SYDDANSK UNIVERSITET
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LIGHT SCALAR PUZZLE IN QCD

MAIN present QUESTION – THE $3.5 × 10^9 QUESTION – FIND THE HIGGS AT LHC.

PRESENT TOPIC – THE 35¢ QUESTION* – FIND (AND UNDERSTAND) THE UR-HIGGS FROM LOW ENERGY ππ SCATTERING DATA

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* COST OF XEROXING A FEW PAGES FROM PHYSICAL REVIEW, ETC. [ππ SCATTERING DATA]

FUNDAMENTAL THEORY OF STRONG INTERACTIONS:

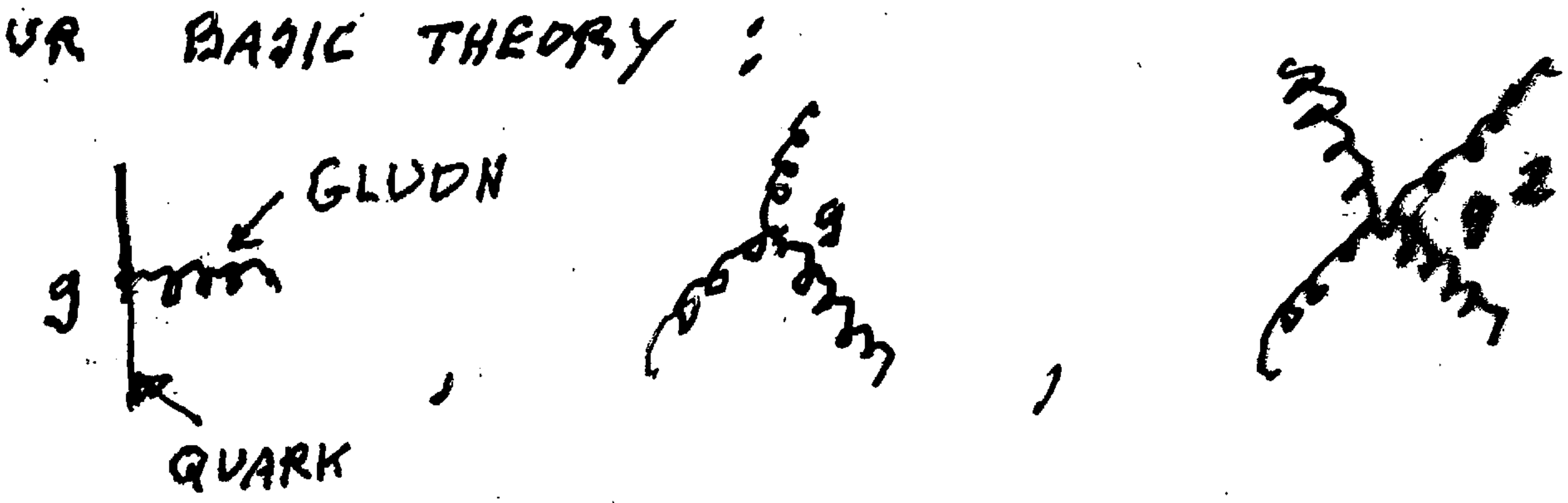
QCD = Quantum Chromo Dynamics

and

CHIRAL SYMMETRY

WELL KNOWN DIFFICULTY FOR QCD CALCULATION

OUR BASIC THEORY:



$$\frac{g^2_{eff}(E)}{4\pi} \sim \frac{1}{\ln \frac{E}{\Lambda}}, \quad E = \text{ENERGY SCALE}, \quad \Lambda \cong 0.25 \text{ GeV}$$

AT LARGE E, $\frac{g^2(E)}{4\pi}$ IS SMALL (ASYMPTOTIC FREEDOM) AND PERTURBATION THEORY IS GOOD.

BUT FOR SMALL E, $\frac{g^2(E)}{4\pi}$ IS BIG! THEN:

PERTURBATION THEORY IN $\frac{g^2}{4\pi}$ CAN NOT BE EXPECTED TO FURNISH A SYSTEMATIC APPROACH

MAYBE STILL HOPE FOR PERTURBATION THEORY,

IF WE USE THE PHYSICAL FIELDS (π AND N) AS FUNDAMENTAL FOR LOW ENERGY WORK.

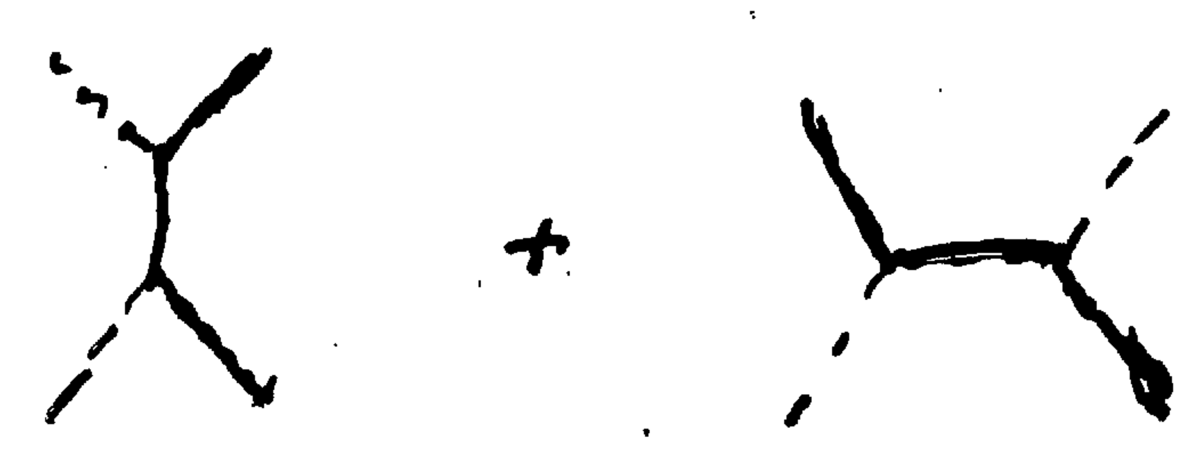
HISTORICALLY

1930's ... YUKAWA THEORY

$g_{\pi NN}$

FOUND FROM LONG RANGE PART OF NN SCATTERING

BUT PERTURBATION THEORY TURNED OUT ALSO TO BE (TERRIBLE) FOR πN SCATTERING LENGTHS (MORE THAN ORDER OF MAGNITUDE TOO LARGE) ETC.



LED TO →

1950's : S-MATRIX THEORY

(UNDERLYING ASSUMPTION : LAGRANGIAN DYNAMICS INCORRECT FOR STRONG INTERACTIONS)

HOWEVER

LATE 1950's, 1960's SAW NEW PHYSICAL INPUT — "V–A" THEORY OF WEAK INTERACTIONS SUGGESTS :

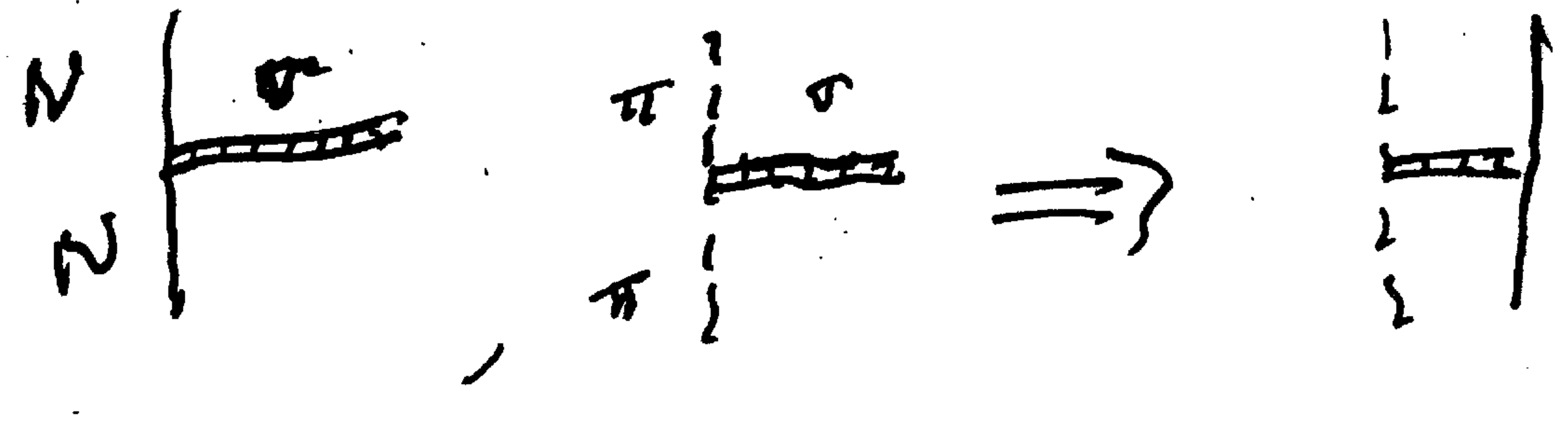
THE FUNDAMENTAL FERMION FIELDS ARE THE (ZERO MASS) L AND R PROJECTIONS OF THE DIRAC FIELDS.

$\Rightarrow$ • ISOSPIN, $SU(2)_{VECTOR}$ SHOULD BE UPGRADED TO $SU(2)_L \times SU(2)_R$

• MASS COMES FROM SPONTANEOUS BREAKDOWN OF THIS SYMMETRY.

$\rightarrow$ (ADLER - WEISSBERGER FORMULA
CURRENT ALGEBRA
EFFECTIVE CHIRAL LAGRANGIANS)

$\rightarrow$ THE SIMPLEST IMPLEMENTATION INTRODUCES A SCALAR "PARTNER" OF THE PIONS, CALLED σ :



NEW CONTRIBUTION TO πN SCATTERING "MAGICALLY" SUBTRACTS AWAY MOST OF THE PREVIOUS ANSWER AND GIVES SCATTERING LENGTHS IN GOOD AGREEMENT WITH EXPERIMENT!

WHAT IS THIS SU(2) LINEAR σ MODEL?

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}(\partial_\mu \underline{\pi})^2 + a(\sigma^2 + \underline{\pi}^2) - b(\sigma^2 + \underline{\pi}^2)^2 + \cdots$$

[Gell-Mann, Levy (1961)]

$F_\pi \simeq 0.131$ GeV $= \sqrt{2}\langle\sigma\rangle$, $a = \frac{1}{4} M^2_{\text{same}}$

WEINBERG FOUND ANOTHER USE FOR THIS MODEL (& A NOBEL PRIZE!)

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} i\pi^+ \\ \frac{\sigma - i\pi^0}{\sqrt{2}} \end{pmatrix}$$

$\sigma \equiv$ HIGGS , $\pi^+ =$ LONGITUDINAL PART OF W_μ^+ ETC.

EVERYONE IS WAITING FOR LHC (OR ?) TO FIND THE HIGGS.

BUT EVERYONE WAS EAGER TO GET RID OF THE "STRONG INTERACTION HIGGS".

IF IT IS HEAVY, "INTEGRATE IT OUT":

$$\sigma = \sqrt{\frac{F_\pi^2}{2} - \underline{\pi}^2} \Rightarrow \mathcal{L} = -\frac{1}{2}(\partial_\mu \underline{\pi})^2 - \frac{1}{2}(\partial_\mu \sqrt{})^2$$

FOR CALCULATIONS: MORE EFFICIENT TO "INTEGRATE OUT" THE σ AND WORK WITH A NON-LINEAR FIELD,

$U = e^{2i\phi/F_\pi}$, $F_\pi = 0.131$ GeV [CHISHOLM'S THEOREM]

ϕ = 2x2 (OR 3x3) MATRIX OF 0^- FIELDS.

THEN $\mathcal{L} = -\frac{F_\pi^2}{8} \mathrm{Tr}(\partial_\mu U \partial_\mu U^\dagger) + \ldots$

GIVES MANY "MAGICALLY GOOD" RESULTS AT LOW ENERGIES:

πN scattering lengths
$\pi\pi$ " "
other "
"SOFT PION" RELATIONS FOR $\Delta I = 1$, SEMI-LEPTONIC, NON-LEPTONIC DECAYS OF MESONS AND BARYONS.

BASIS OF "CHIRAL PERTURBATION THEORY" PROGRAM.

THE σ (= "UR-HIGGS") IS GONE!

THE π STRIKES BACK!

TRY TO UNDERSTAND THE π-π SCATTERING DATA.

R_0^0 = REAL PART OF I=J=0 ππ PARTIAL WAVE AMPLITUDE.

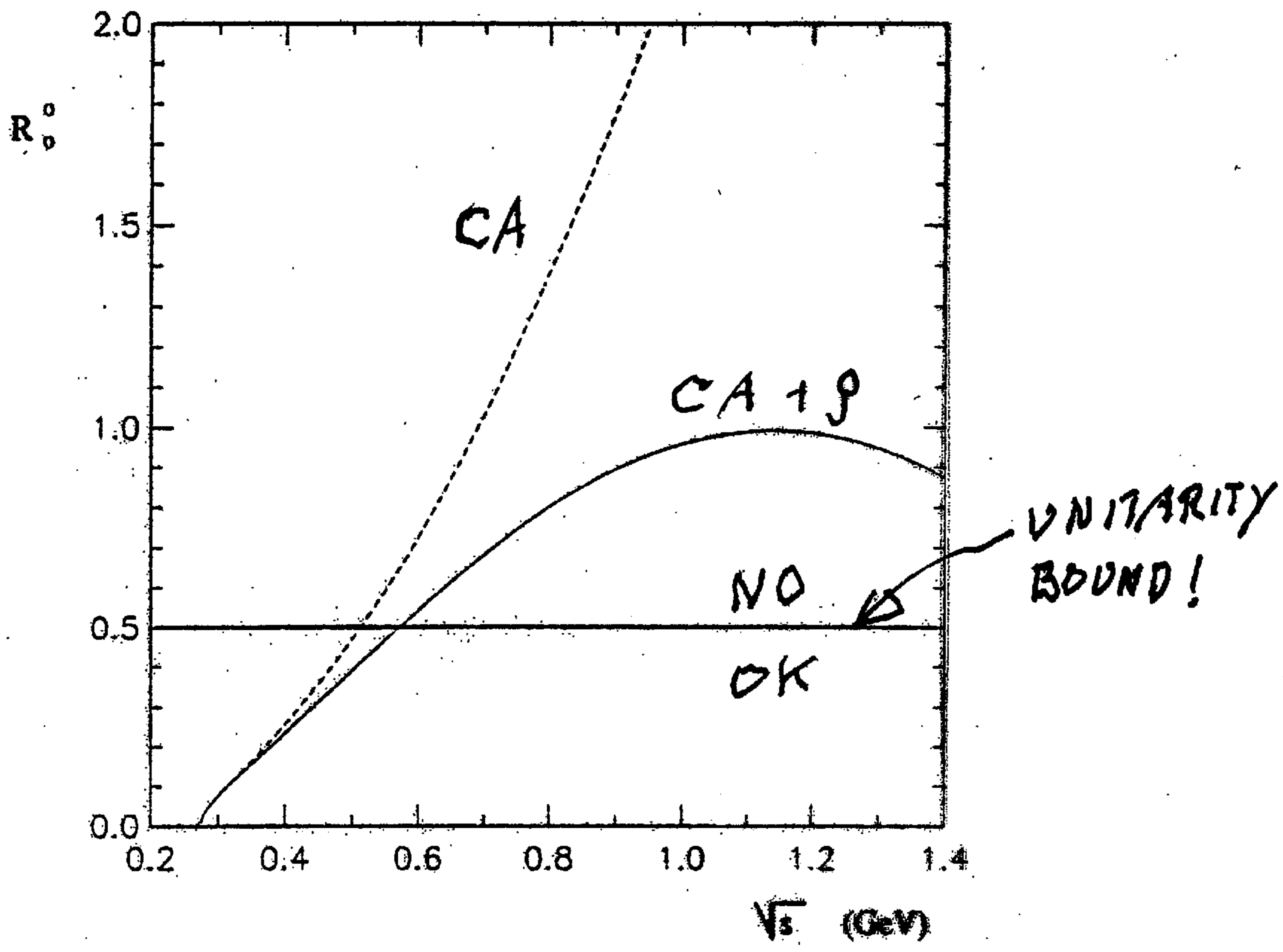


Figure 1: Predicted curves for R_0^0. The solid line which shows the *current algebra* + ρ result for R_0^0 is much closer to the unitarity bound of 0.5 than the dashed line which shows the *current algebra* result alone.

[Picture from M. Harada, F. Sannino, J. Schechter, Phys. Rev D54, 1991 (1996)]

→ USED NON-LINEAR MODEL & "PHENOMENOLOGICAL UNITARIZATION" MOTIVATED BY a) CURRENT ALGEBRA
b) LARGE N_c APPROXIMATION

THE σ SAVES UNITARITY

(NB: $\Re e$ [reson.] = ?)

CA + ρ + σ

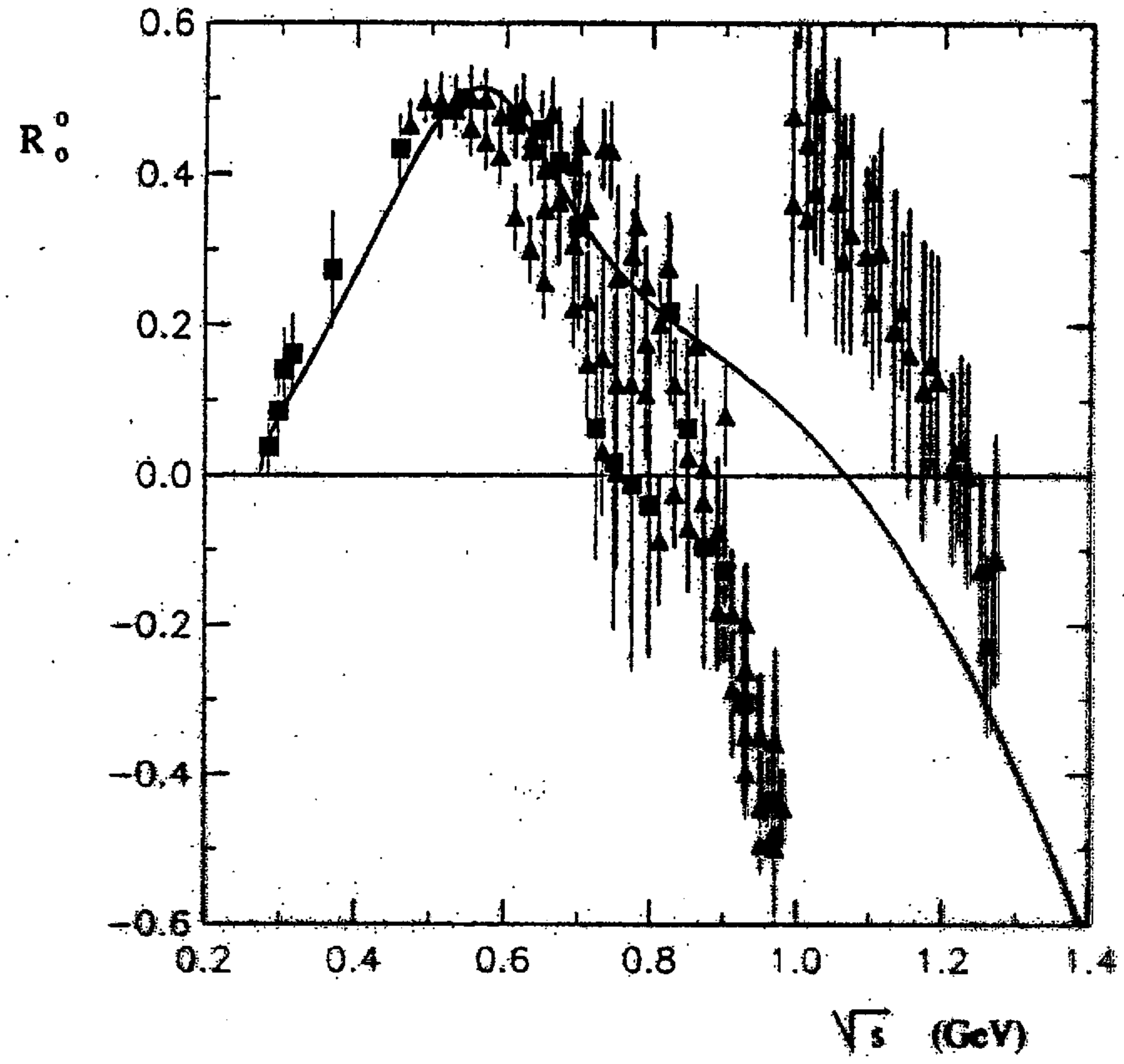
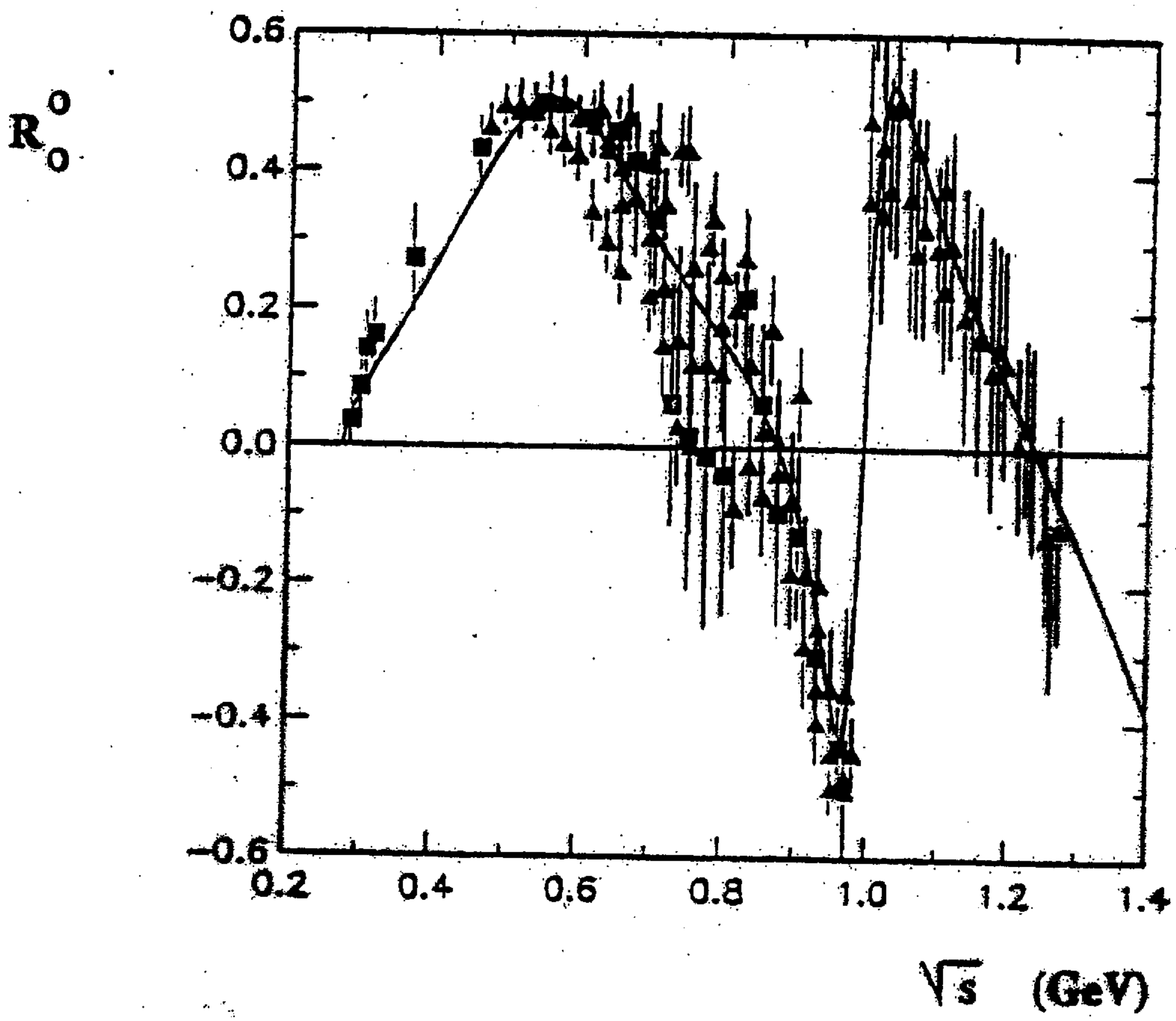


Figure 2: The solid line is the *current algebra* + ρ + σ result for R_0^0. The experimental points, in this and suceeding figures, are extracted from the phase shifts using eq. (A.6) and actually correspond to R_0^0/η_0^0. (■) are extracted from the data of Ref. [8] while (▲) are extracted from the data of Ref. [9]. The predicted R_0^0 is small around the 1 *GeV* region.

■ E.A. ALEKSEEVA ET AL, SOV PHY JETP 55, 591 (1982)
▲ G. GRAYER ET AL, NUCL. PHYS B75, 189 (1974)

ANOTHER LIGHT SCALAR, THE $f_0(980)$, COMPLETES THE JOB



"CURRENT ALGEBRA" + ρ + σ + $f_0(980)$

LAST 15 YEARS OR SO REVIVAL

LIGHT "σ" = $f_0(600)$, LIGHT "κ" ?

POSSIBLE 0^+ LIGHT NONET COMPLETION

$f_0(600) \approx 500$ MeV $(I=0)$

$\kappa(800) \approx 800$ MeV $(I=\frac{1}{2})$

$f_0(980) \approx 980$ MeV $(I=0)$

$a_0(980) \approx 980$ MeV $(I=1)$

↓ MASS

$f_0(980)$, $a_0(980)$ → WELL ESTABLISHED

VECTOR MASS ORDER

$\rho(770)$ $(I=1)$

$\omega(780)$ $(I=0)$

$K^*(880)$ $(I=\frac{1}{2})$

$\phi(1020)$ $(I=0)$

↑ USUAL FOR MOST NONETS

0^+ LIGHT NONET STATES SEEM TO BE <u>FLIPPED</u> !

<u>NATURAL</u> (BASED ON COUNTING THE NUMBER OF HEAVIER s-QUARKS) <u>IF</u> THEY ARE $q\bar{q}-q\bar{q}$ (MOLECULE)

OR $qq-\bar{q}\bar{q}$ (DIQUARK - ANTIDIQUARK)

STATES RATHER THAN $\bar{q}q$ TYPE p-WAVE STATES.

<u>ALSO</u> LIGHTER THAN 1 - 1.5 GeV RANGE OBSERVED FOR OTHER p-WAVE $q\bar{q}$ STATES.

TEMPTING TO TRY TO IDENTIFY 0^- AND 0^+ NONETS AS MEMBERS OF A SINGLE <u>CHIRAL</u> NONET.

THIS IS REASONABLE BUT WE WOULD HAVE TO ASSUME THAT THE 0^+ STATES IN THE LAGRANGIAN SHOULD TRANSFORM AS $q\bar{q}$ STATES UNDER $U(1)_A$ [WHICH WE WOULD ALSO LIKE TO MODEL].

THE PRESENT MODEL CORRESPONDS TO TWO CHIRAL NONETS.

$M \sim$ TRANSFORMS LIKE $q\bar{q}$ UNDER $U(1)_A$ FOR BOTH 0^- AND 0^+ MEMBERS

$M' \sim$ TRANSFORMS LIKE $(qq\bar{q}\bar{q})$ FOR BOTH 0^- AND 0^+ MEMBERS

<u>QUESTION</u>: DOES THE <u>MIXING</u> OF 0^- AND 0^+ FIELDS WITH THE SAME ~~[illegible]~~ QUANTUM NUMBERS RESULT IN:

LIGHT 0^- ~~STATES~~ $\sim q\bar{q}$ MAINLY

LIGHT 0^+ " $\sim (qq\bar{q}\bar{q})$ MAINLY ?

CANDIDATES FOR "USUAL" $q\bar{q}$ p-WAVE SCALARS

$I = 0$: $f_0(1370)$

$I = 1$: $a_0(1474)$

$I = 1/2$: $K_0^*(1414)$

$I = 0$: $f_0(1500)$ → GLUEBALL ?
$f_0(1710)$

WHY IS:
$m[K_0^*] < m[a_0]$?

WHY ARE ALL A BIT ON THE HIGH SIDE ? (MIXING ?)

CANDIDATES FOR POSSIBLE HEAVY 0^- STATES

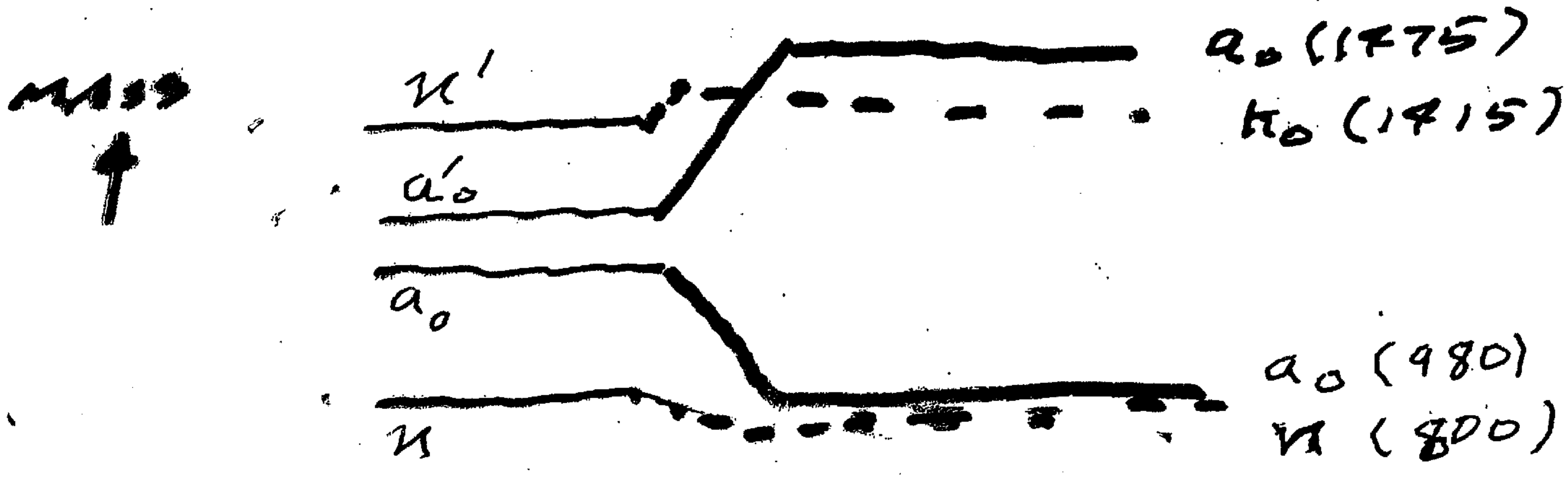
$\pi(1300)$; $I = 1$

$K(1460)$, $K(1830)$: $I = 1/2$

$\eta(1295)$, $\eta(1405)$, $\eta(1475)$, $\eta(1760)$: $I = 0$

4'

FURTHERMORE IT IS REASONABLE TO EXPECT IMPORTANT MIXING WITH "USUAL" p-WAVE SCALARS (D. BLACK, A. FARIBORZ, J.S.) P.R. D61, 074001 (2000)



MORE REPULSION FOR $a_0 - a_0'$

ATTEMPTS TO REALIZE THE 2-CHIRAL NONET MODEL (GENERALIZED SU(3) SIGMA MODEL):

- SECT. V OF D. BLACK, A. FARIBORZ, S. MOUSSA, S. NASRI, J.S.; P.R. D 64, 034005 (2001).
- M. NAPSUCIALE, S. RODRIGUEZ; P.R. D 70, 094043 (2004)
- A. FARIBORZ, R. JORA, J.S, P.R. D 72, 034001 (2005);
  P.R. D 79, 074014 (2009).

FIELDS OF THE "M-M′" MODEL

LINEAR (SU(3)) SIGMA MODEL FIELDS TRANSFORM LIKE THEIR "CONSTITUENT" QUARKS:

$$q_L \to U_L q_L$$
$$q_R \to U_R q_R$$

, U_L, U_R ARE SU(3) UNIMODULAR MATRICES

SCHEMATICALLY:

$$M_a^b = (q_{bA})^\dagger \gamma_4 \frac{1+\gamma_5}{2} q_{aA} \qquad (= M_{ab})$$

$$\boxed{M \to U_L M U_R^\dagger}$$

"$qq\bar{q}\bar{q}$" FIELDS TRANSFORM IDENTICALLY

"MOLECULE" FIELD:

$$M_a^{(2)b} = \epsilon_{acd}\, \epsilon^{bef} (M^\dagger)_e^c (M^\dagger)_f^d$$

DUAL QUARK - DUAL ANTIQUARK FIELD:

$$M_g^{(3)f} = (L^{gA})^\dagger R^{fA} \qquad \text{WITH}$$

$$L^{gE} = \epsilon^{gab} \epsilon^{EAB} q_{aA}^T C^{-1} \frac{1+\gamma_5}{2} q_{bB},$$

$$R^{gE} = \epsilon^{gab} \epsilon^{EAB} q_{aA}^T C^{-1} \frac{1-\gamma_5}{2} q_{bB}$$

ETC. "DUAL QUARK", [$\bar{3}$ FLAVOR, $\bar{3}$ COLOR, SPIN SINGLET]

ALSO:

6-COLOR DIQUARK, $\bar{6}$-COLOR ANTI-DIQUARK

$$M_g^{(4)f} = \left(L^g_{\mu\nu,AB}\right)^{\dagger} R^f_{\mu\nu,AB} \quad , \quad \text{WHERE}$$

$$L^g_{\mu\nu,AB} = L^g_{\mu\nu,BA} = \epsilon^{gab} q^T_{aA} C^{-1} \sigma_{\mu\nu} \frac{1+\gamma_5}{2} q_{bB} ,$$

$$R^g_{\mu\nu,AB} = R^g_{\mu\nu,BA} = \epsilon^{gab} q^T_{aA} C^{-1} \sigma_{\mu\nu} \frac{1-\gamma_5}{2} q_{bB}$$

AT A NAIVE QUARK MODEL LEVEL,
NO DISTINCTION BETWEEN

{ "MOLECULE" FIELD
{ LINEAR COMBINATION OF
DIQUARK-ANTI DIQUARK FIELDS

SINCE (USING FIERZ IDENTITIES)

$$M_a^{(2)b} = \frac{2M_a^{(3)b} - M_a^{(4)b}}{8}$$

UNDER $U(1)_A$:

$$q_{aL} \to e^{i\nu} q_{aL}\,, \quad q_{aR} \to e^{-i\nu} q_{aR}$$

$$\Rightarrow \boxed{M \to e^{2i\nu} M}$$

BUT
$$\boxed{\begin{Bmatrix} M^{(2)} \\ M^{(3)} \\ \vdots \end{Bmatrix} \to e^{-4i\nu} \begin{Bmatrix} M^{(2)} \\ M^{(3)} \\ \vdots \end{Bmatrix}}$$

$U(1)_A$ DISTINGUISHES 2 AND FOUR QUARK STATES!

MAKE A LAGRANGIAN FROM M AND
M' = (UNSPECIFIED LINEAR COMBINATION OF $M^{(2)}$ AND $M^{(3)}$)

$$\boxed{\mathcal{L} = -\frac{1}{2} \mathrm{Tr}\,(\partial_\mu M \partial_\mu M^\dagger) - \frac{1}{2} \mathrm{Tr}\,(\partial_\mu M' \partial_\mu M'^\dagger) - V_0(M, M') - V_{SB}}$$

↑ (V_0): $SU(3)_L \times SU(3)_R$ INVARIANT [$U(1)_A$ VIOLATION IN AGREEMENT WITH ANOMALY]

↑ (V_{SB}): "MOCK UP" QUARK MASS TERMS

COMPLICATED MODEL: **36 STATES**

$$\boxed{\begin{aligned} M &= S + i\phi \\ M' &= S' + i\phi' \end{aligned}}$$

$S = S^\dagger,\ \phi = \phi^\dagger$
$S' = S'^\dagger,\ \phi' = \phi'^\dagger$

$[\pi, \pi']$, $[a_0, a_0']$, $[K, K']$, $[\kappa, \kappa']$, $[4\ \eta\text{'s}]$, $[4\ \sigma\text{'s}]$

HOW MANY ARBITRARY PARAMETERS ?

IF WE RESTRICT V_0 AND V_{SB} TO BE RENORMALIZABLE, THERE ARE 21 IN EACH:

$$
\begin{aligned}
V_0 = & -c_2 \mathrm{Tr}(MM^\dagger) + \tilde{c}_3(\det M + \mathrm{H.c.}) + c_4^a \mathrm{Tr}(MM^\dagger MM^\dagger) + c_4^b(\mathrm{Tr}(MM^\dagger))^2 + d_2 \mathrm{Tr}(M'M'^\dagger) \\
& + d_3(\det M' + \mathrm{H.c.}) + d_4^a \mathrm{Tr}(M'M'^\dagger M'M'^\dagger) + d_4^b(\mathrm{Tr}(M'M'^\dagger))^2 + e_2(\mathrm{Tr}(MM'^\dagger) + \mathrm{H.c}) + e_3^a(\epsilon_{abc}\epsilon^{def} M_d^a M_e^b M_f'^c + \mathrm{H.c} \\
& + e_3^b(\epsilon_{abc}\epsilon^{def} M_d^a M_e'^b M_f'^c + \mathrm{H.c.}) + e_4^a \mathrm{Tr}(MM^\dagger M'M'^\dagger) + e_4^b \mathrm{Tr}(MM'^\dagger M'M^\dagger) + e_4^c[\mathrm{Tr}(MM'^\dagger MM'^\dagger) + \mathrm{H.c.}] \\
& + e_4^d[\mathrm{Tr}(MM^\dagger MM'^\dagger) + \mathrm{H.c.}] + e_4^e[\mathrm{Tr}(M'M'^\dagger M'M^\dagger) + \mathrm{H.c.}] + e_4^f \mathrm{Tr}(MM^\dagger)\mathrm{Tr}(M'M'^\dagger) + e_4^g \mathrm{Tr}(MM'^\dagger)\mathrm{Tr}(M'M^\dagger) \\
& + e_4^h[(\mathrm{Tr}(M'M'^\dagger))^2 + \mathrm{H.c.}] + e_4^i[\mathrm{Tr}(MM^\dagger)\mathrm{Tr}(MM'^\dagger) + \mathrm{H.c.}] + e_4^j[\mathrm{Tr}(M'M'^\dagger)\mathrm{Tr}(M'M^\dagger) + \mathrm{H.c.}]
\end{aligned}
$$

$$
\begin{aligned}
V_{SB} = & +k_1[\mathrm{Tr}(AM) + \mathrm{H.c.}] + k_2[\mathrm{Tr}(AM') + \mathrm{H.c.}] + k_3[\mathrm{Tr}(AMM^\dagger M) + \mathrm{H.c.}] + k_4[\mathrm{Tr}(AMM'^\dagger M') + \mathrm{H.c.}] \\
& + k_5[\mathrm{Tr}(AMM^\dagger M') + \mathrm{H.c.}] + k_6[\mathrm{Tr}(AMM'^\dagger M) + \mathrm{H.c.}] + k_7[\mathrm{Tr}(AM'M'^\dagger M') + \mathrm{H.c.}] + k_8[\mathrm{Tr}(AM'M^\dagger M) + \mathrm{H.c.}] \\
& + k_9[\mathrm{Tr}(AM'M'^\dagger M) + \mathrm{H.c.}] + k_{10}[\mathrm{Tr}(AM'M^\dagger M') + \mathrm{H.c.}] + k_{11}[\mathrm{Tr}(AM) + \mathrm{H.c.}]\mathrm{Tr}(MM^\dagger) + k_{12}[\mathrm{Tr}(AM) + \mathrm{H.c.}] \\
& \times \mathrm{Tr}(M'M'^\dagger) + k_{13}[\mathrm{Tr}(AM)\mathrm{Tr}(MM'^\dagger) + \mathrm{H.c.}] + k_{14}[\mathrm{Tr}(AM)\mathrm{Tr}(M'M^\dagger) + \mathrm{H.c.}] + k_{15}[\mathrm{Tr}(AM') + \mathrm{H.c.}]\mathrm{Tr}(MM^\dagger) \\
& + k_{16}[\mathrm{Tr}(AM') + \mathrm{H.c.}]\mathrm{Tr}(M'M'^\dagger) + k_{17}[\mathrm{Tr}(AM')\mathrm{Tr}(MM'^\dagger) + \mathrm{H.c.}] + k_{18}[\mathrm{Tr}(AM')\mathrm{Tr}(M'M^\dagger) + \mathrm{H.c.}] \\
& + k_{19} A_a^b \epsilon_{bcd}\epsilon^{aef} M_e^c M_f^d + \mathrm{H.c.} + k_{20} A_a^b \epsilon_{bcd}\epsilon^{aef} M_e'^c M_f'^d + \mathrm{H.c.} + k_{21} A_a^b \epsilon_{bcd}\epsilon^{aef} M_e^c M_f'^d + \mathrm{H.c.}
\end{aligned}
$$

$A \propto \mathrm{diag}\{m_u, m_d, m_s\}$

OF COURSE, NO REASON TO RESTRICT AN "EFFECTIVE" LAGRANGIAN TO BE RENORMALIZABLE.

NEED FOR SIMPLIFICATION IF RESULTS ARE TO BE USEFUL,

SIMPLIFICATION NO. 1 [TEMPORARILY]

PRD 77, 034006 (2008)

$V_{SB} = 0$ [LIGHT QUARK MASSES = 0]

REASONABLE:

IN QCD WE EXPECT (APART FROM N-G BOSONS) PARTICLE MASSES TO MOSTLY ARISE FROM SPONTANEOUS BREAKDOWN OF CHIRAL SYMMETRY (IN THE uds SECTOR.)

ASSUME SU(3) INVARIANT VACUUM.

⇒ 4 2×2 MIXING SECTOR'S [RATHER THAN 4: 2×2's AND 2: 4×4's]:

WE ASSUME THE GROUND STATE HAS $SU(3)_V$ SYMMETRY:

$$\langle S_a^{\,b} \rangle = \alpha\, \delta_a^{\,b}$$

$$\langle S'^{\,b}_a \rangle = \beta\, \delta_a^{\,b}$$

$\alpha \sim$ "$\bar{q}q$ CONDENSATE"

$\beta \sim$ "$\bar{q}\bar{q}qq$ CONDENSATE"

2X2 MIXING SECTORS

$(\hat{\phi}, \hat{\phi}')$, (ϕ_0, ϕ_0'), $(\hat{S}, \hat{S}')$, (S_0, S_0')

$(\hat{\phi}, \hat{\phi}')$	(ϕ_0, ϕ_0')	$(\hat{S}, \hat{S}')$	(S_0, S_0')
DEGENERATE SU(3) OCTET	SU(3) SINGLET	SU(3) OCTET	SU(3) SINGLET
0^-	0^-	0^+	0^+

SIMPLIFICATION 2

$U(1)_A$ TRANSFORMATIONS PLAY A SPECIAL ROLE IN THIS MODEL FOR DISTINGUISHING M AND M'.

IN QCD: SPECIAL INSTANTON INDUCED TERM — "'T HOOFT DETERMINANT" BREAKS $U(1)_A$

ONE MIGHT DEMAND NATURALLY THAT A TERM: $\det M + \det M^\dagger$ BE THE <u>ONLY</u> ONE WHICH BREAKS $U(1)_A$, THEN ALL OTHER TERMS WOULD HAVE THIS SYMMETRY

[ACTUALLY $\delta_{U(1)_A} \mathcal{L} \propto$ (gluon anomaly)

SUGGESTS USING A SIMILAR TERM:

$$c_3 \left[\ln \frac{\det M}{\det M^\dagger} \right]^2 \quad \text{IN} \quad V_0 \;]$$

↓ PARAMETER

WITH BOTH M AND M′ PRESENT, THERE IS EQUAL JUSTIFICATION FOR USING

$$C_3\left[\gamma_1 \ln\left(\frac{\det M}{\det M^\dagger}\right) + (1-\gamma_1)\ln\frac{\mathrm{Tr}(MM'^\dagger)}{\mathrm{Tr}(M'M^\dagger)}\right]^2 ,$$

(γ_1 = DIMENSIONLESS PARAMETER)

FOR SATURATING THE $U(1)_A$ ANOMALY.

EXCEPT FOR GIVING CONSTRAINTS ON THE PARAMETER RANGES THE USE OF THESE "ln" TERMS ALLOWS THE η-SINGLET SECTOR TO DECOUPLE FROM THE OTHERS!

CALCULATIONS (WELCOME SIMPLIFICATION)