Effective Lagrangians and New Physics

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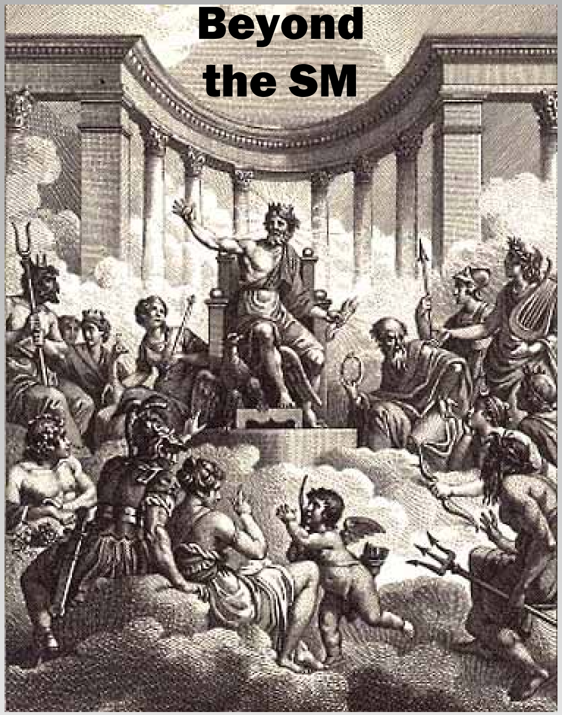
Introduction



Credo:

There is physics beyond the SM…
… but we know nothing about it

Still, its virtual effects cannot be arbitrary,
so we can capitalize on that

$$e^{iS_{\text{eff}}} = \int [d\{\text{heavy fields}\}]\,[d\{k \geq \Lambda\}]\, e^{iS}$$

$$S_{\text{eff}} = \sum c_i \mathcal{O}_i$$

$$\mathcal{O} \sim \prod \{\text{light fields}\}$$

and/or

$$\mathcal{O} \sim \prod \{k < \Lambda \text{ degs. of freedom}\}$$

Effective theories are renormalizable: divergences can be absorbed in the coefficients:

(Divergence $\rightarrow$ local operator with the low-energy symmetries)

But the theory has not predictability **unless** there is a hierarchy of coefficients

… and loop effects do not spoil this

Two modalities of this paradigm have been significantly developed:

- Low-energy theory = SM (sometimes with an extended scalar sector)
- Low-energy theory = SM without physical scalars

All effective operators must obey the SM local symmetries,

… but not necessarily the global ones

Effective theories can mostly be treated as ``usual’’ renormalizable ones

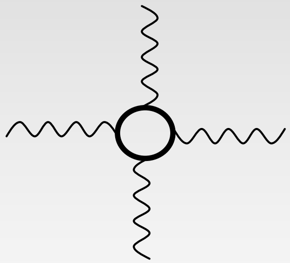
The good: they provide model and process independent paramterization of all new physics effects

The bad: they have a limited range of applicability, the predictions are *not* reliable at scales > Λ

The ugly: the number of unknown parameters grows rapidly as more and more precision is demanded

There are many “old” examples

- Fermi theory of the weak interactions
  - Low energy fields: e, μ , γ , ν
  - Range of applicability energy < M_W
- Landau-Ginzburg theory of superconductivity
  - Low-energy fields: the order parameter
  - Range of applicability: $\Delta T/T_c \sim (T_c/e_F)^4$
- Euler Heisenberg 4-γ Lagrangian
  - Low energy fields: γ
  - Range of applicability: energy < m_e



$$\Rightarrow \quad \mathcal{L}_{\text{eff}} = aF^4 + bF^2(F\tilde{F}) + \cdots$$

$$a, b \sim e^4 \times \frac{1}{16\pi^2} \times \frac{1}{m_e^2}$$

Heavy scale

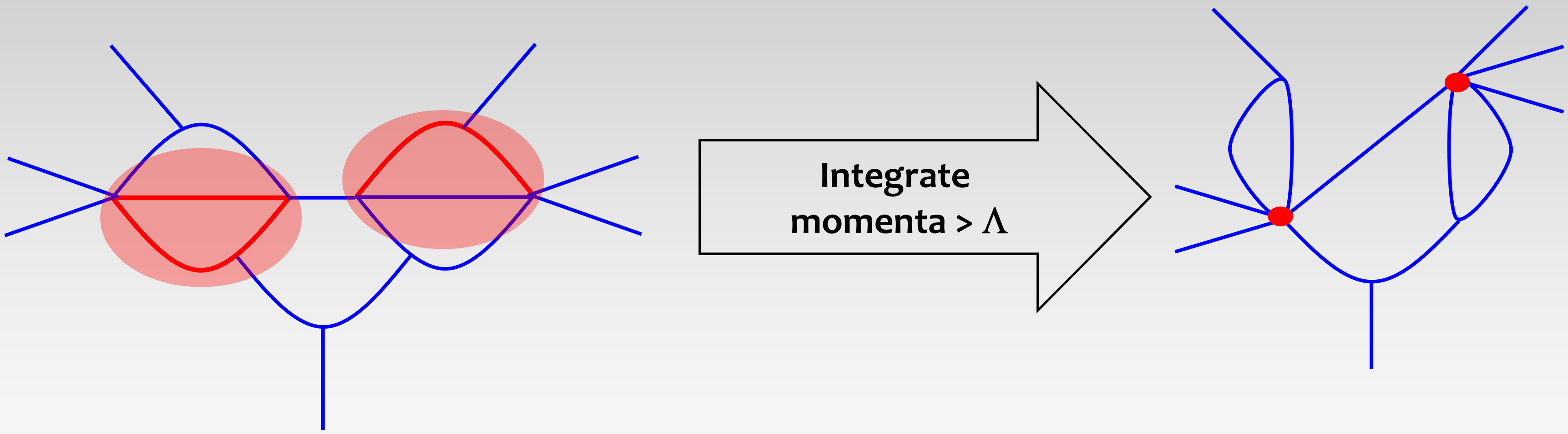
Universal gauge coupling

Loop effect

Note that the heavy scale is **not** necessarily $1/\sqrt{a}$, $1/\sqrt{b}$

Hierarchies

Any graph in the full theory can have
- "Light" lines: masses & momenta < Λ (blue)
- "Heavy" lines: masses or momenta > Λ (red)



Can either
- Do all integrations in the "fundamental" theory
- Do all integrations of momenta < Λ in the effective theory

But ∞ operators contribute to every graph

$\Rightarrow$ need a hierarchy

Notation:

b = # boson lines

f = # fermion lines

d = # derivatives

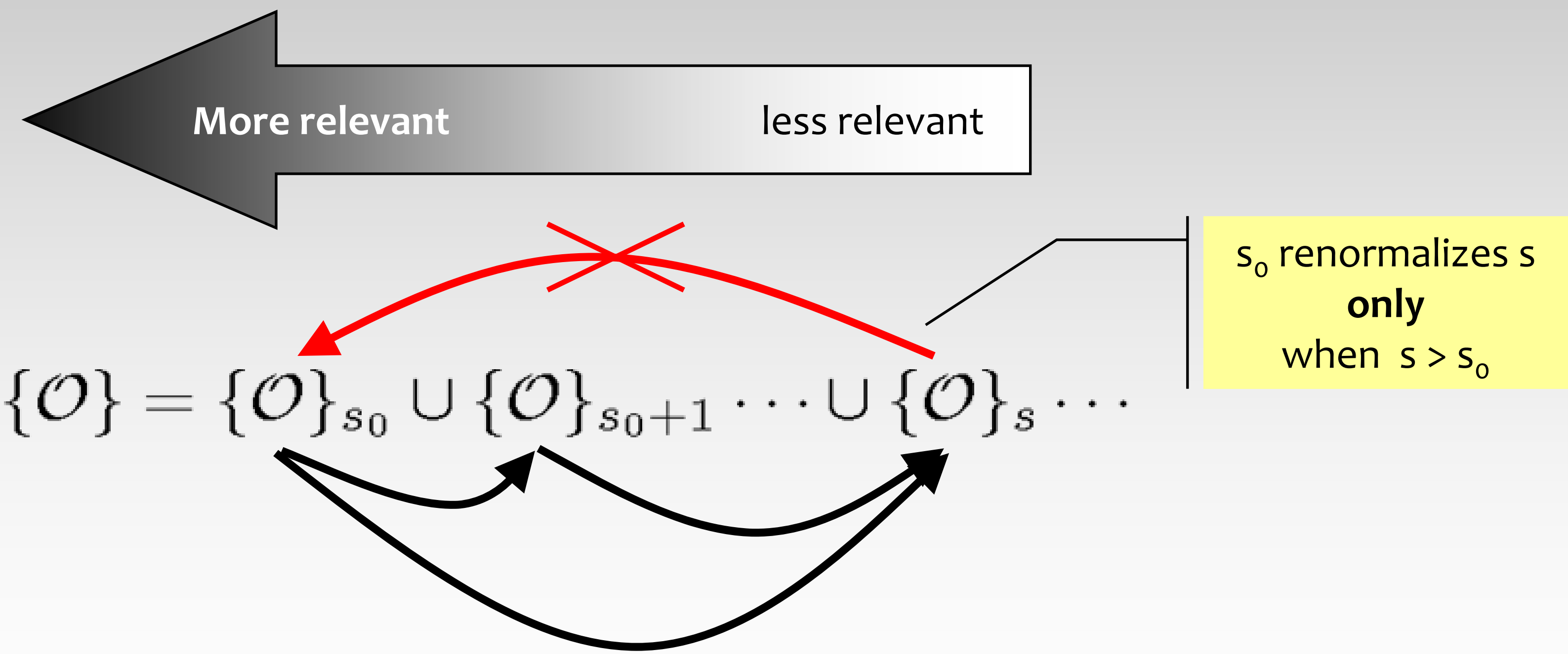
To each operator associate an ***index***:

$$\mathbf{s} = \left(\frac{u-2}{2}\right)\mathbf{b} + \left(\frac{u-1}{2}\right)\mathbf{f} + \mathbf{d} - u$$

Free parameter

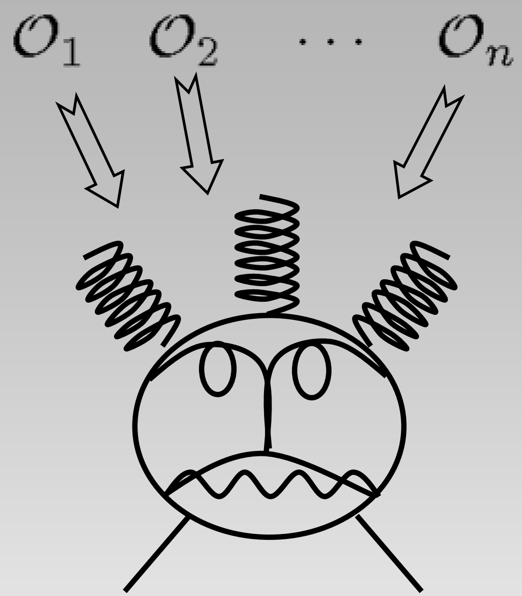
Use ***s*** to order operators: $\mathcal{O}$ more relevant than $\mathcal{O}'$ if ***s < s'***

… and require the order is preserved by the RG



$$\{\mathcal{O}\} = \{\mathcal{O}\}_{s_0} \cup \{\mathcal{O}\}_{s_0+1} \cdots \cup \{\mathcal{O}\}_{s} \cdots$$

Generic Feynman graph:



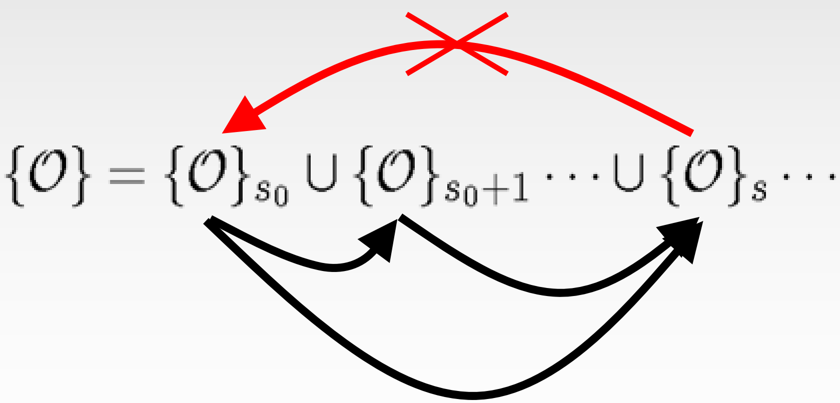
The divergence renormalizes $\mathcal{O}$, with index **s:**

$$\mathbf{s} = (n - u)L + \sum \mathbf{s}_i$$

Dimension of space-time

loops

Index of $\mathcal{O}_i$



$$\{\mathcal{O}\} = \{\mathcal{O}\}_{s_0} \cup \{\mathcal{O}\}_{s_0+1} \cdots \cup \{\mathcal{O}\}_s \cdots$$

Then require $\mathbf{s} \geqslant \mathbf{s}_i$

So, must have

- $\mathbf{s}_i \geqslant 0$
- $\boldsymbol{n} \geqslant \boldsymbol{u}$

$$\mathbf{s} = \left(\frac{u-2}{2}\right)\mathbf{b} + \left(\frac{u-1}{2}\right)\mathbf{f} + \mathbf{d} - u$$

If $u = n$

s = canonical dimension of $\mathcal{O}$

In 4 dimensions (n=4) all interesting operators have **s** > 0

Useful when the heavy physics decouples and is weakly coupled

If $u = 2$

s = **f** + **d** – 2 (independent of **b**)

Useful in chiral theories

If $u = 1$

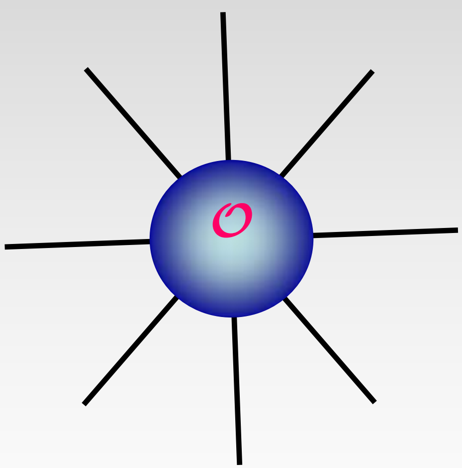
s = **d – b**/2- 1 (independent of **f**)

Useful in theories with no bosons (e.g. non-linear realization of SUSY)

Coefficient estimates

Strongly coupled new physics

$$\mathcal{L} = \frac{f}{\Lambda^{\Delta - n}} \mathcal{O} + \cdots ; \qquad \Delta = \text{dim. of } \mathcal{O}$$



$$\rightarrow \delta f \lesssim f$$

$$\Rightarrow \quad f \lesssim (\ell_n)^{\mathbf{b}+\mathbf{f}-2} ; \quad \ell_n = (4\pi)^{n/2} \Gamma(n/2)$$

n = dimension of space-time

If we also choose the gauge coupling as:

$$g^2 \sim \ell_n \Lambda^{4-n} \quad \xrightarrow{n=4} \quad g \sim 4\pi$$

Operator	coefficient	Operator	coefficient	Operator	coefficient
$(D\phi)^2$	1	$\bar{\psi}\,\not{D}\psi$	1	$(\partial A)^2$	1
ϕ^2	Λ^2	$\bar{\psi}\psi$	Λ		
ϕ^3	$g\Lambda$	$\bar{\psi}\phi\psi$	g	$A^2\partial A$	g
ϕ^4	g^2	$(\bar{\psi}\psi)^2$	$(g/\Lambda)^2$	A^4	g^2

Large oblique parameters

Large FCNC

Large fermion masses

Another application: a 5-dimensional gauge theory with no fermions

choose: u=4

$$
\begin{array}{ll}
s = 0: & F^2\,, \quad \bar{\psi} D \psi \\
s = 1: & A F^2\,, \quad \bar{\psi} F \psi \\
 & \vdots
\end{array}
$$

Operator coefficients (g = gauge coupling constant, $\sim M^{-1/2}$):

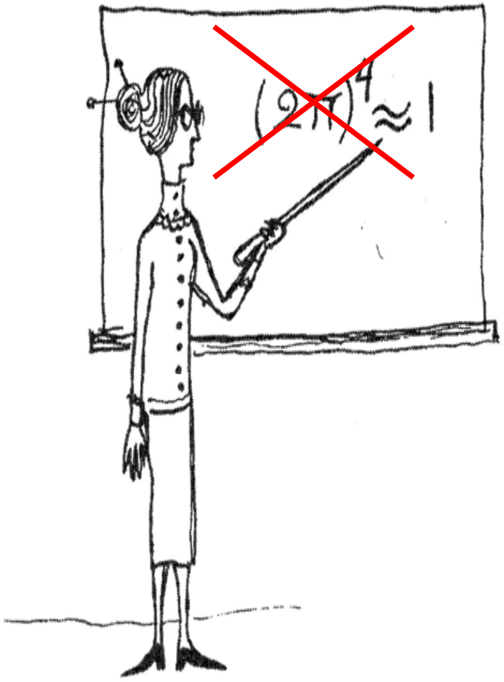
$$\left(\frac{1}{24\pi^3}\right)^s \times (\text{power of } g \text{ to make a } 5-\text{d object})$$

$$
\begin{aligned}
\mathcal{L} \;=\;& -\frac{1}{4}\mathrm{tr} F_{ij}F^{ij} + \bar{\psi}\, \not{D} \psi \\
&+ f' \frac{g^3}{24\pi^3} \bar{\psi}\left(\gamma_i \gamma_j F^{ij}\right)\psi \\
&+ f \frac{g^3}{24\pi^3} \epsilon^{ijklm} \mathrm{tr}\left\{ A_i F_{jk} F_{lm} + \frac{i}{2} A_i A_j A_k F_{lm} - \frac{1}{10} A_i A_j A_k A_l A_m \right\} \\
&+ \cdots
\end{aligned}
$$

CP violation
leptogenesis

Axion-like terms
Strong CP problem

Weakly coupled new physics



$$\mathcal{L} = \frac{f}{\Lambda^{s-n}}\mathcal{O} + \cdots$$

$$f = \begin{cases} \prod \text{couplings} & \text{tree generated} \\ \prod \text{more couplings}/(4\pi)^2 & \text{loop generated} \end{cases}$$

Weakly coupled and decoupling NP will have the strongest effects in operators that

1. Have the lowest dimension
2. Can be generated at tree-level

The factors of 2 π do count!

Two applications

Single top production @ the LHC

Relevant operators

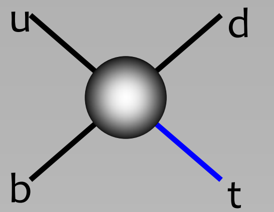
$$\begin{aligned} \mathcal{O}_{\phi q}^{(3)} &= i\left(\phi^\dagger \tau^I D\mu\phi\right)\left(\bar{Q}_L\gamma^\mu\tau^I Q_L\right) \\ \mathcal{O}_{\phi\phi} &= i\left(\phi^\dagger \epsilon D_\mu\phi\right)\left(\bar{t}_R\gamma^\mu b_R\right) \\ \mathcal{O}_{qq}^{(3)} &= \frac{1}{2}\left(\bar{Q}_L\gamma^\mu\tau^I Q_L\right)\left(\overline{Q'_L}\gamma_\mu\tau^I Q'_L\right) \end{aligned}$$

SSB

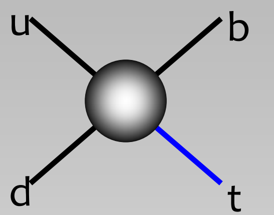
$$\mathcal{L}_{NP} = \frac{\mathcal{G}_{4f}}{v^2}\left(\bar{q}\gamma^\mu P_L q'\right)\left(\bar{t}\gamma_\mu P_L b\right)\frac{g}{\sqrt{2}} + \bar{t}\, W\!\!\!/^{+}(\mathcal{F}_L P_L + \mathcal{F}_R P_R)\, b$$

< 0.004
$b \to s\,\gamma$

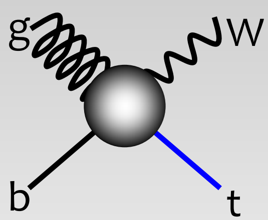
$$\mathcal{F}_{L,R},\ \mathcal{G}_{4f} \sim (v/\Lambda)^2$$



$$\sigma_t^{(0)}\left(1 + 4\mathcal{F}_L - 3.06\mathcal{G}_{4f}\right)$$



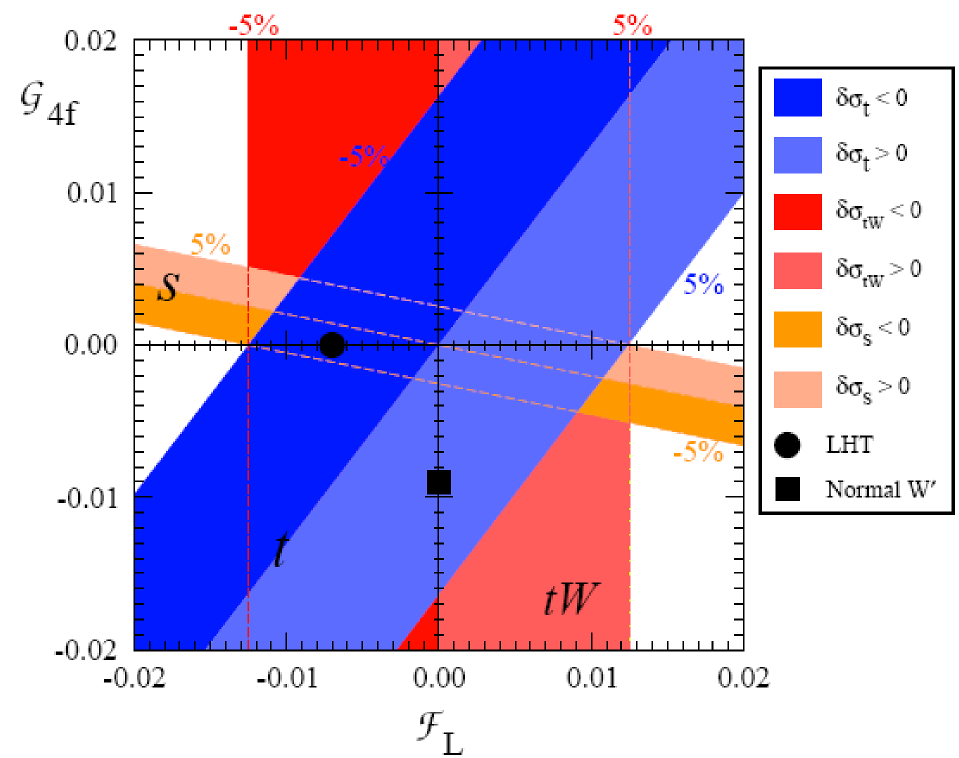
$$\sigma_s^{(0)}\left(1 + 4\mathcal{F}_L + 19.69\mathcal{G}_{4f}\right)$$



$$\sigma_{tW}^{(0)}\left(1 + 4\mathcal{F}_L\right)$$

	ATLAS		CMS	
	$\mathcal{F}_L$	$\mathcal{G}_{4f}$	$\mathcal{F}_L$	$\mathcal{G}_{4f}$
t	0.0029	0.0038	0.0039	0.0051
s	0.0364	0.0074	0.0254	0.0052
W	0.0074	–	0.0118	–

Sensitive to Λ < 3 TeV



Regions where the deviations from the SM are less than 5%

Neutrino physics

ν oscillations *suggest* the presence of ν_R = N:

$$\mathcal{L}_{\nu SM} \equiv \mathcal{L}_{SM} + \left(\frac{1}{2} \bar{N} M N^c - \bar{L} \tilde{\phi} Y N + \text{H.c.} \right)$$

$$m_\nu \sim \frac{(vY)^2}{M}$$

Two examples giving $m_\nu \sim 0.01$ eV:

- $M \sim 10^{15}$ GeV, $m_D \sim m_W$ ($Y \sim 1$) $\Rightarrow$ **too heavy** to be seen
- $M \sim 100$ GeV, $m_D \sim m_{electron}/10$ ($Y \sim 10^{-7}$) $\Rightarrow$ **too weakly coupled** to be seen

$$\mathcal{L}^W_{V-A} = -(g/\sqrt{8}) U_{\ell N} \overline{N^c} \not{W}^+ P_L \ell \quad U_{\ell N} \sim Y$$

Dimension 5 terms

Tree-level generated

$(\bar{L}\tilde{\phi})(\phi^\dagger L^c)$ — ν_L Majorana mass $\sim v^2/\Lambda$ + H interactions

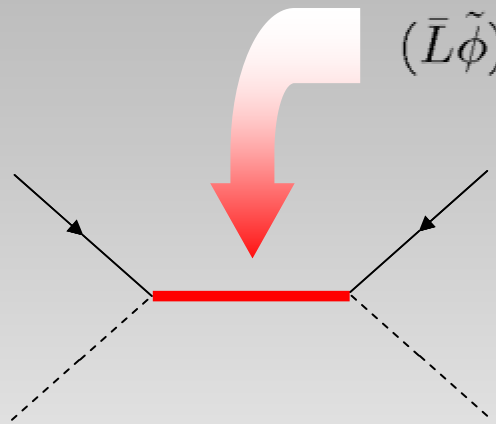
$(\bar{N}N^c)(\phi^\dagger\phi)$ — ν_R Majorana mass $\sim v^2/\Lambda$ + H interactions

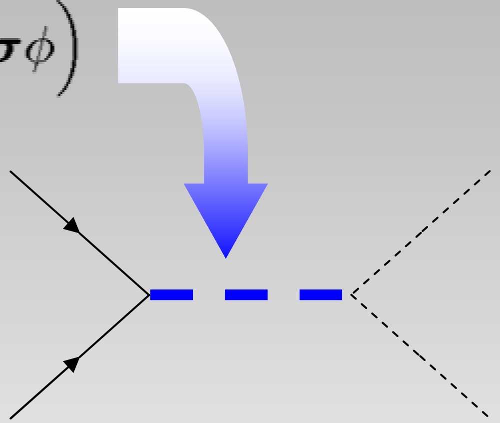
Loop generated

$(\bar{N}\sigma_{\mu\nu}N^c)B^{\mu\nu}$ — ν_R Majorana magnetic moment; Z coupling

Heavy mediators (example):

$$(\bar{L}\tilde{\phi})(\phi^\dagger L^c) = \frac{1}{2}\left(\bar{L}\boldsymbol{\sigma}L\right)\cdot\left(\tilde{\phi}^\dagger\boldsymbol{\sigma}\phi\right)$$





Type 1 see-saw:
Fermion isosinglet,
hypercharge = 0

or

Type III see-saw:
Fermion isotriplet,
hypercharge = 0

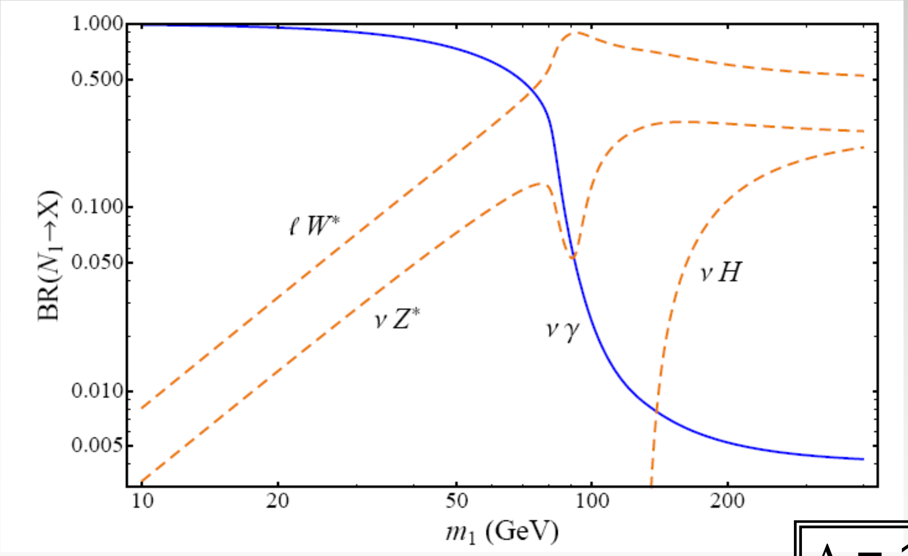
Type 2 see-saw:
Scalar isotriplet,
hypercharge = 1

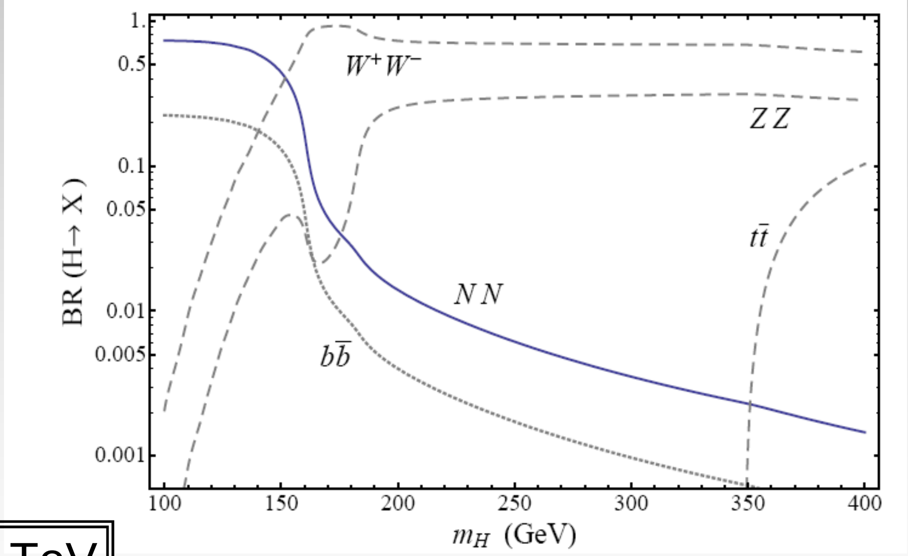
ν_R Majorana magnetic moment; Z coupling

Majorana mass $\sim v^2/\Lambda$ + H interactions

$$\frac{1}{\Lambda}(\bar{N}\sigma_{\mu\nu}N^c)B^{\mu\nu}$$

$$\frac{1}{\Lambda}(\bar{N}N^c)(\phi^\dagger\phi)$$



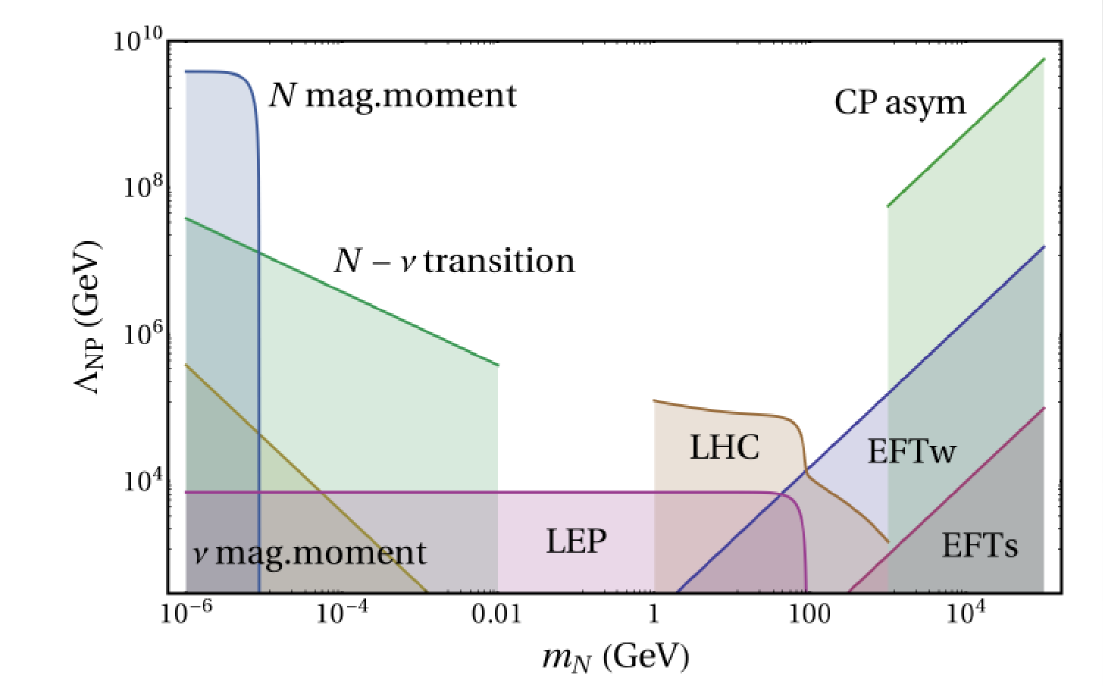


Λ = 10 TeV

Astrophysical & cosmological effects:

- Plasmon decay: $\Lambda > 4 \times 10^6$ TeV for $m_N < 8$ keV
- SN cooling: $4 \times 10^6 \sqrt{m_\nu / m_N}$ TeV; $10\text{keV} < m_N < 30$ MeV
- Leptogenesis

$$\frac{\Gamma(N_2 \to e^- \phi^+) - \Gamma(N_2 \to e^+ \phi^-)}{\Gamma(N_2 \to e^- \phi^+) + \Gamma(N_2 \to e^+ \phi^-)} \sim \frac{g'}{2\pi} \frac{m_1}{\Lambda} \frac{m_2^2 - m_1^2}{m_2^2}$$



Dimension 6 terms

Generated by

- W_R
- Leptoquarks
- Extended scalar sector
- Z', W'
- Heavy fermions

$$
\begin{aligned}
\mathcal{O}_{NN\phi} &= i(\phi^\dagger D_\mu \phi)(\bar{N}\gamma^\mu N), \\
\mathcal{O}_{Ne\phi} &= i(\phi^T \varepsilon D_\mu \phi)(\bar{N}\gamma^\mu e) \\
\mathcal{O}_{duNe} &= (\bar{d}\gamma^\mu u)(\bar{N}\gamma^\mu e) \\
\mathcal{O}_{fNN} &= (\bar{f}\gamma_\mu f)(\bar{N}\gamma^\mu N), \\
\mathcal{O}_{LNLe} &= (\bar{L}N)\varepsilon(\bar{L}e) \\
\mathcal{O}_{LNQd} &= (\bar{L}N)\varepsilon(\bar{Q}d) \\
\mathcal{O}_{QuNL} &= (\bar{Q}u)(\bar{N}L) \\
\mathcal{O}_{QNLd} &= (\bar{Q}N)\varepsilon(\bar{L}d), \\
\mathcal{O}_{LN} &= |\bar{L}N|^2 \\
\mathcal{O}_{QN} &= |\bar{Q}N|^2 \\
\mathcal{O}_{NN} &= (\bar{N}N^c)^2 \\
\mathcal{O}'_{NN} &= |\bar{N}N^c|^2
\end{aligned}
$$

Application: pp → l⁺ l⁺ j j

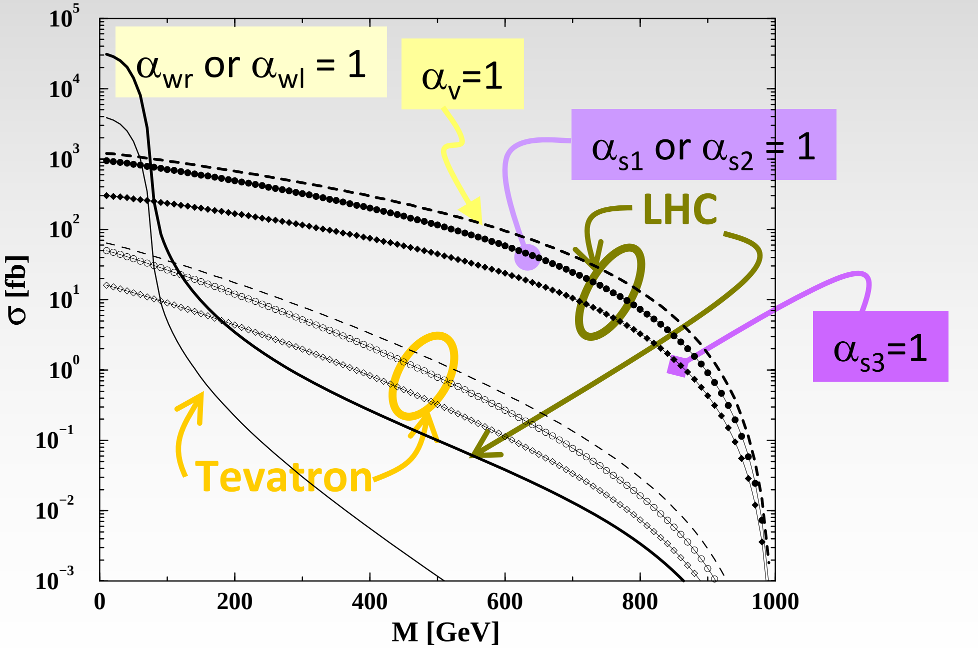
$$\mathcal{L}^{N}_{eff} = \Lambda^{-2}\Big[-\sqrt{2}vm_{\rm w}\alpha_{wl}\overline{N^c}\gamma^\mu e_L W^+_\mu$$
$$-\sqrt{2}vm_{\rm w}\alpha_{wr}\bar{N}\gamma^\mu e_R W^+_\mu$$
$$+\alpha_{\rm v}\,(\bar{d}_R\gamma^\mu u_R)\,(\bar{N}\gamma_\mu e_R)$$
$$+\alpha_{s1}\,(\bar{u}_R d_L)\,(\bar{e}_L N)$$
$$-\alpha_{s2}\,(\bar{u}_L d_R)\,(\bar{e}_L N)$$
$$+\alpha_{s3}\,(\bar{u}_L N)\,(\bar{e}_L d_R) + \text{H.c.}\Big]$$

From $\mathcal{L}_{\nu\,SM}$; small coupling $\sim 10^{-7}$

From $i(\phi^T\varepsilon D_\mu\phi)(\bar{N}\gamma^\mu e)$

$$\left.\begin{array}{l} M \lesssim 200\ \text{GeV}, \\ \Lambda \sim \mathcal{O}(1)\ \text{TeV}, \\ \alpha_{wr} \sim \mathcal{O}(1)\ (\alpha_i = 0\ \text{otherwise}) \end{array}\right\} 5\sigma\ \text{effect@LHC}$$

$$\left.\begin{array}{l} M \lesssim 600\ \text{GeV}, \\ \Lambda \sim \mathcal{O}(1)\ \text{TeV}, \\ \alpha_{\rm v} \sim \mathcal{O}(1)\ (\alpha_i = 0\ \text{otherwise}) \end{array}\right\} \sigma \gtrsim 100\text{fb}\ \ (\text{LHC})$$



Shy new physics

Some current limits are very strict:

$$\mathcal{O} = \frac{f}{\Lambda^2} \left(\bar{q}\gamma^\mu q\right) \left(\bar{\ell}\gamma_\mu \ell\right) \xrightarrow{\text{Tevatron}} \Lambda \gtrsim 10\text{TeV}$$

Three possibilities for hiding new physics:
- Very large scale
- Cancellations:

$$\frac{f}{\Lambda^2} = \sum_{\text{scalars}} \frac{\lambda^2}{M_s^2} - \sum_{\text{vectors}} \frac{g^2}{M_v^2}$$

- A new symmetry

Cancellations

New physics:

$$\begin{aligned} \text{vectors}: \quad & V_{(\text{color},\text{isospin},\text{hypercharge})} \\ \text{scalars}: \quad & \Phi_{(\text{color},\text{isospin},\text{hypercharge})} \\ \text{fermions}: \quad & \Psi_{(\text{color},\text{isospin},\text{hypercharge})} \end{aligned}$$

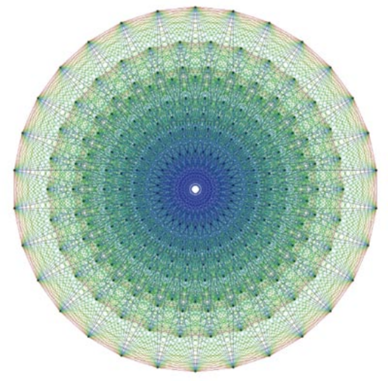
Cancellations for the 4 fermion operators and oblique parameters require:

$$V_{(1,3,0)}, \ V_{(1,1,0)}$$

$$\Phi_{(1,1,0)}, \ \Phi_{(1,1,2)}, \ \Phi_{(1,3,1)}, \ \Phi_{(6,1,4/3)}, \ \Phi_{(6,1,-2/3)}, \ \Phi_{(6,3,1/3)}$$

… all having related masses and couplings !!

New symmetries



All operators generated at tree-level have a vertex of the form
heavy $\times$ light $\times$ light

Simplest possibility: a global symmetry such that

- All SM fields are singlets
- No heavy physics field is a singlet

$\Rightarrow$ *all* tree-level generated operators disappear
$\Rightarrow$ the effective reach in energy is reduced by $\sim$ 1/100

Examples:

- SUSY with R-parity
- Little Higgs models with T parity
- d>4 models with momentum conservation in the extra directions

Outlook

The effective Lagrangian approach is a very useful tool in consistently parameterizing new physics.

It provides coefficient estimates that indicate where new physics effects can be largest

In some cases this suggests Λ > few TeV …

… but symmetry properties might suppress *all* effective operators

… and the LHC might observe new physics without any virtual premonition.