The 1/Nc Expansion at the Hadronic Level

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OUTLINE

- Introduction
- Mesons and Glueballs in $1/N_c$
- Baryons in $1/N_c$
- $1/N_c$ in BChPT
- Comments and outlook

Introduction

- Started in Stat. Mech. in 1968 (Stanley): expansion in 1/N, N: number of internal degrees of freedom.
- Applied to various field theory models to study non-perturbative dynamics.
- Introduced in gauge theories in 1974 ('tHooft): $SU(N_c)$ gauge theory admits expansion in $1/N_c$.
- One "solved" case: QCD_2. Full analysis of meson sector to leading order in $1/N_c$. Sub-leading order still open problem. Too simple to shed light on real world QCD.
- QCD: $1/N_c$ expansion still an open theoretical problem.
- It can be implemented at hadronic level through effective theories!

Basic idea

System with N degrees of freedom ($\phi_i,\ i = 1, \cdots, N$)
Partition function (Euclidean path integral):

$$Z = \int \{D\phi_i\}\ \exp(-S[\{\phi_i\}])$$

Assume the action is of the form: $S[\{\phi_i\}] = N\ S_0[\{\psi_i\}]$,
$\psi_i = \sqrt{N}\phi_i$
Naively expectation is dominance of least action trajectories (semiclassical limit), but PI measure has large entropy
Focus only in "colorless" degrees of freedom

$$\sigma(x, y) = \sum_i \psi_i(x)\psi_i(y)$$

$$Z = \int D\sigma J(\sigma)\ \exp(-N\, S'[\sigma]) = \int D\sigma\ \exp(-N\, S_{\rm eff}[\sigma])$$

Large $N \Rightarrow$ saddle point dominance of $S_{\rm eff}$: semiclassical limit

Solvable models in $N \to \infty$ limit: QM, spin-models, $O(N)$ and CP^N sigma models, etc.
$1/N$ corrections have been calculated in some of these models

$SU(N)$ Gauge Theory

$$\mathcal{L} = \frac{1}{4g^2} G^a_{\mu\nu} G^{\mu\nu\, a}, \qquad a = 1, \cdots, N^2 - 1$$

Large $N: \; g^2 = g_0^2/N$
Wilson loops: loop equations
Other approaches: "master field" reduction (Eguchi-Kawai and variants thereof)
String/Gauge Theory correspondence: not yet clear how it applies to QCD
Bottom up approach to $1/N_c$ expansion still a work in progress.

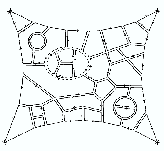
$1/N_c$ Expansion in QCD

Follow 'tHooft's approach of organizing Feynman diagrams
'tHooft expansion: $1/N_c$ at N_f fixed, Veneziano expansion: N_f/N_c fixed.

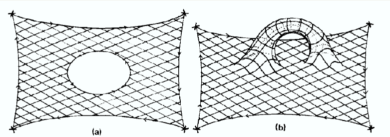
- 'tHooft expansion:

$$\mu\frac{dg}{\mu} = \beta(g) = -\frac{11N_c - 2N_f}{48\pi^2} g^3 = \mathcal{O}(1/\sqrt{N_c})$$

$$\alpha_s = \mathcal{O}(1/N_c) \quad \Lambda_{QCD} \;\; \text{fixed}$$



$1/N_c$



$1/N_c^2$

$1/N_c$ as topological expansion: $1/N_c^{\chi}$
$\chi = -2 + \#\text{ holes} + 2\,\#\text{ handles} + \#\text{ edges}$

Main non-perturbative quantities: look at diagrams which can contribute and deduce the $1/N_c$ power

$$\sigma = \mathcal{O}(N_c^0) \quad \langle \bar{q} q \rangle = \mathcal{O}(N_c) \quad \alpha_s \langle G_{\mu\nu} G^{\mu\nu} \rangle = \mathcal{O}(N_c)$$

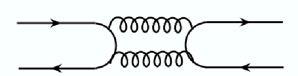
- Key observation: $1/N_c$ expansion can be identified at hadronic level and built into effective theories.
- Two essentially different sectors: glueballs+mesons, and baryons

MESONS & GLUEBALLS

Mesons	Glueballs
$M = \mathcal{O}(N_c^0)$	$M = \mathcal{O}(N_c^0)$
$\Gamma = \mathcal{O}(1/N_c)$	$\Gamma = \mathcal{O}(1/N_c^2)$
$F_M = \mathcal{O}(\sqrt{N_c})$	$F_M = \mathcal{O}(N_c)$
OZI rule: $\Gamma_{OZI} = \mathcal{O}(1/N_c^3)$	OZI rule: $\Gamma_{OZI} = \mathcal{O}(1/N_c^2)$
$\sigma = \mathcal{O}(1/N_c^2)$	G-meson mixing= $\mathcal{O}(1/\sqrt{N_c})$
nonet symmetry	
$M_{\eta'}^2 = \mathcal{O}(N_f/N_c)$ in χ limit	

- Phenomenological evidence: multiple excited mesons with widths $\mathcal{O}(100 \text{ MeV})$
- OZI: in ϕ meson and quarkonia for instance
- Test of $1/N_c^2$ corrections to $m_G/\sqrt{\sigma}$ in Lattice gluodynamics
- $1/N_c$ suppression of certain LECs in ChPT

- **Illustrative tests**
  - OZI suppression: $\Gamma_M^{OZI} = \mathcal{O}(1/N_c^3)$

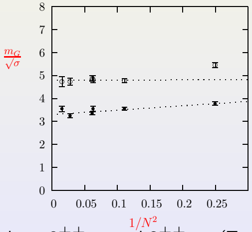


$$\frac{\Gamma(\phi\to\rho\pi+3\pi)}{\Gamma(\phi\to K\bar{K})} \sim 0.2$$

$$\frac{\Gamma(J/\psi\to hadrons)}{\Gamma(J/\psi\to\ell^+\ell^-)} \sim 7.5$$

$$\frac{\Gamma(\rho\to\pi\pi)}{\Gamma(\rho\to\ell^+\ell^-)} \sim 10^4$$

  - Lattice gluodynamics: ratio $\frac{m_G}{\sqrt{\sigma}} = \mathcal{O}(1) + \mathcal{O}(1/N_c^2)$

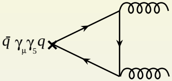


The lightest 0^{++}, $\bullet$, and 2^{++}, $\circ$. (Teper)

Axial Anomaly and the η' Mass

Spontaneous breaking of $U_A(1)$ by $< q\bar{q} >< 0$ condensate
$\Rightarrow$ Goldstone Boson η'

$$< \eta'(k) \mid A^\mu \mid 0 >= i k^\mu \sqrt{N_f} F_0 \qquad F_0 = \mathcal{O}(\sqrt{N_c})$$



Axial anomaly:

$$\partial_\mu A^\mu(x) = N_f\, q(x) \qquad q(x) = \frac{\alpha_s}{8\pi} G\tilde{G}$$

$$\begin{aligned} < \eta'(k) \mid \partial_\mu A^\mu(x) \mid 0 > &= N_f < \eta'(k) \mid q(x) \mid 0 > \\ M^2_{\eta'} &= \frac{\sqrt{N_f}}{F_0} < \eta'(k) \mid q(0) \mid 0 > = \mathcal{O}(1/N_c) \end{aligned}$$

Topological Susceptibility and Witten-Veneziano Formula

θ dependence of vacuum energy:

$$Z^E_{QCD}(\theta) \quad = \quad \int DG\, D\psi\, D\bar{\psi}\, e^{-S_{QCD}+i\,\theta\,Q}$$

$$Q = \int d^4x\; q(x) \in \mathbb{Z} \quad \Rightarrow \quad Z^E_{QCD}(\theta) = Z^E_{QCD}(\theta+2\pi)$$

Massless quarks: θ dependence disappears due to axial anomaly

$$e^{i\alpha} \in U_A(1) \Rightarrow \theta \rightarrow \theta + \alpha$$

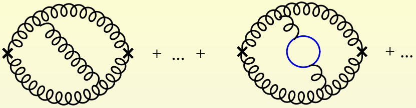
θ dependence $\propto m_u \times m_d \times m_s$

Pure YM part

$$Z^E_{YM}(\theta) = e^{-V_4\, F(\theta)}$$

$$\chi_{YM} = \left.\frac{\partial^2 F(\theta)}{\partial\theta^2}\right|_{\theta=0} \quad = \quad \frac{< Q^2 >}{V_4} = \int \langle q(x) q(0)\rangle\, d^4x$$

In QCD with massless quarks $\chi_{QCD} = 0$:



$$\int d^4x e^{ikx} < q(x) q(0) >_{quarks} \Big|_{large\ N_c}$$

$$= -\int d^4x \sum_n < 0 \mid q(x) \mid n, k >< n, k \mid q(0) \mid 0 > \frac{1}{k^2 + M_n^2}$$

$$= -\sum_n \frac{c_n F_n^2 M_n^4}{k^2 + M_n^2}$$

This cancels χ_{YM}. η' contribution: $-4\,\frac{M_{\eta'}^2 F_{\eta'}^2}{N_f}$

Large N_c limit η' dominates the sum:

$$M_{\eta'}^2 = 4\,\frac{N_f}{F_{\eta'}^2}\,\chi_{YM} \Rightarrow \chi_{YM} \sim (166\ \mathrm{MeV})^4$$

$\chi_{\rm YM}$ in Instanton Model

YM vacuum as ensemble of I + A I carries $Q = 1$

Collective coordinates: (z_μ, U, ρ) $(4, 4N_c - 5, 1)$

Action: $S_0 = \frac{4\pi}{\alpha_s} = \frac{4\pi N_c}{\alpha_0}$

$$\alpha_s \to \alpha_s(\rho) = -\frac{8\pi}{b\ \log(\rho\Lambda)} \qquad b = \frac{11}{3} N_c$$

$$\begin{aligned}
Z_{\rm I+A}(\theta) &= \sum_{N_I, N_A} Z(N_I, N_A, \theta) \\
Z(N_I, N_A, \theta) &= \frac{e^{i\theta(N_I - N_A)}}{N_I!\ N_A!} \int \prod_i (\ d\Omega_i\ e^{-S_0(i)})\ e^{-\sum_{i<j} S_{int}(i,j)} \\
d\Omega &= \left(\frac{4\pi}{\alpha_s}\right)^{2N_c} \rho^{-5}\ d^4 z\ dU\ d\rho
\end{aligned}$$

Model for interaction (Diakonov & Petrov)

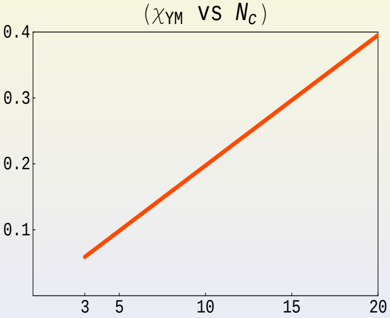
$$S_{int} = -\frac{16\pi}{\alpha_s}\,\frac{\rho^2\rho'^2}{z^4}\,f(U,U')$$

$$\bar{S}_{int} = \frac{4\pi}{\alpha_s}\frac{\kappa^2}{V_4}\,\frac{N_c}{N_c^2-1}\,\rho^2\rho'^2$$

parameter κ^2 contains regularization needed to define $\bar{S}_{int}$
Make a "mean field" approximation:

$$\sum_{i<j}\rho_i^2\rho_j^2 \rightarrow N\bar{\rho^2}\sum_i\ \rho_i^2$$

[Shuryak & Schäfer; Schäfer]



$$\kappa^2 = \frac{27}{4}\pi$$

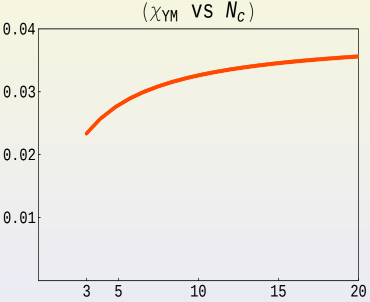
$$\bar{\rho^2} \sim \frac{11}{\Lambda^2}\left(\frac{\alpha_0}{4\pi}\right)^{\frac{12}{11}} = \mathcal{O}(N_c^0)$$

$$\bar{\eta} = \mathcal{O}(N_c)$$

$$\frac{<Q^2>}{V_4} \sim \bar{\eta} \Rightarrow \chi_{YM} = \mathcal{O}(N_c)$$

For $\chi_{YM} = \mathcal{O}(N_c^0)$ we need to modify the relation $\frac{<Q^2>}{V_4} \sim \bar{\eta}$

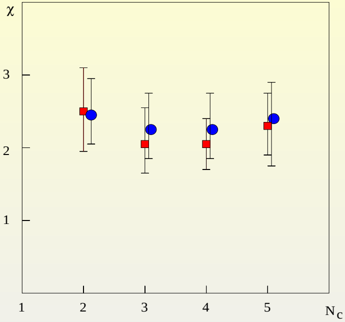
Take $S_{int}(I,I) \neq S_{int}(I,A)$: $\kappa_{II} > \kappa_{IA}$: $\Rightarrow \frac{<Q^2>}{V_4} \propto \bar{\eta}/N_c$



... χ_{YM} too small even at $N_c = 3$
or approach to $N_c \to \infty$ lim too slow

Instanton model has problem reconciling
N_c scaling of χ_{YM} and its magnitude

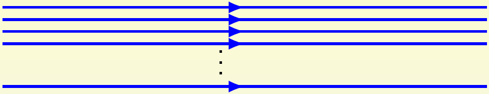
$\chi_{\rm YM}$ from Lattice



[Vicari & Panagopoulos review]

Good agreement with value from W-V formula

BARYONS IN $1/N_c$



- Need for N_c valence quarks to form a color singlet
- $M_B = \mathcal{O}(N_c)$, $r_B = \mathcal{O}(N_c^0)$
- Hartree picture of baryons sufficient to figure out $1/N_c$ countings
- π-baryon couplings: $\frac{g_A}{F_\pi}\partial_i \pi_a G^{ia}$
- Axial currents G^{ia} have $\mathcal{O}(N_c)$ MEs $\Longrightarrow$ $g_{\pi BB} = \mathcal{O}(\sqrt{N_c})$
- $\Gamma_B = \mathcal{O}(N_c^0)$
- Model realizations of all of the above: QM, Skyrme model

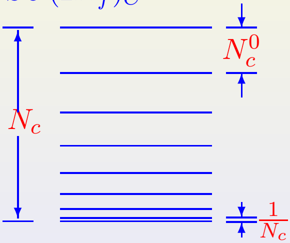
Contracted spin-flavor dynamical symmetry

$\propto \langle B' \mid [G^{jb}, G^{ia}] \mid B \rangle = \mathcal{O}(N_c^0)$

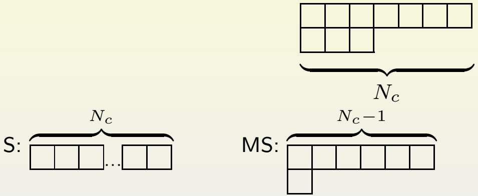
Contracted spin-flavor symmetry: [Gervais & Sakita; Dashen & Manohar]

S^i, T^a, $X^{ia} \equiv G^{ia}/N_c$: generators of *contracted* $SU(2N_f)_C$

- $SU(2N_f)$ group with generators S^i, T^a, G^{ia} can be used to build effective theory
- Justifies $SU(6)$ symmetry introduced in 1960's as dynamical symmetry in large N_c



Spin-flavor multiplets

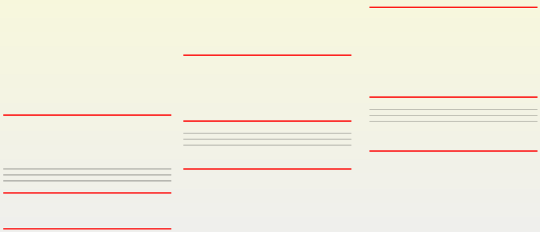


56-plet **70**-plet $N_c = N_f = 3$

Known states fit into **56** and **70**- plets of $SU(6)$
No experimentally established **20**-plet or "pentaquark"type states

Approximate $O(3)$ symmetry

Baryons show approximate $O(3)$ symmetry (small spin-orbit effects)



States classified in multiplets of $O(3) \times SU(2N_f)$ rather than $SU(2N_f)$ alone

- **Effective operators**
  - Effective operators at baryon level

$$\hat{O}_{\rm QCD} \quad \Rightarrow \quad \hat{O}_{\rm eff} = \sum \left(\frac{1}{N_c}\right)^{\nu(j)} c_j \, \hat{O}_j$$

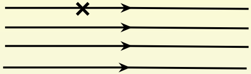
$$\nu(j) = n_j - 1 \quad \text{for} \quad n_j - \text{body} \quad \text{operator}$$

- operator basis $\{\hat{O}_j\}$ can be ordered in powers of $1/N_c$
- c_j effective constants or form factors: determined by the QCD dynamics; expanded also in $1/N_c$ and quark masses

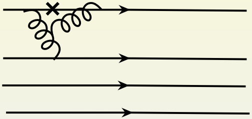
  - Matrix elements

$$\langle B'_{GS} \mid \hat{O}_j \mid B_{GS} \rangle = \mathcal{O}(N_c^{\kappa(j)}) \; ; \quad \langle B_{GS} \mid \hat{O}_j \mid B^* \rangle = \mathcal{O}(N_c^{-\frac{1}{2}+\kappa(j)})$$

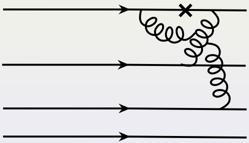
$\kappa(j)$ coherence factor of operator



1-body operator



2-body operator



3-body operator

- Effective operators

$$\hat{O}_j = R_j \otimes \mathcal{G}_j$$

R_j: $O(3)$ tensor operator

$\mathcal{G}_j$: spin-flavor tensor operator made out of product of n_j generators of $SU(2N_f)$

$\kappa(\hat{O}_j) = \#G_{ia}$ or T^8 generators in $\mathcal{G}_j$

- Systematic way of building basis of $\mathcal{G}_j$ operators; numerous "reduction rules"

Baryon Masses

• Ground state baryons mass formula

Most general mass operator after applying various reduction rules valid for MEs in the S representation

Up to $\mathcal{O}(1/N_c)$ and $\mathcal{O}(m_s)$: old Gürsey-Radicati mass formula

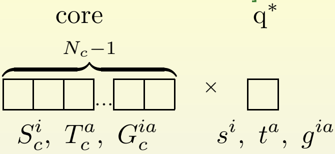
$$M_{GS} = c_1\, N_c + \frac{c_{HF}}{N_c}\,(S^2 - \frac{3}{4}N_c) - c_{\mathcal{S}}\frac{m_s - m_{u,d}}{\Lambda}\,\mathcal{S}$$

Mass relations: deviations $\mathcal{O}(1/N_c^2; m_s^2/N_c)$ and also due to chiral loop corrections

GMO	$\Xi_8 - \Sigma_8 = \frac{1}{2}(3\Lambda - \Sigma_8) - N$	128 vs 141	MeV
ES	$\Sigma_{10} - \Delta = \Xi_{10} - \Sigma_{10}$	153 vs 145	
"	$\Omega^- - \Xi_{10} = \Xi_{10} - \Sigma_{10}$	142 vs 145	
8-10	$\Sigma_{10} - \Sigma_8 = \Xi_{10} - \Xi_8$	212 vs 195	
"	$3\Lambda + \Sigma_8 - 2(N + \Xi_8) = \Xi_{10} + \Sigma_{10} - \Omega^- - \Delta$	26 vs 11	

• Excited baryon mass formulas: $[\mathbf{70}, 1^-]$

[JLG; Carlson et al.; JLG, Schat & Scoccola]



$$O_{eff} = \sum \Lambda_c \times \lambda_{q^*} \times \lambda_\ell \qquad M_{\mathbf{70}} = \underbrace{\sum_n c_n O_n}_{\text{SU(3) singlet}} + \underbrace{\sum_m d_m B_m}_{\text{SU(3) octet}}$$

- States in **70**-plet:
  $2\ \mathbf{8}_{J=1/2},\ 2\ \mathbf{8}_{3/2},\ 1\ \mathbf{8}_{5/2},\ 1\ \mathbf{10}_{1/2},\ 1\ \mathbf{10}_{3/2},\ 1\ \mathbf{1}_{1/2},\ 1\ \mathbf{1}_{3/2}$
- Operators:
  SU(3) singlets: 1 $\mathcal{O}(N_c)$ (SU(6) singlet), 4 $\mathcal{O}(N_c^0)$ and 7 $\mathcal{O}(1/N_c)$
  SU(3) octets: 3 $\mathcal{O}(m_s N_c^0)$

70-plet mass operators

$O_1 = N_c\ 1$	$\mathcal{O}(N_c)$
$O_2 = \ell \cdot s$	
$O_3 = \frac{3}{N_c}\ \ell^{(2)}_{ij}\ g^{ia}\ G_c^{ja}$	$\mathcal{O}(N_c^0)$
$O_4 = \frac{4}{N_c+1}\ \ell^i\ t^a\ G_c^{ia}$	
$O_5 = \frac{1}{N_c} t^a T_c^a - \frac{1}{2\sqrt{3}N_c} O_1$	
$O_6 = \frac{1}{N_c} \ell \cdot\ S_c$	
$O_7 = \frac{1}{N_c} S_c^2$	
$O_8 = \frac{1}{N_c} s \cdot S_c$	
$O_9 = \frac{2}{N_c}\ \ell^{(2)ij} s^i\ S_c^j$	$\mathcal{O}(1/N_c)$
$O_{10} = \frac{3}{N_c^2}\ \ell^i\ g^{ja}\{S_c^j, G_c^{ia}\}$	
$O_{11} = \frac{2}{N_c^2} t^a \{S_c^i, G_c^{ia}\}$	
$O_{12} = \frac{3}{N_c^2}\ \ell^i\ g^{ia}\{S_c^j, G_c^{ja}\}$	
$\bar{B}_1 = T_c^8 - \frac{N_c-1}{2\sqrt{3}N_c} O_1$	
$\bar{B}_2 = \frac{10}{N_c}\ d_{8ab}\ g^{ia}\ G_c^{ib} + \frac{5(N_c^2-9)}{8\sqrt{3}N_c^2(N_c-1)} O_1 + \frac{5}{2\sqrt{3}(N_c-1)} O_6 + \frac{5}{6\sqrt{3}} O_7$	$\mathcal{O}(m_s)$
$\bar{B}_3 = 3\ \ell^i\ g^{i8} - \frac{\sqrt{3}}{2} O_2$	

Fit to 70-plet masses to NLO

GMO: 3346 ± 10 vs 3373 ± 10

State	Expt.	Masses [MeV] $1/N_c$	QM [Isgur-Karl]
$N_{1/2}$	1538 ± 18	1541	1490
$\Lambda_{1/2}$	1670 ± 10	1667	1650
$\Sigma_{1/2}$	(1620)	1637	1650
$\Xi_{1/2}$		1779	1780
$N_{3/2}$	1523 ± 8	1532	1535
$\Lambda_{3/2}$	1690 ± 5	1676	1690
$\Sigma_{3/2}$	1675 ± 10	1667	1675
$\Xi_{3/2}$	1823 ± 5	1815	1800
$N'_{1/2}$	1660 ± 20	1660	1655
$\Lambda'_{1/2}$	1785 ± 65	1806	1800
$\Sigma'_{1/2}$	1765 ± 35	1755	1750
$\Xi'_{1/2}$		1927	1900
$N'_{3/2}$	1700 ± 50	1699	1745
$\Lambda'_{3/2}$		1864	1880
$\Sigma'_{3/2}$		1769	1815
$\Xi'_{3/2}$		1980	1985
$N_{5/2}$	1678 ± 8	1671	1670
$\Lambda_{5/2}$	1820 ± 10	1836	1815
$\Sigma_{5/2}$	1775 ± 5	1784	1760
$\Xi_{5/2}$		1974	1930
$\Delta_{1/2}$	1645 ± 30	1645	1685
$\Sigma''_{1/2}$		1784	1810
$\Xi''_{1/2}$		1922	1930
$\Omega_{1/2}$		2061	2020
$\Delta_{3/2}$	1720 ± 50	1720	1685
$\Sigma''_{3/2}$		1847	1805
$\Xi''_{3/2}$		1973	1920
$\Omega_{3/2}$		2100	2020
$\Lambda''_{1/2}$	1407 ± 4	1407	1490
$\Lambda''_{3/2}$	1520 ± 1	1520	1490

- Parameter free mass relations: true to $\mathcal{O}(1/N_c^2; m_s^2/N_c)$
  5 GMO, 2 ES, 4 new relations involving **8**'s, **10**'s and **1**'s
  Predict masses of several as yet unknown states in **70**-plet
- Mixing angles

	$N_c \to \infty$	Masses	Strong decays	Helicity Amplitudes
θ_1	54.4^o	30 ± 15^o	22 ± 7^o	-
θ_3	65.9^o	175 ± 15^o	161 or 136 ± 7^o	161 ± 7^o

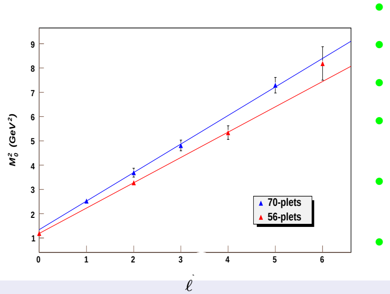
- Hyperfine terms S_c^2 dominate spin-flavor breaking
- All $\mathcal{O}(N_c^0)$ operators in **70**-plets have small coefficients: three spin-orbit type and one of pure flavor type: approximate $O(3)$ symmetry
- All $\mathcal{O}(1/N_c)$ operators other than S_c^2 give small corrections
- $SU(3)$ breaking dominated by strangeness operator

A new take on baryon Regge trajectories $\mathrm{M}_0(\ell)^2$ vs ℓ

[JLG & Matagne]

Spin-flavor singlet component of baryon masses from analyses of all well and not so well known **56**-plets and **70**-plets:

$$[\mathbf{56}, 0^+]_{GS},\ [\mathbf{56}, (2^+, 4^+, 6^+)],\ [\mathbf{70}, (1^-, 2^+, 3^-, 5^-)]$$



- $M_0^2[\mathbf{56}, \ell] = [(1.18 \pm 0.003) + (1.05 \pm 0.01)\,\ell]\ \mathrm{GeV}^2$
- $M_0^2[\mathbf{70}, \ell] = [(1.13 \pm 0.02) + (1.18 \pm 0.02)\,\ell]\ \mathrm{GeV}^2$
- $(M_0[\mathbf{70}, \ell] - M_0[\mathbf{56}, \ell])^2 \simeq (5.7 + 4.2\,\ell) \times 10^{-4}\ \mathrm{GeV}^2$
- Splitting between trajectories $\mathcal{O}(N_c^0)$: due to exchange interaction. In magnitude smaller than expected.
- Regge trajectories with physical masses include contributions which do not have linear behavior.
- Strong indication of small **56**-**70** configuration mixings and good approximate $O(3)$ symmetry

$1/Nc$ Expansion in HBChPT: case of $SU(2)$ to 1-loop

[Flores-Mendieta, Hofmann, Jenkins & Manohar; JLG work in progress]

Heavy Baryon expansion: remove the spin-flavor singlet mass component M_0 from baryon field

Baryon field is a $\frac{(N_c+1)(N_c+2)(N_c+3)}{6}$-plet of $SU(4)$ (20-plet for $N_c = 3$)

$$B = \begin{pmatrix} N \\ \Delta \end{pmatrix}$$

Two possible counting schemes:

i) $1/N_c = \mathcal{O}(p) = \mathcal{O}(\xi)$ ii) $1/N_c = \mathcal{O}(p^2) = \mathcal{O}(\xi^2)$

Since $M_\Delta - M_N > M_\pi$ scheme i) is correct one

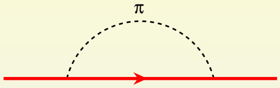
Effective Lagrangian can be ordered in powers of ξ

$$\mathcal{L} = \overbrace{iB^\dagger\, D_0\, B}^{\mathcal{O}(\xi)} + \overbrace{\frac{6}{5}\, g_A\, B^\dagger\, u_{ia} G^{ia}\, B}^{\mathcal{O}(\xi^{1/2})} - \overbrace{\frac{C_{HF}}{N_c} B^\dagger\, \vec{S}^2\, B}^{\mathcal{O}(\xi)} + \overbrace{c_1\; N_c \langle \chi_+ \rangle B^\dagger B}^{\mathcal{O}(\xi)} + \cdots$$

$$\begin{aligned} D_0 &= \partial_0 + \frac{i}{4F_\pi^2} \epsilon_{abc} \pi^b \partial_0 \pi^c I^a + \cdots \qquad u_{ja} = -\frac{i}{F_\pi} \partial_j \pi^a + \cdots \\ \langle \chi_+ \rangle &= 4M_\pi^2 \qquad F_\pi = 92.4\mathrm{MeV} \qquad g_A = 1.267 \end{aligned}$$

Self-energy to 1-loop

ξ-counting: $\nu = \sum_n (\nu_n - 1) + 3L + \frac{N_\pi}{2}$



$= \mathcal{O}(\xi^2)$

$$\delta\Sigma^{1-loop} = i\frac{36}{25}\frac{g_A^2}{F_\pi^2}\frac{1}{3-2\epsilon}\sum_n G^{ia}\mathcal{P}_n G^{ia} I_\Sigma(p_0 - \delta m_n, M_\pi)$$

$$\begin{aligned} I_\Sigma(q,M) &= \frac{i}{16\pi^2}\Big(q(2q^2-3M^2)(\lambda_\epsilon + \frac{1}{3} - \log(M^2-q^2) + 2\pi(M^2-q^2)^{3/2} \\ &+ 3q(q^2-M^2)\frac{\partial}{\partial\nu}\ {}_2F_1(\frac{1}{2},\nu,\frac{3}{2},\frac{q^2}{q^2-M^2})_{\nu\to 1}\Big) \\ \lambda_\epsilon &= \frac{1}{\epsilon} - \gamma + \log 4\pi \end{aligned}$$

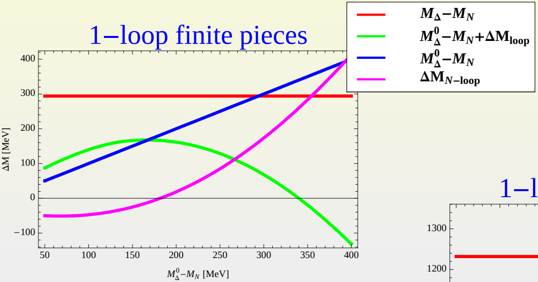
Renormalization

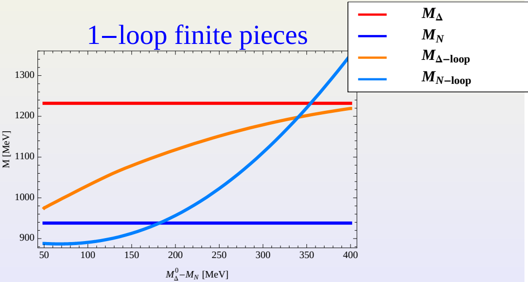
Consistency with $1/N_c$ counting: $p_0 \to p_0 + \delta m_{\rm in}$; expanding in p_0

$$
\begin{aligned}
\delta\Sigma_{UV}^{1-loop} \quad = & \quad -\frac{36}{25}\frac{g_A^2}{48F_\pi^2}\lambda_\epsilon \\
\times & \quad \Big\{3C_{HF}M_\pi^2(\frac{3}{8}(N_c+4)+\frac{5}{N_c}\vec{S}^2)+\frac{C_{HF}^3}{N_c^2}(-\frac{3}{2}(N_c+4)+(24-\frac{5}{2}N_c(N_c+4)\vec{S}^2)) \\
+ & \quad p_0\left(3M_\pi^2(\frac{3}{16}N_c(N_c+4)-\frac{1}{2}\vec{S}^2)+\frac{C_{HF}^2}{N_c^2}(\frac{9}{2}N_c(N_c+4)+3(N_c+6)(N_c-2)\vec{S}^2)\right)\Big\}
\end{aligned}
$$

- All mass UV divergencies induced by $\Delta - N$ mass splitting
- All finite mass terms $\mathcal{O}(\xi^2)$ induced by $\Delta - N$ mass splitting
- Finite $\mathcal{O}(\xi^3)$ terms but different than those as in ordinary BChPT due to Δ in loop

Masses: 1-loop finite contributions $\mu = m_\rho$





COMMENTS and OUTLOOK

- $1/N_c$ expansion is very useful providing an additional ordering principle for hadron phenomenology
- This has been shown in both mesons and baryons
- Multiple applications to excited baryons: masses, strong decays, helicity amplitudes. More to be done
- Interesting to further investigate applications in BChPT
- Possible applications analyzing lattice results, especially for baryons: e.g. understand quark mass dependencies of effective coefficients in mass formulas