Relativistic dynamics in graphene:

Magnetic Catalysis & Quantum Hall Effect

Igor Shovkovy

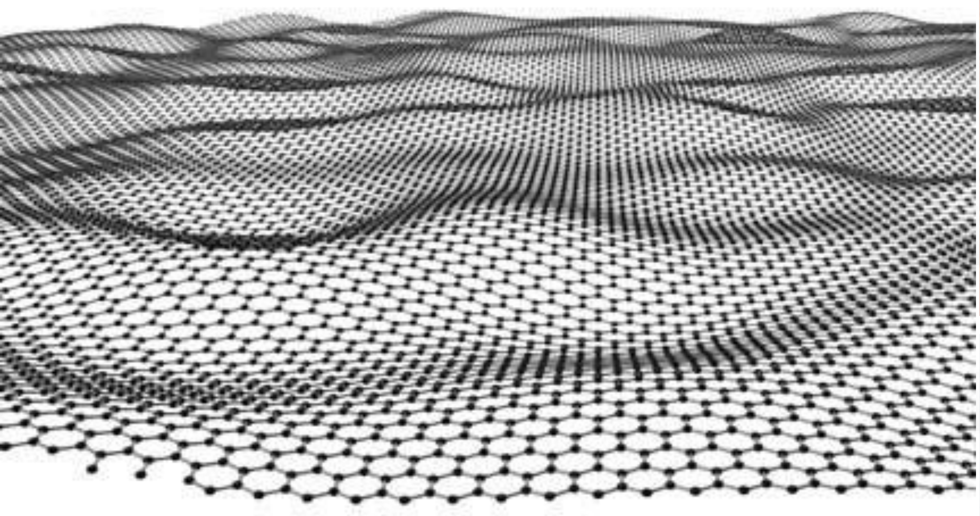
Arizona State University
POLYTECHNIC CAMPUS

XII MEXICAN WORKSHOP ON PARTICLES AND FIELDS
NOVEMBER 9-14, 2009, MAZATLÁN, MÉXICO

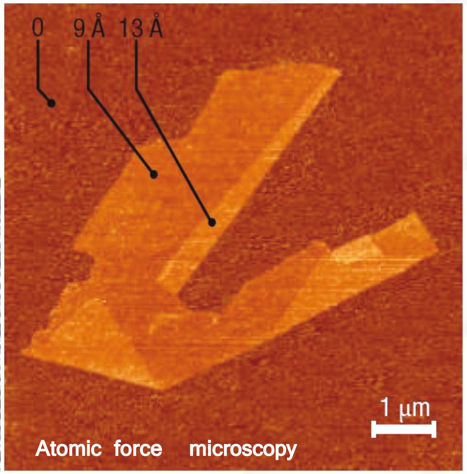
What is graphene?

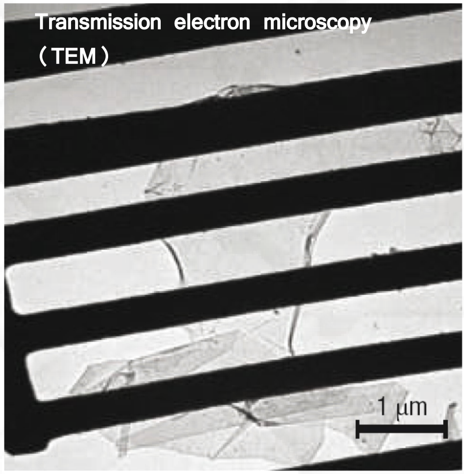
- It is a single atomic layer of graphite, see

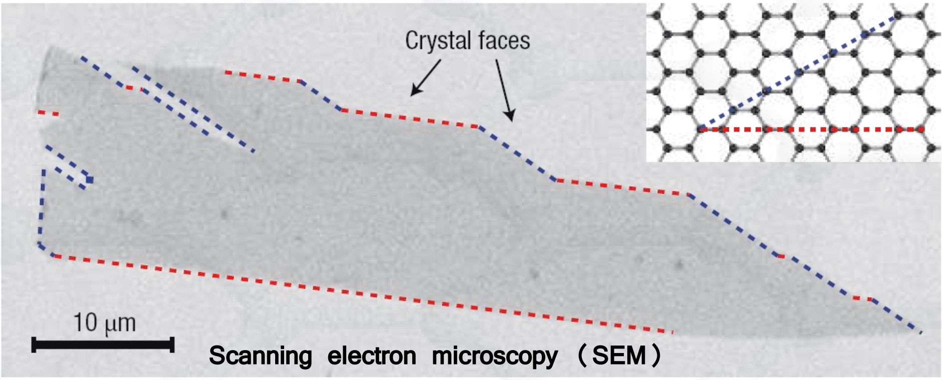
  [Novoselov et al., Science **306**, 666 (2004)]



2D crystal with hexagonal lattice of carbon atoms



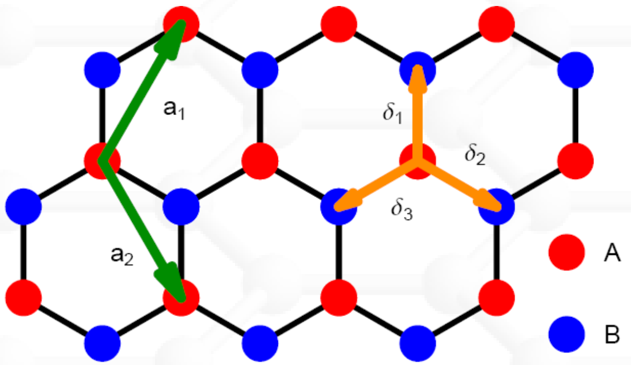




Lattice in coordinate/reciprocal space

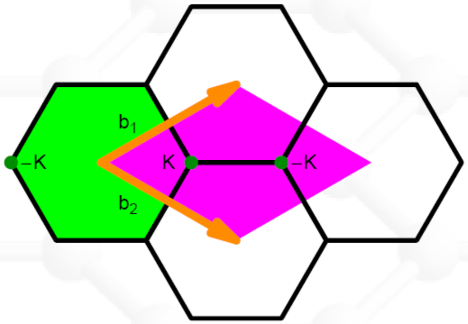
- Two carbon atoms per primitive cell
- Translation vectors

$$\mathbf{a}_1 = a\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right), \quad \mathbf{a}_2 = a\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$



where a is the lattice constant

- Reciprocal lattice vectors



$$\mathbf{b}_1 = 2\pi/a(1, 1/\sqrt{3}), \; \mathbf{b}_2 = 2\pi/a(1, -1/\sqrt{3})$$

Tight binding model

- There are strong covalent sigma-bonds between nearest neighbors
- Hamiltonian

$$H = -t \sum_{\mathbf{n},\boldsymbol{\delta}_i,\sigma} \left[a^{\dagger}_{\mathbf{n},\sigma} \exp\left(\frac{ie}{\hbar c} \boldsymbol{\delta}_i \mathbf{A} \right) b_{\mathbf{n}+\boldsymbol{\delta},\sigma} + \mathrm{c.c.} \right]$$

where $a_{\mathbf{n},\sigma}$ and $b_{\mathbf{n}+\delta,\sigma}$ are the annihilation operators of electrons with spin $\sigma=\uparrow,\downarrow$

- The nearest neighbor vectors are

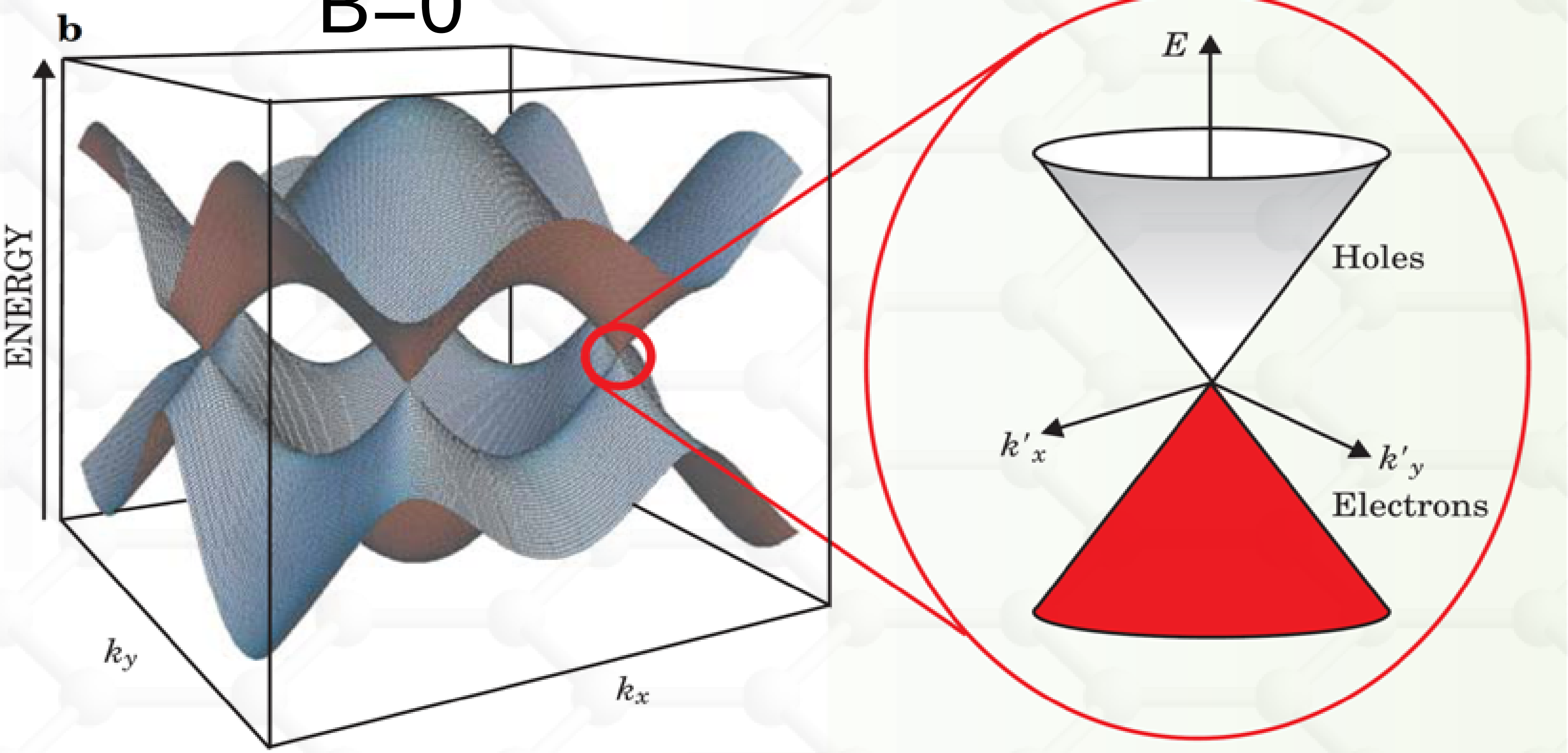
$$\boldsymbol{\delta}_1 = (\mathbf{a}_1 - \mathbf{a}_2)/3\,, \quad \boldsymbol{\delta}_2 = \mathbf{a}_1/3 + 2\mathbf{a}_2/3\,,$$
$$\boldsymbol{\delta}_3 = -\boldsymbol{\delta}_1 - \boldsymbol{\delta}_2 = -2\mathbf{a}_1/3 - \mathbf{a}_2/3$$

Low energy Dirac fermions

$$\mathcal{L} = \sum_{\sigma=\pm 1} \bar{\Psi}_\sigma(t,\mathbf{r})[i\gamma^0(\hbar\partial_t - i\mu_\sigma) + i\hbar v_F\gamma^1 D_x + i\hbar v_F\gamma^2 D_y]\Psi_\sigma(t,\mathbf{r})$$

P. R. Wallace, Phys. Rev. **71,** 622 (1947)
G.W. Semenoff, Phys. Rev. Lett. **53**, 2449 (1984)

B=0



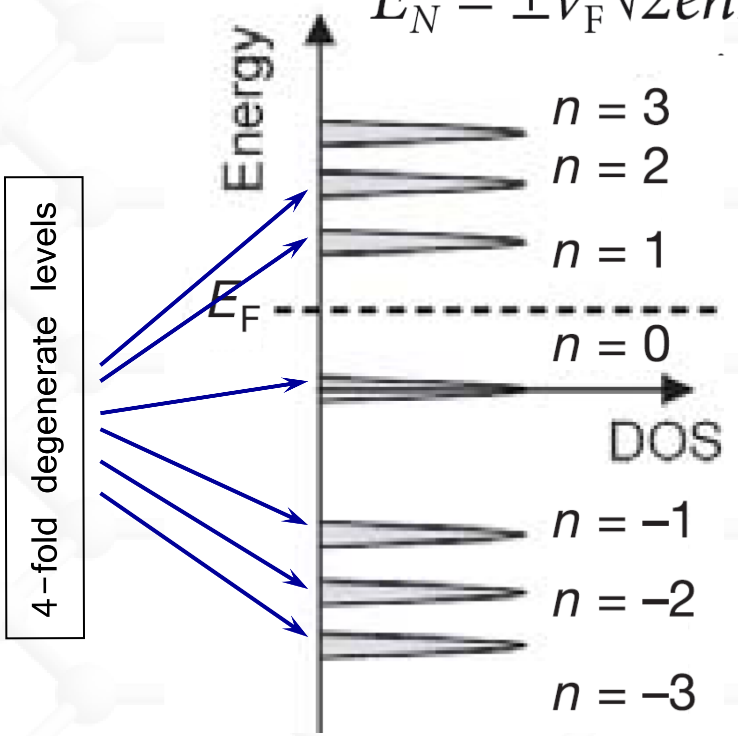
Quantum Hall effect in graphene

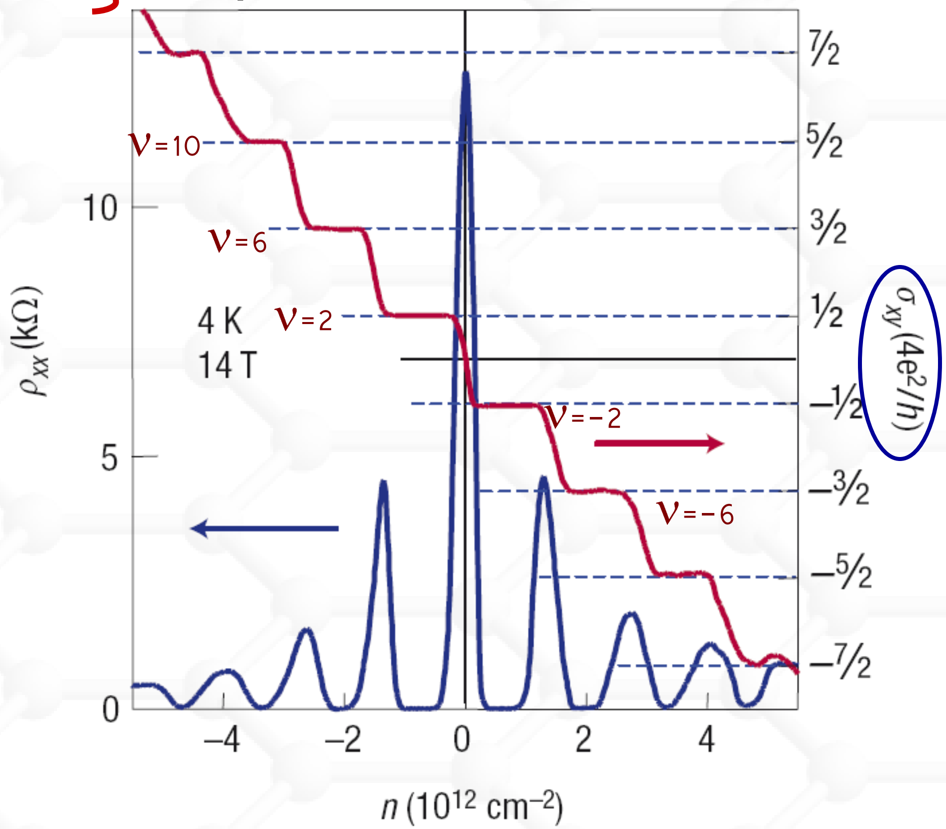
[1] Zheng & Ando, PRB **65**, 245420 (2002)
[2] Gusynin & Sharapov, PRL **95**, 146801 (2005)
[3] Peres, Guinea, & Castro Neto, PRB **73**, 125411 (2006)
[4] Novoselov et al., Nature **438**, 197 (2005)
[5] Zhang et al., Nature **438**, 201 (2005)

$$\sigma_{xy} = \frac{\nu e^2}{h} = \frac{4e^2}{h}\left(n + \frac{1}{2}\right)$$

$$\left.\begin{array}{l}[4]\\[5]\end{array}\right\}\ \text{Experiment}$$

$$E_N = \pm v_F \sqrt{2e\hbar B N}$$





Quantum Hall Effect at large *B*

There are new plateaus at

$\nu=\pm 0, \nu=\pm 1, \nu=\pm 4$

i.e., the degeneracy of some Landau levels is lifted

Abanin et al., PRL **98**, 196806 (2007)

Jiang et al., PRL **99**, 106802 (2007)

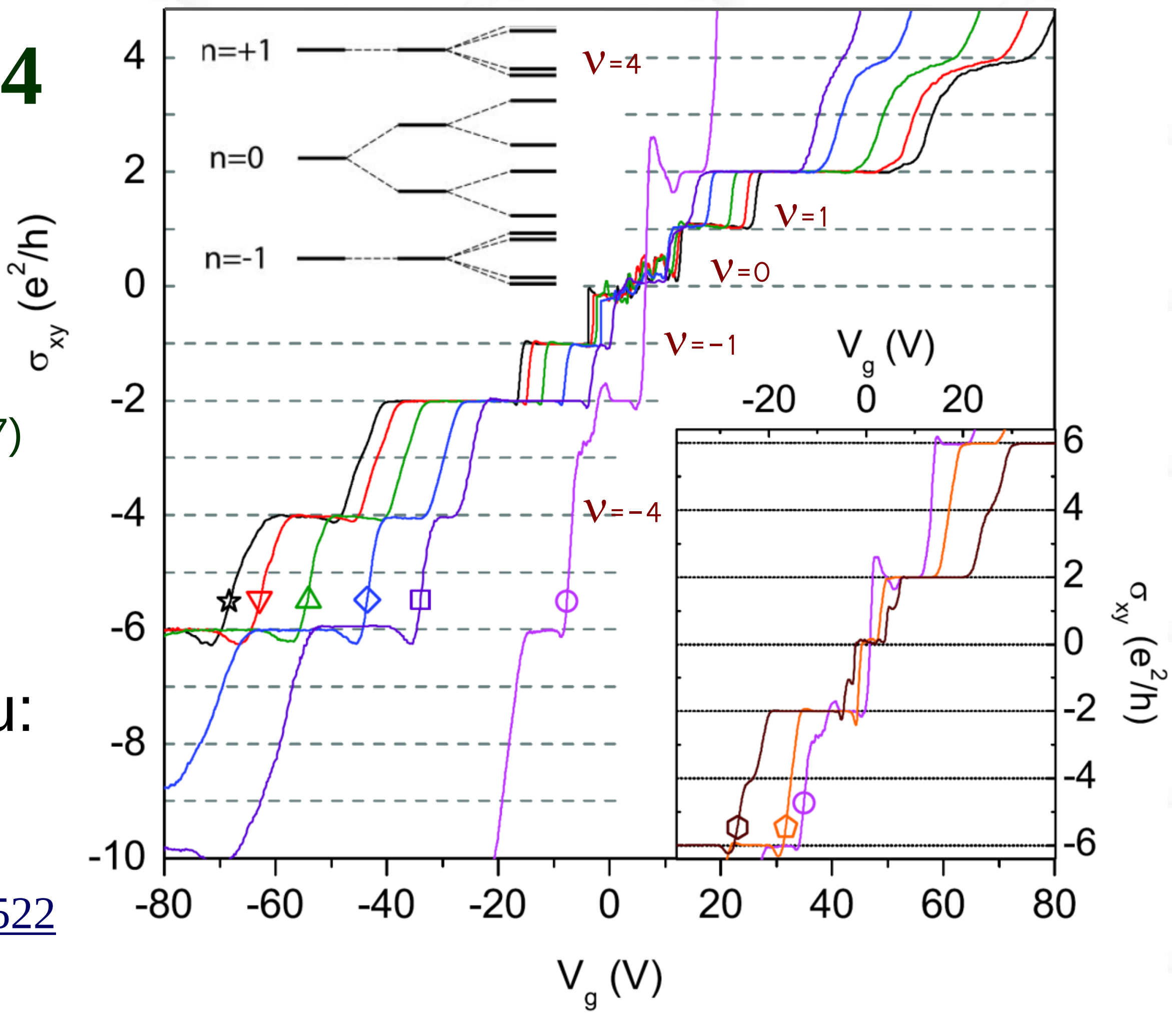
Checkelsky et al., PRL 100, 206801 (2008)

Most recent new plateau:

$\nu=3$ (as well as $\nu=⅓$)

Andrei et al., doi:10.1038/nature08522

Zhang et al., PRL **96**, 136806 (2006)



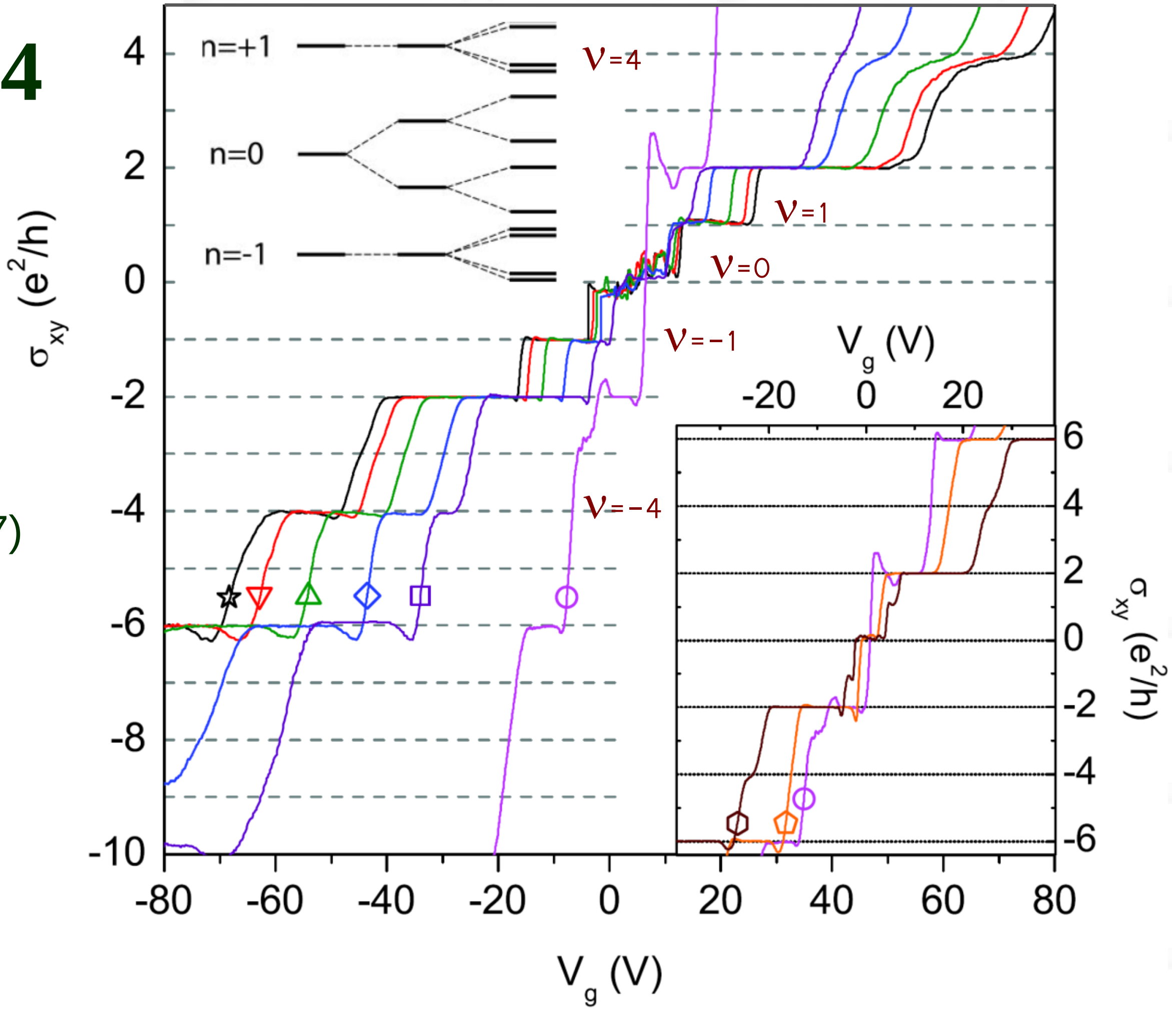
Quantum Hall Effect at large *B*

There are new plateaus at

$\nu=\pm 0, \nu=\pm 1, \nu=\pm 4$

i.e., the degeneracy of some Landau levels is lifted

Zhang et al., PRL **96**, 136806 (2006)



Abanin et al., PRL **98**, 196806 (2007)

Jiang et al., PRL **99**, 106802 (2007)

Checkelsky et al., PRL 100, 206801 (2008)

Latest Quantum Hall Plateaus

Suspended graphene

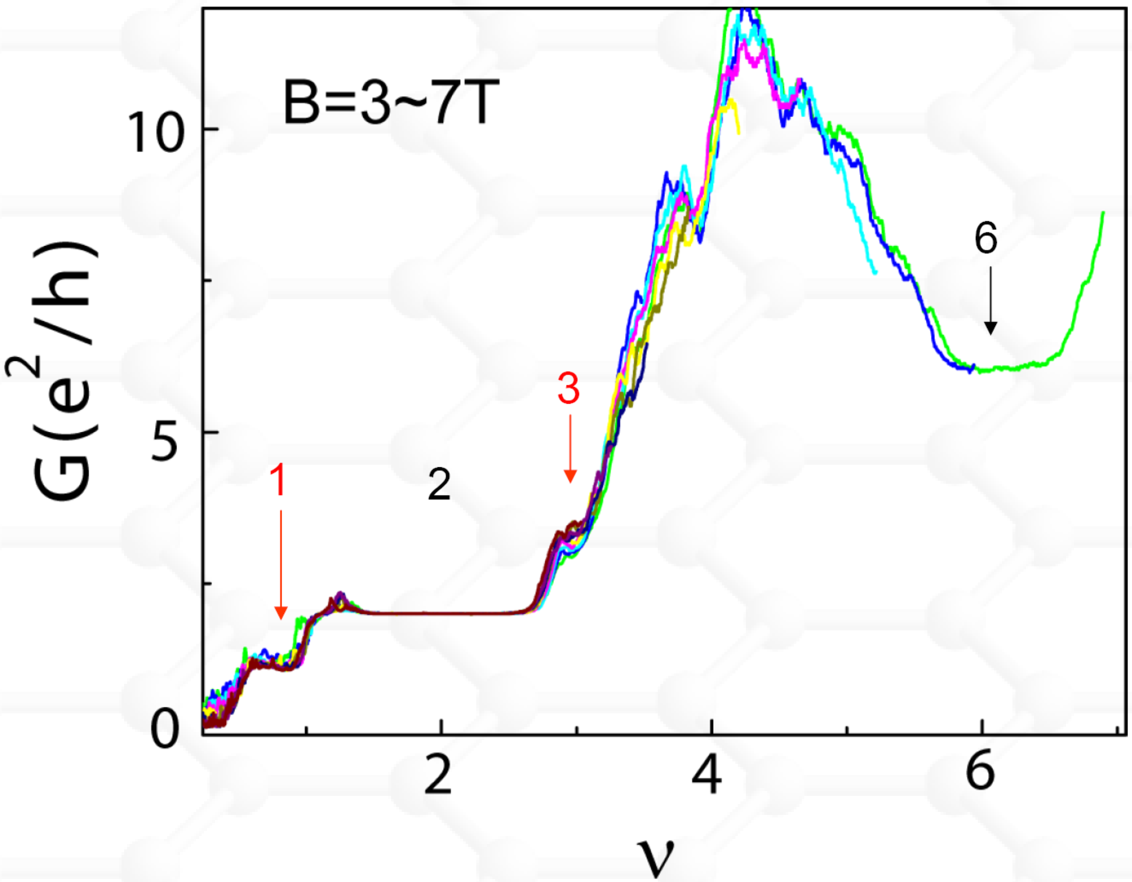
Andrei et al., doi:10.1038/nature08522

The most recent new (integer) plateau:

$$\nu=3$$

Also, the first fractional QH plateau:

$$\nu=\frac{1}{3}$$



Magnetic catalysis (MC) scenario

VOLUME 73, NUMBER 26 PHYSICAL REVIEW LETTERS 26 DECEMBER 1994

Catalysis of Dynamical Flavor Symmetry Breaking by a Magnetic Field in 2 + 1 Dimensions

V. P. Gusynin,[1] V. A. Miransky,[1,2] and I. A. Shovkovy[1]
[1]*Bogolyubov Institute for Theoretical Physics, 252143 Kiev, Ukraine*
[2]*Institute for Theoretical Physics, University of California, Santa Barbara, California 93106-4030*
(Received 11 May 1994)

It is shown that in 2 + 1 dimensions, a constant magnetic field is a strong catalyst of dynamical flavor symmetry breaking, leading to generating a fermion dynamical mass even at the weakest attractive interaction between fermions. The effect is illustrated in the Nambu–Jona-Lasinio model in a magnetic field. The low-energy effective action in this model is derived, and the thermodynamic properties of the model are established.

$$E_n = \sqrt{2n|eB|} \Rightarrow E_n = \sqrt{2n|eB| + \Delta_0^2}$$

where $\Delta_0 \sim \sqrt{|eB|}$ $\Longrightarrow$ $\nu=0$

In relation to graphene (before discovery of graphene!):

Khveshchenko, Phys. Rev. Lett. **87**, 206401 (2001); ibid. **87**, 246802 (2001)
Gorbar, Gusynin, Miransky, & Shovkovy, Phys. Rev. B **66**, 045108 (2002)

Quantum Hall Ferromagnetism (QHF)

Arovas, Karlhelde, & Lilliehook, Phys. Rev. B **59**, 13147 (1999)
Ezawa & Hasebe, Phys. Rev. B **65**, 075311 (2002)
Nomura & MacDonald, Phys. Rev. Lett. **96**, 256602 (2006)
Alicea & Fisher, Phys. Rev. B 74, 075422 (2006)

- Spin/valley degeneracy of the half-filled Landau level is lifted by the exchange (repulsive Coulomb) interaction
- This is similar to the **Hund's Rule(s)** in atomic physics
- Lowest energy state: the wave function is **antisym-metric** in coordinate space (electrons are as far apart as possible), i.e., it is **symmetric** in spin (or valley) indices
- This is nothing else but ferromagnetism

General Approach

Model Hamiltonian

[Gorbar, Gusynin, Miransky, Shovkovy, **arXiv:0806.0846**, Phys. Rev. B **78** (2008) 085437]

$$H = H_0 + H_C + \int d^2\mathbf{r}\,[\underbrace{\mu_B B \Psi^\dagger \sigma^3 \Psi}_{\textit{Zeeman term}} - \mu_0 \Psi^\dagger \Psi]$$

where

$$H_0 = v_F \int d^2\mathbf{r}\,\overline{\Psi}\left(\gamma^1 \pi_x + \gamma^2 \pi_y\right)\Psi,$$

is the Dirac Hamiltonian, and

$$H_C = \frac{1}{2}\int d^2\mathbf{r} d^2\mathbf{r}' \Psi^\dagger(\mathbf{r})\Psi(\mathbf{r}) U_C(\mathbf{r}-\mathbf{r}')\Psi^\dagger(\mathbf{r}')\Psi(\mathbf{r}')$$

is the Coulomb interaction term.

Note that $\Psi_s^T = (\psi_{KAs}, \psi_{KBs}, \psi_{K'Bs}, \psi_{K'As})$

Spin index

$v_F \approx 10^6$ m/s

Symmetry

- The Hamiltonian $H = H_0 + H_C$ possesses "flavor" *U(4)* symmetry
- 16 generators read (*spin* ⊗ *pseudospin*)

$$\frac{\sigma^\alpha}{2} \otimes I_4, \quad \frac{\sigma^\alpha}{2i} \otimes \gamma^3, \quad \frac{\sigma^\alpha}{2} \otimes \gamma^5, \quad \text{and} \quad \frac{\sigma^\alpha}{2} \otimes \gamma^3\gamma^5.$$

- The Zeeman term breaks *U(4)* down to $U(2)_+ \times U(2)_-$
- Dirac mass breaks $U(2)_s$ down to $U(1)_s$

Energy scales in the problem

- Landau energy scale

$$\epsilon_B \equiv \sqrt{2\hbar|eB_\perp|v_F^2/c} \simeq 424\sqrt{|B_\perp[\mathrm{T}]|}\ \mathrm{K}$$

- Zeeman energy

$$Z \simeq \mu_B B = 0.67 B[\mathrm{T}]\ \mathrm{K}$$

- Dynamical mass scales ($Z \ll A \le M \ll \epsilon_B$)

$$A \equiv \frac{G_{\mathrm{int}}|eB_\perp|}{8\pi\hbar c} = \frac{\sqrt{\pi}\lambda\epsilon_B^2}{4\Lambda}$$

- In the model of Ref. [Phys. Rev. B **78** (2008) 085437]

$$M = 4.84 \times 10^{-2}\epsilon_B \ \text{and}\ A = 3.90 \times 10^{-2}\epsilon_B$$

Full propagator

- We use the following general ansatz:

$$iG_s = \left[(i\hbar\partial_t + \mu_s + \tilde{\mu}_s\gamma^3\gamma^5)\gamma^0 - v_F(\boldsymbol{\pi}\cdot\boldsymbol{\gamma}) - \tilde{\Delta}_s + \Delta_s\gamma^3\gamma^5\right]^{-1}$$

Electron chemical potential

"Pseudospin" chemical potential

Dirac mass

T-odd mass

- Physical meaning of the order parameters

$$\Delta_s: \qquad \bar{\Psi}\gamma^3\gamma^5 P_s\Psi = \psi^\dagger_{KAs}\psi_{KAs} - \psi^\dagger_{K'As}\psi_{K'As} - \psi^\dagger_{KBs}\psi_{KBs} + \psi^\dagger_{K'Bs}\psi_{K'Bs}$$

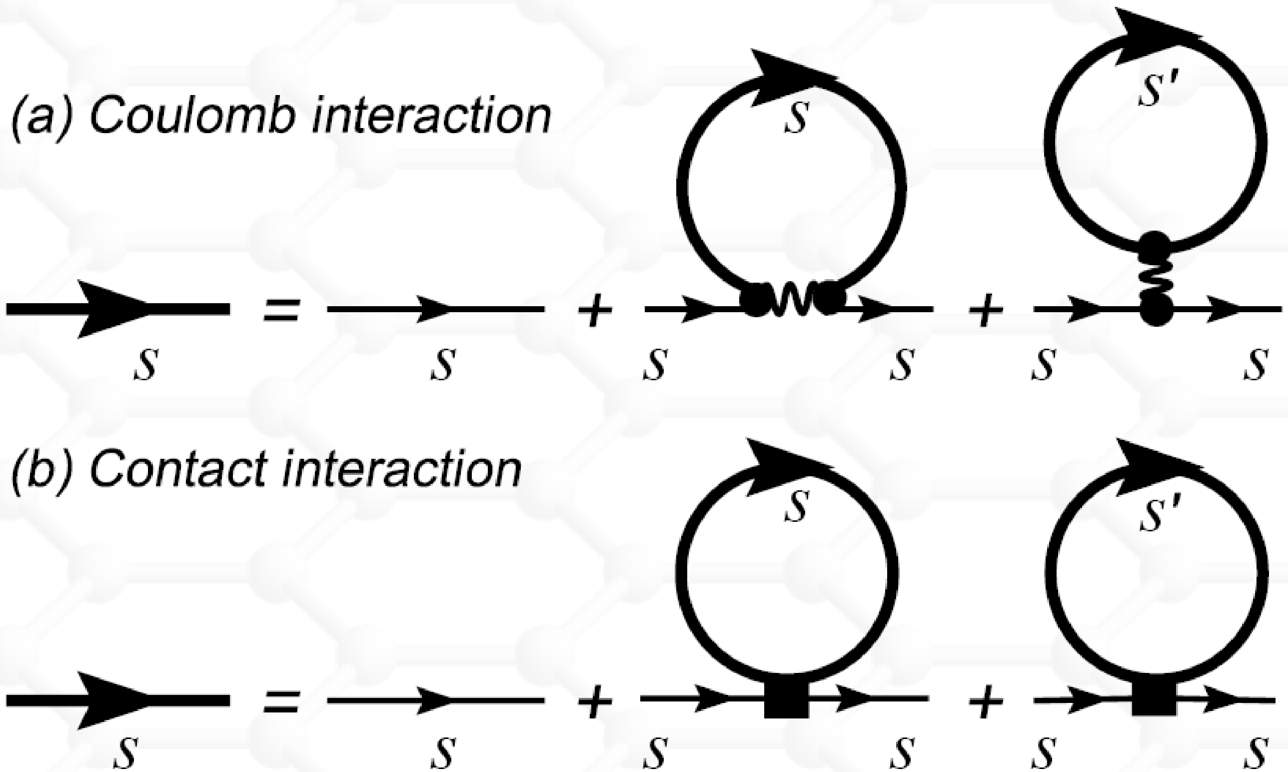
$$\tilde{\Delta}_s: \qquad \bar{\Psi}P_s\Psi = \psi^\dagger_{KAs}\psi_{KAs} + \psi^\dagger_{K'As}\psi_{K'As} - \psi^\dagger_{KBs}\psi_{KBs} - \psi^\dagger_{K'Bs}\psi_{K'Bs}$$

$$\mu_3: \qquad \Psi^\dagger\sigma^3\Psi = \frac{1}{2}\sum_{\kappa=K,K'}\sum_{a=A,B}\left(\psi^\dagger_{\kappa a+}\psi_{\kappa a+} - \psi^\dagger_{\kappa a-}\psi_{\kappa a-}\right)$$

$$\tilde{\mu}_s: \qquad \Psi^\dagger\gamma^3\gamma^5 P_s\Psi = \psi^\dagger_{KAs}\psi_{KAs} - \psi^\dagger_{K'As}\psi_{K'As} + \psi^\dagger_{KBs}\psi_{KBs} - \psi^\dagger_{K'Bs}\psi_{K'Bs}$$

Schwinger Dyson equation

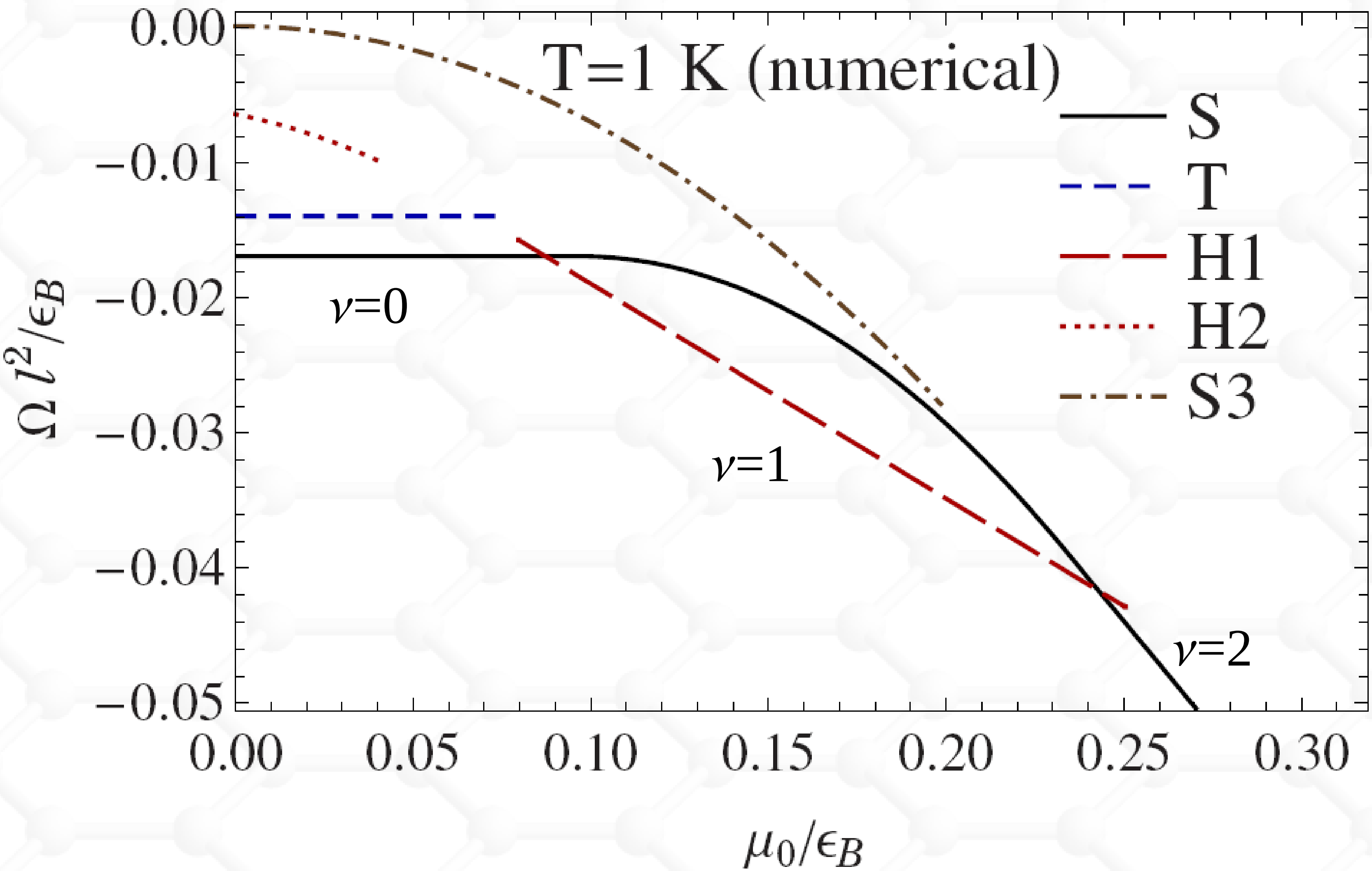
- Hartree-Fock (mean field) approximation:



Three types of solutions

- $\boldsymbol{S}$ (*singlet* with respect to $U(2)_s$ where $s=\uparrow,\downarrow$)
  - Order parameters: μ_3 and/or Δ_s
  - Symmetry: $U(2)_+ \times U(2)_-$
- $\boldsymbol{T}$ (*triplet* with respect to $U(2)_s$)
  - Order parameters: $\tilde{\mu}_s$ and/or $\tilde{\Delta}_s$
  - Symmetry: $U(1)_+ \times U(1)_-$
- $\boldsymbol{H}$ (*hybrid*, i.e., singlet + triplet)
  - Order parameters: mixture of $\boldsymbol{S}$ and $\boldsymbol{T}$ types
  - Symmetry: $U(2)_+ \times U(1)_-$ or $U(1)_+ \times U(2)_-$

Solutions at LLL ($\mu_0 \ll \epsilon_B$)



Singlet solution vs. *T* (ν=0 QHE state)

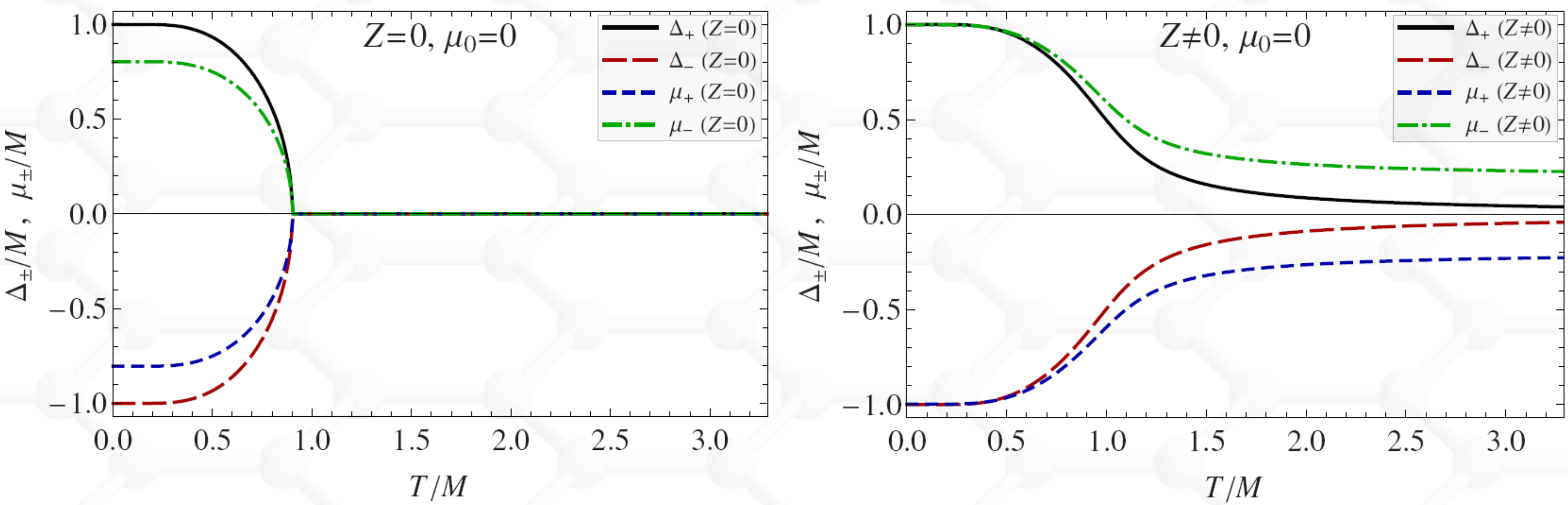
- *T=0:*

$$\tilde{\Delta}_+ = \tilde{\mu}_+ = 0, \qquad \boxed{\mu_+ = \bar{\mu}_+ - A,} \qquad \Delta_+ = s_\perp M,$$

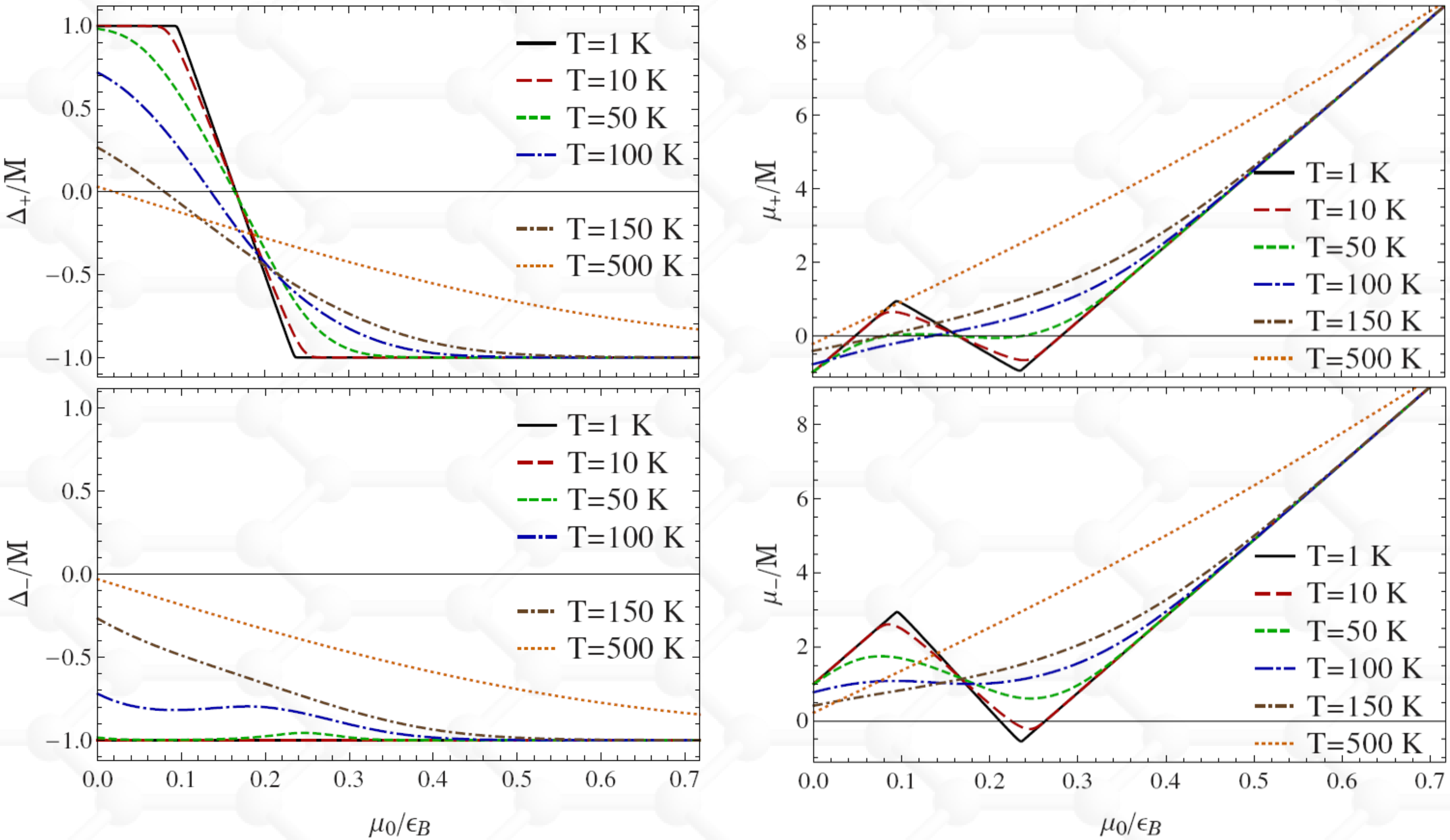
$$\tilde{\Delta}_- = \tilde{\mu}_- = 0, \qquad \boxed{\mu_- = \bar{\mu}_- + A,} \qquad \Delta_- = - s_\perp M$$

- "Flavor" symmetry:

Z=0: $U(4) \to U(2)_+ \times U(2)_-$ **Z≠0**: $U(2)_+ \times U(2)_-$



Singlet solution (ν=0 & 2 QHE states)



Solutions for ν=1 and ν=2 QHE states

- *T=0* hybrid solution for ν=1 state

$$\tilde{\Delta}_+ = M, \quad \tilde{\mu}_+ = A s_\perp, \quad \mu_+ = \bar{\mu}_+ - 4A, \quad \Delta_+ = 0$$

$$\boxed{\tilde{\Delta}_- = \tilde{\mu}_- = 0,} \quad \mu_- = \bar{\mu}_- - 3A, \quad \Delta_- = -s_\perp M$$

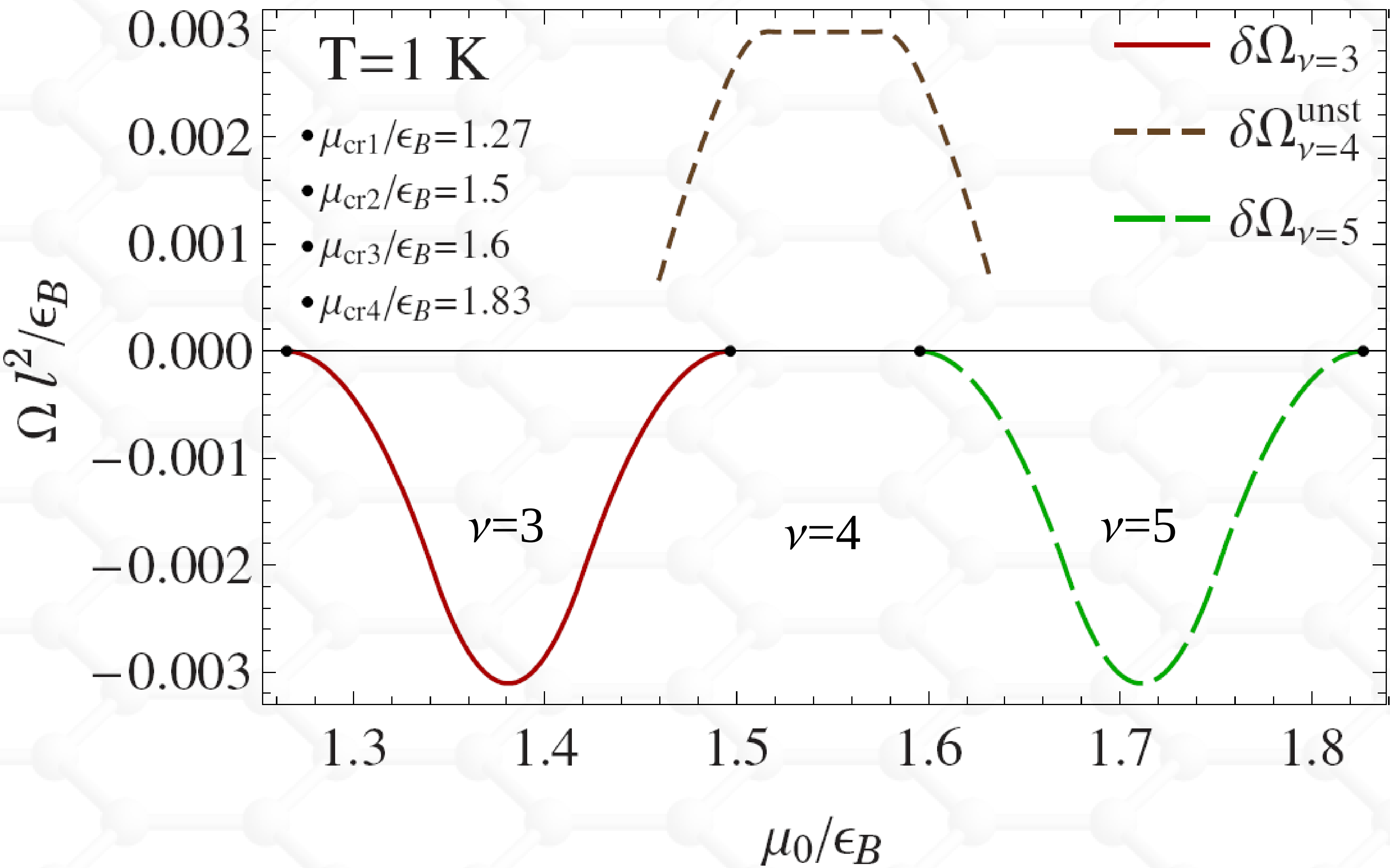
Symmetry: $U(1)_+ \times U(2)_-$

- *T=0* singlet solution for ν=2 state

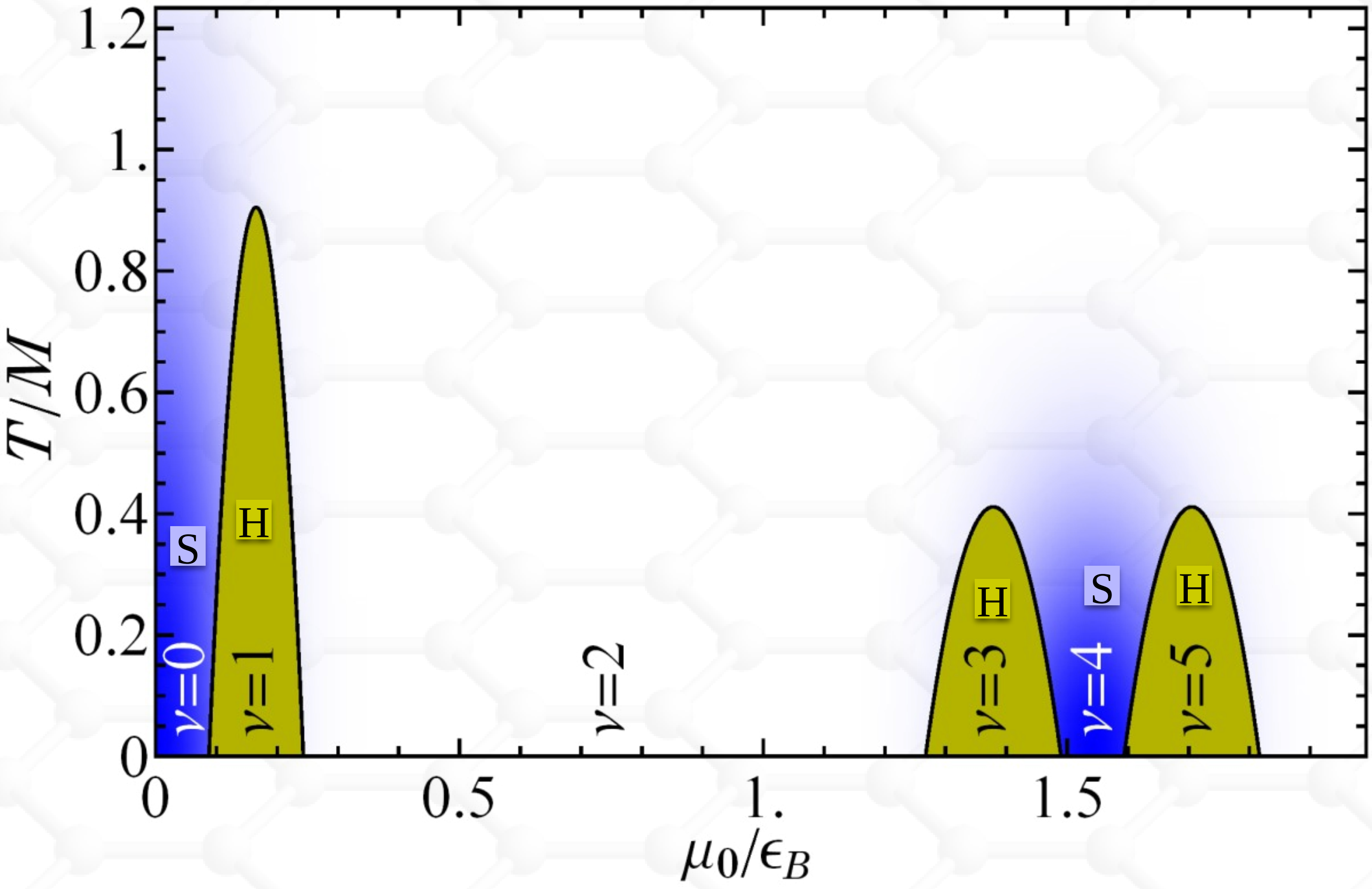
$$\boxed{\begin{array}{l} \tilde{\Delta}_+ = \tilde{\mu}_+ = 0, \\ \tilde{\Delta}_- = \tilde{\mu}_- = 0, \end{array}} \quad \begin{array}{ll} \mu_+ = \bar{\mu}_+ - 7A, & \Delta_+ = -s_\perp M \\ \mu_- = \bar{\mu}_- - 7A, & \Delta_- = -s_\perp M \end{array}$$

Symmetry: $U(2)_+ \times U(2)_-$

Hybrid solutions at 1st Landau level



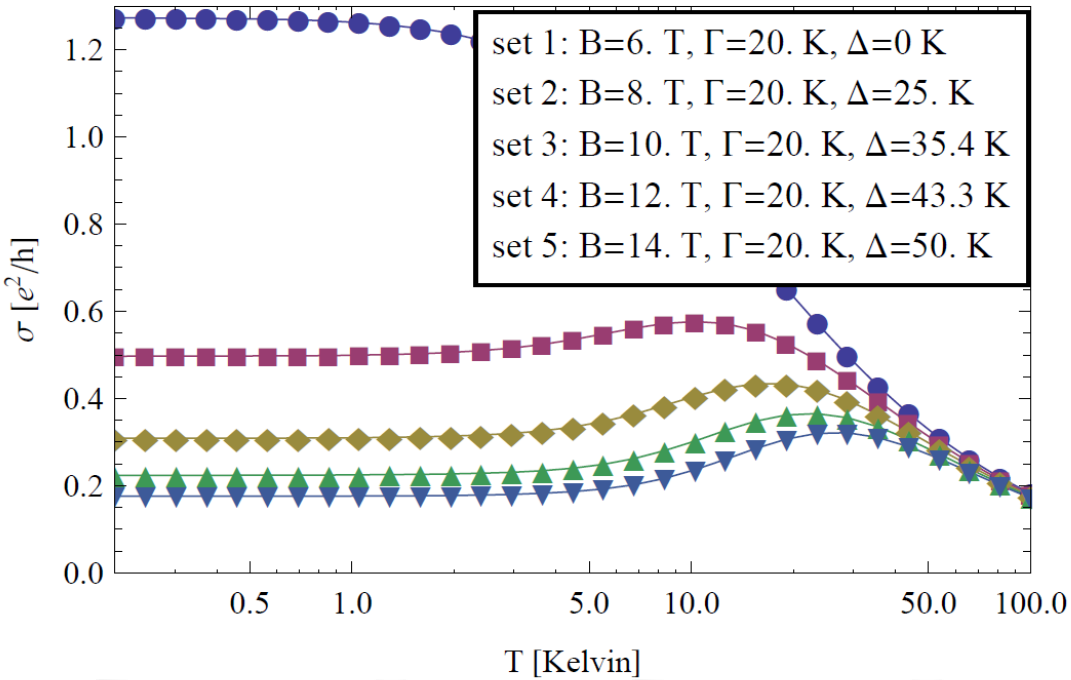
Phase diagram



Theory vs. experiment (1)

- Theory predicts all "new" QHE plateaus ($\nu=0$, $\nu=\mp1$, $\nu=\mp4$) observed in a strong magnetic field
- The plateaus $\nu=\mp3$, $\nu=\mp5$ are also predicted (now the $\nu=3$ plateau has also been seen!)
- Weak plateaus $\nu=\mp3$, $\nu=\mp5$ are in qualitative agreement with a suggested large width of higher Landau levels [Giesbers et al., Phys. Rev. Lett. **99**, 206803 (2007)]

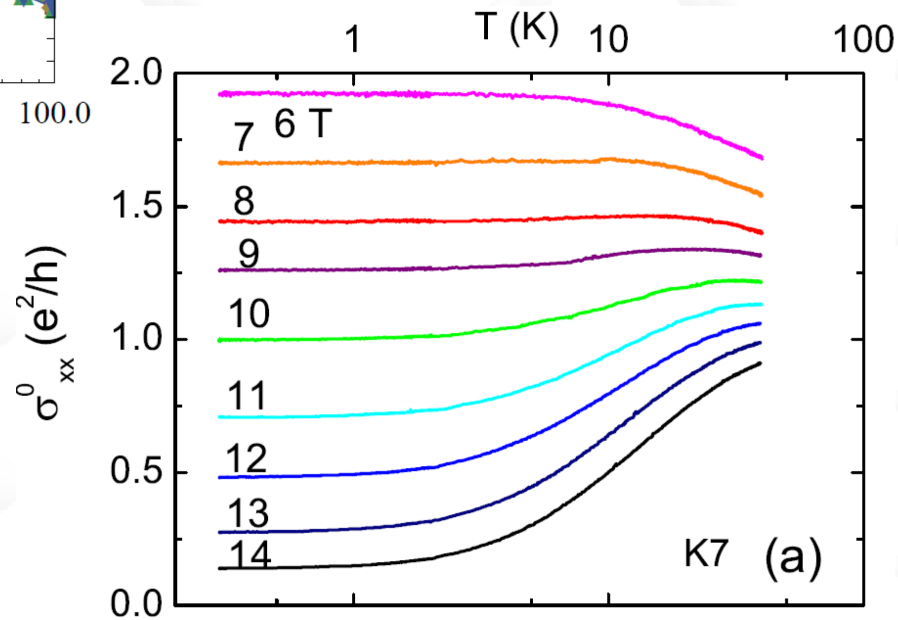
Theory vs. experiment (2)



⇐Theory

First try with a "reasonable" set of parameters

Experiment ⇒



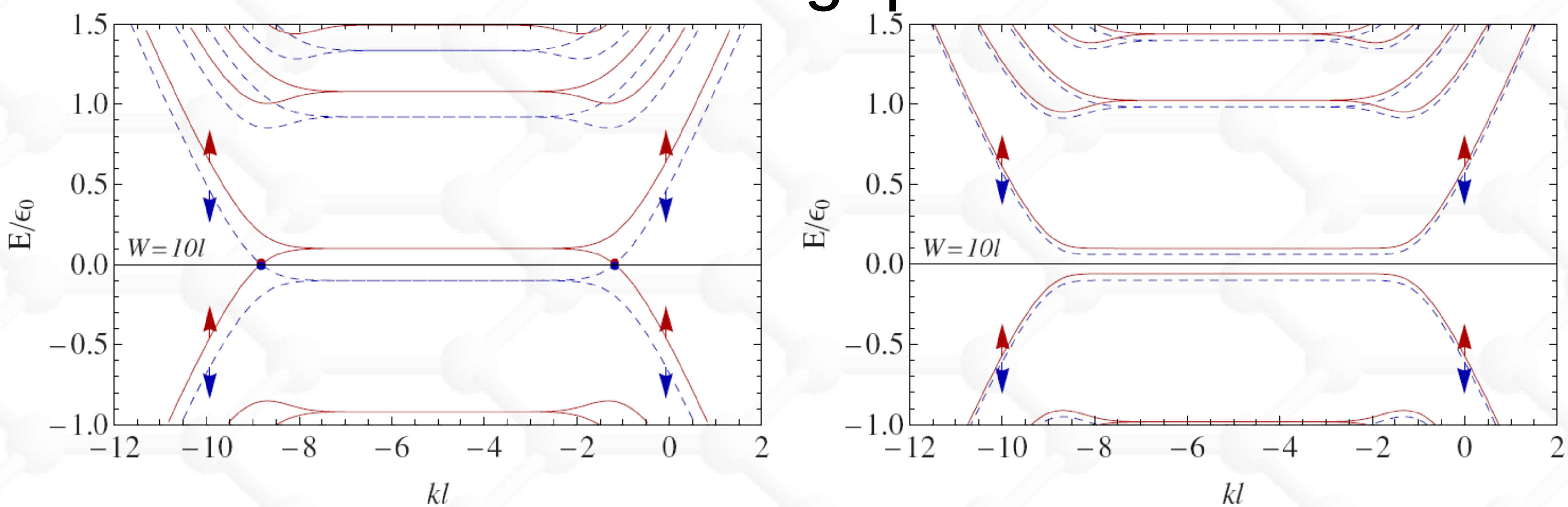
Checkelsky, Li, Ong, Phys. Rev. Lett. **100**, 206801 (2008)

The edge state puzzle

- ν=0 state: is it a quantum Hall metal or insulator?
- In other words: are there gapless edge states?
- Abanin et al [Phys. Rev. Lett. **96**, 176803 (2006)] suggested that

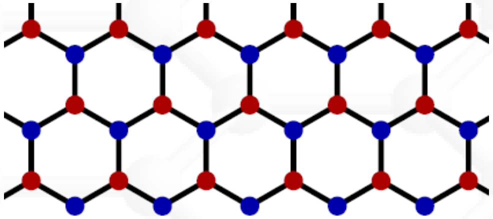
QHF $\Rightarrow$ gapless modes present

MC $\Rightarrow$ no gapless modes



Gapless edge states

- General criteria for the existence of gapless modes among the edge states are [Gusynin et al., Phys. Rev. B **77**, 205409 (2007); Phys. Rev. B **79**, 115431 (2009)]
- Zigzag edges:



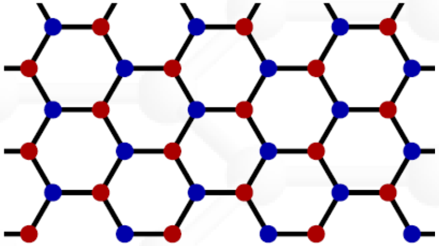
zigzag edge

➢ $|\mu_s^{(\pm)}| > |\Delta_s^{(\mp)}|$

where $\mu_s^{(\pm)} \equiv \mu_s \pm \tilde{\mu}_s$

and $\Delta_s^{(\pm)} \equiv \Delta_s \pm \tilde{\Delta}_s$

- Armchair edges:



armchair edge

➢ always when some **singlet** gaps are present

➢ $|\mu_s| > |\tilde{\Delta}_s|$ if only **triplet** gaps are present

Summary

- Insight into non-perturbative dynamics of QHE in graphene comes from relativistic physics
- A rich phase diagram of graphene is proposed
- Both MC and QHF necessarily coexist (“two sides of the same coin”) and lift the degeneracy of Landau levels in graphene
- Qualitative agreement with experiments is already evident (details are to be worked out)
- Edge state puzzle can be resolved