TOPICS ON SCALAR MESONS IN THE 1 GeV REGION

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CONTENT

- Isospin Violation

$$R = \frac{\Gamma(\phi \to K^+ K^-)}{\Gamma\phi \to K^0 K^0)}$$

$f_0(980) - a_0(980)$ MIXING

- Wave Function Renormalization

$\mathbf{M}, \pi, \vartheta'_s \to f_0(980), f_0(600)$

Threshold singularities

- Summary

The Ratio Phi ---> K+ K- / K0 anti-K0
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On the mass, width and coupling constants of the f(0)(980).
R. Escribano, A. Gallegos, J.L. Lucio Martínez ,G. Moreno, J. Pastieau.
Eur.Phys.J.C28:107-114,2003

Resonance propagation and threshold singularities
J.L. Lucio Martínez, J. Pestieau.
Mod.Phys.Lett. A.16:1315-1320,2001.

Charge asymmetries in e+ e- ---> pi+ pi- gamma at the phi resonance.
Armando Gallegos, J.L. Lucio, G. Moreno, M. Napsuciale

Scalar meson dynamics in chiral perturbation theory
Albert Bramon, Rafel Escribano, J.L. Lucio Martínez.
Phys.Rev.D69:0704008,2004.

pi pi invariant mass spectrum in V-prime ---> V pi pi and the f(0)(600) pole.
A. Gallegos, J.L.Lucio Martínez, J. Pestieau.
B. Phys.Rev.D69:074033,2004.

Chiral loops and a(0)(980) exchange in phi ---> pi0 eta gamma.
A. Bramon, R. Escribano, J.L. Lucio Martínez, Mauro Napsuciale, G. Pancheri. Phys.Lett.B494:221-228,2000.

A Consistent Scenario for B ---> PS Decays.
D. Delepine , J.L. Lucio Martínez, J. A. Mendoza S.
Phys.Rev.D78:114016,2008.

Annihilation contribution and B ---> a(0)pi, f(0)K decays.
D. Delepine , J.L. Lucio Martínez, Carlos A. Ramírez.
Eur.Phys.J.C45:693-700,2006.

The ratio $\Phi \to K^+K^-/K^0\bar{K}^0$

VEPP-2M

$$\mathrm{BR}(\phi \to K^+K^-) = (49.2 \pm 1.2)\% \ ,$$

$$\mathrm{BR}(\phi \to K^0\bar{K}^0) = (33.5 \pm 1.0)\% \ ,$$

$$R_{\mathrm{exp}} \equiv \frac{\mathrm{BR}(\phi \to K^+K^-)}{\mathrm{BR}(\phi \to K^0\bar{K}^0)} = 1.47 \pm 0.06 \ .$$

measured in a variety of independent experiments

PDG edition [5]

$$\left.\begin{array}{l}\mathrm{BR}(\phi \to K^+K^-) = (49.1 \pm 0.8)\% \\ \mathrm{BR}(\phi \to K^0\bar{K}^0) = (34.1 \pm 0.6)\%\end{array}\right\} \Longrightarrow R_{\mathrm{exp}} = 1.44 \pm 0.04 \ ,$$

Naïve result $R = 1$

$\phi \to K\bar{K}$ isospin symmetry in strong interaction dynamics

good isospin limit: $m_u = m_d$

ignore phase space diferences: $m_{k^+} = m_{k^0}$

Electromagnetic radiative corrections:

- isospin breaking effects $m_u - m_d$
- Vector-meson dominated electromagnetic form-factors
- Rescattering effects through $f_0(980)$ and $a_0(980)$

- Isospin symmetry valid only for the S.I.D. :

vicinity of the ϕ mass to the $K\bar{K}$ thresholds: Phase-space correction .

$$R = \frac{\left(1 - \frac{4m_{K^+}^2}{M_\phi^2}\right)^{3/2}}{\left(1 - \frac{4m_{K^0}^2}{M_\phi^2}\right)^{3/2}} = 1.528 \ ,$$

- Electromagnetic Radiative corrections:

Charged decay mode but not the neutral

Gemmer and Gourdin: positive correction ≅ 4% enlarging discrepancy.

Pilkuhn: **R ≅ 1.52 - 1.61**

- **Isosspin breaking in vertices**

$\phi K^+ K^-$ and $\phi K^0 \bar{K}^0$ $SU(2)$- breaking in *Leff* via quark mass differences. Bijnens *et al*

$$\frac{g_{\phi K^+K^-}}{g_{\phi K^0\bar{K}^0}} \simeq 1 + 4\sqrt{2}\frac{f_\chi}{g_V}\frac{m_{K^+}^2 - m_{K^0}^2|_{m_u \neq m_d}}{M_\phi^2} \simeq 1.01 \ ,$$

$$\frac{g_{\phi K^+K^-}}{g_{\phi K^0\bar{K}^0}} \simeq 1 - \frac{m_{K^+}^2 - m_{K^0}^2|_{m_u \neq m_d}}{m_K^2 - m_\pi^2} c_A \simeq 1.01 \ .$$

- **ρ − ϕ mixing:** $e^+e^- \to \phi \to \pi^+\pi^-$

$$F(s)\left(1 - \frac{Z M_\phi \Gamma_\phi}{M_\phi^2 - s - iM_\phi\Gamma_\phi}\right)$$

Average Z

- Further attempts

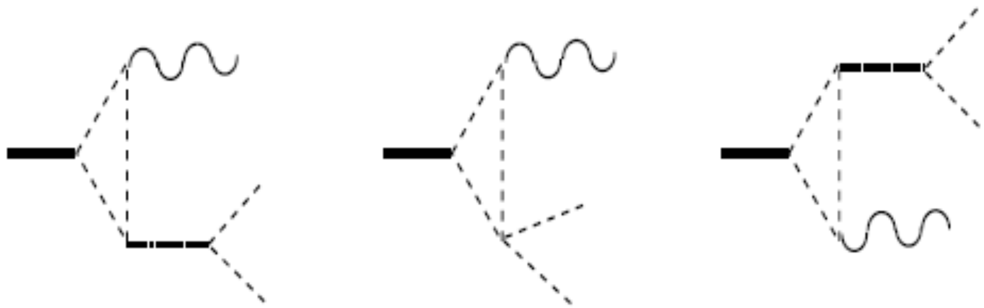
$\gamma - kk$ Couplings: vector-meson dominated: R changes in per mille

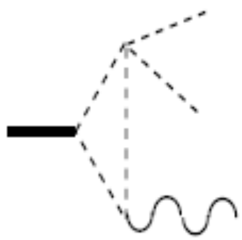
$\phi \to K^+K^-$ soft photons. Loop Model

K^+K^- system $J^{PC} = 0^{++}$ or 2^{++}

$J^{PC} = 0^{++}$ scalar resonances $f_0(980)$ and $a_0(980)$

LOOP MODEL





+ *seagull diagrams*

effects from 2^{++} states are suppressed $f_2(1270)$ $a_2(1310)$, are well above the ϕ mass

Exchange of the f_0 and a_0 : mixing!

- Summary

$$R_{\text{exp}} = 1.44 \pm 0.04$$

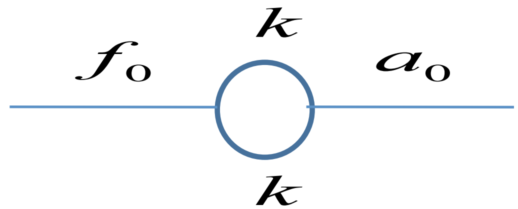
We have failed to reproduce the value R_{exp}

- Isospin breaking in vertices
- u –d mass difference
- RC including vector meson dominated vertices
- $\rho-\phi$ mixing

poor knowledge on the SU (2)-breaking

$f_0(980)$ $a_0(980)$ mixing

$$m_{01}^2$$



- **non diagonal mass matrix can be considered an interaction term**

$$\Delta_f^c = \frac{s - m_a^2}{(s - m^2 F)(s - m_A^2)}$$

$$\Delta_{af}^c = \frac{-m_{01}^2}{(s - m^2 F)(s - m_A^2)}$$

$$m_f^2 + m_a^2 = m_F^2 + m_A^2$$

$$m_f^2 m_a^2 - m_{01}^4 = m_F^2 m_A^2$$

- Basis of mass eigenstate: physical quantities involve mixing angle

  - **Mixing: $\Gamma = 0$**

$$\begin{pmatrix} m_f^2 & m_{01}^2 \\ m_{01}^2 & m_a^2 \end{pmatrix} \qquad \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\tan 2\theta = \frac{2m_{01}^2}{m_f^2 - m_a^2}.$$

  - **Mixing Γ 0**

$$\begin{pmatrix} m_f^2 - i m_f \Gamma_f & m_{01}^2 \\ m_{01}^2 & m_a^2 - i m_a \Gamma_a \end{pmatrix} \qquad \begin{pmatrix} \cos(\phi + i\theta) & \sin(\phi + i\theta) \\ -\sin(\phi + i\theta) & \cos(\phi + i\theta) \end{pmatrix}$$

$$\tan(2\phi) = \frac{1}{1+r^2}\frac{2m_{01}^2}{m_f^2 - m_a^2},$$
$$\tanh(2\theta) = \frac{r^2}{1+r^2}\frac{2m_{01}^2}{m_f\Gamma_f - m_a\Gamma_a},$$

$$r^2 = \frac{2}{d^2}\left(-\left(1+\frac{d^2}{4}-\frac{D^2}{4}\right)+RC\right),$$
$$RC = \sqrt{\left(1+\frac{d^2}{4}-\frac{D^2}{4}\right)^2+\frac{d^2D^2}{4}},$$
$$d = \frac{m_f^2 - m_a^2}{m_{01}^2} \text{ y } D = \frac{m_f\Gamma_f - m_a\Gamma_a}{m_{01}^2}.$$

Model dependent and effected by large uncertainties propagators $D_{f_0/a_0}(m)$

$g_{SK\bar{K}}$ the nature of the scalar mesons.

$\mathbf{M}, \Gamma, G'_s$ *of the* $f_0(980)$

$\pi\pi$ Data, phase shift, inelasticity, invariant mass spectrum of the $J/\psi \to \phi\pi\pi$ and $\phi K\bar{K}$ decays. Single pole describes the $f_0(980)$

Mass and width defined in terms of the phase shift:

$$\delta(s = M_\delta^2) = 90^\circ \ , \quad \Gamma_\delta = \frac{1}{M_\delta}\left[\frac{d\delta(s)}{ds}\right]^{-1}_{s=M_\delta^2} \ ,$$

$$\tan\delta(s) = -\frac{M_\delta \Gamma_\delta(s)}{s - M_\delta^2} \ ,$$

$$a = \frac{e^{2i\delta} - 1}{2i} = -\frac{M_\delta \Gamma_\delta(s)}{s - M_\delta^2 + iM_\delta\Gamma_\delta(s)} \ ,$$

POLE APPROACH

$$a = \frac{R}{s - s_p} + B \ , \qquad s_p = m_p^2 - i m_p \Gamma_p \ .$$

Process and background Independent Gauge inv.!

Resonance coupled to several channels. Unitary constraint:

$$T_{ab} = \frac{e^{2i\delta_a(s)} - 1}{2i\sqrt{\beta_a \beta_b}}\delta_{ab} - \frac{e^{i(\delta_a(s)+\delta_b(s))}}{\sqrt{\beta_a\beta_b}} \frac{\sqrt{G_a G_b}}{F(s) + iG(s)} \ ,$$

$\beta_{a,b} = \sqrt{1 - 4m_{a,b}^2/s}$ $\quad m_{a,b}$ masses two-body decays,

$F(s)$ and $G(s)$ arbitrary real functions

$G_i(s) > 0 \qquad G(s) = \sum_{i=a,b} G_i(s)$

$F(s) = s - m_p^2 \qquad m_p \Gamma_p$

$F(s)$ and $G(s)$ real and imaginary parts one-loop propagator

Effective field theory to calculate the $f_0(980)$ propagator

$$f_0(980)\ \Gamma_{f_0}/m_{f_0} \approx \ 0.04\text{–}0.1$$

One-loop propagator : consistent description of the analytical Properties

$K\bar{K}$ threshold and f_0 mass

$$\Delta(p^2) = \frac{i}{p^2 - m_0^2 + \Pi(p^2)} \ ,$$

$$\Delta(p^2) = \frac{iZ}{p^2 - m_R^2 + im_R\Gamma_R} + \cdots \ ,$$

on-shell definition of the resonance width is inadequate

$$\frac{i}{p^2 - m_R^2 + \mathrm{Re}\Pi(p^2) - \mathrm{Re}\Pi(m_R^2) + i\mathrm{Im}\Pi(p^2)} \, ,$$

$$\frac{e^{2i\delta_a(s)} - 1}{2i\sqrt{\beta_a\beta_b}}\delta_{ab} - \frac{e^{i(\delta_a(s)+\delta_b(s))}}{16\pi}\frac{g_a g_b}{s - m_R^2 + \mathrm{Re}\Pi(s) - \mathrm{Re}\Pi(m_R^2) + i\mathrm{Im}\Pi(s)}$$

m_R and $g_{a,b}$ parameters to be fitted *pole mass* m_p *pole width* Γ_p obtained from

$$D(s_p) = s_p - m_R^2 + \mathrm{Re}\Pi_+(s_p) - \mathrm{Re}\Pi_+(m_R^2) + i\mathrm{Im}\Pi_+(s_p) = 0 \, ,$$

For arbitrary complex s $\quad s_p = m_p^2 - i m_p \Gamma_p$

$$\Pi_+(s) = \mathrm{Re}R(s) - \mathrm{Im}I(s) + i\left[\mathrm{Im}R(s) + \mathrm{Re}I(s)\right] \, .$$

$$D(s) = D[s, p_a(s), p_b(s)]$$

Resonance coupled to channels (a,b) $\longrightarrow$ poles in the four different Riemann sheets

$$\begin{aligned}
&\text{sheet I} && (++): && (\mathrm{Im}p_a > 0, \mathrm{Im}p_b > 0)\ , \\
&\text{sheet II} && (-+): && (\mathrm{Im}p_a < 0, \mathrm{Im}p_b > 0)\ , \\
&\text{sheet III} && (--): && (\mathrm{Im}p_a < 0, \mathrm{Im}p_b < 0)\ , \\
&\text{sheet IV} && (+-): && (\mathrm{Im}p_a > 0, \mathrm{Im}p_b < 0)\ .
\end{aligned}$$

$J/\psi \to \phi\pi^+\pi^-$ $\quad$ $J/\psi \to \phi K^+K^-$

$\pi\pi$ phase shift inelasticity

Fit	set B	set D	set DF
m_R^2 (GeV^2)	0.966 ± 0.003	0.982 ± 0.001	0.978 ± 0.003
$g^2_{f_0\pi\pi}/16\pi$ (GeV^2)	0.071 ± 0.007	0.065 ± 0.005	0.11 ± 0.01
$g^2_{f_0K\bar{K}}/16\pi$ (GeV^2)	0.25 ± 0.03	0.16 ± 0.01	0.31 ± 0.04

$$m_p^D = 999 \pm 2 \text{ MeV} \ , \qquad \Gamma_p^D = 39 \pm 8 \text{ MeV} \ ,$$

Pole mass is always larger than the renormalized mass

$$i.e.\ m_p^{II} > m_R.$$

poles in sheet III *only* for $m_R > 1020$ MeV $\quad m_p^{\check{I}II} < m_R$

RESONANCE PROPAGATION AND THRESHOLD SINGULARITIES

$$P(s) = \frac{1}{s - m_0^2 - A(s)},$$

On shell renormalization scheme

$$M^2 = m_0^2 + Re\ A(M^2),$$

$$M\Gamma^{os} = - \frac{Im\ A(M^2)}{1 - Re\ A'(M^2)},$$

$$Z_2^{os} = \frac{1}{1 - Re\ A'(M^2)}.$$

$$\Pi(p^2) = ik^{2L+1} f(p^2) + g(p^2)$$

$$k = \frac{1}{2\sqrt{p^2}} \left(p^2 - (m_1 + m_2)^2\right)^{1/2} \left(p^2 - (m_1 - m_2)^2\right)^{1/2}$$

$$k \to \pm i0 \qquad \mathrm{Re}\Pi' \to k^{-1}$$

On-shell renormalization scheme $\longrightarrow$ Threshold singularities

$$Re\ A'(m^2)\ \text{-}$$

• Scalar or vector resonance.

• Mass of the decaying particle approaches from below the mass threshold of the poduced particles

Kniehl, Palisoc and Sirlin KPS   (Valid at one loop) :

$$Re\ A'(m^2)\ =\ \frac{Im\ A^{(1)}(m^2)\ -\ Im\ A^{(1)}(s_p)}{m\Gamma}.$$

Superscript reter to the number of quantum loops. KPS propose

$$\frac{1}{Z_2^{KPS}}\ =\ 1-\frac{Im\ A(m^2)-Im\ A(s_p)}{m\Gamma}.$$

Stable particles: Gauge invariant definition of the mass particle.

When resonance width can not be neglected, pole of the propagator.

$$s_p = m_0^2 + A(s_p) = m^2 - im\Gamma$$

consider $Re\ A(s) = R(s), \qquad Im\ A(s) = I(s).$

$$\mathrm{T(s)} = \frac{I(s)}{s - m_0^2 - R(s) - iI(s)}.$$

T(s) fulfills the unitarity relation:

$$Im\ T(s) = T(s)T^{\dagger}(s) \qquad\qquad P(s) = \mathrm{T(s)}/I(s)$$

$$\mathrm{T(s)} = \frac{-m\Gamma}{s - m^{2} - i\, m\Gamma} e^{2i\delta(s)} + \frac{-y(s)}{1 + iy(s)}$$

$$e^{2i\delta(s)} = \frac{1 - iy(s)}{1 + iy(s)}$$

$$y(s) = \frac{I(s)(s - m^2) + F(s)m\Gamma}{I(s)m\Gamma - F(s)(s - m^2)},$$

$$F(s) = s - m_0^2 - R(s) = s - m^2 + Re\ R(s_p) - R(s) - Im\ I(s_p).$$

$$\mathrm{T(s)} = \frac{-m\Gamma - (s - m^2)y(s)}{(s - s_p)(1 + iy(s))},$$

$$P(s) = \frac{Z_2(s)}{(s - s_p)(1 + iy(s))},$$

$$Z_2(s) = \frac{-m\Gamma - (s - m^2)y(s)}{I(s)}$$

Renormalization: free of threshold singularities

- Valid to arbitrary order of perturbation theory
- Any point can be chosen to expand the Green function

$s_0 = m^2$: $\quad Z_2(m^2) = -\frac{m\Gamma}{I(m^2)} \quad P(s) = \frac{Z_2(m^2)}{s - s_p}\frac{1}{1 + iy(m^2)} + \ldots$

Z_2 found by KPS $\quad$ term $1 + iy(m^2)$ is not considered by KPS

$$y(m^2) = \frac{m^2 - m_0^2 - R(m^2)}{I(m^2)}$$

$y(m^2)$ Vanishes only to leading order.

Width and Partial Widths of Unstable Particles

$$i \to Z \to f,$$

$$\mathcal{D}_{\mu\nu} = -i \frac{Q_{\mu\nu}}{s - \bar{s} - A(s) + A(\bar{s})}.$$

$$\mathcal{A}_{fi}(s) = -i \frac{Q_{\mu\nu} V_f^{\mu}(\bar{s}) V_i^{\nu}(\bar{s})}{(s - \bar{s} \sum_f \hat{\Gamma}_f \neq \Gamma_2{}'(\bar{s})]} + N_{fi},$$

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A gauge-independent definition of partial width

$$m_2 \hat{\Gamma}_f = -\frac{1}{6} \sum_{\text{spins}} \int d\Phi_f \frac{Q_{\mu\nu} V_f^{\mu *}(\bar{s}) V_f^{\nu}(\bar{s})}{|1 - A'(\bar{s})|},$$

Integration is over the phase sáce of the final-state particles $\sum_f \hat{\Gamma}_f \neq \Gamma_2$

SUMMARY

THRESHOLD PROBLEMS

- **ISOSPIN VIOLATION IN F DECAYS** IN PAIR OF KAONS NOT UNDERSTOOD

  f(980) – *a*(980) mixing

- **T HRESHOLD SINGULARITIES** IN W.F. RENORMALIZATION

  Scalar and vector mesons

  Gauge invariance and unitarity

  Gauge invariant definition of partial widths?

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