta^\alpha_\beta)H^\beta \\ &\quad + \tfrac{h^{ij}}{4}\varepsilon_{\alpha\beta\gamma\delta\epsilon}\psi_i^{\alpha\beta}\psi_j^{\gamma\delta}H^\epsilon + \sqrt{2}f^{ij}\psi_i^{\alpha\beta}\phi_{j\alpha}\bar{H}_\beta, \end{aligned} \tag{18}$$

where $i, j = 1, 2, 3$ are family indices,and α, β, γ... are SU(5) indices. The chiral superfields $\psi(\mathbf{10})$, $\phi(\bar{\mathbf{5}})$ are matter multipletes.

- The Higgs multiplet content is

$$\begin{aligned}
\Sigma &= \Sigma^a T^a = \begin{pmatrix} \Sigma_8 & \Sigma_{(3,2)} \\ \Sigma_{(3^*,2)} & \Sigma_3 \end{pmatrix} + \frac{1}{2\sqrt{15}}\Sigma_{24}\begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}, \\
H^T &= (H_C, H_C, H_C, H_f^+, H_f^0), \qquad (19) \\
\bar{H}^T &= (\bar{H}_C, \bar{H}_C, \bar{H}_C, \bar{H}_f^-, -\bar{H}_f^0),
\end{aligned}$$

and the matter multiplets:

$$\begin{aligned}
\psi &= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & u^c & -u^c & u & d \\ -u^c & 0 & u^c & u & d \\ u^c & -u^c & 0 & u & d \\ -u & -u & -u & 0 & e^c \\ -d & -d & -d & -e^c & 0 \end{pmatrix} \qquad (20) \\
\phi^T &= (d^c, d^c, d^c, e, -\nu)
\end{aligned}$$

- Then, from (15) and (18), the 1-loop RGE for the Yukawa parameters are [7]:

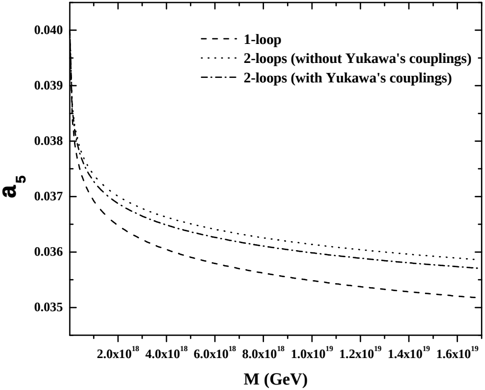
$$\begin{aligned} \mu\frac{d\lambda}{d\mu} &= \frac{1}{(4\pi)^2}\left(-\frac{98}{5}g_5^2 + \frac{53}{10}\lambda^2 + \frac{21}{40}f^2 + 3(h^t)^2\right)\lambda, \\ \mu\frac{df}{d\mu} &= \frac{1}{(4\pi)^2}\left(-30g_5^2 + \frac{3}{2}\lambda^2 + \frac{63}{40}f^2)^2\right)f, \\ \mu\frac{dh^t}{d\mu} &= \frac{1}{(4\pi)^2}\left(-\frac{96}{5}g_5^2 + \frac{12}{5}\lambda^2 + 6(h^t)^2\right)h^t, \end{aligned} \tag{21}$$

- while the 2-loop RGE for gauge coupling is given by:

$$\begin{aligned} \mu\frac{dg_5}{d\mu} &= \frac{1}{(4\pi)^2}(-3g_5^3) + \frac{1}{(4\pi)^4}\frac{794}{5}g_5^5 - \frac{1}{(4\pi)^4}\left\{\frac{49}{5}\lambda^2\right. \\ &\left. + \frac{21}{4}f^2 + 12(h^t)^2\right\}g_5^3. \end{aligned} \tag{22}$$

[7] J. Hisano, H. Murayama and T. Yanagida, Nucl. Phys. B 402, 46 (1993) [arXiv:hepph/ 9207279].

- We used values of the coefficients λ, h^t and f that are themselves safe at the Planck scale, and looked for their effects on the unified gauge coupling.
- The resulting gauge coupling evolution is shown in one of the lines in the following figure , where we show the 1-loop results, as well as the the 2-loop results with and without the Yukawas 1-loop contributions.



The parameters used in the plots are $M_{GUT} = 1.28 \times 10^{16} GeV$, $\alpha(M_{GUT}) = 0.040$, $h^t(M_{GUT}) = 0.6572$, $\lambda(M_{GUT}) = 0.6024$, and $f(M_{GUT}) = 1.7210$.

- We have studied the problem of anomalies in SUSY gauge theories, in order to search for alternatives to the usual vector -like representations used in extended Higgs sector.
- The known results have been extended to include higher-dimensional Higgs representations, which in turn have been applied to discuss anomaly cancellation within the context of realistic GUT models of SU(5) type.

- We have succed in identifying ways to replace the $\mathbf{45} + \bar{\mathbf{45}}$ models within SU(5) SUSY GUTs. Then, we have studied the β functions for all the alternatives, and we find that they are not equivalent in terms of their values of their β functions.
- We have also considered the RGE effect associated with the Yukawa coupling that involve the additional Higgs representations. We found that there are appreciable differences for the evolution of the gauge coupling when going from the one to the two-loop RGE, but this difference is reduced when one includes the 1-loop Yukawa couplings at the 2-loop level.

Thank you!