Fundamental Landau Levels in a Strongly Coupled Plasma

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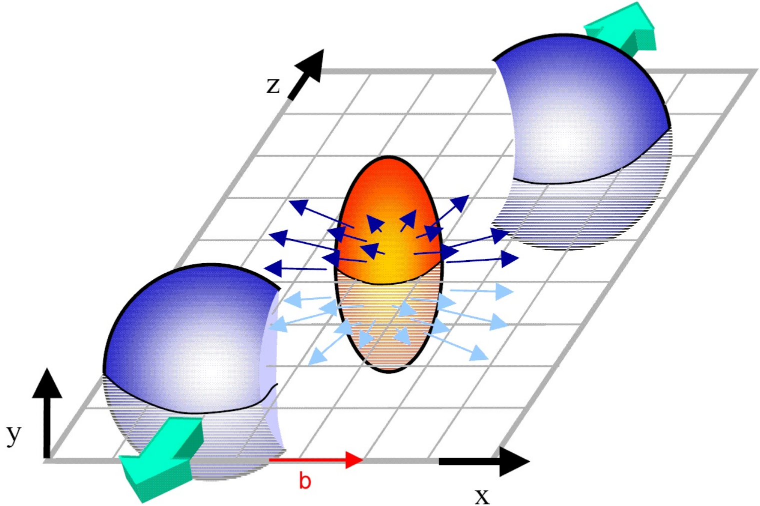
The Origin of the Magnetic Field



Collision is not central

Observations Supporting the Existence of an Intense Magnetic Field

PHENIX experiments report an excess of direct photons in comparison with the expectations based on the individual partons collisions.

They also report an azimuthal anisotropy v_2 higher than expected.

It was suggested that a conformal anomaly in the presence of a strong magnetic field could cause the current not to be conserved.

Just Some Words About the Gauge Gravity Correspondence

Motivated by the decoupling limit of one physical system in two perspectives.

It has not been proven.

Establishes a correspondence between a 4D gauge theory and ST in certain backgrounds.

Physical Quantities We Have Studied

- Direct photon production.
- DC conductivity parallel and perpendicular to the field.
- Drag force experienced by a quark in the plasma.
- Running of double trace operators coupling.
- Meson Physics.
- Phase diagrams.

Main result for FLL

$$\chi(\mathsf{k},\mathsf{x}) = -2\mathrm{Im}[\tilde{G}^R(\omega)]|\psi_{\mathsf{x}}(\mathsf{x})|^2 \quad . \qquad (1)$$

where the function $\psi_{\mathsf{x}}(\mathsf{x})$ is a solution to the Schrödinger equation that describes Landau levels in ordinary Quantum Mechanics with ω taking the place of the mass m, so the energy spectrum in terms of the Landau frequency

$$\omega_L = \frac{e\,B}{\omega},$$

is

$$\mathcal{E}_n = \left(n + \frac{1}{2}\right)\omega_L. \qquad (2)$$

Main result for FLL

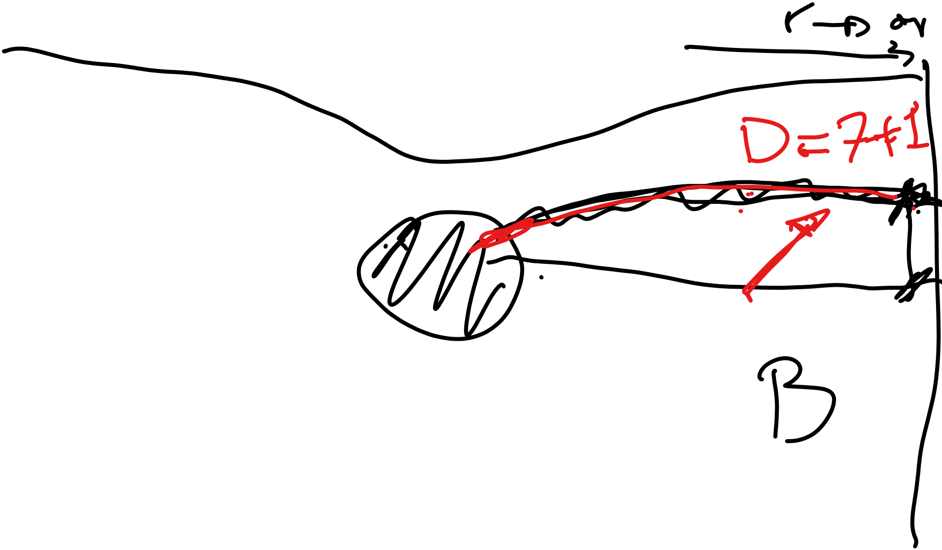
In Landau gauge we need to set the origin of the coordinate system so that the guiding center parameter is equal to zero.

In the symmetric gauge we only recover states with canonical angular momentum equal to zero, not implying the vanishing of the kinetic angular momentum, for which we provide an spectrum.

Steps to follow

- Build a 10-D background with a magnetic field.
- Embed a D7-brane in it to add fundamental degrees of freedom.
- Study the perturbations of such embedding.
- Compute the spectral function.
- Do some checks.

Steps to follow



The Background We Used

The action is

$$S = \frac{1}{16\pi G_5}\Big[-\int d^5x\sqrt{-g}\Big(R + F^{MN}F_{MN} - \frac{12}{L}\Big) + \frac{8}{3\sqrt{3}}\int A\wedge F\wedge F\Big], \tag{3}$$

with the general ansatz

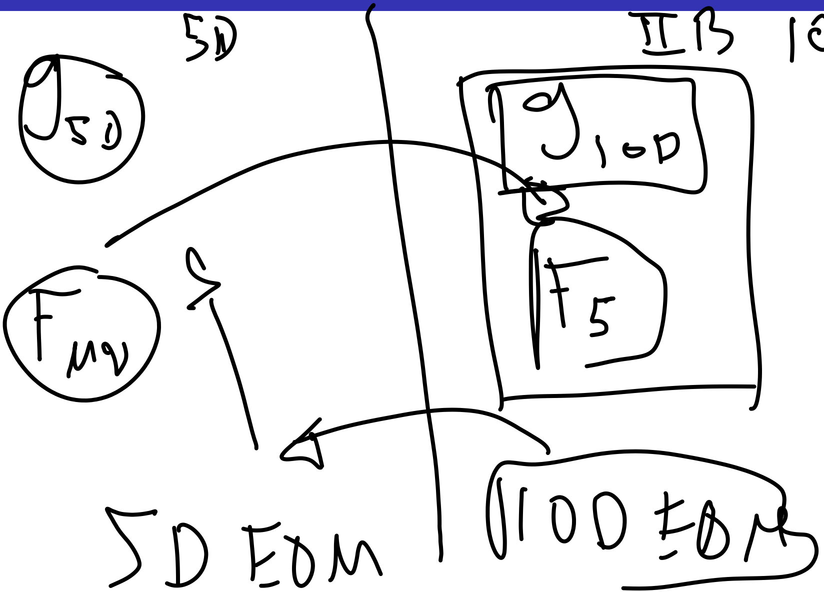
$$ds^2 = -U(r)dt^2 + \frac{1}{U(r)}dr^2 + V(r)(dx^2+dy^2) + W(r)dz^2,$$

for the metric and

$$F_{BG} = B dx\wedge dy,$$

with constant B for the field strength.

The uplift



The uplift

$$
\begin{aligned}
ds_{10}^2 = ds_5^2 + {}^2 \Bigg\{ & d\theta^2 + \sin^2\theta \left(d\phi_1 + \frac{2}{\sqrt{3}} A_\mu dx^\mu \right)^2 \\
& + \cos^2\theta \left[d\vartheta^2 + \sin^2\vartheta \left(d\phi_2 + \frac{2}{\sqrt{3}} A_\mu dx^\mu \right)^2 \right. \\
& \left. + \cos^2\vartheta \left(d\phi_3 + \frac{2}{\sqrt{3}} A_\mu dx^\mu \right)^2 \right] \Bigg\},
\end{aligned} \tag{4}
$$

The uplift

$$F_{(5)} = G_{(5)} + *G_{(5)}, \tag{5}$$

with

$$\begin{aligned} G_{(5)} = -4\epsilon_{(5)} + \frac{2}{\sqrt{3}} \Big[&\sin\theta\cos\theta d\theta \wedge \left(d\phi_1 + \frac{2}{\sqrt{3}} A_\mu dx^\mu \right) \\ &- \cos\theta \sin\theta \sin^2\vartheta d\theta \wedge \left(d\phi_2 + \frac{2}{\sqrt{3}} A_\mu dx^\mu \right) \\ &+ \sin\vartheta\cos\vartheta\cos^2\theta d\vartheta \wedge \left(d\phi_2 + \frac{2}{\sqrt{3}} A_\mu dx^\mu \right) \\ &- \cos\theta\sin\theta\cos^2\vartheta d\theta \wedge \left(d\phi_3 + \frac{2}{\sqrt{3}} A_\mu dx^\mu \right) \\ &- \cos\vartheta\sin\vartheta\cos^2\theta d\vartheta \wedge \left(d\phi_3 + \frac{2}{\sqrt{3}} A_\mu dx^\mu \right) \Big] \wedge \bar{*}F, \end{aligned} \tag{6}$$

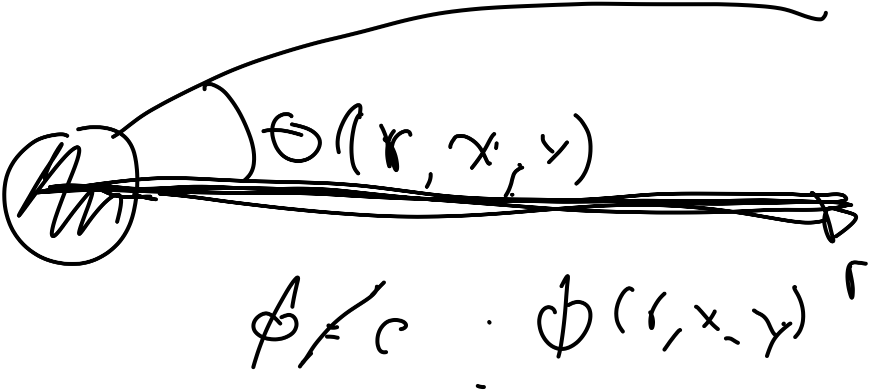
The uplift

Once (10) and $F = Bdx \wedge dy$ have been explicitly placed in (4) and (5), the substitution of the resulting

$$ds_{10}^2 = -U(r)dt^2 + \frac{1}{U(r)}dr^2 + V(r)\left(dx^2 + dy^2\right) + W(r)dz^2 + \left[d\theta^2 + \sin^2\theta d\widetilde{\phi}_1^2 + \cos^2\theta\left(d\vartheta^2 + \sin^2\vartheta d\widetilde{\phi}_2^2 + \cos^2\vartheta d\widetilde{\phi}_3^2\right)\right], \quad (7)$$

and 5-form into the equations of type IIB supergravity in ten dimensions indeed leads to the 5-D equations and nothing else, regardless of whether we use $d\widetilde{\phi}_i = d\phi_i + \frac{2}{\sqrt{3}}B\,x\,dy$ or $d\widetilde{\phi}_i = d\phi_i + \frac{1}{\sqrt{3}}B(x\,dy - y\,dx)$.

Embedding



Embedding

The embedding is dictated by the DBI action

$$S_{DBI} = -T_{D7}N_f \int d^8x\sqrt{-\det(g_{D7})}, \qquad (8)$$

where g_{D7} is the induced metric on the D7-brane, and its tension is given by

$$T_{D7} = \frac{1}{(2\pi l_s)^7 l_s g_s} = \frac{1}{16\pi^6}\lambda N_c, \qquad (9)$$

while the constants N_c and N_f respectively indicate the number of D3-branes sourcing the background and D7-branes embedded in it.

Embedding

We express the determinant of the metric induced on the worldvolume in terms of θ_{emb} and $\phi_{1\,emb}$ as functions of r, x, and y.

For the calculations ahead, it is more convenient to use $\psi(r, x, y) \equiv \sin\theta_{emb}(r, x, y)$ and we will refer to $\phi_{1\,emb}$ as φ.

Embedding

The direct substitution of $\psi(r,x,y)=0$ and $\varphi(r,x,y)=\varphi_0$ into the embedding equations shows that these functions satisfy them for any real constant φ_0 regardless of the norm chosen for A.

Embedding

Perturbing the embedding function ψ as $\psi + \delta\psi$ makes $\delta\psi$ dual to scalar excitations of the fundamental fields in the gauge theory, while perturbations $\delta\varphi$ of φ correspond to psudoscalar ones.

Choosing Landau gauge

$$A = B\, x\, dy$$

to describe the constant magnetic field

$$\mathrm{B} = B\, dx \wedge dy$$

corresponds to setting

$$d\widetilde{\phi}_i = d\phi_i + \frac{2}{\sqrt{3}} B\, x\, dy.$$

Landau gauge

Taking their Fourier transformation in the directions that preserve translational invariance we write

$$\delta\psi = \int \frac{d\omega dq_y dq_z}{(2\pi)^3} e^{-i\omega t + iq_y y + iq_z z} \delta\psi(\mathrm{k}, r, x), \qquad (10)$$

where $\mathrm{k} = (\omega, q_y, q_z)$.

Landau gauge

Working in a frame where $q_y = q_z = 0$, and writing

$$\delta\psi(\mathrm{k}, r, x)) = \psi_r(r)\psi_x(x),$$

and

$$\delta\varphi = 0,$$

the equation from the variation of (8) with respect to $\delta\varphi(r, t, x, y, z)$ is satisfied to all orders, while at leading order in the perturbation the one from varying with respect to $\delta\psi(r, t, x, y, z)$ reduces to

$$\psi_x \left[6VW \left(3 + \frac{\omega^2}{U} \right) \psi_r + 3UVW'\psi_r' + 6W \left(VU'\psi_r' + UV'\psi_r' + UV\psi_r'' \right) \right] + W\psi_r \left(6\partial_x^2\psi_x - 8B^2x^2\psi_x \right) = 0. \tag{11}$$

The latter can be separated into

$$\left(3\frac{U}{\omega}+\omega\right)VW\psi_r+\frac{1}{2}\frac{U^2}{\omega}VW'\psi_r' + \frac{U}{\omega}W\left(VU'\psi_r'+UV'\psi_r'+UV\psi_r''\right)=2\mathcal{E}UW\psi_r, \quad (12)$$

and

$$\frac{1}{2\omega}\left[-\partial_x^2\psi_x+e^2B^2x^2\psi_x\right]=\mathcal{E}\psi_x. \quad (13)$$

The writing of the corresponding coupling as $e = \frac{2}{\sqrt{3}}$ is an accurate reading of the 1-forms $d\widetilde{\phi}_i = d\phi_i + \frac{2}{\sqrt{3}} B\, x\, dy$.

Landau gauge

Demanding $\delta\psi$ to remain bounded for all values of x leads to the quantization

$$\mathcal{E}_n = \left(n + \frac{1}{2}\right)\omega_L, \qquad (14)$$

of the constant $\mathcal{E}$ in terms of the Landau frequency $\omega_L = \frac{e\,B}{\omega}$ and the Landau level number n.

In the symmetric gauge

$$A = \frac{B}{2}(x\,dy \; - \; y\,dx),$$

implemented by setting

$$d\widetilde{\phi}_i = d\phi_i + \frac{1}{\sqrt{3}}B(x\,dy \; - \; y\,dx),$$

we write

$$\delta\psi = \int \frac{d\omega dq_z}{(2\pi)^2} e^{-i\omega t + iq_z z}\delta\psi(\mathrm{k}, r, x, y), \tag{15}$$

with $\mathrm{k} = (\omega, q_z)$, and factoring

$$\delta\psi(\mathrm{k}, r, x, y)) = \psi_r(r)\psi_{xy}(x, y),$$

while keeping

$$\delta\varphi = 0,$$

as before.

The perturbative equation from the variation with respect to $\delta\psi$ separates as before with the only difference that now we have

$$\frac{1}{2\omega}\left[-\partial_x^2\psi_{xy} - \partial_y^2\psi_{xy} + \frac{1}{4}e^2B^2(x^2+y^2)\psi_{xy}\right] = \mathcal{E}\psi_{xy}, \qquad (16)$$

instead of (13).

The symmetric gauge

The equation from varying $\delta\varphi$ at leading perturbative order reduces to

$$y\partial_x\psi_{xy} - x\partial_y\psi_{xy} = 0, \tag{17}$$

which left hand side is interestingly proportional to the canonical angular momentum operator

$$L_z \equiv -i(x\partial_y - y\partial_x). \tag{18}$$

The symmetric gauge

Any function ψ_{xy} that simultaneously solves (16) and (17) is also a solution to

$$\frac{1}{2\omega}\Big[-\partial_x^2\psi_{xy}-\partial_y^2\psi_{xy}-e\,B\,L_z\psi_{xy} + \frac{1}{4}e^2B^2(x^2+y^2)\psi_{xy}\Big]=\mathcal{E}\,\psi_{xy}. \tag{19}$$

The symmetric gauge

The kinetic angular momentum is given by the operator

$$\mathcal{L}_z \equiv x(-i\partial_y - e\,A_y) - y(-i\partial_x - e\,A_x), \tag{20}$$

which in our setting, and given the $L_z\psi_{xy} = 0$ restriction, reduces to

$$\mathcal{L}_z = -\frac{e\,B}{2}(x^2 + y^2), \tag{21}$$

when acting over our set of solutions ψ_{xy}.

The symmetric gauge

The expectation value

$$\langle x^2 + y^2 \rangle,$$

in the nth Landau level with $l_z = 0$ is given by

$$\frac{2}{e\,B}(n+1).$$

The solutions we just found therefor have an associate a kinetic angular momentum equal to $n+1$, in $\hbar$ units, and a cyclotron frequency given again by $\omega_L = \frac{e\,B}{\omega}$.

The spectral function

The retarded Green function for the scalar associated to $\delta\psi$ is given by

$$G^R(\mathrm{k},\mathrm{x}) = \lim_{r\to\infty} \mathcal{Q}(r,\mathrm{x}) \frac{\delta\psi^*(\mathrm{k},r,\mathrm{x})\partial_r\delta\psi(\mathrm{k},r,\mathrm{x})}{\delta\psi^*(\mathrm{k},\infty,\mathrm{x})\delta\psi(\mathrm{k},\infty,\mathrm{x})}$$
$$= \lim_{r\to\infty} \mathcal{Q}(r,\mathrm{x})|\psi_{\mathrm{x}}(\mathrm{x})|^2 \frac{\partial_r\psi_r(r)}{\psi_r(r)}, \quad (22)$$

where in the last equality we substituted $\delta\psi(\mathrm{k},r,\mathrm{x})) = \psi_r(r)\psi_{xy}(x,y)$.

The spectral function

The expression above is obtained by finding the boundary term that remains after evaluating the action on the solution $(\psi + \delta\psi, \varphi + \delta\varphi)$, and taking its second variation with respect to the dual of the source $\mu(x)$.

The spectral function

As $r \to \infty$, $\psi_r(r)$ can be approximated by

$$\psi_r(r) \simeq {\psi_r}^{(-1)} \left[\frac{1}{r} - \frac{U_1}{2r^2} + \left(\omega^2 - 2\mathcal{E}\right) \frac{\log r}{2r^3} - 3U_1 \left(\omega^2 - 2\mathcal{E}\right) \frac{\log r}{4r^4} + 3U_1 \left(\omega^2 - 2\mathcal{E} + {U_1}^2\right) \frac{1}{4r^4} \right] + {\psi_r}^{(-3)} \left[\frac{1}{r^3} - 6U_1 \frac{1}{4r^4} \right] + \mathcal{O}\left(\frac{1}{r^5}\right), \tag{23}$$

from where we can read the source term

$$\mu(x) = -\frac{1}{2}\sqrt{\lambda} T {\psi_r}^{(-1)} \psi_x(x), \tag{24}$$

for the scalar operator dual to $\delta\psi$.

This boils down to $\mathcal{Q}(r, \mathrm{x})$ being the term of first order in the perturbation found in what results from computing

$$-T_{D7}N_f\frac{1}{(-\frac{1}{2}\sqrt{\lambda}T)^2r^2}\partial_{\delta\psi'}\sqrt{-\det(g_{D7})}. \qquad (25)$$

The spectral function

Upon substitution we obtain

$$\mathcal{Q}(r,\mathrm{x}) = -T_{D7}N_f \frac{1}{(-\frac{1}{2}\sqrt{\lambda}T)^2 r^2} 2\pi^2 UV\sqrt{W}, \tag{26}$$

and the final Green function is

$$G^R(\omega,\mathrm{x}) = -\frac{N_c N_f}{2\pi^4 T^2} \lim_{r\to\infty} r^{-2} UV\sqrt{W} |\psi_{\mathrm{x}}(\mathrm{x})|^2 \frac{\partial_r \psi_r(r)}{\psi_r(r)} \tag{27}$$
$$= |\psi_{\mathrm{x}}(\mathrm{x})|^2 \tilde{G}^R(\omega).$$

The spectral function

We conclude that the density of quasinormal modes with any given frequency ω is dependent on the position in the gauge theory directions only through $|\psi_x(x)|^2$.

The radial profile

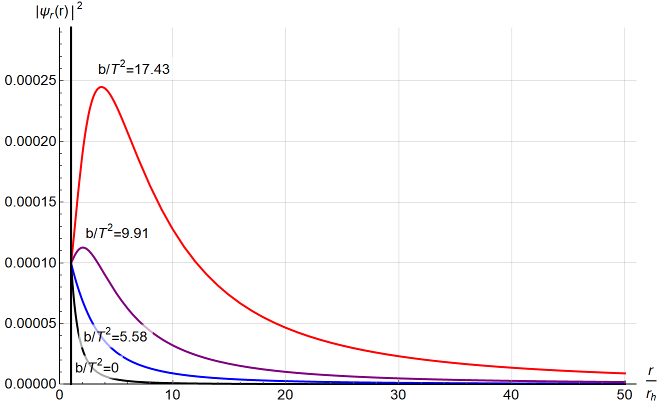


Figure: $n = 5, \omega/2\pi T = 1$

The radial profile

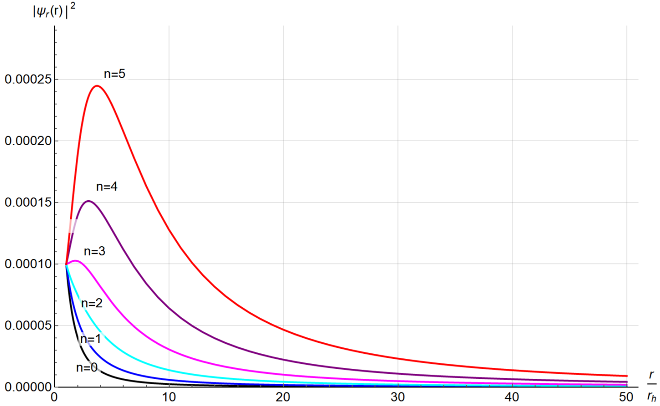


Figure: $b/T^2 = 17.43, \omega/2\pi T = 1$

The radial profile

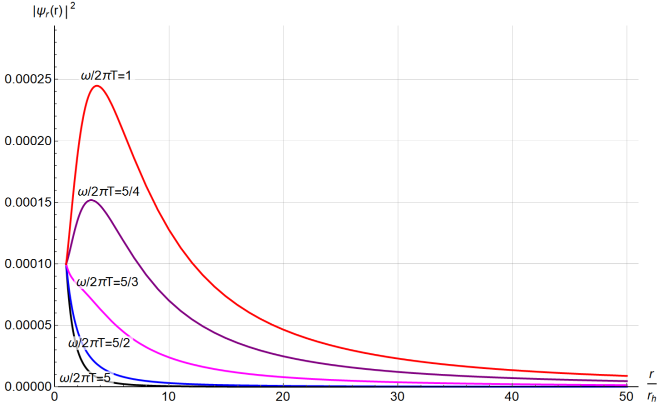


Figure: $b/T^2 = 17.43, n = 5$

The radial profile

We have numerically confirmed that the coefficient ${\psi_r}^{(-1)}$ is entirely real, while the first imaginary contribution to the series comes from ${\psi_r}^{(-3)}$.

From the expressions above we see that this is the correct asymptotic behavior to lead to a well defined spectral function.

Thank You All Very Much