Soft and Hard Scale QCD Dynamics in Mesons

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Topics

- Overview of DSE modeling of meson physics—mainly soft scale
  - Masses, decays, form factors
- Including a hard scale:
  - DIS: quark distributions in π, K mesons
  - Mesons involving a heavy quark
- Summary

Lattice-QCD and DSE-based modeling

- Lattice: $\langle \mathcal{O} \rangle = \int \; D\bar{q}qG \;\; \mathcal{O}(\bar{q}, q, G) \;\; e^{-\mathcal{S}[\bar{q},q,G]}$
  - Euclidean metric, x-space, Monte-Carlo
  - Issues: lattice spacing and vol, sea and valence m_q, fermion Det
  - Large time limit $\Rightarrow$ nearest hadronic mass pole
- EOMs (DSEs): $0 = \int \; D\bar{q}qG \;\; \frac{\delta}{\delta q(x)} \; e^{-\mathcal{S}[\bar{q},q,G]+(\bar{\eta},q)+(\bar{q},\eta)+(J,G)}$
  - Euclidean metric, p-space, continuum integral eqns
  - Issues: truncation and phenomenology—not full QCD
  - Analtyic contin. $\Rightarrow$ nearest hadronic mass pole
  - Can be quick to identify systematics, mechanisms, $\cdots$

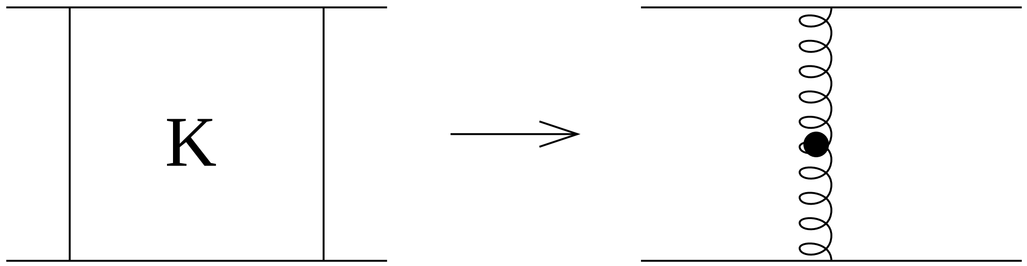
DSE-based modeling of Hadron Physics

- Soft physics: truncate DSEs to min: 2-pt, 3-pt fns
- Should be relativistically covariant—-convenient for decays, Form Factors, etc
  - No boosts needed on wavefns of recoiling bound st.
  - ∞ d.o.f. $\rightarrow$ few quasi-particle effective d.o.f.
- Do not make a 3-dimensional reduction
- Preserve 1-loop QCD renorm group behavior in UV
- Preserve global symmetries, conserved em currents, etc
- Preserve PCAC $\Rightarrow$ Goldstone's Thm
- Can't preserve local color gauge covariance—-just choose Landau gauge [RG fixed pt]
- Parameterize the deep infrared (large distance) QCD coupling

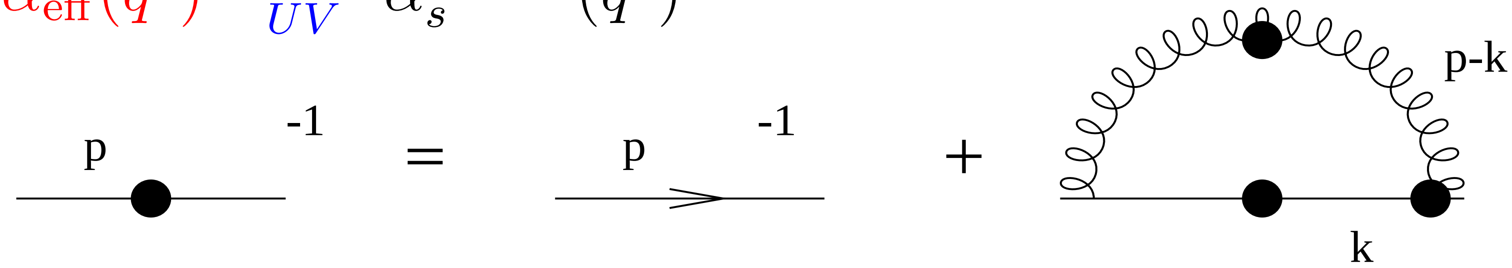
Constraints on Modeling

- Preserve vector WTI, and axial vector WTI
  E.g.
  $$-iP_\mu \Gamma_{5\mu}(k;P) = S^{-1}(k_+)\gamma_5 \frac{\tau}{2} + \gamma_5 \frac{\tau}{2} S^{-1}(k_-) \\ -2\, m_q(\mu)\, \Gamma_5(k;P)$$
- $\Rightarrow$ kernels of DSE$_q$ and $K_{\rm BSE}$ are related
- Ladder-rainbow is the simplest implementation
- Goldstone Theorem preserved, ps octet masses good, indep of model details
- DCSB $\Rightarrow \pi$: $\quad \Gamma^0_\pi(p^2) = \frac{i\gamma_5}{f^0_\pi} \; [\, \frac{1}{4}\, {\rm tr}\, S_0^{-1}(p^2)] \;\; + \cdots$
- Here, 1-body and 2-body systems are the same

Ladder-Rainbow Model



- $K_{\rm BSE} \rightarrow -\gamma_\mu \frac{\lambda^a}{2} \;\; 4\pi\alpha_{\rm eff}(q^2) \;\; D_{\mu\nu}^{\rm free}(q)\; \gamma_\nu \frac{\lambda^a}{2}$
- $\alpha_{\rm eff}(q^2) \;\; \underset{IR}{\rightarrow} \;\; \langle \bar{q}q \rangle_{\mu=1\ \rm GeV} = -(240\text{MeV})^3$, incl vertex dressing
- $\alpha_{\rm eff}(q^2) \;\; \underset{UV}{\rightarrow} \;\; \alpha_s^{1-\rm loop}(q^2)$



- P. Maris & P.C. Tandy, PRC60, 055214 (1999)
  $M_\rho, M_\phi, M_{K^\star}$ good to 5%, $\quad f_\rho, f_\phi, f_{K^\star}$ good to 10%

Summary of light meson results

$m_{u=d} = 5.5$ MeV, $m_s = 125$ MeV at $\mu = 1$ GeV

Pseudoscalar (PM, Roberts, PRC56, 3369)

	expt.	calc.
$-\langle \bar{q}q \rangle^0_\mu$	$(0.236\ \text{GeV})^3$	$(0.241^\dagger)^3$
m_π	0.1385 GeV	$0.138^\dagger$
f_π	0.0924 GeV	$0.093^\dagger$
m_K	0.496 GeV	$0.497^\dagger$
f_K	0.113 GeV	0.109

Charge radii (PM, Tandy, PRC62, 055204)

r^2_π	0.44 fm^2	0.45
$r^2_{K^+}$	0.34 fm^2	0.38
$r^2_{K^0}$	-0.054 fm^2	-0.086

$\gamma\pi\gamma$ transition (PM, Tandy, PRC65, 045211)

$g_{\pi\gamma\gamma}$	0.50	0.50
$r^2_{\pi\gamma\gamma}$	0.42 fm^2	0.41

Weak K_{l3} decay (PM, Ji, PRD64, 014032)

$\lambda_+(e3)$	0.028	0.027
$\Gamma(K_{e3})$	$7.6 \cdot 10^6\ \text{s}^{-1}$	7.38
$\Gamma(K_{\mu 3})$	$5.2 \cdot 10^6\ \text{s}^{-1}$	4.90

bsampl

Vector mesons (PM, Tandy, PRC60, 055214)

$m_{\rho/\omega}$	0.770 GeV	0.742
$f_{\rho/\omega}$	0.216 GeV	0.207
$m_{K^\star}$	0.892 GeV	0.936
$f_{K^\star}$	0.225 GeV	0.241
m_ϕ	1.020 GeV	1.072
f_ϕ	0.236 GeV	0.259

Strong decay (Jarecke, PM, Tandy, PRC67, 035202)

$g_{\rho\pi\pi}$	6.02	5.4
$g_{\phi KK}$	4.64	4.3
$g_{K^\star K\pi}$	4.60	4.1

Radiative decay (PM, nucl-th/0112022)

$g_{\rho\pi\gamma}/m_\rho$	0.74	0.69
$g_{\omega\pi\gamma}/m_\omega$	2.31	2.07
$(g_{K^\star K\gamma}/m_K)^+$	0.83	0.99
$(g_{K^\star K\gamma}/m_K)^0$	1.28	1.19

Scattering length (PM, Cotanch, PRD66, 116010)

a^0_0	0.220	0.170
a^2_0	0.044	0.045
a^1_1	0.038	0.036

DSE kernel constrained from Lattice QCD

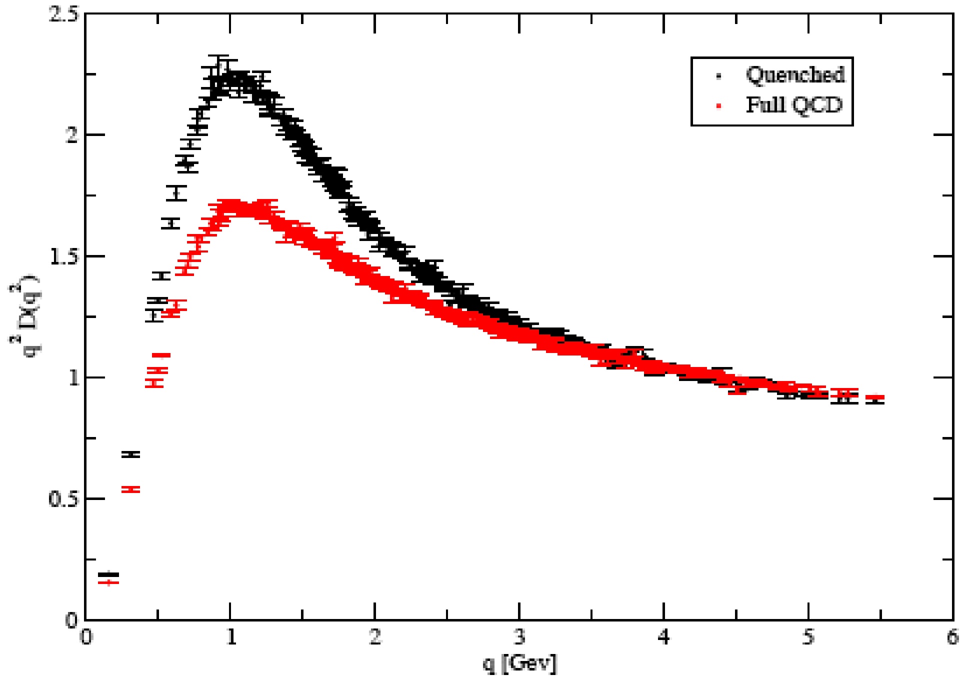
— Bhagwat, Pichowsky, Roberts, Tandy, PRC68, 015203 (03)

- Qu-lattice $D_{\rm gluon}(q)$

  Leinweber, Bowman et al

  PRD60, hep-lat/9811027

- Find $\Gamma_\nu^{\rm eff}(q,p)$ so DSE produces $S_{\rm latt}(p)$ from $D_{\rm latt}(q)$

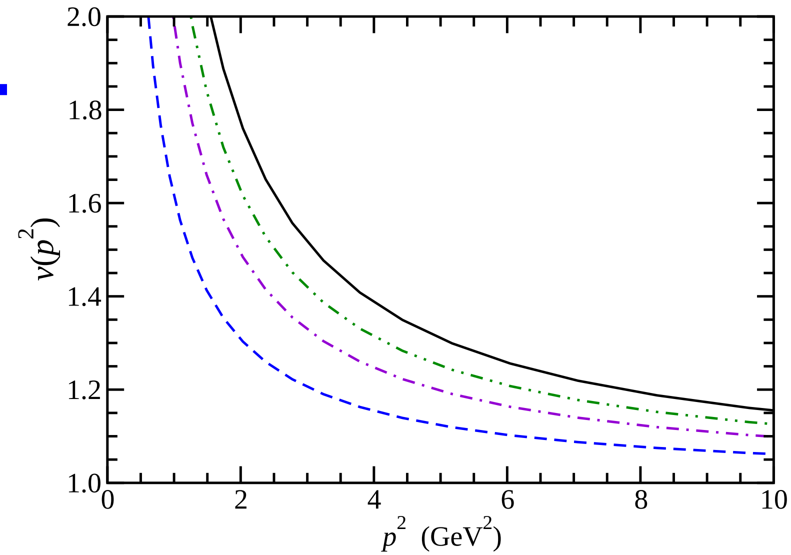


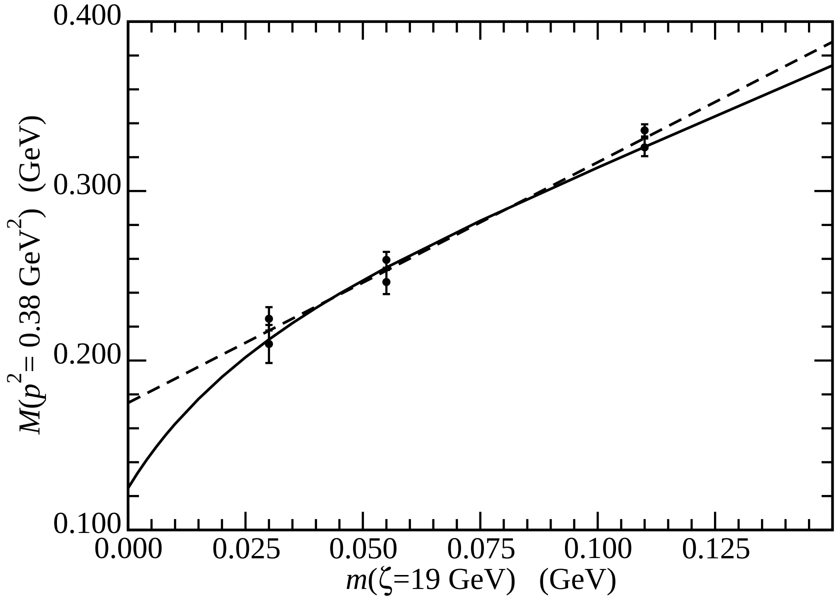
$$g^2\gamma_\mu D(p-q)\, Z_{1F}(\mu,\Lambda)\,\Gamma_\nu(q,p) \rightarrow \gamma_\mu g^2 D(p-q)\gamma_\nu\, V(p-q)$$

UV limit: $$g^2\, D(k^2)\, V(k^2) \rightarrow \frac{4\pi\alpha_s^{1-\rm loop}(k^2)}{k^2}$$

Lattice-assisted DSE Results

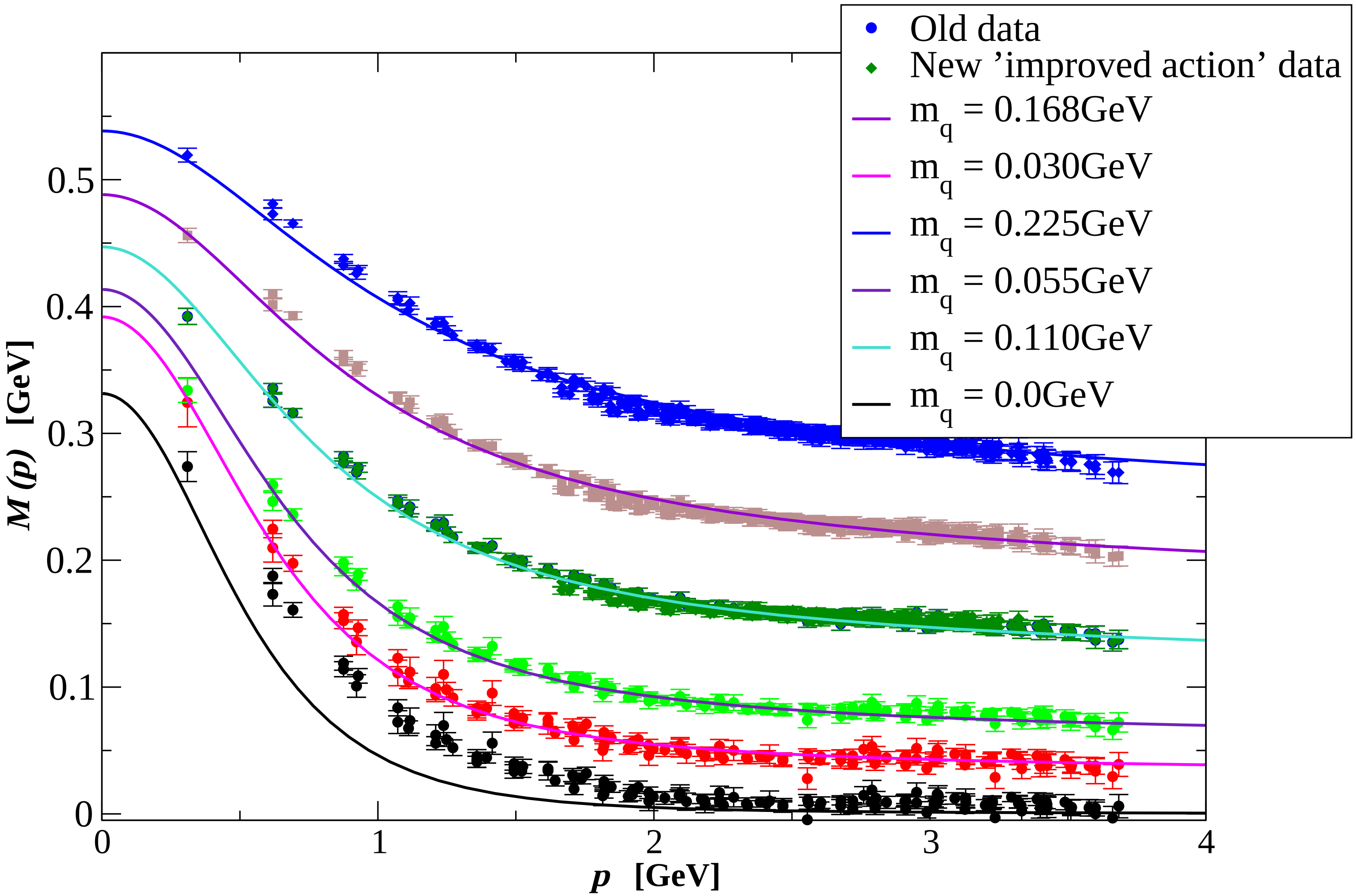
- Evident vertex enhancement
- Curvature in low m_q depn
- $M^{\rm IR}(p^2)$ 40% below linear
- Chiral Extrapolation
- $\langle \bar q q \rangle^{\rm qu-lat}_{\mu=1\ {\rm GeV}} = -(190\ {\rm MeV})^3$
- $\langle \bar q q \rangle^{\rm qu-lat} \approx \langle \bar q q \rangle^{\rm expt}/2$
- f_π 30% low



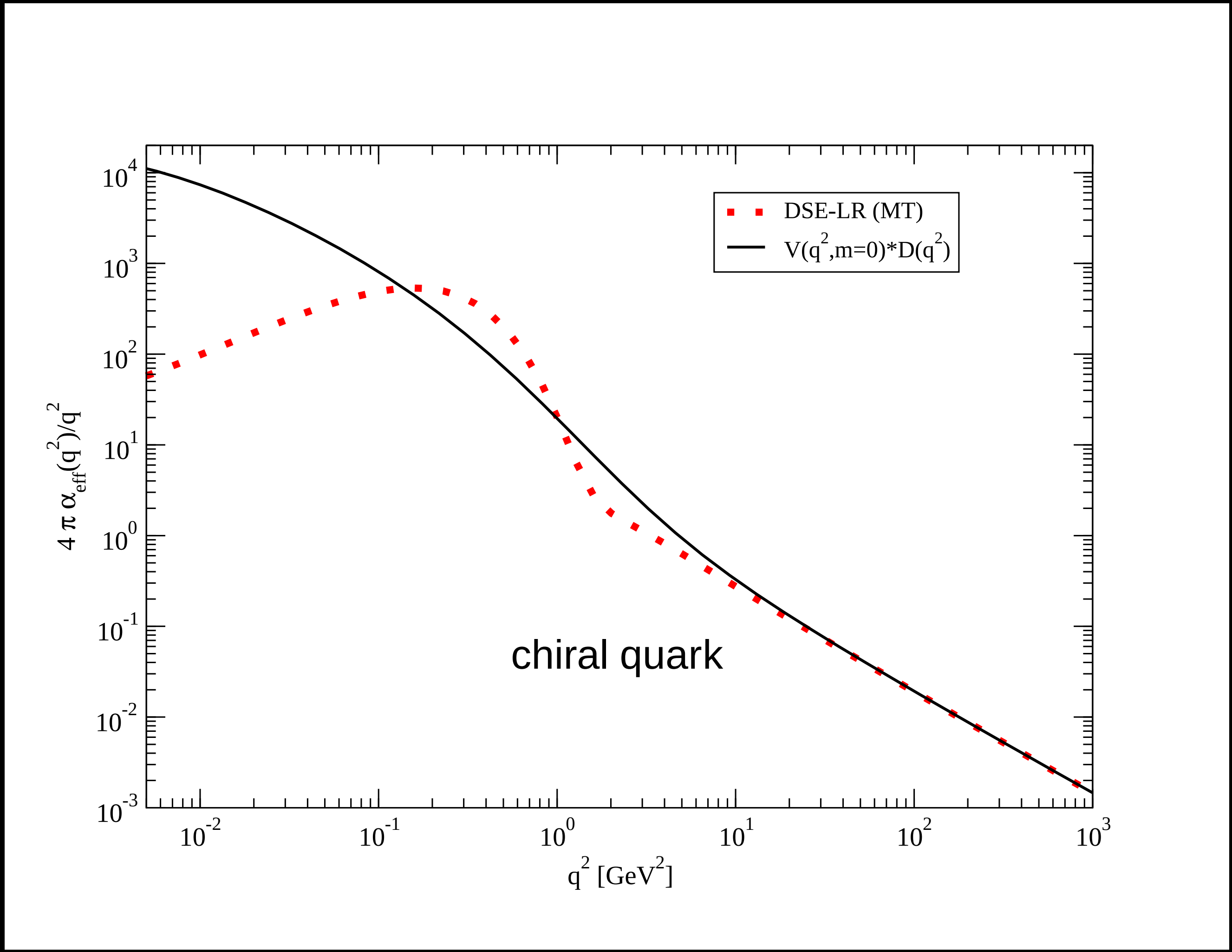


Qu-lattice $S(p), D(q)$ mapped to a DSE kernel

$$S(p) = Z(p)\,[i\, \not p + M(p)]^{-1}$$

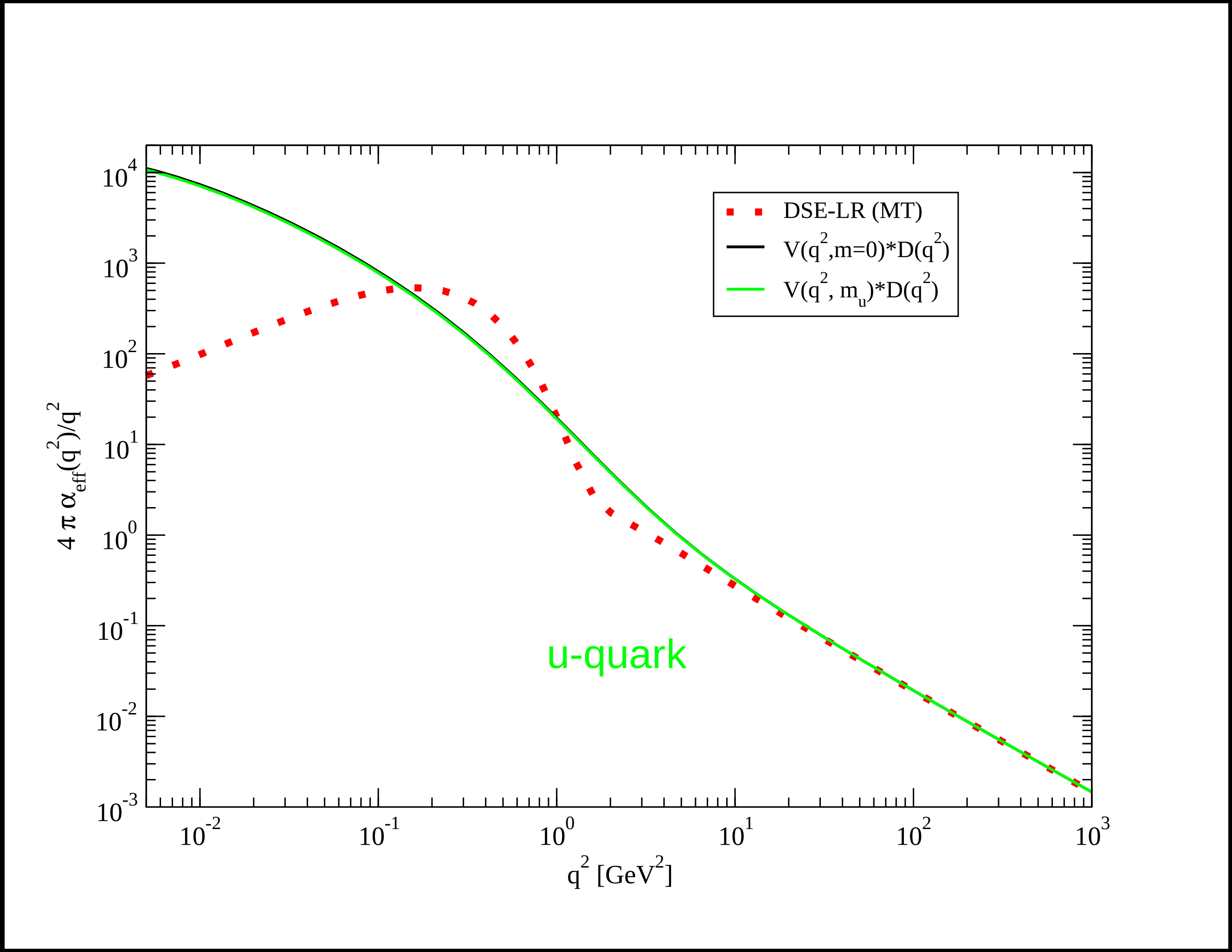


Quenched lattice $\Rightarrow m_q$ *Depn of DSE Kernel*



Bhagwat,Pichowsky,Roberts,Tandy, PRC68, 015203 (2003)

Quenched lattice $\Rightarrow m_q$ ***Depn of DSE Kernel***



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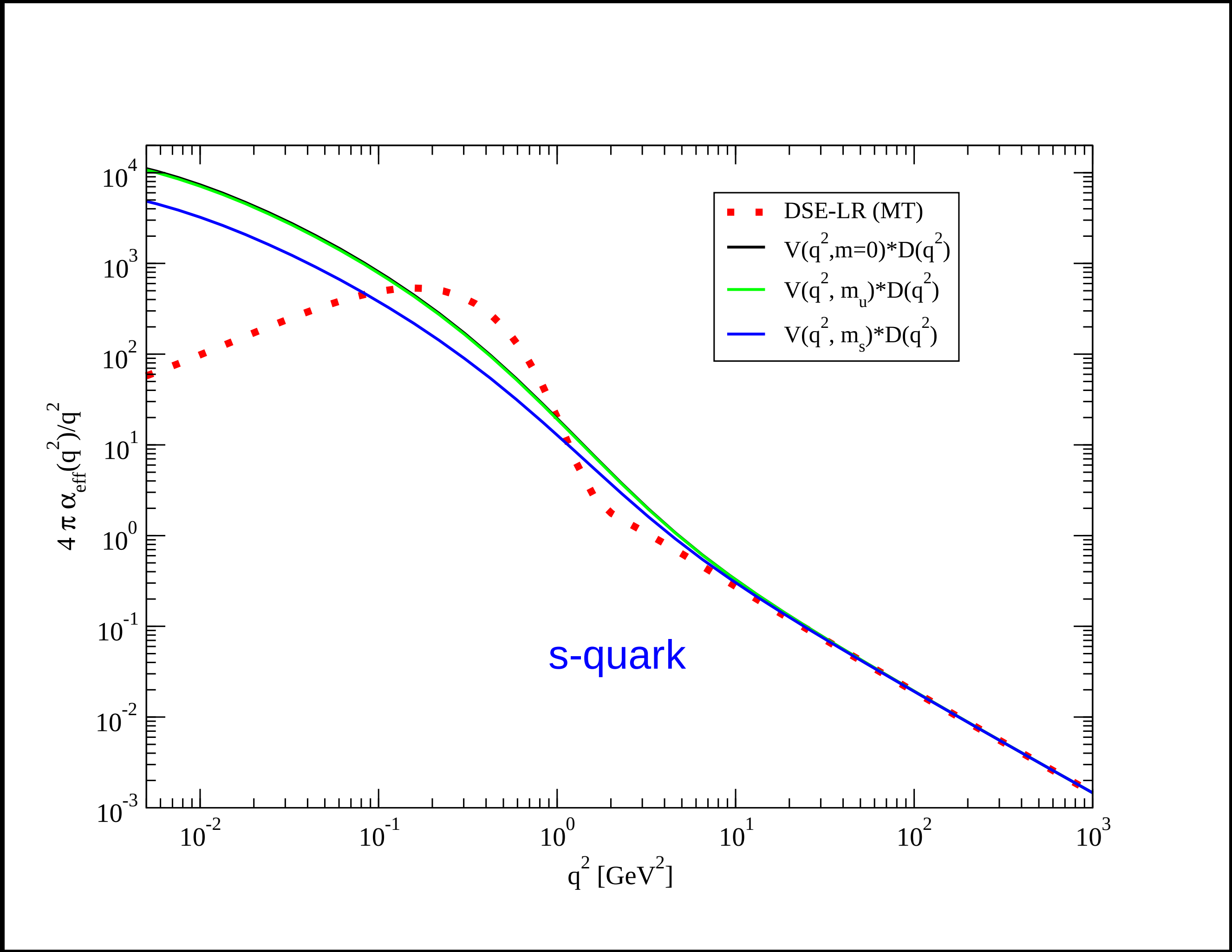
Quenched lattice ⇒ m_q Depn of DSE Kernel



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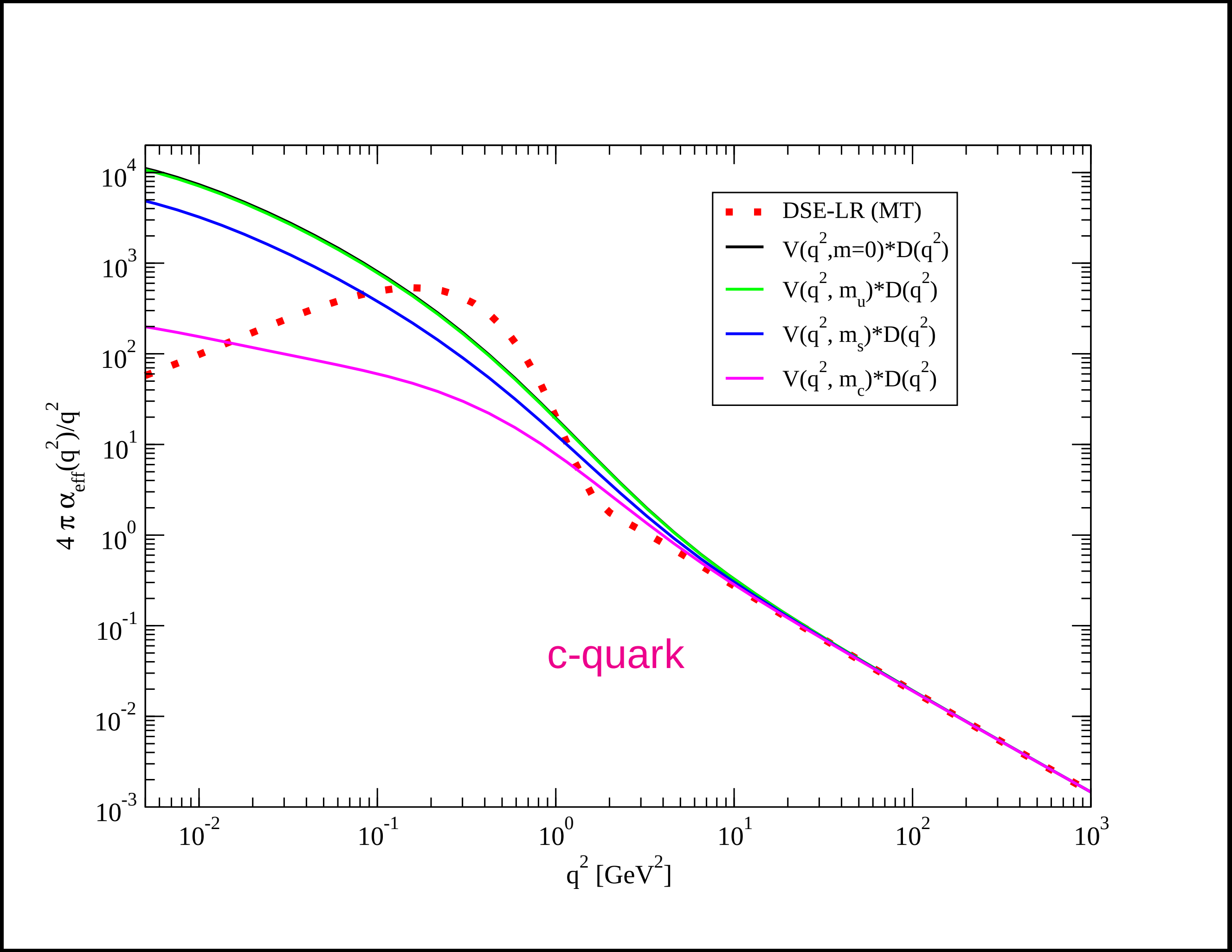

Bhagwat,Pichowsky,Roberts,Tandy, PRC68, 015203 (2003)

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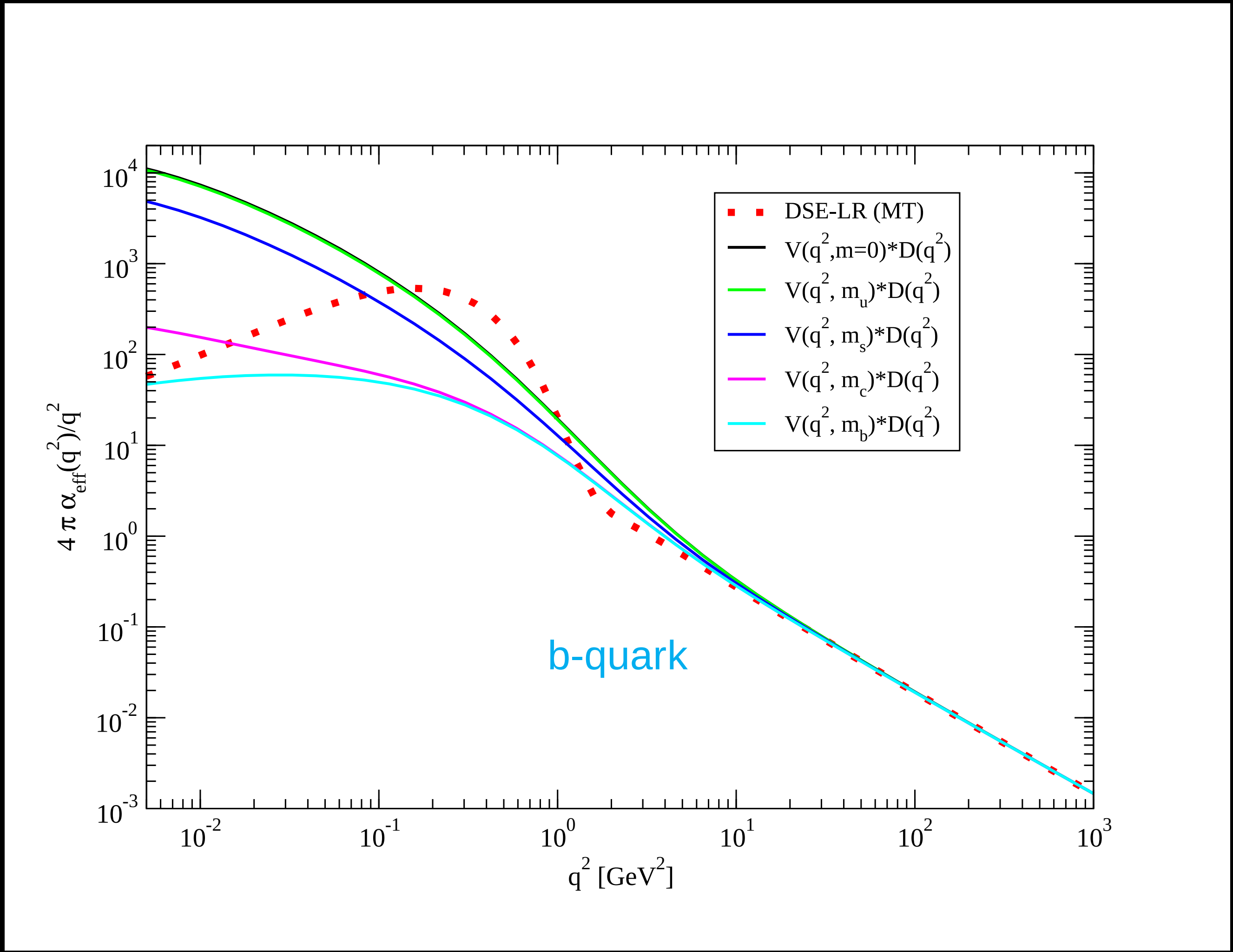
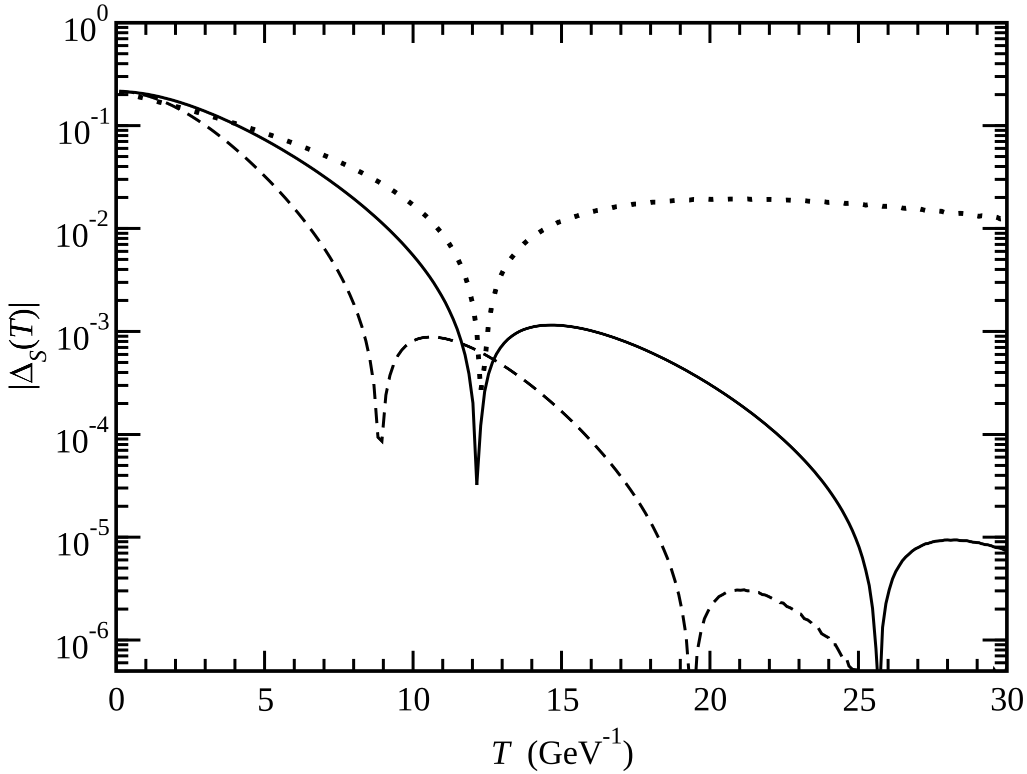


Bhagwat,Pichowsky,Roberts,Tandy, PRC68, 015203 (2003)

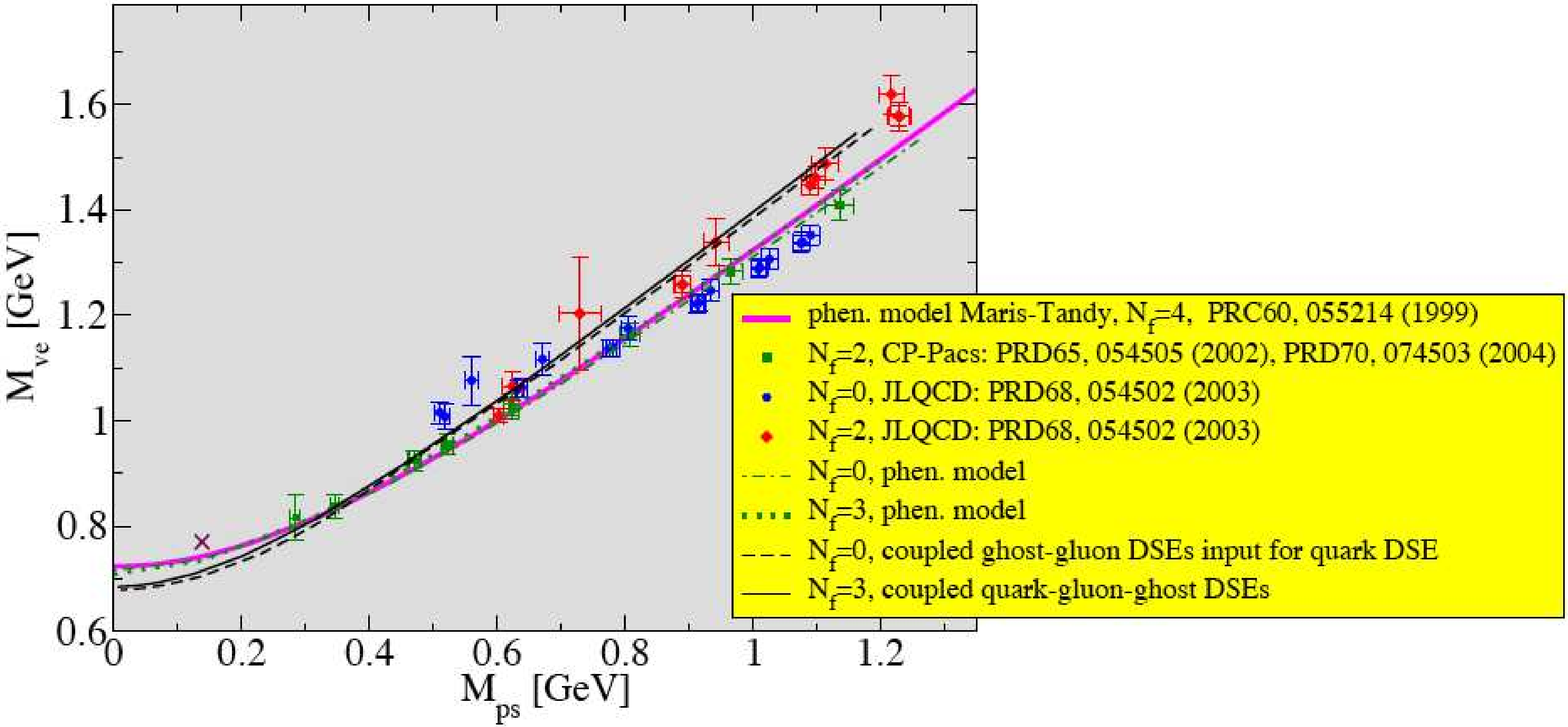
Quark Confinement—positivity violation

- Confinement/positivity analysis (Osterwalder-Schrader axiom No. 3)
- Fourier transf $\sigma_S(p_4, \vec{p} = 0)$ to Eucl time T



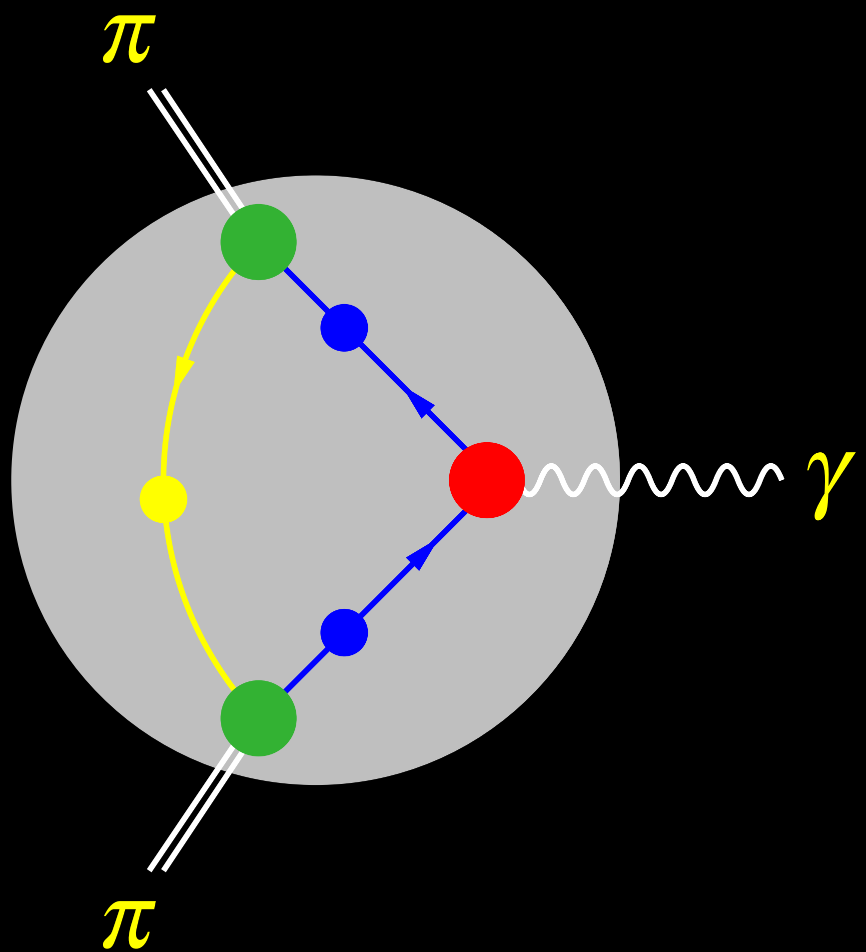
solid = lattice prop, dashed = MT DSE, dotted = cc pole eg

DSE and Lattice results for M_V and M_{ps}



Pion electromagnetic form factor

$$\Lambda_\mu \;=\; (P' + P)_\mu \, F_\pi(Q^2) \quad = \quad N_c \int \frac{d^4q}{(2\pi)^4} \,\mathrm{Tr}\big[\bar{\Gamma}^\pi \, S \, i\Gamma_\mu \, S \, \Gamma^\pi \, S\big]$$

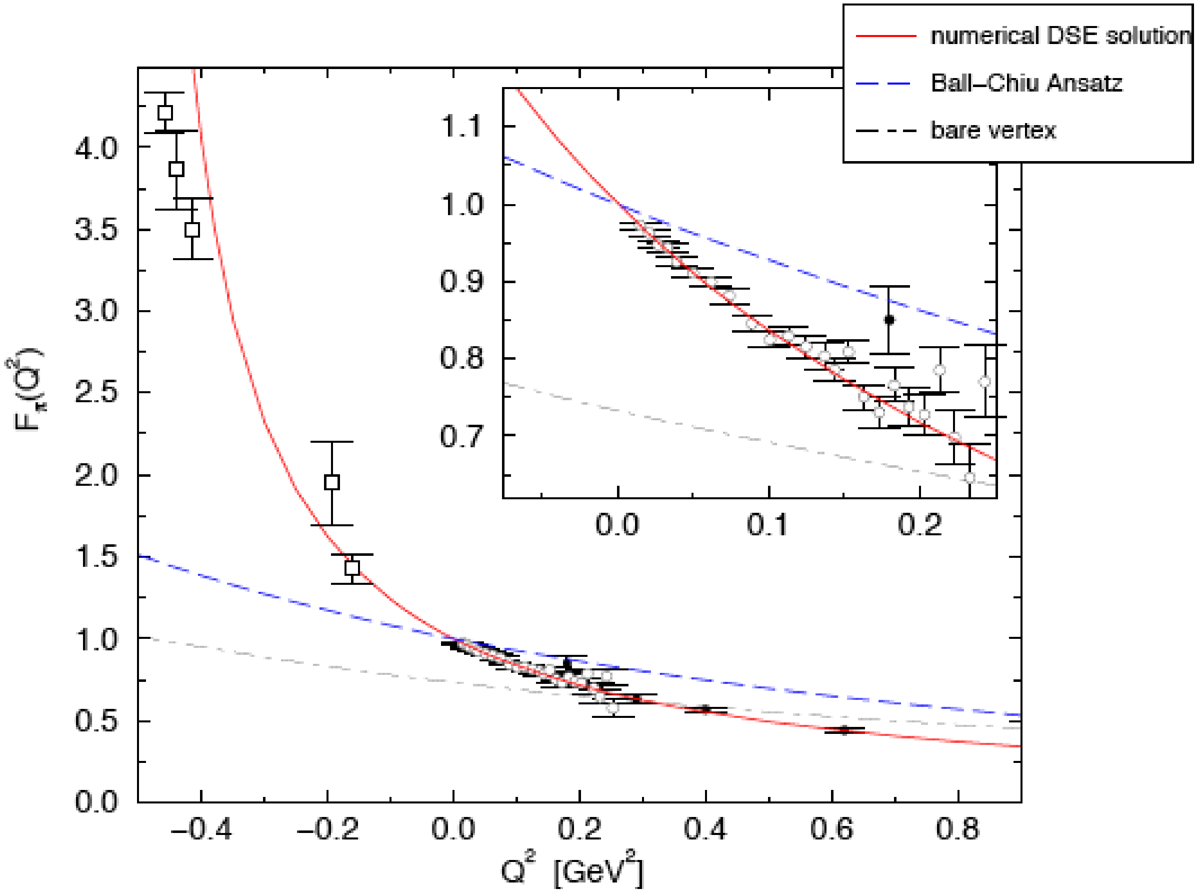


Pion $F(Q^2)$: Low Q^2

(P Maris & PCT, PRC 61, 045202 (2000)

(P. Maris & PCT, PRC 62, 0555204 (2000)

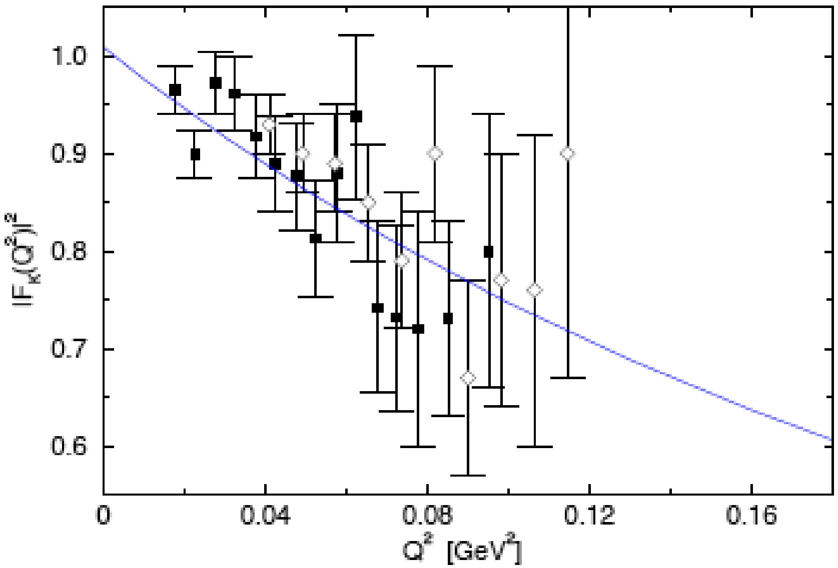
$r_\pi^{\rm DSE} = 0.68$ fm $\qquad r_\pi^{\rm expt} = 0.663 \pm .006$ fm



Kaon $F(Q^2)$: Low Q^2

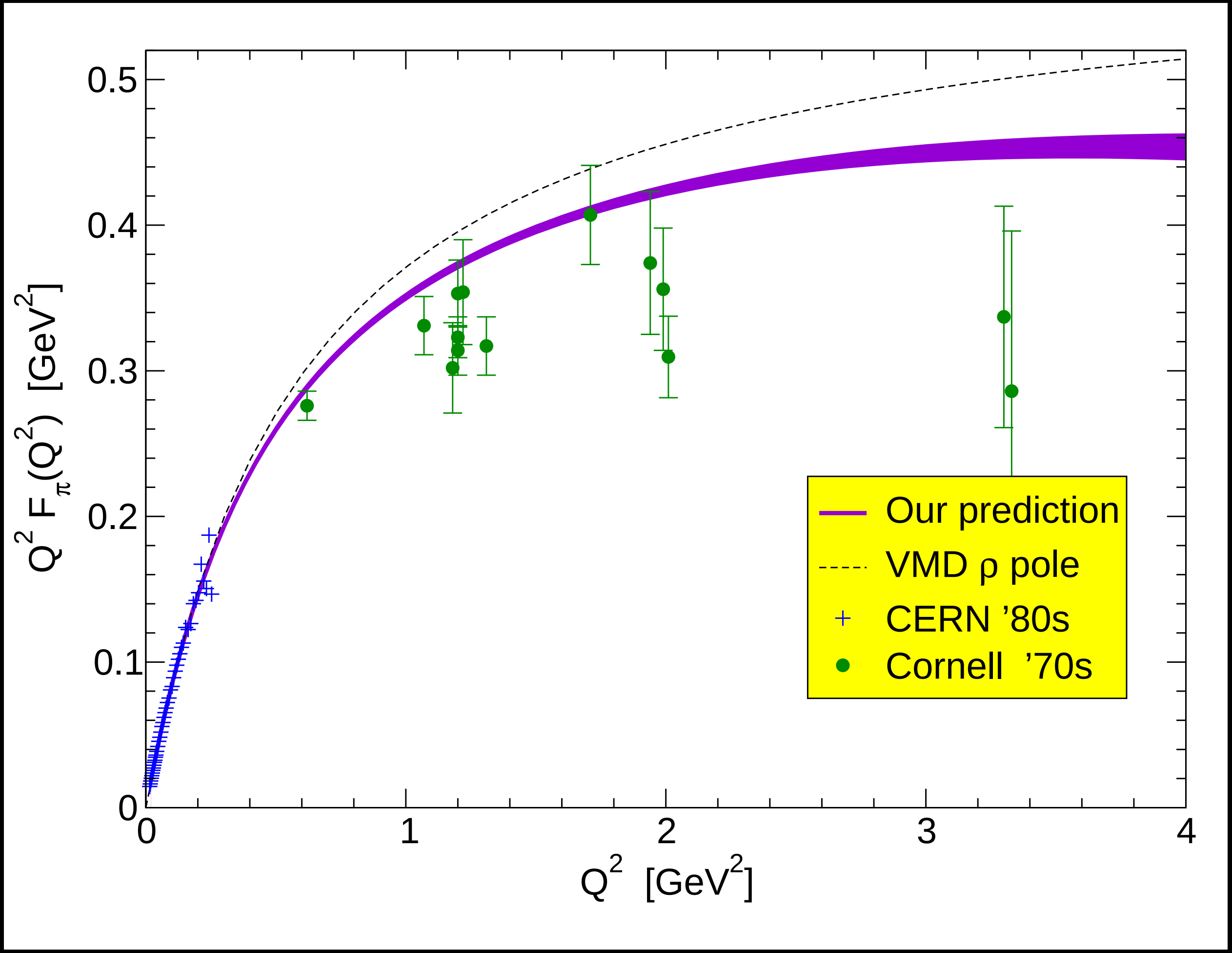
- Impulse approx + rainbow/ladder $\Rightarrow$

  conserved em current, correct charge of K^+ and K^0



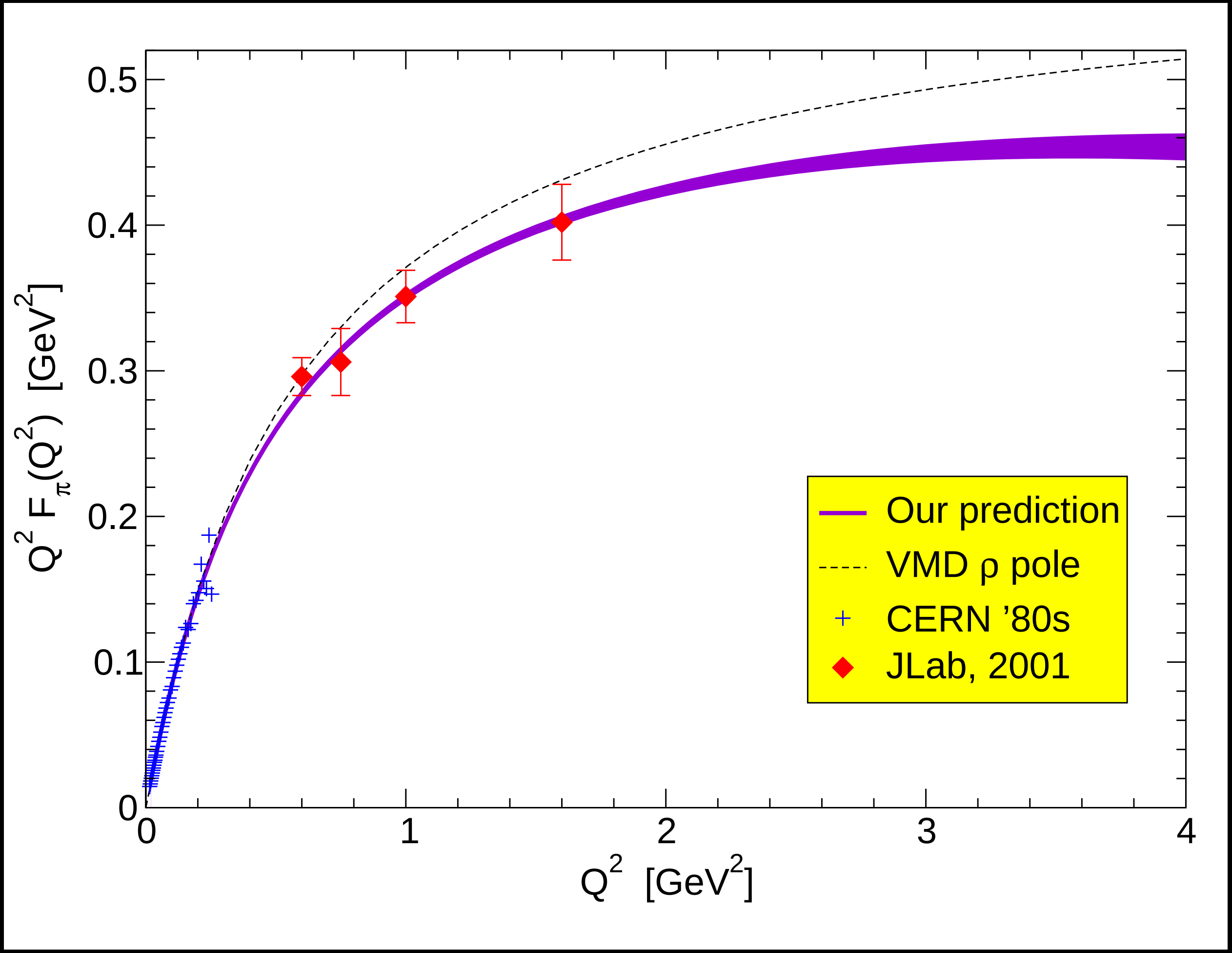
charge radii	experiment	DSE calc
r_π^2	$0.44 \pm 0.01\ \mathrm{fm}^2$	$0.45\ \mathrm{fm}^2$
r_{K+}^2	$0.34 \pm 0.05\ \mathrm{fm}^2$	$0.38\ \mathrm{fm}^2$
$r_{K^0}^2$	$-0.054 \pm 0.026\ \mathrm{fm}^2$	$-0.086\ \mathrm{fm}^2$

Pion electromagnetic form factor



PM and Tandy, PRC62,055204 (2000) [nucl-th/0005015]

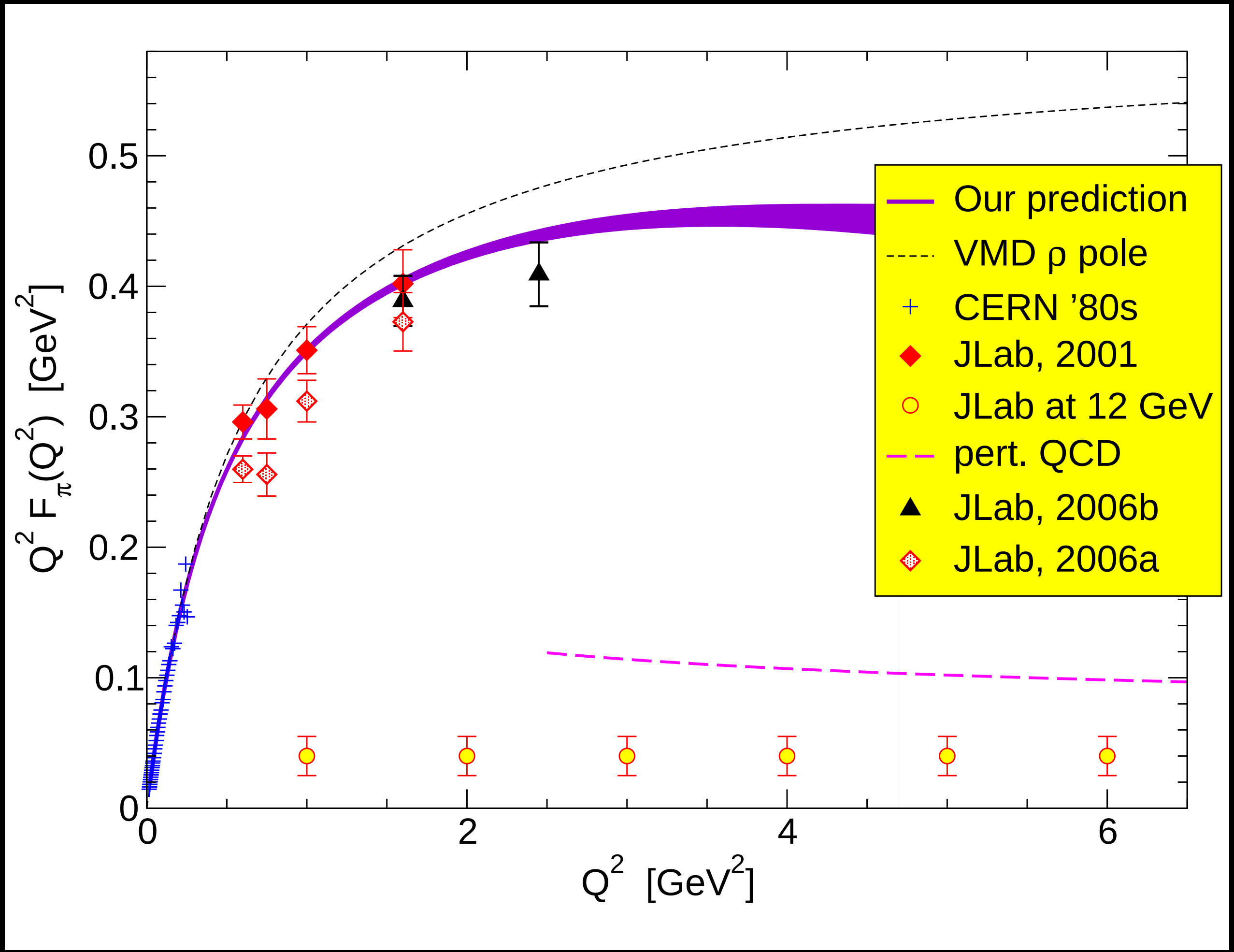
Pion electromagnetic form factor



JLab data from Volmer *et al*, PRL86, 1713 (2001) [nucl-ex/0010009]

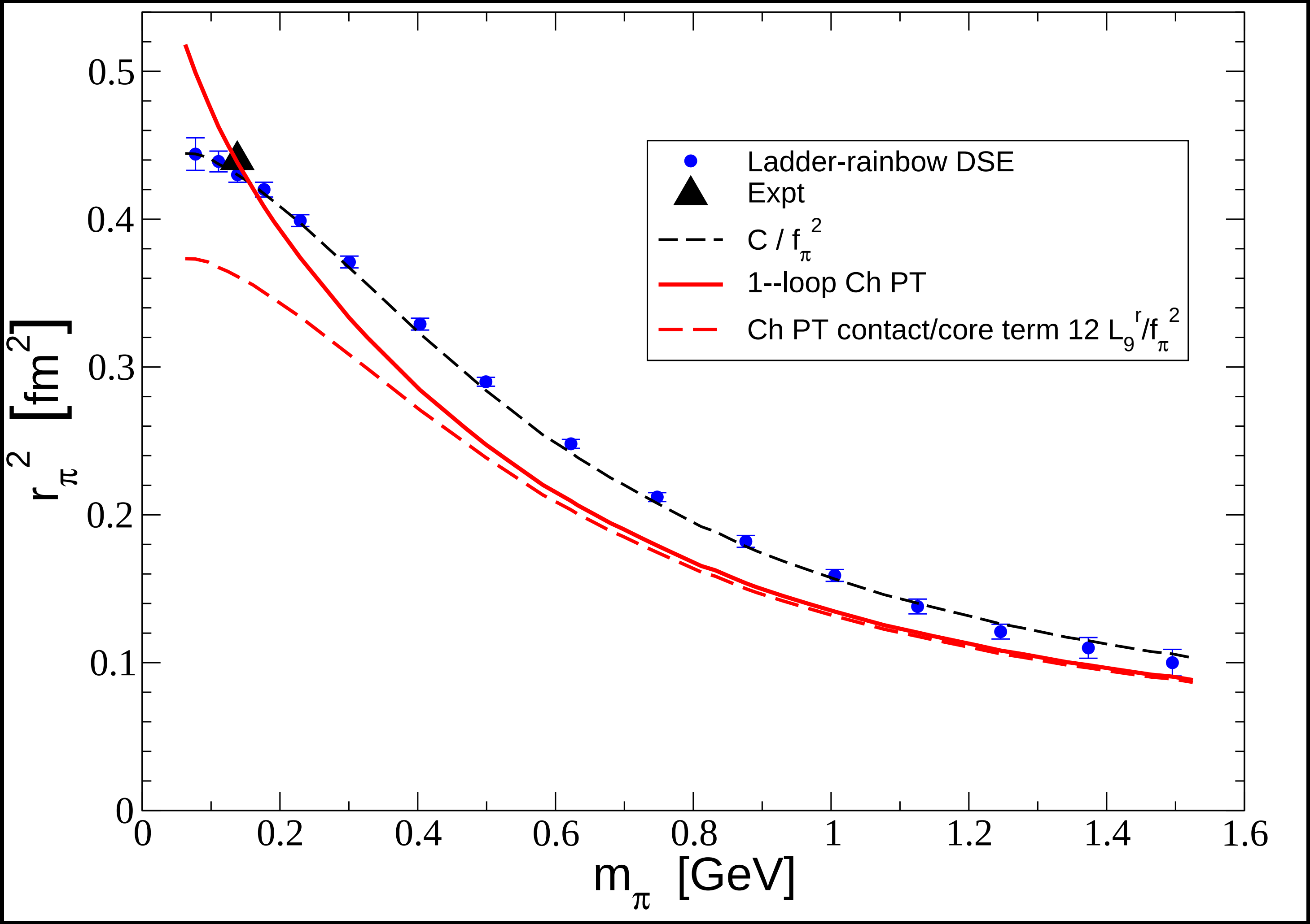
PM and Tandy, PRC62,055204 (2000) [nucl-th/0005015]

Pion electromagnetic form factor



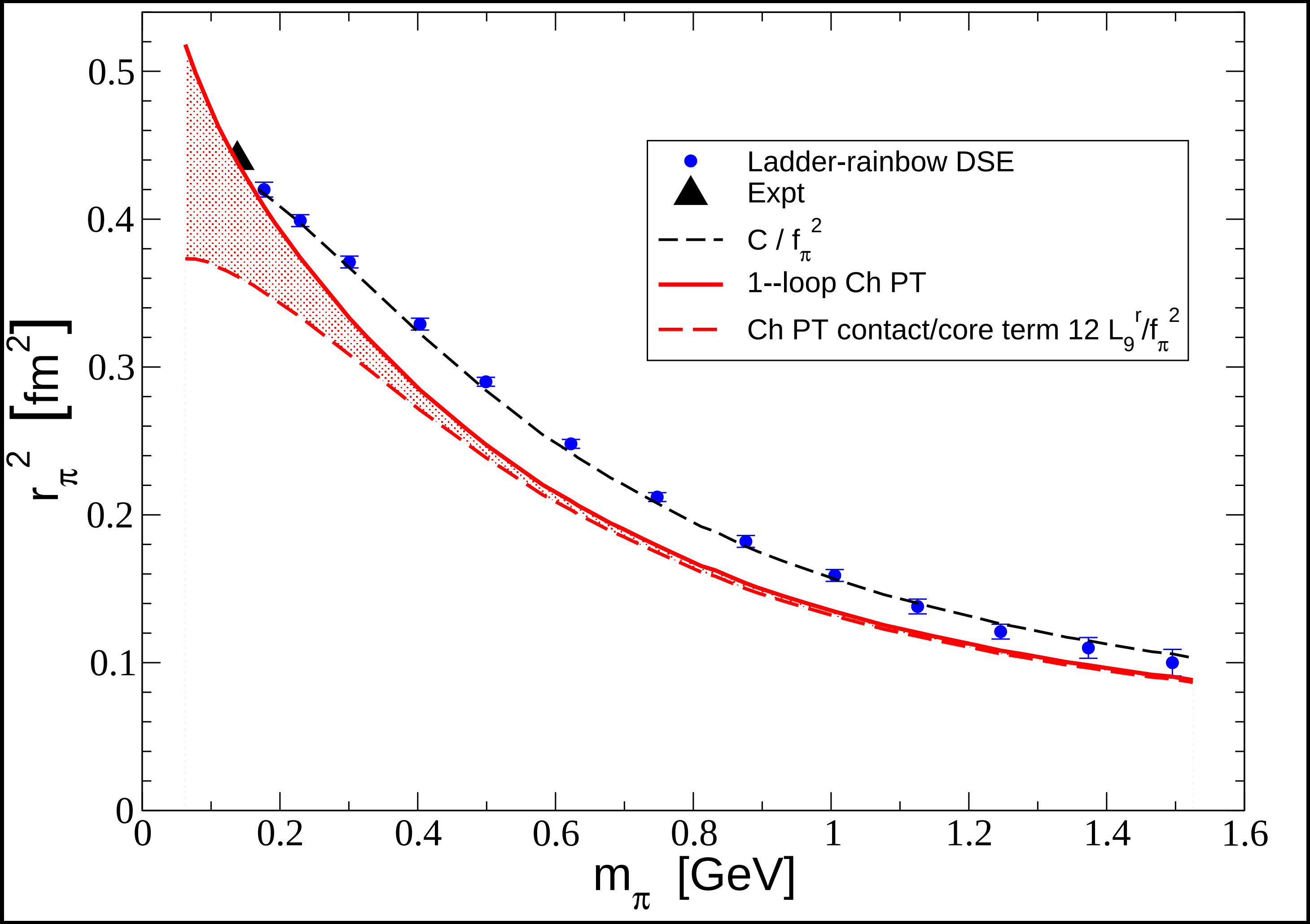
PM and Tandy, PRC62,055204 (2000) [nucl-th/0005015]
2006a: V. Tadevosyan *et al*, [nucl-ex/0607007], 2006b: T. Horn *et al*, [nucl-ex/0607005]

1-loop chiral correction to r_π vs m_π



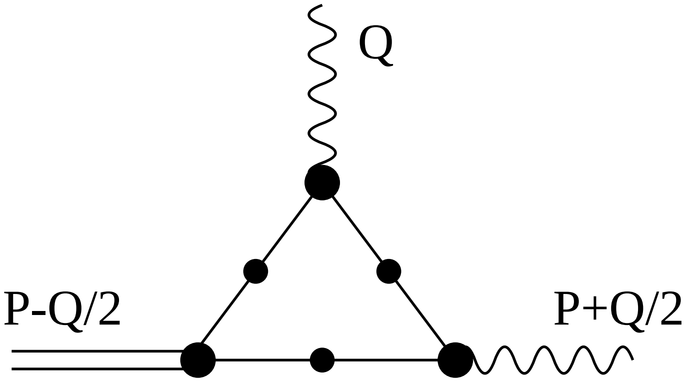
P. Maris and PCT, in preparation

1-loop chiral correction to r_π vs m_π

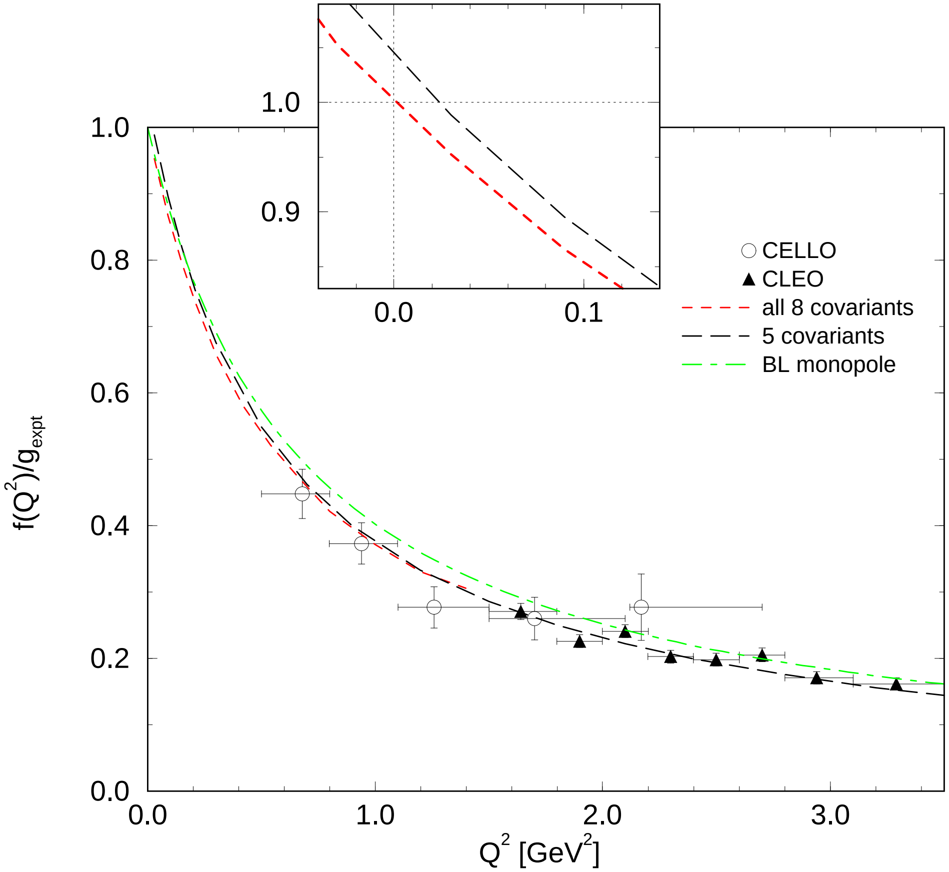


P. Maris and PCT, in preparation

$\gamma^{\star}\pi^{0} \rightarrow \gamma$ *Transition Form Factor*

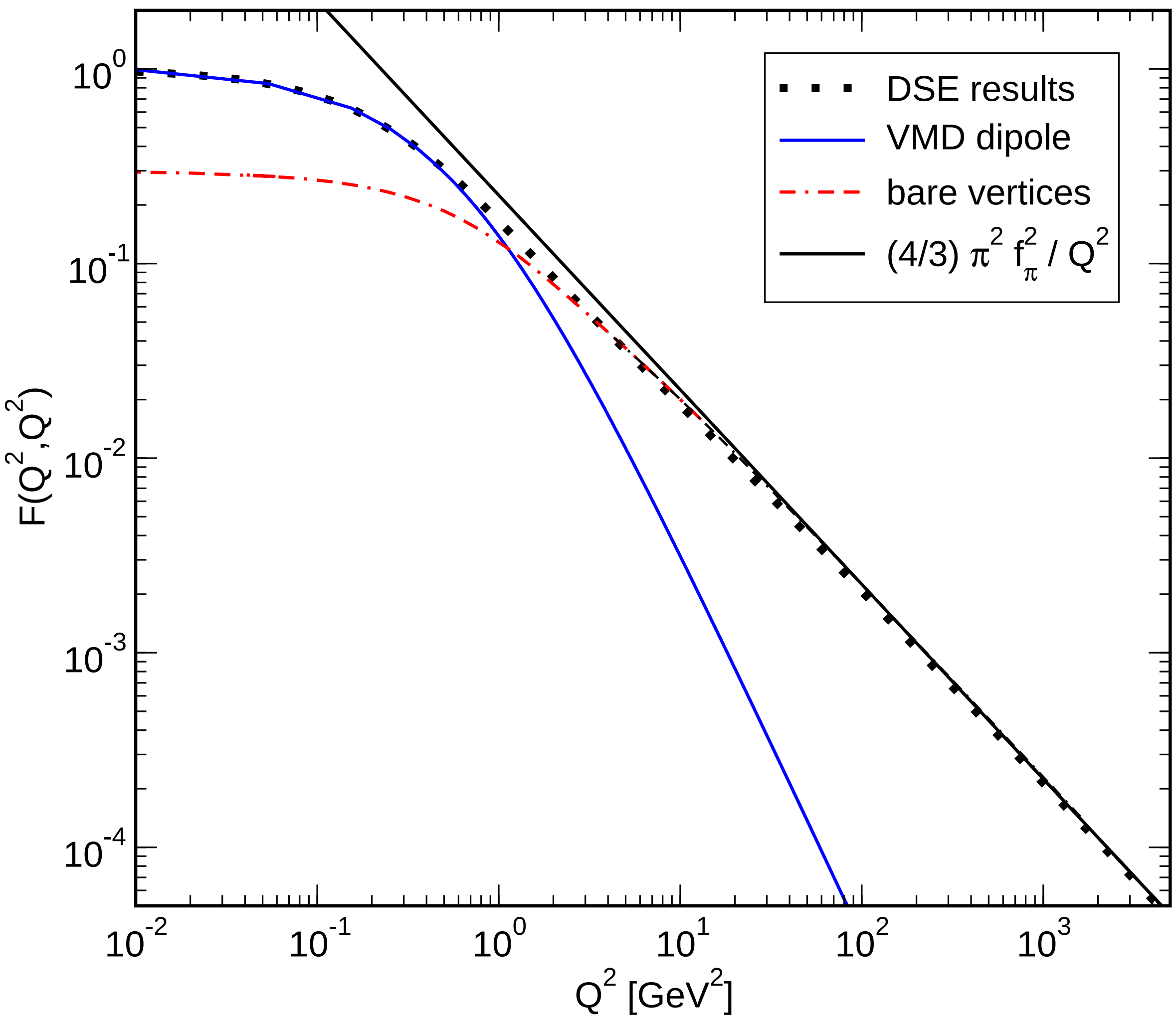


- Abelian axial anomaly $+\ \pi$ pole in $\Gamma_{5\mu} \Rightarrow G(0,0)$
- Chiral limit $G(0,0) = \dfrac{1}{2}$ $\Rightarrow \Gamma_{\pi\gamma\gamma}$ to 2%



$\gamma^\star \pi \gamma^\star$ *Asymptotic Limit*

Lepage and Brodsky, PRD22, 2157 (1980): LC-QCD/OPE $\Rightarrow$



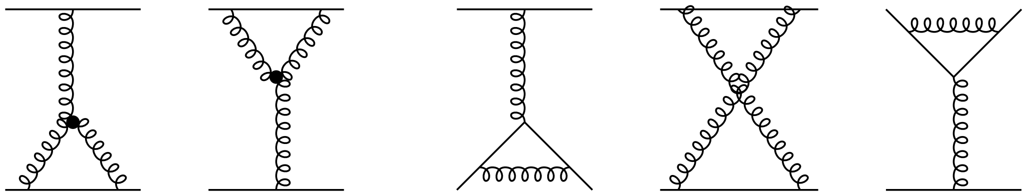
LR: Successes, Problems, Resolutions

- Successes:
  - S-wave mesons, PS and V, light quarks and QQ, no spurious thresholds
  - Exact PS mass formula, Goldstone Thm, ΔM_{HF} from DCSB
  - f_{EW}, strong decays, radiative decays, form factors, $Q^2 < 5GeV^2$
- Problems:
  - Axial vector ($L > 0$) mesons ($a_1, b_1, \cdots$) too light
  - Physical diquarks, no physical V or PS qQ states
  - Excited states are difficult
- Probable Resolution:
  - Quark-gluon vertex: $\Gamma_\mu \Rightarrow \Sigma_q \Rightarrow K_{BSE}$
  - Use analysis of spacelike correlators, 3-pt functions

From Gluon vertex to BSE Kernel

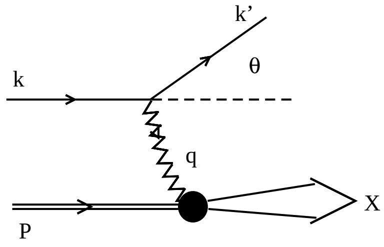
- A symmetry-preserving procedure [Bender, Roberts, von Smekal, PLB380, (1996), nucl-th/9602012; Munczek 1995] ; Axial vector and vector WTIs, and Goldstone Thm preserved
- $K_{\rm BSE}(x', y'; x, y) = -\frac{\delta}{\delta S(x,y)} \Sigma(x', y')$
- Vertex $\Gamma_\mu(p, q) = \sum$ diagrams $\Rightarrow K_{\rm BSE} = \sum$ diagrams
- If Σ contains:

- $K_{\rm BSE}$ contains:



- Independent of model parameters. Model does not fight chiral symmetry, use light vector mesons to fix parameters

Deep Inelastic Lepton Scattering

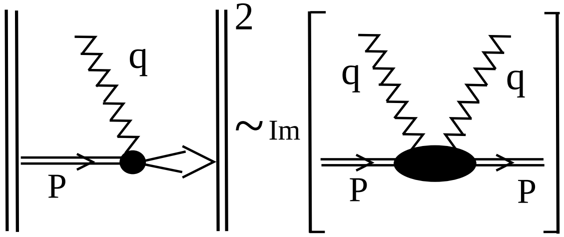


Bjorken limit:

$$\nu = q \cdot P/M \to \infty \quad ; \quad -q^2 = Q^2 \to \infty$$

$$0 < x = \frac{Q^2}{2P\cdot q} < 1$$

$W^{\alpha\beta} =$



$= \frac{1}{2\pi}\,\mathrm{Disc}\, T^{\alpha\beta}(\nu)$

$$W^{\alpha\beta} = -(g^{\alpha\beta} - \frac{q^\alpha q^\beta}{q^2})\, F_1 + \frac{P_T^\alpha(q)\, P_T^\beta(q)}{P\cdot q}\, F_2$$

$$F_1(x) = \Sigma_q \frac{e_q^2}{2} f_q(x) + \cdots$$

Deep Inelastic Lepton Scattering

Convenient basis in Bj lim:

$n^\nu = \frac{M}{2\omega}(1,\ -1;\ \vec{0}_\perp)\ ;\ \ n^2 = 0 = p^2\ ;\ \ p\cdot n = 2\ .;\ \ \omega = M/2$ (rest frame) , $\omega = \infty$ (IMF)

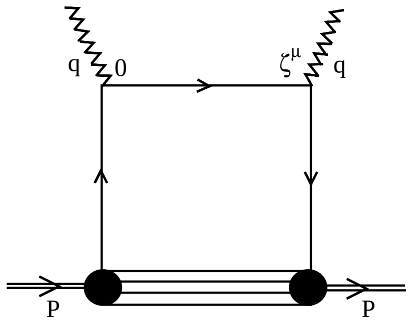
$$P^\mu = \frac{M}{2}(n^\mu + p^\mu)\ ;\quad q^\mu \to \nu\, n^\mu + \frac{Mx}{2}(n^\mu - p^\mu) + \mathcal{O}(\tfrac{1}{\nu})$$

$$W^{\alpha\beta} \to (a\,\nu + b)\,(F_2 - 2x\,F_1) + (-g^{\alpha\beta} + n^\alpha \frac{P^\beta}{M} + \frac{P^\alpha}{M} n^\beta)\,F_1 + \mathcal{O}(\tfrac{1}{\nu})$$

$$\{W^{\alpha\beta}\,q_\beta\}_{LO} = 0 = W^{\alpha\beta}\,n_\beta$$

handbag diagram $\Rightarrow W^{\alpha\beta}_{HB}\,n_\beta = 0$, (LO current consv)

Deep Inelastic Lepton Scattering

$T^{\mu\nu}(\mathrm{LO}) = T^{\mu\nu}_{GHB} =$ 

$q^+ = q\cdot n = -Mx, \ |\xi^-| \sim \frac{1}{Mx}$

$q^- = q\cdot p = 2\nu, \ |\xi^+| \sim 0$

DIS is hard and fast—confinement is soft and slow $\Rightarrow S(k+q) \to \frac{\gamma^+}{2\,(k^+ - P^+ x) + i\epsilon}$

$W^{\mu\nu} \propto \{T^{\mu\nu}(\epsilon) - T^{\mu\nu}(-\epsilon)\} \Rightarrow$ Euclidean model elements can be continued

$$\mathrm{EG}, \pi^+ \mathrm{target}: \ f_q(x) = \frac{1}{4\pi}\int d\xi^- e^{iq^+\xi^-} \langle \pi(P)|\bar{q}(\xi^-)\gamma^+ q(0)|\pi(P)\rangle_c = -f_{\bar{q}}(-x)$$

$$f_q(x) = \frac{1}{2}\mathrm{tr}\int \frac{d^4k}{(2\pi)^4}\,\delta(k^+ - P^+ x)\; S(k)\gamma^+ S(k)\; T(k,P)$$

General $T(k,P) = \bar{u}\pi^+$ scattering amplitude:

s-channel structure $\to$ "spectator $\bar{d}$" $\Rightarrow f_u(x), \ \ 0 < x < 1$ — correct x

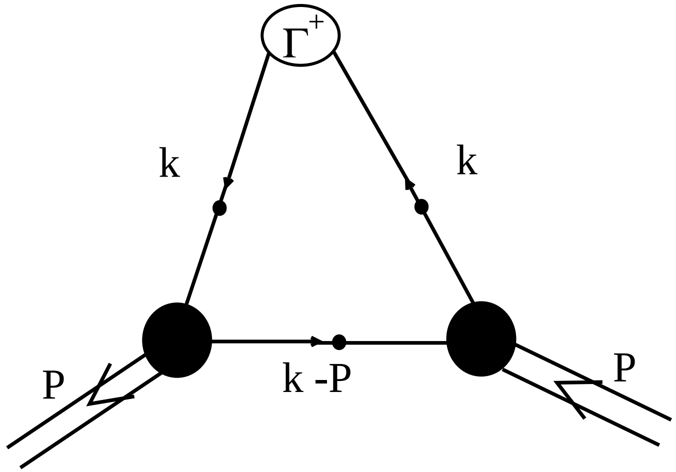
u-channel structure $\to$ "spectator $uu\bar{d}$" $\Rightarrow f_{\bar{u}}(-x), \ \ 0 < x < 1$ — support

Deep Inelastic Lepton Scattering

Quark number sum: $N_q^V = \int_0^1 dx \, \{f_q(x) - f_{\bar{q}}(x)\} = \frac{1}{2P^+} \langle \pi(P) | J^+(0) | \pi(P) \rangle = 1$

DSE calculation: $u_\pi(x), \;\; u_K(x), \;\; s_K(x)$ [T. Nguyen, PCT, (2009)]

- BSE $q\bar{q}$ solutions for π, K
- DSE solns for dressed quark $S(k)$
- Constituent mass approx for spectator propagator
- Vertex approx via Ward Id

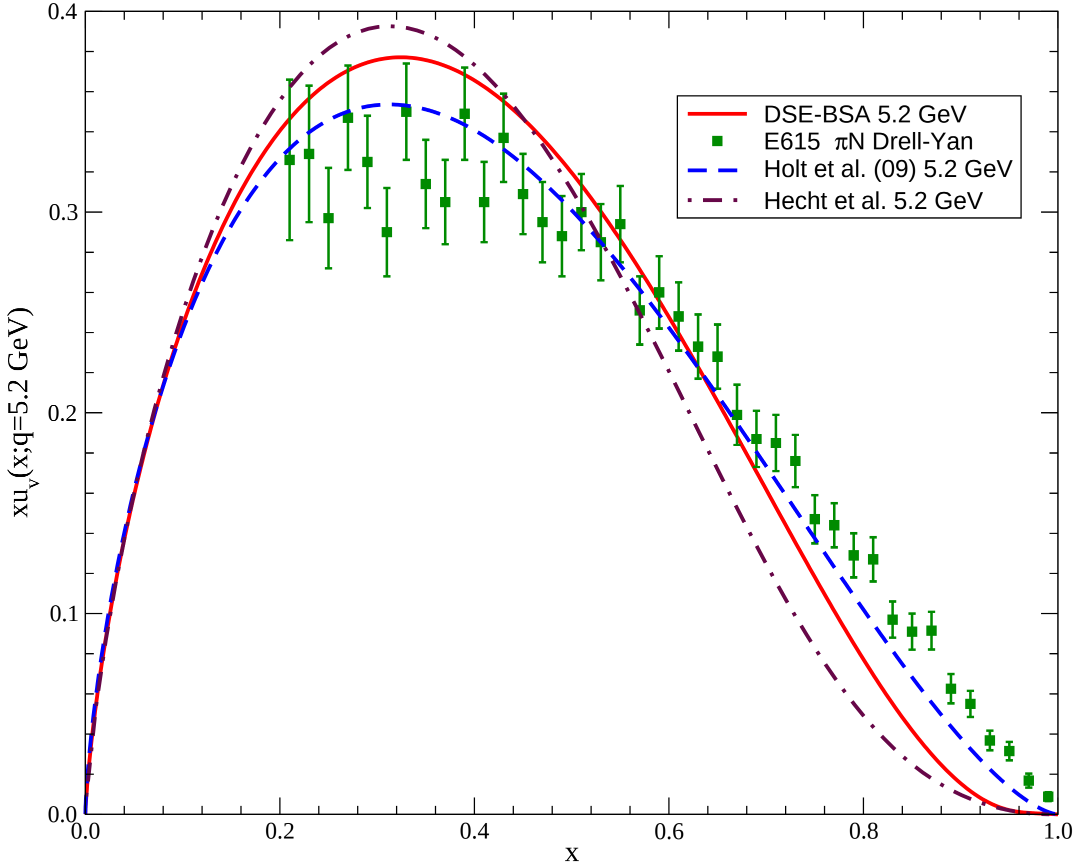


DIS on pion: from DSE-BSE solutions

- Valence quarks, handbag diagram, γ^+, $\Gamma^+_{WI}(k)$

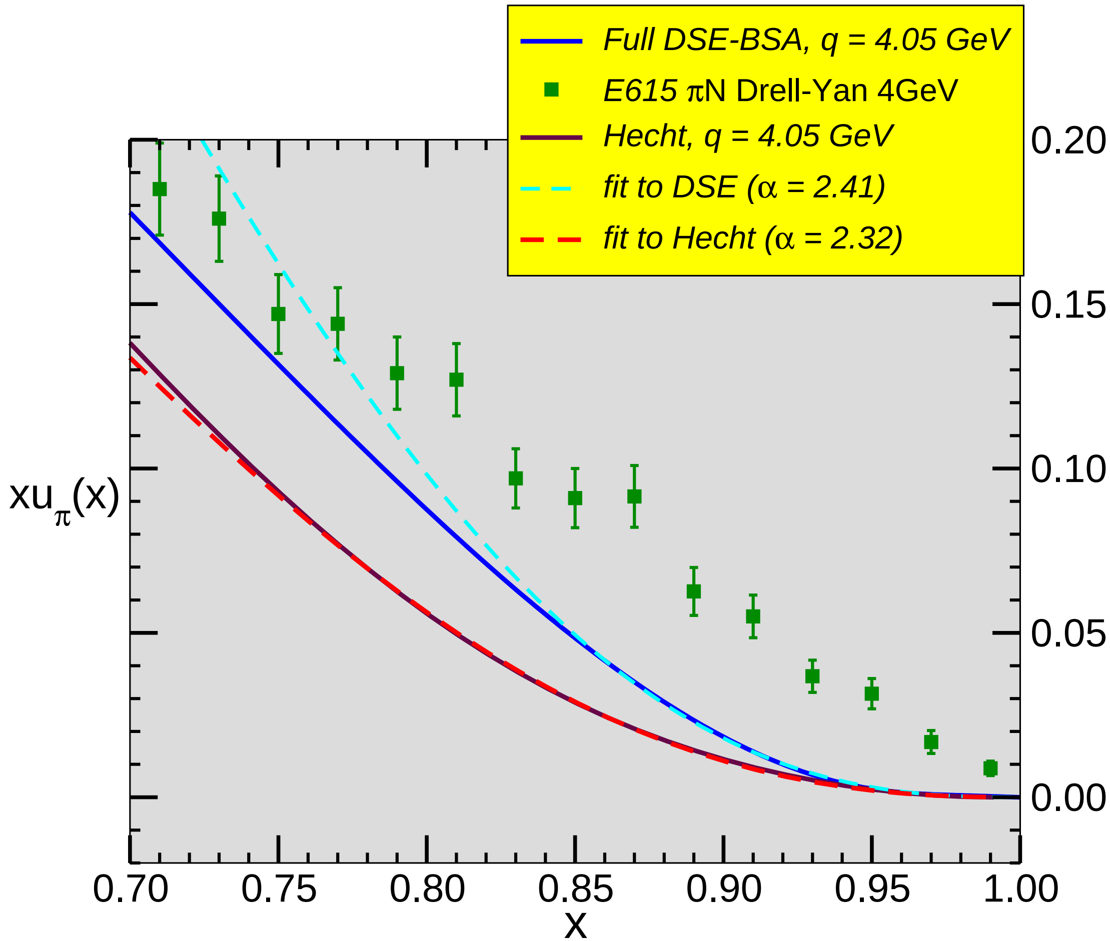
- Data: J. S. Conway et al, PRD39, 92 (1989)
  $M_{l\bar{l}} = 4.05$ GeV

- Previous: Hecht, Roberts, Schmidt, PRC63, 025213 (2001)
  $\Gamma_\pi(k,P) \approx i\gamma_5\, B_0(k^2)/f^0_\pi + \cdot\cdot$
  $S(p)$ fit to data



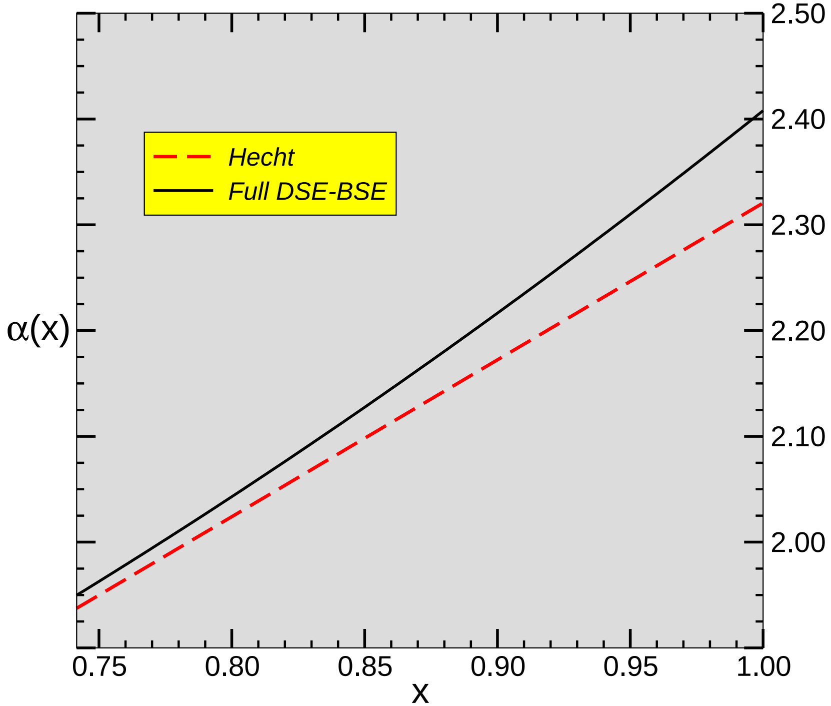
- Large x behavior: $(1-x)^\alpha$, $\alpha =$?

DIS on pion: large x behavior?



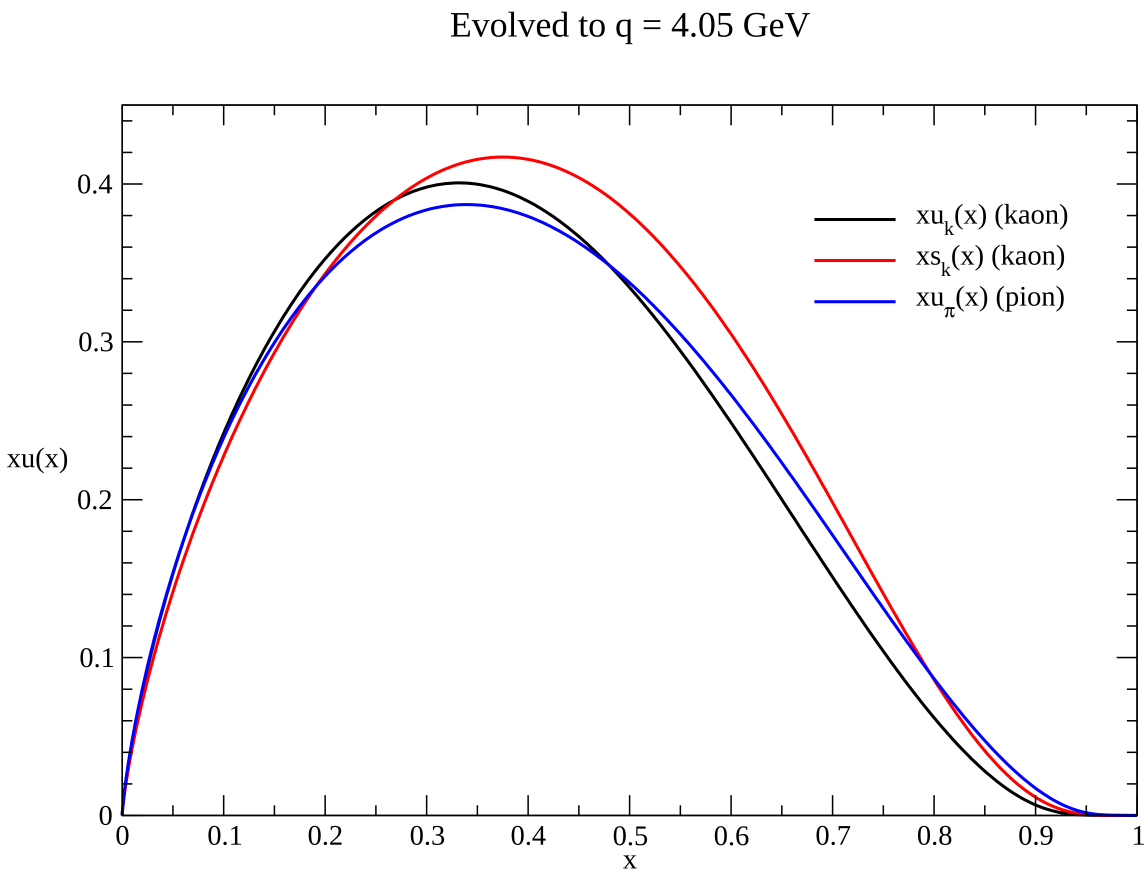
- Fit: $a\,x\,(1-x)^{\alpha(x)}$
- BSE ampls: pQCD behavior sets in at a larger scale

DIS on pion: large x behavior?



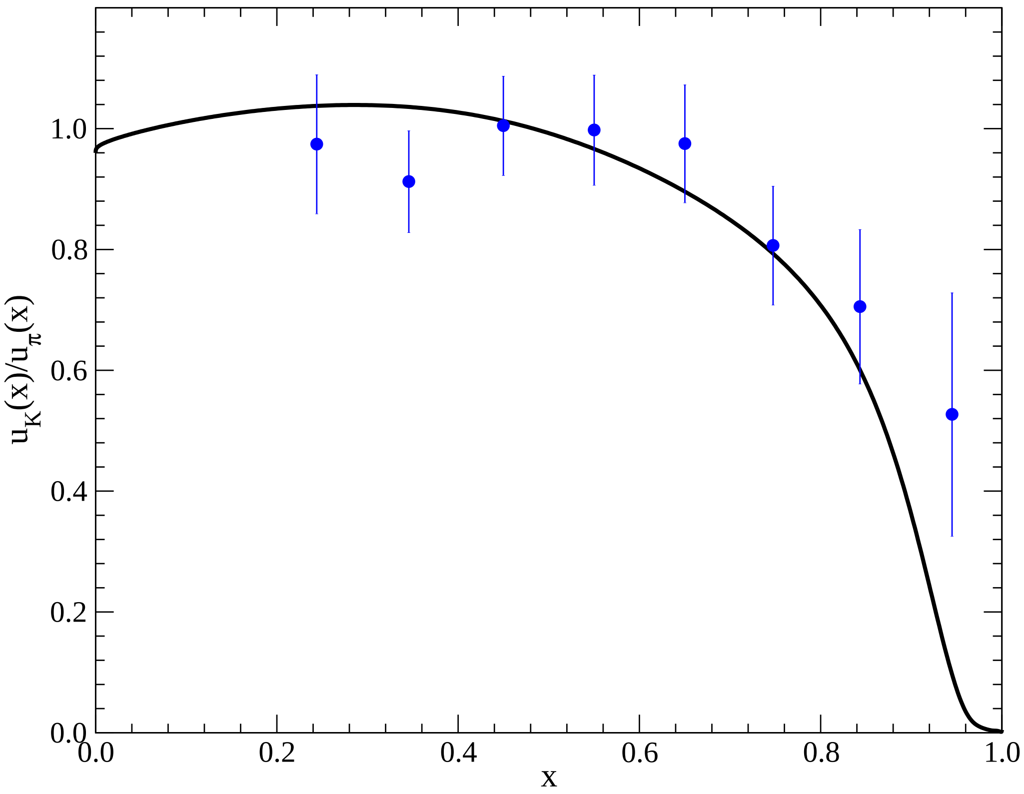
- Global fits to (limited) DIS data produce $\alpha \sim 1.5$
- Parton model (F-J), pQCD (Brodsky, Ezawa), DSEs, $\Rightarrow \alpha \sim 2+$
- Constituent q models, NJL, duality, etc $\Rightarrow \alpha \sim 1$

Quark Distributions in π and K



- Environmental depn of $u(x)$ in accordance with effective quark mass
- $u(x), \;\; s(x)$ difference in K in accordance with effective quark mass

$u_K(x)/u_\pi(x)$ **Ratio**



- Data: (Drell-Yan, CERN-SPS) J. Badier it al., PLB **93**, 354 (1980); $M_{l\bar{l}} = 4 - 8$ GeV
- u has greater fraction of P_π than it has of P_K, in accord with effective quark mass

Axial anomaly and $\eta - \eta'$ states

- Ch symm: $\partial_\mu(z)\langle j^\alpha_{5\mu}(z)\, q(x)\bar{q}(y)\rangle$ involves $2\,\mathrm{tr_f}(\mathcal{F}^\alpha)\langle Q_t(z)q(x)\bar{q}(y)\rangle$
- Matrix elements, amputated $\Rightarrow$ AV-WTI

$$P_\mu \Gamma^\alpha_{5\mu}(k;P) = -2i\,\mathcal{M}^{\alpha\beta}\Gamma^\beta_5(k;P) - \delta_{\alpha,0}\,\Gamma_A(k;P)$$
$$+S^{-1}(k_+)\,i\gamma_5\mathcal{F}^\alpha + i\gamma_5\mathcal{F}^\alpha S^{-1}(k_-)$$

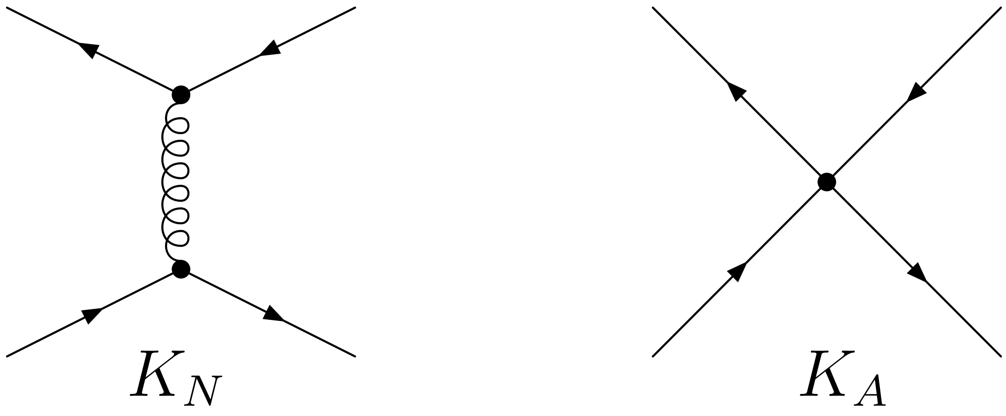
- Residues at PS poles $\Rightarrow$ PS mass formula for arbitrary m_q, any flavor:

$$\boxed{m_p^2 f_p^\alpha = 2\,\mathcal{M}^{\alpha\beta}\rho_p^\beta + \delta^{\alpha,0}\,n_p} \quad , \quad n_p = 2\,\mathrm{tr_f}(\mathcal{F}^0)\,\langle 0|Q_t|p\rangle$$

$$\rho_p^\alpha(\mu) = \langle 0|\bar{q}\,\gamma_5\mathcal{F}^\alpha\,q|p\rangle\,, \quad p = \text{any PS}$$

——[Bhagwat, Chang, Liu, Roberts, PCT, PRC (**76), 2007; arXiv:0708.1118]**

A Schematic Model: Flavor mixing, η, η'



- [Bhagwat, Chang, Liu, Roberts, PCT, PRC (**76), 2007; arXiv:0708.1118]**
- Structure: $K_N =$ LR vector gluon exch,
  $K_A = \mathcal{F}(\gamma_5, \not{P}\gamma_5) \otimes (\gamma_5, \not{P}\gamma_5)\mathcal{F}\ , \quad \mathcal{F} = \mathrm{diag}(1/M_f)$
- (Munczek-Nemirovsky) t-channel $\delta^4(k)$ for K_N **and** K_A
- 2 strength parameters: $\rho^0 \Rightarrow K_N\ , \quad m_{\eta'} \Rightarrow K_A$.
- Fix $m_u, m_d, m_s \cdots$ via vector mesons

$\pi^0 - \eta - \eta'$ mixing: 3 flavors

- $m_u - m_d$ causes π^0 to be mixed in:

$$\begin{aligned} 135\ \mathrm{MeV}: &\quad |\pi^0\rangle \sim 0.72\,\bar{u}u - 0.69\,\bar{d}d - 0.013\,\bar{s}s \\ 455\ \mathrm{MeV}: &\quad |\eta\rangle \sim 0.53\,\bar{u}u + 0.57\,\bar{d}d - 0.63\,\bar{s}s \\ 922\ \mathrm{MeV}: &\quad |\eta'\rangle \sim 0.44\,\bar{u}u + 0.45\,\bar{d}d + 0.78\,\bar{s}s \end{aligned}$$

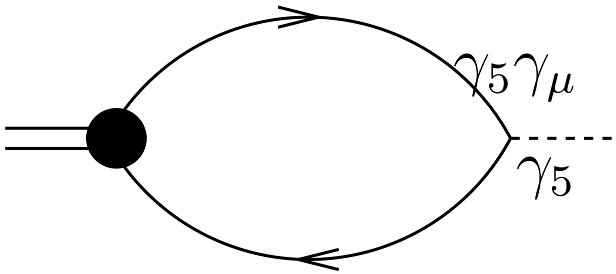
- $m_u = m_d \Rightarrow$

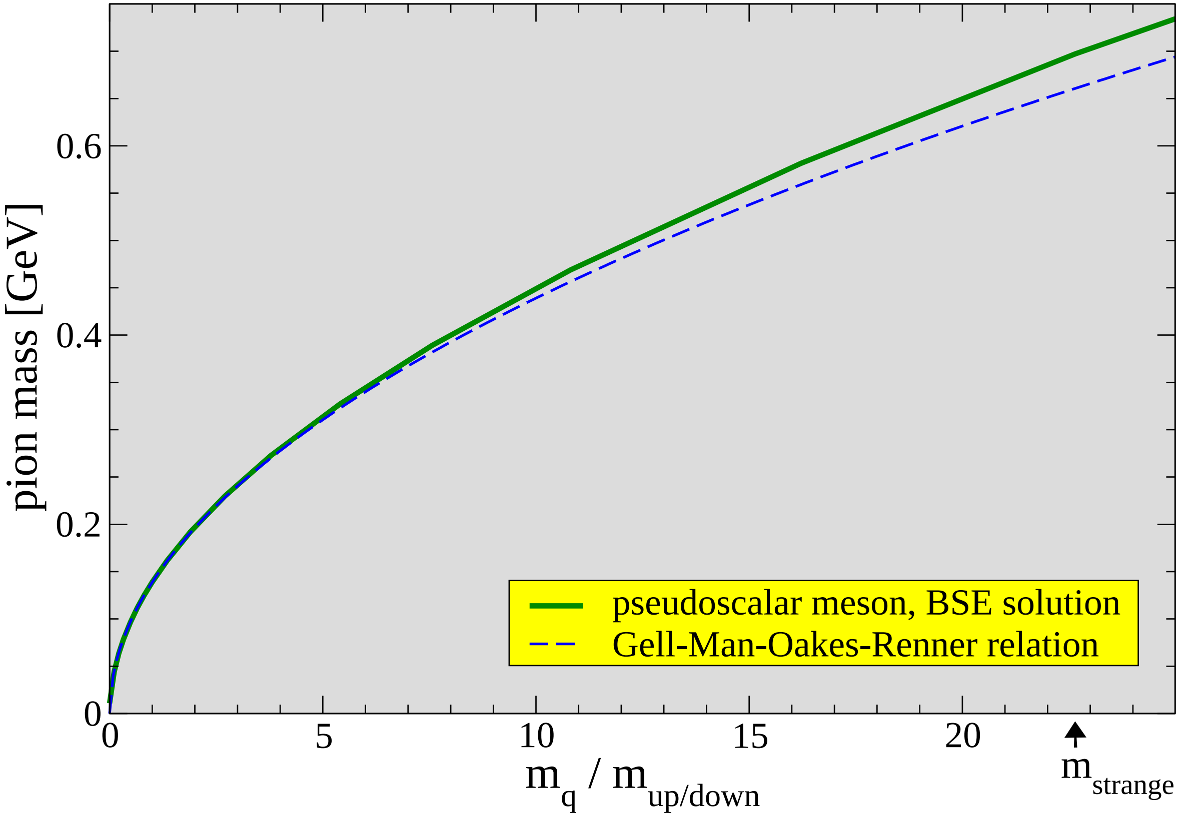
$$\begin{aligned} 455\ \mathrm{MeV}: &\quad |\eta\rangle \sim 0.55\,(\bar{u}u + \bar{d}d) - 0.63\,\bar{s}s, \quad \theta_\eta = -15.4^\circ \\ 924\ \mathrm{MeV}: &\quad |\eta'\rangle \sim 0.45\,(\bar{u}u + \bar{d}d) + 0.78\,\bar{s}s, \quad \theta_{\eta'} = -15.7^\circ \end{aligned}$$

- Chiral limit: $m^2_{\eta'} = (0.852\ \mathrm{GeV})^2 \equiv 2\mathrm{tr_f}(\mathcal{F}^0)\ \langle 0|Q_t|\eta'\rangle / f^0_{\eta'}$
- cf Witten-Veneziano a-v ghost scenario $\Rightarrow m^2_{\eta'} = h^2 + m^2_{\mathrm{GB}}$
- It is worth extending to a realistic LR model for K_N with separable K_A: one obtains access to decay constants, residues, and details of the mass relations

Flavor Non-singlet PS Mass Relation

$$f_H\ m_H^2 = 2\,m_q(\mu)\ \rho_H(\mu)$$

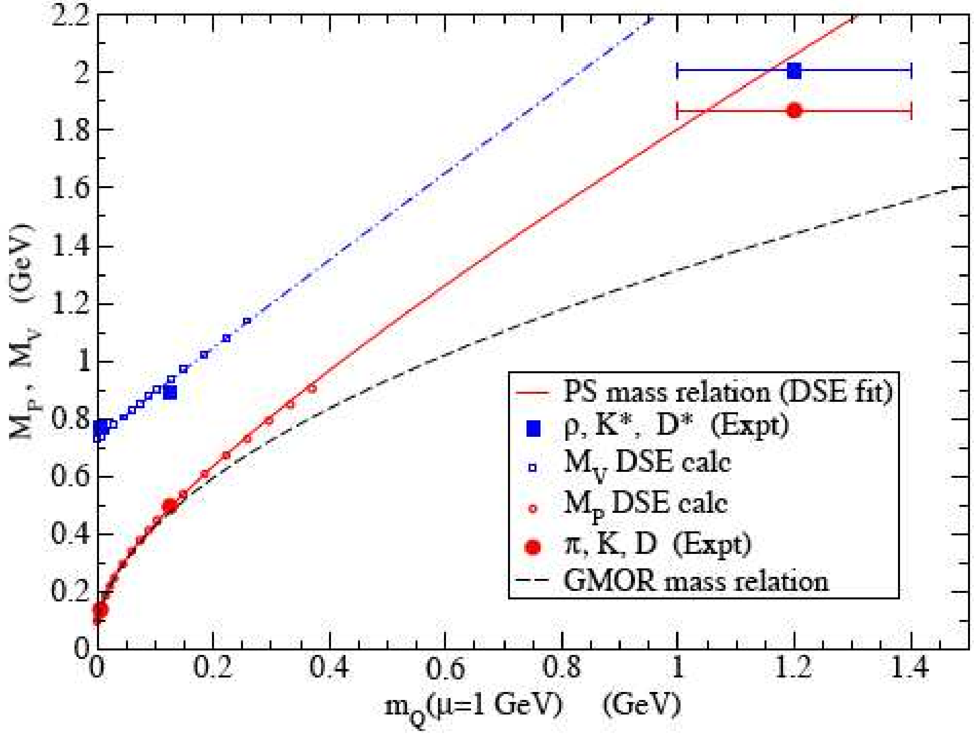




PM, Roberts, Tandy, PLB420, 267 (1998) [nucl-th/9707003]

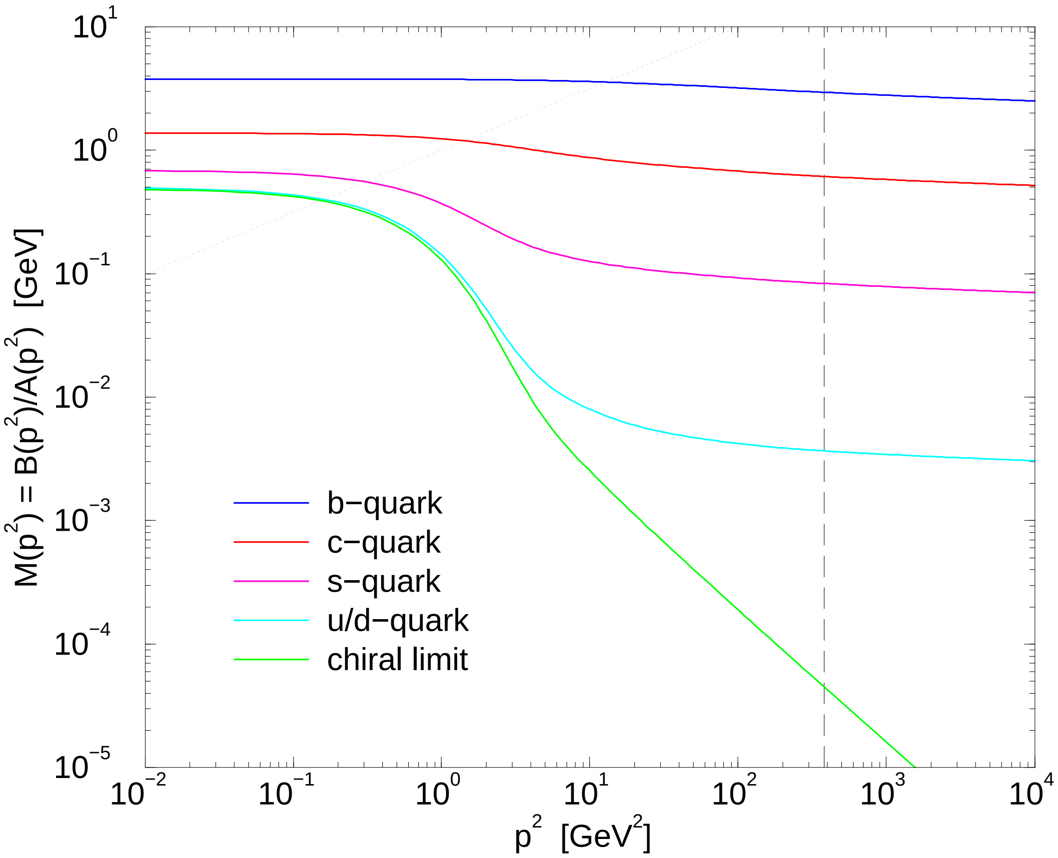
Inaccuracy of GMOR

qQ case:



GMOR: 0.2%(π); 4%(K); 14%(0.4GeV); 30%(D)

Quark mass functions from DSE solutions



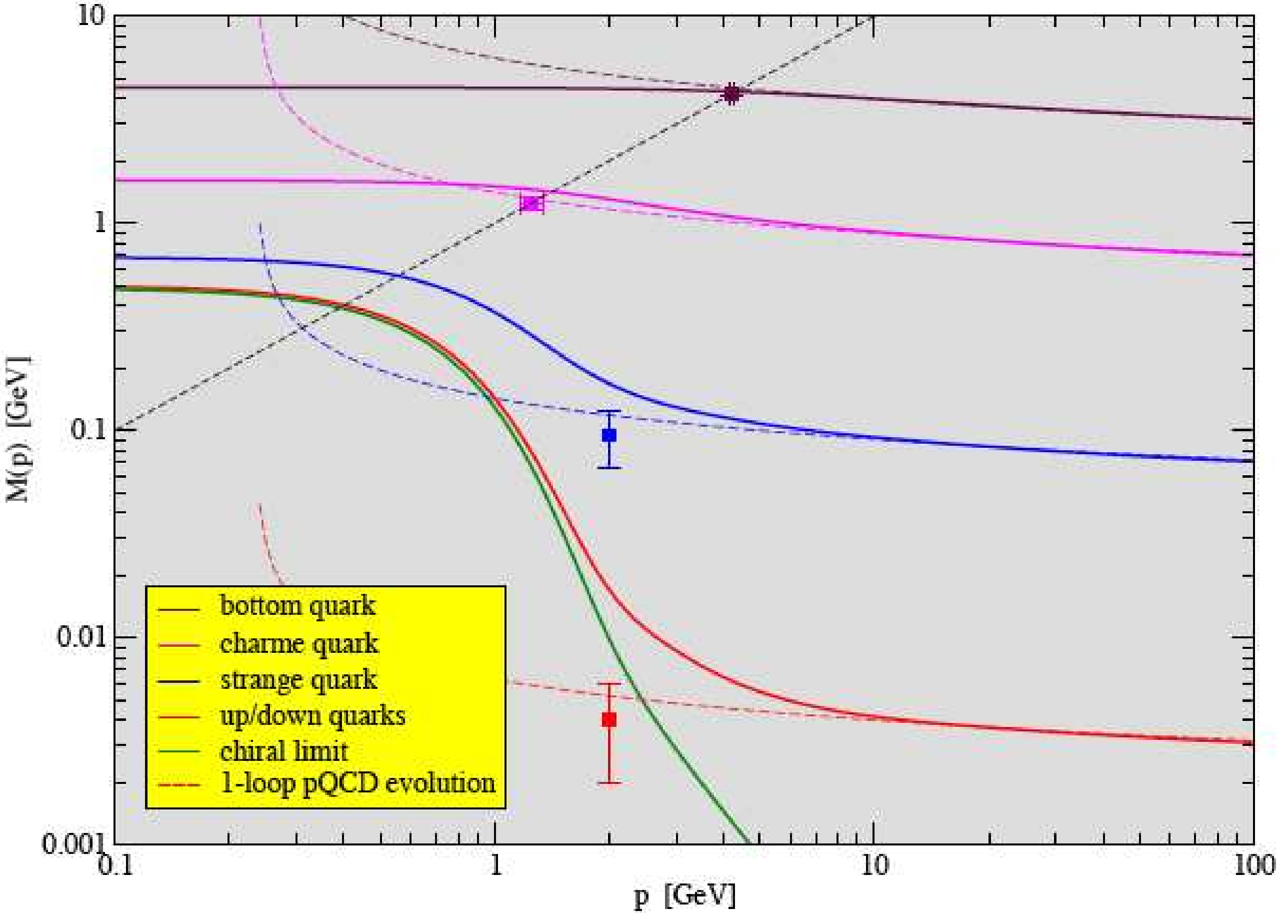
Constituent Mass Concept for c- and b-quarks

All GeV	D(uc)	D^*(uc)	D_s(sc)	D_s^*(sc)
expt M	1.86	2.01	1.97	2.11
calc M	1.85(FIT)	2.04	1.97	2.17
expt f	0.222	?	0.294	?
calc f	0.154	0.160	0.197	0.180

All GeV	B(ub)	B^*(ub)	B_s(sb)	B_s^*(sb)	B_c(cb)	B_c^*(cb)
expt M	5.28	5.33	5.37	5.41	6.29	?
calc M	5.27(FIT)	5.32	5.38	5.42	6.36	6.44
expt f	0.176	?	?	?	?	?
calc f	0.105	0.182	0.144	0.20	0.210	0.18

- Fit $\Rightarrow$ constituent masses: $M_c^{\rm cons} = 2.0$ GeV, $M_b^{\rm cons} = 5.3$ GeV
- Consistent with $M^{DSE}(p^2 \sim -M^2)$ generated by $m_c = 1.2 \pm 0.2, \;\; m_b = 4.2 \pm 0.2$, [PDG, $\mu = 2$ GeV]
- Does heavy quark dressing contribute anything? Too much in this DSE model—no mass shell !

Compare Quark Masses with PDG



Quarkonia

All GeV	M_{η_c}	f_{η_c}	$M_{J/\psi}$	$f_{J/\psi}$
expt	2.98	0.340	3.09	0.411
calc with $M_c^{\rm cons}$	3.02	0.239	3.19	0.198
calc with $\Sigma_c^{\rm DSE}(p^2)$	3.04	0.387	3.24	0.415

All GeV	M_{η_b}	f_{η_b}	M_{Υ}	f_{Υ}
expt	9.4 ?	?	9.46	0.708
calc with $M_b^{\rm cons}$	9.6	0.244	9.65	0.210
calc with $\Sigma_b^{\rm DSE}(p^2)$	9.59	0.692	9.66	0.682

- QQ and qQ decay constants too low by 30-50% in constituent mass approximation
- Quarkonia decay constants much better for DSE dressed quarks (within 5% of expt.)
- IR sector (gluon k below ~ 0.8 GeV) contribute little for bb or cc quarkonia in DSE, BSEs
- QQ states are more point-like than qq or qQ states

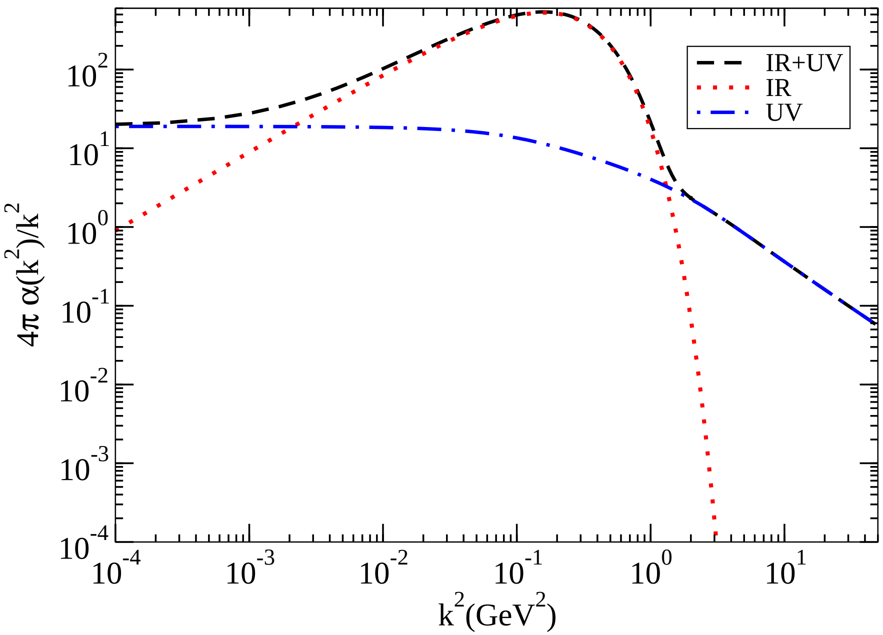
Recovery of a qQ Mass Shell

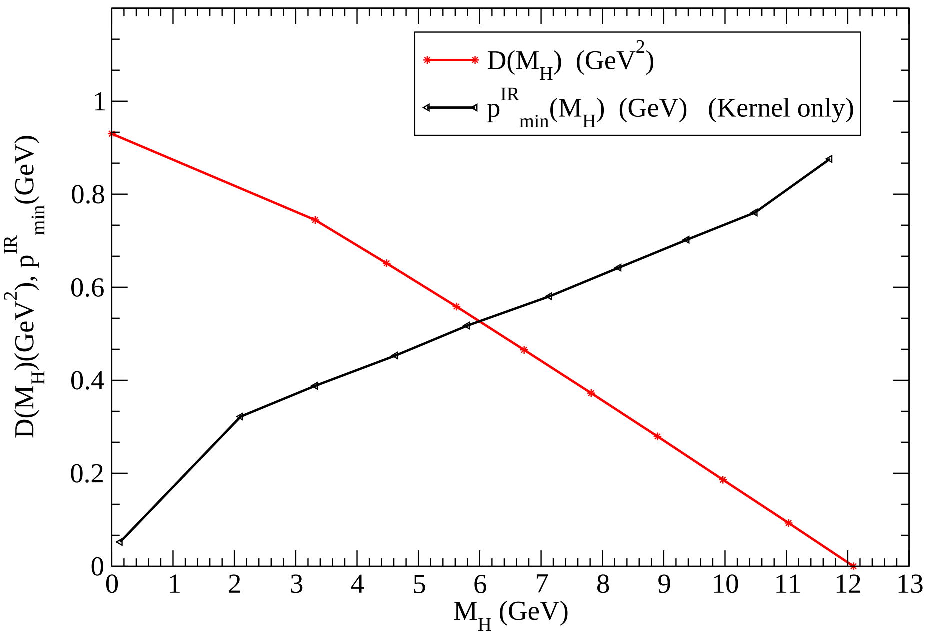
- Suppress gluon k below ~ 0.8 GeV in DSE dressing of b propagator
- Retain IR sector for dressed "light" quark and BSE kernel
- Now a mass shell is produced

All GeV	B(ub)	B^*(ub)	B_s(sb)	B_s^*(sb)	B_c(cb)	B_c^*(cb)
expt M	5.28	5.33	5.37	5.41	6.29	?
calc M	4.66	–	4.75	–	5.83	—
expt f	0.176	?	?	?	?	?
calc f	0.133	–	0.164	–	0.453	–

- Masses are ~ 10 % low
- It makes sense that $R_b < R_{qQ} \Rightarrow$ greater limit on low k in Σ_b
- May be partial confirmation of Brodsky and Shrock's suggestion of universal maximum wavelength for quarks/gluons in hadrons [Phys. Lett. B666, (2008)]

IR Suppression of Kernel



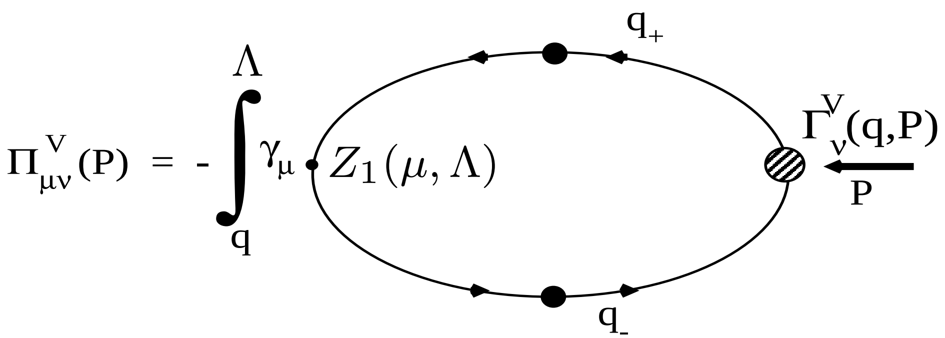


The V-A Current Correlator

- $\Pi^V_{\mu\nu}(x) = \langle 0|\, T\, j_\mu(x) j^\dagger_\nu(0)\, |0\rangle$, isovector currents $j_\mu = \bar{u}\gamma_\mu d, \quad j^5_\mu = \bar{u}\gamma_5\gamma_\mu d$

$$\Pi^V_{\mu\nu}(P) = (P^2\delta_{\mu\nu} - P_\mu P_\nu)\, \Pi^V(P^2)$$

$$\Pi^A_{\mu\nu}(P) = (P^2\delta_{\mu\nu} - P_\mu P_\nu)\, \Pi^A(P^2) + P_\mu P_\nu\, \Pi^L(P^2)$$



- $m_q = 0: \quad \Pi^V - \Pi^A = 0$, to all orders in pQCD
- $\Pi^V - \Pi^A$ probes the scale for onset of non-perturbative phenomena in QCD

The 4-quark Condensates

- Operator product expansion $\Rightarrow$ leading uv behavior

$$\Pi^{V-A}(P^2) = \frac{32\pi\alpha_s\langle\bar{q}q\bar{q}q\rangle}{9\,P^6}\left\{1 + \frac{\alpha_s}{4\pi}\left[\frac{247}{4\pi} + \ln\left(\frac{\mu^2}{P^2}\right)\right]\right\} \;+\; \mathcal{O}\left(\frac{1}{P^8}\right)$$

- Often vacuum saturation ($\langle\bar{q}q\bar{q}q\rangle \approx \langle\bar{q}q\rangle^2$) is assumed for QCD Sum Rules. Validity not known.
- Extract $\langle\bar{q}q\bar{q}q\rangle$ from $\lim|_{P^2\to\infty}\; P^6\Pi^{V-A}(P^2)$

Model	$-<\bar{q}q>_{\mu=19}\;(GeV)^3$	$<\bar{q}q\bar{q}q>_{\mu=19}\;(GeV)^6$	$R(\mu=19)$
Set A	$(0.5682)^3$	$(0.619)^6$	1.67
Set B	$(0.1734)^3$	$(0.1902)^6$	1.74
Set C	$(0.2469)^3$	$(0.2695)^6$	1.69
Set D	$(0.216)^3$	$(0.235)^6$	1.65

——T. Nguyen, PCT, in preparation, 2008

DSE Calculation: Weinberg Sum Rules

- I: $\frac{1}{4\pi^2} \int_0^\infty ds[\rho_v(s) - \rho_a(s)] = [P^2 \, \Pi^{V-A}(P^2)]_{P^2 \to 0} = -f_\pi^2$
- II: $P^2 \, [P^2 \, \Pi^{V-A}(P^2)]|_{P^2 \to \infty} = 0$
- DGMLY: $\int_0^\infty dP^2 \, [P^2 \, \Pi^{V-A}(P^2)] = -\frac{4\pi f_\pi^2}{3\alpha}[m_{\pi^\pm}^2 - m_{\pi^0}^2]$

Model	$f_\pi^2 \, (GeV^2)$	$f_\pi \, (MeV)$	f_π^{exp}/f_π^{num}	$\Delta m_\pi \, (MeV)$	$(\Delta m_\pi)_{exp}$
Set A	0.00456291	67.5	1.37	4.86	
Set B	0.00538895	73.4	1.26	5.2	4.43 ± 0.03
Set C	0.00518379	72.0	1.28	4.88	

Summary

- Effective ladder-rainbow model based on QCD -DSEs; $\langle \bar{q}q \rangle_\mu \Rightarrow$ 1 IR parameter
- Convenient and covariant approach to hadronic form factors: N, π, various transitions
- Ground state qQ and QQ mesons (V & PS) up to b-quark region
- Dynamical dressing in $S(p)$ at each stage increases the value of the decay constant [factor of 3 for $\bar{b}b$, factor of 2 for $\bar{c}c$] !
- First combination of BSE-DSE solutions for pion and kaon DIS distributions $u(x), s(x)$
- Used $\langle J\ J \rangle$, V-A, to estimate $\langle \bar{q}q\bar{q}q \rangle$ as $\sim$ 70% greater than vac saturation, and npQCD enters at scale 0.5 fm.

Collaborators

- Craig Roberts, Argonne National Lab
- Pieter Maris, Iowa State University
- Yu-xin Liu, Lei Chang, Peking University
- Nick Souchlas, Trang Nguyen, Kent State University

Thankyou!

DIS on pion: from DSE-BSE solutions

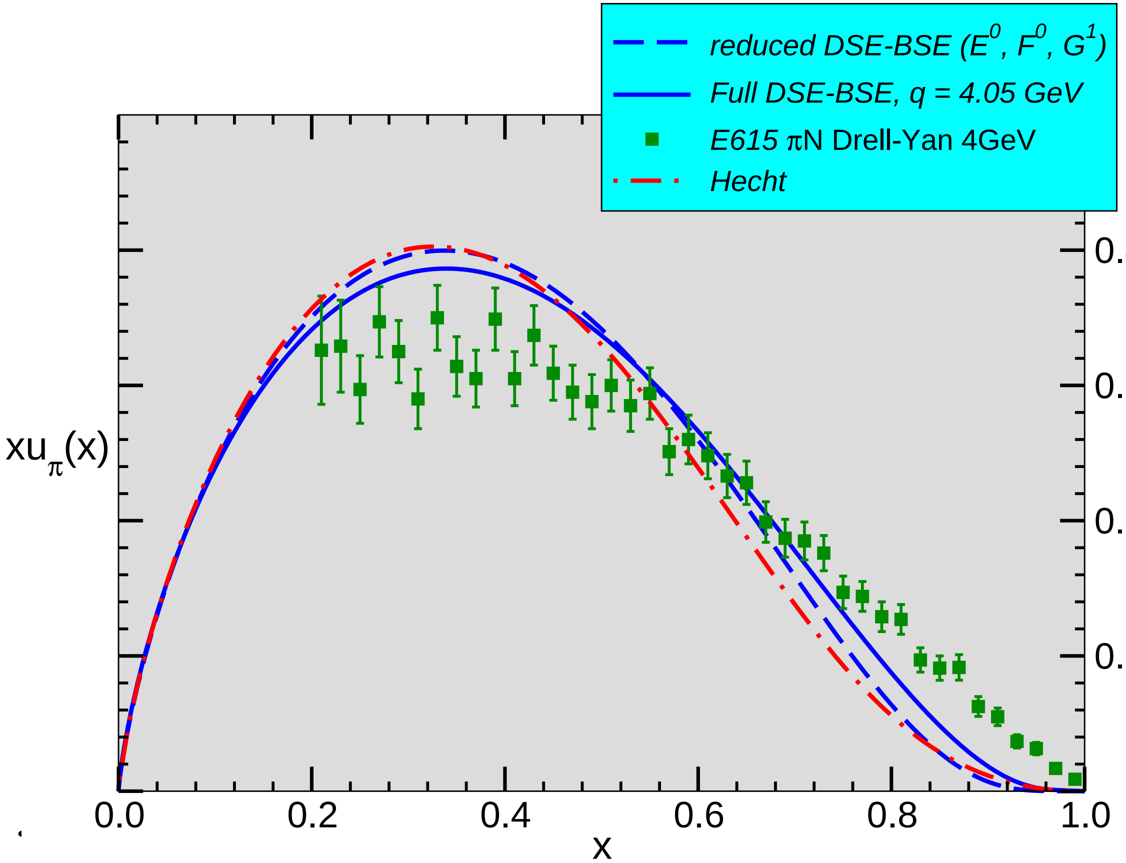
- Valence quarks, handbag diagram, γ^+, $\Gamma^+_{WI}(k)$

- Data: J. S. Conway et al, PRD39, 92 (1989)

- Previous: Hecht, Roberts, Schmidt, PRC63, 025213 (2001)

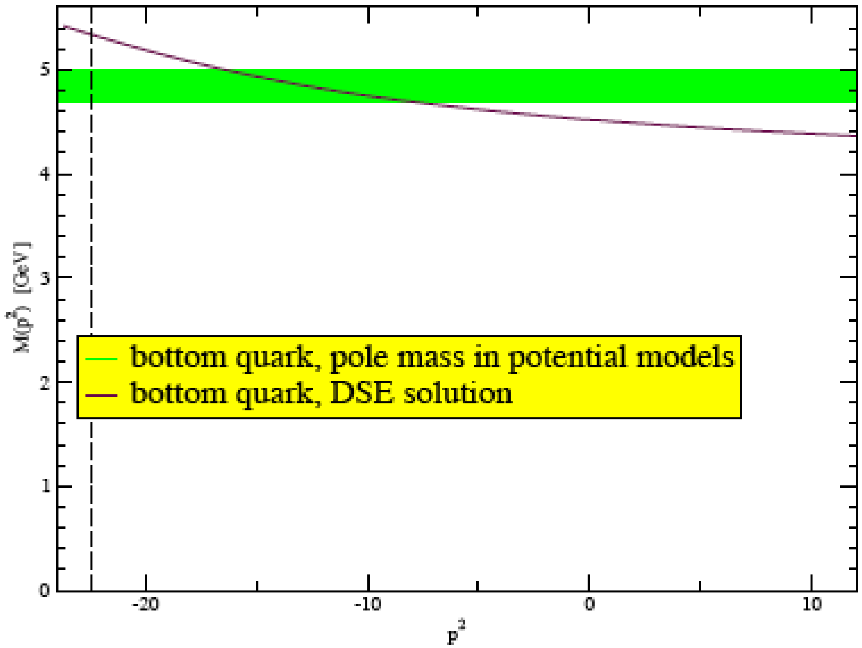
$\Gamma_\pi(k,P) \approx i\gamma_5 \; B_0(k^2)/f^0_\pi + \cdots$

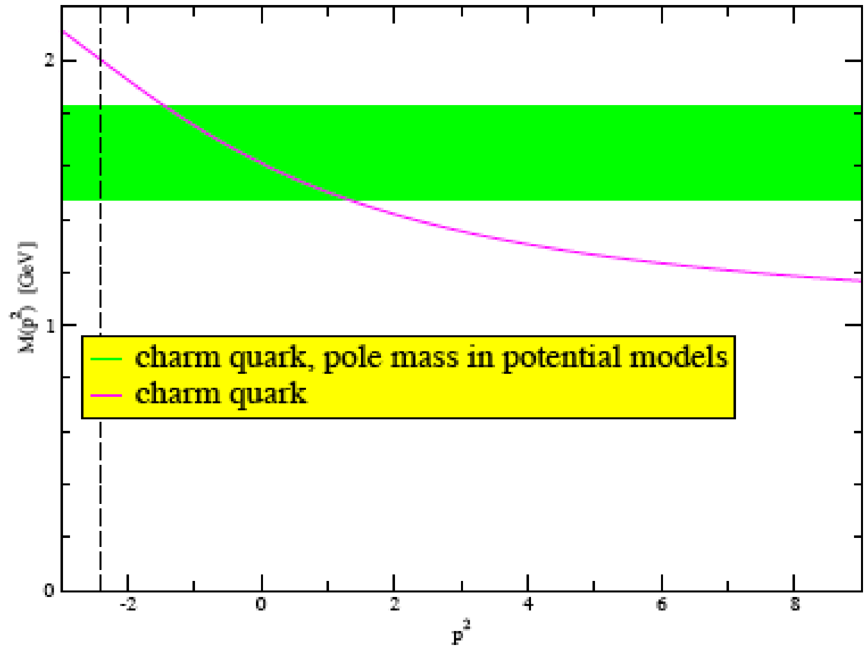
$S(p)$ fit to data



- Large x behavior: $(1-x)^\alpha \quad , \alpha = ?$

Constituent Quark-like Behavior for c, b-quarks





- Mass shell positions marked for $\bar{b}b$ and $\bar{c}c$ quarkonia
- qQ mesons sample $M_Q(p^2) \sim$ 4 times further into timelike region
- The same constituent or pole mass is unlikely to suffice for QQ and qQ mesons

General Pseudoscalar Mass Formula

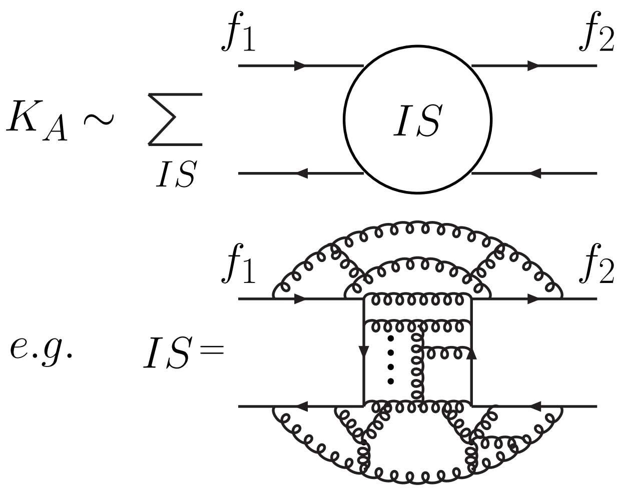
- $N_f = 3$, charge neutral states: $p = \pi^0, \eta, \eta'$

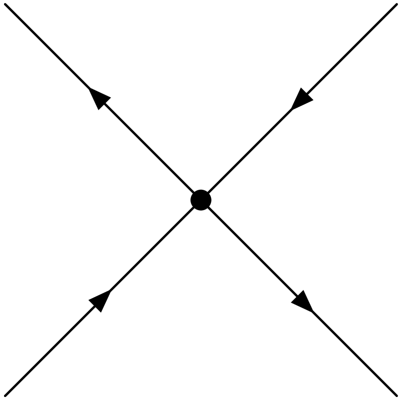
$$m_p^2 \begin{bmatrix} f_p^3 \\ f_p^8 \\ f_p^0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ n_p \end{bmatrix} + \left[2\, \mathcal{M}_{3\times 3} \right] \begin{bmatrix} \rho_p^3 \\ \rho_p^8 \\ \rho_p^0 \end{bmatrix}$$

- Isospin breaking: $m_u \neq m_d$ allows anomaly, $\mathcal{F}^0$, and $s\bar{s}$ into π^0
- η' in $SU(N_f)$ limit: $\boxed{m_{\eta'}^2 f_{\eta'}^0 = n_{\eta'} + 2m\; \rho_{\eta'}^0}$

Model Bethe-Salpeter Kernel for flavor singlet?

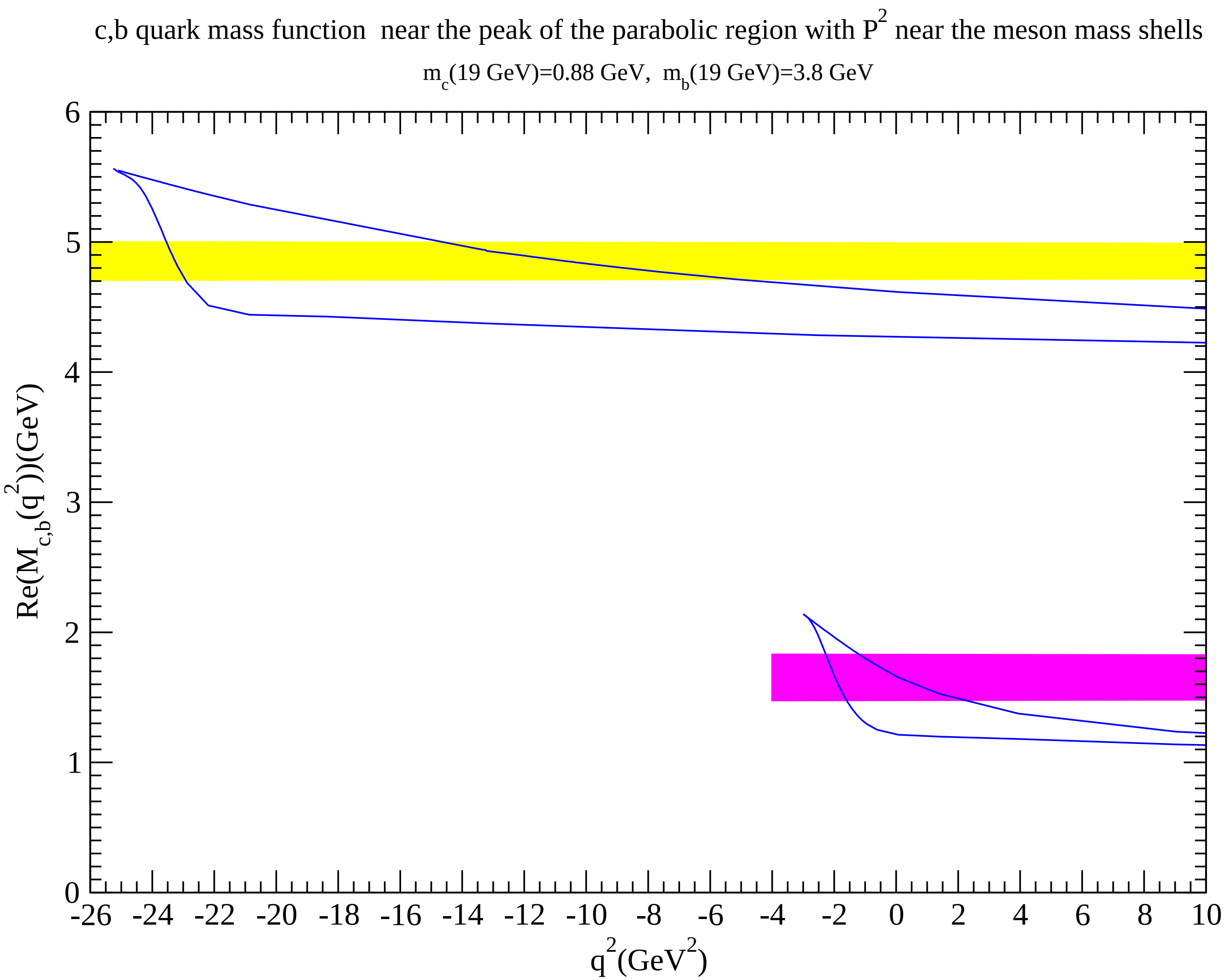
- Vertex integral eqns do not involve $Q_t(x)$ explicitly:
  $\Gamma^{\alpha}_{5\mu}(k;P) = Z_2\,\gamma_5\gamma_\mu\mathcal{F}^{\alpha} + \int^{\Lambda} K\; S_+\Gamma^{\alpha}_{5\mu}S_-$
- DSE models need: $K_{\rm BSE} = K_{\rm N} + K_{\rm A}$, both are $\bar{q}q$ irreducible, $K_{\rm N}$ is also n-gluon irreducible



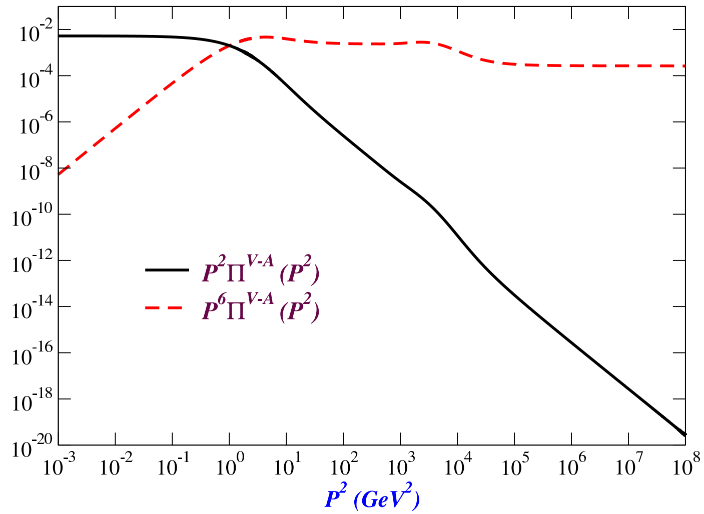


- A scenario that works: Witten-Veneziano massless axial-vector ghost linking pseudoscalar GBs

c- and b-Quark Mass Function for BSE



DSE Calculation: Estimated 4 quark condensate



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