Manifestation of conformal symmetry in the spectra of the light flavor baryons

M. Kirchbach (IF-UASLP)

1. Clustering phenomenon in N and Δ spectra
2. The AdS/CFT correspondence concept and Wilson loop potentials. The sech^2 potential.
3. Spectra from $\mathbf{R}^1 \times S^3$ trapping
4. Dressing function of the gluon propagator
5. AdS_{2+1} confinement by the sech^2 potential
6. Comparison to the light-cone spectra
7. Conclusions

1. Clustering phenomenon in N and Δ spectra

Observation: *M.K., MPLA* **12** *(1997)*

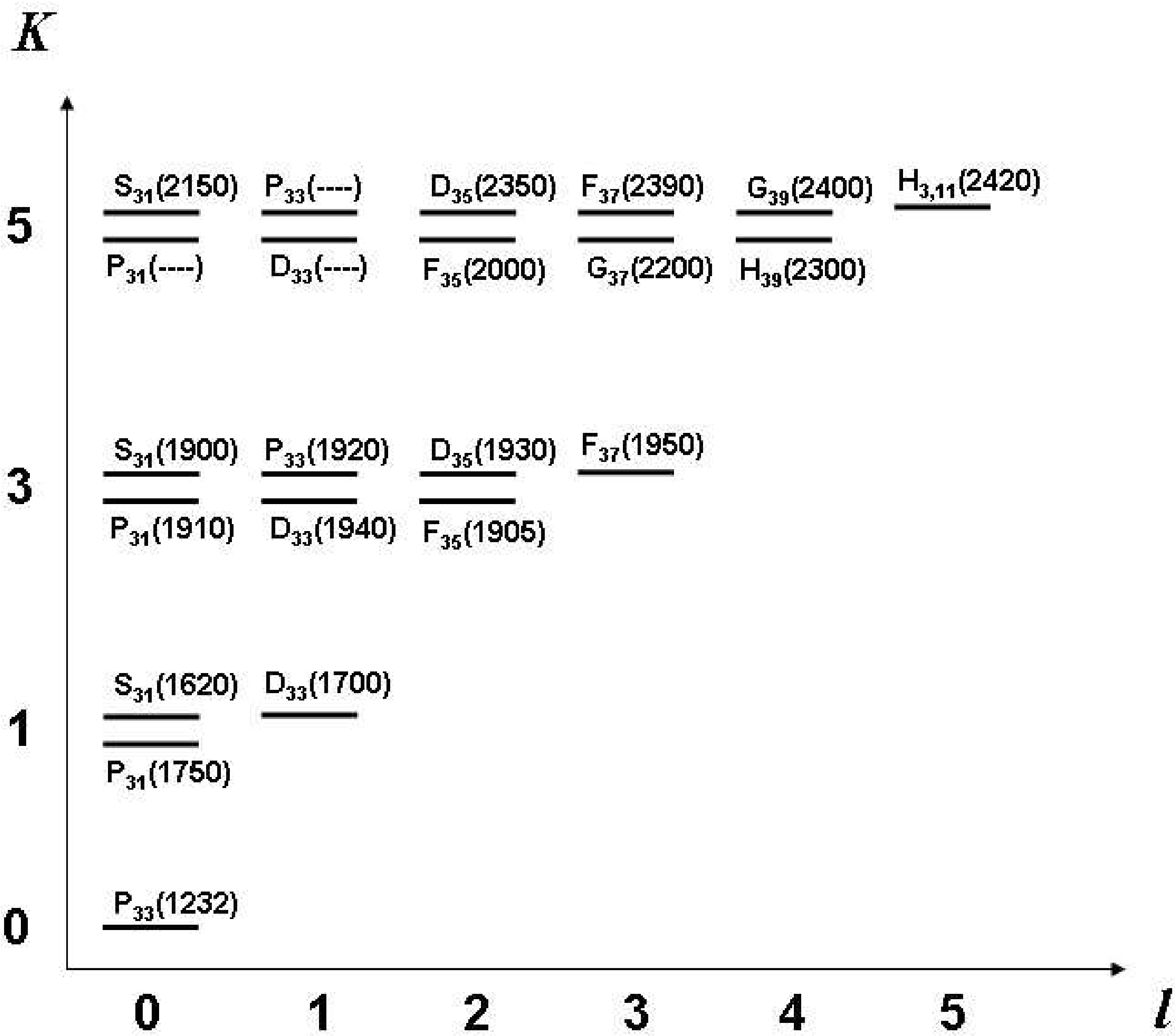
- N and $\Delta(1232)$ spectra reveal identical prominent clustering patterns of the states with masses below 2500 MeV.
- The clusters are located in three mass regions, well separated from each other by gaps of the order of 100-150 MeV.
- A cluster consists of one state of maximal spin,

$$J^\pi = \left(K + \frac{1}{2}\right)^\pi, \quad \pi = \begin{cases} - & \text{for} \quad K = 1, \\ + & \text{for} \quad K = 3, 5. \end{cases}$$

and K parity dyads,

$$J^\pm = \frac{1}{2}^\pm, ..., \left(K - \frac{1}{2}\right)^\pm, \quad K = 1, 3, 5,$$

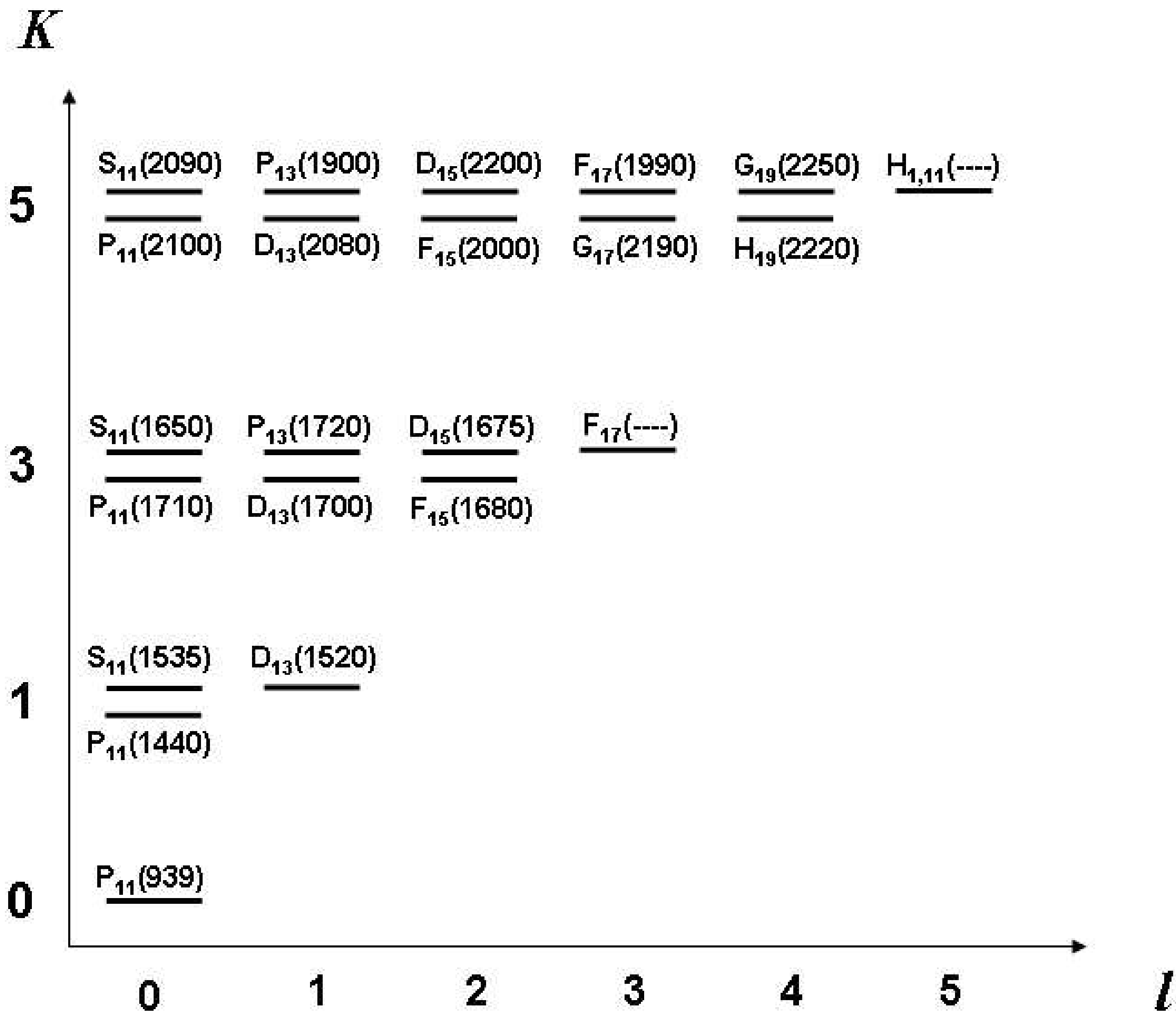
a total of $(1 + 2K)$ states in a cluster (K=1,3,5)



Clustering phenomenon in the Δ spectrum.

One loner front spin + K pillion spins,

a total of $(1 + 2K)$ states in each cluster.



Clustering phenomenon in the N spectrum.
One loner front spin + K pillion spins,
a total of $(1 + 2K)$ states in each cluster.

GOAL:

Design a quark potential model that describes this phenomenon.

Search for a potential that:

1. respects the space-time symmetries of the QCD Lagrangian,
2. captures correctly the quark-gluon dynamics in all three regimes,
   - perturbative ($\sim 1/r$ potential)
   - non-perturbative ($\sim r$ potential)
   - asymptotic freedom (free quarks at small distances, trapped at long distances, as in the infinite radial well),
3. can be placed within the context of the AdS/CFT correspondence concept

The space-time symmetries of the QCD Lagrangian

- $\sim SO(2,4)$ conformal symmetry of the light-flavor sector of QCD.
  Conformal symmetry would require the N and Δ spectra to fall each into an ∞d unitary SO(2,4)representation in parallel to the conformally invariant Maxwell equations which place the spectrum of the H atom (as a whole) in a SO(4,2) irrep of this type.
- $SO(2,1)/SO(4)$ symmetries of the perturbative regime: In the perturbative regime the $1g$ exchange gives raise to a 1/r interaction known to have $SO(2,1)/SO(4)$ as potential algebras:

$$SO(2,4) \supset SO(4) \supset SO(3) \supset SO(2)$$

$$SO(2,4) \supset SO(2,1) \supset SO(2)$$

2. The AdS/CFT correspondence concept...

D3 branes ((3+1)d world-volume) solve superg. eq. m.

D3 brane surrounded by 6 transversal dimensions, 1 radial + 5 angular (S^5)

Near horizon geometry: D3 brane theory reduces to a 4d super Yang-Mills on $AdS_{4+1} \times S^5$

Conformal boundary of $AdS_{4+1} \times S^5$ at ∞ :

$\mathbf{R}^{1,3}$ Minkowski space

Maldacena's conjecture:

Zero-T super 4d Yang-Mills on the conformal boundary equivalent to high-T 3d QCD.

Lüscher, Mack (1975): Confinement as trapping on finite volume and spectrum discretization. Compactify $\mathbf{R}^{1+3}$ conformally to $\mathbf{R}^1 \times S^3$ and study hadrons there.

...and Wilson loop potentials

Derive 3d quark confinement potentials
from AdS/CFT

Technique: Wilson loop (generalization of the Ahronow-Bohm loop integral to non-Abelian theories)

$$< W(C) >= N_c e^{-\sigma S_{min}}$$

S_{min}: minimal area inside the contour C,
σ string tension

- rectangular loop + cut off: Coulombic +linear potential (Cornell pt.)
- deformed gravitational bulks: non-perturbative corrections to the Cornell potential
- open strings with ends on Rindler space (portion of Minkowski space adapted to an observer at constant acceleration) produces the sech^2 potential.

3. Spectra from $\mathbf{R}^1 \times S^3$ trapping

Factorizing $\mathbf{R}^1$ time, e^{-iEt}, reduces the problem to the stationary Schrödinger equation on S^3

Geodesic motion on S^3 is described in terms of the squared 4D-angular momentum, $\mathcal{K}^2$,

$$\Box_{E_4} = -\frac{1}{R^2}\mathcal{K}^2, \quad 1/R^2 \equiv \kappa$$

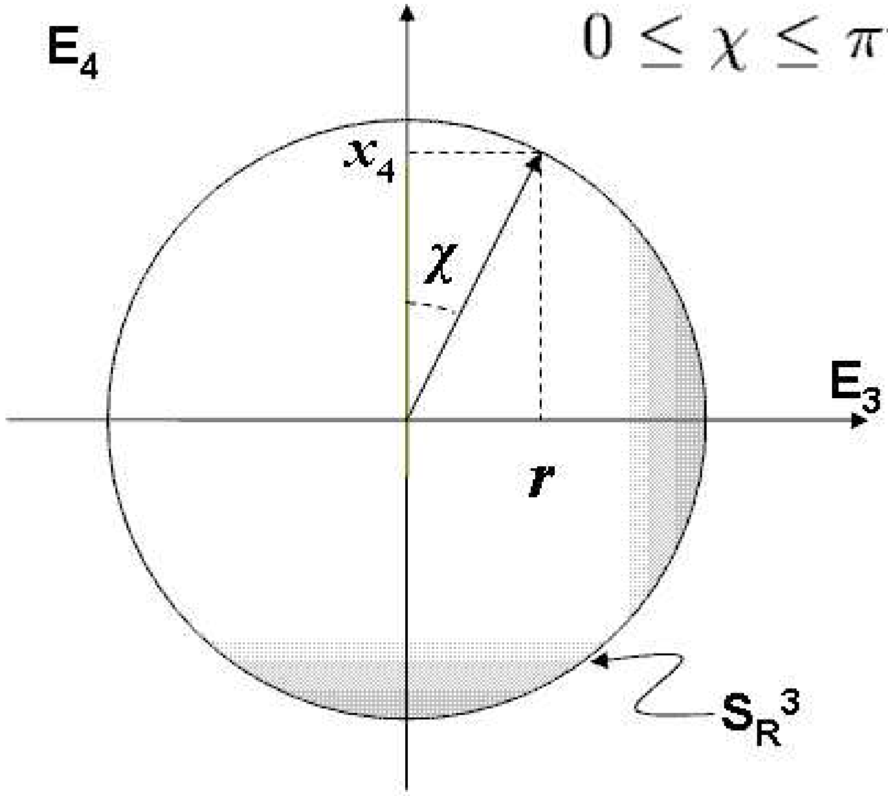
$$\mathcal{K}^2 = \left[\frac{1}{\sin^2\chi}\frac{\partial}{\partial\chi}\sin^2\chi\frac{\partial}{\partial\chi} - \frac{L^2}{\sin^2\chi}\right],$$

κ: constant positive curvature

$$-\frac{\hbar^2}{2\mu}\kappa\mathcal{K}^2|Klm> = \frac{\hbar^2}{2\mu}\kappa K(K+2)|Klm>,$$

$$E_K(\kappa) = \frac{\hbar^2\kappa}{2\mu}[(K+1)^2 - 1]$$

$l = 0, 1, 2, ...K$ degeneracy: SO(4) potential algebra



Robertson-Walker (polar) parametrization of S^3.

Different reading of the $\mathcal{K}^2$ eigenvalue problem on S^3 upon changing to $\psi(\chi,\kappa) = \sin\chi\,\mathcal{S}(\chi,\kappa)$:

$$\left[-\kappa\frac{\hbar^2}{2\mu}\frac{\mathrm{d}^2}{\mathrm{d}\chi^2} + U_l(\chi,\kappa)\right]\mathcal{S}(\chi,\kappa) = E_K(\kappa)\mathcal{S}(\chi,\kappa),$$

$$U_l(\chi,\kappa) = \kappa\frac{\hbar^2}{2\mu}l(l+1)\csc^2\chi,$$

$\csc^2$ as standard centrifugal barrier on S^3

The potential algebra does not change upon adding to the $\csc^2\chi$ the harmonic $\cot\chi$ function, $\mathcal{K}^2 \cot\chi = 0$.

$$\left[-\kappa \frac{\hbar^2}{2\mu}\mathcal{K}^2 + G\sqrt{\kappa}\cot\chi \, - E(\kappa)\right] X(\chi,\kappa) = 0$$

Solved in:

[Compean, M.K., J. Phys. A:Math.Gen. **39** *(2006)]*

$\cot\chi$ is the exactly solvable extension of the Wilson loop/Lattice Coulombic +linear potential by phenomenological non-perturbative corrections:

$$\mathcal{V}(\chi,\kappa) = 2G\sqrt{\kappa}\left(-\frac{1}{\chi} + \frac{1}{3}\chi + \frac{\chi^3}{45} + \frac{2\chi^5}{945} + ...\right)$$

$$+\kappa\frac{\hbar^2}{2\mu}\frac{l(l+1)}{\chi^2} + ..., \quad \chi = r\sqrt{\kappa}$$

Solutions require non-classical

Romanovski polynomials

$$R_n^{(\alpha,\beta)}(x)=e^{\alpha\cot^{-1}x}(1+x^2)^{-\beta+1}$$
$$\times\frac{\mathrm{d}^n}{\mathrm{d}x^n}e^{-\alpha\cot^{-1}x}(1+x^2)^{\beta-1+n},$$

where $x=\cot\chi$

Reviewed in:

[Raposo, Weber, Alvarez-Castillo, M.K., C. Eur. J. Phys. (2007)]

The $\cot+\csc^2$ spectrum is

$$E_K(\kappa)=-\frac{G^2}{\frac{\hbar^2}{2\mu}}\frac{1}{(K+1)^2}+\kappa\frac{\hbar^2}{2\mu}((K+1)^2-1),$$
$$l=0,1,2,...,K.$$

and reveals $SO(4)$ as potential algebra again.

4. Dressing function of the gluon propagator

Gluon propagator in Landau gauge:

$$G^{ab}_{\mu\nu}{=}{-}i[(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2})\frac{G(q^2)}{q^2}]\delta^{ab},$$

$$G(q^2){=}\left(1+i\frac{\Pi(q)}{q^2}\right)^{-1},$$

"Dressing" function $G(q^2)\sim q^2$, finite in the infrared

Fourier transform of $\cot\chi$ with the S^3 integration volume

$$\Pi(|\mathbf{q}|){=}{-}2G\sqrt{\kappa}\int_0^\infty d|x||x|^3\delta(|x|-R)\int_0^{2\pi}d\varphi$$

$$\int_0^\pi d\theta\sin\theta\int_{0/\frac{\pi}{2}}^{\frac{\pi}{2}/\pi}d\chi\sin^2\chi e^{i|\mathbf{q}|\frac{\sin\chi}{\sqrt{\kappa}}|\cos\theta}\cot\chi$$

with

$$e^{iq\cdot x}=e^{i|\mathbf{q}||\mathbf{r}|\cos\theta}{=}e^{i|\mathbf{q}|\frac{\sin\chi}{\sqrt{\kappa}}\cos\theta},$$

$$|\mathbf{r}|{=}R\sin\chi=\frac{\sin\chi}{\sqrt{\kappa}}.$$

Result:

$$G(\mathbf{q}^2)=2c\sin^2\frac{\mathbf{q}}{2}=c(\frac{\mathbf{q}^2}{2!}-\frac{\mathbf{q}^4}{4!}+\frac{\mathbf{q}^6}{6!}-...),$$

$$c=2G\frac{2\mu}{\hbar^2\kappa}.$$

$\frac{G(\mathbf{q}^2}{\mathbf{q}^2}$ finite at origin, in accord with lattice QCD and Dyson-Schwinger approaches

Compean, Kirchbach, J.Phys. A:Math.Theor. **42** *(2009)*

5. AdS_{2+1} confinement by the sech^2 potential

Change variables in the $-\frac{\hbar^2}{2\mu}\mathcal{K}^2$ eigenvalue problem to

$$\sin\chi = \frac{1}{\cosh y}, \quad \cos\chi = \tanh y, \quad R(\chi) = Y(y).$$

and obtain 1D Schr. eq. with the Pöschl-Teller pot.

$$-\left[\frac{d^2}{dy^2} + \frac{\epsilon - \frac{1}{4}}{\cosh^2 y}\right] Y(y) = -\left(l + \frac{1}{2}\right)^2 Y(y),$$

$$\epsilon = \frac{2\mu E_K^{G=0}(\kappa)}{\hbar^2\kappa} = (K+1)^2$$

COMPARE THIS TO:

$$\left[-\frac{d^2}{dy^2} + \frac{\mathcal{J}_3^2 - \frac{1}{4}}{\cosh^2 y}\right] Y(y) \equiv \left[-\mathcal{J}^2 - \frac{1}{4}\right]_{AdS_{2+1}} Y(y)$$

$$= -\left(j - \frac{1}{2}\right)^2 Y(y)$$

- $\mathcal{J}^2$: squared 3D pseudo-angular (SO(2,1)) mom.
- $\mathcal{J}_3$: pseudo-magnetic (SO(2,1)) quantum number

Equalizing both eqs. amounts to

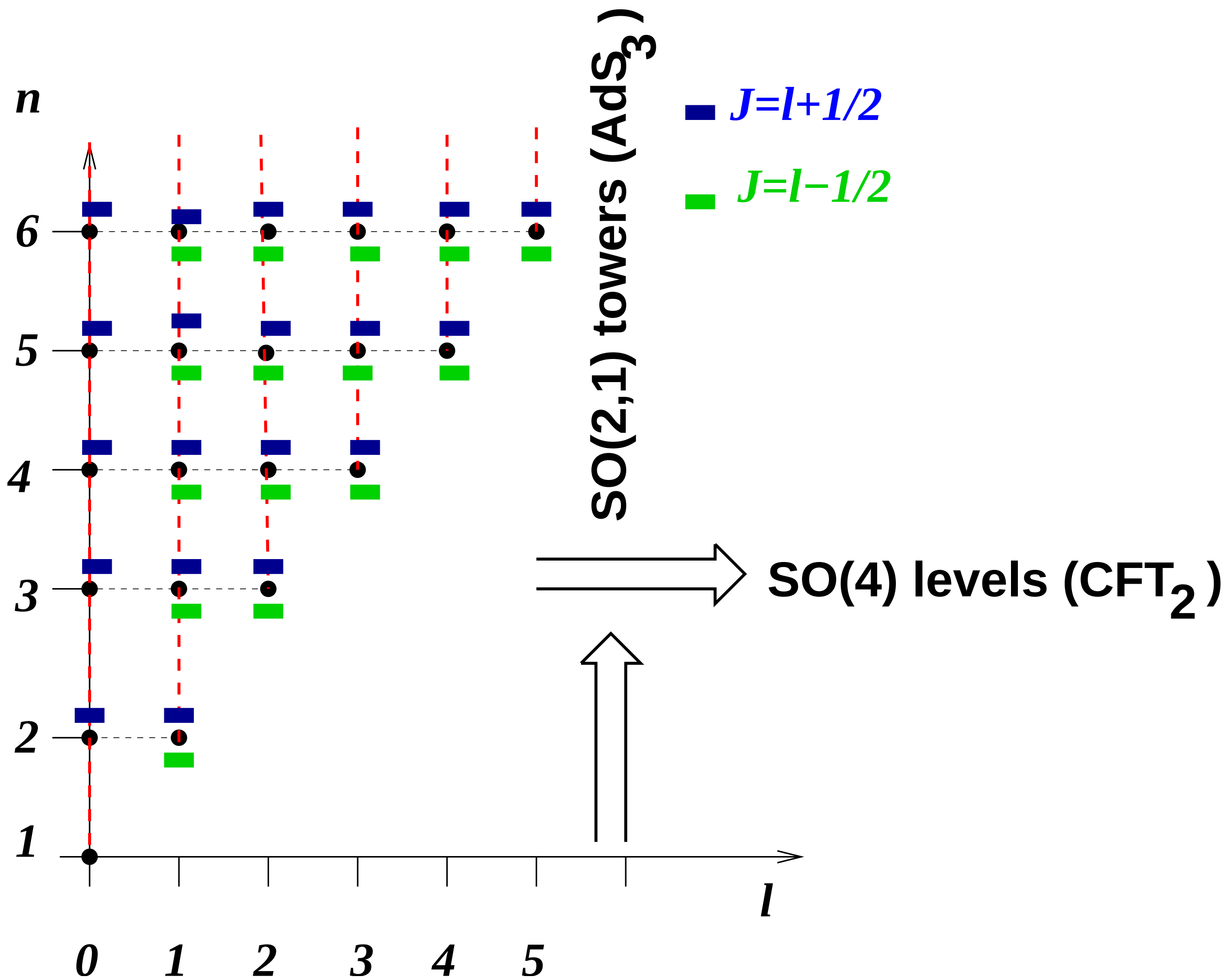
- $\mathcal{J}^2 Y(y) = j(j-1)Y(y) = l(l+1)Y(y)$, implying $j = l+1$,
- $\mathcal{J}_3^2 Y(y) = (m')^2 Y(y) = \epsilon Y(y)$, meaning, $m' = j + n_r = l + 1 + n_r$=K+1,

which recovers the cot spectrum on $\mathbf{R}^1 \times S^3$ now as J_3^2 spectrum on the one-sheet AdS_{2+1} hyperboloid $\mathbf{H}_1^{1+1}$:

$$\mathbf{H}_1^{1+1}: \quad \xi_0^2 - \xi_1^2 - \xi_2^2 = -R^2$$

Pseudo-magnetic quantum number, m', limited from below, unlimited from above:

$$\mathcal{J}_z Y(y) = m' Y(y), \quad m' = j + n_r, \quad n_r = 0, 1, 2, 3, \ldots$$



∞d $SO(2,1)$ unitary irreps $D^{+(m'=j+n_r)}_{j=l+1}$

put up to a SO(2,4) unitary irrep

Identify $m' = K + 1$ with n, the principal qnt. nmbr.

$$n = \begin{cases} K+1, \quad \text{for} \quad SO(4), \quad \text{with} \quad K+1 = l+1+n_r, \\ m' \quad \text{for} \quad SO(2,1), \ \text{with } m' = j + n_r, \ j = l+1. \end{cases}$$

Energy of geodesic motion on AdS_{2+1} given in terms of $\mathcal{J}_3^2$ eigenvalues as

$$\epsilon_n = n^2 = (j+n_r)^2 = (l+1+n_r)^2,$$

$$l = 0, 1, 2, ..., n-1 \quad \text{degeneracy.}$$

Geodesic motions on S^3/H_1^{1+1} characterized by same degeneracies

Recapitulate:

- $\csc^2$ potential on S^3 dual to Wilson loop potential sech^2 on the AdS$_{2+1}$ hyperboloid H_1^{1+1}
- $\mathbf{R}^1 \times S^3$ /AdS$_{2+1}$ duality implies dual SO(2,1) and SO(4) potential algebras.
- Spectrum as a whole falls into a SO(4,2) irrep in accord with the ∼conformal inv. of QCD Lagrangian.
- Symmetries of perturbative regime promoted to degeneracy symmetries.

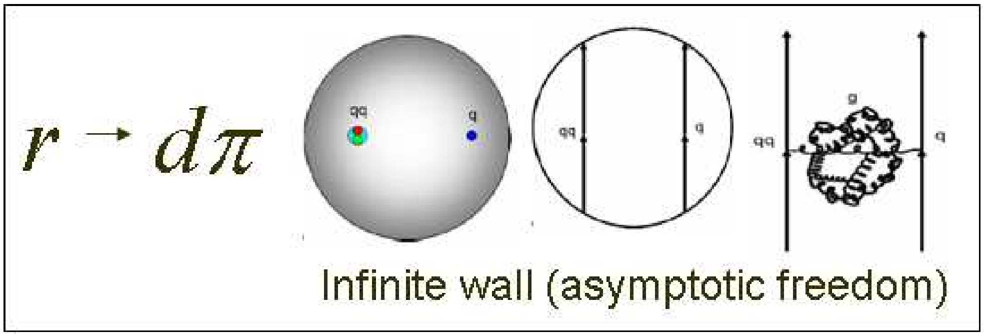
Inclusion of cot conserves the SO(4)/SO(2,1) dual potential algebras and the dynamical SO(2,4) algebra as well as visible from the energy solutions.

3. N and Δ spectra from $\cot + \csc^2$

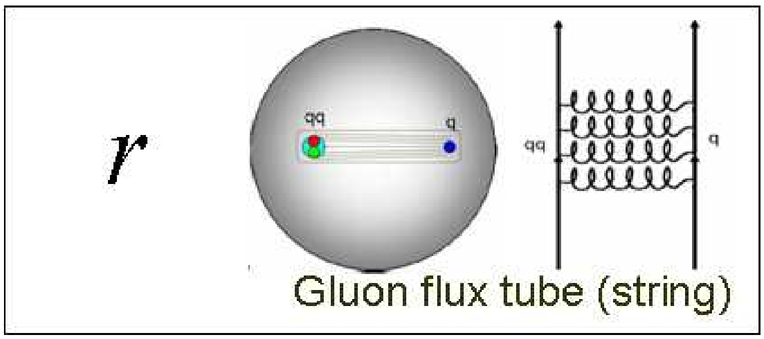
$$\mathcal{V}(\chi,\kappa)=\kappa\frac{\hbar^2}{2\mu}l(l+1)\csc^2\chi+\frac{G}{\sqrt{\kappa}}\cot\chi\,,$$

Virtues of $\cot + \csc^2$:

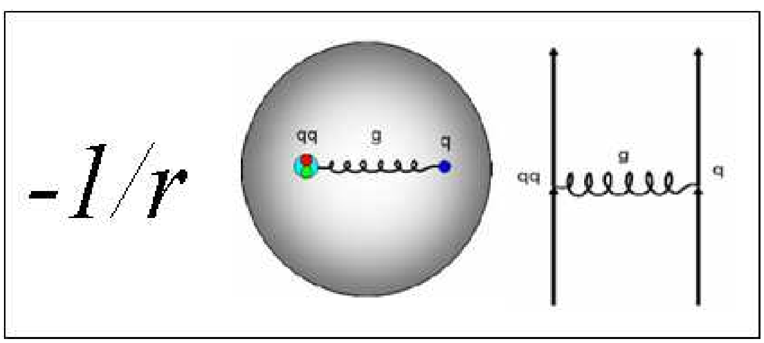
- describes interaction between effective quark-diquark degrees of freedom on curved (S^3/H_1^{1+1}) spaces with the curvature absorbing many-body effects,
- embeds the Cornell potential as small angle approximation,
- provides a phenomenological non-perturbative corrections beyond the Coulombic+linear terms,
- captures adequately the quark-gluon dynamics of all three regimes of QCD.



Infinite well piece $\Longrightarrow$ asymptotic freedom.

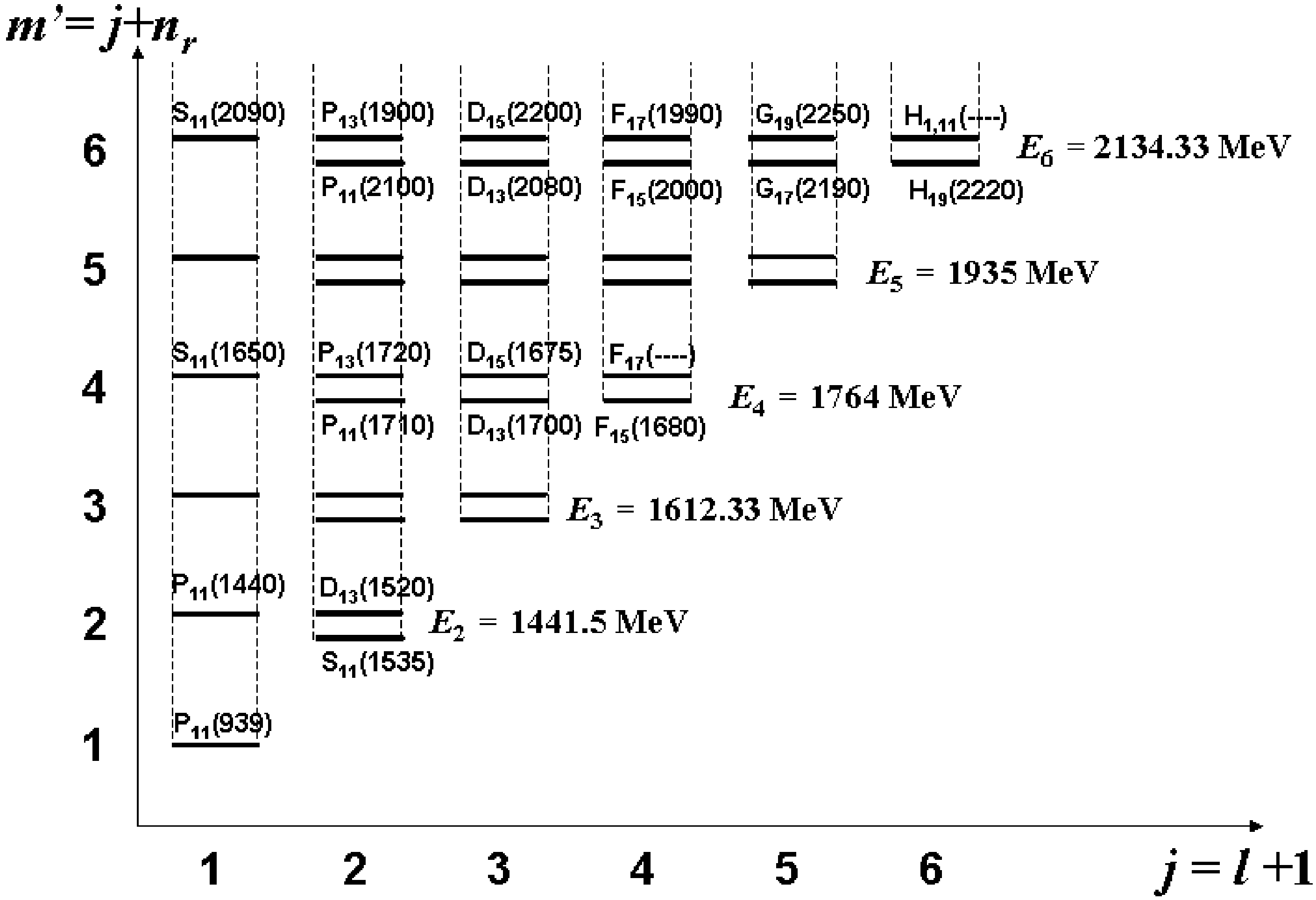


Linear piece$\Longrightarrow$ flux-tube interactions.



Coulomb piece $\Longrightarrow$ $1g$ exchange

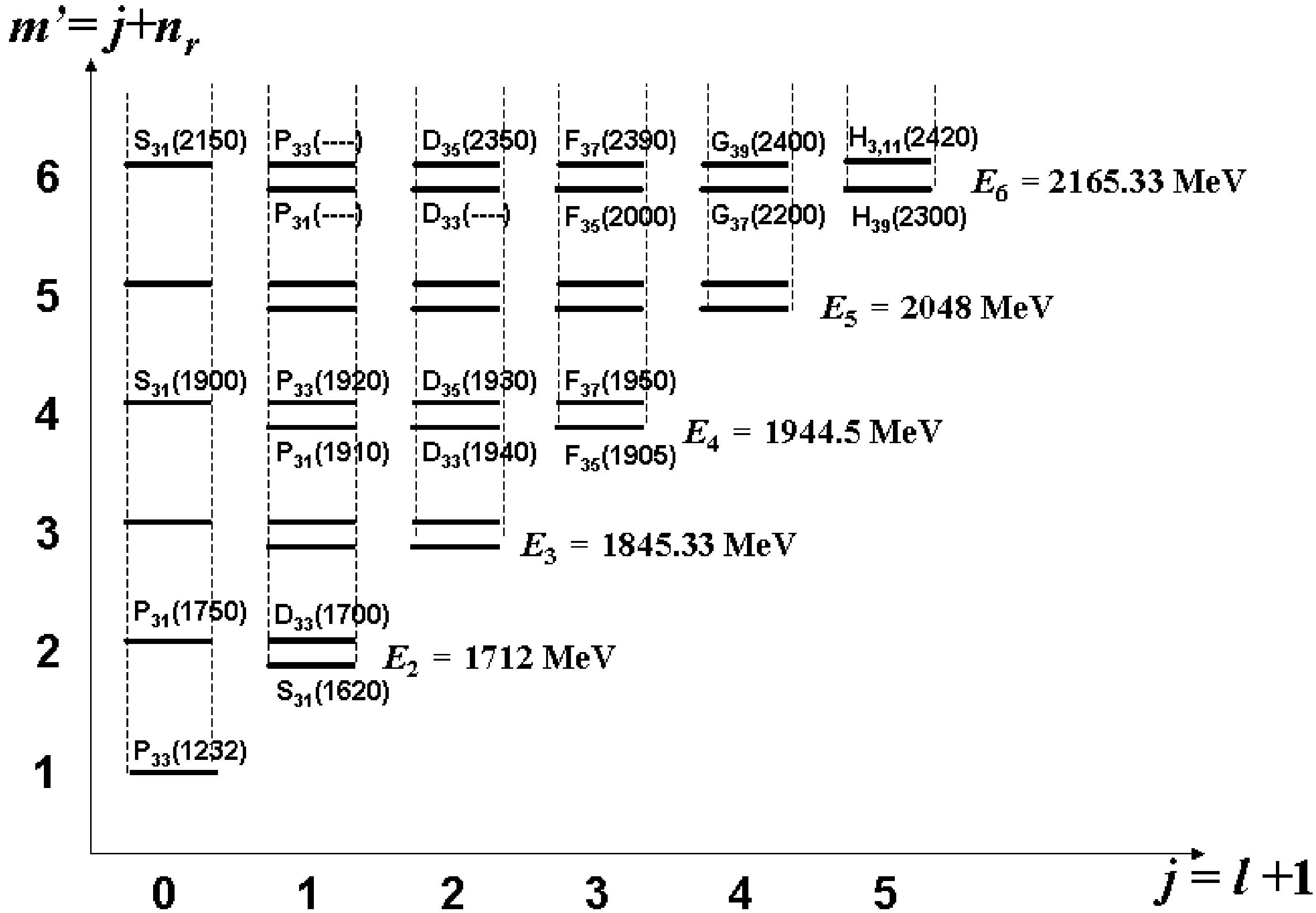
Fitting potential parameters to the N spectrum gives $G = 237.55$ MeV$\cdot$ fm, $\kappa = 0.019$ fm $^{-2}$, $\mu = 1.057\,\mathrm{fm}^{-1}$.



$SO(2,1)/SO(4)$ dual symmetry patterns in the N spectrum.

Prominent in the complete data set.

The completely missing $m' = 3, 5$ for any l are built on top of a scalar diquark.



$SO(2,1)/SO(4)$ dual symmetry patterns in the Δ spectrum,

$\kappa = 0.011\text{fm}^{-2}$. Prominent in the complete data set.

The completely missing $m' = 3, 5$ for any l

are built on top of a scalar diquark.

A total of 33 "missing" N and Δ resonances.

6. Comparison to the light cone spectra

"holographic" principle:

AdS/CFT conjectures correspondence between gravity on AdS_{d+1} and CFT_d. For $d = 1$ this amounts to light-ray holography. In parallel, CFT_d is viewed as "holographic" projection of AdS_{d+1}. A "holographic" relation may exist also between any QFT and some of its lower dimensional CFT's.

Simplest example:

- Chiral conformal holographic image of 2d massive QFT, so called light-ray restriction quantization, amounts to light-cone quantization formalism.

[*Teramond, Brodsky, PRL* **84**, *201601 (2005)*]

Amounts to solving 2d Schrödinger equation

$$-\frac{d^2\Psi_+}{d\zeta^2}+\frac{\nu^2-\frac{1}{4}}{\zeta^2}\Psi_+ +(\kappa^4\zeta^2+2\kappa^2(\nu+1))\Psi_+ = \mathcal{M}^2\Psi_+.$$

Wave-functions:

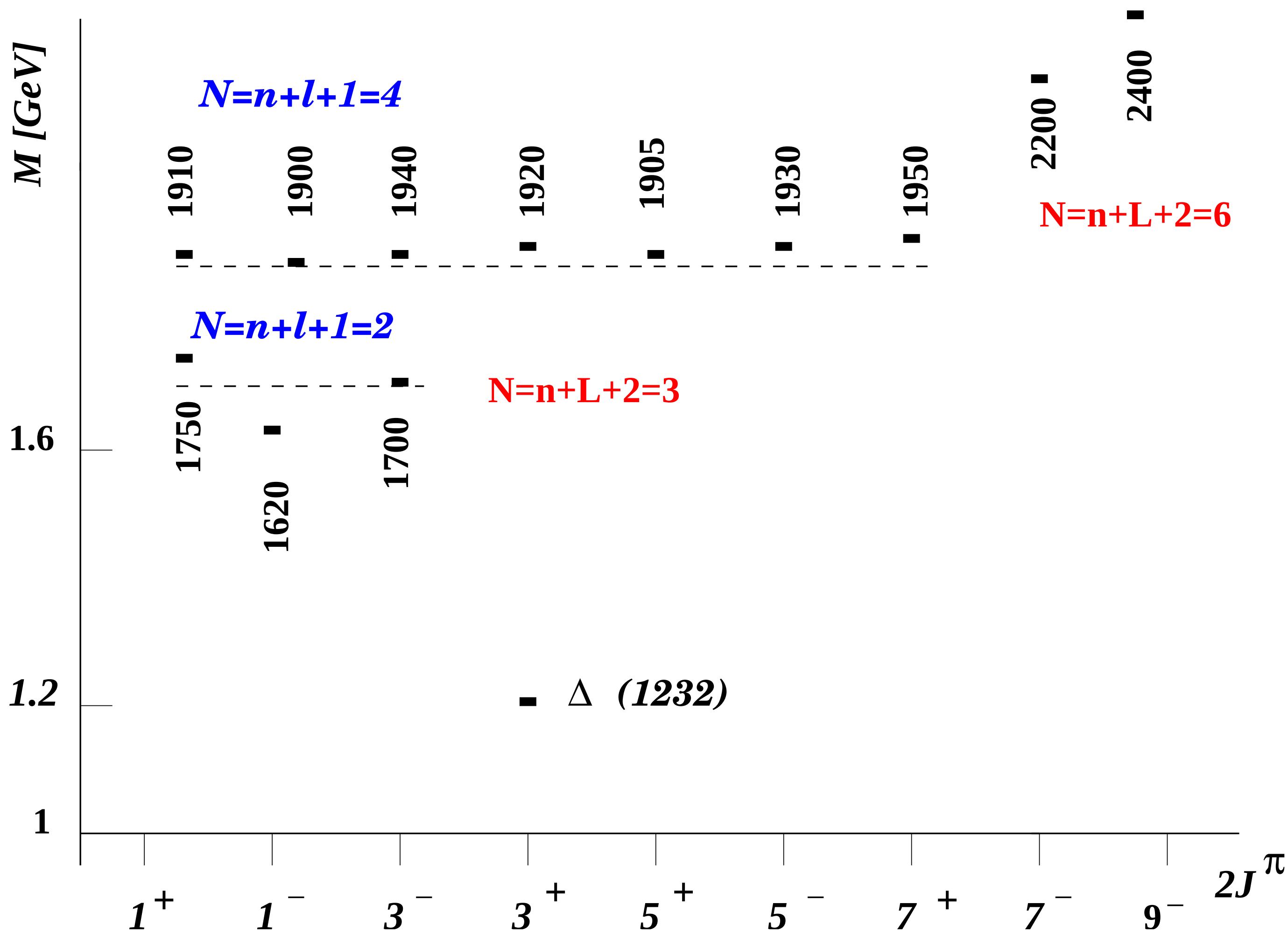
$$\Psi_+ = y^{\frac{\nu+\frac{1}{2}}{2}}e^{-\frac{y}{2}}L_n^\nu(y), \quad y=\kappa^2\zeta^2,$$

Mass spectrum:

$$\mathcal{M}^2 = 4\kappa^2 N, \quad N=n+\nu+1$$

Degeneracy $\nu = 0,1,2,...,N-1$

Identification" $\nu = L+1$ with L from $SU(6)_{SF}\times O(3)_L$. implying $L=0,1,2,N-2$. In coupling to this $S=\frac{1}{2},\frac{3}{2}$ strong overlaps between $SU(6)_{SF}\times O(3)_L$ multipelts found.



Clustering phenomenon in the light-cone Δ spectrum. $N = l + n + 1 = 4$ and $N = l + n + 1 = 6$ conformal SO(4) levels get mixed up to a $N = L + n + 1 = 6$ non-conformal level. Artifact of the non-conformal potential employed.

5. Curvature parameter and deconfinement

$\cot + \csc^2$ is a three-parameter potential, the strength G, the reduced mass μ, and the curvature, κ as a driver of the confinement-deconfinement transition

When curvature goes down, High-Lying S^3 states approach "flat" scattering states of the $1/r$ piece for both the energy and wave functions.

Two limits:

- $\kappa \longrightarrow 0$,
- $K\sqrt{\kappa} \longrightarrow k, \quad k$ constant,

$$E_K(\kappa) \xrightarrow{\kappa \to 0} -\frac{G^2}{\frac{\hbar^2}{2\mu}} \frac{1}{n^2},$$

$$E_K(\kappa) \xrightarrow{K\sqrt{\kappa} \to k} -\frac{G^2}{\frac{\hbar^2}{2\mu}} \frac{1}{n^2} + \frac{\hbar^2}{2\mu}\mathrm{k}^2.$$

Possible because of the common $SO(4)/SO(2,1)$ symmetries shared by the $\cot + \csc^2$ and $1/r$ potentials.

- Barut,Wilson, J.Phys.A:Math.Gen. **20**, 6271 (1987)

As curvature can go down because of a possible density dependence, confinement fades away, an observation that is suggestive of a deconfinement scenario controlled by the curvature parameter of the $\cot + \csc^2$ potential.

Deconfinement as flattening of space considered by

- F. Takagi, PRD **35**, 2226 (1987)

within the context of a AdS_5 black hole universe as bag scenario.

Advantage of our scheme:

Density dependent space flattening paralleled by regression of the "curved" $\cot + \csc^2$ – to the flat $1/r$ potential, and correspondingly, by regression of the $\cot + \csc^2$ wave functions from the confined to the $1/r$ wave-functions from the deconfined phases.

5. Proton electric charge form factor as a 4D Fourier transform

$$e^{iq\cdot x} = e^{i|\mathbf{q}||\mathbf{r}|\cos\theta} = e^{i|\mathbf{q}|\frac{\sin\chi}{\sqrt{\kappa}}\cos\theta},$$

$$|\mathbf{r}| = R\sin\chi = \frac{\sin\chi}{\sqrt{\kappa}}.$$

of the ground state charge density:

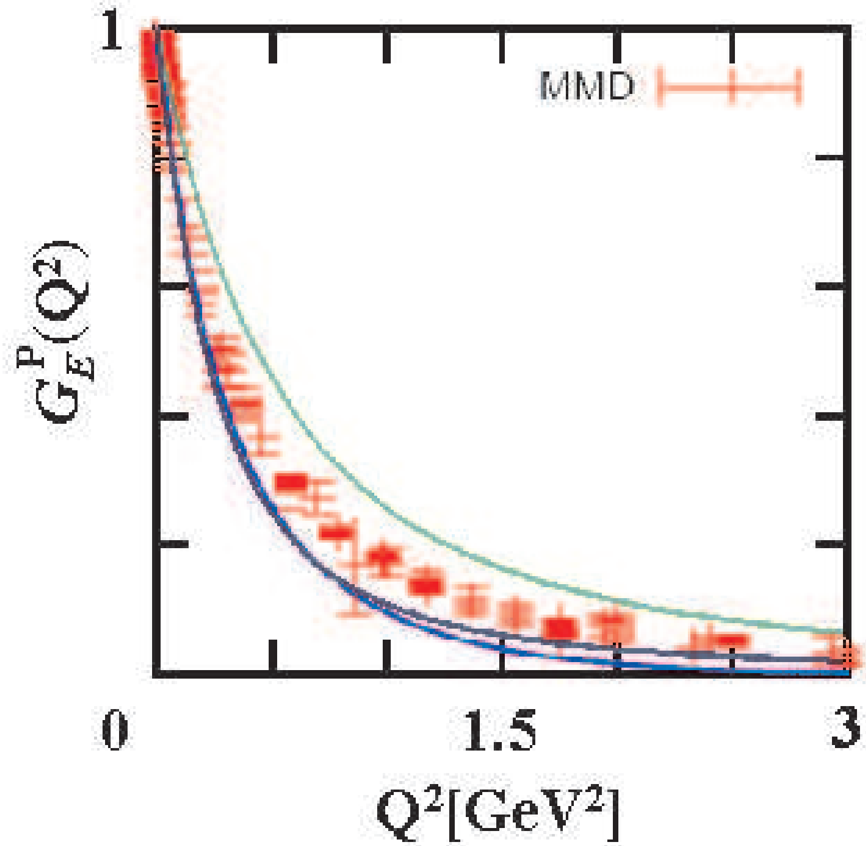
$$J_0(\mathbf{r}) = e(\mathcal{X}_{00}(\chi,\kappa))^2,$$

$$\mathcal{X}_{00} = \frac{4b(b^2+1)}{1-e^{-2\pi b}}\sin\chi e^{-b\chi}, \quad b = \frac{2\mu G}{\sqrt{\kappa}\hbar^2},$$

using the 4D int. vol., $R^3\sin^2\chi\sin\theta dRd\chi d\theta d\varphi$:

$$G_{\mathrm{E}}^{p}(|\mathbf{q}|) = \int_0^\infty d|x||x|^3\delta(|x|-R)\int_0^{2\pi}d\varphi\int_0^\pi d\theta\sin\theta$$

$$\int_0^\pi d\chi\sin^2\chi e^{i|\mathbf{q}|\frac{\sin\chi}{\sqrt{\kappa}}|\cos\theta}\mathcal{X}_{(00)}(\chi,\kappa)|^2$$



green line:

Compean, M.K., EPJA **39**, *1(2007),[hep-ph:0805.2404]* corresponding to $\kappa = 0.019$ fm^{-2} fitted to spectra.
middle blue line: fit to the proton mean square charge radius, $\sqrt{< \mathbf{r}^2 >} = 0.87$ fm , $\kappa = 0.009$ fm^{-2}
lowest blue line: Bethe-Salpeter calculation by
`Ch. Haupt, Ph.D. thesis,`
`www.itkp.uni−bonn.de/∼haupt/talks/Internal/2005.pdf`

7. Conclusions

- Convenient description of QCD confinement as trapping on $\mathbf{R}^1 \times S^3$, the conformally compactified $\mathbf{R}^{1+3}$ infinite boundary of $AdS_5 \times S^5$,
- Spectra of the light-flavor baryons N and Δ well understood in terms of a conformal spectrum, i.e. ∞d irrep of SO(2,4).
- The S^3 potential, $\cot + \csc^2$, provides a phenomenological modeling for non-perturbatve corrections to AdS/CFT Wilson loop potential beyond the Coulombic+linear interactions.
- Model describes pretty well proton electric charge form-factor and dressing function of the (effective) gluon propagator predicting it finite in the infrared.