Quark mass matrices in the physical basis

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Introduction

The quark mixing matrix V can be viewed in three basis.

1. Physical basis: M_u, M_d diagonal.
2. M_u diagonal
3. M_d diagonal

$V \rightarrow$ 3 angles, 1 phase.

Basic formulas

The 3×3 hermitian quark mass matrix $M_{\rm q}$ is diagonalized by $V_{\rm q}$ so that $M_{\rm q} = V_{\rm q}^{\dagger} \hat{M}_{\rm q} V_{\rm q}$, q=u,d.
The eigenvalues are denoted by ($\lambda_{\rm u}$,$\lambda_{\rm c}$,$\lambda_{\rm t}$) and ($\lambda_{\rm d}$,$\lambda_{\rm s}$,$\lambda_{\rm b}$) for the up and down quark mass matrices.

Each mass matrix can be expressed in terms of its projectors

$$M_{\rm u} = \sum_{\alpha={\rm u,c,t}} \lambda_\alpha N_\alpha \qquad \text{and} \qquad M_{\rm d} = \sum_{j={\rm d,s,b}} \lambda_j N_j . \tag{1}$$

Basic formulas

$$|V_{\alpha\,j}|^2 = \mathrm{Tr}[N_\alpha N_j], \tag{2}$$

$$N_\alpha = \frac{(\lambda_\beta - M_\mathrm{u})(\lambda_\gamma - M_\mathrm{u})}{(\lambda_\beta - \lambda_\alpha)(\lambda_\gamma - \lambda_\alpha)} \tag{3}$$

$$N_j = \frac{(\lambda_k - M_\mathrm{d})(\lambda_l - M_\mathrm{d})}{(\lambda_k - \lambda_j)(\lambda_l - \lambda_j)}, \tag{4}$$

with (α,β,γ) and (j,k,l) any permutation of (u,c,t) and (d,s,b).

Basic formulas

The Jarlskog invariant $J(V)$, which is a measure of CP-violation can be directly expressed in terms of $M_{\rm u}$ and $M_{\rm d}$.

$$\mathrm{Det}([M_{\rm u}, M_{\rm d}]) = 2i\, D(\lambda_\alpha) D(\lambda_j) J(V),$$

$$D(\lambda_\alpha) = (\lambda_{\rm c} - \lambda_{\rm u})(\lambda_{\rm t} - \lambda_{\rm u})(\lambda_{\rm t} - \lambda_{\rm c})$$

$$D(\lambda_j) = (\lambda_{\rm s} - \lambda_{\rm d})(\lambda_{\rm b} - \lambda_{\rm d})(\lambda_{\rm b} - \lambda_{\rm s}).$$

If one mass matrix is diagonal then

$$J(V) = Im\ \left(M_{12} M_{23} M_{13}^*\right) / D(\lambda)$$

Fritzsch type Mass Matrices

$$M_{\rm u} = \begin{pmatrix} 0 & A & 0 \\ A^* & 0 & B \\ 0 & B^* & C \end{pmatrix} , \qquad M_{\rm d} = \begin{pmatrix} 0 & A' & 0 \\ A'^* & 0 & B \\ 0 & B'^* & C' \end{pmatrix}$$

Without lack of generality we can take C and C' to be positive and A and B to be real and positive. Then, $M_{\rm u}$ and $M_{\rm d}$ have eight real parameters A, B, C, C', $|A'|$, $|B'|$ and the phases $\phi_{A'}$ and $\phi_{B'}$.

Gupta-Rajpoot type mass matrices

For the three families of quarks the Gupta-Rajpoot mass matrices are of the form

$$M_{\rm u} = \begin{pmatrix} 0 & A & 0 \\ A^* & |D| & B \\ 0 & B^* & |C| \end{pmatrix} , \qquad M_{\rm d} = \begin{pmatrix} 0 & A' & 0 \\ A'^* & |D'| & B \\ 0 & B'^* & |C'| \end{pmatrix}$$

Without lack of generality we can make $M_{\rm u}$ to be real and positive. Then, $M_{\rm u}$ and $M_{\rm d}$ have ten real parameters A, B, C, D, $|A'|$, $|B'|$, $|C'|$, $|D'|$, and the phases $\phi_{A'}$ and $\phi_{B'}$. To reduce the number of parameters to nine we shall take $|D'| = 0$.

CGS type mass matrices

$$M = \begin{pmatrix} 0 & a & d \\ a^* & 0 & b \\ d^* & b^* & c \end{pmatrix}$$

In the physical basis with the CGS-type mass matrix it is enough to consider the case where $M_{\rm u}$ is Fritzsch-type while $M_{\rm d}$ is CGS-type

$$M_{\rm u} = \begin{pmatrix} 0 & a & 0 \\ a & 0 & b \\ 0 & b & c \end{pmatrix}, \qquad M_{\rm d} = \begin{pmatrix} 0 & a' & i|d'| \\ a' & 0 & b' \\ -i|d'| & b' & c' \end{pmatrix}$$

Fits

1. For a given choice of the mass matrices we form a χ^2-function which contains eleven summands.
2. The first five compare the experimental values the four best measured moduli ($|V_{\rm ud}|$, $|V_{\rm us}|$, $|V_{\rm cd}|$, $|V_{\rm cs}|$) and $J(V)$ with their theoretical expressions.
3. The last six summands constrain the eigenvalues of quark mass matrices to the experimentally deduced quark masses at a specified energy scale.

Quark masses at various energy scales.

Quark mass	Scale		
	2 GeV	$M_Z = 91{,}1876\,\mathrm{GeV}$	PDG(06)
m_u	$2{,}2^{+0{,}8}_{-0{,}7}$	$1{,}28^{+0{,}50}_{-0{,}43}$	$2{,}25 \pm 0{,}75$
m_d	$5{,}0 \pm 2{,}0$	$2{,}91^{+1{,}24}_{-1{,}20}$	$5{,}0 \pm 2{,}0$
m_s	95 ± 25	55^{+16}_{-15}	95 ± 25
m_c	$1{,}07 \pm 0{,}12$	$0{,}624 \pm 0{,}083$	$1{,}25 \pm 0{,}09$
m_b	$5{,}04^{+0{,}16}_{-0{,}15}$	$2{,}89 \pm 0{,}09$	$4{,}20 \pm 0{,}07$
m_t	$318{,}9^{+13{,}1}_{-12{,}3}$	$172{,}5 \pm 3{,}0$	$174{,}2 \pm 3{,}3$

Observed values of CKM parameters.

Observable	Exp. value
$\vert V_{\mathrm{ud}}\vert$	$0{,}97383 \pm 0{,}00024$
$\vert V_{\mathrm{us}}\vert$	$0{,}2272 \pm 0{,}0010$
$\vert V_{\mathrm{cd}}\vert$	$0{,}2271 \pm 0{,}0010$
$\vert V_{\mathrm{cs}}\vert$	$0{,}97296 \pm 0{,}00024$
$J(V)$	$(3{,}08 \pm 0{,}18) \times 10^{-5}$

Fit for quark masses at M_Z energy scale.

Type of mass matrix	Basis	Number of parameters	$\chi^2/(\mathrm{dof})$
Fritzsch	Physical ($\phi_{A'}$ and $\phi_{B'}$ free)	8	4.23/3 = 1.41
	Physical ($\phi_{A'} = -\pi/2$ and $\phi_{B'} = 0$)	6	4.84/5= 0.97
Gupta-Rajpoot	Physical ($\phi_{A'}$ and $\phi_{B'}$ free)	9	0.80/2 = 0.40
	Physical ($\phi_{A'} = -\pi/2$ and $\phi_{B'} = 0$)	7	2.62/4= 0.66
M_u Fritzsch-type and M_d CGS-type	Physical	7	1.89/4 = 0.47

Fit for quark at 2 GeV energy scale.

Type of mass matrix	Basis	Number of parameters	$\chi^2/(\mathrm{dof})$
Fritzsch	Physical ($\phi_{A'}$ and $\phi_{B'}$ free)	8	4.80/3 = 1.60
	Physical ($\phi_{A'} = -\pi/2$ and $\phi_{B'} = 0$)	6	5.49/5= 1.10
Gupta-Rajpoot	Physical ($\phi_{A'}$ and $\phi_{B'}$ free)	9	0.87/2 = 0.44
	Physical ($\phi_{A'} = -\pi/2$ and $\phi_{B'} = 0$)	7	2.76/4= 0.69
M_{u} Fritzsch-type and M_{d} CGS-type	Physical	7	2.47/4 = 0.62

Fit of PDG quark masses.

Type of mass matrix	Basis	Number of parameters	$\chi^2/(\mathrm{dof})$
Fritzsch	Physical ($\phi_{A'}$ and $\phi_{B'}$ free)	8	3.32/3 = 1.11
	Physical ($\phi_{A'} = -\pi/2$ and $\phi_{B'} = 0$)	6	4.27/5= 0.85
Gupta-Rajpoot	Physical ($\phi_{A'}$ and $\phi_{B'}$ free)	9	0.78/2 = 0.39
	Physical ($\phi_{A'} = -\pi/2$ and $\phi_{B'} = 0$)	7	3.77/4= 0.94
M_u Fritzsch-type and M_d CGS-type	Physical	7	0.80/4 = 0.20

Conclusion

1. The stability of this type of analysis with respect to evolution of the quark masses is important. As can be seen the results for χ^2/dof for the different cases at 2 GeV scale are very similar to results at M_Z scale.
2. The CGS type of mass matrix with 7 parameters is favored over the F type or the choice of the G-R type.